

## Chapter 4

# *Assessment of Performance of Students Taking Advanced Courses in Mathematics*

### **Overview**

The NAEP 1996 assessment of mathematics included a special study to examine the performance of students at the eighth- and twelfth-grade levels who were taking or had taken advanced courses in mathematics (hereafter referred to as the Advanced Study). The motivation for the Advanced Study was an observation that the main NAEP assessment did not include enough advanced mathematics questions to allow students with appropriate preparation to demonstrate the full extent of their proficiency.

For example, the main eighth-grade NAEP assessment covers the standard content of the basic eighth-grade curriculum. This means that most topics in the Algebra and Functions content strand of the NAEP framework currently are assessed at the pre-algebra level — that is, with limited use of literal terms to represent variables and limitations on the types of applications and formulas on which students can be assessed. However, a substantial percentage of eighth-grade students are currently enrolled in first-year algebra. The main NAEP assessment does not provide these students with sufficient opportunity to display what they know and are able to do in algebraic situations. A similar situation exists at the twelfth-grade level, where many students have taken pre-calculus or calculus, but are limited in what mathematical proficiency they can display on the main NAEP assessment.

As a result, the National Assessment Governing Board (NAGB) and the National Center for Educational Statistics (NCES) decided to undertake a special study to assess what these “advanced” students could do as part of the NAEP 1996 mathematics assessment. At the eighth-grade level, the questions in the Advanced Study focused on algebra, with a special emphasis on examining what students know about various representations — graphical, numerical, symbolic, and written — for algebraic concepts and relationships, and the transformations among the various representations. At grade 12, the Advanced Study included a more even distribution of questions from all of the content strands of the NAEP mathematics framework.

## ***Administration of the Advanced Study***

To qualify for the Advanced Study, eighth-grade students had to be currently enrolled in or already have taken first-year algebra or a more advanced course such as geometry. Twelfth-grade students had to be enrolled in or have already taken a pre-calculus course, a course equivalent to pre-calculus, or a more advanced course such as calculus. These qualifying students represented 21 percent of the student population at grade 8 and 24 percent of the student population at grade 12. Students at participating schools who qualified but were not selected for the Advanced Study were eligible for the main NAEP assessment, where their performance could be directly compared to that of students with less rigorous preparation. (Appendix A provides more detail on the sampling for the Advanced Study.)

At both grade levels, each student in the Advanced Study completed a special NAEP assessment booklet that consisted of three blocks of mathematics questions: two Advanced blocks and a block comprising questions selected from the main NAEP assessment. At the eighth-grade level, the questions from the main NAEP were mostly from the Algebra and Functions content strand, along with some questions from the Data Analysis, Statistics, and Probability strand. At the twelfth-grade level, the questions from the main assessment covered the more advanced content from the content strands of Algebra and Functions; Data Analysis, Statistics, and Probability; and Geometry and Spatial Sense. The block of questions from the main NAEP assessment was 15 minutes in length for both grade levels and was the first cognitive block in the Advanced Study assessment booklet. At the eighth-grade level, the two special blocks containing Advanced Study questions were each 20 minutes in length, whereas at the twelfth-grade level, the Advanced blocks were each 30 minutes long. All students participating in the Advanced Study were told they could bring their own calculators; students who did not bring a calculator were provided with a scientific calculator.

## ***Grade Eight Advanced Study***

### ***Student background characteristics***

Table 4.1 compares the background characteristics of the grade 8 students in the Advanced Study with the characteristics of the students who took the national NAEP assessment but were not eligible for the Advanced Study. Table 4.2 includes additional data about students' school types and their eligibility for income-related, school-based intervention programs.

The data in Table 4.1 reflect some differences between students who qualified for the Advanced Study and those who did not. The percentage of female students was higher in the Advanced Study than among the students who were not eligible for the Advanced Study. In terms of racial/ethnic origins, greater percentages of White students and smaller percentages of Hispanic students were in the Advanced Study. In addition, Advanced Study students were more likely to have parents who had graduated from college.

Table 4.1

**Student Demographic Distributions, Grade 8, 1996**

Grade 8	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
Gender		
Males	48	53
Females	52	47
Students who Indicated Their Race/Ethnicity as...		
White	71	62
Black	14	18
Hispanic	6	13
Asian/Pacific Islander	6	6
American Indian	2	2
Students who Reported Their Parents' Highest Level of Education as...		
Did Not Finish High School	3	8
Graduated From High School	14	23
Some Education After High School	20	19
Graduated From College	58	38
I Don't Know	5	12
Home Environment Contains...		
0-2 Types of Educational Materials	12	23
3 Types of Educational Materials	26	32
4 Types of Educational Materials	62	45
Students From...		
Northeast	27	18
Southeast	17	22
Central	30	28
West	26	31
Students Live in...		
Center City	36	34
Urban Fringe/Small City	39	35
Rural	25	31

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Paralleling the findings for parental education, the data on types of educational materials in the home (newspapers, books, encyclopedias, magazines, etc.) show that students included in the Advanced Study were more likely to have 4 such materials in their homes, while students not eligible for the Advanced Study were more likely to have 0–2 or 3 of these materials in their homes.

There were no differences in the proportions of Advanced Study or not eligible students based on region or type of community.

### ***Student/school demographics***

The data in Table 4.2 describe the types of schools students attended and present categorical information about special programs within schools for which the students may have been eligible. The data for type of school do not show any differences in the proportions of students participating in the Advanced Study relative to those eighth-grade students who were not eligible. However, the data for special programs do show differences; specifically, fewer students in the Advanced Study participated in Title I programs or were eligible for the federal Free/Reduced-Price Lunch program.

	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
<b>Grade 8</b>		
Students who Attend...		
Public Schools	84	90
Nonpublic Schools	16	10
Title I Participation...		
Participated	3	14
Did Not Participate	97	86
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	16	32
Not Eligible	63	52
Information Not Available	22	16

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### ***Classroom content emphases***

NAEP also collected information from students' teachers concerning the instructional emphases they placed on content areas in the curriculum and on developing students' mathematical process abilities. Questionnaires were used to collect this information, as well as information on the use of calculators during mathematics instruction. Data from the teachers' responses are shown in Tables 4.3 through 4.5. The percentages in the tables reflect the percentages of students whose teachers responded as shown.

The data in Table 4.3 indicate that eighth-grade students in the Advanced Study were more likely to receive "some" emphasis on the area of Numbers and Operations and less likely to receive "a lot" of emphasis on this area than students who were not eligible for the Advanced Study. In addition, the Advanced Study students were more likely to receive "a little" emphasis on Measurement in their programs and less likely to receive "a lot" of emphasis on Geometry and Spatial Sense. There were no differences in student exposure to the area of Data Analysis, Statistics, and Probability.

Grade 8 students in the Advanced Study also were reported as receiving more emphasis in the area of Algebra and Functions than students not qualifying for the Advanced Study. This is consistent with the curricular prerequisites on which study selection was based.

Table 4.3

**Content Emphases in Mathematics Classes,  
Grade 8, 1996**



	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
<b>Grade 8</b>				
In This Mathematics Class How Often Do You Address...				
<b>Numbers and Operations</b>				
Advanced Study	0!	6	21	73
Non-Eligibles	0!	1	9	90
<b>Measurement</b>				
Advanced Study	4	39	45	13
Non-Eligibles	1	19	59	21
<b>Geometry</b>				
Advanced Study	6	26	54	14
Non-Eligibles	2	20	52	26
<b>Data Analysis, Statistics, and Probability</b>				
Advanced Study	9	33	46	12
Non-Eligibles	6	30	48	16
<b>Algebra and Functions</b>				
Advanced Study	0!	1	7	92
Non-Eligibles	2	9	40	49

NOTE: Row percentages may not total 100 due to rounding.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### **Classroom process emphases**

The data in Table 4.4 relate to four general mathematical processes: problem solving, reasoning, connections, and communication. The data indicate that the students in the Advanced Study received much the same instructional emphases as other grade 8 students on learning mathematical facts and solving routine problems. However, the students qualifying for the Advanced Study were given more emphasis on solving unique problems and communicating ideas in mathematics. This finding substantiates the opportunity-to-learn differences that arise from the curricular tracking of students in the upper middle grades/junior high schools.<sup>1</sup>

<sup>1</sup> Kifer, E. (1993). Opportunities, talents, & participation. In L. Burstein (Ed.), *The IEA Study of Mathematics III: Student growth and classroom processes* (pp. 279–308). New York: Pergamon Press; Schmidt, W. H., et al. (1998). *Facing the consequences: Using TIMSS for a closer look at United States mathematics and science education*. Dordrecht, Netherlands: Kluwer Academic Publishers.

Table 4.4

**Process Emphases in Mathematics Classes,  
Grade 8, 1996**



	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
<b>Grade 8</b>				
In This Mathematics Class How Often Do You Address...				
Learning Mathematical Facts and Concepts				
Advanced Study	2	7	13	78
Non-Eligibles	0	5	16	79
Learning Skills and Procedures Needed to Solve Routine Problems				
Advanced Study	0!	4	13	82
Non-Eligibles	1!	1	18	80
Developing Reasoning and Analytical Ability to Solve Unique Problems				
Advanced Study	0!	5	28	68
Non-Eligibles	1	8	44	46
Learning How to Communicate Ideas in Mathematics Effectively				
Advanced Study	0!	10	34	55
Non-Eligibles	1	17	43	39

NOTE: Row percentages may not total 100 due to rounding.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### **Calculator access and usage**

NAEP also collected information on the extent to which students use calculators as part of their normal mathematics experience in schools. Table 4.5 contains the data reported by grade 8 teachers relative to student access to and use of calculators as part of their regular instructional programs. The data reflect that students in the Advanced Study had fewer restrictions on the use of calculators in their regular school mathematics programs than students not eligible for the Advanced Study. Similar results were found regarding the use of calculators on classroom tests; more than 80 percent of the Advanced Study students, compared to about two-thirds of the non-eligible students, had teachers who allowed calculators to be used in these circumstances. Responses to questions dealing with access to school-owned calculators and with instruction on the use of calculators, however, showed no differences between eighth-grade students in the Advanced Study and those who were not eligible for the study.

	Percentage of Students Whose Teachers Responded	
	Yes	No
<b>Grade 8</b>		
Do You Permit Students in This Class Unrestricted Use of Calculators...		
Advanced Study	61	39
Non-Eligibles	42	58
Do You Permit Students in This Class to Use Calculators for Tests...		
Advanced Study	82	18
Non-Eligibles	64	36
Do the Students in This Class Have Access to Calculators Owned by the School...		
Advanced Study	81	19
Non-Eligibles	81	19
Do You Provide Instruction to Students in This Class in the Use of Calculators...		
Advanced Study	80	20
Non-Eligibles	84	16

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.





## Performance on the Main Assessment

The results from the Advanced Study did not lend themselves to either the development of a separate proficiency scale or equating to the main NAEP mathematics scale (see Appendix A). However, as noted above, some of the students who were eligible for the Advanced Study actually participated in the main NAEP assessment. This affords the opportunity to compare directly the performance of students who were eligible for the Advanced Study and those who were not eligible, using the composite NAEP scale. The data in Table 4.6 show that students in the eighth grade who were eligible for the Advanced Study had an average NAEP score of 300, while non-eligible students had an average score of 264.

Table 4.6

**Average Mathematics Scale Scores by Eligibility for Advanced Study, Grade 8, 1996**



		Average Scale Score
Grade 8		
	All Students	272
	Eligible	300
	Non-Eligible	264


SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### General description of Advanced Study questions

The two blocks of Advanced Study questions at the eighth-grade level consisted of 10 and 12 questions, respectively. The questions were classified primarily in the Algebra and Functions content strand of the NAEP mathematics framework, although a few questions designed to assess concepts and procedures from the Measurement and Data Analysis, Statistics, and Probability strands also were included among the 22 questions. Furthermore, some of the questions classified as Algebra and Functions were open to solutions via Number Sense, Properties, and Operations approaches.

Six of the algebra questions required students to make an interpretation or complete an operation based on understanding symbolic representations of algebraic relationships. Nine of the questions required students to begin by considering written descriptions of algebraic relationships, possibly translate these relationships into numerical or algebraic representations, and then find solutions for the situations described. Another question was designed to measure students' understanding of and ability to use the Pythagorean theorem in an applied setting.

Table 4.7 displays the breakdown of questions in the Grade 8 Advanced Study by content strand and response structure. As noted previously, the Advanced Study at grade 8 differed from the main NAEP assessment in the distribution across content strands.<sup>2</sup> In addition, the Advanced Study contained a far greater percentage of questions that required students to show or explain their work than were found in the main NAEP.

<b>Table 4.7</b>		<b>Distribution of Questions by Content Strand and Response Format, Grade 8, 1996</b>	
			
<b>Content Strand Coverage</b>		<b>Number of Items</b>	
Number Sense, Properties, and Operations		1	
Measurement		0	
Geometry and Spatial Sense		1	
Data Analysis, Statistics, and Probability		2	
Algebra and Functions		18	
<b>Response Formats for Items</b>			
Multiple-Choice		9	
Short Constructed-Response		11	
Extended Constructed-Response		2	

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

<sup>2</sup> Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states*. Washington, DC: National Center for Education Statistics.

## Performance on the Advanced Study

Students' overall performance on the Advanced Study, measured by average percentage correct scores, is presented in Table 4.8. Students' average percentage correct score was 36 percent, and male students outperformed females. White, Hispanic, and Asian/Pacific Islander students outperformed Black students, and White and Asian/Pacific Islander students also outperformed Hispanic students. The sample of American Indian students was too small to permit reliable estimates of the performance on the Advanced Study blocks or on individual questions.

**Table 4.8**

**Average Percentage Correct Scores, Advanced Study, Grade 8, 1996**



Grade 8		Percentage Correct
		All Students
	Males	38
	Females	34
	White	40
	Black	19
	Hispanic	28
	Asian/Pacific Islander	50
	American Indian	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

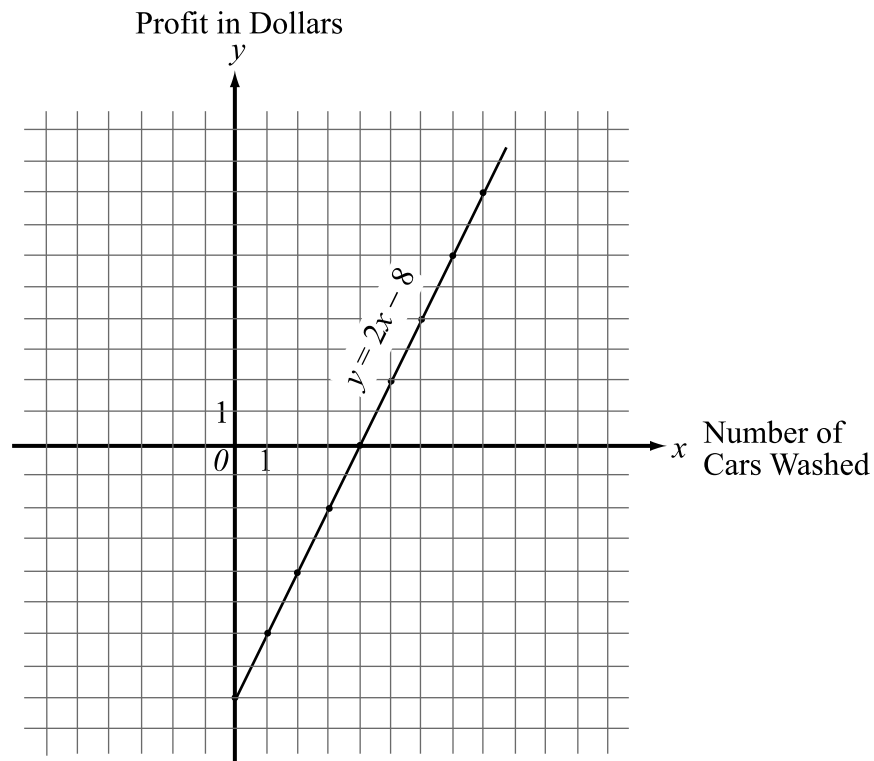
### Examples of Advanced Study questions and student performance

Some sample questions from the Grade 8 Advanced Study are presented below to show how well the Advanced Study students could perform on specific tasks and to explicate the types of knowledge and skills students at the eighth-grade level needed to have to solve these problems. Information on how students performed on these questions is presented in terms of percentages correct on the individual questions. Performance information is provided for all students and by gender and race/ethnicity subgroups.

The first three sample questions constitute a family of questions based on a car wash being conducted by an eighth-grade class. The problem stimulus provided students with important data in various forms: written, graphical, and symbolic. Based on this information, students were asked to complete three different short constructed-response questions.

**Questions 4–6** refer to the following information and graph.

The eighth-grade class at Carter School is going to hold a car wash to raise money for a class trip. The class determined that it had purchased enough supplies to wash at most 50 cars. The graph below shows the relationship between the number of cars washed and the profit earned in dollars. The line that can be drawn through these points is represented by the equation  $y = 2x - 8$ .



Profit is defined as the amount of money collected from the people whose cars are washed minus the amount of money that was spent on car wash supplies.

The first question asked students to interpret the information in the graph to find the fixed costs, or amount spent on supplies, before the first car was washed. In order to answer the question correctly, students needed to consider the  $y$ -intercept on the graph (i.e., the profit value associated with the situation before the first car was washed). They could do this either by inspecting the graph and finding the  $y$ -intercept at  $y = -8$ , or by substituting  $x = 0$  into the equation shown in the stimulus and solving to find  $y = -8$ .

4. According to the graph, how much money did the class spend on car wash supplies?

\_\_\_\_\_

Explain how you found your answer.

Student responses were rated “correct,” “partial,” or “incorrect.” The scoring rubric for the question required that students’ responses indicate an understanding that initially the number of cars washed was zero (i.e., that the  $y$ -intercept must be used). That is, to receive a rating of “correct,” students were required to give both the correct response of 8 and an explanation that showed a correct understanding of the  $y$ -intercept in the graph or in the equation. Two sample “correct” responses are shown below. These responses clearly indicate both a correct answer and a rationale for why that answer is correct. The first response indicates recognition of the values associated with zero cars washed. The second response contains evidence of understanding the meaning of the  $y$ -intercept in the symbolic equation presented in the stimulus of the problem.

**Sample “correct” response 1**

4. According to the graph, how much money did the class spend on car wash supplies?

8 DOLLARS

Explain how you found your answer.  $(0, -8)$

### Sample “correct” response 2

4. According to the graph, how much money did the class spend on car wash supplies?

\$ 8.00

Explain how you found your answer.

because profit equals money collected  
minus money spent.  
The line is  $y = 2x - 8$

Students responding with the value 8 and no explanation or an incorrect explanation or students responding without a value or with an incorrect value but with an explanation that indicated that they knew that the value of the y-intercept answered the question received “partial” ratings. The sample “partial” response shown below indicates a correct value, but lacks an acceptable rationale for the answer.

### Sample “partial” response

4. According to the graph, how much money did the class spend on car wash supplies?

8

Explain how you found your answer.

The last point on the graph is 8.

Students simply responding – 8 for the amount spent on car wash supplies received a rating of “incorrect.” Student responses that contained an incorrect value and an explanation that indicated a lack of understanding that the answer should be found in the value of the y-intercept also received an “incorrect” rating. The “incorrect” response shown demonstrates such a lack of understanding.

**Sample “incorrect” response**

4. According to the graph, how much money did the class spend on car wash supplies?

\$ 9.75

Explain how you found your answer.

I counted up the y axis, there were 9 boxes and about  $3/4$  left over.

Table 4.9 presents the breakdown of student performance according to gender and race/ethnicity. Overall, 13 percent of the student responses were rated “correct,” 26 percent were rated “partial,” and 36 percent were rated “incorrect.” Twenty-one percent of the students did not respond to the question, and an additional four percent, not shown in the table, only wrote off-task remarks on their papers.<sup>3</sup>

<sup>3</sup> Student responses for this and all other constructed-response questions were scored as “off task” if the student provided a response that was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be “I don’t like this test.” In contrast, responses scored as “incorrect” were valid attempts to answer the question that were simply wrong.

Table 4.9

**Score Percentages for "Car Wash Supplies,"  
Grade 8**



	Correct	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	13	26	36	21
Males	15	30	32	20
Females	10	23	41	21
White	15	30	34	19
Black	3!	14!	46!	32!
Hispanic	10	23	40	23
Asian/Pacific Islander	14	33	35	17
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second question in this three-question family asked students to find the number of cars that must be washed before the class would start to earn a profit. In order to answer the question correctly, students needed to find the value of  $x$  for which  $y = 0$ . This value could be found by locating the  $x$ -intercept of the graph (i.e.,  $x = 4$ ), or by solving  $y = 2x - 8$  for  $y = 0$ . Some students also responded with the value of 5 cars, as this is the first point at which there is a positive profit. Values of either 4 or 5 were accepted as correct numerical answers for the value portion of the question, and correct corresponding explanations were accepted for the explanation portion of the question.

5. How many cars do the students have to wash before the class would start to earn a profit?

\_\_\_\_\_

Explain your answer.



