Derivation of Humidty and NOx Humidty Correction Factors

(This document follows the presentations in
"Vapor Pressure Equation for Water in the Range 0 to 100 °C", by Arnold Wexler and Lewis Greenspan, February 19, 1971
JOURNAL OF RESEARCH of the National Bureau of Standards Vol. 75A, No. 3, May-June 1971

"Vapor Pressure Formulation for Water in Range 0 to 100 °C. A Revision", by Arnold Wexler, July 15, 1976
JOURNAL OF RESEARCH of the National Bureau of Standards Vol. 80A, Nos. 5 and 6, September-December 1976

"Vapor Pressure Formulation for Ice", by Arnold Wexler, September 23, 1976
JOURNAL OF RESEARCH of the National Bureau of Standards Vol. 81A, No. 1, September 23, 1976

"New Equations for Computing Vapor Pressure and Enhancement Factor", 11 May 1981 and 15 August 1981
by Arden L. Buck, 1982 Journal of Applied Meterology, 11 May 1981 and 15 August 1981.)

Input of Ambient Conditions

$Baro_{mmHg} := 750.0612$

$Drybulb_{degC} := 20$

$Dewpoint_{degC} := 10$

$Baro_{inHg} := \frac{Baro_{mmHg}}{25.4}$ $\qquad Baro_{inHg} = 29.53$

$Baro_{mb} := Baro_{inHg} \cdot 33.86389$ $\qquad Baro_{mb} = 1000$

$Drybulb_{degF} := Drybulb_{degC} \cdot \frac{9}{5} + 32$ $\qquad Drybulb_{degF} = 68$

$Drybulb_{degK} := 273.15 + \left(\frac{5}{9}\right) \cdot \left(Drybulb_{degF} - 32\right)$ $\qquad Drybulb_{degK} = 293.15$

$Dewpoint_{degF} := Dewpoint_{degC} \cdot \frac{9}{5} + 32$ $\qquad Dewpoint_{degF} = 50$

$Dewpoint_{degK} := 273.15 + \left(\frac{5}{9}\right) \cdot \left(Dewpoint_{degF} - 32\right)$ $\qquad Dewpoint_{degK} = 283.15$

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Derivation of Humidty and NOx Humidty Correction Factors

Compute Corrected Saturation Vapor Pressure for Dry Bulb Temperature above and below ice point. This value will be used to compute Relative Humidty.

The following are Wexler's most accurate formulations: Eq. (15) (Wexler, 1976) for vapor pressure over water and Eq. (5) (Wexler, 1977) for vapor pressure over ice. Wexler's formulations are given below. These calculations are most clearly presented in Buck's Eqs. (5a) and (5b) (Buck, 1981).

Absolute Temperature in K

$\theta := Drybulb_{degK}$ $\qquad \theta = 293.15$

Vapor pressure over water

$$e_w := 0.01\cdot\exp\left[2.858487\cdot\ln(\theta) + \frac{-2991.2729}{\theta^2} + \frac{-6017.0128}{\theta} + 18.87643854 + -0.028354721\cdot\theta + 1.7838301\cdot10^{-5}\cdot\theta^2 + -8.4150417\cdot10^{-10}\cdot\theta^3 + 4.4412543\cdot10^{-13}\cdot\theta^4\right]$$

$e_w = 23.3854$

Vapor pressure over ice

$$e_i := 0.01\cdot\exp\left[0.69186510\cdot\ln(\theta) + \frac{-5865.3696}{\theta} + 22.24103300 + 1.3749042\cdot10^{-2}\cdot\theta + -3.4031775\cdot10^{-5}\cdot\theta^2 + 2.6967687\cdot10^{-8}\cdot\theta^3\right]$$

$e_i = 28.3145$

The following will select the appropriate derived vapor pressure for the temperature range.

$PsatDry_{mb} := if\left(Drybulb_{degC} \le 0, e_i, e_w\right)$ $\qquad PsatDry_{mb} = 23.3854$

Wexler's formulations relate the saturation vapor pressure of pure water to temperature; a slight correction must be made when one is dealing with moist air rather than pure water vapor. This correction, called the enhancement factor, is a weak function of temperature and pressure. These calculations are in Buck's Eq. (6) and Table 3 (Buck, 1981).

Baormetric Pressure in millibars

$P := Baro_{mb}$ $\qquad P = 1000$

Enhancement for vapor pressure over water.

$f_w := 1 + 0.00041 + P\cdot\left[3.48\cdot10^{-6} + 7.4\cdot10^{-10}\cdot\left[(\theta - 273.15) + 30.6 - 0.038\cdot P\right]^2\right]$ $\qquad f_w = 1.004$

Enhancement for vapor pressure over ice.

