Anatomy of price change

The commodity substitution effect in CPI data, 1982–91

Recent analysis of the substitution effects in CPI-like Laspeyres measures of price change indicate small differences across index number formulas

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The Consumer Price Index (CPI) is a measure of the average price change of a fixed market basket of goods and services purchased by the average household. The market basket contains a sample of items—food, clothing, shelter, fuels, and other goods and services—that people buy for day-to-day living. While it is often referred to as a "cost-of-living index" (CLI), and indeed was so titled until 1945, the CPI is not in general a CLI. A CLI is defined as the ratio of the minimum expenditure required to attain a particular level of satisfaction in two price situations; a comparison period and the base period.[1] The CPI, a modified Laspeyres index, holds the standard of living constant (in the span between major revisions) by keeping quantities fixed, but allows prices to vary. In addition, over an extended period of time the CPI is a chain index, because the expenditure pattern of consumers is updated and sequentially linked into the index at approximately 10-year intervals.[2] The restriction imposed on the CPI, by keeping the quantities fixed and not allowing substitution among goods in response to relative price change, results in a substitution effect,[3] or a divergence between the CPI (or any other index with fixed quantity weights) and the CLI. In the case of a Laspeyres index, the effect is such that it is greater than or equal to the true cost of living. Indeed, as is well known a Laspeyres index is an upper bound to the true CLI.

Prior research

Several empirical studies have examined the extent of the substitution effect between a Laspeyres measure and the CLI. On an aggregate level, these include studies by Steven D. Braithwait[4], Mary F. Kokoski[5], and Marilyn E. Manser and Richard J. McDonald.[6] Braithwait estimated the complete set of demand equations corresponding to three specifications of preferences, the linear expenditure system[7] the generalized linear expenditure system,[8] and the indirect addilog.[9] Then, he measured the substitution effect of a Laspeyres index against the CLI's calculated for each of these consumer demand systems. In his study, Braithwait utilized annual price and quantity data on personal consumption expenditures from the National Income and Product Accounts for 53 commodities. He found that for the 15-year period, 1958–73, the Laspeyres index overstated the CLI by 1.5 percent, or about 0.1 percent a year. Later studies utilized the theoretical results of W. Erwin Diewert,[10] who showed that there exists a class of index numbers allowing for substitution, termed "superlative," which could be calculated using only price and quantity data. The study by Kokoski, which was primarily designed to measure group specific price and cost-of-living indexes, also yielded an estimate of the substitution effect. Utilizing data from the 1972 and 1980 Surveys of Consumer Expenditures for 54 components, Kokoski estimated the aggregate substitution effect of the Laspeyres index between these periods to be 1.3 percent of the index level or 0.16 percent per year. The fixed-base price indexes—Laspeyres and Paasche—were based on CPI data and the CLI measures were indexes of the Tornqvist and Fisher type.[11] The CLI measures were derived without estimation of the complete set of demand equations by utilizing

Diewert's results on exact and superlative indexes. The definitive study on the substitution effect is the work by Manser and McDonald, which covered from the 1959–85 period and utilized data on personal consumption expenditures for 101 commodities. Fixed-base Laspeyres and Paasche indexes were constructed and compared with the superlative indexes of the Tornqvist and Fisher type. The amount of the substitution effect was estimated for both fixed base and chained specifications over selected time intervals. For the entire sample period, the Laspeyres index overstates the CLI by 15.7 index points or about 5 percent of the index level, when compared with the fixed-base Tornqvist measure. This resulted in an estimate of the substitution effect of 0.19 percent per year. In addition to the numerical estimate of substitution effect, the Manser and McDonald study established the upper and lower bounds for the amount of substitution effect in the Laspeyres index—between 0.14 percent and 0.22 percent per year in the 1959–85 period. The study also added support for the contention that disaggregated data were necessary to measure the amount of the substitution effect and that the magnitude of the effect in the Laspeyres index was positively related to the amount of inflation and the length of the period covered.