Student responses were rated as “correct,” “partial,” or “incorrect” according to the following rubric, or rating scale. To receive a rating of “correct” a response had to contain both the correct answer and an explanation of why it was the right value. Responses received a rating of “partial” when they contained a numerical answer of 4 or 5 without a supporting explanation, or when they contained a number other than 4 or 5 but demonstrated an understanding that the  $x$ -intercept must be used. Responses were rated “incorrect” when students failed to give either the correct answer or an explanation of how to get a correct answer.

The “correct” response shown below reflects the work of a student who recognized that washing four cars would pay for the supplies and that washing a fifth car would be needed to achieve a positive profit.

### **Sample “correct” response**

5. How many cars do the students have to wash before the class would start to earn a profit?

4 cars to break even 5 to have some profit

Explain your answer.

$x$  = the amount of cars you washed

$z$  = how much they were paid for each car

To make up for the money they spent on supplies ~~they would~~ have to wash 4 cars at 2 dollars each (\$8 total) Then to gain a profit they would have to wash another car (5 cars total washed) and have a \$2 profit.

The “partial” response that follows shows a correct numerical answer but has an explanation that contains errors.

**Sample “partial” response**

5. How many cars do the students have to wash before the class would start to earn a profit?

5

Explain your answer.

Because when the line on the graph comes above 0 they have made a profit and the line comes up to 0 they have washed 5 cars

Information on student performance is presented in Table 4.10. Overall, 21 percent of students submitted responses rated “correct,” 19 percent submitted responses rated “partial,” and 39 percent turned in responses rated “incorrect.” Another 18 percent of the students omitted the question. Performance on this question was somewhat better than performance on the previous problem. In this case, compared to the preceding problem, students had to make less of a conceptual leap to interpret what the problem was asking them to do. The problem also was more open to being solved through numerical reasoning,  $2 \times [ ] = 8$ , and therefore, was more straightforward than the fixed-costs problem. There was some ambiguity in the phrasing of the question, however, which required acceptance of either 4 or 5 as the correct answer.

Table 4.10

**Score Percentages for  
"Begin to Earn Profit," Grade 8**



	Correct	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	21	19	39	18
Males	23	36	16	
Females	19	16	42	20
White	24	22	36	15
Black	7!	6!	53!	32!
Hispanic	16	14	49	19
Asian/Pacific Islander	30	14	38	17
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The third question in the family, also rated on a three-point scale, asked students to find the greatest profit (in dollars) that the class could expect to make from the car wash. To solve this problem, students needed to find the profit associated with  $x = 50$ , the maximum number of cars that could be washed. By substituting into the equation  $y = 2x - 8$ , they could find that the maximum profit that could be expected was \$92.

6. What is the greatest profit (in dollars) that the class can expect to earn?

\_\_\_\_\_

Show your work.

To receive a rating of “correct,” a student only had to give the correct answer of \$92. A supporting rationale was not required for the response to be rated “correct” even though the question asked students to show their work. The scoring rubric allowed raters to give partial credit for an incorrect value if the correct process was shown. While many of the students who received full credit simply wrote in \$92 or showed something like  $2(50) - 8 = 92$ , the “correct” response shown below is from a student who clearly communicated his or her approach as well as giving the correct solution.

**Sample “correct” response**

6. What is the greatest profit (in dollars) that the class can expect to earn?

\$ 92.00

Show your work. equation:  $y = 2x - 8$

They expect to wash at the most 50 cars. When they wash 50 cars they make \$100. But it cost them \$8.00 to buy supplies, so to find their profit you subtract the money they used for supplies from the money they collected from washing cars.

$$100 - 8 = \boxed{92} \rightarrow \text{profit}$$

Students who used the equation and made a correct substitution, but made an error in calculation, were awarded a rating of “partial.” For example, some divided the 92 by 2 or incorrectly multiplied  $2 \times 50$  to get 200 and then subtracted 8 to arrive at an answer of 192, as in the “partial” response shown.

**Sample “partial” response**

6. What is the greatest profit (in dollars) that the class can expect to earn?

192

Show your work.

$$y = 2x - 8$$
$$y = 2(50) - 8$$
$$y = 200 - 8$$
$$y = 192$$

Student responses were rated “incorrect” if they neither found the value nor gave a method for finding that value. Some students receiving “incorrect” ratings omitted the start-up costs and simply multiplied  $50 \times 2$  to arrive at an answer of \$100; others attempted to estimate the answer, as shown in the following “incorrect” response.

**Sample “incorrect” response**

6. What is the greatest profit (in dollars) that the class can expect to earn?

\$80.00

Explain your answer.  
 The graph on the ... “y” axis is positive  
 it goes up about 8 squares assum-  
 ing they each represent 10.  
 $8 \cdot 10 = 80.00$

Information on student performance is presented in Table 4.11. Overall, 27 percent of responses were rated “correct,” 4 percent were rated “partial,” and 40 percent were rated “incorrect.” Twenty-six percent of the students omitted the problem. On this question, a higher percentage of students submitted responses that were rated “correct,” but there was a slight decrease in overall performance (“correct” and “partial”) compared with the two previous questions. The increase in responses receiving a “correct” rating could be due to the fact that the student could read the problem and then simply substitute into the equation to find the answer. While the question is open to solution by numerical patterns or an extension of the graph, these are less likely approaches. It also is possible that the score distribution reflects the decision to give full credit for a correct numerical answer without requiring that work be shown.

Table 4.11

**Score Percentages for  
"Greatest Profit Expected," Grade 8**



	Correct	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	27	4	40	26
Males	29	4	39	24
Females	25	4	41	27
White	30	4	40	22
Black	12!	2!	40!	44!
Hispanic	20	5	44	28
Asian/Pacific Islander	36	3	33	26
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to responses rated "Off Task" or to rounding, or both.

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The fourth sample question presented here for grade 8 is one that asked students to consider a situation in which one hot-air balloon is ascending and another balloon is descending. Students were given the rate of change in elevation and the initial starting point for each balloon and were asked to find the time required for the two balloons to reach the same altitude. The problem is most readily solved through the use of a single equation  $3t = 1000 - 5t$  or the system of equations:  $y = 3t$  and  $y = 1000 - 5t$ . However, a student could solve the problem through a table of values and estimation or through experimentation.

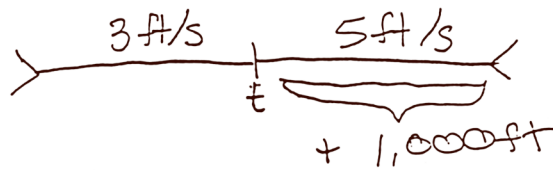
7. A hot-air balloon begins rising at the rate of 3 feet per second. At the same time, a second hot-air balloon that is 1,000 feet above the first balloon begins to descend at the rate of 5 feet per second. In how many seconds will the balloons reach the same altitude?

Students' solutions to this problem were rated on a three-point scale. "Correct" responses gave the correct numeric response of 125 seconds with or without an explicit rationale. Responses that reflected work that did more than restate the problem but failed to achieve a correct answer for the time in seconds were rated "partial." The rating of "incorrect" was given to responses that contained neither a correct value nor a correct equation or formulation of the problem.

While many “correct” responses resulted from students simply solving  $3t = 1000 - 5t$ , the sample “correct” response shows both the modeling that led to the development of the equation and the equation-solving approach employed.

**Sample “correct” response**

7. A hot-air balloon begins rising at the rate of 3 feet per second. At the same time, a second hot-air balloon that is 1,000 feet above the first balloon begins to descend at the rate of 5 feet per second. In how many seconds will the balloons reach the same altitude?



	d	r	t
A	$3t$	3	$t$
D	$-5t + 1,000$	-5	$t$

$$3t = -5t + 1,000$$

$$\frac{8t}{8} = \frac{1,000}{8}$$

$$t = 125 \text{ s.}$$



The following “partial” response demonstrates some effort toward working out the problem but a failure to obtain the correct answer.

**Sample “partial” response**

7. A hot-air balloon begins rising at the rate of 3 feet per second. At the same time, a second hot-air balloon that is 1,000 feet above the first balloon begins to descend at the rate of 5 feet per second. In how many seconds will the balloons reach the same altitude?

1,000 feet above

<u>ascend</u>		<u>descend</u>
3ft per 1second		5 feet per 1second
6ft " 2 "		10 " " 2 "
9ft " 3 "		15 " " 3 "
12ft " 4 "		20 " " 4 "
15ft " 5 "		25 " " 5 "
18ft " 6 "		30 " " 6 "

The results, presented in Table 4.12, show that 19 percent of the responses received a “correct” rating, 7 percent received a “partial” rating, and 58 percent received an “incorrect” rating. In addition, 13 percent of the students omitted the question. Students used a number of approaches in answering this question. While the use of a system of equations or a single equation that equated the two ways of representing height at the time desired,  $3t = 1000 - 5t$ , were most common, a number of numerical approaches were noted among the Advanced Study students.

<b>Table 4.12</b>		<b>Score Percentages for “Hot Air Balloon,” Grade 8</b>			
		<b>Correct</b>	<b>Partial</b>	<b>Incorrect</b>	<b>Omit</b>
<b>Grade 8</b>					
	All Students	19	7	58	13
	Males	27	6	52	13
	Females	12	8	64	13
	White	23	8	57	10
	Black	2!	4!	68!	25!
	Hispanic	14	4	60	18
	Asian/Pacific Islander	24	10	51	12
	American Indian	***	***	***	***



NOTE: Row percentages may not total 100 due to responses rated “Off Task” or to rounding, or both.

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

Mathematics Assessment.

## **Grade Twelve Advanced Study**

### **Student background characteristics**

The data in Table 4.13 describe the student background characteristics of the grade 12 students in the Advanced Study compared to those of students in the main NAEP assessment who had not taken advanced courses and were therefore not eligible for the Advanced Study. Compared to the non-eligible group, higher percentages of Asian/Pacific Islander and White students and a lower percentage of Black students participated in the Advanced Study. In addition, students in the Advanced Study were more likely to have parents who had graduated from college.

Similar to the findings at grade 8, the data on types of educational materials in the home (newspapers, books, encyclopedias, magazines, etc.) show that students included in the Advanced Study were more likely to have 4 such materials in their homes, while students not eligible for the Advanced Study were more likely to have 0–2 of these materials in their homes. There were no significant differences in Advanced Study participation based on region or type of community.

Table 4.13

**Student Demographic Distributions, Grade 12, 1996**

Grade 12	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
<b>Gender</b>		
Males	51	48
Females	49	52
<b>Students who Indicated Their Race/Ethnicity as...</b>		
White	74	68
Black	7	15
Hispanic	8	11
Asian/Pacific Islander	10	4
American Indian	0	2
<b>Students who Reported Their Parents' Highest Level of Education as...</b>		
Did Not Finish High School	4	7
Graduated From High School	13	22
Some Education After High School	23	26
Graduated From College	59	41
I Don't Know	2	3
<b>Home Environment Contains...</b>		
0-2 Types of Educational Materials	13	20
3 Types of Educational Materials	25	27
4 Types of Educational Materials	62	53
<b>Students From...</b>		
Northeast	25	22
Southeast	21	23
Central	28	24
West	26	30
<b>Students Live in...</b>		
Center City	32	31
Urban Fringe/Small City	38	38
Rural	29	31

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Student/school demographics

Information on type of school and student participation in categorical programs is shown in Table 4.14. When these data are considered, more differences between the groups begin to emerge. A larger proportion of grade 12 students participating in the Advanced Study were from nonpublic schools than were students not eligible for the Advanced Study. The two groups did not differ in terms of participation in Title I programs, but fewer Advanced Study students were eligible for the federal Free/Reduced-Price Lunch program.

**Table 4.14**

### **Student/School Demographic Distributions, Grade 12, 1996**



	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
<b>Grade 12</b>		
Students who Attend...		
Public Schools	82	91
Nonpublic Schools	18	9
Title I Participation...		
Participated	2	2
Did Not Participate	98	98
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	7	15
Not Eligible	56	62
Information Not Available	37	23

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### **Classroom content emphases**

Table 4.15 contains the data from teachers' responses to questions requesting an indication of the amount of emphasis they placed on each of a number of subject matter content areas within the grade 12 curriculum. Note that the data shown here are only from the teachers who were teaching students in the Advanced Study; teachers of twelfth-grade students who took the main NAEP assessment were not surveyed. The data shown in Table 4.15 indicate that, in classes taken by Advanced Study participants, there was more "heavy emphasis" placed on Functions than on any other topic. Algebra and Trigonometry ranked next and did not differ much from each other in frequency of "heavy emphasis." Geometry, at 23 percent, received "heavy emphasis" less often than Functions, Algebra, or Trigonometry, but more often than Statistics, Probability, or Discrete Mathematics.

**Table 4.15**

**Content Emphases in Classes Taken by Advanced Study Students, Grade 12, 1996**



	Percentage of Students Receiving		
	Little Emphasis	Moderate Emphasis	Heavy Emphasis
<b>Grade 12</b>			
How Much Emphasis on...			
Algebra	4	23	73
Geometry	14	63	23
Trigonometry	5	28	67
Functions	2	13	85
Statistics	65	25	10
Probability	66	25	9
Discrete Mathematics	60	34	6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### **Classroom process emphases**

In Table 4.16, a second set of data from classroom teachers of Grade 12 Advanced Study students describes the emphases teachers reported placing on process-related activities. Specifically, teachers were questioned about four common mathematical processes: problem solving, reasoning, connections, and communication. Most students had teachers who reported that they placed "heavy emphasis" on the first three areas: facts and concepts, skills and procedures for solving routine problems, and reasoning and analysis for solving nonroutine problems. However, fewer students had teachers who reported a "heavy emphasis" on

preparing students to communicate the results of their mathematical endeavors. This latter finding is consistent with classroom practices data reported by twelfth-grade students who participated in the main NAEP assessment.<sup>4</sup>

**Table 4.16**

**Process Emphases in Classes Taken by Advanced Study Students, Grade 12, 1996**



	Percentage of Students Receiving		
	Little Emphasis	Moderate Emphasis	Heavy Emphasis
<b>Grade 12</b>			
How Much Emphasis on...			
Learning Mathematical Facts and Concepts	2	20	78
Learning Skills and Procedures Needed to Solve Routine Problems	1	22	77
Developing Reasoning and Analytical Ability to Solve Unique Problems	2	25	73
Learning How to Communicate Ideas in Mathematics Effectively	10	47	43

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Calculator access and usage**

Data on calculator access and usage for students in the Advanced Study also were collected from their teachers. Table 4.17 presents findings about calculator access. The data reflect an even level of access to both scientific and graphing calculators in the classroom, but greater access to scientific calculators outside the classroom. From a different perspective, the data indicate that 80 percent of Advanced Study students were permitted unlimited access to calculators in their mathematics classes. However, the percentage of students having access to calculators during assessment sessions was somewhat lower, with teachers of 71 percent of the students reporting that their students always could use calculators on tests; the remainder of the teachers reported that their students were sometimes permitted to use calculators on tests.

<sup>4</sup> Reese, et al., (1997). op. cit.

Table 4.17

**Calculator Access in Classes Taken by Advanced Study Students, Grade 12, 1996**



Grade 12	Percentage of Students
Do You Permit Students in This Class Unrestricted Use of Calculators...	
Yes	80
No	20
Do Students in This Class Have Access to Scientific Calculators in Class...	
Yes	83
No	17
Do Students in This Class Have Access to Scientific Calculators Out of Class...	
Yes	82
No	18
Do Students in This Class Have Access to Graphing Calculators in Class...	
Yes	84
No	16
Do Students in This Class Have Access to Graphing Calculators Out of Class...	
Yes	56
No	44
Do You Permit Students in This Class to Use Calculators on Tests...	
Yes All	71
Yes Some	28
No	0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### ***Calculator instruction***

Table 4.18 contains more detail about calculator instruction and graphing calculators. The responses indicate that more students received instruction in how to use graphing calculators than scientific calculators. The larger extent of training being devoted to graphing calculators may be due partially to their special uses in these classes and partially to their newness in grade 12 classrooms.

Over 70 percent of the students in Advanced Study classrooms had full access to a graphing calculator for their study of mathematics, either from a complete set in the classroom or through a personally owned graphing calculator. Reportedly these calculators were primarily utilized for their unique capability — that is, graphing functions.



Table 4.18

**Calculator Usage and Instruction in Classes Taken  
by Advanced Study Students, Grade 12, 1996**




Grade 12	Percentage of Students
In This Class Students are Provided Instruction in the Use of Scientific Calculators...	
Yes	65
No	35
In This Class Students are Provided Instruction in the Use of Graphing Calculators...	
Yes	87
No	13
Which of the Following Best Describes the Availability of Graphing Calculators in This Class...	
One	1
Less Than Six	3
Complete Set	48
Some Students Have One	8
Most Students Have One	9
All Students Have One	24
No Student Has One	6
If Graphing Calculators are Used in This Class, What is Their Primary Usage...	
Calculating	7
Graphing	76
Tables	0
Statistics	1
Symbolic Manipulation	3
Not Used	13

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Performance on the Main Assessment

Table 4.19 compares performance on the composite NAEP mathematics scale for grade 12 students in the main assessment who were, or were not, eligible for the Advanced Study. As can be seen, students who were eligible for the Advanced Study had an average scale score of 327, while non-eligible students had a lower average score of 297.

<b>Table 4.19</b>		<b>Average Mathematics Scale Scores by Eligibility for Advanced Study, Grade 12, 1996</b>		<b>THE NATION'S REPORT CARD</b> 
<b>Grade 12</b>		<b>Average Scale Score</b>		
	All Students		304	
	Eligible		327	
	Non-Eligible		297	

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### **General description of the Grade 12 Advanced Study questions**

The two blocks of Advanced Study questions for grade 12 students each consisted of 11 questions. Although the questions were advanced in nature, no questions required calculus. Most of the questions could be solved by several methods.

The 22 questions were balanced among situations that were presented in symbolic, graphical, written, and visual formats. Many of the questions required students to make connections between two or more forms of representation in order to complete the requirements set by the question.

Table 4.20 displays the breakdown of questions by content strand and response structure. All five NAEP mathematics content strands were represented in the Advanced Study. Compared to the main NAEP assessment, however, the Advanced Study included considerably higher percentages of questions classified in the Algebra and Functions; Data Analysis, Statistics, and Probability; and Geometry and Spatial Sense strands; there also was a far greater percentage of questions that required students to show or explain their work.<sup>5</sup>

**Table 4.20**

***Distribution of Questions by Content Strand and Response Format, Grade 12, 1996***



Content Strand Coverage	Number of Items
Number Sense, Properties, and Operations	1
Measurement	2
Geometry and Spatial Sense	7
Data Analysis, Statistics, and Probability	5
Algebra and Functions	7
Response Formats for Items	
Multiple-Choice	7
Short Constructed-Response	10
Extended Constructed-Response	5

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### ***Performance on the Advanced Study***

Students' overall performance on the Grade 12 Advanced Study, measured by average percentage correct scores, is presented in Table 4.21. For the group as a whole, the average percentage correct score was 30 percent. Male students outperformed female students, and White and Asian/Pacific Islander students outperformed Black and Hispanic students. The sample of American Indian students was too small to permit reliable estimates of the performance on the Advanced Study blocks or on individual questions. Students who were currently taking mathematics or who were taking, or had taken, an AP course in mathematics outperformed students who were not currently taking a mathematics course or who had not taken an AP course in this subject area.

<sup>5</sup> Ibid.

Table 4.21

**Average Percentage Correct Scores, Advanced Study, Grade 12, 1996**



		Percentage Correct
<b>Grade 12</b>		
	All Students	30
	Males	32
	Females	27
	White	32
	Black	14
	Hispanic	19
	Asian/Pacific Islander	32
	American Indian	***
	Are Students Presently Enrolled in Mathematics...	
	Yes	30
	No	18
	Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...	
	Yes	37
	No	26

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### **Examples of individual questions and student performance**

Four sample questions from the Grade 12 Advanced Study are presented below to show the types of knowledge and skills students at the twelfth-grade level needed to solve the Advanced Study problems. As before, student performance information is presented in terms of percentages correct on the individual questions. The subgroup comparisons for grade 12 include gender, race/ethnicity, whether students are presently enrolled in mathematics, and whether students are enrolled in or have taken an AP mathematics course.

The first sample question asked students to find a third value for a linear function,  $f$ , given two other values for the function. Students were asked to show their work. In order to solve the problem, students first had to realize that they were being asked to find the third point on a straight line, given two other points on the line. Students then could have proceeded to find the slope of the line connecting the two given points graphically or by using the equation for a straight line. The value of the slope, along with the coordinates of the given points, one of which is the  $y$ -intercept, allows for an algebraic construction of an equation for the line. Solving this equation for the value  $x = 3.8$  results in the correct answer of 1.58 for  $f(3.8)$ . Other students might have approached the problem graphically and more geometrically and used proportions such as  $0.35/1 = h/3.8$  to find  $h$ , which they then added to the value of the  $y$ -intercept to get the required value of 1.58.

3. If  $f$  is a linear function such that  $f(0) = 0.25$  and  $f(1) = 0.6$ , what is the value of  $f(3.8)$ ? Show your work.