$f_i := 1 + 0.00048 + P\cdot\left[3.47\cdot10^{-6} + 5.9\cdot10^{-10}\cdot\left[(\theta - 273.15) + 23.8 - 0.031\cdot P\right]^2\right]$ $\qquad f_i = 1.004$

The following will select the appropriate vapor pressure enhancement for the temperature range.

$EnhanceDry := if\left(Drybulb_{degC} \le 0, f_i, f_w\right)$ $\qquad EnhanceDry = 1.004$

Apply the correction to finally obtain the enhanced partial saturation vapor pressure.

$PsatDryCorrected_{mb} := PsatDry_{mb}\cdot EnhanceDry$ $\qquad PsatDryCorrected_{mb} = 23.4792$

$PsatDryCorrected_{inHg} := \dfrac{PsatDryCorrected_{mb}}{33.86389}$ $\qquad PsatDryCorrected_{inHg} = 0.693$

Derivation of Humidty and NOx Humidty Correction Factors

Compute Corrected Saturation Vapor Pressure for Dew Point Temperature above and below ice point. This value will be used in Relative and Specific Humidity derivations.

The following are Wexler's most accurate formulations: Eq. (15) (Wexler, 1976) for vapor pressure over water and Eq. (5) (Wexler, 1977) for vapor pressure over ice. Wexler's formulations are given below. These calculations are most clearly presented in Buck's Eqs. (5a) and (5b) (Buck, 1981).

Absolute Temperature in K

$\theta := Dewpoint_{degK}$ $\qquad \theta = 283.15$

Vapor pressure over water

$$e_w := 0.01 \cdot \exp\left[2.858487 \cdot \ln(\theta) + \frac{-2991.2729}{\theta^2} + \frac{-6017.0128}{\theta} + 18.87643854 + -0.028354721 \cdot \theta + 1.7838301 \cdot 10^{-5} \cdot \theta^2 + -8.4150417 \cdot 10^{-10} \cdot \theta^3 + 4.4412543 \cdot 10^{-13} \cdot \theta^4\right]$$

$e_w = 12.2794$

Vapor pressure over ice

$$e_i := 0.01 \cdot \exp\left[0.69186510 \cdot \ln(\theta) + \frac{-5865.3696}{\theta} + 22.24103300 + 1.3749042 \cdot 10^{-2} \cdot \theta + -3.4031775 \cdot 10^{-5} \cdot \theta^2 + 2.6967687 \cdot 10^{-8} \cdot \theta^3\right]$$

$e_i = 13.5203$

The following will select the appropriate derived vapor pressure for the temperature range.

$PsatDew_{mb} := if\left(Dewpoint_{degC} \le 0, e_i, e_w\right)$ $\qquad PsatDew_{mb} = 12.2794$

Wexler's formulations relate the saturation vapor pressure of pure water to temperature; a slight correction must be made when one is dealing with moist air rather than pure water vapor. This correction, called the enhancement factor, is a weak function of temperature and pressure. These calculations are in Buck's Eq. (6) and Table 3 (Buck, 1981).

Baormetric Pressure in millibars

$P := Baro_{mb}$ $\qquad P = 1000$

Enhancement for vapor pressure over water.

$f_w := 1 + 0.00041 + P \cdot \left[3.48 \cdot 10^{-6} + 7.4 \cdot 10^{-10} \cdot \left[(\theta - 273.15) + 30.6 - 0.038 \cdot P\right]^2\right]$ $\qquad f_w = 1.004$

Enhancement for vapor pressure over ice.

$f_i := 1 + 0.00048 + P \cdot \left[3.47 \cdot 10^{-6} + 5.9 \cdot 10^{-10} \cdot \left[(\theta - 273.15) + 23.8 - 0.031 \cdot P\right]^2\right]$ $\qquad f_i = 1.004$

The following will select the appropriate vapor pressure enhancement for the temperature range.

$EnhanceDew := if\left(Dewpoint_{degC} \le 0, f_i, f_w\right)$ $\qquad EnhanceDew = 1.0039$

Apply the correction to finally obtain the enhanced partial saturation vapor pressure.

