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CONCERNING PERMEABILITY UNITS FOR SOILS

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IT is becoming increasingly apparent that the permeability of soil is one of its most important physical properties, and the study of soil permeability is making the transition from a qualitative to a quantitative stage. It should facilitate progress for both scientific investigations and engineering applications if there were a permeability unit that would be acceptable for use by the majority of soils workers.

The Soil Physics Section of the Soil Science Society of America[3] is sponsoring consideration of the advantages and disadvantages of the various permeability units in the hope that more uniformity of usage may result. Since many more permeability measurements will be made in the future than have been made in the past, new units should be considered if this seems desirable, although units established in the literature should be carefully considered and given precedence.

The following appear to be desirable features to be kept in mind during any attempt to select a permeability unit :

A. It should be a practical unit chosen for maximum convenience and usefulness in laboratory and field work.

B. It should be simple as regards definition, defining equation, and dimensional relations so as to facilitate the use of either metric or English units.

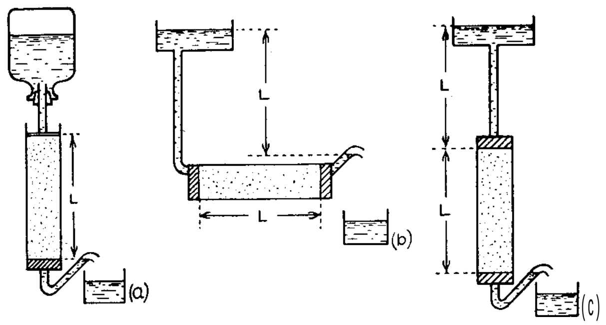
C. It is desirable to have a permeability unit that is usable for all flow cases following Darcy's law, i.e., stream line flow of liquids or gases in porous media, but it may be questioned whether this should be accomplished at the sacrifice of the first feature mentioned above.

DARCY'S LAW

The readiness with which a porous medium transmits fluids is customarily expressed by means of a permeability coefficient or transmission constant, the numerical calculation being based on an application of the Darcy law to actual flow data. The Darcy law, however, has been variously quoted and paraphrased in the literature, often incorrectly, and many different permeability units having different physical dimensions and numerical size have been used.

Darcy, on the basis of his observations on water moving vertically through horizontal beds of filter sands, summarized his findings (2)[4] as follows :

> "Ces experiences demontrent positivement que le volume d'eau qui passe à travers une couche de sable d'une nature donnée est proportionnel à la pression et en raison inverse de l'epaisseur des couches traversées; ainsi, en appelant s la superficie d'un filtre, k un coefficient d&pendant de la nature du sable, e l'épaisseur de la couche de sable, P-H, la pression sous la couche filtrante, P +H la pression atmosphérique augmentée de la hauteur d'eau sur le filtre ; on a pour le debit de ce dernier $Q = \frac{ks}{e}(H + e + H,)$, qui se réduit à $Q = \frac{ks}{e}(H + e)$ quand $H_o = o$ ou quand la pression sous le filtre est kgale au poids de. l'atmosphère."



PRESSURE GRADIENT (cgs UNITS) $\nabla p^{(a)} = 0$ $\nabla p^{(b)} = 980$ $\nabla p^{(c)} = 980$
HYDRAULIC GRADIENT $i = 1$ $i = I$ i -2
DISCHARGE VELOCITY(RELATIVE) V_a : I $V_b = 1$ v_c 2

FIG. I.-Permeameters to illustrate various flow conditions. The Mariotte bottle is used to maintain a thin layer of water over the soil for permeameter (a). The symbol L represents an arbitrary length which is the same throughout the figure.

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[3]This discussion of permeability units was prepared at the request of the Executive Committee of Section I of the Soil Science Society of America for presentation at the December, 1940, annual meeting of the Society. No independent or immediate action by the Society toward the designation of a permeability unit is contemplated. It is expected that a report summarizing the views of members of various interested societies will be prepared and published at a later date. The author has been encouraged and assisted by conversations and correspondence with various colleagues during the preparation of this paper, but must himself accept responsibility for any errors that may exist in the analysis of the problem.

'Figures in parenthesis refer to "Literature Cited", p. 53.

