NATIONAL CENTER FOR EDUCATION STATISTICS

Estimation Skills, Mathematics-in-Context, and Advanced Skills in Mathematics

Results from Three Studies of the National Assessment of Educational Progress 1996 Mathematics Assessment

THE NATION'S REPORT CARD naep

U.S. Department of Education
Office of Educational Research and Improvement

NCES 2000-451

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THE NATION'S REPORT CARD, the National Assessment of Educational Progress (NAEP), is the only nationally representative and continuing assessment of what America's students know and can do in various subject areas. Since 1969, assessments have been conducted periodically in reading, mathematics, science, writing, history/geography, and other fields. By making objective information on student performance available to policymakers at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement is collected under this program. NAEP guarantees the privacy of individual students and their families.

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The National Assessment Governing Board

NATIONAL CENTER FOR EDUCATION STATISTICS

Estimation Skills, Mathematics-in-Context, and Advanced Skills in Mathematics

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Due to the confidential nature of NAEP surveys, the photographs on the cover of this report do not portray actual students who participated in the NAEP mathematics assessment. The photographs used are from the PhotoDisc stock library.

The work upon which this publication is based was performed for the National Center for Education Statistics, Office of Educational Research and Improvement, by Educational Testing Service.

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Chapter 1

Introduction

This report presents information from three special studies conducted as part of the National Assessment of Educational Progress (NAEP) 1996 mathematics assessment. For more than a quarter of a century, NAEP has been the only nationally representative and continuing assessment of what students in the United States know and can do in various academic subjects. Each NAEP assessment is guided by a framework that specifies important learning outcomes in that subject area; the 1996 mathematics framework was an enhancement of the one used for the mathematics assessments in 1990 and 1992. The goal of the new framework was to define a 1996 assessment that would: (1) more adequately reflect current curricular emphases and objectives, and yet (2) maintain a connection with the 1990 and 1992 assessments to measure trends in student performance.[1]

In addition to the main NAEP assessment, NAEP periodically conducts special studies focused on areas of interest to educators and others. Topics for some of these studies arise as a result of how students performed on NAEP; others are generated simply from research questions about teaching, learning, and assessment of student achievement. This report focuses on studies in mathematics; special studies have also been conducted in, for example, reading and writing.

Purpose and Audience for the Report

This report is intended primarily for mathematics educators and others concerned with mathematics education, such as curriculum specialists, teachers, and university faculty in schools of education. The three studies reported here were designed to provide greater detail on how students perform on particular types of mathematics questions. They include: the Estimation Study; the Study of Mathematics-in-Context, which will be referred to as the Theme Study; and the Study of Students Taking Advanced Courses in Mathematics, which will be referred to as the Advanced Study. The Theme Study and the Advanced Study were administered for the first time in 1996. The Estimation Study, on the other hand, had been administered twice before, in 1990 and 1992.

[1] For a more in-depth description of the 1996 mathematics framework and how it guided the development of cognitive items, see the following: National Assessment Governing Board (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress*. Washington, DC: Author; Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states*. Washington, DC: National Center for Education Statistics.

Major Findings of the Report

The first study was designed to explore students' skills in estimation. It was implemented at three grade levels and was the only one of the studies that provided trend information. Findings from the Estimation Study include the following:

- Although there has been significant improvement in mathematics performance overall since 1990 at all grade levels, the trend for student performance in estimation over the 6 years since the inception of the Estimation Study in 1990 is less clear.
- Student performance in Estimation at grades 4 and 12 was stronger in 1996 than in 1990.
- Student performance in Estimation at grade 8 appears to be level across the 3 years of the assessment.

The Theme Study was administered at three grade levels and was designed to assess problem-solving abilities within contexts that allow students to make connections across mathematics content areas. Findings from the Theme Study include:

- At the fourth-grade level, with the exception of the first problem, most students attempted to answer the questions posed, even though large percentages produced responses that were scored as "incorrect." Although not definitive, this may be evidence that the thematic context of the block of questions encouraged students' attention to the task of solving problems, even ones that proved to be difficult for most students.
- At grade 8, unlike grade 4, many students did not attempt to answer the more complex questions that required them to write explanations or apply concepts in problem settings.
- The response rate to the Theme questions at grade 12 was somewhere between the rates observed for grades 4 and 8, with most questions being attempted by at least 90 percent of the students.
- At all grade levels, students appear to have difficulty with complex multistep problems, even those that require only simple computational skills at each step of the problem.
- At all grade levels, many students seemed to lack the mathematical knowledge needed to solve problems. Other students, however, appeared to understand the underlying mathematics but provided incorrect or incomplete responses as a result of carelessness, inexperience in writing out solutions to problems, or confusion over the wording of the question.
- At all grade levels, no positive relationship was seen between the frequency with which students engaged in writing a few sentences about how to solve a mathematics problem, or writing reports or doing mathematics projects, and student performance on the Theme blocks.