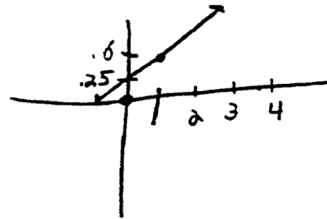
Student responses were rated “correct,” “partial,” or “incorrect.” Responses of 1.58 or 1.6 with correct accompanying work were rated “correct.” If no work was shown, the answer had to be 1.58 in order to be considered “correct.” Responses were rated “partial” if they contained some form of the equation  $y = 0.35x + 0.25$  or correctly found the values of  $f(3) = 1.3$  and  $f(4) = 1.65$  or correctly found the slope,  $m = 0.35$ . All other responses were rated “incorrect.”

One example of a “correct” student response is shown below. Here the student correctly calculated the slope and then used the slope value and one of the given ordered pairs to find an equation of the line. Finally, the value of  $f(3.8)$  was computed correctly.

**Sample “correct” response**

3. If  $f$  is a linear function such that  $f(0) = 0.25$  and  $f(1) = 0.6$ , what is the value of  $f(3.8)$ ? Show your work.

$$\begin{array}{l} (0, .25) \\ (1, .6) \\ \hline .35 \\ 1 \end{array}$$



$$\begin{aligned} y - .6 &= .35(x - 1) \\ y - .6 &= .35x - .35 \\ y &= .35x + .25 \\ y &= .35(3.8) + .25 \end{aligned}$$

$$\boxed{1.58}$$

One response that received a “partial” rating follows. The student correctly calculated the slope as  $\frac{7}{20}$  but did not complete the problem.

**Sample “partial” response**

3. If  $f$  is a linear function such that  $f(0) = 0.25$  and  $f(1) = 0.6$ , what is the value of  $f(3.8)$ ? Show your work.

$$\begin{array}{l} (0, \frac{1}{4}) \quad (1, \frac{3}{5}) \quad \frac{\frac{3}{5} - \frac{1}{4}}{1 - 0} = \frac{\frac{2}{20}}{1} = \frac{7}{20} \end{array}$$

$$(3.8, y) = \frac{7}{20}$$

The data in Table 4.22 reflect student performance on the linear functions question. Twenty percent overall submitted a response rated “correct,” and another 11 percent received partial credit. This performance is somewhat disappointing, given that the question covers basic concepts of linear functions. Common errors included reversing values for the independent and dependent variables, perhaps indicating a fundamental misunderstanding of the functional notation, and reversing the definition of a slope. Some students incorrectly defined the slope as the change in  $x$  over the change in  $y$ .

**Table 4.22**

**Score Percentages for “Use Linear Function,”  
Grade 12**



	Correct	Partial	Incorrect	Omit
<b>Grade 12</b>				
All Students	20	11	43	23
Males	25	10	38	23
Females	16	11	48	23
White	23	11	42	21
Black	4	8	44	43
Hispanic	11!	6!	46!	32!
Asian/Pacific Islander	22	12	42	22
American Indian	***	***	***	***
Are Students Presently Enrolled in Mathematics...				
Yes	21	11	43	23
No	9!	4!	46!	37!
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...				
Yes	31	12	38	17
No	13	10	46	27

NOTE: Row percentages may not total 100 due to responses rated “Off Task” or to rounding, or both.

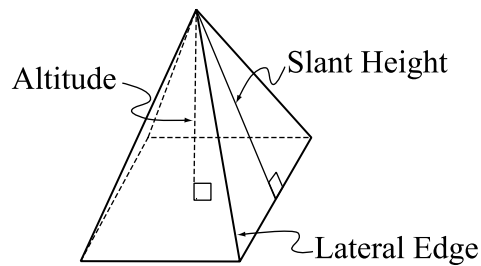
\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second sample question from the Grade 12 Advanced Study asked students to rank order the volumes of three square pyramids from smallest to largest and supply a rationale for their ranking. As data, students were given one of the measurements of altitude, slant height, or lateral edge for each pyramid and told that all pyramids resided on a square base with sides of 10 units. Students also were given the general formula for the volume of a pyramid.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.



**10.** The figure above shows the altitude, lateral edge, and slant height of a square pyramid. Each of three square pyramids  $P$ ,  $Q$ , and  $R$  (not shown) has a base with side 10. In each pyramid the lateral edges are of equal length. Pyramids  $P$ ,  $Q$ , and  $R$  have the following characteristics.

- a. The altitude of pyramid  $P$  is 10.
- b. The lateral edge of pyramid  $Q$  is 10.
- c. The slant height of pyramid  $R$  is 10.

List the pyramids by order of volume from smallest volume to largest volume.

Support your conclusion with mathematical evidence.

[The formula for the volume of a pyramid with base area  $B$  and height (altitude)  $h$  is  $V = \frac{1}{3}Bh$ .]



Since the area of the base was the same for all three pyramids, the solution to the problem was dependent on the height, or altitude ( $h$ ), of the pyramids. The height of pyramid  $P$  was given as 10 units. The height of pyramid  $R$  could be found by recognizing that the distance from the foot of the segment representing the slant height to the middle of the base was 5 units and then using the Pythagorean theorem to find the height of the pyramid, namely  $5\sqrt{3}$ , or approximately 8.66 units. The height of pyramid  $Q$  also could be found by using the Pythagorean theorem. In this case, however, students had first to solve for either the slant height or the length of the segment running from the base of the lateral edge to the foot of the altitude. Once either of these dimensions had been determined, it was possible to solve for the altitude, or height. The correct solution was  $5\sqrt{2}$ , or approximately 7.07 units.

Since all three pyramids had the same base area, and their volumes could be found by multiplying the area of the bases by one-third of the heights, the volumes were ordered from smallest to largest in the same order as the heights were ordered from shortest to longest. That is,  $V_Q < V_R < V_P$ , since  $h_Q < h_R < h_P$ .

A few students noted that there is a quicker way to solve the problem without doing any calculations. Because the hypotenuse of a right triangle is the side of greatest length, it can be argued that the lateral edge is greater than the slant height, which in turn is greater than the altitude. Consequently, the pyramid with a lateral edge of 10 is “shorter” than the pyramid with a slant height of 10, which is “shorter” than the pyramid with an altitude of 10. Since “shorter” relates to the height of the pyramids, and since all of the pyramids are on the same base, the correct ordering follows.

Responses were rated “extended,” “satisfactory,” “partial,” “minimal,” and “incorrect.” The percentage of students achieving each rating is shown in Table 4.23, which follows the presentation of the sample responses.

Only responses that gave the correct ordering of the volumes of the three pyramids along with some supporting evidence received a rating of “extended.” In addition, if the student did not indicate that the answer was only dependent on height, the numerical values of all three volumes were required to be correct. Only five percent of the students participating in the Advanced Study reached this level of performance. The following response is an example of an “extended” response.

**Sample “extended” response**

$$V_{of\ P} = \frac{1}{3} B h \quad B = 100 \quad h = 10$$

$$V_p = \frac{1000}{3} = 333$$

$$U_{of\ Q} = \frac{1}{3} B h \quad B = 100 \quad L_s = 10 \text{ so}$$

$$\frac{100 \sqrt{50}}{3} = 230 \quad S_h = \sqrt{50}, \text{ so } h = \sqrt{50}$$

$$V_r = \frac{1}{3} B h \quad B = 100 \quad S_h = 10 \text{ so } h = \sqrt{10}$$

$$V_r = 289$$

$$V_Q < V_R < V_P$$

Because ~~is~~ the area of the base is equal, it can be cancelled as can the  $\frac{1}{3}$ . So, therefore, the volume is <sup>directly</sup> proportional to the height.

Responses that failed to order the three pyramids correctly but that provided correct solutions for the volumes of two or three pyramids, or for the heights of  $Q$  and  $R$ , or that correctly compared the heights of  $Q$ ,  $P$ , and  $R$ , were rated “satisfactory.” The following response was rated “satisfactory” because the volumes of pyramids  $P$  and  $R$  are correct.

**Sample “satisfactory” response**

**P**

$V_P = \frac{1}{3}Bh$   
 $V_P = \frac{1}{3}(100)(10)$   
 $V_P = 333.\bar{3}$

**Q**

$a^2 + 14^2 = 10^2$   
 $a^2 + 196 = 100$   
 $a^2 = -96$   
 $V_Q = \frac{1}{3}(100)(9.8)$   
 $V_Q = 326.\bar{6}$

**R**

$10^2 + 5^2 = 2^2$   
 $100 + 25 = 4$   
 $125 = 4$   
 $11 = c$

$14^2 + b^2 = 10^2$   
 $196 + b^2 = 100$   
 $b^2 = -96$   
 $b = 8.6$

$V_R = \frac{1}{3}(100)(8.6)$   
 $V_R = 286.\bar{6}$

Smallest to largest  
 $Q, P, R$

Responses were rated “partial” if they showed a correct solution for either the height or the volume of pyramid *Q* or *R*. The following response was rated “partial” because 288.7 is the volume of pyramid *R*.

**Sample “partial” response**

$x^2 + 25 = 50$   
 $x = 5$

$5^2 + 5^2 = h^2$   
 $h = 5\sqrt{2}$

$100 + 25 = h^2$   
 $125 = h^2$   
 $h = 5\sqrt{5}$

$V = \frac{1}{3}(100)(5)$   
 $V = 166\frac{2}{3} \Rightarrow Q$

$25 + x^2 = 100$   
 $x^2 = 75$   
 $x = 5\sqrt{3}$

$V = \frac{1}{3}(100)(10)$   
 $V = 33\frac{1}{3} \Rightarrow P$

$V = \frac{1}{3}(100)(5\sqrt{3})$   
 $V = 288.7$

smallest = P  
 Q  
 R

Responses that showed or stated that the three pyramids had the same base area or that  $\frac{1}{3}$  (area of the base) was the same for all three pyramids were rated “minimal.” Responses that contained a correct solution for the volume of pyramid  $P$  also received a “minimal” rating. The following response was rated “minimal,” since it clearly states that the bases of the three pyramids are equal. The area of the bases, however, should have been shown as 100, not 10.

**Sample “minimal” response**

$V = \frac{1}{3} Bh$

$B = 10$

Base of pyramid Q = Base of pyramid P = Base of pyr. R  
 Lateral edge " " lateral " " lateral " " "

Pyramid P: Base = 10  
 Alt = 10  
 lateral = 10

$V = \frac{1}{3} BH$

$V = \frac{1}{3} \cdot 10 \cdot 10$   
 $V = 33.\bar{3}$

Pyramid Q

$V = \frac{1}{3} \cdot 10 \cdot 10$   
 $= 33.3$

Pyramid R

$V = \frac{1}{3} \cdot 10 \cdot 10$   
 $= 33.3$

Responses that attempted to answer the question, but did not satisfy the conditions for at least a “minimal” response, were rated “incorrect.”

Performance data on the square pyramid question are presented in Table 4.23. This question was very challenging for students because it involved a significant number of steps. Students showed a number of approaches to solving the problem. Most students whose responses were rated at least “satisfactory” approached the problem analytically, similar to the sample responses shown previously. The fact that only 10 percent of the students received “satisfactory” or above is somewhat disappointing but may be explained by the fact that many students might not have worked with geometric volume problems since they took geometry, possibly as long as 2 or 3 years before the assessment.

**Table 4.23**

**Score Percentages for “Compare Volumes of Pyramids,” Grade 12**



	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 12</b>						
All Students	5	5	5	11	52	20
Males	6	6	5	12	46	22
Females	4	3	5	9	59	18
White	5	5	5	12	52	18
Black	0!	1!	1	6	58	32
Hispanic	3!	2!	1!	5!	57!	30!
Asian/Pacific Islander	8	6	5	9	51	18
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	5	5	5	11	53	19
No	2!	2!	1!	9!	46!	35!
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	8	8	6	13	47	16
No	2	3	4	9	56	22

NOTE: Row percentages may not total 100 due to responses rated “Off Task” or to rounding, or both.

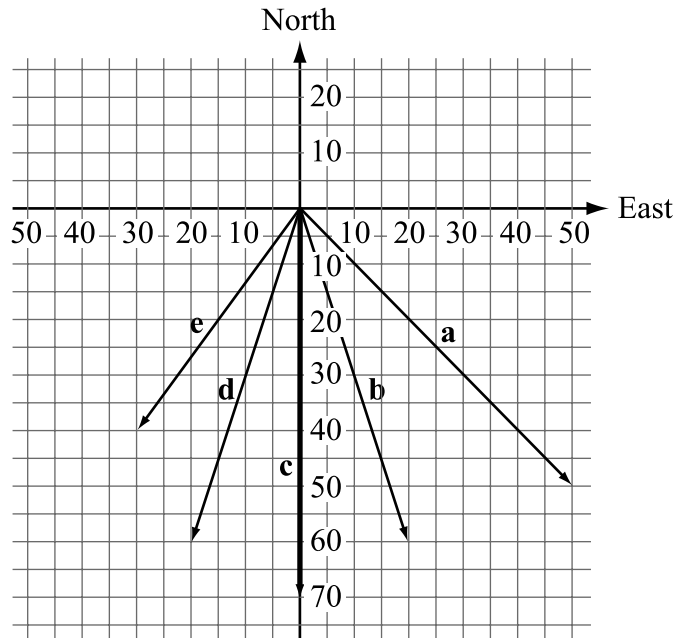
\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

Mathematics Assessment.

The third sample question from the Grade 12 Advanced Study is a multiple-choice question assessing students' ability to select the vector that correctly represented the result of a ship's movement from the origin of a coordinate grid to a position described in terms of the sum of two direction vectors. Students had to recognize that the resultant sum, or position, of a trip due south for 40 miles followed by a trip of 30 miles southwest could be represented by the vector associated with Option D. Sixty-three percent of the grade 12 students correctly selected Option D.



1. A ship travels due south for 40 miles and then southwest for 30 miles. Which of the vectors in the figure above best represents the result of the ship's movement from its starting point?

- (A) a
- (B) b
- (C) c
- (D) d
- (E) e

The only other option that was selected by more than five percent of the students was Option E. Twenty-seven percent of the students made this incorrect choice. These students selected a vector representing the sum of movement due south and due west rather than the sum of movement due south and southwest.

The problem could be solved in a variety of ways. Most students would not have had this type of question (representing and solving problems using vector methods) as part of their geometry classroom experience, but those who had studied physics or pre-calculus would have studied vectors. The result of 63 percent correct, shown in Table 4.24, is a measure of students' problem-solving skills related to interpreting a situation, representing it, and solving it. However, fewer students might have been successful if the task had been to draw the vector rather than simply to recognize it, or if they had been required to discriminate between two vectors with the same direction but different lengths.

**Table 4.24** **Percentage Correct for "Find Resultant Vector,"**  
**Grade 12**



Grade 12		Percentage Correct
All Students		63
Males		67
Females		58
White		66
Black		51
Hispanic		50
Asian/Pacific Islanders		55
American Indian		***
Are Students Presently Enrolled in Mathematics...		
Yes		63
No		52
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...		
Yes		67
No		60

\*\*\* Sample size is insufficient to permit a reliable estimate.

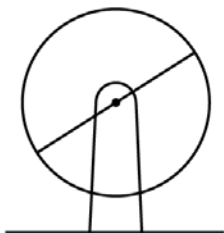
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.



The final sample question from the Grade 12 Advanced Study was classified as an Algebra and Functions question. It presented students with a written description of the motion of a Ferris wheel. Using the given information, students were to draw a graph depicting the height, over 45 seconds of time, of an individual on a wheel that rotates once every 15 seconds, if the individual is at the bottom of the wheel at time 0. In order to answer the question correctly, students needed to attend to three conditions:

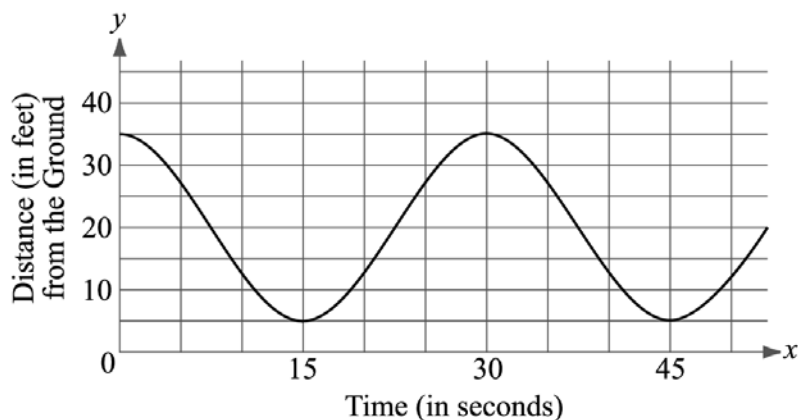
- at time  $t = 0$ ,  $h = 5$ ;
- the *height* can only vary from 5 feet to 35 feet throughout the graph; and
- the *period* in the solution graph is 15 seconds (i.e., the graph repeats itself every 15 seconds).

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.



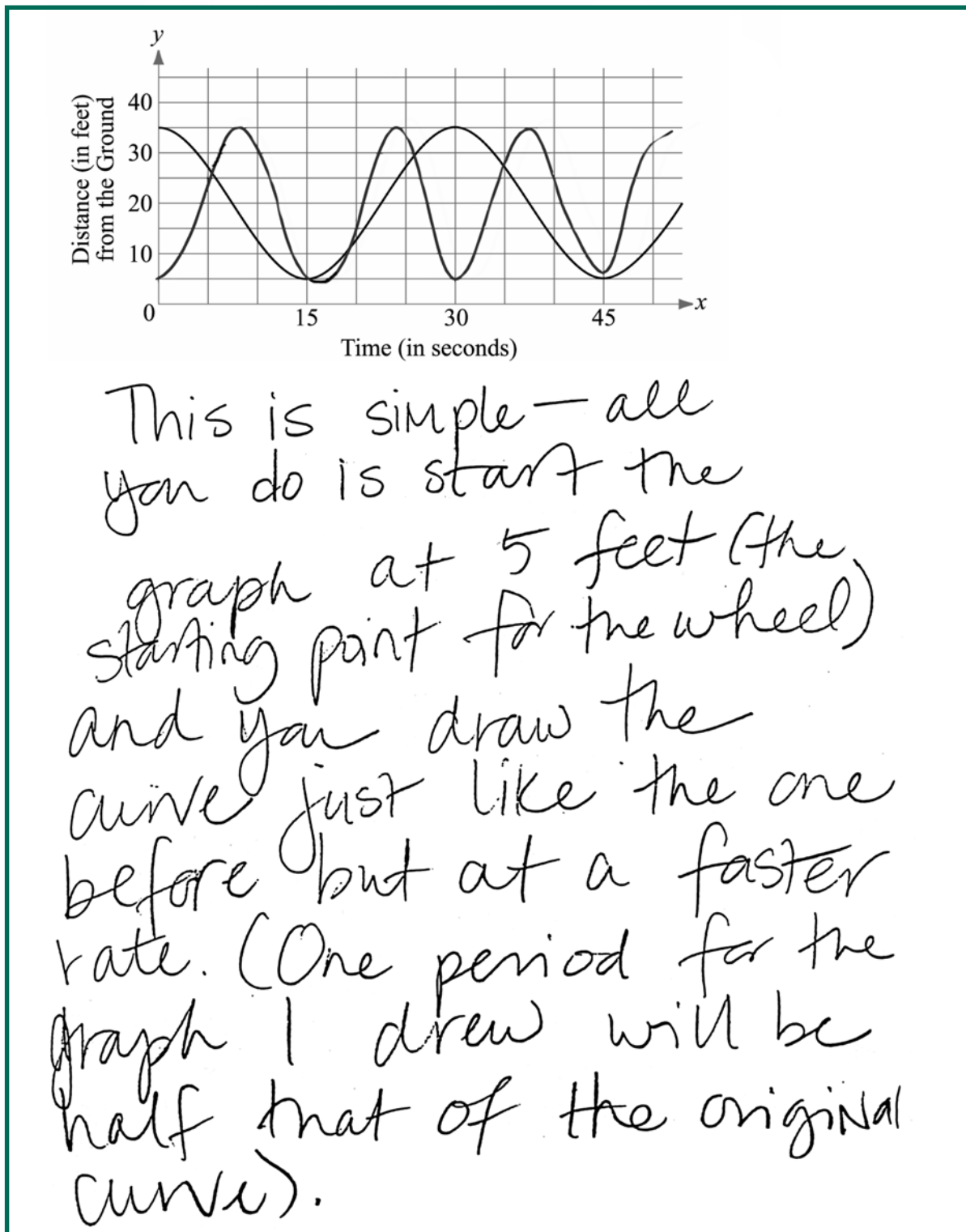
- 10.** The Ferris wheel above is 30 feet in diameter and 5 feet above the ground. It turns at a steady rate of one revolution each 30 seconds. The graph below shows a person's distance from the ground as a function of time if the person is at the top of the Ferris wheel at time 0.

On the same graph, draw a second curve that shows a person's distance from the ground, as a function of time, if that person is at the bottom of the Ferris wheel at time 0 and if the Ferris wheel turns at a steady rate of one revolution each 15 seconds. Sketch the graph from time equals 0 to time equals 45 seconds.



