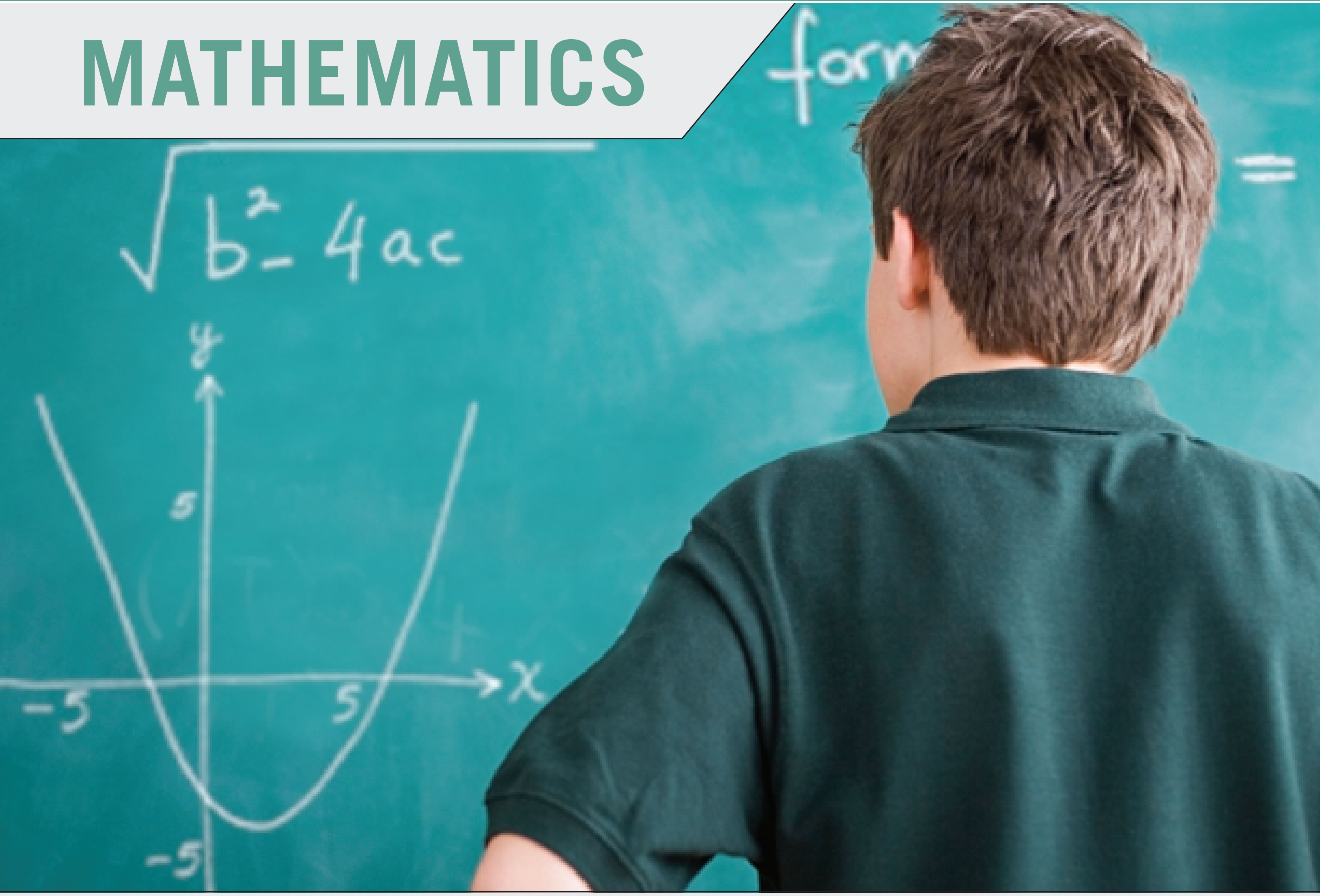
MATHEMATICS



New Mathematics Assessment

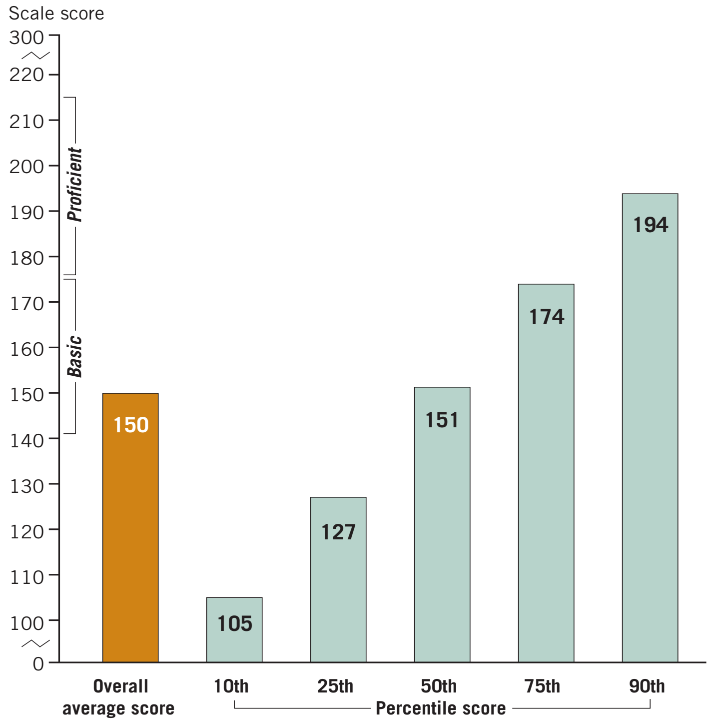
Sixty-one percent of students nationwide performed at or above the *Basic* achievement level in 2005, and 23 percent performed at or above *Proficient* on the new 12th-grade mathematics assessment. Asian/Pacific Islander students outperformed those in all other racial/ethnic groups. The average for White students was 31 points higher than for Black students and 24 points higher than for Hispanic students. Overall, male students scored higher on average than female students, but in two of the four content areas measured, there was no significant difference by gender in average scores.

Because of changes in assessment content and administration, the results for 2005 could not be directly compared to those from previous years.

Less than one-quarter of 12th-graders perform at the *Proficient* level or higher

Figure 11

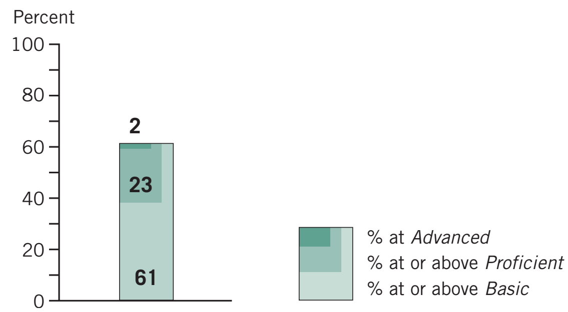
Average 12th-grade NAEP mathematics score and percentile scores in 2005



While the overall average mathematics score in 2005 was set at 150, students in grade 12 exhibited a wide range of performance as shown in figure 11. Scores ranged from 105 for lower-performing students at the 10th percentile, to 194 for higher-performing students at the 90th percentile. Sixty-one percent of 12th-graders performed at or above the *Basic* level in 2005, and 23 percent performed at or above the *Proficient* level (figure 12).

Figure 12

Twelfth-grade NAEP mathematics achievement-level results in 2005



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Changes to the NAEP mathematics assessment in 2005

For 2005, the National Assessment Governing Board adopted a new mathematics framework for grade 12 to reflect changes in high school standards and coursework. In addition, changes were made in booklet design and calculator-use policy for the one-third of the assessment in which calculators were allowed. Major differences from previous assessments are highlighted in the table. As a result of these changes, the 2005 results could not be placed on the previous NAEP scale and are not compared to results from previous years in this report. There were, however, some questions from the 2000 assessment that fit the requirements in the new framework and were used again in 2005. A special analysis was done to see how students' performance on this set of items differed between the two years. More information about this analysis can be found at http://nces.ed.gov/nationsreportcard/mathematics/interpret-results.asp.