While previous studies examine data from an earlier time period (pre-1985), there is interest in examining the substitution effect issue using more recent data. Furthermore, the data used in previous studies (except Kokoski) is personal consumption expenditure data at the national level provided by the Bureau of Economic Analysis. In contrast, this article examines the substitution effect issue using expenditure data from the Bureau of Labor Statistics Consumer Expenditure Survey, the source used for the official CPI. Consumer expenditure data include a finer level of disaggregation in the commodity classes as well as geographic detail than do data from the Bureau of Economic Analysis, and also allows the different formulas to be applied at a lower level of aggregation. The importance of allowing for interarea differences in price trends has been documented in recent studies by Brent R.Moulton[12] and Diane F. Primont and Mary F .Kokoski.[13] Finally, the time period covered in the analysis is 1982–91 which allows an assessment of the potential substitution effect in more recent years.

Exhibit 1. Comparison of the formulas used to measure price change

Fixed-base index number formulas

Laspeyres: $I^L_{t,o} = \sum_{i=1,n} P^t_i X^o_i / \sum_{i=1,n} P^o_i X^o_i$
$= \sum_{i=1,n} [RI^o_i \; (P^t_i / P^o_i)\;]$

where: $RI^s_i = P^s_i X^s_i / \sum_{i=1,n} P^s_i X^s_i$

Paasche: $I^P_{t,o} = \sum_{i=1,n} P^t_i X^t_i / \sum_{i=1,n} P^o_i X^t_i$
$= 1/\sum_{i=1,n} RI^t_i \; (P^o_i / P^t_i)$

Fisher Ideal: $I^F_{t,o} = [I^L_{t,o} \times I^P_{t,o}\;]^{1/2}$

Tornqvist: $I^T_{t,o} = \prod_i [P^t_i / P^o_i\;]^{w^t_i}$

where: $w^t_i = (^1/_2) RI^o_i + (^1/_2)(RI^t_i)$

Chained index number formulas

Laspeyres: $CI^L_{t,o} = \prod_{s=o,t-1} [\; \sum_{i=1,n} [RI^s_i \; (P^{s+1}_i / P_i\;)\;]]$

Paasche: $CI^P_{t,o} = \prod_{s=o,t-1} [1/[\; \sum_{i=1,n} [RI^{s+1}_i (P^s_i / P^{s+1}_i\;)\;]]]$

Fisher Ideal: $CI^F_{t,o} = [CI^L_{t,o} \times CI^P_{t,o}\;]^{1/2}$

Tornqvist: $CI^T_{t,o} = \prod_{s=o,t-1} [\prod_i [P^{s+1}_i / P^s_i\;]^{w^s_i}$

where: $w^s_i = (^1/_2) RI^o_i + (^1/_2)(RI^s_i)$

NOTE: Subscripts *i* denote each of *n* (number of cells) commodity/geographic area classes.

Constructing the measures

As noted, in this study, estimates of the substitution effect are based on data from the annual Survey of Consumer Expenditures and the CPI. These estimates have been calculated at the lowest level of detail consistent with the CPI classification structure and data collected from the Consumer Expenditure Survey—207-item strata. The CPI has a two-tiered weighting structure,[14] the basic weights (derived from the Point of Purchase Survey), and the expenditure weights (derived from the Consumer Expenditure Survey data). (The effect of the functional form on the calculation of the price relatives at the elementary aggregate level, the area in which substitution is most likely to be observed, is discussed by Brent Moulton on pages 13–24, this issue.) The analysis of the substitution effect utilizes the price relatives derived by the official CPI procedure, that is, a base-weighted or Laspeyres-type calculation, and examines the possibility of substitution effect in the upper tier of aggregation. (The appendix details the sources and calculation of data.)