$PsatDewCorrected_{mb} := PsatDew_{mb} \cdot EnhanceDew$ $\qquad PsatDewCorrected_{mb} = 12.3272$

$PsatDewCorrected_{inHg} := \frac{PsatDewCorrected_{mb}}{33.86389}$ $\qquad PsatDewCorrected_{inHg} = 0.364$

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Derivation of Humidty and NOx Humidty Correction Factors

Compute Relative Humidity, Specific Humidity and NOx Humidity Corrections Factors per 40 CFR 86.144, 86.344, 86.544, and 86.1342

$$RelativeHumidity_{pct} := 100 \cdot \frac{PsatDewCorrected_{inHg}}{PsatDryCorrected_{inHg}} \qquad RelativeHumidity_{pct} = 52.503$$

The following equations have the Dry Bulb Vapor Pressure factored out and are presented in English Units..

$$SpecificHumidity_{grperlb} := \frac{4347.8 \cdot PsatDewCorrected_{inHg}}{Baro_{inHg} - PsatDewCorrected_{inHg}} \qquad SpecificHumidity_{grperlb} = 54.265$$

$$NOxCorrection_{gasoline} := \frac{1}{\left[1 - 0.0047 \cdot \left(SpecificHumidity_{grperlb} - 75\right)\right]} \qquad NOxCorrection_{gasoline} = 0.9112$$

86.1342 for Diesel Engines

$$NOxCorrection_{diesel} := \frac{1}{\left[1 - 0.0026 \cdot \left(SpecificHumidity_{grperlb} - 75\right)\right]} \qquad NOxCorrection_{diesel} = 0.9488$$

86.344 Calcualtion of Specific Humidity - SI Units

$$Ra := RelativeHumidity_{pct}$$

Grams of H2O per gram of dry air

$$K := 0.6220$$

$$P_a := Baro_{mb} \cdot 100$$

$$P_v := PsatDewCorrected_{mb} \cdot 100$$

Specific Humidity, Grams H2) per gram of dry air

$$H := \frac{K \cdot P_v}{P_a - P_v} \qquad H = 0.00776$$

86.144 and 86.1342 SI Unts

$$SpecificHumidity_{grperkg} := \frac{6.211 \cdot RelativeHumidity_{pct} \cdot PsatDryCorrected_{mb}}{Baro_{mb} - PsatDryCorrected_{mb} \cdot \frac{RelativeHumidity_{pct}}{100}} \qquad SpecificHumidity_{grperkg} = 7.752$$

$$NOxCorrection_{gasoline} := \frac{1}{\left[1 - 0.0329 \cdot \left(SpecificHumidity_{grperkg} - 10.71\right)\right]} \qquad NOxCorrection_{gasoline} = 0.9113$$

86.1342 for Diesel Engines

$$NOxCorrection_{diesel} := \frac{1}{\left[1 - 0.0182 \cdot \left(SpecificHumidity_{grperkg} - 10.71\right)\right]} \qquad NOxCorrection_{diesel} = 0.9489$$

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Derivation of Humidty and NOx Humidty Correction Factors

The follwing derivations support plotting of vapor pressure, humidity and NOx correction factors.

$i := 0 .. 90$

$t_i := (i - 40) + 273.15$

$$e_i := \text{if}\left[t_i > 273.15,\ \left[2.858487 \cdot \ln(t_i) + \frac{-2991.2729}{(t_i)^2} + \frac{-6017.0128}{t_i} + 18.87643854 + -0.028354721 \cdot t_i + 1.7838301 \cdot 10^{-5} \cdot (t_i)^2 + -8.4150417 \cdot 10^{-10} \cdot (t_i)^3 + 4.4412543 \cdot 10^{-13} \cdot (t_i)^4\right],\ 0\right]$$

$$e_i := \text{if}\left[t_i > 273.15,\ e_i,\ \left[0.69186510 \cdot \ln(t_i) + \frac{-5865.3696}{t_i} + 22.24103300 + 1.3749042 \cdot 10^{-2} \cdot t_i + -3.4031775 \cdot 10^{-5} \cdot (t_i)^2 + 2.6967687 \cdot 10^{-8} \cdot (t_i)^3\right]\right]$$