2005 mathematics assessment	Previous mathematics assessments
Content areas	
Four content areas, with measurement and geometry combined into one because the majority of 12th-grade measurement topics are geometric in nature	Five content areas, with measurement and geometry represented as separate areas
Distribution of questions across content areas	
10% Number properties & operations	20% Number sense, properties, & operations
30% Measurement & geometry	15%; 20% Measurement; geometry & spatial sense
25% Data analysis & probability	20% Data analysis, statistics, & probability
35% Algebra	25% Algebra & functions
Reporting scale	
0–300 single-grade scale	0–500 cross-grade scale
Calculators	
Students given the option to bring their own graphing or scientific calculator, or are provided with a scientific calculator	Students provided with standard model scientific calculator
Booklet design	
Two 25-minute blocks	Three 15-minute blocks

Male students score higher than female students overall, but not in every content area

In 2005, male students scored higher on average than female students as shown in figure 13. When the results were examined by each of the four content areas, the scores for male students were only higher than the scores for female students in two content areas: the number properties and operations and the measurement and geometry content areas (table 3). Apparent differences in the other content areas were not statistically significant.

Figure 13

Average 12th-grade NAEP mathematics scores in 2005, by gender

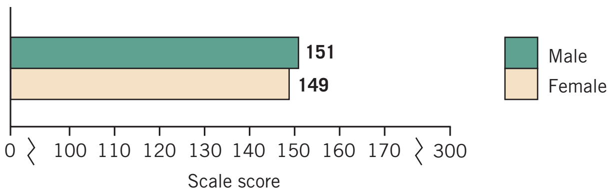


Table 3

Average 12th-grade NAEP mathematics scores in 2005, by gender and content area

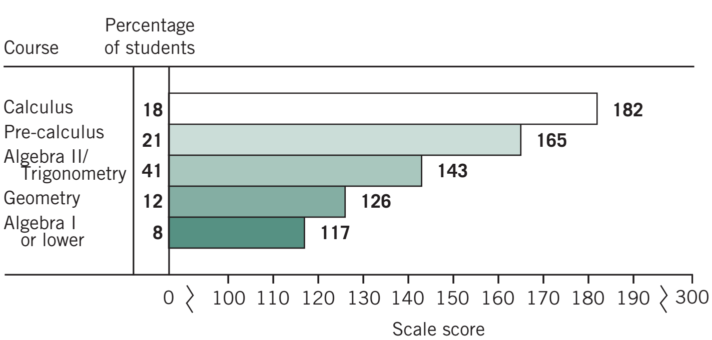
Content area	Male	Female
Number properties and operations	152	148
Measurement and geometry	152	148
Data analysis and probability	151	149
Algebra	151	150

Higher scores associated with advanced mathematics courses

Figure 14 shows the percentages of students and their average scores by the highest level mathematics course they reported having taken. The five categories, from highest to lowest level, were calculus, pre-calculus, algebra II/trigonometry, geometry, and algebra I or lower. The results show that taking higher level mathematics courses was associated with higher mathematics scores.

Figure 14

Percentages of 12th-grade students and average NAEP mathematics scores in 2005, by highest reported mathematics course



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

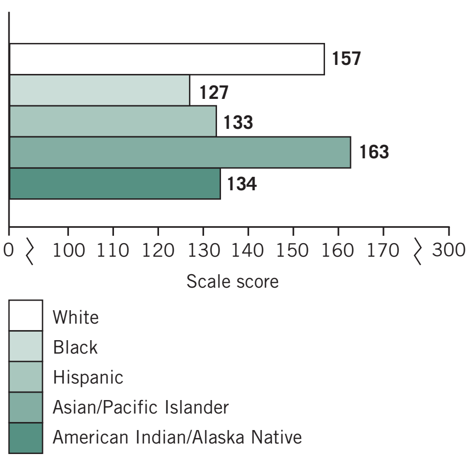
Asian/Pacific Islander students outperform students in other racial/ethnic groups

As shown in figure 15, Asian/Pacific Islander students scored higher on average in 2005 than the other four racial/ethnic groups. The average score for White students was higher than the scores for Black, Hispanic, and American Indian/Alaska Native students. Hispanic students scored higher on average than Black students.