As in previous studies, the measures of the substitution effect are derived by comparing Laspeyres measures of price change to the superlative indexes: the Fisher Ideal and Tornqvist measures. To construct the Fisher Ideal and Tornqvist indexes, as measures of the true cost of living, one

Table 1. **Long-term measures of price change in fixed-base and chained indexes, 1982–91**

Year	Laspeyres	Fisher	Tornqvist	Paasche	Substitution effect
Fixed weight:					
1982	100.0	100.0	100.0	100.0	0.0
1983	103.8	103.7	103.7	103.5	.2
1984	108.1	107.9	107.8	107.6	.2
1985	112.5	112.1	112.1	111.7	.4
1986	114.4	113.6	113.6	112.9	.8
1987	119.6	118.7	118.7	117.8	.9
1988	125.0	123.7	123.7	122.5	1.3
1989	130.9	129.3	129.4	127.7	1.6
1990	139.2	137.3	137.0	135.5	2.2
1991	143.8	141.3	141.2	138.9	2.6
Chained:					
1982	100.0	100.0	100.0	100.0	0.0
1983	103.8	103.7	103.7	103.5	.1
1984	108.1	107.8	107.8	107.5	.3
1985	112.6	112.1	112.1	111.6	.5
1986	114.8	113.9	113.9	113.0	.9
1987	120.0	118.8	118.8	117.5	1.2
1988	125.4	123.7	123.7	122.0	1.7
1989	131.2	129.2	129.2	127.1	2.0
1990	139.3	136.9	136.6	134.6	2.7
1991	144.3	141.3	140.9	138.5	3.4

may calculate either fixed-based or chained indexes. This article uses both indexes because each is designed to answer different questions,[15] and also because it is interesting to know the effect of various formulas on both long-and short-term measures of price change.

Fixed-base index formulas. Exhibit 1 presents formulas for measuring price change from some base period (o) to period t where the prices and quantities are indexed by i to represent each commodity/geographic cell. Note that the difference between a Paasche and Laspeyres index formula lies in which period expenditure weights are used to aggregate over commodities: the Laspeyres uses the base period weights while the Paasche uses weights for the current period.

The popularity of the Laspeyres measure is partly because it does not have the data requirements of other formulas. To illustrate, note that to calculate a Laspeyres index, expenditure data are required only for the base period. Calculation of the index in subsequent years requires only relative prices [P^{s+1} / P^s]. For the other formulas in exhibit 1, expenditure data (as well as the price relatives) must be collected each period.

Chained index formulas. One interpretation of the fixed-base measures is that they are obtained first by "chaining" period-to-period price relatives within each cell to obtain a cell-specific measure of price change from the base period o to period t (P^t / P^o) and then aggregating over cells to obtain an index. That is, the month-to-month price relatives from period o to period t are "chained" using the following formula:

$$P_i^t / P_i^o = \prod_{s=o,t-1} (P^{s+1}_i / P_i^s)$$
$$= (P^{o+1}_i / P_i^o) (P^{o+2}_i / P^{o+1}_i) \ldots (P_i^t / P^{t-1}_i).$$

Given this long-term price relative, a fixed-base index is obtained by aggregating over cells using the formulas in exhibit 1. The salient feature of the fixed-base approach is that chaining is done within the disaggregate cells followed by aggregation over cells using the expenditure weights.

$$RI_i^s = P_i^s X_i^s / \sum_{i=1,n} (P_i^s X_i^s) .$$

Another approach reverses the order of chaining and aggregation. The chaining method first aggregates period-to-period price relatives (P^{s+1}_i / P_i^s) over all cells for each intermediate period to form an individual period index, and then chains these intermediate period indexes to obtain a measure of long-term price change. For example, the Laspeyres period-to-period price index is written as:

$$I^L_{s+1,s} = \sum_{i=1,n} [RI_i^s (P^{s+1}_i / P_i^s)],$$

where n equals the number of cells. Given one period-to-period Laspeyres index for each intermediate period from time o to time t, the chained Laspeyres measure of price change over the period is obtained by "chaining" the intermediate period indexes as follows:

$$I^L_{t,o} = \prod_{s=o,t-1} I^L_{s+1,s} .$$

Therefore, the difference between the fixed-base and chained indexes lies in that the fixed-base

Chart 1. **Comparison of Laspeyres, Paasche, and Tornqvist fixed-weight indexes, 1982–91**

[1982=100]