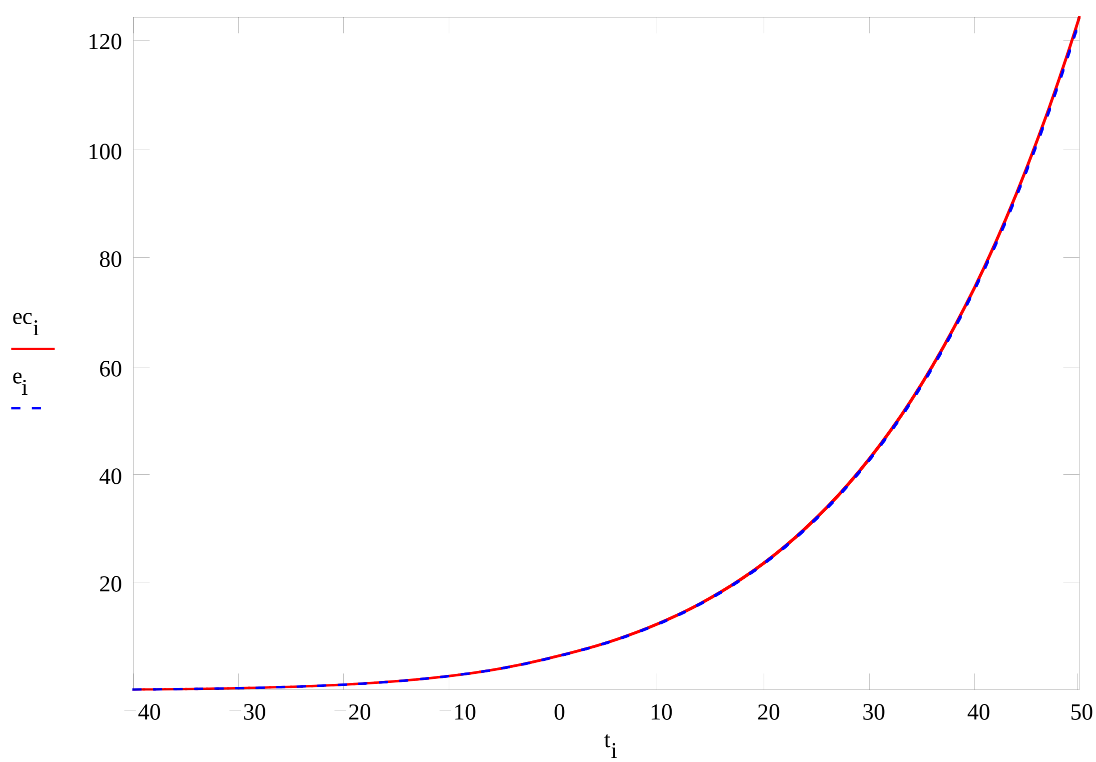
$e_i := 0.01\ \exp(e_i)$

$$f_i := \text{if}\left[t_i > 273.15,\ 1 + 0.00041 + P \cdot \left[3.48 \cdot 10^{-6} + 7.4 \cdot 10^{-10} \cdot \left[(t_i - 273.15) + 30.6 - 0.038 \cdot P\right]^2\right],\ 0\right]$$

$$f_i := \text{if}\left[t_i > 273.15,\ f_i,\ 1 + 0.00048 + P \cdot \left[3.47 \cdot 10^{-6} + 5.9 \cdot 10^{-10} \cdot \left[(t_i - 273.15) + 23.8 - 0.031 \cdot P\right]^2\right]\right]$$

$ec_i := e_i \cdot f_i$

$t_i := t_i - 273.15$



Corrected and Uncorrected Partial Saturation Vapor Pressure -40 to 50 °C

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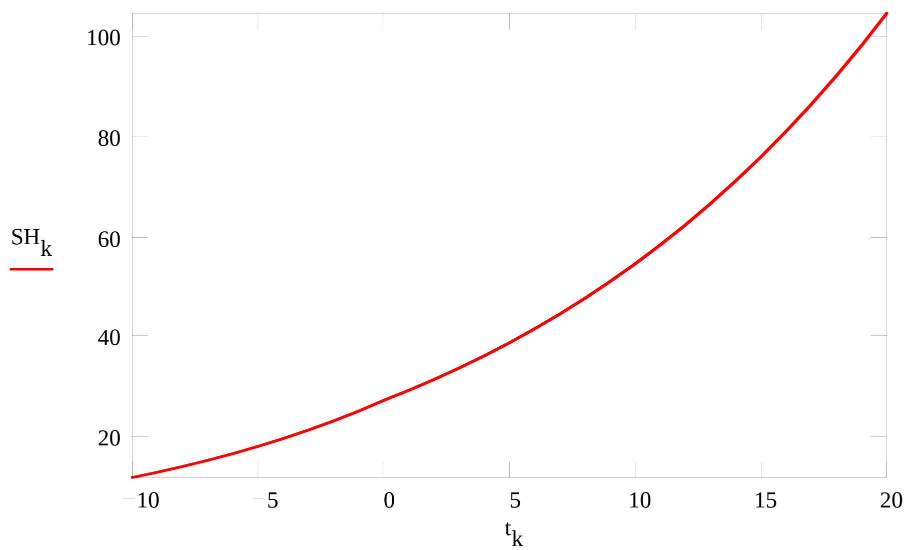
Derivation of Humidty and NOx Humidty Correction Factors

$$SH_i := \frac{4347.8 \cdot \frac{ec_i}{33.86389}}{Baro_{inHg} - \frac{ec_i}{33.86389}}$$