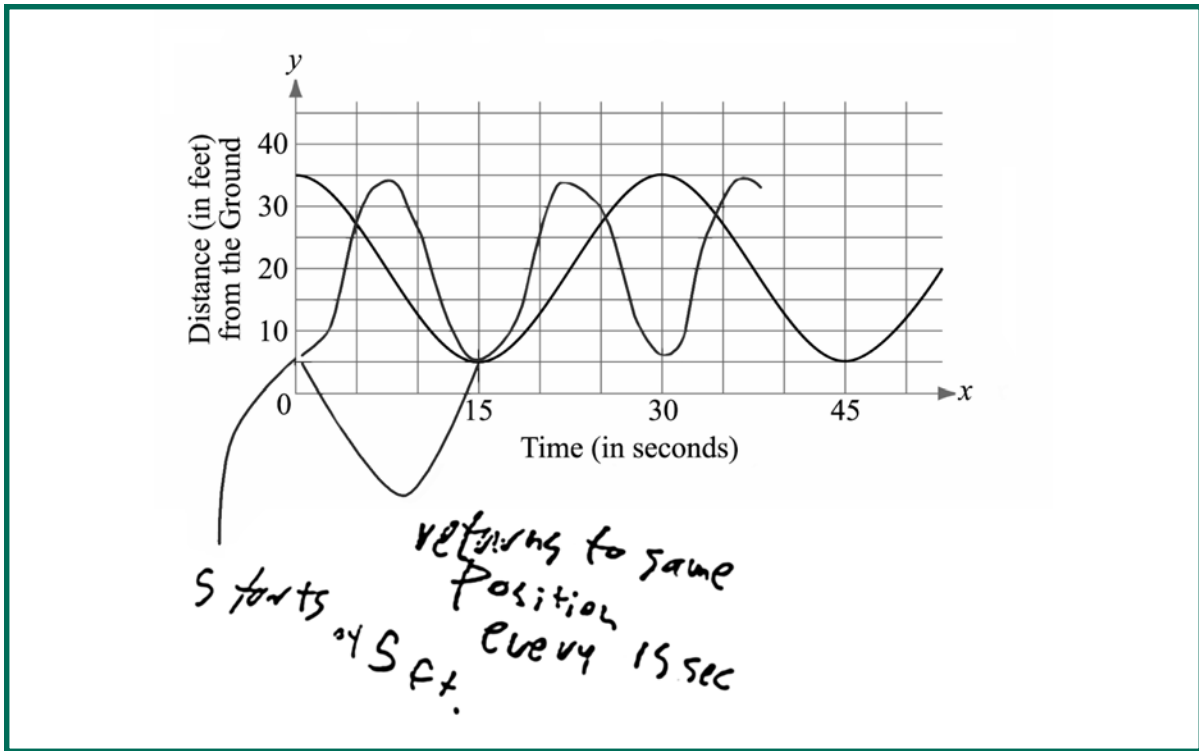
Responses were rated “extended,” “satisfactory,” “partial,” “minimal,” or “incorrect.” The rating of “extended” was awarded to responses that showed a complete and correct graph extending over the 45-second period, as shown below.

**Sample “extended” response**



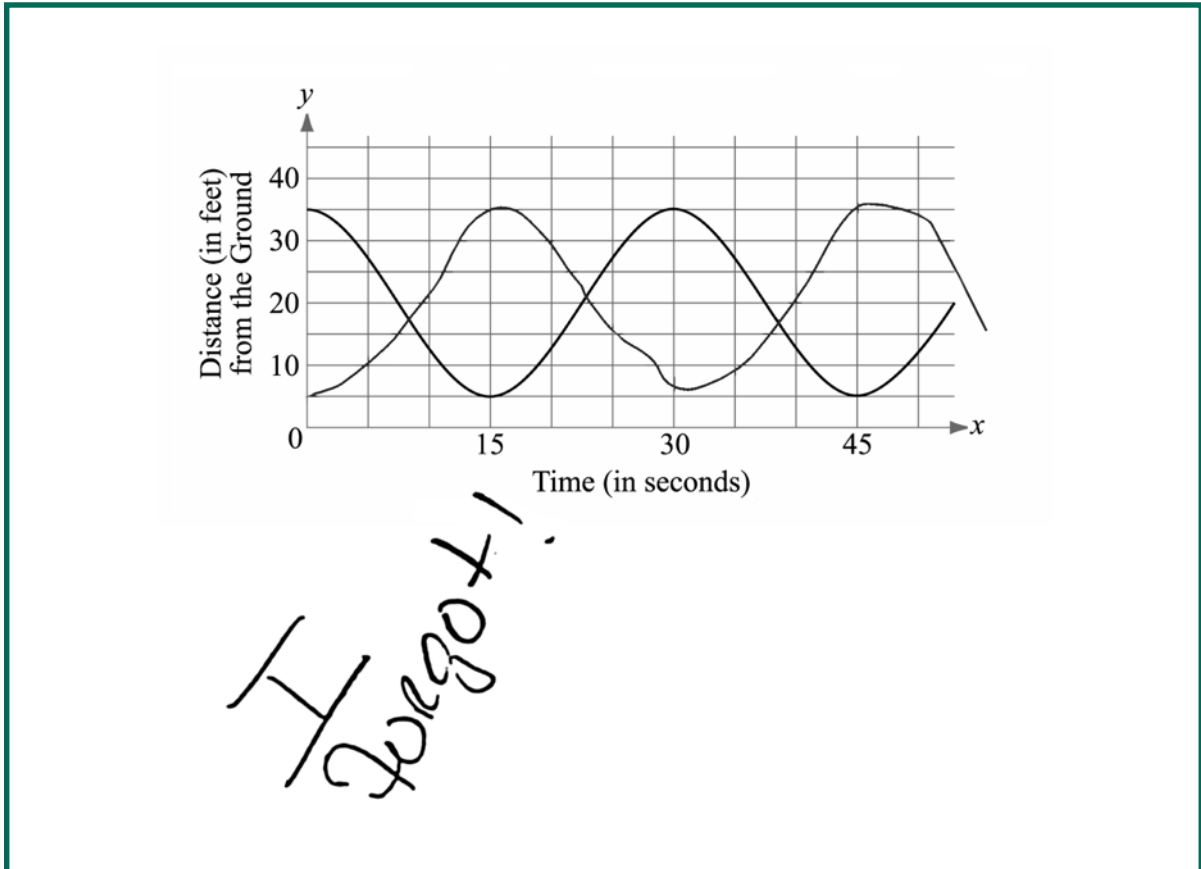
Responses containing a graph of *three* of the conditions but that failed to extend the graph from time 0 to 45 seconds were rated “satisfactory.” The sample “satisfactory” response shown satisfied three conditions, but the sketch was not shown for the entire time from  $t = 0$  to  $t = 45$  seconds. Very few students received “satisfactory” scores. In the sample response shown, the response error suggests inattention to the full requirements of the problem rather than a lack of understanding.

**Sample “satisfactory” response**



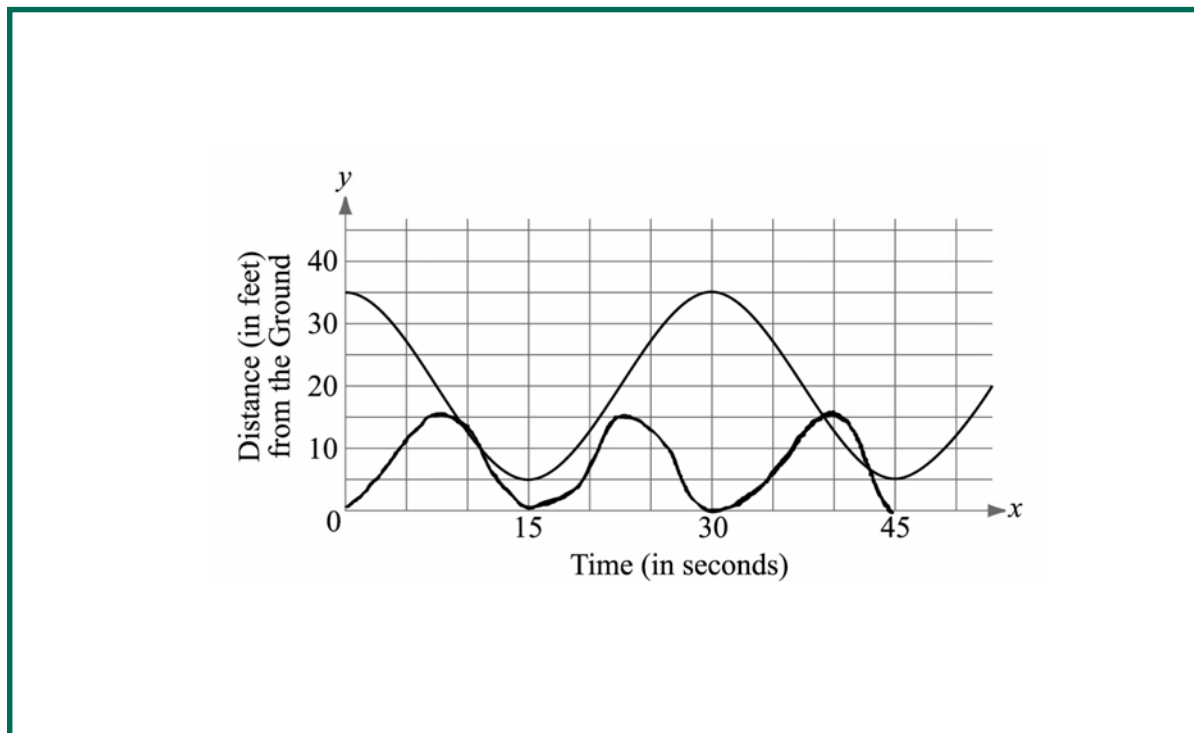
Ratings of “partial” were given to responses showing a graph of *two* of the conditions. The sample “partial” response shown satisfied the conditions that at time  $t = 0$ ,  $h = 5$  and that the height vary from 5 feet to 35 feet throughout the graph, but did not portray the correct period of 15 seconds per rotation.

**Sample “partial” response**



“Minimal” ratings were awarded to responses that correctly depicted a graph of *one* of the conditions. The sample “minimal” response shown has the 15-second period graphed correctly.

**Sample “minimal” response**



Responses that did not move beyond a simple restatement of the problem were rated “incorrect.”

Table 4.25 presents student performance data on the Ferris wheel problem. The data show that students did relatively well on this question; 39 percent of the responses were rated as “extended.” The percentage of students receiving the “extended” rating is somewhat surprising, considering student performance on the other problems. This higher level of performance may be a reflection of how recently students in the Grade 12 Advanced Study had dealt with similar content. The height and vertical variation in the graph indicate the amplitude of a periodic function. The compression of the graph horizontally represents the period of the graph. Both of these topics, amplitude and periodicity, receive a great deal of attention in the study of trigonometric, or circular, functions in pre-calculus and calculus.

**Table 4.25**

**Score Percentages for “Ferris Wheel,” Grade 12**



	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 12</b>						
All Students	39	0	18	13	19	10
Males	44	1	18	11	14	10
Females	33	0!	18	16	23	9
White	44	0	19	13	16	7
Black	13	0!	13	12	34	25
Hispanic	22!	0!	12!	12!	30!	20!
Asian/Pacific Islander	33	0!	19	14	22	11
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	40	0	18	13	18	9
No	24!	0!	19!	17!	21!	14!
Are Students Presently Enrolled in or Have Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	50	0!	17	12	14	8
No	32	0	20	14	22	10

NOTE: Row percentages may not total 100 due to responses rated “Off Task” or to rounding, or both.

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## ***The Effects of Grade 12 Mathematics and Advanced Placement***

An analysis of the data related to the performance of students in the Advanced Study who reported that they were currently taking mathematics or who were, or had been, enrolled in an AP mathematics course shows that in each case these students outperformed students in the study who were not currently taking a mathematics course or who had not taken such an AP course. Some of those not currently taking a course may have taken mathematics during the first semester of their senior year as part of a block scheduling arrangement at their schools. Others may simply have completed their schools' mathematics curriculum earlier and were "lying fallow" mathematically during the assessment semester. Finally, it also is possible that some of the students who, by self-report, were not currently enrolled in mathematics had been misclassified by the individuals responsible for the NAEP samples at their schools and were not really eligible for the Advanced Study.

### ***Summary***

The background and demographic data collected along with the achievement data indicate that both grade 8 and grade 12 students participating in the Advanced Study were different from those who did not qualify for the study. They tended to come from homes providing a stronger educational context, both in materials and in parental education. Further, these students appeared to have somewhat better financial means, as a smaller proportion qualified for the federal Free/Reduced-Price Lunch program. Similarly, fewer of them participated in Title I programs.

The Advanced Study was designed to provide students who were taking or had taken advanced courses in mathematics with an opportunity to demonstrate their full mathematical proficiency. Students at both grade levels who met the criterion for inclusion in the Advanced Study performed substantially better than other students on the main NAEP mathematics assessment. However, student performance on the Advanced Study itself shows that the study questions were quite difficult, even for students who were taking the more challenging mathematics courses that were prerequisite to qualify. Overall performance, measured by percentage correct, was 36 percent at grade 8 and 30 percent at grade 12, and, at both grade levels, most of these students were unable to solve problems that required two or three successive steps to achieve the desired result.





# Appendix A

## Procedures

### **The NAEP 1996 Mathematics Assessment**

The 1996 assessment was the first update of the NAEP mathematics assessment framework since the release of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*.<sup>1</sup> This update reflected refinements in the specifications governing the development of the NAEP 1996 mathematics assessment while ensuring comparability of results across the 1990, 1992, and 1996 assessments.

### **Special Studies in the 1996 NAEP Mathematics Assessments**

In addition to the main NAEP 1996 mathematics assessment, three special studies were conducted: an Estimation Study, a Theme Study, and an Advanced Study. In each of the studies, students were presented with an assessment booklet consisting of blocks of cognitive questions. The Estimation and Advanced Studies had a booklet for each grade level, while the Theme Study had two different booklets at each grade level.

The booklets consisted of blocks of cognitive questions and blocks of background questions. The cognitive block structure of the special study booklets is displayed in Table A.1. The Estimation and Theme booklets began with block M4, a Balanced Incomplete Block (BIB) linking block taken from the main assessment. In the Estimation booklets, the linking block was followed by two Estimation blocks: a trend block, M16, and a new block, M17. In the Theme booklets, the linking block was followed by a single Theme block, either M21 or M22. The Advanced booklet began with a nonadvanced block, M20, which was composed of questions from various BIB (main assessment) blocks; this was followed by two Advanced blocks, M18 and M19. In addition to the cognitive blocks, all of the special study booklets had three blocks in common with the main assessment: a general student background block, a mathematics background block, and a motivation block.

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<sup>1</sup> National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Table A.1

**Block Structure of the Special Study Booklets**

Study	Booklet	Cognitive Blocks
Estimation	127	M4, M16, M17
Theme	128	M4, M21
Theme	129	M4, M22
Advanced Mathematics*	130	M20, M18, M19

\* Grades 8 and 12 only

The cognitive portion of the Estimation booklet was administered in two sections: the first (BIB) block was self-paced; and the two Estimation blocks were administered by a paced audio tape. The Theme and Advanced booklets were entirely self-paced.

The special studies were not part of the main assessment. However, the Estimation and Theme samples were drawn from the same population as the main assessment; that is, they were chosen to be representative of all students at the target grade level. The grades 8 and 12 Advanced samples, by contrast, were drawn from populations of students who were considered high mathematics achievers based on their enrollment in advanced mathematics classes during the 1995–1996 school year.

### ***Sampling Procedures for the Special Studies***

The populations for the special studies were sampled as part of the complex multistage sample design used for the main national sample. The design involved sampling students from selected schools within 94 selected geographic areas, called primary sampling units (PSUs), across the United States. There were five steps in the selection process:

1. Selection of geographic PSUs (counties or groups of counties).
2. Selection of schools within PSUs.
3. Assignment of sample types to schools.
4. Assignment of session types to schools.
5. Selection of students for session types within schools.

The samples for the main and special studies were drawn for grades 4, 8, and 12. In addition, separate age-based samples were selected for the long-term trend studies in mathematics. Table A.2 shows the numbers of students and schools that were assigned to each condition. In addition to representing the respective populations as a whole, the main and special study samples involved oversampling of nonpublic schools, and of public schools with moderate or high enrollment of Black or Hispanic students. This oversampling was undertaken to increase the sample sizes of nonpublic school students and minority students, so as to increase the reliability of estimates for those groups of students. The assessment period for the main assessment and special studies was in the winter of 1996, between January 3 and March 29.

Table A.2

### Number of Students Per School for Each Session Type

	Number of Assessed Students	Number of Schools	Mean Number of Students Per Assessment Per School	Mean Number of Students Per Item Per School
<b>Sample</b>				
Age Class 9				
Long-Term Trend				
Print Booklets 51–56	5,019	215	23.3	3.9–7.8*
Tape Booklet 91	1,852	127	14.6	14.6
Tape Booklet 92	1,721	116	14.8	14.8
Tape Booklet 93	1,840	125	14.7	14.7
Grade 4 Main				
Print Mathematics	10,830	445	24.3	5.6
Print Science	11,578	421	27.5	4.5–7.4*
Tape Mathematics Estimation	2,115	120	17.6	17.6
Print Mathematics Theme	4,004	230	17.4	8.7
Age Class 13				
Long-Term Trend				
Print Booklets 51–56	5,493	221	24.9	4.1–8.3*
Tape Booklet 91	1,928	128	15.1	15.1
Tape Booklet 92	1,866	125	14.9	14.9
Tape Booklet 93	1,864	124	15.0	15.0
Grade 8 Main				
Print Mathematics	11,521	411	28.0	6.5
Print Science	11,971	346	34.6	5.6–9.4*
Tape Mathematics Estimation	2,244	104	21.6	21.6
Print Mathematics Theme	4,227	175	24.2	12.1
Print Advanced Mathematics	2,365	253	9.3	9.3
Age Class 17				
Long-Term Trend				
Print Booklets 51–56	4,669	186	25.1	4.2–8.4*
Tape Booklet 84	1,848	133	13.9	13.9
Tape Booklet 85	1,691	122	13.9	13.9
Grade 12 Main				
Print Mathematics	10,600	430	24.8	5.7
Print Science	11,481	401	28.6	4.6–7.7*
Tape Mathematics Estimation	1,889	96	19.7	19.7
Print Mathematics Theme	3,860	196	19.7	9.8
Print Advanced Mathematics	2,965	207	14.3	14.3
Print Advanced Science	2,431	222	11.0	11.0

\* This number varied because some item blocks appeared more often than others in the set of booklets used for this sample.

Following is a brief summary of the general features of the sampling procedure. Many details and adjustments left out of this description can be found in Chapter 3 of *The NAEP 1996 Technical Report*.<sup>2</sup>

## ***PSU Selection***

Of the population of 1,000 PSUs in the country, the 22 largest PSUs were included in the sample with certainty. The remaining PSUs were partitioned into 72 strata (defined by region and various socioeconomic characteristics), and one PSU was chosen from each stratum with probability proportional to size (of the population of the PSU).

## ***Selection of Schools***

In the second-stage sampling a list was created, for each grade, of all public and nonpublic schools in the 94 selected PSUs. Schools were selected (without replacement) across all PSUs with probabilities proportional to assigned measures of size. Details of the assignment of school size can be found in Section 3.3 of *The NAEP 1996 Technical Report*.

## ***Assignment of Sample Type to Schools***

In order to determine the effect of using different guidelines for excluding or assessing students with disabilities or limited English proficiency, three different sample types were assigned to schools selected for the main or special study assessments. These sample types were:

- S1, in which students were subject to the exclusion criteria used in 1990 and 1992;
- S2, in which students were subject to new 1996 exclusion criteria; and
- S3, in which students were subject to new 1996 exclusion criteria and accommodations were offered to students with disabilities (SD) and limited English-proficient (LEP) students.<sup>3</sup>

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<sup>2</sup> Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). *The NAEP 1996 technical report*. Washington, DC: National Center for Education Statistics.

<sup>3</sup> See Chapter 5 of the *NAEP 1996 Technical Report* for a fuller description of sample types, exclusion criteria, and accommodations.

Sample type was assigned to schools separately for each grade. For schools that were not also selected for the State Assessment, sample type was assigned to schools according to the percentages shown in Table A.3.

**Table A.3** *Percent of Schools Assigned to Sample Type in the 1996 Assessment*



		Sample Type		
		S1	S2	S3
Grade	4	20	45	35
	8	16	42	42
	12	15	46	39

### **Sampling Students**

The first step in sampling students was to compile a list of all grade-eligible students in a school. Along with this list, schools at grades 8 and 12 were asked to provide course enrollment information to be used in determining eligibility for the Advanced Study. Specifically, eligibility was based on reported enrollment in Algebra 1 or beyond at grade 8 and on reported enrollment in Algebra 3, Pre-Calculus, Calculus and Analytical Geometry, Calculus, or AP Calculus at grade 12.

Systematic selection of students from the grade-eligible list was made to obtain the target sample size for each school. Westat, Inc. district supervisors then assigned sampled students to one of the various session types according to specified procedures. Session types included the various subject areas and, within the subject area of mathematics, whether the students would be included in the main (BIB), long-term trend, Estimation, Theme, or Advanced assessments. Obviously, only students who were eligible for the Advanced Study could be assigned to Advanced sessions, but students eligible for the Advanced Study also were assigned proportionally to all other session types.