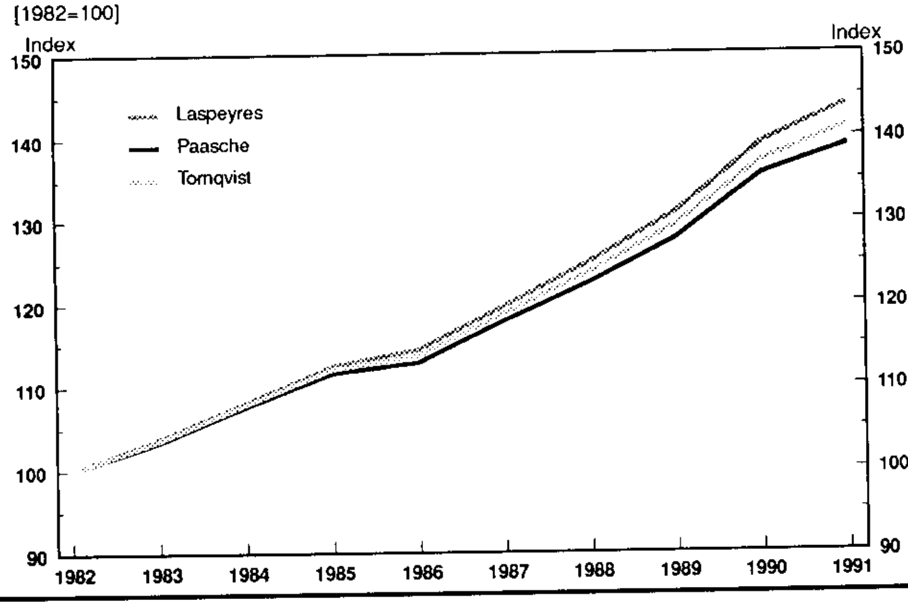
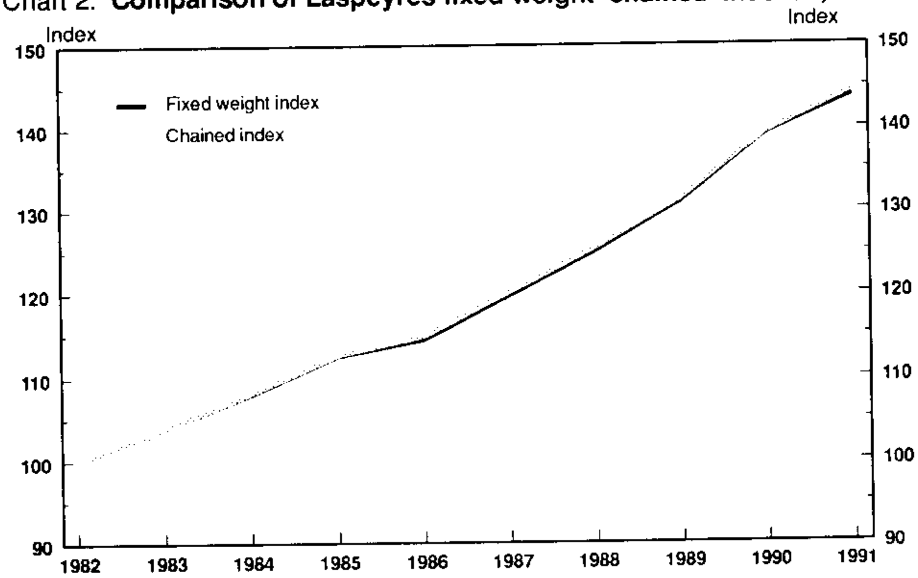


Chart 2. **Comparison of Laspeyres fixed weight chained indexes, 1982–91**



method chains price relatives, while the chaining method chains indexes.

Exhibit 1 also shows how chained formulas for the remaining indexes are obtained.

Substitution effect

Two measures of price change are utilized in this article. First, we use long-term indexes (such as the CPI) for which price change is measured relative to the price level in December 1982. Second, because the CPI is often used to measure changes in prices over shorter periods of time, we also utilize short-run price changes to further examine the potential effects in using Laspeyres measures to make inferences about year-to-year price movement. (See appendix table A-1 for the results.)