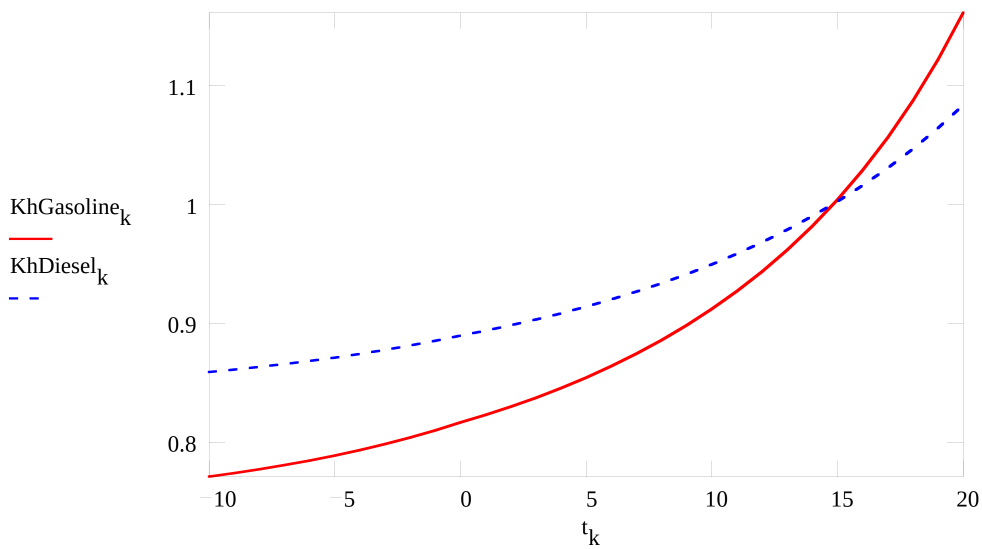
$k := 30 .. 60$



Specific Humidy from -10 through 20 °C

$$KhGasoline_i := \frac{1}{\left[1 - 0.0047 \cdot \left(SH_i - 75\right)\right]}$$

$$KhDiesel_i := \frac{1}{\left[1 - 0.0026 \cdot \left(SH_i - 75\right)\right]}$$



Oxides of Nitrogen Correction Factors from -10 through 20 °C

Derivation of Humidty and NOx Humidty Correction Factors

The following may be compared to Buck, 1981, Table 1 to verify agreement.
(40 is added to each subscript to bring it into a useable range of values.)

Vapor pressure pure water from -40 through 50 °C

$e_{-40+40} = 0.128486$

$e_{-30+40} = 0.380239$

$e_{-20+40} = 1.032761$

$e_{-10+40} = 2.599229$

$e_{0+40} = 6.111536$

$e_{10+40} = 12.279396$

$e_{20+40} = 23.385445$

$e_{30+40} = 42.45202$

$e_{40+40} = 73.812731$

$e_{50+40} = 123.447791$

Enhancement factor @ 1000 mb from -40 through 50 °C

$f_{-40+40} = 1.005264$

$f_{-30+40} = 1.004766$

$f_{-20+40} = 1.004387$

$f_{-10+40} = 1.004125$

$f_{0+40} = 1.003981$

$f_{10+40} = 1.003895$

$f_{20+40} = 1.004007$

$f_{30+40} = 1.004268$

$f_{40+40} = 1.004676$

$f_{50+40} = 1.005233$

Vapor pressure of moist air at 1000 mb from -40 through 50 °C

$ec_{-40+40} = 0.129163$

$ec_{-30+40} = 0.382051$

$ec_{-20+40} = 1.037291$

$ec_{-10+40} = 2.60995$

$ec_{0+40} = 6.135863$

$ec_{10+40} = 12.327225$

$ec_{20+40} = 23.479161$

$ec_{30+40} = 42.633204$

$ec_{40+40} = 74.157912$

$ec_{50+40} = 124.093784$

Derivation of Humidty and NOx Humidty Correction Factors

The follwing derivations support plotting of vapor pressure, humidity and NOx correction factors.

