Presentation to the National Mathematics Advisory Panel
Chicago, IL, April 20, 2007
By Dr. Henry Borenson, President
Borenson and Associates, Inc.

Mr. Chairman and members of the panel: I thank you for this opportunity. My name is Dr. Henry Borenson, President of Borenson and Associates, Inc. Some twenty years ago, as a middle school math teacher, I was concerned with the difficulty students were having learning algebra abstractly. I determined to find a way to simplify the concepts, to make them concrete and visual, and to make them accessible to grade school students.

After two years of experimentation, working with children, including LD children, I developed a system known as Hands-On Equations. This is a system which uses games pieces, a flat laminated scale, and a specific progression of ideas to enable students as early as the third grade to physically represent and solve algebraic linear equations, the type of equations which until then were typically taught in the 8th or 9th grade.

Since 1995, Borenson and Associates has conducted more than 1500 Making Algebra Child's Play workshops throughout the United States. In these workshops, teachers of grades 3 to 8 see learn how to introduce the concept of a variable, the concept of an equation, the subtraction and addition properties of equality, and other key algebraic concepts.

A key part of these workshops is a student demonstration with local 4th and 5th grade students. More than 1500 times since 1995, the teachers attending our seminar have seen how in three lessons 4th and 5th grade students, even so called "low ability" students, can learn to solve an algebraic linear equation such as $4x + 3 = 3x + 9$.

In a study to determine teacher confidence level to teach algebraic linear equations to their lowest achieving students, Barber and Borenson (2006) discovered that only 16% of 751 teachers from grades 3 to 8 attending a Making Algebra Child's Play workshop felt they would be successful using the traditional abstract teaching methods, while 98% expressed confidence of success if they were to use the Hands-On Equations materials. See Appendix A.

In an ongoing series of studies involving multiple student characteristics and multi-site replication, supervised by Dr. Larry W. Barber, formerly Director of Research for Phi Delta Kappa, we have found significant pre test to post test gains for 2nd grade gifted students, regular 6th grade students and 9th and 10th grade low achieving students.

Recently we completed a study involving four 5th grade inner-city classes comprising a total of 111 students. The pre to post test results showed a large and highly significant increase in scores. The combined mean increased, in percentage terms, from 44.8 % on the pre test to 85.3% on the post test. On a three week retention test, provided three weeks after the post test—with no Hands-On Equations instruction in the interim—the mean was 78.6%. When compared with the pre test score of 44.8%, this increase was found to be statistically significant with a t-value of 13.71. We are talking about 5th grade inner city students succeeding with important algebraic concepts. This study may be found in Appendix B.

We believe we have provided evidence that Hands-On Equations system of instruction significantly and positively impacts upon teacher self-confidence in their ability to introduce algebraic linear equations to their students, and evidence that the program makes a measurable difference in student learning. We believe it is possible and it is important for students to gain the perception that mathematics is a subject they can <u>understand,</u> and a subject at which they can excel. In Hands-On Equations the students need not memorize a set of procedures in order to obtain an answer. They can use their creativity to apply general algebraic principles in the manner that best suits them. We ask the Panel to consider recommending Hands-On Equations as a supplementary program that is effective in introducing grade school students to basic algebra.

Thank you.

Appendix A

The Teacher Study Results*

First Problem: To obtain a measure of teacher confidence in teaching algebraic concepts to 80% or more of the students in their lowest achieving class using traditional instructional vs. HOE, and to compare the results.

We wished to identify whether or not teachers attending the **Making Algebra Child's Play seminar**, the large majority of whom are elementary and middle school teachers, are confident that they can successfully teach algebraic concepts to at least 80% of the teacher's lowest achieving class, via two different modes of instruction: the traditional mode of instruction and HOE, and then to compare these responses to see if there is a significant difference. The response to the traditional mode was obtained on the pre-test, prior to the beginning of the seminar treatment. The HOE response was obtained at the conclusion of the 6^{th} lesson of the seminar, approximately 2 ½ hours into the full-day **Making Algebra Child's Play** seminar.

To accomplish this objective, the teachers were asked to respond anonymously to the following question on their onsite questionnaire (see Appendix), prior to the beginning of the seminar:

Please indicate "Yes" or "No" to the following question.