To ensure an adequate sample size of SD/LEP students, oversampling procedures were applied to these students in sample type 2 (S2) and sample type 3 (S3). In general, SD/LEP students were sampled (within schools) at twice the rate at which non-SD/LEP students were sampled. For grades 8 and 12, because of the way sessions were assigned, oversampling of SD/LEP students took place only among students who were not eligible for the Advanced assessment.

## Collected and Reporting Samples

The exclusion and accommodation conditions that were applied in S3 are anticipated to be similar to those that will be used in future NAEP assessments. Consequently, the data from S3 constitute a statistical bridge that can be used in maintaining future trend lines. However, in order to allow timely reporting of 1996 results, data from S3 were generally not included in the reporting sample, but were set aside for further analysis of the impact of the availability of accommodations on group achievement estimates.

The following tables — A.4 through A.6 — illustrate how the special studies were distributed across the S1, S2, and S3 samples. In the tables, the notation “B” refers to that subset of the participating students who were identified as having either disabilities or limited English proficiency. “A” refers to all other participating students. As can be seen, the data reported for the Estimation Study were generally drawn from S1, but S2 students were added at grade 12 in order to obtain a sufficient sample size. However, to insure comparability across grades, results from S2 students with disabilities or limited English proficiency were not reported. The Theme Study sample was drawn from S2. The Estimation and Theme Studies also were administered to S3, as a statistical bridge to future administrations of these studies. The Advanced Study sample was taken from S3 at grade 12 and, in order to obtain a sufficient sample size, from both S2 and S3 at grade 8. Due to the nature of the Advanced Study selection criteria, accommodations were infrequent in this sample, particularly at grade 12. Consequently, it was judged acceptable to report Advanced Study results from S3. As a precaution, however, data from grade 8 students with disabilities or limited English proficiency (where some accommodations did occur) were held back from reporting.

		THE NATION'S REPORT CARD		
		S1	S2	S3
<b>Grade 4</b>				
	A	R	X	NR
	B	R	X	NR
<b>Grade 8</b>				
	A	R	X	NR
	B	R	X	NR
<b>Grade 12</b>				
	A	R	R	NR
	B	R	NR	NR

\* An “R” indicates that this cell was part of the reporting sample. An “NR” cell indicates that data were collected but not included in the reporting sample. An “X” indicates that no data were collected from that category.

Table A.5

### Collected and Reporting Samples for the Theme Assessment Grades 4, 8, and 12\*



	S1	S2	S3
A	X	R	NR
B	X	R	NR

\* An "R" indicates that this cell was part of the reporting sample. An "NR" cell indicates that data were collected but not included in the reporting sample. An "X" indicates that no data were collected from that category.

Table A.6

### Collected and Reporting Samples for the Advanced Assessment, Grades 8 and 12 Only\*



	S1	S2	S3
<b>Grade 8</b>			
A	X	R	R
B	X	R	NR
<b>Grade 12</b>			
A	X	X	R
B	X	X	R

\* An "R" indicates that this cell was part of the reporting sample. An "NR" cell indicates that data were collected but not included in the reporting sample. An "X" indicates that no data were collected from that category.

## Participation Rates

In order for the data to be reported, NCES school and student participation rates must be met. School nonparticipation or student nonparticipation in the form of absenteeism creates a potential for bias to be introduced in the reporting of the data. The participation rates of schools and students included in the 1996 assessments were inspected for any differences. NCES standards regarding acceptable potentials for bias are expressed in terms of weighted participation rates. Table A.7 shows the weighted participation rates by grade and session type for the main reporting samples. For the main samples, the student participation rates are similar for different session types at grades 4 and 8, but the student participation rates at grade 12 and the school participation rates at all grades vary by session type. The differential school participation rates show that different session types include different schools. This is due to the assignment of schools to sample type, the fact that all session types were not assessed in all sample types, and the specific sample types included in the reporting populations for each session type.

Table A.7

**Weighted Participation Rates (in %), by Grade and Session Type, 1996 Main NAEP Reporting Samples**


	Mathematics Print	Science Print	Mathematics Estimation Print	Mathematics Theme Print	Advanced Mathematics Print	Advanced Science Print
<b>Grade 4</b>						
School Participation	82.3	77.8	93.5	77.9	—	—
Student Participation	95.3	94.9	96.7	95.4	—	—
Overall Participation	78.4	73.8	90.4	74.4	—	—
<b>Grade 8</b>						
School Participation	81.5	79.7	85.3	86.8	77.0	—
Student Participation	92.9	93.1	93.8	92.7	95.6	—
Overall Participation	75.7	74.3	80.0	80.4	73.6	—
<b>Grade 12</b>						
School Participation	76.2	77.4	63.9	78.4	77.6	77.7
Student Participation	82.3	77.5	81.0	78.2	85.8	86.5
Overall Participation	62.7	60.0	51.7	61.3	66.6	67.2

### ***Evaluation of Potential for Bias***

Although school and student nonresponse adjustments are intended to reduce the potential for nonparticipation to bias the assessment results, they cannot completely eliminate this potential bias with certainty. The extent of bias remains unknown since there are no assessment data for the nonparticipating schools and students.

Some insight can be gained about the potential for residual nonresponse bias by examining the weighted school- and student-level distributions of characteristics known for both participants and nonparticipants, especially for those characteristics known or thought likely to be related to achievement on the assessment. If the distributions for the full sample of schools (or students) without the use of nonresponse adjustments are close to those for the participants with nonresponse adjustments applied, there is reason to be confident that the bias from nonparticipation is small.

A nonresponse bias analysis completed on the reporting population for the science assessment can be found in the *NAEP 1996 Technical Report*.<sup>4</sup> Science was chosen because it contained the largest number of students and could, therefore, provide the most precise estimates of student distributions across several demographic characteristics (i.e., age, race, gender, type of school and school size). Generally, the findings show that the student distributions before and after school and/or student nonresponse adjustments were similar, with a few exceptions. Most of these exceptions were at grade 12 due to its relatively high nonresponse rate (20.3% for grade 12 students). An additional nonresponse bias analysis was completed on the

<sup>4</sup> Allen, et al., (1999). op. cit.



reporting populations of Mathematics, Mathematics Theme, Mathematics Estimation and Advanced Mathematics. Each of these reporting populations have smaller numbers of schools and students in the sample. Thus, the variance of the estimated distributions will be larger for these subjects, than for science.

Within the NAEP data, there are several school-level characteristics available for both participating and nonparticipating schools. The tables that follow show the combined impact of nonresponse and of the nonresponse adjustments on the distributions of schools (weighted by the estimated number of eligible students enrolled) and students, by the type of school (public, Catholic, other nonpublic), the size of the school (measured by the estimated number of eligible students enrolled) and whether the school is located in an urban/rural place. Three size classes have been defined for each grade. The data are for the 1996 samples excluding the science assessment.

Several student-level characteristics are available for both absent and assessed students. The tables that follow show the impact of school nonresponse and nonresponse adjustments, and student nonresponse and nonresponse adjustments on the distributions of eligible students for each grade. The distributions are presented by age category (at or below modal age, and above modal age), race/ethnicity category (White, Black, Hispanic, and other), gender, SD and LEP.

Table A.8 shows the weighted marginal distributions of students for each of the three classification variables for grade 12, using weighted eligible schools. The distributions before school nonresponse adjustments are based on the full sample of in-scope schools for each assessment — those participating, plus those refusals for which no substitute participated. The distributions after school nonresponse adjustments are based only on participating schools for each assessment, with school nonresponse adjustments applied to them. The weighted school-level nonparticipation rates at grade 12 are as follows: Mathematics, 27.8 percent; Mathematics Theme, 21.6 percent; Mathematics Estimation, 36.1 percent; and Advanced Mathematics, 22.4 percent. For more detail, see the *NAEP 1996 Technical Report*.<sup>5</sup>

**Table A.8** *Distribution (in %) of Populations of Eligible Students Based on Full Weighted Sample of Eligible Schools, Before and After School Nonresponse Adjustments, 1996 Main NAEP Samples (Excluding Science), Grade 12*



Population	Mathematics		Mathematics Theme		Mathematics Estimation		Advanced Mathematics		Advanced Science	
	Before	After	Before	After	Before	After	Before	After	Before	After
<i>School Type</i>										
Catholic	5.8	6.6	4.5	5.5	7.5	7.1	4.5	5.2	5.4	5.5
Other Nonpublic	3.6	3.1	4.5	3.8	3.9	4.8	4.1	3.4	3.8	3.1
Public	90.6	90.3	91.0	90.8	88.5	88.1	91.4	91.4	90.7	91.4
<i>School Size<sup>1</sup></i>										
1 (1–49)	6.0	6.2	5.7	5.1	6.8	8.3	7.1	7.3	5.5	5.0
2 (50–399)	67.9	66.0	71.6	71.4	72.1	66.4	72.7	72.9	69.4	65.7
3 (400+)	26.1	27.7	22.7	23.5	21.1	25.3	20.2	19.9	25.1	29.2
<i>School Location</i>										
Large City	14.8	15.1	14.9	17.5	14.7	15.4	13.5	15.1	15.4	16.0
Midsize City	18.6	21.2	14.6	15.6	20.8	23.4	17.3	18.2	16.3	18.4
Urban Fringe/ Large City	22.4	19.1	27.5	23.5	18.5	19.5	25.0	22.6	23.8	22.5
Urban Fringe/ Midsize City	14.5	15.2	13.3	15.2	17.5	14.4	14.3	15.2	15.0	13.9
Large Town	1.1	1.2	0.7	0.9	0.7	1.1	0.5	0.7	1.5	0.9
Small Town	12.1	13.1	14.5	14.6	11.5	11.5	16.0	16.1	14.7	15.9
Rural MSA	3.8	3.0	6.0	5.7	4.5	2.1	5.6	4.5	4.1	4.1
Rural nonMSA	12.8	12.0	8.4	7.0	11.8	12.6	7.7	7.6	9.1	8.4

<sup>1</sup>Distributions by school size are not comparable to previous assessments, since students were eligible by grade only (instead of by grade or age) in 1996. School size refers to the number of eligible students enrolled.

<sup>5</sup> Allen, et al., (1999). op. cit.

It can be seen from Table A.8 that, overall, the distributions for school type, school size and school location remain similar. For Mathematics, even though the nonresponse rate is 27.8 percent, the only exceptions may be midsize cities and urban fringe of large cities; for Mathematics Theme, the exceptions may be large cities and their urban fringe; for Mathematics Estimation, exceptions may be at medium and large schools, midsize cities and their urban fringe, and rural MSAs (Metropolitan Statistical Areas). For Advanced Mathematics, even though the school nonparticipation rate is 22.4 percent, the only exception may be urban fringes of large cities. The potential for bias is greatest for Mathematics Estimation, which also has the highest nonparticipation rate for schools.

Table A.9 shows the distributions of two school-level characteristics — school type and school location, plus additional distributions of student-level characteristics, using weighted eligible students. The distributions before student nonresponse adjustments are based on assessed and absent students (with base weights adjusted for school nonparticipation). The distributions after student nonresponse adjustments are based on assessed students only, with the student nonresponse adjustments also applied to them. The rates of student nonparticipation for the five subjects at grade 12 are as follows: Mathematics, 17.7 percent; Mathematics Theme, 21.8 percent; Mathematics Estimation, 19.0 percent; Advanced Mathematics, 14.2 percent. For more detail, see the *NAEP 1996 Technical Report*.<sup>6</sup>

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<sup>6</sup> Allen, et al., (1999). op. cit.

Table A.9

**Distribution (in %) of Populations of Eligible Students Before and After Student Nonresponse Adjustments, 1996 Main NAEP Samples (Excluding Science), Grade 12**



Population	Mathematics		Mathematics Theme		Mathematics Estimation		Advanced Mathematics		Advanced Science	
	Before	After	Before	After	Before	After	Before	After	Before	After
<b>School Type</b>										
Catholic	7.2	8.0	5.9	7.5	7.9	9.2	9.1	10.6	10.6	11.7
Other Nonpublic	3.3	3.8	3.8	4.1	5.2	6.1	7.1	7.9	4.9	5.4
Public	89.5	88.1	90.3	88.3	86.9	84.7	83.7	81.5	84.5	82.9
<b>School Location</b>										
Large City	15.3	14.6	17.2	17.3	15.0	14.5	15.5	16.9	15.5	15.2
Midsize City	21.3	20.8	15.8	15.4	23.4	23.5	20.4	19.8	16.7	15.6
Urban Fringe/ Large City	19.2	18.8	23.8	23.6	19.2	19.4	19.5	18.8	27.1	27.6
Urban Fringe/ Midsize City	14.4	15.2	15.0	15.7	14.3	15.1	15.5	14.8	12.1	12.9
Large Town	1.3	1.2	0.9	0.9	1.3	1.2	0.7	0.7	0.8	0.7
Small Town	13.2	13.1	14.6	14.3	11.5	10.7	14.4	14.6	13.9	13.5
Rural MSA	2.9	3.1	5.9	5.8	2.0	2.2	6.0	6.6	5.6	5.9
Rural nonMSA	12.5	13.1	6.8	7.0	13.2	13.4	7.9	7.9	8.3	8.7
<b>Age Category</b>										
At Modal Age or Younger	65.9	66.0	65.4	65.7	67.0	66.6	74.0	73.7	72.5	72.3
Older than Modal Age	34.1	34.0	34.6	34.3	33.0	33.4	26.0	26.3	27.5	27.7
<b>Race/Ethnicity</b>										
White	70.0	70.0	69.1	68.6	70.5	70.9	74.8	74.1	74.8	74.1
Black	13.1	13.4	13.1	13.0	12.7	12.8	7.3	7.2	8.9	9.1
Hispanic	10.1	9.6	10.6	11.2	11.6	10.9	7.5	7.9	6.8	7.0
Other	6.8	7.0	7.1	7.2	5.1	5.4	10.4	10.8	9.5	9.7
<b>Gender<sup>1</sup></b>										
Male	47.9	47.6	49.8	49.1	47.4	47.3	50.2	50.3	49.4	49.2
Female	52.1	52.4	50.2	50.8	52.6	52.7	49.1	49.0	50.5	50.8
<b>SD</b>										
Yes	2.7	2.7	2.5	2.2	2.7	2.4	0.6	0.7	0.4	0.3
No	97.3	97.3	97.5	97.8	97.3	97.6	99.4	99.3	99.6	99.7
<b>LEP</b>										
Yes	1.1	1.1	1.8	1.6	0.9	1.0	1.2	1.2	1.1	1.0
No	98.9	98.9	98.2	98.4	99.1	99.0	98.8	98.8	98.9	99.0
<b>SD, LEP</b>										
SD Yes; LEP Yes	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SD Yes; LEP No	2.7	2.7	2.4	2.2	2.7	2.4	0.6	0.7	0.4	0.3
SD No; LEP Yes	1.1	1.1	1.8	1.6	0.9	1.0	1.2	1.2	1.1	1.0
SD No; LEP No	96.2	96.2	95.7	96.2	96.3	96.5	98.2	98.1	98.5	98.6

<sup>1</sup> Gender is unknown for a small percentage of students.

Table A.9 shows that with two exceptions at grade 12, for the distributions of type of school attended and place where the school is located, the effect of the student nonparticipation adjustment has resulted in very little change in distribution. The changes in the distribution of school type for Mathematics Estimation and Advanced Mathematics reflect the relatively high nonresponse rate of grade 12 public school students (22.7% versus 8.5% for nonpublic school students; from the *NAEP 1996 Technical report*).<sup>7</sup>

Table A.10 shows the weighted distributions of eligible students in participating schools, using the base weights of assessed and absent students unadjusted for school-level nonresponse. Tables A.9 and A.10 show that both school and student-level nonresponse and nonresponse adjustments have little effect on the distributions of eligible students by age, race/ethnicity, gender, and SD and LEP status. All of the distributions in the tables are similar.

**Table A.10**

***Distribution (in %) of Populations of Eligible Students Before School and Student Nonresponse Adjustment, 1996 Main NAEP Samples (Excluding Science), Grade 12***



	Mathematics	Mathematics Theme	Mathematics Estimation	Advanced Mathematics	Advanced Science
<b>Population</b>					
<i>Age Category</i>					
At Modal Age or Younger	65.6	64.9	66.3	74.0	72.1
Older than Modal Age	34.4	35.1	33.7	26.0	27.9
<i>Race/Ethnicity</i>					
White	71.0	69.1	70.8	75.4	76.0
Black	12.8	13.4	12.6	7.5	8.8
Hispanic	9.5	10.3	11.6	7.3	6.3
Other	6.8	7.1	5.1	9.8	9.0
<i>Gender<sup>1</sup></i>					
Male	47.9	49.9	47.8	49.8	49.6
Female	52.1	50.0	52.2	49.4	50.3
<i>SD</i>					
Yes	2.6	2.5	2.8	0.5	0.4
No	97.4	97.5	97.2	99.5	99.6
<i>LEP</i>					
Yes	1.0	1.8	1.0	1.1	1.0
No	99.0	98.2	99.0	98.9	99.0
<i>SD, LEP</i>					
SD Yes; LEP Yes	0.0	0.0	0.0	0.0	0.0
SD Yes; LEP No	2.6	2.5	2.8	0.5	0.4
SD No; LEP Yes	1.0	1.8	1.0	1.1	1.0
SD No; LEP No	96.3	95.7	96.2	98.4	98.6

<sup>1</sup> Gender is unknown for a small percentage of students.

<sup>7</sup> Allen, et al., (1999). op. cit.

When comparing the distributions in Table A.9 before and after student nonresponse adjustments, we expect the distributions by age category and race/ethnicity to be similar because these variables were used to determine student nonresponse adjustment classes. However, the distributions by gender and SD and LEP status are also similar. To the extent that nonrespondents would perform like respondents with the same characteristics (defined by the characteristics in the tables), the bias in the assessment data is small.

Further information about potential nonresponse bias can be gained by studying the absent students. NAEP proficiency estimates are biased to the extent that assessed and absent students within the same weighting class differ in their distribution of proficiency. It seems likely that the assumption that absent students are similar in proficiency to assessed students is reasonable for some absent students — namely, those whose absence can be characterized as random. Conversely, it seems likely that students with longer and more consistent patterns of absenteeism — such as truants, dropouts, near dropouts, and the chronically ill — are unlikely to be as proficient as their assessed counterparts.

In the 1996 assessments, schools were asked to classify each absent student into one of nine categories. The results of this classification for the assessments are shown in Table A.11. Table A.11 shows that, as anticipated, the majority of absenteeism from the assessment was the result of an absence from school of a temporary and unscheduled nature. The table shows that among the two Advanced sessions, the absenteeism rate is lower than among the ‘non-Advanced’ sessions. The proportion of absenteeism classified as temporary is similar across subjects, including science (63.6%).

Table A.11

**Weighted Distribution (in %) of Absent Students,  
by Nature of Absenteeism, 1996 Assessments  
(Excluding Science), Grade 12**



	Mathematics	Mathematics Theme	Mathematics Estimation	Advanced Mathematics	Advanced Science
<b>Nature of Absenteeism</b>					
Temporary Absence <sup>1</sup>	65.2	59.9	58.9	60.9	64.5
Long-term Absence <sup>2</sup>	1.3	1.0	2.0	1.0	0.0
Chronic Truant	1.0	1.4	1.5	0.1	0.0
Suspended or Expelled	0.4	0.2	0.3	0.1	0.3
Parent Refusal	8.1	5.8	12.8	11.1	11.7
Student Refusal	10.8	14.7	6.9	14.3	12.9
In School, Did not Attend Session	8.2	10.6	9.3	9.3	5.7
In School, not Invited <sup>3</sup>	0.0	0.0	0.0	0.0	0.0
Other	4.9	6.3	8.3	3.2	4.8
Missing	0.0	0.0	0.0	0.0	0.0
Total Absentee Sample	2,598	1,112	449	460	379
Total Sample Size	13,258	4,972	2,338	3,425	2,810
Overall Absentee Rate	19.6	22.4	19.2	13.4	13.5

<sup>1</sup> Absent less than two weeks due to illness, disability, or excused absence.

<sup>2</sup> Absent more than two weeks due to illness or disability.

<sup>3</sup> In school, but not invited to assessment session due to disruptive behavior.

For each subject in grade 12, a significant component of absenteeism is not temporary or due to parental refusal. Chronic truants, those suspended, and those in school but not attending, constitute the obvious candidates for potential bias. These groups comprise 6.0 to 9.5 percent of absent students in the Advanced sessions (or 0.8% to 1.3% of each total sample). Among the non-Advanced sessions, the groups comprise 9.6 to 12.2 percent of absent students (or 1.8% to 2.7% of each total sample). Thus their potential for introducing significant bias under the current procedures is minor.

As with all NAEP assessments, data collection was conducted by trained Westat field staff. Materials collected as part of the 1996 assessment were shipped to National Computer Systems, where trained staff evaluated the responses to the constructed-response questions using scoring rubrics or guides prepared by the Educational Testing Service (ETS).