Long term price change. Year-end values for fixed-base indexes of long-term price change relative to December 1982 are shown in chart 1. As was found in earlier studies, the gap between the Laspeyres and Paasche indexes increases over time. The Tornqvist and Fisher Ideal indexes are generally very close over the period (within 0.1 index points except for 1990). (See table 1.) The magnitude of the substitution effect also increases over time, and at an increasing rate. For the entire period—1982–91—the amount of the substitution effect reaches 2.6 index points, or an average rate of about 0.2 percent per year. This annual estimate of the substitution effect is substantially higher than that reported by Braithwait, where he found a 0.1-percent annual difference between the Laspeyres and his measures of the CLI over a 15-year period (1958–73).[16] The annual estimate is about the same as the 0.19-percent difference estimated by Manser and McDonald, but over a substantially shorter time span.[17] This variation in the estimates of the substitution effect is most likely to be due to the level of detail at which the substitution effect is being measured. As noted earlier, Braithwait used 53 national aggregates, and Manser and MacDonald used 101 national categories. Another potential cause is that the 1982–91 period was characterized by greater price variability than that during the periods covered by the other studies. This, however, seems unlikely in view of the average rates of inflation during the different periods; 3.1 percent in the 1958–73 period, 5.2 percent during the 1959–85 period and 3.9 percent in the 1982–91 period. In this study, we have used over 8,000 area/item categories. The potential effects of differences due to substitution are greater as the level of detail becomes finer.

The degree of the substitution effect in chained indexes is somewhat larger than that found using the fixed-base indexes. (See table 1.) Using the Tornqvist index as the basis for comparison,[18] the estimated degree of the substitution effect is 3.4 percentage points by 1991, or an annual rate of 0.27 percentage points, compared with 2.6 index points using the fixed-weight index, or an annual rate of 0.2. Finally, as shown in table 1, the chained and unchained superlative indexes provide virtually identical estimates.

Given that the substitution effect problem arises because the expenditure weights are fixed, one would think that the chained Laspeyres—where weights change every year—would provide an index closer to the Tornqvist than would the fixed-base index. However, a comparison of the Laspeyres indexes in table 1 reveals that the chained Laspeyres increases at a faster rate over the 1982–91 period than does the fixed-base Laspeyres: by December 1991, the chained Laspeyres was 144.3, compared with 143.8 for the fixed-base Laspeyres.[19] Second, the differences are numerically small. (See charts 1 and 2.) Finally, other empirical studies of price indexes have reported similar findings.[20] One should contrast this result with the findings of Mary Lynn Schmidt (pages 59–62, this issue) which show that for each year of the 1987–91 period, the annual reweighting of the CPI with the most recent 3-year average of data from the Consumer Expenditure Survey data results in a lower index than that using earlier consumer expenditure patterns. The use of 3 years of consumer expenditure data, the procedure utilized for the official CPI, has two effects. First, it reduces the sampling variance of the consumer expenditure data. Second, it should reduce what is called "chain drift," the effect of the correlation between price relatives and the quantity relatives. If the price relatives are negatively correlated with the quantity relatives, the chain index tends to be higher than the fixed-base measure. This is less likely to occur when changes in expenditures are collected over longer time intervals.

Short-run price change. Short-term price movement in Laspeyres measures (such as the CPI) is normally measured by forming ratios of the reported index for the period of interest. For example, the change in prices from time t to time $t+k$ is obtained by dividing the $t+k$ Laspeyres index by the time t index, both of which use the same reference month, o. Using the traditional formula for the Laspeyres ($I^L_{t+k,o} = [\sum_{i=1,n} (P^{t+k}_i X^o_i) / \sum_{i=1,n} (P^o_i X^o_i)]$), the resulting measure of price change ($PCI^L_{t+k,t}$) is:

$$PCI^L_{t+k,t} = \frac{[\sum_{i=1,n}(P^t_i X^o_i)] / [\sum_{i=1,n}(P^o_i X^o_i)]}{[\sum_{i=1,n}(P^t_i X^o_i)] / [\sum_{i=1,n}(P^o_i X^o_i)]}$$

$$= \frac{\sum_{i=1,n} P^t_i X^o_i}{\sum_{i=1,n} P^t_i X^o_i}$$

which may be called a "modified-Laspeyres" because the base period for expenditures (o) does not coincide with the reference month for prices (t). In this article this method of obtaining short-term price change is called the "ratio" method.