Generally, the comparisons between groups were similar in each of the four content areas (table 4). Scores for Asian/Pacific Islander students and White students were not significantly different in the number properties and operations and the data analysis and probability content areas. While there was no significant difference in scores for Black and American Indian/Alaska Native students overall, American Indian/Alaska Native students scored higher on average than Black students in measurement and geometry.

Figure 15

Average 12th-grade NAEP mathematics scores in 2005, by race/ethnicity



Achievement-level profiles

To help in understanding differences in performance among student groups, this section shows the percentage of students in each of several groups who performed at or above the *Proficient* level. For example, 29 percent of White students performed at or above *Proficient*. The percentage of Black students at or above this level was 6 percent.

Percentage of students at or above *Proficient*

- 29% of White students; 6% of Black students; 8% of Hispanic students; 36% of Asian/Pacific Islander students; 6% of American Indian/Alaska Native students
- 7% of students who reported neither parent finished high school; 34% of students who reported at least one parent graduated from college
- 15% of students who reported never taking a mathematics Advanced Placement course; 55% of students who reported taking a mathematics Advanced Placement course

Table 4

Average 12th-grade NAEP mathematics scores in 2005, by race/ethnicity and content area

Content area	White	Black	Hispanic	Asian/Pacific Islander	American Indian/ Alaska Native
Number properties and operations	158	126	132	160	132
Measurement and geometry	158	124	134	163	141
Data analysis and probability	158	126	132	157	134
Algebra	157	130	134	167	129

NOTE: Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Mathematics achievement levels at grade 12

The following mathematics achievement levels describe what 12th-graders should know and be able to do in mathematics at each level. The cut score indicating the lower end of the score range for each level is noted in parentheses.

***Basic* (141):** Twelfth-grade students performing at the *Basic* level should be able to solve mathematical problems that require the direct application of concepts and procedures in familiar situations. For example, they should be able to perform computations with real numbers and estimate the results of numerical calculations. These students should also be able to estimate, calculate, and compare measures and identify and compare properties of two- and three-dimensional figures, and solve simple problems using two-dimensional coordinate geometry. At this level, students should be able to identify the source of bias in a sample and make inferences from sample results, calculate, interpret, and use measures of central tendency and compute simple probabilities. They should understand the use of variables, expressions, and equations to represent unknown quantities and relationships among unknown quantities. They should be able to solve problems involving linear relations using tables, graphs, or symbols; and solve linear equations involving one variable.

***Proficient* (176):** Students in the twelfth grade performing at the *Proficient* level should be able to select strategies to solve problems and integrate concepts and procedures. These students should be able to interpret an argument, justify a mathematical process, and make comparisons dealing with a wide variety of mathematical tasks. They should also be able to perform calculations involving similar figures including right triangle trigonometry. They should understand and apply properties of geometric figures and relationships between figures in two and three dimensions. Students at this level should select and use appropriate units of measure as they apply formulas to solve problems. Students performing at this level should be able to use measures of central tendency and variability of distributions to make decisions and predictions; calculate combinations and permutations to solve problems, and understand the use of the normal distribution to describe real-world situations. Students performing at the *Proficient* level should be able to identify, manipulate, graph, and apply linear, quadratic, exponential, and inverse proportionality ($y = k/x$) functions; solve routine and non-routine problems involving functions expressed in algebraic, verbal, tabular, and graphical forms; and solve quadratic and rational equations in one variable and solve systems of linear equations.

***Advanced* (216):** Twelfth-grade students performing at the *Advanced* level should demonstrate in-depth knowledge of the mathematical concepts and procedures represented in the framework. They can integrate knowledge to solve complex problems and justify and explain their thinking. These students should be able to analyze, make and justify mathematical arguments, and communicate their ideas clearly. *Advanced* level students should be able to describe the intersections of geometric figures in two and three dimensions, and use vectors to represent velocity and direction. They should also be able to describe the impact of linear transformations and outliers on measures of central tendency and variability; analyze predictions based on multiple data sets; and apply probability and statistical reasoning in more complex problems. Students performing at the *Advanced* level should be able to solve or interpret systems of inequalities; and formulate a model for a complex situation (e.g., exponential growth and decay) and make inferences or predictions using the mathematical model.