The Advanced Study was administered at grades 8 and 12 and was designed to provide students who were taking or had taken advanced courses in mathematics an opportunity to demonstrate their full mathematical proficiency. Findings from the Advanced Study include:

- Students participating in the Advanced Study differed from those who did not qualify for the study in that they tended to come from homes providing a stronger educational context, both in materials and in level of parental education. In addition, based on their participation in Title I programs or qualification for the federal Free/Reduced-Price Lunch program, fewer Advanced Study students appeared to come from low-income homes.
- As would be expected, students at both grade levels who met the criterion for inclusion in the Advanced Study performed substantially better than other students on the main NAEP mathematics assessment.
- The results show that Advanced Study questions were quite difficult, even for students who were taking the more challenging mathematics courses that were prerequisite for participation in the study. Overall performance, measured by average percentage correct, was 36 percent at grade 8 and 30 percent at grade 12. At both grade levels, moreover, most of these students were unable to solve problems that required two or three successive steps to achieve the desired result.
- At grade 12, students who were currently taking mathematics or who were, or had been, enrolled in an Advanced Placement (AP) mathematics course outperformed students in the study who were not currently taking a mathematics course or who had not taken an AP course in mathematics.

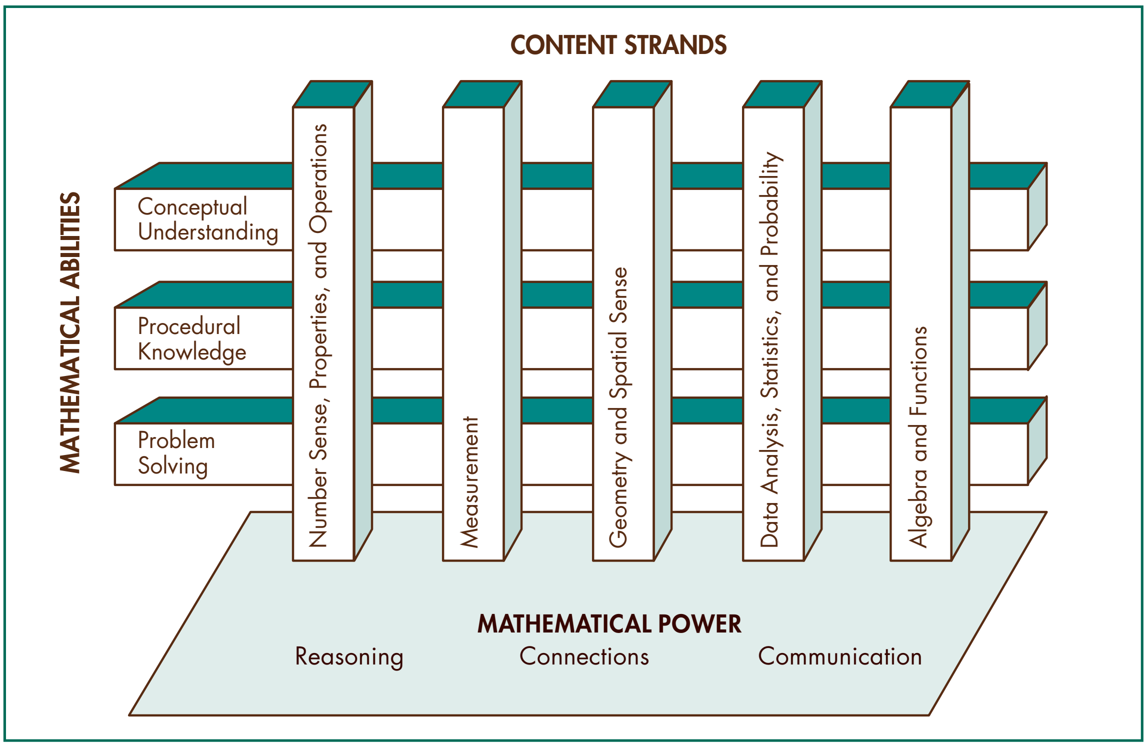
The 1996 NAEP Mathematics Assessment

To provide a context for the special studies that are the focus of this report, the following sections give additional information about the NAEP 1996 mathematics assessment and about the manner in which the design and execution of the special studies relate to the main mathematics assessment.

NAEP Mathematics Framework

The NAEP mathematics framework encompasses three cross-cutting domains: a content domain, a domain of mathematical abilities, and a domain of mathematical power. The content domain has five strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions.[2] The domain of mathematical abilities describes the nature of the knowledge or processes that are involved in successfully handling mathematical tasks or problems; it includes Conceptual Understanding, Procedural Knowledge, and Problem Solving. The domain of mathematical power refers to students' ability to reason, to communicate, and to make connections of concepts and skills across mathematics strands, or from mathematics to other curricular areas. Figure 1.1 summarizes the structure of the framework for the NAEP 1996 mathematics assessment.

Figure 1.1 ***Mathematics Framework for the 1996 Assessment***



SOURCE: National Assessment Governing Board, *Mathematics Framework for the 1996 National Assessment of Educational Progress.*

[2] The content strand *Number Sense, Properties, and Operations* was called *Numbers and Operations* in the 1990 and 1992 assessments. The content strand *Geometry and Spatial Sense* was called *Geometry* in the 1990 and 1992 assessments.