Jack Triplett has pointed out that although this practice is valid for Laspeyres measures, one cannot do the same for other index number formulas.[21] This is because although the ratio of two Laspeyres indexes is still a (modified) Laspeyres and therefore provides a measure of price change between two periods, this relationship does not hold for other index number formulas such as the Paasche. Therefore, the formulas used to measure short-term price change are standard Paasche, Fisher, and Tornqvist indexes rather than taking ratios of the long-term indexes to obtain the short-run price change. For example, applying the formulas in exhibit 1, a Paasche measure of price change between the two periods t and $t+k$ is given by:

$$PCI^P_{t,t+k} = \frac{\sum_{i=1,n} P^{t+k}_i X^{t+k}_i}{\sum_{i=1,n} P^t_i X^{t+k}_i}$$

A true (not modified) Laspeyres measure of short-

Table 2. **Annual price changes in fixed-base indexes, December 1983–91**

Year	Laspeyres		Tornqvist	Paasche
	Ratio	True		
1983	3.8	3.8	3.7	3.5
1984	4.1	4.1	4.0	3.9
1985	4.1	4.1	4.0	3.8
1986	1.7	1.9	1.6	1.3
1987	4.5	4.6	4.3	4.0
1988	4.5	4.4	4.1	3.8
1989	4.7	4.7	4.4	4.2
1990	6.3	6.2	5.7	5.9
1991	3.3	3.6	3.1	2.9

term price change is also calculated to examine differences in price change measured through the "ratio" method.

Year-to-year changes in the Laspeyres index. Table 3 provides several measures of 12-month price changes, calculated at December of each year. These measures indicate how the cost of living changed over the current year, rather than over a 9-year period. In common parlance, these measures represent 1-year inflation rates.

The ratio method is the measure typically used (modified Laspeyres) to calculate the percent change in the price indexes which is obtained by taking ratios of the Laspeyres long-term indexes for the two periods in question. For example, the modified Laspeyres measure for the price increase over the December 1986-December 1987 period is 4.5 percent. The remaining columns provide fixed-base indexes for each of the formulas in exhibit 1. So, for example, the fixed-base Laspeyres measures shows a price change of 4.6 percent from December 1986 to December 1987. This measure uses 1986 expenditure data and is, therefore, a true Laspeyres measure of price change.

A comparison of the estimates provided by the "ratio" method and the Laspeyres method suggests short-term price measures may be more sensitive to the choice of formula. For the 12 months ended in December 1986, for example, the difference between the Laspeyres and modified Laspeyres measures is 0.2 index points (1.9 minus 1.7), or more than 10 percent of the Laspeyres measure. The differences in the two measures range from 0.0 to 0.3 index points. While the period is too short to make any conclusions, the periods with the largest relative differences (1986 and 1991) are those characterized by a sharp deceleration in prices, particularly those for petroleum-based energy.

Table 3. **Substitution effect in annual measures of fixed base, Laspeyres-type indexes, December 1983–91**

Year	Laspeyres (ratio)	Substitution effect (percent)	Laspeyres (true)	Substitution effect (percent)
1983	3.8	.2	3.8	.2
1984	4.1	.1	4.1	.1
1985	4.1	.1	4.1	.2
1986	1.7	.1	1.9	.3
1987	4.5	.2	4.6	.3
1988	4.5	.4	4.4	.3
1989	4.7	.2	4.7	.2
1990	6.3	.5	6.2	.4
1991	3.3	.1	3.6	.4

If one takes the view that the superlative index number formulas provide the best measure of price change, it is interesting to compare the "ratio" calculations to those provided by the Fisher and Tornqvist measures. The focus on the "ratio" method stems from the fact that this is the method typically used. The question, then, is: How much is the annual percent change in the true CLI (approximated by the superlative indexes) overstated by the "ratio" method? Table 3 gives the calculated measures of the substitution effect in the annual measures.