One interpretation of his introductory statement would be that the flow rate is proportional to the pressure difference. This, of course, is not generally true, for soil moisture flow experiments are frequently carried on in the laboratory under the condition illustrated in Fig. 1 (a), where there is zero pressure difference and zero pressure gradient. The need for including gravity effects in the flow equation at low heads is further illustrated by the permeameters (b) and (c) in Fig. 1. Assuming identical soil columns for the three cases, the pressure gradients, hydraulic gradients and relative discharge velocities are given in the figure.

It should be clear from these examples that the discharge velocity is simply related to the hydraulic gradient, but not to the pressure gradient. The final equation given in the above quotation removes any ambiguity and makes clear that both gravity and the pressure gradient force were included in Darcy's original formulation of the flow problem.

Although only the vertical direction of flow was considered by Darcy, his equation may be generalized to hold for flow in any direction as follows:

$$v=ki \qquad 1$$

In this equation the hydraulic gradient i is the change in static hydraulic head (potential head) per unit distance along the average flow line. The macroscopic or discharge velocity v is the volume of water crossing unit area in the soil in unit time and k is referred to as the Darcy coefficient of permeability.

Although considerable work has been done toward developing an equation for fluid flow through porous media based on the number and shape of the flow channels and the equation of motion of a viscous fluid, no result of general usefulness has as yet been obtained. It therefore appears that any descriptive analytic theory for fluid flow through porous media will, at least for the present, be based on an empirical relation like the Darcy law.

ESSENTIAL ELEMENTS OF FLOW EQUATIONS

Differences among the many flow equations that have been used for expressing flow in porous media may be ascribed to (a) whether the equation is for use with liquids or gases; (b) the method used for expressing the driving force; (c) the method used for taking into account the effect of viscosity on flow; or (d) the units used.

Wyckoff, Botset, Muskat, and Reed (10) have shown that for certain media the same value for permeability is obtained whether the measurement is made with a liquid or a gas, providing appropriate flow equations are used. For such media, it is usually less troublesome to use gas for the permeability determinations. For most agronomic and engineering work where it is desired to know the permeability of soils to water, the presence of hydrophilic colloids precludes the use of anything but water in the permeability measurements. Consequently this paper will be concerned primarily with the water flow case.

The driving force that moves liquids through porous media may be expressed in a number of ways. The equation of motion for a slow moving viscous liquid is ordinarily written (8) as

$$\rho\frac{dv}{dt} = \rho F - \nabla p + \eta \nabla \cdot \nabla v \qquad 2$$

where ρ is the density, F the body force per unit mass, p the pressure, η the viscosity and ∇ the gradient operator. The left-hand side of the equation is the product of the mass of unit volume and the acceleration, while the terms on the right-hand side represent the forces which determine the acceleration. For soil moisture flow, F represents the gravity force and is numerically equal to the acceleration of gravity, g. The second term represents the force arising from the pressure gradient and the third term represents the viscous retarding force. Darcy's law is simply a statement of the observed fact that for the steady viscous flow of liquids in porous media under conditions commonly met, the rate of flow of the liquid through the medium is proportional to and in the direction of the vector sum of the first two force expressions on the right side of the above equation. Dividing equation 2 by ρ would have changed the expressions from force "per unit volume" to force "per unit mass". Choosing whether driving force "per unit volume" or "per unit mass" is to be used in the flow equation is an arbitrary matter to be settled on the basis of convenience.

The relation between the hydraulic gradient and the force expressions given in equation 2 may be indicated by considering the example shown in Fig. 2.

The average driving force acting on the water between c and d may be represented in several ways. The hydraulic gradient may be taken as a dimensionless vector of such magnitude that

$$i=\frac{h}{L}i_1 \qquad 3$$

i_1 being a unit vector along the line cd.

Viscous flow of liquids in isotropic porous media

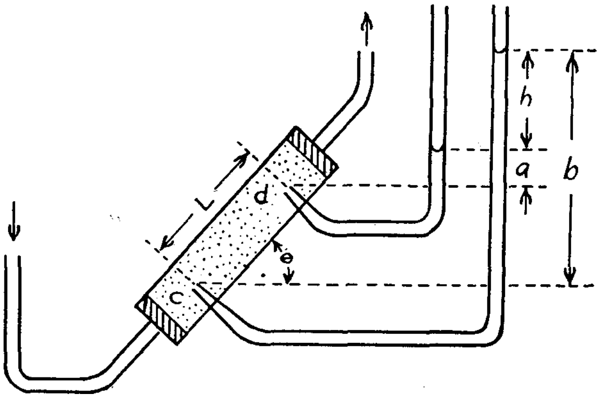

FIG. a.-Inclined permeameter to illustrate the relation of the hydraulic gradient to the vector sum of the pressure gradient and gravitational forces.