Mathematics Framework

The framework calls for the assessment of mathematics within four content areas and at different levels of complexity. The framework specifies that 10 percent of assessment questions should be devoted to number properties and operations, 30 percent to measurement and geometry, 25 percent to data analysis and probability, and 35 percent to algebra.

The level of complexity of a question is determined by the demands it places on students. According to the framework, the ideal balance for the assessment is that one-half of the score is based on items of moderate complexity, with the remainder of the score based equally on items of low and high complexity.

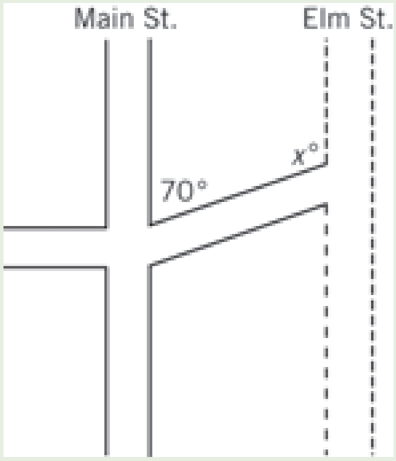
Assessment design

Each student received a booklet containing two 25-minute sections of 17 to 21 mathematics questions. Multiple-choice questions required students to select an answer from five options, while constructed-response questions required students to write either short or extended answers.

Calculators could be used for approximately one-third of the assessment. Students were permitted to bring whatever calculator they were accustomed to using in the classroom (including a graphing calculator) or were provided with scientific calculators. Graphing calculators were not needed to complete any question on the assessment.

Sample Multiple-Choice Mathematics Question

The following multiple-choice question comes from the measurement and geometry content area. The question required students to determine an angle formed by a cross street between two parallel streets.



In the figure above, Elm Street is to be constructed parallel to Main Street. What is the value of x ?

- (A) 70
- (B) 110 (filled in)
- (C) 120
- (D) 130
- (E) 140

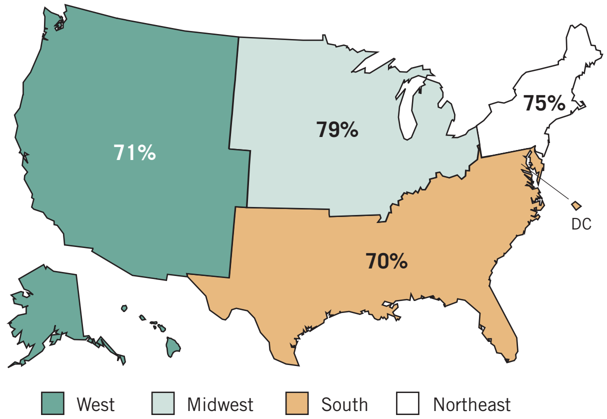
In addition to the overall percentage of students who answered the question correctly, the percentage of students at each achievement level who answered correctly is presented.

Percentage correct overall and at achievement levels in 2005

Overall	Below *Basic*	At *Basic*	At *Proficient*	At *Advanced*
73	49	83	96	‡

‡ Reporting standards not met. Sample size was insufficient to permit a reliable estimate.

Percentage of correct 12th-grade student responses in 2005, by region



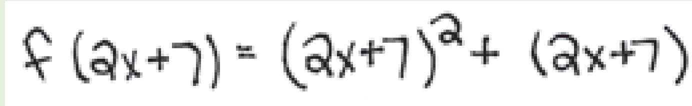
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Full information is available for NAEP mathematics questions of various types and difficulty levels at http://nces.ed.gov/nationsreportcard/itmrls.

Sample Short Constructed-Response Mathematics Question

The following is a short constructed-response question from the algebra content area. The question asked students to determine the composition $f \circ g$ of a quadratic function f and a linear function g. Responses were rated as "Correct," "Partial," or "Incorrect." The sample student response below was rated "Correct."