The development of the questions for the special studies, although guided by the 1996 NAEP mathematics framework, naturally focused on the goal of each individual study. Questions for the Estimation Study cut across the five content strands, but the main intent was to assess estimation skills. Questions for the Theme Study tended to emphasize problem-solving abilities within the context of real-life types of experiences. Finally, questions for the Advanced Study tended to include more content in Algebra and Functions than did questions in the main NAEP assessment.

In addition to cognitive achievement questions, student assessment booklets for the special studies contained blocks of background questions. The background questions asked students to provide information about themselves, their classroom instruction, and their motivation to expend effort on the assessment. Teachers and school administrators of students participating in NAEP also responded to background questionnaires. Teachers provided information about their education, professional careers, curricular practices, and instructional approaches, as well as the resources available to them for teaching mathematics. School administrators answered questions about school policies and practices.

Samples

The NAEP 1996 mathematics assessment was conducted nationally at grades 4, 8, and 12. As mentioned earlier, both the Estimation Study and the Theme Study also were conducted at grades 4, 8, and 12, while the Advanced Study was conducted at grades 8 and 12 only. Students for the Estimation and Theme Studies were selected through the same sampling design as students for the main NAEP assessment and were representative of all U.S. public and nonpublic school students.[3] Students selected for the Advanced Study were representative of students who had taken, or were enrolled in, more advanced mathematics courses. Specifically, to qualify for the Advanced Study, eighth-grade students had to be currently enrolled in, or already have taken, first-year algebra or a more advanced course in mathematics; and twelfth-grade students had to be currently enrolled in, or already have taken, a pre-calculus or pre-calculus-equivalent course or a more advanced course such as calculus.

Following the model of the main NAEP data collection, school administrators of students participating in the special studies were surveyed at all grade levels, but mathematics teachers of participating students only were surveyed at grades 4 and 8. The exception was the Advanced Study, which included surveys of grade 12 mathematics teachers.

[3] See Appendix A for detailed information on sample selection.

Reporting NAEP Results

Student performance on NAEP assessments has been reported using a variety of measures. Results for the main NAEP mathematics assessment are reported using the NAEP composite mathematics scale, which summarizes performance across five separate subscales — one for each of the five content strands. In addition to the NAEP mathematics scale, results also are reported using the mathematics achievement levels as authorized by the NAEP legislation[4] and as adopted by the National Assessment Governing Board. The achievement levels are performance standards based on collective judgments about what students should be expected to know and to do. Viewing students' performance from this perspective provides some insight into the adequacy of students' knowledge and skills and the extent to which they achieved expected levels of performance. The Board reviewed and adopted the recommended achievement levels derived from the judgments of a broadly representative panel that included teachers, education specialists, and members of the general public.

For each grade tested, the Board has adopted three achievement levels: *Basic*, *Proficient*, and *Advanced*. For reporting purposes; the achievement level cut scores for each grade represent the boundaries between four ranges on the NAEP mathematics scale: below *Basic*, *Basic*, *Proficient*, and *Advanced*. The generic policy definitions of the achievement levels are shown below in Figure 1.2. The text of the descriptions of expected mathematics performance at each achievement level at each grade can be found in the *NAEP 1996 Mathematics Report Card*.[5]

Figure 1.2 ***Policy Definitions of NAEP Achievement Levels***

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Basic	This level denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade.
Proficient	This level represents solid academic performance for each grade assessed. Students reaching this level have demonstrated competence in challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
Advanced	This level signifies superior performance.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

[4] The National Education Statistics Act of 1994 requires that the National Assessment Governing Board develop "appropriate student performance levels" for reporting NAEP results.

[5] Reese, et al., (1997). op. cit.

The NAEP legislation requires that the achievement levels be used on a developmental basis until the Commissioner of Education Statistics determines, as the result of a Congressionally mandated evaluation by one or more nationally recognized evaluation organizations that the achievement levels are "reasonable, valid, and informative to the public." Upon review of the available information, the Commissioner of Education Statistics agrees with the recent recommendation of the National Academy of Science that caution needs to be exercised in the use of the current achievement levels, since in the opinion of the Academy "... appropriate validity evidence for the cut scores is lacking; and the process has produced unreasonable results."[6] Therefore, the Commissioner concludes that these achievement levels should continue to be considered developmental and should continue to be interpreted and used with caution. The Commissioner and the Governing Board believe that the achievement levels are useful for reporting on trends in the educational achievement of students in the United States.

Reporting Results for the Special Studies

None of the special study assessment questions contributes to the NAEP composite mathematics scale. However, in 1990, the first year the composite scale and its component subscales were used, a separate scale was established that summarized performance on questions used in the Estimation Study. Each scale was constructed separately, and the metrics of the scales are arbitrary. Therefore, although each scale ranges from 0 to 500 across the three grade levels assessed, it is not possible to conclude that a student who performed at level 300 on the estimation scale, for example, and one who performed at level 300 on the main mathematics scale had both mastered the same proportion of their respective content domains.[7] The value of the estimation scale, like the value of the composite scale, is that it allows for trend analysis across years as well as making it possible to report the results using achievement levels.

The results from the Advanced Study and the Theme Study did not lend themselves to either the development of separate proficiency scales or equating to the main NAEP mathematics scales.[8] Consequently, the overall results from the Theme Study and the Advanced Study are reported simply in terms of the percentages of questions that students answered correctly. Student performance on individual items also is highlighted for each of these studies.