Each constructed-response question had a unique scoring rubric that defined the criteria used to evaluate students' responses. The extended constructed-response questions were evaluated with four- or five-level rubrics (e.g., no evidence of understanding, evidence of minimal understanding, evidence of partial understanding, and evidence of satisfactory or extended understanding). Short constructed-response questions first appearing in the 1996 assessment were rated according to three-level rubrics that permitted partial credit (e.g., evidence of little or no understanding, evidence of partial understanding, and evidence of full

understanding). Other short constructed-response questions that were carried over from previous assessments were scored as either “correct” or “incorrect.” For more information, see the *NAEP 1996 Technical Report*.<sup>8</sup>

Student responses for constructed responses also were scored as “off task” if the student provided a response that was deemed not related in content to the question asked. A simple example of this type of response is, “I don’t like this test.” In contrast, responses scored as “incorrect” were valid attempts to answer the question that were simply wrong.

## **Analysis Procedures**

The results from the Estimation Study were analyzed and reported in terms of the NAEP proficiency scale metric, while the results from the Theme Study and the Advanced Study were reported simply in terms of block percent correct scores. It was possible to scale the results from the Estimation Study because all of the questions were presumed to measure a relatively unidimensional mathematical trait: the ability to estimate and work with estimated data. However, no such single trait was hypothesized to underlie the questions from either the Theme Study or the Advanced Study. Rather, Theme Study questions were characterized by being presented in a single practical context, and Advanced Study questions were characterized by their more challenging content, some of which might only be covered in advanced courses.

For the Theme and Advanced Studies, another alternative might have been to scale the items onto the mathematics scales used in the main NAEP assessment. However, this would have required a very large number of linking questions because the main reporting scale in mathematics is actually a composite of five separately scaled subscales representing the various mathematics content areas. Theme Study and Advanced Study questions therefore would have had to be disaggregated onto five subscales for analysis and scaling, and stable links would have been required for each subscale.

### **Estimation Study**

The Estimation booklet at each grade was scaled with an IRT analysis, and proficiencies were calculated for students and put on the scale of the 1992 Mathematics Estimation assessment. The calculation of proficiencies for each grade involved a number of steps. First, the Estimation data were scaled, using the PARSCALE program to estimate item parameters. This scaling was entirely separate from the analysis of the main BIB data, and data from the non-estimation block, M4, were not used.

Next, conditioned proficiency scores were created. Here, the same background variables and the same resulting contrasts as in the main assessment were used. However, because the principal components of the contrasts are sample dependent, these principal components were calculated separately for the Estimation and the main data.

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<sup>5</sup> Allen, et al., (1999). op. cit.




Conditioned scores are initially in an arbitrary metric, with a mean approximately zero and a standard deviation approximately one. Therefore, the conditioned Estimation scores were linearly transformed in order to place them onto the 1992 Estimation proficiency scale. That is, each score was multiplied by one constant and then another constant was added to it, in the form:

$$P_i = A * x_i + B,$$

where  $x_i$  is the score for student  $i$  in the arbitrary metric,  $P_i$  is the score for student  $i$  in the 1992 Estimation scale metric, and  $A$  and  $B$  are appropriate constants. These constants, called transformation constants, also are sample dependent and therefore specific to the Estimation samples.

Table A.12 shows the  $A$  and  $B$  constants used to transform the initial 1996 Estimation scores from an arbitrary metric to the 1992 metric.

Table A.12		<b>Constants Used to Transform 1996 Estimation Scale Scores to the 1992 NAEP Estimation Proficiency Metric</b>		THE NATION'S REPORT CARD 
Grade		A	B	
	Grade 4	37.9729	207.2608	
	Grade 8	28.0702	269.9908	
	Grade 12	27.9881	295.8688	

For this report, the following Estimation Study performance data were presented for each grade and for selected subgroups within each grade:

- average proficiency means, with standard errors reflecting sampling and measurement error;
- average proficiency means at percentile levels — 5, 10, 25, 50, 75, 90, and 95; and
- the percentage of students at or above achievement levels of *Advanced*, *Proficient*, *Basic*, and below *Basic*.

## **Theme Study**

**Block Percent Correct Scores for Each Student.** The linking block, M4, was dropped from the analysis, and a block percent correct score for each student was calculated using the Theme block (M21 or M22). The block percent correct score was calculated as a straight percentage correct with not-reached items counted as wrong. Dichotomous items were scored 0 (wrong) and 1 (correct), and polytomous items were scored from 0 to  $m - 1$  ( $m$  being the number of categories in a given polytomous item), so 0 = 0, the first partial credit category = 1, the second = 2, ..., and so on to  $m - 1$ .

The student percent correct scores were broken down by:

- gender;
- race/ethnicity;
- frequency of writing a few sentences about how to solve a mathematics problem; and
- frequency of writing reports or doing a mathematics project.

For the “frequency of writing” variables, teacher responses were used at grades 4 and 8. Because teachers of twelfth-grade students were not surveyed, student responses were used at grade 12.

Significance tests were run for families of tests defined by the contrasts among levels of each variable. This was accomplished by using standard almanac programs.

**Item-by-Item Statistics for Questions in the Released Block.** For the individual item statistics for questions in the released Theme blocks at each grade, the following procedures were used:

- For dichotomous items, percentages responding to each alternative were calculated, but generally only the percentages for the correct response were included in the report.
- For polytomous items, percentages responding to each partial credit category were calculated and generally reported.
- The percentages responding in categories were reported for the whole group and were also broken down by categories of four variables:
  1. gender;
  2. race/ethnicity;
  3. frequency of writing a few sentences about how to solve a mathematics problem; and
  4. frequency of writing reports or doing a mathematics project.

## ***Advanced Study***

Booklet percent correct scores were calculated for each student in a manner similar to the block percents created for the Theme analysis. The difference is that the two Advanced blocks were analyzed together to create a single booklet percent correct score for each student. The linking block, M20, was deleted from the analysis.

The booklet percent correct scores were reported for the whole group and for subgroups defined by gender and race/ethnicity. In addition, for grade 12, a breakdown of the booklet percents was done by two self-report variables: (1) whether the student was currently taking mathematics and (2) whether the student was taking or had taken an AP mathematics course.

Response category frequencies and percentages were reported on various background, course-taking, and classroom practices variables. These responses also were reported for a comparison sample of students from the main assessment who were not eligible for the Advanced Study.

For each group, information on the following demographic variables was reported:

- gender;
- race/ethnicity;
- parent education;
- home resources;
- region;
- location;
- type of school;
- Title I participation; and
- eligibility for the federal Free/Reduced-Price School Lunch program.

In addition, information was reported on the following classroom practices variables:

- emphasis on mathematics topics;
- emphasis on mathematics skills;
- unrestricted use of calculators;
- permitted use of calculators on tests;
- access to calculators;
- provision of instruction in use of calculators;

- availability of graphing calculators; and
- primary use of graphing calculators.

The classroom practices variables are based on teacher responses. At grade 12, these were only available for the Advanced sample because grade 12 teachers in the main sample were not surveyed.

In addition, the report provides information on the average proficiency of the Advanced-eligible and not-Advanced-eligible students in the main sample. Because the Advanced-eligible students in the main sample were selected by exactly the same criterion as the students who actually participated in the Advanced Study, this affords a direct comparison of the mathematics achievement of the two groups.

Item-by-item statistics for items in both Advanced blocks were reported for grades 8 and 12.

- For dichotomous items this included percentages responding to each alternative, with the key (correct) alternative marked.
- For polytomous items this included percentages responding to each partial credit category.
- The percentages responding in categories were shown for the whole group and broken down by the variables:
  - gender, and
  - race/ethnicity.
- In addition for grade 12, the breakdown variables also included:
  - whether the student was currently taking mathematics; and
  - whether student had taken an AP course in mathematics.

## **NAEP Reporting Groups**

In this report, results are provided for groups of students defined by shared characteristics — gender, race/ethnicity, parental education, type of school, region of the country, participation in Title I programs, and eligibility for the federal Free/Reduced-Price School Lunch program. Results are reported for subpopulations only when sufficient numbers of students and adequate school representation are present. For public and nonpublic school students in the national assessment, the minimum requirement is at least 62 students in a particular subgroup from at least 5 primary sampling units (PSUs).<sup>5</sup> However, the data for all students, regardless of whether their subgroup was reported separately, were included in computing overall results. Definitions of the subgroups referred to in this report are presented below.

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<sup>5</sup> For the national assessment, a PSU is a selected geographic region (a county, group of counties, or metropolitan statistical area).

## **Gender**

Results are reported separately for males and females.

## **Race/ethnicity**

The race/ethnicity variable is derived from school records and two questions asked of students from the set of general student background questions.

If you are Hispanic, what is your Hispanic background?

- I am not Hispanic
- Mexican, Mexican American, or Chicano
- Puerto Rican
- Cuban
- Other Spanish or Hispanic background

Students who responded to this question by filling in the second, third, fourth, or fifth oval were considered Hispanic. For students who filled in the first oval, did not respond to the question, or provided information that was illegible or could not be classified, responses to the following question were examined to determine their race/ethnicity.

Which best describes you?

- White (not Hispanic)
- Black (not Hispanic)
- Hispanic (“Hispanic” means someone who is Mexican, Mexican American, Chicano, Puerto Rican, Cuban, or of other Spanish or Hispanic background.)
- Asian or Pacific Islander (“Asian or Pacific Islander” means someone who is from a Chinese, Japanese, Korean, Filipino, Vietnamese, or other Asian or Pacific Islander background.)
- American Indian or Alaskan Native (“American Indian or Alaskan Native” means someone who is from one of the American Indian tribes or is one of the original people of Alaska.)
- Other (specify) \_\_\_\_\_

Students’ race/ethnicity was then assigned on the basis of their responses. For students who filled in the sixth oval (“Other”), provided illegible information or information that could not be classified, or who did not respond at all, race/ethnicity was assigned as determined by school records.<sup>6</sup>

Race/ethnicity could not be determined for students who did not respond to either of the demographic questions and whose schools did not provide information about race/ethnicity.

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<sup>6</sup> The procedure for assigning race/ethnicity was modified for Hawaii. See the *NAEP 1996 Technical Report* for details.

Details of how race/ethnicity classifications were derived are presented so that readers can determine how useful the results are for their particular purposes. Also, some students indicated that they were from a Hispanic background (e.g., Puerto Rican or Cuban) and that a racial/ethnic category other than Hispanic best describes them. These students were classified as Hispanic based on the rules described above. Furthermore, information from the schools did not always correspond to how students described themselves. Therefore, the racial/ethnic results presented in this report attempt to provide a clear picture based on several sources of information.

### ***Parents' highest level of education***

The variable representing the level of parental education is derived from responses to two questions from the set of general student background questions. Students were asked to indicate the extent of their mother's education and their father's education.

How far in school did your mother go?

- She did not finish high school.
- She graduated from high school.
- She had some education after high school.
- She graduated from college.
- I don't know.

How far in school did your father go?

- He did not finish high school.
- He graduated from high school.
- He had some education after high school.
- He graduated from college.
- I don't know.


The information was combined into one parental education reporting variable determined through the following process. If a student indicated the extent of education for only one parent, that level was included in the data. If a student indicated the extent of education for both parents, the higher of the two levels was included in the data. If a student did not know the level of education for both parents or did not know the level for one parent and did not respond to the other, the parental education level was classified as "I don't know." If the student did not respond for either parent, the student was recorded as having provided no response.

### **Type of school**

Results are reported by the type of school that the student attended — public or nonpublic. Nonpublic schools include Catholic and other private schools. Although Bureau of Indian Affairs (BIA) schools and Department of Defense Domestic Dependent Elementary and Secondary Schools (DDESS) are not included in either the public or nonpublic categories, they are included in the overall national results.

### **Region**

Results are reported for four regions of the nation: Northeast, Southeast, Central, and West. Figure A.1 shows how states are subdivided into these regions.

<b>Figure A.1</b>		<b>Regions of the Country</b>		<b>THE NATION'S REPORT CARD</b> 
<b>Northeast</b>	<b>Southeast</b>	<b>Central</b>	<b>West</b>	
Connecticut	Alabama	Illinois	Alaska	
Delaware	Arkansas	Indiana	Arizona	
District of Columbia	Florida	Iowa	California	
Maine	Georgia	Kansas	Colorado	
Maryland	Kentucky	Michigan	Hawaii	
Massachusetts	Louisiana	Minnesota	Idaho	
New Hampshire	Mississippi	Missouri	Montana	
New Jersey	North Carolina	Nebraska	Nevada	
New York	South Carolina	North Dakota	New Mexico	
Pennsylvania	Tennessee	Ohio	Oklahoma	
Rhode Island	Virginia*	South Dakota	Oregon	
Vermont	West Virginia	Wisconsin	Texas	
Virginia*			Utah	
			Washington	
			Wyoming	

\* Note: The part of Virginia that is included in the Washington, DC metropolitan area is included in the Northeast region; the remainder of the state is included in the Southeast region.

### **Title I participation**

Based on available school records, students were classified either as currently participating in a Title I program or receiving Title I services, or as not receiving such services. The classification applies only to the school year when the assessment was administered (i.e., the 1995–96 school year) and was not based on participation in previous years. If the school did not offer Title I programs or services, all students in that school would be classified as not participating.

### ***Eligibility for the federal Free/Reduced-Price School Lunch program***

Based on available school records, students were classified as either currently eligible for the Free/Reduced-Price School Lunch component of the Department of Agriculture’s National School Lunch Program or not eligible. The classification applies only to the school year when the assessment was administered (i.e., the 1995–96 school year) and was not based on eligibility in previous years. If school records were not available, the student was classified as “Information not available.” If the school did not participate in the program, all students in that school were classified as “Information not available.”

### ***Cautions in Interpretation***

This report describes students’, teachers’, and principals’ responses to background questions as well as mathematics performance for fourth-, eighth-, and twelfth-grade students. The report also compares the performance results for various groups of students within these populations (e.g., those who responded to a specific background question in a particular way or by individual course-taking groups as described above). However, it does not include an analysis of the relationships among combinations of these subpopulations or background questions. In interpreting these data, it is important to understand that a relationship that exists between achievement and another variable does not reveal its underlying cause, which may be influenced by a number of other variables. Similarly, the assessments do not capture the influence of unmeasured variables. The results are most useful when they are considered in combination with other knowledge about the student population and the educational system, such as trends in instruction, changes in the school-age population, and societal demands and expectations.

### ***Estimating Variability***

Because the statistics presented in this report are estimates of group and subgroup performance based on samples of students rather than the estimates that could be calculated if every student in the nation answered every question, the degree of uncertainty associated with the estimates should be taken into account. Two components of uncertainty are accounted for in the variability statistics based on student ability: (1) the uncertainty due to sampling only a relatively small number of students and (2) the uncertainty due to sampling only a relatively small number of cognitive questions. The first component accounts for the variability associated with the estimated percentages of students who had certain background characteristics or who answered a certain cognitive question correctly.

Because NAEP uses complex sampling procedures, conventional formulas for estimating sampling variability that assume simple random sampling are inappropriate. NAEP uses a jackknife replication procedure to estimate standard error. The jackknife standard error provides a reasonable measure of uncertainty for any student information that can be observed without error. However, because each student typically responds to only a few questions within any



content strand, the scale score for any single student would be imprecise. In this case, plausible values technology can be used to describe the performance of groups or subgroups of students, but the underlying imprecision involved in this step adds another component of variability to statistics based on NAEP scale scores.<sup>7</sup>

Typically, when the standard error is based on a small number of students or when the group of students is enrolled in a small number of schools, the amount of uncertainty associated with the standard error may be quite large. Throughout this report, estimates of standard errors subject to a large degree of uncertainty are designated.

The reader is reminded that, like findings from all surveys, NAEP results are subject to other kinds of error, including the effects of imperfect adjustments for student and school nonresponse and unknowable effects associated with the particular instrumentation and data collection methods. Nonsampling errors can be attributed to a number of sources — inability to obtain complete information about all selected schools in the sample (some students or schools refused to participate, or students participated but answered only certain questions); ambiguous definitions; differences in interpreting questions; inability or unwillingness to give correct information; mistakes in recording, coding, or scoring data; and other errors in collecting, processing, sampling, and estimating missing data. The extent of nonsampling error is difficult to estimate, and, because of their nature, the impact of such errors cannot be reflected in the data-based estimates of uncertainty provided in NAEP reports.

## ***Drawing Inferences from the Results***

As noted, because the percentages of students and their average scale scores are based on samples rather than on the entire population of fourth-, eighth-, or twelfth-graders in the nation or a jurisdiction, the numbers reported are estimates. As such, they are subject to a measure of uncertainty, reflected in the standard error of the estimate. When the percentages or average scale scores of certain groups are compared, the standard error should be taken into account, and observed similarities or differences should not be relied on solely. Therefore, the comparisons discussed in this report are based on statistical tests that consider the standard errors of those statistics and the magnitude of the difference among the averages or percentages.

The results from the sample, taking into account the uncertainty associated with all samples, are used to make inferences about the population. Using confidence intervals based on the standard errors provides a way to make inferences about the population averages and percentages in a manner that reflects the uncertainty associated with the sample estimates. An estimated sample average scale score  $\pm 2$  standard errors approximates a 95 percent confidence interval for the corresponding population quantity. This statement means that one can conclude with approximately a 5 percent level of confidence that the average performance of the entire population of interest (e.g., all fourth-grade students in public schools in a jurisdiction) is within  $\pm 2$  standard errors of the sample average.

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<sup>7</sup> For more details, see Johnson, E. G. & Rust, K. F. (1992). Population inferences and variance estimation for NAEP data. *Journal of Educational Statistics*, 17(2), 175–190.

As an example, suppose that the average mathematics scale score of the students in a particular group was 256, with a standard error of 1.2. A 95 percent confidence interval for the population quantity could be described in any of the following ways:

$$\begin{aligned} &\text{Average} \pm 2 \text{ standard errors} \\ &256 \pm 2 \times 1.2 \\ &256 \pm 2.4 \\ &253.6, 258.4 \end{aligned}$$

Thus, one can conclude with a 5 percent level of confidence that the average scale score for the entire population of students in that group is between 253.6 and 258.4.

Similar confidence intervals can be constructed for percentages, if the percentages are not extremely large or extremely small. For extreme percentages, confidence intervals constructed in the above manner may not be appropriate, and accurate confidence intervals can be constructed only using procedures that are quite complicated.

Extreme percentages, defined by both the magnitude of the percentage and the size of the sample from which it was derived, should be interpreted with caution. The *NAEP 1996 Technical Report* contains a more complete discussion of extreme percentages.<sup>8</sup>

## **Analyzing Group Differences in Averages and Percentages**

The statistical tests determine whether the evidence, based on the data from the groups in the sample, is strong enough to conclude that the averages or percentages are actually different for those groups in the population. If the evidence is strong (i.e., the difference is statistically significant), the report describes the group averages or percentages as being different (e.g., one group performed higher than or lower than another group), regardless of whether the sample averages or percentages appear to be approximately the same. If the evidence is not sufficiently strong (i.e., the difference is not statistically significant), the averages or percentages are described as being not significantly different, regardless of whether the sample averages or percentages appear to be approximately the same or widely discrepant.

The reader is cautioned to rely on the results of the statistical tests rather than on the apparent magnitude of the difference between sample averages or percentages when determining whether the sample differences are likely to represent actual differences among the groups in the population.

To determine whether a real difference exists between the average scale scores (or percentages of a certain attribute) for two groups in the population, one needs to obtain an estimate of the degree of uncertainty associated with the difference between the averages (or percentages) of these groups for the sample. This estimate of the degree of uncertainty, called the standard error of the difference between the groups, is obtained by taking the square of each group's standard error, summing the squared standard errors, and taking the square root of that sum.

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<sup>8</sup> Allen, et al., (1999). op. cit.

$$\text{Standard Error of the Difference} = SE_{A-B} = \sqrt{(SE_A^2 + SE_B^2)}$$

Similar to how the standard error for an individual group average or percentage is used, the standard error of the difference can be used to help determine whether differences among groups in the population are real. The difference between the averages or percentages of the two groups  $\pm 2$  standard errors of the difference represents an approximate 95 percent confidence interval. If the resulting interval includes zero, there is insufficient evidence to claim that a real difference between the groups is statistically significant (different) at the 0.05 level. In this report, differences among groups that involve poorly defined variability estimates or extreme percentages are not discussed.

As an example, to determine whether the average mathematics scale score of Group A is higher than that of Group B, suppose that the sample estimates of the average scale score and standard errors were as follows:

<u>Group</u>	<u>Average Scale Score</u>	<u>Standard Error</u>
A	218	0.9
B	216	1.1

The difference between the estimates of the average scale scores of Groups A and B is 2 points (218–216). The standard error of this difference is:

$$\sqrt{(0.9^2 + 1.1^2)} = 1.4$$

Thus, an approximate 95 percent confidence interval for this difference is:

$$\begin{aligned} &\text{Difference} \pm 2 \text{ standard errors of the difference} \\ &2 \pm 2 \times 1.4 \\ &2 \pm 2.8 \\ &-0.8, 4.8 \end{aligned}$$

The value zero is within the confidence interval; therefore, there is insufficient evidence to claim that Group A outperformed Group B.