"I am confident that using the traditional method of teaching algebra, I am able to teach 80% or more of the students in my lowest class how to understand and solve these two questions.

$$2x + x + x + 2 = 2x + 10$$

and

$$2(x + 4) + x = x + 16$$

Following the conclusion of Lesson #6, they were asked to respond, also anonymously, to the following question on the same questionnaire:

Please indicate "YES" or "NO" to the following question:

*" I am confident that using the **Hands-On Equations** system of instruction, with each student having their own set of game pieces, that I would be able to teach 80% or more of the students in my lowest class how to understand and solve the two equations shown above."*

Both questions essentially asked the teacher, "Are you confident that you can teach these algebraic equations to 80% or more of the students in your lowest class using this specific method of instruction?" The teachers had to select either a "Yes" or a "No" response.

For this line of research we found 751 teachers who filled out both the pre-test and the post-test. All but 46 of these respondents were elementary and middle school teachers. In order to quantify the data we assigned a 1 to each "yes" response on the pre and post-test and assigned a 0 to each "no" response. The comparison was between the mean of the yeses on the pre-test and the mean of the yeses on the post-test. The pre-test mean was .162. The post-test mean was .984. We then calculated a t value (we used the t for paired observations) between the two means: the t value was 57.81. The post-test mean was over 6 times larger than the pre-test mean.

* This study is taken from the report titled, **"Borenson Hands-On Equations Research Designs and Interim Results: December 2006. Effect of Making Algebra Child's Play® Seminar on Teacher Self-Concept and Student Achievement"** by Larry W. Barber and Henry Borenson. This study may be found in full at www.borenson.com

Teacher Study Data Results: Problem #1

	X1	**X2**	**D**	**D2**
Sum	122	740	618	622
Mean	.162	.984		
N =	751			
t =	57.81			

The statistic used above was the difference between the Means for Paired Observation and Equated Groups ("t-test for paired observation," Edwards 1963) (Barber et. al 1988). The formula used came from the Edwards book (p. 281) and was applied to test the difference between the group mean on the pre-training self-report and the group mean on the post-training self-report from the same teachers. For this first analysis we simply analyzed the data on all teachers who gave both a pre and post response.

Conclusion: *We note that only 16% of teachers coming to the* ***Making Algebra Child's Play*** *seminar expressed confidence that they would be able to teach 80% or more of the students in their lowest classes the solution to equations such as* $\mathbf{2\mathit{x} + \mathit{x} + \mathit{x} + 2 = 2\mathit{x} + 10}$ **and** $\mathbf{2(\mathit{x} + 4) + \mathit{x} = \mathit{x} + 16}$ *using the traditional teaching methods.* In light of the significant relationship which research shows to exist between teacher efficacy, i.e. teacher belief, and student achievement (see page one of this study), this result is important. If this result turns out to be representative of teachers nationwide, it would suggest that the use of the traditional methods of instruction is not likely to accomplish the goal of successfully teaching the above concepts to 80% or more of the students in our lowest achieving classes. On the other hand, *by the end of the 6th lesson of the seminar, 98% of the participants at this seminar, the majority of who were elementary and middle school teachers, expressed confidence that, using the Hands-On Equations system of instruction, with each child having their own set of manipulatives, they would be able to teach these concepts to 80% or more of the students in their lowest class.*

Appendix B

Hands-On Equations® Research
Interim Results, Study # 33b, Mar. 30, 2007.
The Effect of Hands-On Equations on the Learning of Algebra
By Title I Inner City Students in the 5th grade.

By Larry W. Barber and Henry Borenson

Hands-On Equations (HOE), developed by Dr. Henry Borenson, uses numbered cubes to represent the constants, and blue pawns to represent the variable x. It also uses a scale representation on which the students "set up" the equation. The students then proceed to use "legal moves" which are the mathematical counterpart of the abstract algebraic methods which are used to solve these linear equations. The system thus makes abstract linear equations visual and understandable, and further provides students with the means of solution through a kinesthetic approach which makes sense to them

The program is unique, in that the abstract knowledge base needed by students to solve these equations is transformed into an easily understood and manageable set of verbal, visual and kinesthetic responses using manipulatives. The program teaches algebraic principles which students in grade 3 to 8 can apply in any sequence desired to solve the given equation, as is the case in more traditional instruction. Rather they feel empowered to use their thinking and understanding of basic principles to solve the problem at hand.

The research study uses a Multi-Site Replications Design and studies the effect of the HOE program on many groups of students with different characteristics (regular education, special ed., gifted, elementary, middle-school, high school, etc). All of these groups of students will be studied separately. Presently, we have student achievement data on 24 classrooms.

This particular study (# 33b) includes four separate studies, each a Title I classroom in a large inner-city school district on the west coast. The teachers of these students had been trained in HOE teaching procedures in Feb. 2007 and began teaching their students (the subjects in this study) during the second semester of the 2006-2007 school year.