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[6] Pellegrino, J. W., Jones, L. R., & Mitchell, K. J. (Eds.). (1999). *Grading the nation's report card: Evaluating NAEP and transforming the assessment of educational progress*. Committee on the Evaluation of National and State Assessments of Educational Progress, Board on Testing and Assessment, Commission on Behavioral and Social Sciences and Education, National Research Council. (p. 182). Washington, DC: National Academy Press.

[7] In the initial year of use, each scale was set to have a mean of 250 and a standard deviation of 50.

[8] See Appendix A for more detail.

Organization of the Report

Each special study is presented in a separate chapter. The second chapter of this report describes the Estimation Study, the third chapter depicts the Theme Study, and the fourth chapter characterizes the Advanced Study. This report also includes two appendices. The first provides additional information on the procedural and technical aspects of these special studies, and the second includes standard error tables for the data presented in the body of the report.

Chapter 2

Estimation Study

Estimation is a process whereby one approximates, through rough calculations, the worth, size, or amount of an object or quantity that is present in a given situation. The approximation, or estimate, is a value that is deemed close enough to the exact value or measurement to answer the question being posed. Beginning with recommendations issued in 1975, estimation has been seen as an important, even necessary, daily living skill that all students should have, whether they are working with paper and pencil, doing mental mathematics, or using technology.[1] Estimation is now viewed as a necessary component of the school mathematics curriculum.

Estimation in the school curriculum can range from making quick ad hoc estimates to applying "rules of thumb." "Rule of thumb" operations include the application of mental checkmarks to a situation; for example, knowing that the circumference of a circle is slightly larger than 3 times the diameter. Other examples of estimation strategies include:

- adding only the leading digits to approximate the sum of a long column of numbers that all have the same number of digits (e.g., approximating the sum of 23 + 74 + 81 + 19 + 37 as 2 + 7 + 8 + 2 + 4 or about 23 tens, or 230);

- using compatible numbers for proportional problems (e.g., approximating the number of cups of oats that are needed to make 12 cups of granola, given that 1¾ cups of oats are needed to make 5 cups, by reasoning that it takes about 2 cups of oats for 6 cups of granola, so it will take about 4 cups for 12); and

- breaking or separating an uncommon shape into parts with known attributes to approximate a quality of the less common shape (e.g., approximating the area of an irregularly shaped region by separating it into 8 squares, each of which has an area of 4 square centimeters, and computing the area of the region as 8 × 4, or 32 square centimeters).[2]

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[1] National Advisory Committee on Mathematical Education. (1975). *Overview and analysis of school mathematics: Grades K-12*. Washington, DC: Conference Board of the Mathematical Sciences; National Council of Supervisors of Mathematics. (1978). Position paper on basic mathematical skills. *The Mathematics Teacher, 71*, pp. 147–152; National Council of Teachers of Mathematics. (1980). *An agenda for action*. Reston, VA: Author; National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

[2] Reys, B. (1988). Estimation. In Thomas R. Post (Ed.), *Teaching mathematics in grades K-8*. (pp. 262–284). Newton, MA: Allyn & Bacon, Inc.

The importance of estimation in the school curriculum was acknowledged in 1986 when the National Council of Teachers of Mathematics (NCTM) devoted its annual yearbook to the topic.[3] The NCTM also featured estimation as a central area within the *Curriculum and Evaluation Standards* in 1989.[4] Estimation is seen as a central mathematical process applicable across a wide range of mathematical content areas, including all five of the NAEP content strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions.

Design of the Estimation Study

It is because of the acknowledged importance of estimation that, since 1990, the national NAEP mathematics assessment has included blocks of questions focused on this topic. State data, however, were only collected in the 1992 Estimation Study.[5] The Estimation blocks include questions that require students to make estimates in a wide range of mathematical settings, from problems involving basic numeric operations to questions assessing students' knowledge of chance and growth. Some questions also require students to make use of spatial estimation skills, such as looking at pictures of dots in a region or marbles in a jar and making numerical estimates of the total.

Unlike the blocks in the main mathematics assessment or either of the other special studies, the Estimation blocks are administered using a paced audio tape. That is, students are paced through the Estimation Study questions by an audio tape that moves students along as they read and respond to the questions. This mode of administration is intended to minimize the time students have available to make actual computations or to give undue consideration to one question over another within the block. The test developers hope thus to increase the probability that students will actually use their estimation skills to respond to the questions. Furthermore, the paced audio tape reads the questions to the students, facilitating access to the problems for students with limited reading skills.

Because of these differences in administration procedures, the 1996 Estimation blocks were presented to students in separate test booklets rather than being randomly spiraled with the blocks from the main assessment. There was a separate Estimation booklet for each grade level, each containing three blocks of cognitive questions. The first block consisted of non-estimation questions taken from the main assessment. The second block (also referred to as the trend block) was based on the Estimation block used for that grade level in 1990 and 1992. Although some questions in the trend blocks were modified to improve their measurement characteristics, care was taken to maintain a sufficient core of unmodified questions from previous assessments to ensure the integrity of the trend data.