$i := 0 .. 90$

$t_i := (i - 40) + 273.15$

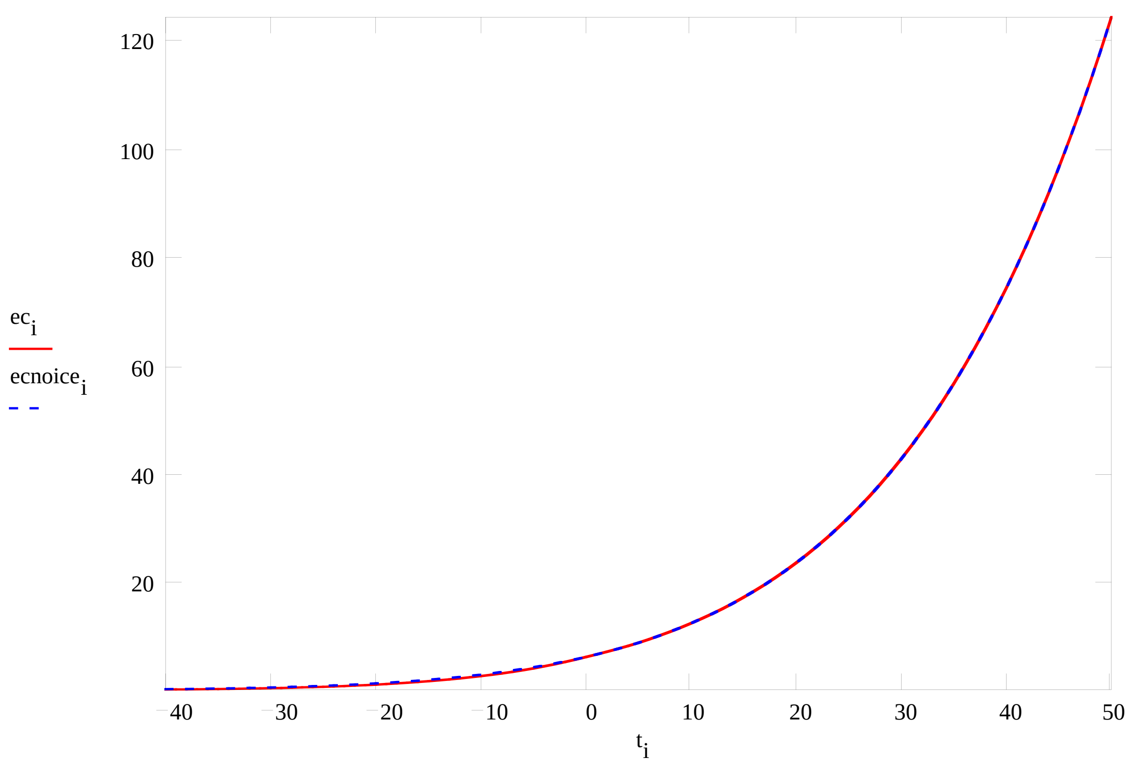
$$enoice_i := \mathrm{if}\left[t_i > 0,\ \left[2.858487 \cdot \ln(t_i) \ldots + \frac{-2991.2729}{(t_i)^2} + \frac{-6017.0128}{t_i} + 18.87643854 + -0.028354721 \cdot t_i \ldots + 1.7838301 \cdot 10^{-5} \cdot (t_i)^2 + -8.4150417 \cdot 10^{-10} \cdot (t_i)^3 \ldots + 4.4412543 \cdot 10^{-13} \cdot (t_i)^4\right],\ 0\right]$$

$enoice_i := 0.01\ \exp(enoice_i)$

$$fnoice_i := \mathrm{if}\left[t_i > 0,\ 1 + 0.00041 + P \cdot \left[3.48 \cdot 10^{-6} + 7.4 \cdot 10^{-10} \cdot \left[(t_i - 273.15) + 30.6 - 0.038 \cdot P\right]^2\right],\ 0\right]$$