The procedures described in this section and the certainty ascribed to intervals (e.g., a 95 percent confidence interval) are based on statistical theory that assumes that only one confidence interval or test of statistical significance is being performed. However, in Chapters 2–4 of this report, many different groups are being compared (i.e., multiple sets of confidence intervals are being analyzed). In sets of confidence intervals, statistical theory indicates that the certainty associated with the entire set of intervals is less than that attributable to each individual comparison from the set. To hold the significance level for the set of comparisons at a particular level (e.g., 0.05), adjustments (called multiple comparison procedures) must be made to the methods described in the previous section. One such procedure, the Bonferroni method, was used in the analyses described in this report to obtain confidence intervals for the differences among groups when sets of comparisons were considered.<sup>9</sup> Thus, the confidence intervals for the sets of comparisons in the text are more conservative than those described on the previous pages.

<sup>9</sup> Miller, R. G. (1966). *Simultaneous statistical inference*. New York: Wiley.

Most of the multiple comparisons in this report pertain to relatively small sets or families of comparisons. For example, for discussions concerning comparisons of parents' level of education, six comparisons were conducted — all pairs of the four parental education levels. In these situations, Bonferroni procedures were appropriate.

## Appendix B

### *Standard Error Tables*

The comparisons presented in this report are based on statistical tests that consider the magnitude of the difference between group averages or percentages and the standard errors of those statistics. The following appendix contains the standard errors for the averages and percentages discussed in Chapters 2, 3, and 4. For ease of reference, the format and headings of each table in this appendix match the corresponding chapter table, although the numbers that appear are actually standard errors.

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Table B2.4

**Standard Errors for Average Scale Scores for  
National NAEP and Estimation Studies,  
Grades 4, 8, and 12**



	Assessment Year	Average Overall Scale Score in Mathematics NAEP	Average Estimation Scale Score
<b>Grade 4</b>	1996	0.9	2.1
	1992	0.7	1.5
	1990	0.9	1.5
<b>Grade 8</b>	1996	1.1	1.2
	1992	0.9	1.3
	1990	1.3	1.2
<b>Grade 12</b>	1996	1.0	1.2
	1992	0.9	1.2
	1990	1.1	1.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B2.5

**Standard Errors for Scale Scores in  
Estimation by Background Variables,  
Grades 4, 8, and 12, 1996**



	Average Scale Score		
	Grade 4	Grade 8	Grade 12
<b>Gender</b>			
Males	2.3	1.7	1.6
Females	2.3	1.6	1.4
<b>Students who Indicated Their Race/Ethnicity as...</b>			
White	2.5	1.6	1.4
Black	4.2	2.0	3.0
Hispanic	2.8	2.8	3.1
Asian/Pacific Islander	9.2!	3.0	6.1
American Indian	***	***	***
<b>Students who Reported Their Parents' Highest Level of Education as...</b>			
Did Not Finish High School	5.1	2.6	4.2
Graduated From High School	3.6	1.7	1.7
Some Education After High School	5.6	1.5	1.7
Graduated From College	2.9	1.9	1.6
I Don't Know	2.7	2.0	***
<b>Students who Attend...</b>			
Public Schools	2.2	1.3	1.2
Nonpublic Schools	4.9!	3.6!	4.4!
<b>Title I Participation...</b>			
Participated	3.2	3.2	4.9!
Did Not Participate	2.3	1.4	1.3
<b>Free/Reduced-Price Lunch Program Eligibility...</b>			
Not Eligible	2.1	1.9	1.4
Eligible	2.8	1.7	3.1
Information Unavailable	5.8	2.8!	2.8

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

Mathematics Assessment.

Table B2.6

**Standard Errors for Average Scale Scores in  
Estimation at Different Percentile Levels,  
Grades 4, 8, and 12**



	Assessment Year	10th Percentile	25th Percentile	50th Percentile	75th Percentile	90th Percentile
<b>Grade 4</b>	1996	5.5	2.6	1.5	3.0	3.6
	1992	1.5	2.4	1.8	1.5	3.1
	1990	2.4	2.0	2.6	1.9	2.4
<b>Grade 8</b>	1996	1.2	1.4	1.7	1.1	2.5
	1992	2.1	2.0	1.6	1.9	2.5
	1990	1.3	1.8	2.3	2.3	1.2
<b>Grade 12</b>	1996	1.2	1.9	1.6	1.2	1.7
	1992	1.4	1.9	1.5	0.9	2.1
	1990	1.5	1.6	2.2	1.3	1.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B2.7

**Standard Errors for National Percentages Attaining  
Achievement Levels in Estimation,  
Grades 4, 8, and 12**



	Percentage of Students				
	Assessment Year	Advanced	At or Above Proficient	At or Above Basic	Below Basic
<b>Grade 4</b>					
	1996	0.4	2.4	1.3	1.3
	1992	0.3	1.7	1.1	1.1
	1990	---	1.3	1.8	1.8
<b>Grade 8</b>					
	1996	0.3	1.6	1.9	1.9
	1992	0.5	1.8	2.0	2.0
	1990	0.5	1.6	2.3	2.3
<b>Grade 12</b>					
	1996	1.0	2.5	1.2	1.2
	1992	0.7	2.0	1.7	1.7
	1990	0.9	2.1	1.6	1.6

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B2.8

**Standard Errors for Percent of Students Reaching  
at Least Proficient Level in Estimation by  
Background Variables, Grades 4, 8, and 12, 1996**



	Percentage of Students Achieving Proficient or Better		
	Grade 4	Grade 8	Grade 12
<b>Gender</b>			
Males	3.1	2.0	2.7
Females	2.7	2.1	3.2
<b>Students who Indicated Their Race/Ethnicity as...</b>			
White	3.1	2.1	3.1
Black	2.6	1.1	3.9
Hispanic	2.4	2.4	3.6
Asian/Pacific Islander	9.9!	3.5	8.2
American Indian	9.9!	---	---
<b>Students who Reported Their Parents' Highest Level of Education as...</b>			
Did Not Finish High School	4.2	3.1	4.6
Graduated From High School	4.0	2.3	3.1
Some Education After High School	5.9	2.9	3.5
Graduated From College	4.0	2.7	3.0
I Don't Know	2.9	2.1	---
<b>Students who Attend...</b>			
Public Schools	2.6	1.7	2.2
Nonpublic Schools	5.8!	5.2!	7.4!
<b>Title I Participation...</b>			
Participated	2.3	---	4.2!
Did Not Participate	2.6	1.7	2.6
<b>Free/Reduced-Price Lunch Program Eligibility...</b>			
Not Eligible	3.1	2.8	2.7
Eligible	2.4	1.9	3.4
Information Unavailable	6.1	2.7!	4.4

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

Mathematics Assessment.

Table B3.1

**Standard Errors for Student Demographic Distributions by Assessment, Grade 4, 1996**



Grade 4	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
Gender			
Males	0.7	1.4	1.4
Females	0.7	1.4	1.4
Students who Indicated Their Race/Ethnicity as...			
White	0.4	0.9	0.9
Black	0.2	0.6	0.6
Hispanic	0.4	0.6	0.7
Asian/Pacific Islander	0.2	0.3	0.4
American Indian	0.2	0.3	0.3
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	0.3	0.5	0.8
Graduated From High School	0.6	1.0	1.0
Some Education After High School	0.4	0.7	0.7
Graduated From College	1.2	1.3	1.4
I Don't Know	0.9	1.1	1.2
Students who Attend...			
Public Schools	1.6	1.8	1.6
Nonpublic Schools	1.6	1.8	1.6
Title I Participation...			
Participated	1.4	1.9	1.9
Did Not Participate	1.4	1.9	1.9
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	1.4	1.6	1.3
Not Eligible	2.5	2.7	2.6
Information Not Available	3.0	2.4	2.3

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.2

**Standard Errors for Percentage of Students by  
Teachers' Reports on Classroom Practices,  
Grade 4, 1996**



	Percentage of Students		
	Main Assessment	Theme Block 1 The Butterfly Booth	Theme Block 2 Recycling
<b>Grade 4</b>			
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	1.4	1.2	0.9
Once or Twice a Week	2.1	2.1	2.4
Once or Twice a Month	2.6	2.3	2.7
Never or Hardly Ever	2.4	2.8	3.0
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...			
Nearly Every Day	0.5	0.1	0.1
Once or Twice a Week	0.8	0.6	0.5
Once or Twice a Month	2.3	2.7	2.6
Never or Hardly Ever	2.4	2.7	2.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.3

**Standard Errors for Average Percentage Correct Scores by Theme Block, Grade 4, 1996**



	The Butterfly Booth	Recycling
<b>Grade 4</b>		
All Students	0.7	0.6
Gender		
Males	0.9	0.8
Females	0.8	0.6
Race/Ethnicity		
White	0.8	0.7
Black	0.7	1.1
Hispanic	0.9	1.1
Asian/Pacific Islander	2.4	2.1
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	3.3	1.9
Once or Twice a Week	1.2	1.3
Once or Twice a Month	1.2	1.4
Never or Hardly Ever	1.2	1.3
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	***	***
Once or Twice a Month	1.4	1.6
Never or Hardly Ever	0.8	0.7

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.



Table B3.4

**Standard Errors for Score Percentages for  
"Draw Symmetrical Figure," Grade 4**



	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 4</b>					
All Students	1.2	0.8	0.9	0.7	1.4
Males	1.6	1.1	1.3	1.1	1.8
Females	1.8	1.3	1.2	0.9	1.7
White	1.4	0.9	1.1	0.9	1.5
Black	1.7	1.8	2.1	1.9	2.6
Hispanic	1.7	2.0	1.5	1.5	2.9
Asian/Pacific Islander	7.6	4.1	3.1	2.8	5.5
American Indian	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.5

**Standard Errors for Score Percentages for  
"Measure Length Using Ruler," Grade 4**



	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 4</b>						
All Students	1.4	0.8	1.0	0.5	1.3	0.5
Males	2.2	1.2	1.4	0.5	2.0	0.8
Females	1.7	1.4	1.1	0.9	1.4	0.6
White	1.7	1.0	1.2	0.7	1.6	0.4
Black	2.0	2.3	2.0	1.3	2.7	1.5
Hispanic	2.7	1.5	2.2	0.9	3.2	2.3
Asian/Pacific Islander	6.0	2.8	2.5	1.3	5.3	1.6
American Indian	***	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.6

**Standard Errors for Score Percentages for  
"Solve Packing Problems," Grade 4**



Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0.7	1.0	1.1	1.4	1.3	0.2
Males	1.2	1.3	1.2	1.8	1.7	0.4
Females	0.9	1.3	1.6	1.7	1.8	0.2
White	0.9	1.3	1.5	1.9	1.8	0.3
Black	1.0	0.9	1.4	2.6	2.4	0.6
Hispanic	0.4	1.6	1.8	2.9	3.2	---
Asian/Pacific Islander	1.4	6.4	4.3	5.8	4.4	---
American Indian	***	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.7

**Standard Errors for Score Percentages for  
"Determine Number of Models," Grade 4**



Grade 4	Satisfactory	Partial	Minimal	Incorrect	Omit
All Students	0.4	1.0	1.0	1.7	0.4
Males	0.5	1.3	1.3	2.0	0.7
Females	0.5	1.5	1.4	2.2	0.4
White	0.6	1.3	1.4	2.2	0.5
Black	0.6	1.2	1.7	2.5	1.0
Hispanic	0.4	1.5	2.0	2.3	1.1
Asian/Pacific Islander	2.6	4.4	5.0	6.0	1.9
American Indian	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B3.8**

**Standard Errors for Score Percentages for  
"Determine Number of Leaves," Grade 4**



	Complete	Partial	Incorrect	Omit
<b>Grade 4</b>				
All Students	0.6	0.8	0.9	0.3
Males	0.9	0.9	1.1	0.4
Females	0.7	1.1	1.4	0.4
White	0.7	1.1	1.3	0.3
Black	1.0	0.9	1.8	1.1
Hispanic	0.8	0.8	1.4	0.8
Asian/Pacific Islander	3.2	3.3	4.9	3.0
American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B3.9**

**Standard Errors for Score Percentages for  
"Interpret Pattern of Figures," Grade 4**



	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 4</b>					
All Students	0.3	0.4	0.5	0.8	0.2
Males	0.4	0.6	0.7	1.2	0.3
Females	0.3	0.6	0.6	1.0	0.4
White	0.4	0.6	0.7	1.1	0.3
Black	---	0.7	1.2	1.6	0.6
Hispanic	---	0.5	1.1	2.1	0.6
Asian/Pacific Islander	---	---	3.2	3.7	2.2
American Indian	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.10

**Standard Errors for Student Demographic  
Distributions by Assessment, Grade 8, 1996**



Grade 8	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
<b>Gender</b>			
Males	0.8	1.0	1.2
Females	0.8	1.0	1.2
<b>Race/Ethnicity</b>			
White	0.2	0.7	0.6
Black	0.2	0.6	0.5
Hispanic	0.1	0.5	0.5
Asian/Pacific Islander	0.2	0.3	0.3
American Indian	0.2	0.3	0.3
<b>Students who Reported Their Parents' Highest Level of Education as...</b>			
Did Not Finish High School	0.4	0.7	1.1
Graduated From High School	0.8	1.2	1.1
Some Education After High School	0.7	1.0	1.0
Graduated From College	1.3	1.6	1.7
I Don't Know	0.6	0.7	0.7
<b>Students who Attend...</b>			
Public Schools	1.1	1.9	1.9
Nonpublic Schools	1.1	1.9	1.9
<b>Title I Participation...</b>			
Participated	1.6	1.7	1.7
Did Not Participate	1.6	1.7	1.7
<b>Free/Reduced-Price Lunch Program Eligibility...</b>			
Eligible	1.4	1.5	2.0
Not Eligible	2.4	2.5	2.8
Information Not Available	2.9	2.9	3.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B3.11**

**Standard Errors for Percentage of Students by Teachers' Reports on Classroom Practices, Grade 8, 1996**



	Percentage of Students		
	Main Assessment	Theme Block 1 Building a Doghouse	Theme Block 2 Flooding
<b>Grade 8</b>			
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	1.1	1.1	1.2
Once or Twice a Week	2.8	3.2	3.7
Once or Twice a Month	2.8	3.2	3.6
Never or Hardly Ever	3.3	3.4	3.6
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...			
Nearly Every Day	---	---	---
Once or Twice a Week	1.1	1.3	1.4
Once or Twice a Month	3.2	2.7	2.8
Never or Hardly Ever	3.3	2.5	2.8

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.12

**Standard Errors for Average Percentage Correct Scores by Theme Block, Grade 8, 1996**



	Build a Doghouse	Flooding
<b>Grade 8</b>		
All Students	0.5	0.7
Gender		
Males	0.6	0.8
Females	0.7	0.9
Race/Ethnicity		
White	0.7	1.0
Black	0.8	0.5
Hispanic	0.8	0.8
Asian/Pacific Islander	2.5	2.8
American Indian	***	***
Students Whose Teachers Report Asking Students to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	3.1	3.0
Once or Twice a Week	1.6	2.4
Once or Twice a Month	1.1	1.3
Never or Hardly Ever	1.0	1.4
Students Whose Teachers Report Asking Students to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	2.6	2.1
Once or Twice a Month	1.1	1.4
Never or Hardly Ever	0.9	1.0

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.13

**Standard Errors for Score Percentages for  
"Identifying Needed Information," Grade 8**


	5 Correct	4 Correct	3 Correct	2 Correct	1 Correct	0 Correct	Omit
<b>Grade 8</b>							
All Students	1.5	1.0	0.7	0.7	0.7	0.3	0.1
Males	1.8	1.3	1.2	0.9	1.2	0.5	0.1
Females	2.1	1.5	0.9	0.9	0.8	0.2	0.1
White	1.9	1.1	0.8	0.7	0.7	0.4	---
Black	2.5	2.0	2.4	1.4	1.2	0.5	0.5
Hispanic	2.5	2.8	2.0	1.7	1.5	---	0.3
Asian/Pacific Islander	5.2	4.4	---	2.5	---	---	---
American Indian	***	***	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.14

**Standard Errors for Percentages Correct for  
"Identifying Needed Information," Grade 8**


	1a — Yes	1b — Yes	1c — No	1d — Yes	1e — Yes
<b>Grade 8</b>					
All Students	0.9	0.9	1.2	1.0	1.2
Males	1.6	1.3	1.5	1.6	1.6
Females	0.9	1.1	1.6	0.9	1.3
White	1.1	1.1	1.7	1.2	1.5
Black	2.2	1.9	1.9	2.4	2.5
Hispanic	1.8	1.6	3.0	2.6	2.7
Asian/Pacific Islander	---	3.1	4.1	4.5	4.1
American Indian	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B3.15**

**Standard Errors for Percentage Correct for  
"Determine Minimum Measuring Needed," Grade 8**



Grade 8		Percentage Correct
All Students		1.3
Males		1.3
Females		2.2
White		1.6
Black		2.6
Hispanic		2.4
Asian/Pacific Islander		7.4
American Indian		***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B3.16**

**Standard Errors for Score Percentages for  
"Measure Lengths Using Ruler," Grade 8**



Grade 8		Complete	Partial	Incorrect	Omit
All Students		1.5	1.2	1.2	1.3
Males		1.8	1.4	1.5	1.2
Females		2.0	1.7	1.6	1.6
White		1.8	1.5	1.4	1.4
Black		2.4	3.1	4.1	2.7
Hispanic		3.0	3.0	2.9	1.5
Asian/Pacific Islander		7.3	5.8	8.5	---
American Indian		***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.



Table B3.17

**Standard Errors for Score Percentages for  
"Apply Concept of Ratio," Grade 8**



	Complete	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	1.1	1.2	1.5	1.1
Males	1.4	1.5	1.7	1.1
Females	1.2	1.5	2.0	1.7
White	1.4	1.5	1.8	1.4
Black	0.9	1.1	2.8	3.3
Hispanic	1.9	1.7	3.1	2.5
Asian/Pacific Islander	4.8	6.6	7.2	4.5
American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.18

**Standard Errors for Percentage Correct for  
"Understand Concept of Ratio (I)," Grade 8**



	Percentage Correct
<b>Grade 8</b>	
All Students	1.3
Males	1.9
Females	1.3
White	1.6
Black	2.3
Hispanic	3.1
Asian/Pacific Islander	9.0
American Indian	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.19

**Standard Errors for Percentage Correct for  
"Understand Concept of Ratio (II)," Grade 8**



Grade 8		Percentage Correct
	All Students	1.2
	Males	1.7
	Females	1.7
	White	1.4
	Black	2.4
	Hispanic	2.8
	Asian/Pacific Islander	8.2
	American Indian	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.20

**Standard Errors for Score Percentages for  
"Correctly Position Door," Grade 8**



Grade 8		Complete	Partial	Incorrect	Omit
	All Students	1.3	1.2	1.3	0.8
	Males	2.0	1.8	2.0	1.3
	Females	1.7	1.4	1.6	0.9
	White	1.8	1.4	1.7	0.9
	Black	1.1	2.9	2.7	2.7
	Hispanic	1.9	3.3	3.2	2.4
	Asian/Pacific Islander	4.5	8.3	6.6	6.8
	American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.21

**Standard Errors for Score Percentages for  
"Visualize Cut-Outs on Grid," Grade 8**


	Complete	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	1.5	0.6	1.4	0.8
Males	1.7	0.9	1.5	1.1
Females	2.1	1.2	2.1	1.1
White	1.9	0.8	1.7	0.8
Black	2.2	0.8	2.3	2.6
Hispanic	2.6	1.3	2.7	3.0
Asian/Pacific Islander	8.6	5.0	6.4	4.8
American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.22

**Standard Errors for Score Percentages for  
"Apply Geometry in Model," Grade 8**


	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 8</b>					
All Students	0.5	0.7	0.8	1.1	1.6
Males	0.4	1.0	1.3	2.1	2.3
Females	0.8	0.9	1.2	1.6	1.5
White	0.7	1.0	1.1	1.2	2.0
Black	---	---	1.4	3.4	3.1
Hispanic	---	1.1	2.0	3.8	3.5
Asian/Pacific Islander	***	***	***	***	***
American Indian	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.23

**Standard Errors for Score Percentages for  
"Find Maximum Area When Perimeter is Fixed,"  
Grade 8**



	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 8</b>						
All Students	0.1	0.0	1.2	0.6	1.7	1.3
Males	0.1	---	1.8	1.0	1.7	1.7
Females	0.2	0.1	1.5	0.4	2.2	2.0
White	0.1	---	1.5	0.8	1.9	1.4
Black	---	---	1.7	0.9	3.3	3.1
Hispanic	---	---	2.6	1.0	3.2	4.0
Asian/Pacific Islander	***	***	***	***	***	***
American Indian	***	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.24