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[3] Schoen, H., & Zweng, M. (Eds.). (1986). *Estimation and mental calculations: The 1986 Yearbook of the NCTM.* Reston, VA: National Council of Teachers of Mathematics.

[4] National Council of Teachers of Mathematics. (1989). op. cit.

[5] Results from the state administration of the 1992 Estimation Study can be found in the following publications: Mullis, I. V. S., Dossey, J. A., Owen, E. H., & Phillips, G. W. (1993). *NAEP 1992 mathematics report card for the nation and the states.* Washington, DC: National Center for Education Statistics; National Center for Education Statistics (1993). *Data compendium for the NAEP 1992 mathematics assessment of the nation and the states.* Washington, DC: Author.

The third block in each booklet contained new estimation questions written for the 1996 assessment. In contrast to the trend blocks, the new Estimation blocks utilized constructed-response as well as multiple-choice questions, and they also included questions that were designed to assess students' ability to distinguish between situations in which it is more appropriate to over- or underestimate. In addition, the new blocks were written specifically for each grade level, whereas the trend blocks included a great deal of overlap across grades. As shown in Table 2.1, nearly half of the trend questions were used across all three grades, and, among the remaining questions, only those for grade 4 were unique. All of the questions in the new blocks, on the other hand, were unique to the grade level for which they were written.

Table 2.1 — ***1996 Estimation Questions by Grade Level and Block***

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	Grade Level					
	Only 4	Only 8	Only 12	4-8	4-8-12	8-12
Trend	10	0	0	0	10	12
New	13	15	16	0	0	0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Content of the Estimation Blocks

Unfortunately, unlike the Theme Study and Advanced Study questions, which are discussed in later chapters of this report, none of the Estimation Study questions have been released to the public. This is because the number of Estimation Study questions is limited, and NAEP expects to readminister them in future NAEP assessments in order to continue to track trends over time. A few mock questions, based on actual assessment questions, were written for this report to provide the reader with a sense of the kinds of questions included in the Estimation blocks. The following paragraphs and mock questions provide a description of the content of the Estimation Study.

As shown in Table 2.2, multiple-choice questions predominated at each grade level, despite the introduction of some constructed-response questions in the new blocks written for the 1996 assessment.

Table 2.2 ***1996 Estimation Questions, by Grade Level and Question Type***

	Multiple Choice	Constructed Response	Total
Grade 4	30	3	33
Grade 8	31	6	37
Grade 12	38	0	38

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The Estimation Study questions drew upon content from each of the five strands identified in the NAEP mathematics framework. However, as can be seen in Table 2.3, content from the Number Sense, Properties, and Operations strand and the Measurement strand predominated.

Table 2.3 ***1996 Estimation Questions, by Grade Level and Content Strand***

	Number Sense, Properties, and Operations	Measurement	Geometry and Spatial Sense	Data Analysis, Statistics, and Probability	Algebra and Functions
Grade 4	21	9	0	3	0
Grade 8	20	9	2	4	2
Grade 12	15	13	1	3	6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In addition to questions in which the correct response was an actual estimate of the quantity posed, some questions asked students to indicate an appropriate *method* for arriving at an estimate, to identify the one response that could *not* satisfy the specified conditions, or to label possible responses as being either "reasonable" or "unreasonable" estimates. Some questions emphasized visual, or "sight" estimation, and a few required knowledge of whether over- or underestimation would be more prudent under a range of specified circumstances. In general, the questions designed for the higher grade levels exhibited increased complexity; for example, they required the combination of more facts or factors, involved more complex concepts or operations, or were presented in more general or abstract settings.

Number Sense, Properties, and Operations

Questions involving estimation that were classified in the Number Sense, Properties, and Operations strand required students to estimate quantities such as the following:

- the result of a single mathematical computation;
- the largest or smallest among the results of different arithmetic operations based on the same numbers;
- a number rounded to a specified decimal place value;
- the approximate total cost of several purchases;
- the total monetary value of several piles of coins;
- the amount of material needed for several articles, given the amount of material needed for one;
- the number of offspring after several cycles of reproduction;
- the number of individuals in a subgroup, given information about the size of the total group and the percentage represented by the subgroup;
- the mean score on a group of tests, given information about the number of tests and the minimum and maximum scores;
- the number of persons present at the end of a sequence of events in which people both come and go; and
- the number of words on a typewritten page, given an enlarged image of a portion of the page.

In addition to whole numbers, questions in this content strand involved fractions, decimals, money (including groups of coins), and time. At grade 12, exponents and roots also were included. A mock Number Sense, Properties, and Operations question similar to those that were asked of eighth-grade students is presented below. (As noted earlier, no Estimation Study items were released to the public, so the items shown in this chapter are not actual NAEP items.)

> As they were riding along a highway, Chris and Lee noticed that all of the cars in the opposite lane were stopped due to road construction. Driving at an average speed of 48 miles per hour, it took Chris and Lee 11 minutes to get from the beginning of the stopped cars to the end. If the average length of a car is 20 feet, show how you could round the numbers and compute in order to estimate how many cars were stopped in the opposite lane. (1 mile = 5,280 feet)
>
> *Not an operational NAEP question*

This question is relatively difficult because it requires more than one operation or comparison to reach a solution, whether or not the computational burden is reduced through appropriate rounding. Students also need to understand the relationship between time, distance, and rate in order to arrive at a correct solution.