TESTING PROCEDURES;

Six questions were used for the pretest, the two posttests and the three-week retention test*. The pretest was administered to the students prior to the introduction of the HOE program and was designed to measure the level of student knowledge on the concepts to be covered in the first six lessons of the program. A posttest was administered at the completion of the first lesson (the reason for this posttest is explained below) and at the completion of the 6th lesson. A retention test was administered three weeks after the completion of the 6th lesson.

These sets of 6 questions were randomly selected out of a pool of 8 sets of equivalent questions, that is, the first question of each set were considered to be questions which tested the same concept and, questions of equal difficulty. Likewise the second question on each set were considered equivalent and of equal difficulty, and so on for all six questions. The first and second question of each set tested different concepts. The third question of each set was more difficult than either the first or second question. The fourth question of each set was considered more difficult than the first three questions of the set, and so on.

Students were allowed up to 15 minutes to complete each test. The students were provided with their student kits for both posttests as well as the retention test.

*See the Appendix for the questions used on each of the tests.

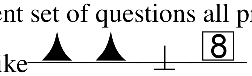
RESULTS

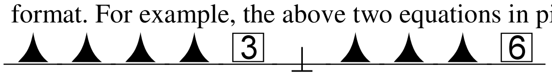
Four classrooms were included in this study (combined n = 111). Each classroom's data was analyzed independently to provide each teacher with feed back about their own and their students' performance. T-tests were run between the means of the pretest and the posttest after Lesson 1, between the pretest and the posttest after Lesson #6, between the pretest and the retention test three weeks after the teaching of Lesson #6. In all four classrooms the size of the gain between the pretest and each of the posttests as well as between the pretest and the retention test was large and highly significant. The analysis of the difference between the posttest and the retention test showed a loss of test points in three of the four classrooms.

This report is primarily interested in the combined classroom analysis, given that this is a multi-site replicated experiment that pools data from all sites that are similar, i.e., 5th grade Title I students from an urban school district. (Note: we will add similar classrooms to this study over time as data arrives and rerun the data as many times as needed into a larger study. These "meta analyses" will be done in accordance with the Multiple Replication Methodology of these studies.) This study (# 33b, N = 111) shows large effect sizes between the pretest and the Lesson 1 posttest (t= 10.90, mean x1= 2.69 mean x2 = 4.50), between the pretest and the Lesson 6 posttest (t= 18.l8, mean x1= 2.69, mean x6= 5.12) between the pretest and the retention test (t= 13.71, mean x1=2.69, mean retention= 4.72). There was a loss of .40 points between the posttest following Lesson #6 and the retention test (t= -3.4).

ANALYSIS

Posttest Following Lesson #1

The pretest consisted of six questions all written in the traditional abstract algebraic notation such as for example 2x = 8 and $4x + 3 = 3x + 6$. The posttest following Lesson #1 consisted of an equivalent set of questions all presented in pictorial format. For example, the above two equations in pictorial format would look like  and



Once the equivalent problems to those presented on the pretest were presented in pictorial format in the posttest following Lesson #1, a large and highly significant increase in test scores was noticed (t= 10.90, mean x1= 2.69 mean x2 = 4.50). This translates to an increase in mean score from 44.8% on the pretest to 75% on the posttest following Lesson #1.

In explaining this increase, we need to understand that in Lesson #1 of HOE not only were the students taught the meaning of this pictorial notation, but they were introduced to the trial and error method of testing values for the blue pawn (shaded triangle) in order to find the value that would make both sides balance. This testing procedure involved only the operation of addition. (In the pictorial notation the students did not need to use the operation of multiplication to test their guesses. Indeed the operation of multiplication was not required anywhere in these six problems, as is it not required in the rest of the HOE program.) Hence, the pictorial notation along with the trial and error approach to testing possible values for the blue pawn enabled the students to show a large and highly significant gain as compared with the pretest where they were provided with an equivalent set of problems in the traditional abstract algebraic notation.

Posttest Following Lesson #6

The posttest following Lesson #6 showed a large and highly significant increase in score (t= 18.l8, mean x1= 2.69, mean x6= 5.12) over the pretest. This posttest used the traditional abstract algebraic notation, the same notation as was used for the pretest. The students used the HOE approach of translating or "setting up" the equation using their game pieces. At that point they were free to use the "legal moves" they had learned in the program to find the value of the blue pawn. In percentage terms, the mean increased from 44.8% on the pretest to 85.3% on the posttest.