takes place in the direction of the net driving force which is the vector sum of the gravity and pressure gradient forces. Since, in the example illustrated, c and d are on a stream line, this vector sum will be equal to the sum of the components of the two forces along the line c d. On the basis of force per unit volume, this vector sum becomes

$$\rho F - \nabla p = (A+B)\,i_1 = \rho g i \quad , \qquad 4$$

where A and B are the scalor magnitudes of the components of ρF and $-\nabla p$ along the line c *d. We* have

$$A = -\rho g \sin\theta \qquad 4a$$

and

$$B = \rho g \left(\frac{b-a}{L}\right) = \rho g\,(h + a + L \sin\theta - a) = \rho g\,\frac{h}{L} + \rho g \sin\theta \qquad 4b$$

From equation 4 it follows that the relation between force per unit mass and the hydraulic gradient is

$$F - \frac{\nabla p}{\rho} = g i \qquad 5$$

To summarize then, we may say that the driving force for viscous flow of liquids through porous media may be expressed in terms of hydraulic gradient, force per unit mass, or force per unit volume, using any consistent set of units. For the special case of flow in the horizontal direction or when the pressure gradient force is large compared with gravity, it is permissible to omit the gravity term from the flow equation. Since these special conditions are not satisfied for the majority of soil moisture flow cases in the field or laboratory, we must conclude that permeability units defined in flow equations which do not include gravity in the driving force are not suitable for soils work,

VISCOSITY AND THE FLOW EQUATION

Since the viscous flow of liquids in saturated porous media is proportional to the driving force and inversely proportional to the viscosity, there has been an increasing tendency to have viscosity appear explicitly in the flow equation.

The range of values for the permeability of various soils is so large that for many engineering purposes the change in the value of k in equation 1 caused by variation in the temperature of water may be neglected. However, for precise work with water at various temperatures or for flow problems with other liquids, it seems desirable to include a viscosity factor in the flow equation so that permeability will be independent of viscosity effects.

FLOW EQUATION UNITS

Any attempt to designate a standard permeability unit inevitably involves the units to be used for expressing the various factors in the flow equation. A number of considerations may have influenced particular choices that have been made in the past. Standard usage in a particular field must be considered. For instance, in geological work gallons per day seems to be a preferred unit for expressing flow. Also, permeability values of more convenient size may be obtained by choosing large time and force units. The writer is of the opinion, however, that there is a better chance for agreement on a permeability unit for use in such diverse fields as soil mechanics, ceramics, agronomy, soil physics, geology, civil engineering, and agricultural engineering if a consistent set of units, either standard or gravitational, is used in the flow equation.

If reasonably general agreement on a permeability unit could be attained, there would be definite advantages in assigning a name to the unit. Numbers expressing measured permeability values can be adjusted to convenient numerical size by the use of prefixes such as milli or micro appended to the name of the permeability unit.

PERMEABILITY UNITS FOR SOILS

The following is a list of permeability units that appear to merit special consideration for soils use :

1. Both from the standpoint of historical significance and present usage, the permeability unit deserving first mention is the "Darcy coefficient" of equation 1. Since it has the dimensions of velocity, transformation from metric to English units is not

difficult and it is usually expressed in feet per second or cm per second.[5]

2. Dividing the right-hand side of equation 1 by η makes the permeability independent of viscosity. Such a step, however, complicates dimensional relations and makes it less convenient to transpose from cgs to English units. In effect, most users of equation 1 introduce a viscosity ratio into the flow equation at the time permeability calculations are made; thus,

$$v = k\frac{\eta_s}{\eta}i \qquad 6$$

where η_s is the viscosity of a standard liquid at a standard temperature, namely water at 68°F. This equation has advantages which are worthy of mention. Viscosity effects are taken into account. The simplicity of the dimensions and physical significance of k remain the same as for the Darcy coefficient, and the permeability constant defined in this equation is the same as that used by many workers who now convert the Darcy coefficient to the standard temperature of 68°F (20°C).