One possible solution to this problem follows. However, because different students might approximate or round the values in different ways, credit must be based on appropriate identification and application of a solution strategy rather than a specific numeric outcome. For example, some students might approximate 11 minutes as 1/6 of an hour rather than 1/5 of an hour. Similarly, since the number of place values was not specified, the number of feet per mile might well be rounded up to 5,300 rather than down to 5,000.

One possible solution

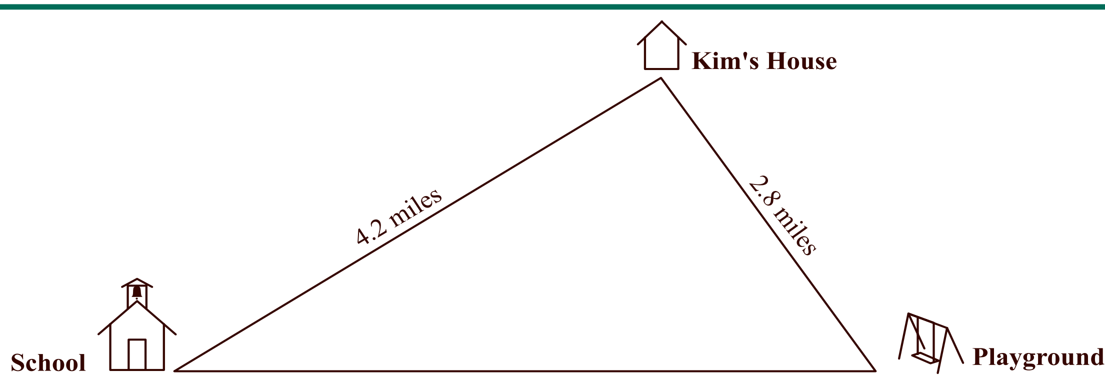
> 48 mph is almost 50 mph and 11 minutes is about $\frac{1}{5}$ hour, so the length of the stopped traffic is about $50 \times \frac{1}{5}$ or 10 miles. Since 5,280 feet rounds to 5,000, the number of cars stopped is approximately $\frac{10(5000)}{20}$, which is 2,500.

Measurement

Questions about estimation that were classified in the Measurement strand involved estimates of quantities such as the following:

- the length, in centimeters or inches, of an object shown to actual size;
- the relationship between readings of numbers on two different vertical scales;
- the number of objects that can be held in a one-quart container, given the number that fit into a one-cup container;
- the number of smaller objects that could be drawn into the interior of a larger object of the same shape;
- the length of a specified path between two points, given information about the shape of the path and the straight-line distance between the two points;
- the time difference between travel on two different routes, given rates and graphs drawn to scale for each route;
- the area of an irregularly shaped object, given a basic square unit of measure; and
- the area of an irregularly shaped object, given information about the dimensions of two rectangles, one larger and one smaller than the object.

A mock Measurement question similar to those that might have been asked of fourth-grade students is presented below.



Kim rides her bicycle from her house to school, then from school to the playground, and finally back home. She travels a total of 12.1 miles. About how many miles apart are the playground and the school?

(A) 4 miles

(B) 5 miles

(C) 7 miles

Not an operational NAEP question

The correct response is Option B, 5 miles. To answer the question, a student might read the graphic representation of Kim's journey, subtract the mileage for each of the two labeled segments from the total mileage for the journey, and round the answer to whole miles. If a student rounded all of the mileage values before doing the computations, the same numeric answer would be obtained. Alternatively, a student simply might reason from the picture that the third side is slightly longer than the side measuring 4.2 miles; hence, the answer must be 5 miles. (As noted earlier, all of the Estimation Study questions were presented via audio tape in order to establish the pace for the assessment and discourage students from undertaking precise calculations.)

Geometry and Spatial Sense

Only a few of the questions in the Estimation blocks tapped content classified as Geometry and Spatial Sense. Those that did required students to estimate the size of angles, given either a visual representation or information about the size of other angles in the same figure.

Data Analysis, Statistics, and Probability

Questions involving estimation that were classified in the Data Analysis, Statistics, and Probability strand all required the students to make estimates based on data presented in graphic or tabular format. Examples of the types of values they were asked to estimate include the following:

- the approximate value of a data point on a graph;
- the median of several values presented in a bar graph;
- which of two politicians performed best overall, given a tabular display of votes earned in each of several election districts; and
- the slope of the line that best fits data in a scatter plot.

An example of a Data Analysis, Statistics, and Probability question similar to those that might have been asked of twelfth-grade students follows.

MONTHLY COSTS FOR TELEPHONE CALLING OPTIONS
AT PHONEBELL TELEPHONE COMPANY

Calling Option	Cost per Month
Call Blocking	$4.04
Call Forwarding	$2.30
Call Waiting	$4.59
Caller ID	$6.55
Caller ID Deluxe	$7.50

The table above shows the monthly costs for certain calling options offered by Phonebell Telephone Company. Stephan chooses the following options: call blocking, call waiting, and caller ID. Approximately how much would Stephan pay over a 6-month period for these options?