Testing the Power of the "Legal Moves" to Improve Student Performance

In order to determine if the concept of "legal move" provided the students with a significant advantage over that of using trial and error to obtain the answer, a t-test was conducted on the four classrooms combined to see if there was a significant difference between the posttest following Lesson #1 and the posttest following Lesson #6. We will recall that in the posttest following Lesson #1 the students were provided with the pictorial equations and had available trial and error methods of solution. In the posttest following Lesson #6, the students had available the concept of legal move to simplify the physical equations.

The gain obtained was found to be significant (t= 3.05, mean x2=4.50, mean x6=5.12). In percentage terms, the mean average increased from 75% to 85.3%. If we recall that the posttest for Lesson #1 already provided the students with the pictorial notation, whereas the posttest for lesson #6 required the students to first set up the pictorial notation (or the equivalent physical representation) and then use legal moves, it appears clear that the actual effect of the legal move concept is actually greater than that shown above.

Three-Week Retention Test

The retention test, using the traditional algebraic notation, was administered three weeks after the teaching of Lesson #6. During the three weeks prior to the retention test the teachers were asked not to do any HOE work with the students. The intent of the measurements conducted with the retention test was two-fold: first, to determine how the results of the retention test would compare with the pretest scores; secondly, to determine the extent of loss of knowledge, if any, that the students would show after three weeks of not having any HOE work. Regarding the first question, the gain from the pretest to the posttest (t= 13.71, mean x1=2.69, mean retention= 4.72) was large and highly significant. In percentage terms, the mean increased from 44.8% on the pretest to 78.6% on the retention test.

Regarding the second question, it is noted that the achievement level of the students decreased .40 points from the posttest following Lesson #6 to the three-week retention test (t = -3.40, mean x6=5.12, mean retention=4.72,). In percentage terms, the score decreased from a mean of 85.3% on the Lesson #6 posttest to 78.6% on the three week retention test. This was a drop in score of 7.8% of the Lesson #6 posttest results.

CONCLUSIONS

Four inner city, 5th grade, Title I classes, comprising a total of 111 students, participated in this study. Their teachers were trained in the use of the program in February 2007. The implementation of the first six lessons of the program, and the accompanying testing, took place shortly thereafter.

The combined group of 111 students showed a large and significant increase from the pretest to each of the subsequent tests: the posttest following Lesson #1, the posttest following Lesson #6 and the three-week retention test. In percentage terms, the average mean on the pretest was 44.8%. The average mean on the posttest following Lesson #1 was 75%; the average mean on the posttest following Lesson #6 was 85.3%; and the average mean on the three-week retention test was 78.6%.

The representation of algebraic linear equations in the pictorial notation, along with the strategy of trial and error using the operation of addition, produced the large and significant increase from the pretest to the posttest following Lesson #1. This result suggests that it is advantageous to present grade school students with pictorial equations and the trial and error method of solution using addition to enable them to experience success with more advanced algebraic linear equations such as those presented to the students in this study. The pictorial notation is one which the students can understand, and it obviates the need to use the operation of multiplication in using trial and error to test the proposed solutions.

The posttest following Lesson #6 and the three-week retention test three weeks later enabled the students to demonstrate their ability to represent the abstract algebraic linear equations in a physical format using their game pieces, and then to solve using the legal moves. This HOE methodology produced the large and highly significant increase from 44.8% on the pretest to 85.3% on the posttest following Lesson #6. The three-week retention test saw a drop in score to 78.6%. Nonetheless, this result of 78.6%, as compared with the pretest score, also constituted a large and highly significant increase.

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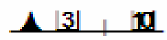
APPENDIX

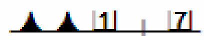
PRE TEST

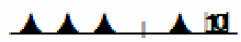
1. $2x = 8$
2. $x + 3 = 8$
3. $2x + 1 = 13$
4. $3x = x + 12$
5. $4x + 3 = 3x + 6$
6. $2(2x + 1) = 2x + 6$

POST TEST AFTER LESSON #1

1. 

2. 

3. 

4. 

5. 

6. 

POST TEST AFTER LESSON #6

1. $2x = 10$
2. $x + 3 = 8$
3. $2x + 2 = 10$
4. $3x = x + 4$
5. $4x + 3 = 3x + 9$
6. $2(2x + 1) = 2x + 8$

THREE-WEEK RETENTION TEST

1. $2x = 12$
2. $x + 3 = 10$
3. $2x + 1 = 9$
4. $3x = x + 10$
5. $4x + 3 = 3x + 10$
6. $2(2x + 2) = 2x + 18$