3. Taking no account of viscosity, a number of soils workers and engineers (1, 4, 5, 6, 9) have used the flow equation

$$v = kgi \equiv k\left(F - \frac{\nabla p}{\rho}\right) \equiv -k\nabla\Phi \qquad 7$$

in which the permeability constant has the dimensions of time. Introducing viscosity effects by means of a ratio

$$v = k\frac{\eta_s}{\eta}gi \qquad 8$$

maintains this dimensional simplicity. Therefore, using this equation, calculations based on experimental data expressed in any consistent set of units for length, mass, and force, will give the same numerical value for the permeability. In this respect the unit is well suited for international use. Moreover, in engineering soils work, it is often convenient to use metric units in the laboratory and English units in field work. This permeability unit has been briefly discussed elsewhere (3).

4. Looking toward generality of application of Darcy's law both for liquids and gases, there is an advantage in using force per unit volume in the flow equation. Thus

$$v = \frac{k}{\eta}(\rho F - \nabla p) \equiv \frac{k}{\eta}\rho gi \qquad 9$$

For gaseous flow where, in general, body force may be neglected, equation 9 becomes

$$v = -\frac{k}{\eta}\nabla p \qquad 10$$

which leads directly to equations given by Muskat (7) for permeability determination with gaseous flow. The permeability constant in equations 9 and 10 has the dimensions of length squared.

5. A permeability unit called the "darcy" has been proposed by Wyckoff, Botset, Muskat, and Reed (10). The flow equation they use is the same as equation 10. In accordance with their definitions, if the permeability k is to be expressed in darcys, then the macroscopic velocity must be expressed in cm sec^{-1}, the viscosity in centipoises, and the pressure gradient in atmos cm^{-1}. This equation, though, is only a special case of Darcy's law applying to horizontal flow and is not usable for most soil moisture work where gravity must be considered. To preserve the use of the "darcy" unit and include gravity, the above equation could be written

$$v = 9.87 \times 10^{-9}\frac{k}{\eta}\rho gi \equiv 9.87 \times 10^{-9}\frac{k}{\eta}(\rho F - \nabla p) \qquad 11$$

in which c g s units are used for the various quantities involved, including η. It appears that any flow equation taking gravity into account will contain a numerical constant depending on the units if the permeability is expressed in darcys.

VARIOUS MEANINGS OF *PERMEABILITY*

The term *permeability* is often used in a general sense to indicate that fluids can pass into and through certain porous media. It is also variously used to designate a quantitative specific property characteristic of each unit of volume of such media. Perhaps a logical procedure in discussing units for measuring and expressing *permeability* would be first to set forth various definitions that might be given for the exact meaning of the term. This procedure would lead directly to the question: Is it desirable to define permeability in such a way that it is a property of the porous medium alone and independent of the properties of the fluid and the acceleration of gravity?

From one commonly used meaning for permeability associated with the k in equation 1, it follows that the permeability of a given medium, let us say a certain sand, is greater for warm water than for cold, or is greater for gases than for liquids. Basing

[5]For agronomic soils work there are certain advantages in expressing macroscopic velocity or discharge velocity in inches per hour. Infiltration capacity measurements are now commonly expressed in this unit and it is simply related to irrigation and rainfall data.

a definition for permeability on equation 8 would make the unit independent of changes in the acceleration of gravity and the viscosity but not independent of the density of the liquid. For example, if the permeaters in Fig. 1 contained identical columns of a material whose pore system is not affected by water or oil, the discharge velocity in each case would be greater for water than for a petroleum oil having the same viscosity because of the difference in the density of the two liquids.

A permeability unit acceptable for general use in different lines of work must be based on a flow equation which is adequate to cover a variety of flow cases and in this connection equation 9 seems to have certain advantages.[6] The *k* in this equation could be referred to as the cgs unit of permeability or in view of the various meanings associated with permeability it might be desirable from analogy with the treatment of thermal and electrical flow in physics to designate this *k* as "fluid conductivity". There is precedence for such a name in the literature of both soil science and engineering.

If on the basis of feature A mentioned at the beginning of this paper persons doing a given type of work prefer to use their own particular permeability unit, they might be willing to publish their results in both their own unit and the standard or fundamental unit.

[6]It is entirely possible, as suggested by Anderson (3), that equation 9 will not be sufficiently general for expressing moisture movement in highly colloidal soils, especially where solution concentration gradients exist. Preliminary experiments by the writer with porous porcelain media, however, indicate that in the absence of semi-permeable membranes even large gradients in the concentration of dissolved material do not produce an appreciable force tending to cause "streaming" of the liquid through the medium. At present also it would seem equation 9 is adequate for both saturated and unsaturated flow of liquids in porous media (9).

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