(A) $15

(B) $35

(C) $90

Not an operational NAEP question

The correct answer is Option C, $90. Students would need to use the tabular data display to locate the monthly costs of the three specified options, sum them, and then multiply by six to obtain the 6-month total. These computations would be facilitated by using leading digits to sum the costs or by rounding the costs of the options before beginning. However, given the magnitude of the differences between response options, adept students would not have to perform any computations. They could see very quickly that only Option C was large enough to represent a plausible 6-month cost for three options, the cheapest of which costs more than $4.00 per month.

Algebra and Functions

Questions about estimation that were classified in the Algebra and Functions strand occurred only at grades 8 and 12. They required students to estimate quantities such as the following:

- the cube root of a four-digit number;
- the solution to a linear equation with decimal coefficients; and
- the value of a trigonometric function of a fractional number of radians, given a graph of the function.

Student Performance on Estimation

Overall performance on the NAEP mathematics scale

The tables that follow contain information on student performance on the NAEP Estimation Study over the past three assessments. The first table, Table 2.4, also includes performance data from the main NAEP mathematics assessment. These data show that, for all three grades on the main mathematics assessment, there was a pattern of continuous improvement between 1990 and 1992 and between 1992 and 1996 (although the improvement at grade 12 between 1992 and 1996 was not statistically significant). The trend for student performance on the Estimation Study, however, is not so clear. In part, this may reflect the smaller number of estimation items in the assessment, and the correspondingly greater error of measurement. In any event, student performance in Estimation at grade 8 appears to be level across the three years of the assessment, while performance at grades 4 and 12 was stronger in 1996 than in 1990.

In reviewing these data, it is important to remember that, because the Estimation Study was scaled separately from the main assessment, it is not appropriate to make direct comparisons of the average scale values obtained in a given year across the two scales. On the other hand, the Estimation Study scale, like the scale used for the main assessment, is a cross-grade scale. This means that all three grades are portrayed on a single scale, and it is possible to obtain rough estimates of the performance differences across grade levels.

Table 2.4 **Average Scale Scores for National NAEP and Estimation Studies, Grades 4, 8, and 12**

THE NATION'S REPORT CARD naep

	Assessment Year	Average Overall Scale Score in Mathematics NAEP	Average Estimation Scale Score
Grade 4			
	1996	224*†	206*
	1992	220*	208*
	1990	213	200
Grade 8			
	1996	272*†	270
	1992	268*	271
	1990	263	269
Grade 12			
	1996	304*	297*
	1992	299*	294
	1990	294	292

* Significant difference from 1990.

† Significant difference from 1992.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Other findings related to Estimation proficiency and student characteristics

Table 2.5 contains information from analyses made of the relationship between students' 1996 performance on the Estimation Study and their background characteristics. At grades 4 and 8, males and females performed about the same, while twelfth-grade boys outperformed twelfth-grade girls. At all three grades, White students and Asian/Pacific Islander students outperformed Black and Hispanic students. On average, students whose parents had at least some education after high school had higher scale scores than those whose parents did not finish high school, while at grades 4 and 12, students whose parents had a high school education also outperformed those who reported that their parents did not finish high school. Finally, students who did not participate in Title 1 and were not eligible for the federal Free/Reduced-Price Lunch program performed better than those who did participate or were eligible for these programs.

Data from earlier years are not included in Table 2.5 because there were few significant changes in subgroup performance in Estimation over the 6-year span from 1990 to 1996. The two significant changes that were observed were in the performance of males at grade 12 (where performance increased from 296 in 1990 to 303 in 1996) and in the performance of fourth-grade White students (where performance increased from 208 in 1990 to 216 in 1996).[6]

[6] The sources of these data are the 1990 and 1996 NAEP mathematics assessments.

Table 2.5 Scale Scores in Estimation by Background Variables, Grades 4, 8, and 12, 1996

	Average Scale Score		
	Grade 4	Grade 8	Grade 12
Gender			
Males	209	272	303*†
Females	203	268	292
Students who Indicated Their Race/Ethnicity as…			
White	216*	278	303
Black	181	246	276
Hispanic	183	253	279
Asian/Pacific Islander	213!	271	318
American Indian	***	***	***
Students who Reported Their Parents' Highest Level of Education as…			
Did Not Finish High School	179	257	275
Graduated From High School	203	262	289
Some Education After High School	213	275	291
Graduated from College	217	278	306
I Don't Know	197	253	***
Students who Attend…			
Public Schools	204	269	295
Nonpublic Schools	221!	278!	308!
Title I Participation…			
Participated	178	248	271!
Did Not Participate	217	273	298
Free/Reduced-Price Lunch Program Eligibility…			
Eligible	183	254	273
Not Eligible	217	276	299
Information Unavailable	218	275!	304

* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 2.6 presents data on students' average Estimation scores at different percentile levels for 1990, 1992, and 1996. Here also, little change has occurred since 1990. At grade 4, students at the median and above have made progress since 1990. No significant changes have taken place at grade 8. At grade 12, there has been some improvement for students at the low end of the distribution — the 10th percentile. These data indicate that the pattern of limited or no improvement noted earlier for performance on the Estimation Study questions is fairly widespread across the full range of student performance, with the possible exception of students at the upper end of the grade 4 distribution.