$ecnoice_i := enoice_i \cdot fnoice_i$

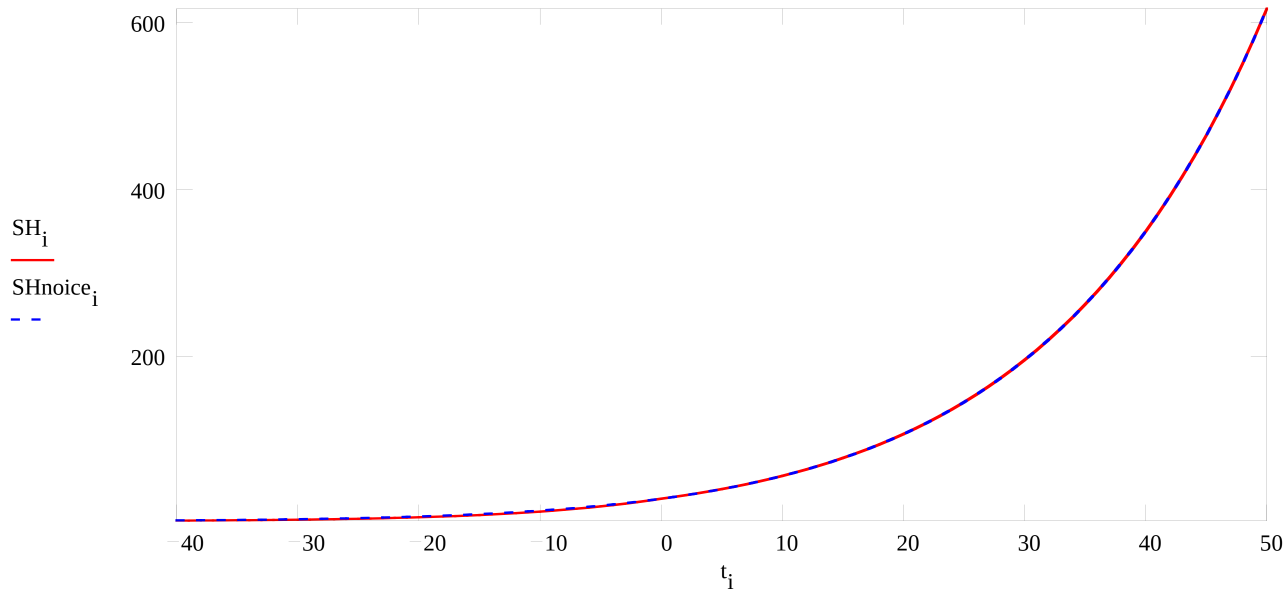
$t_i := t_i - 273.15$



Ice Point and No Ice Point Partial Saturation Vapor Pressure -40 to 50 °C

Derivation of Humidty and NOx Humidty Correction Factors

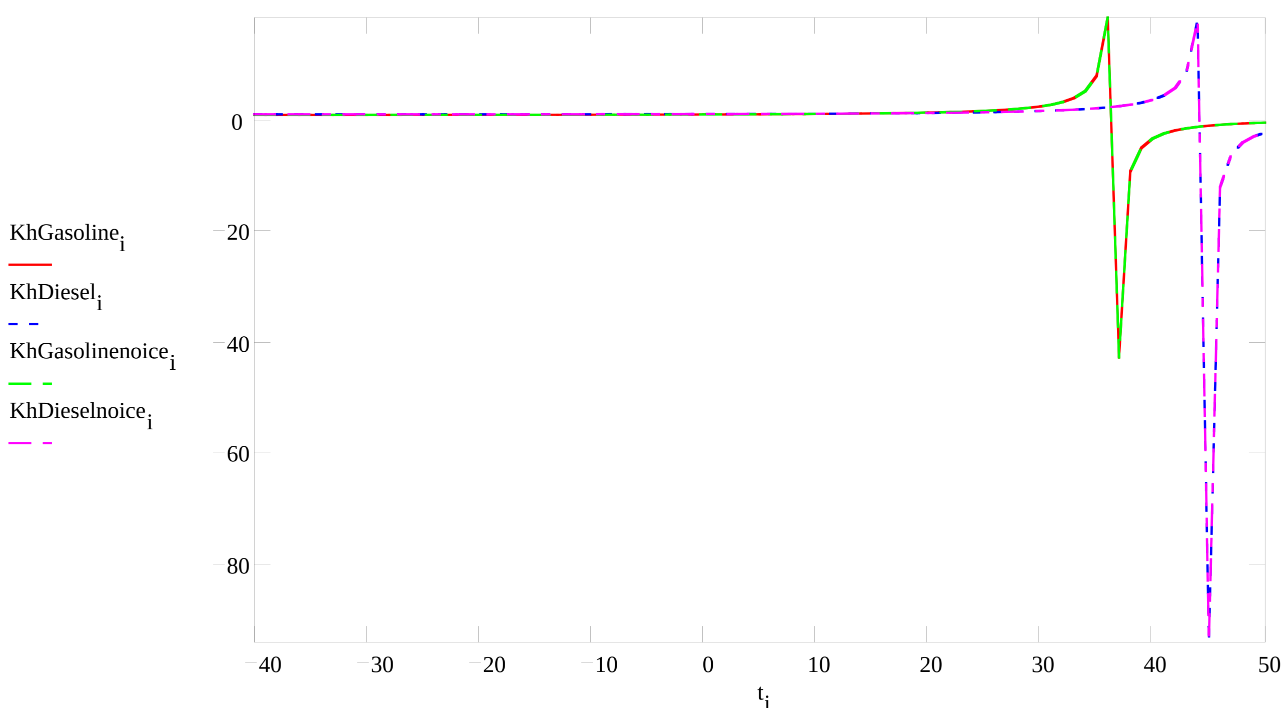
$$SHnoice_i := \frac{4347.8 \cdot \frac{ecnoice_i}{33.86389}}{Baro_{inHg} - \frac{ecnoice_i}{33.86389}}$$