**Standard Errors for Student Demographic Distributions by Assessment, Grade 12, 1996**



	Percentage of Students		
	Main Assessment	Theme Block 1 Buying a Car	Theme Block 2 Flooding
<b>Grade 12</b>			
Gender			
Males	0.9	1.4	1.3
Females	0.9	1.4	1.3
Race/Ethnicity			
White	0.5	0.8	0.8
Black	0.4	0.6	0.6
Hispanic	0.4	0.6	0.6
Asian/Pacific Islander	0.4	0.3	0.4
American Indian	0.6	0.2	0.2
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	0.5	0.8	0.6
Graduated From High School	0.8	1.6	1.3
Some Education After High School	0.8	1.1	1.3
Graduated From College	1.5	1.8	1.8
I Don't Know	0.2	0.5	0.5
Students who Attend...			
Public Schools	1.6	1.9	2.0
Nonpublic Schools	1.5	1.9	2.0
Title I Participation...			
Participated	0.6	0.8	0.8
Did Not Participate	0.6	0.8	0.8
Free/Reduced-Price Lunch Program Eligibility...			
Eligible	1.3	1.2	1.1
Not Eligible	3.7	3.5	3.7
Information Not Available	3.8	3.5	3.6

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.25

**Standard Errors for Percentage of Students by Reports on Classroom Practices, Grade 12, 1996\***



	Percentage of Students		
	Main Assessment	Buying a Car	Flooding
<b>Grade 12</b>			
Students who Report Writing a Few Sentences About How to Solve a Mathematics Problem...			
Nearly Every Day	0.4	0.7	0.7
Once or Twice a Week	0.5	0.8	0.9
Once or Twice a Month	0.6	1.4	0.9
Never or Hardly Ever	0.8	1.6	1.6
Students who Report Writing Reports or Doing a Mathematics Project...			
Nearly Every Day	0.2	---	---
Once or Twice a Week	0.3	0.5	0.5
Once or Twice a Month	1.0	1.6	1.1
Never or Hardly Ever	1.0	1.8	1.3

\* Teachers of twelfth-grade students were not surveyed in the Main Assessment.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.26

**Standard Errors for Average Percentage Correct Scores by Theme Block, Grade 12, 1996**



	Buying a Car	Flooding
<b>Grade 12</b>		
All Students	1.1	0.8
Gender		
Males	1.4	1.1
Females	1.2	1.1
Race/Ethnicity		
White	1.3	1.0
Black	1.3	1.1
Hispanic	2.1	1.9
Asian/Pacific Islander	3.8	3.3
American Indian	***	***
Students Whose Teachers Ask Them to Write a Few Sentences About How to Solve a Mathematics Problem...		
Nearly Every Day	4.4	1.9
Once or Twice a Week	2.3	1.8
Once or Twice a Month	2.1	1.5
Never or Hardly Ever	1.1	1.1
Students Whose Teachers Ask Them to Write Reports or Do a Mathematics Project...		
Nearly Every Day	***	***
Once or Twice a Week	2.9	2.7
Once or Twice a Month	2.0	1.4
Never or Hardly Ever	1.4	1.0

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.27

**Standard Errors for Percentage Correct for  
"Find Amount of Down Payment," Grade 12**



Grade 12		Percentage Correct
	All Students	1.1
	Males	1.6
	Females	1.6
	White	1.0
	Black	3.1
	Hispanic	3.3
	Asian/Pacific Islander	4.1
	American Indian	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.28

**Standard Errors for Percentage Correct for  
"Find Total Amount Paid for Car," Grade 12**



Grade 12		Percentage Correct
	All Students	1.1
	Males	1.5
	Females	1.2
	White	1.2
	Black	3.4
	Hispanic	3.8
	Asian/Pacific Islander	4.5
	American Indian	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.



Table B3.29

**Standard Errors for Percentage Correct  
for "Find Difference Between Total  
Amount Paid and Price," Grade 12**



Grade 12		Percentage Correct
	All Students	1.2
	Males	1.9
	Females	1.4
	White	1.3
	Black	3.6
	Hispanic	3.1
	Asian/Pacific Islander	4.1
	American Indian	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.30

**Standard Errors for Score Percentages for  
"Find Amount To Be Financed," Grade 12**



Grade 12		Complete	Partial	Incorrect	Omit
	All Students	1.4	1.5	1.4	0.6
	Males	2.0	1.8	1.7	1.2
	Females	1.6	1.8	1.6	0.6
	White	1.7	1.6	1.8	0.6
	Black	2.5	3.2	2.9	2.2
	Hispanic	3.8	3.5	3.3	2.0
	Asian/Pacific Islander	5.9	5.1	3.5	3.2
	American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.31

**Standard Errors for Score Percentages for  
"Use Formula to Find Total Cost," Grade 12**



	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 12</b>						
All Students	0.9	1.1	0.9	0.8	1.5	0.8
Males	1.0	1.1	1.0	1.2	1.9	1.2
Females	1.2	1.6	1.2	1.1	2.0	1.1
White	1.2	1.4	1.2	1.0	1.9	1.0
Black	0.8	0.9	1.3	1.9	2.9	2.0
Hispanic	1.6	2.7	1.2	2.8	4.0	1.9
Asian/Pacific Islander	3.9	4.1	2.5	2.3	5.8	2.9
American Indian	***	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.32

**Standard Errors for Score Percentages for  
"Find Amount Saved if Leased," Grade 12**



	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 12</b>					
All Students	1.6	1.0	1.3	1.3	0.5
Males	2.0	1.6	1.5	1.9	0.9
Females	1.9	1.6	1.9	1.7	0.7
White	1.8	1.2	1.5	1.3	0.6
Black	1.7	2.6	4.2	3.3	2.6
Hispanic	3.7	3.2	3.6	3.9	2.2
Asian/Pacific Islander	4.3	3.9	4.9	7.1	0.8
American Indian	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.33

**Standard Errors for Score Percentages for  
"Price Lease vs. Buy," Grade 12**



	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 12</b>					
All Students	1.5	0.4	0.9	1.7	0.9
Males	1.9	0.7	1.4	2.7	1.1
Females	1.7	0.4	1.4	1.9	1.2
White	1.8	0.6	1.1	1.9	0.9
Black	1.4	---	1.8	4.1	4.0
Hispanic	2.8	---	1.3	3.9	2.7
Asian/Pacific Islander	4.2	1.5	4.8	7.0	1.1
American Indian	***	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.1

**Standard Errors for Student Demographic  
Distributions, Grade 8, 1996**



	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
<b>Grade 8</b>		
Gender		
Males	1.2	0.9
Females	1.2	0.9
Students who Indicated Their Race/Ethnicity as...		
White	2.6	0.7
Black	2.7	0.5
Hispanic	0.9	0.4
Asian/Pacific Islander	0.8	0.5
American Indian	0.8	0.5
Students who Reported Their Parents' Highest Level of Education as...		
Did Not Finish High School	0.4	0.5
Graduated From High School	1.2	0.8
Some Education After High School	1.0	0.7
Graduated From College	1.6	1.2
I Don't Know	0.5	0.8
Home Environment Contains...		
0–2 types of educational materials	1.1	0.7
3 types of educational materials	1.2	1.0
4 types of educational materials	1.4	1.0
Students From...		
Northeast	3.8	1.4
Southeast	2.2	1.8
Central	4.0	1.7
West	2.7	1.7
Students Live in...		
Center City	4.2	2.8
Urban Fringe/Small City	4.6	3.1
Rural	3.8	3.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.2

**Standard Errors for Student/School  
Demographic Distributions, Grade 8, 1996**



	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
<b>Grade 8</b>		
Students who Attend...		
Public Schools	2.3	1.0
Nonpublic Schools	2.3	1.0
Title I Participation...		
Participated	0.8	1.9
Did Not Participate	0.8	1.9
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	2.0	1.6
Not Eligible	3.3	2.3
Information Not Available	3.4	2.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.3

**Standard Errors for Content Emphases in  
Mathematics, Grade 8, 1996**



	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
<b>Grade 8</b>				
In This Mathematics Class How Often Do You Address...				
Numbers and Operations				
Advanced Study	---	1.8	3.7	3.7
Non-Eligibles	---	0.5	1.8	1.9
Measurement				
Advanced Study	1.6	4.0	4.5	2.5
Non-Eligibles	0.3	2.7	3.7	3.1
Geometry				
Advanced Study	3.1	3.3	3.6	2.8
Non-Eligibles	0.7	3.2	3.4	3.0
Data Analysis, Statistics, and Probability				
Advanced Study	3.1	4.9	5.0	2.7
Non-Eligibles	1.4	3.4	3.5	2.5
Algebra and Functions				
Advanced Study	---	0.5	2.2	2.2
Non-Eligibles	0.4	1.4	3.5	3.8

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.4

**Standard Errors for Process Emphases  
in Mathematics, Grade 8, 1996**



	Percentage of Students Whose Teachers Responded			
	None	A Little	Some	A Lot
<b>Grade 8</b>				
In This Mathematics Class How Often Do You Address...				
Learning Mathematical Facts and Concepts				
Advanced Study	1.4	2.5	2.3	3.8
Non-Eligibles	0.0	1.9	2.6	3.0
Learning Skills and Procedures Needed to Solve Routine Problems				
Advanced Study	---	1.6	2.2	2.7
Non-Eligibles	---	0.3	2.5	2.5
Developing Reasoning and Analytical Ability to Solve Unique Problems				
Advanced Study	---	1.7	3.8	4.4
Non-Eligibles	0.3	2.0	3.5	3.4
Learning How to Communicate Ideas in Mathematics Effectively				
Advanced Study	---	2.9	3.1	4.6
Non-Eligibles	0.2	2.3	3.3	3.2

--- Standard errors could not be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.5**

**Standard Errors for Calculator Emphases  
in Mathematics, Grade 8, 1996**



	Percentage of Students Whose Teachers Responded	
	Yes	No
<b>Grade 8</b>		
Do You Permit Students in This Class Unrestricted Use of Calculators...		
Advanced Study	5.2	5.2
Non-Eligibles	3.2	3.2
Do You Permit Students in This Class to Use Calculators for Tests...		
Advanced Study	3.3	3.3
Non-Eligibles	2.8	2.8
Do Students in This Class Have Access to Calculators Owned by the School...		
Advanced Study	3.6	3.6
Non-Eligibles	3.2	3.2
Do You Provide Instruction to Students in This Class in the Use of Calculators...		
Advanced Study	3.8	3.8
Non-Eligibles	2.7	2.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.6**

**Standard Errors for Average Mathematics Scale  
Scores by Eligibility for Advanced Study, Grade 8**



	Average Scale Score
<b>Grade 8</b>	
All Students	1.1
Eligible	2.0
Not Eligible	1.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.



**Table B4.8****Standard Errors for Average Percentage Correct Scores, Advanced Study, Grade 8, 1996**

		Percentage Correct
Grade 8		
	All Students	1.1
	Males	1.3
	Females	1.1
	White	0.9
	Black	2.3
	Hispanic	2.1
	Asian/Pacific Islander	1.7
	American Indian	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.9

**Standard Errors for Score Percentages for  
"Car Wash Supplies," Grade 8**



	Correct	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	1.2	1.8	1.7	1.3
Males	1.6	2.1	1.9	1.4
Females	1.2	2.0	2.1	1.6
White	1.4	1.9	2.0	1.7
Black	1.2!	3.3!	3.9!	1.9!
Hispanic	2.7	3.6	4.0	3.4
Asian/Pacific Islander	3.2	4.4	3.9	3.7
American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.10

**Standard Errors for Score Percentages for  
"Begin to Earn Profit," Grade 8**



	Correct	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	1.6	1.3	1.8	1.5
Males	2.0	1.8	2.2	1.5
Females	1.6	1.2	2.1	1.9
White	1.8	1.5	2.1	1.8
Black	2.3!	0.9!	1.8!	2.3!
Hispanic	3.5	2.6	4.8	3.2
Asian/Pacific Islander	3.9	2.8	4.3	3.4
American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

Mathematics Assessment.

**Table B4.11**

**Standard Errors for Score Percentages for  
"Greatest Profit Expected," Grade 8**



	Correct	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	1.9	0.7	1.8	2.1
Males	2.2	0.9	2.4	2.4
Females	2.2	0.8	1.9	2.2
White	2.0	0.8	2.4	2.5
Black	3.1!	0.7!	2.1!	3.6!
Hispanic	3.2	1.3	3.4	4.0
Asian/Pacific Islander	5.9	1.4	6.1	4.2
American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.12

**Standard Errors for Score Percentages for  
"Hot Air Balloon," Grade 8**



	Correct	Partial	Incorrect	Omit
<b>Grade 8</b>				
All Students	1.3	1.2	1.6	1.3
Males	1.7	1.1	2.0	1.6
Females	1.3	1.6	2.0	1.3
White	1.6	1.2	1.9	1.3
Black	0.9!	1.7!	2.8!	3.5!
Hispanic	3.2	1.4	2.9	3.2
Asian/Pacific Islander	5.0	4.2	5.6	3.6
American Indian	***	***	***	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.13

**Standard Errors for Student  
Demographic Distributions, Grade 12, 1996**



	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
<b>Grade 12</b>		
Gender		
Males	1.8	0.9
Females	1.8	0.9
Students who Indicated Their Race/Ethnicity as...		
White	2.2	0.9
Black	1.0	0.6
Hispanic	1.5	0.5
Asian/Pacific Islander	1.6	0.6
American Indian	0.1	1.0
Students who Reported Their Parents' Highest Level of Education as...		
Did Not Finish High School	0.7	0.6
Graduated From High School	1.0	0.9
Some Education After High School	1.1	0.9
Graduated From College	1.7	1.6
I Don't Know	0.3	0.3
Home Environment Contains...		
0–2 types of educational materials	1.0	0.9
3 types of educational materials	1.0	0.8
4 types of educational materials	1.3	1.2
Students From...		
Northeast	3.8	1.4
Southeast	3.4	2.3
Central	2.9	1.7
West	3.0	2.6
Students Live in...		
Center City	4.3	3.9
Urban Fringe/Small City	4.9	3.9
Rural	4.1	3.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.14

**Standard Errors for Student/School  
Demographic Distributions, Grade 12, 1996**



	Percentage of Students	
	Advanced Study	Not Eligible for Advanced Study
<b>Grade 12</b>		
Students who Attend...		
Public Schools	3.6	1.2
Nonpublic Schools	3.6	1.2
Title I Participation...		
Participated	1.0	0.7
Did Not Participate	1.0	0.7
Free/Reduced-Price Lunch Program Eligibility...		
Eligible	1.4	1.5
Not Eligible	4.5	3.6
Information Not Available	4.7	3.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.15**

**Standard Errors for Content Emphases in Classes  
Taken by Advanced Study Students,  
Grade 12, 1996**



		Percentage of Students Receiving		
		Little Emphasis	Moderate Emphasis	Heavy Emphasis
<b>Grade 12</b>				
How Much Emphasis on...				
	Algebra	1.3	3.6	3.3
	Geometry	2.1	4.1	3.6
	Trigonometry	1.4	2.8	2.7
	Functions	0.8	2.1	2.1
	Statistics	2.9	2.7	1.9
	Probability	2.4	2.6	2.1
	Discrete Mathematics	3.2	3.1	1.4

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.



Table B4.16

**Standard Errors for Process Emphases in Classes  
Taken by Advanced Study Students,  
Grade 12, 1996**



	Percentage of Students Receiving		
	Little Emphasis	Moderate Emphasis	Heavy Emphasis
<b>Grade 12</b>			
How Much Emphasis on...			
Learning Mathematical Facts and Concepts	0.9	3.3	3.3
Learning Skills and Procedures Needed to Solve Routine Problems	0.4	3.1	3.2
Developing Reasoning and Analytical Ability to Solve Unique Problems	1.0	2.9	3.1
Learning How to Communicate Ideas in Mathematics Effectively	1.9	3.7	3.5

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.17

**Standard Errors for Calculator Access in Classes  
Taken by Advanced Study Students,  
Grade 12, 1996**



Grade 12	Percentage of Students
Do You Permit Students in This Class Unrestricted Use of Calculators...	
Yes	2.4
No	2.4
Do Students in This Class Have Access to Scientific Calculators in Class...	
Yes	2.8
No	2.8
Do Students in This Class Have Access to Scientific Calculators Out of Class...	
Yes	2.6
No	2.6
Do Students in This Class Have Access to Graphing Calculators in Class...	
Yes	3.0
No	3.0
Do Students in This Class Have Access to Graphing Calculators Out of Class...	
Yes	3.4
No	3.4
Do You Permit Students in This Class the Use of Calculators on Tests...	
Yes All	3.3
Yes Some	3.3
No	0.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.18

**Standard Errors for Calculator Usage and Instruction  
Classes Taken by Advanced Study Students,  
Grade 12, 1996**



Grade 12		Percentage of Students
In This Class Students are Provided Instruction in the Use of Scientific Calculators...		
Yes		3.0
No		3.0
In This Class Students are Provided Instruction in the Use of Graphing Calculators...		
Yes		2.6
No		2.6
Which of the Following Best Describes the Availability of Graphing Calculators in This Class...		
One		0.8
Less than Six		1.0
Complete Set		4.0
Some Students Have One		1.9
Most Students Have One		2.3
All Students Have One		3.5
No Student Has One		1.7
If Graphing Calculators are Used in This Class, What is Their Primary Usage...		
Calculating		1.8
Graphing		3.0
Tables		0.2
Statistics		0.5
Symbolic Manipulation		1.4
Not Used		2.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.19

### Standard Errors for Average Mathematics Scale Scores by Eligibility for Advanced Study, Grade 12



Grade 12		Average Scale Score
	All Students	1.0
	Eligible	1.6
	Not Eligible	0.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.21

### Standard Errors for Average Percentage Correct Scores, Advanced Study, Grade 12, 1996



Grade 12		Percentage Correct
	All Students	0.4
	Males	1.0
	Females	0.9
	White	0.8
	Black	1.6
	Hispanic	1.4
	Asian/Pacific Islander	2.2
	American Indian	***
	Are Students Presently Enrolled in Mathematics...	
	Yes	0.8
	No	1.4
	Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...	
	Yes	1.1
	No	0.9

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.22

**Standard Errors for Score Percentages for "Use Linear Function," Grade 12**



	Correct	Partial	Incorrect	Omit
<b>Grade 12</b>				
All Students	1.1	0.6	1.2	1.3
Males	1.3	0.9	1.5	1.6
Females	1.3	0.9	1.7	1.4
White	1.2	0.7	1.4	1.3
Black	1.2	1.7	3.3	3.4
Hispanic	2.4!	1.7!	3.5!	3.2!
Asian/Pacific Islander	4.2	1.7	4.0	2.6
American Indian	***	***	***	***
Are Students Presently Enrolled in Mathematics...				
Yes	1.1	0.7	1.2	1.3
No	3.2!	---	5.6!	5.2!
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...				
Yes	1.9	0.9	1.6	1.6
No	1.4	0.9	1.7	1.7

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.23

**Standard Errors for Score Percentages for  
"Compare Volumes of Pyramids," Grade 12**



	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 12</b>						
All Students	0.5	0.5	0.7	0.9	1.2	1.2
Males	0.7	0.8	0.8	1.1	1.5	1.7
Females	0.6	0.7	0.7	1.1	1.8	1.3
White	0.5	0.6	0.8	1.2	1.3	1.2
Black	---	---	0.6	1.5	5.4	5.2
Hispanic	1.2!	0.7!	0.5!	1.5!	5.1!	4.5!
Asian/Pacific Islander	1.9	1.7	2.0	1.6	2.5	2.4
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	0.5	0.5	0.7	0.9	1.3	1.2
No	1.1!	---	0.4!	4.4!	5.9!	5.1!
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	1.1	0.9	0.9	1.3	1.7	1.3
No	0.5	0.6	0.9	1.2	1.6	1.7

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

Mathematics Assessment.

Table B4.24

**Percentage Correct for "Find Resultant Vector,"  
Grade 12**



Grade 12		Percentage of Students
All Students		1.0
Males		1.5
Females		1.3
White		1.1
Black		4.1
Hispanic		3.3!
Asian/Pacific Islander		3.1
American Indian		***
Are Students Presently Enrolled in Mathematics...		
Yes		1.0
No		4.2
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...		
Yes		1.7
No		1.2

\*\*\* Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

Mathematics Assessment.

Table B4.25

**Standard Errors for Score Percentages for  
"Ferris Wheel," Grade 12**



	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
<b>Grade 12</b>						
All Students	1.5	0.1	0.9	0.8	1.2	0.9
Males	1.9	0.3	1.1	1.1	1.3	1.1
Females	1.7	---	1.3	1.1	1.6	1.0
White	1.6	0.2	1.1	0.9	1.4	0.8
Black	2.9	---	2.8	2.3	2.8	4.3
Hispanic	4.1!	---	1.7!	2.0!	2.7!	3.1!
Asian/Pacific Islander	3.2	---	2.4	2.2	2.7	2.3
American Indian	***	***	***	***	***	***
Are Students Presently Enrolled in Mathematics...						
Yes	1.5	0.1	0.9	0.8	1.2	1.0
No	4.2!	---	3.7!	4.3!	4.0!	3.3!
Are Students Presently Enrolled in or Have They Previously Taken an Advanced Placement (AP) Mathematics Course...						
Yes	1.7	---	1.4	1.0	1.1	1.1
No	2.1	0.2	1.1	1.2	1.6	1.1

\*\*\* Sample size is insufficient to permit a reliable estimate.

--- Standard errors could not be accurately determined.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.



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