Table 2.6 ***Average Scale Scores in Estimation at Different Percentile Levels, Grades 4, 8, and 12***

THE NATION'S REPORT CARD naep

	Assessment Year	10th Percentile	25th Percentile	50th Percentile	75th Percentile	90th Percentile
Grade 4						
	1996	153	180	208	235*	256*
	1992	160	184	210*	234*	253
	1990	158	178	200	223	243
Grade 8						
	1996	232	251	272	290	304
	1992	233	251	272	291	306
	1990	232	249	269	288	305
Grade 12						
	1996	259*	278	298	317	333
	1992	258*	276	295	314	328
	1990	252	273	295	313	327

* Significant difference from 1990.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Performance on NAEP achievement levels

Along with the development of the NAEP Estimation Study score scale, *Advanced*, *Proficient*, and *Basic* achievement levels also were established for the estimation questions. These achievement levels were first reported in 1990 and were used again, though in slightly modified form, in 1992 and 1996.[7] This section includes information on the percentages of students scoring at or above the achievement levels set on the Estimation Study questions.

The data, presented in Table 2.7, reflect much the same picture as was seen in the earlier proficiency scale results — that is, there has been little change since 1990. The only area where a significant increase in student achievement appears to have taken place is at the fourth-grade level, where there was a significant increase from 1990 to 1996 in the percentage of students reaching at least the *Proficient* level of performance.

Table 2.7 National Percentages Attaining Achievement Levels in Estimation, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

	Percentage of Students				
	Assessment Year	Advanced	At or Above Proficient	At or Above Basic	Below Basic
Grade 4					
	1996	2	30*	88	12
	1992	1	30*	91	9
	1990	0!	20	90	10
Grade 8					
	1996	1	19	68	32
	1992	1	20	67	33
	1990	1	18	64	36
Grade 12					
	1996	6	38	83	17
	1992	4	34	82	18
	1990	4	33	79	21

* Significant difference from 1990.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

[7] Mullis, I. V. S., Dossey, J. A., Owen, E. H., & Phillips, G. W. (1991). *The state of mathematics achievement: NAEP's 1990 assessment of the nation and the trial assessment of the states.* Washington, DC: National Center for Education Statistics; Mullis, et al., (1993). op. cit.

An overall analysis of the percentages of students reaching the various achievement levels in 1996 reflects a differentiated pattern across the grades. Grade 12 had the greatest percentage of students reaching the *Advanced* level (6%), while only one or two percent reached this level at the lower grades. The *Proficient* level was reached by almost one-third of the students at grades 4 and 12, while one-fifth of the eighth-grade students reached this level. At the same time, over 80 percent of the students at grades 4 and 12 reached at least the *Basic* level, compared to two-thirds of the grade 8 students.

The data in Table 2.8 provide information parallel to that provided in Table 2.5. Here one can see the percentages of students in different subgroups reaching at least the *Proficient* level on the Estimation Study for each of the three grade levels in 1996.

The data for 1990 and 1992 were not reported in the table due to the small number of significant differences in performance on estimation over the time period of the three assessments. However, at grade 4, male students, female students, White students, and students attending public schools all exhibited significant increases since 1990 in percentages reaching at least the *Proficient* level, indicating that this increase in proficiency occurred broadly across the largest subgroups of students.[8]

[8] The sources of these data are the 1990 and 1996 NAEP mathematics assessments.

Table 2.8 Percent of Students Reaching at Least Proficient Level in Estimation by Background Variables, Grades 4, 8, and 12, 1996

	Percentage of Students Achieving Proficient or Better		
	Grade 4	Grade 8	Grade 12
Gender			
Males	32*	22	47
Females	28*	16	31
Students who Indicated Their Race/Ethnicity as...			
White	38*	25	45
Black	9	3	13
Hispanic	11	6	18
Asian/Pacific Islander	38!	16	70
American Indian	16!	6!	18!
Students who Reported Their Parents' Highest Level of Education as...			
Did Not Finish High School	7	6	14
Graduated From High School	27	10	25
Some Education After High School	36	19	27
Graduated From College	40	28	51
I Don't Know	21	6	18!
Students who Attend...			
Public Schools	28*	18	36
Nonpublic Schools	43!	29!	53!
Title I Participation...			
Participated	8	2!	7!
Did Not Participate	38	21	40
Free/Reduced-Price Lunch Program Eligibility...			
Not Eligible	38	24	39
Eligible	12	6	12
Information Unavailable	40	24!	49

* Significant difference from 1990.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

The overall results presented in this chapter, indicate a lack of significant progress over the 6 years since the inception of the Estimation Study in 1990. In terms of the achievement expectations set for this area of mathematics, the greatest change appears to have taken place at grade 4, although smaller score increases were also found at grade 12. The differences in grade 4 performance may reflect action on the recommendation by the National Council of Teachers of Mathematics that emphasis be placed on estimation in the primary grades.[9] The lack of equivalent increases at the upper two grades may be an indication of the lack of focused instruction and opportunity to practice estimation concepts and skills in either classroom or assessment settings.

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[9] National Council of Teachers of Mathematics. (1989). op. cit.