If $f(x) = x^2 + x$ and $g(x) = 2x + 7$, what is an expression for $f(g(x))$?

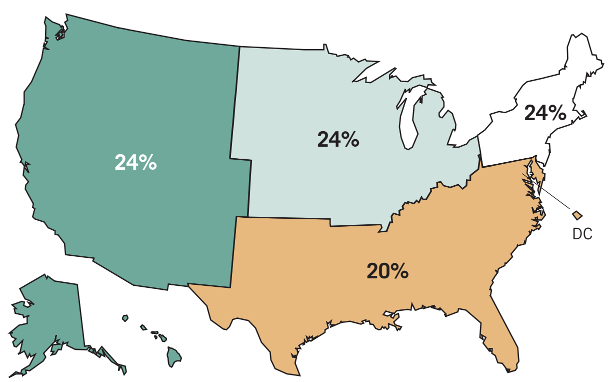


Percentage "Correct" overall and at achievement levels in 2005

Overall	Below *Basic*	At *Basic*	At *Proficient*	At *Advanced*
23	1	16	60	‡

‡ Reporting standards not met. Sample size was insufficient to permit a reliable estimate.

Percentage of "Correct" 12th-grade student responses in 2005, by region



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Range of Mathematics Performance

The item map is a useful tool for better understanding what it means to perform at different levels on the mathematics scale. The left side of the map shows the scores that define the lower boundaries of the *Basic, Proficient,* and *Advanced* achievement levels. The right side lists descriptions of some selected assessment questions that fall at various levels on the 0–300 scale. Using the Pythagorean Theorem to determine the length of a hypotenuse (160 on the scale) is an example of the knowledge and skills demonstrated by students performing at the *Basic* achievement level. An example of the knowledge and skills demonstrated by students performing at the *Proficient* level is using trigonometric ratios to determine length (205). Note that several examples of performance below the *Basic* level are included. For example, students who perform below *Basic* are likely to be able to identify a graph representing data given in a table (100) and to determine which spinner probably produced given results (133).

NAEP Mathematics Item Map

Achievement level	Scale	Score	Question description
Advanced	300		
	240		
		234	*Calculate a weighted average for two groups*
	230		
		222	Use survey results to make an inference
	220	221	*Identify graph of solution for absolute value inequality*
216			
Proficient		215	*Determine which triangle is not a 30-60-90 right triangle*
	210	210	Solve a multi-step problem with a rectangle and sphere ○
		205	Use trigonometric ratios to determine length ○
		204	Use a spinner to generate data and answer question
	200	202	*Determine the initial number of bacteria in a sample* ○
		199	*Read and interpret a boxplot*
		199	Determine proportional enlargement of a photograph ○
		196	*Identify algebraic expressions to represent a situation*
	190	195	Determine composition of two functions (page 20)[1]
		184	*Find fourth term in numerical sequence* ○
	180		
		177	*Determine the type of graph to best represent a situation*
176			
Basic		172	*Compute with values from step-function graph*
		172	*Identify effect of changing certain digits on a number's value*
	170	170	*Divide numbers given in scientific notation*
		165	*Find the length of the sides of a square*
		163	*Given the graph of f(x), identify the graph of -f(x)*
		163	Identify the 3-D figure resulting from folding paper
	160	160	*Use Pythagorean Theorem to determine length of hypotenuse* ○
		153	Convert a decimal to a fraction
	150		
		144	*Identify solution to graphical system of equations*
		142	*Find the measure of an angle between streets* (page 19)[1]
141			
	140		
		133	*Determine which spinner probably produced given results*
	130		
	120		
		118	*Identify figure with incorrect line of symmetry shown*
	110		
	100	100	*Identify graph representing data given in a table*
	0		

○ Calculator available

[1] Page numbers refer to the location in the report where the question described is presented.

NOTE: The position of a question on the scale represents the average scale score attained by students who had a 65 percent probability of correctly answering a constructed-response question, or a 72 percent probability of correctly answering a five-option multiple-choice question. For constructed-response questions, the question description represents students' performance rated as completely correct. Regular type denotes a constructed-response question. *Italic* type denotes a multiple-choice question.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.