Specific Humidy from -40 through 50 °C

$$KhGasolinenoice_i := \frac{1}{\left[1 - 0.0047 \cdot \left(SHnoice_i - 75\right)\right]}$$

$$KhDieselnoice_i := \frac{1}{\left[1 - 0.0026 \cdot \left(SHnoice_i - 75\right)\right]}$$



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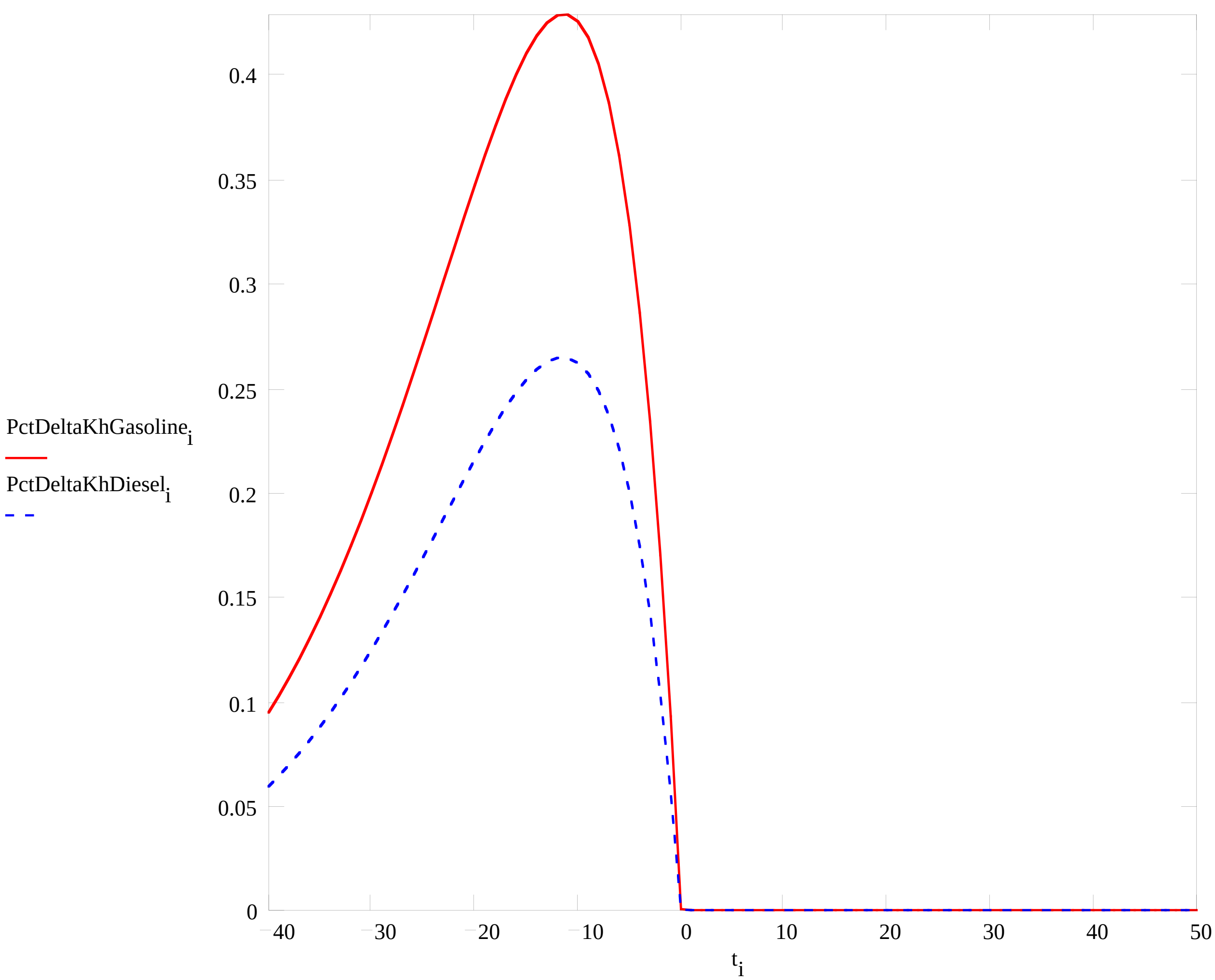
Derivation of Humidty and NOx Humidty Correction Factors

Oxides of Nitrogen Correction Factors from -40 through 50 °C

$$PctDeltaKhGasoline_i := 100 \cdot \frac{\left(KhGasolinenoice_i - KhGasoline_i\right)}{KhGasoline_i}$$

$$PctDeltaKhDiesel_i := 100 \cdot \frac{\left(KhDieselnoice_i - KhDiesel_i\right)}{KhDiesel_i}$$



Percent Difference Between NOx Factors Computed with and without Ice Point Saturation Vapor Preseure Equations

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