

# FHWA Workshop over the Web for Travel Model Development

## Session 1 Homework

### Multinomial Logit Mode Choice Model

In this exercise, a multinomial logit mode choice model will be applied (without a computer!) using the attached sheet. As a reminder, in the multinomial logit model, the probability of choosing an alternative is given by the formula:

$$P(\text{alt } j) = \frac{\exp(V_j)}{\sum_j \exp(V_j)}$$

where:

$V_j$  = Deterministic component of utility of alternative  $j$

exp = exponential function ( $e^x$ )

The utility function  $V_j$  is given by:

$$V_j = B_{0j} + B_{1j} X_{1j} + B_{2j} X_{2j} + \dots + B_{nj} X_{nj}$$

where:

$V_j$  = Utility (deterministic component) of alternative  $j$

$X_{kj}$  = Attributes ( $k = 1, n$ ) for alternative  $j$

$B_{kj}$  = Estimated coefficients for attribute  $k$  for alternative  $j$

$B_{0j}$  = Alternative-specific constant for alternative  $j$

In this model there are four modes: drive alone, shared ride, bus, and rail. The utility equation for each mode is:

Utility for mode  $i$  = - 0.03 (in vehicle time, minutes) - 0.06 (out of vehicle time, minutes)  
- 0.400 (cost, dollars) + constant for mode  $i$

The alternative-specific constants for the four modes are:

Drive alone constant	0.000
Shared ride constant	-0.400
Bus constant	-0.900
Rail constant	-0.600

The input data for the model application are shown in the following table:

	<b>Drive Alone</b>	<b>Shared Ride</b>	<b>Bus</b>	<b>Rail</b>
In vehicle time (minutes)	15	20	30	15
Out of vehicle time (minutes)	8	10	20	20
Cost (dollars)	\$2.00	\$1.00	\$1.00	\$1.50

On the following sheet, each column is used to fill in components of the utility for one of the modes. Under the “Variables” section, in each box fill in the value of the variable  $X_{kj}$ . For example, for drive alone, the value of in-vehicle time is 15 minutes, from the above table.

Under the “utility calculation” section, in each box fill in the utility component  $B_{kj} X_{kj}$  (the product of the coefficient  $B_{kj}$  and the value of the variable  $X_{kj}$ ). For example, for drive alone, the in-vehicle time utility component = -0.45 (15 minutes times the in-vehicle time coefficient of -0.03).

The “total utility” row for each mode should be the sum of the utility components in the utility calculation section for the mode. The exponentiated utility in the next row is the exponent ( $e^x$ ) of this value. In the total column for this row, sum the exponentiated utilities for all modes.

The probability of mode in the next row is the exponentiated utility for each mode divided by the sum of the exponentiated utilities. These should sum to 1.0.

## Multinomial Logit Model Calculations

Calculations	Drive alone	Shared ride	Bus	Rail	Total
<i>Variables</i>					
In-vehicle time	15				
Out of vehicle time					
Cost					
<i>Utility calculation</i>					
In-vehicle time	-0.45				
Out of vehicle time					
Cost					
Constant					
<i>Total utility</i>					
exp(utility)					
Probability of mode					

## Answer Sheet

Calculations	Drive alone	Shared ride	Bus	Rail	Total
<i>Variables</i>					
In-vehicle time	15	20	30	15	
Out of vehicle time	8	10	20	20	
Cost	\$2.00	\$1.00	\$1.00	\$1.50	
<i>Utility calculation</i>					
In-vehicle time	-0.45	-0.60	-0.90	-0.45	
Out of vehicle time	-0.48	-0.60	-1.20	-1.20	
Cost	-0.80	-0.40	-0.40	-0.60	
Constant	0	-0.40	-0.90	-0.60	
<i>Total utility</i>	-1.73	-2.00	-3.40	-2.85	
exp(utility)	0.177	0.135	0.033	0.058	0.404
Probability of mode	43.9%	33.5%	8.3%	14.3%	100%