The estimates of the "ratio" method substitution effect range from 0.1 percentage point to 0.5 percentage point. The greatest difference in the two measures occurs in 1990, where the discrepancy in the two measures is about 0.5 percentage point. Using the true Laspeyres measure yields estimates of the substitution effect within the same general range from 0.1 percentage to 0.4 percentage point, but peaking in different periods. The largest relative substitution effect occurs in the 12-month periods ended in December of 1986 and 1991—periods characterized, as noted earlier, by a sharp deceleration in energy prices.

In conclusion, the calculated aggregate price indexes were not particularly sensitive to the choice of formulas over the 1982–91 period. A comparison of Laspeyres measures to the calculated superlative indexes provided measures of the substitution effect only slightly higher than those found in earlier studies.

The results suggest that the substitution effect in the CPI-like Laspeyres measures of price change was small. Because of the similarities in these data to those used in calculating the official CPI, the results suggest that the latter is likely to display a substitution effect of similar magnitudes over the 1982–91 period.

It would be useful to have some criteria for judging whether the differences in the Laspeyres and superlative measures is in some sense "significant." The data used in calculating the CPI relies on statistical surveys which are estimated with some variance. Because the reported differences across index number formulas are small in magnitude, it is important to know whether these differences may be attributed to the randomness inherent in using survey data. Efforts are underway to calculate variance measures for the indexes reported here so that one can test the statistical significance of the substitution effect estimates. □

Footnotes

ACKNOWLEDGMENT: Kim Zieschang, Charles Mason, and other members of the BLS staff provided useful comments to earlier versions of this paper.

[1] See Robert A. Pollak, *The Theory of the Cost-of-Living Index*. (England, Oxford University Press, 1989).

[2] For example, the present expenditure pattern reflected in the official CPI is a 3-year average—1982–84—and was introduced in December 1986. Thus, the period immediately prior to and including December 1986 was based on 1972–73 expenditures, while the period from December 1986 forward was based on 1982–84 expenditures.

[3] Economic literature generally refers to this effect as "substitution bias." We have chosen to characterize the difference between the CPI and CLI as the substitution effect to avoid confusion with other types of biases associated with price indexes, such as sampling bias.

[4] Steven D. Braithwait, "Substitution Bias of the Laspeyres Price Index: An Analysis Using Estimated Cost-of-Living Indexes," *American Economic Review*, March 1980, pp. 64–77.

[5] Mary F. Kokoski, "Consumer Price Indexes by Demographic Group," working paper 167 (Bureau of Labor Statistics, 1987).

[6] Marilyn E. Manser and Richard J. McDonald, "An Analysis of Substitution Bias in Measuring Inflation, 1959–85," *Econometrica*, July 1988, pp. 909–30.

[7] See Lawrence R. Klein and Rubin Herman, "A Constant Utility Index of the Cost of Living." *Review of Economic Studies*, 1948, vol. 15, pp. 84–87.

[8] See Robert A. Pollak, "Additive Utility Functions and Linear Engle Curves," *Review of Economic Studies*, 1971, vol. 38, pp. 401–14.

[9] See Hendrik S. Houthakker, "Additive Preferences," *Econometrica*, 1960, vol. 28, pp. 244–57.

[10] W. Erwin Diewert, "Exact and Superlative Index Numbers," *Journal of Econometrics*, May 1976, pp. 115–45.

[11] See exhibit 1 for the index number formula for these measures.

[12] Brent R. Moulton, "Interarea Indexes of the Cost of Shelter Using Hedonic Quality Adjustment Techniques," working paper 238 (Bureau of Labor Statistics, 1993).

[13] Diane F. Primont and Mary F. Kokoski, "Comparing Prices Across Cities: A Hedonic Approach," working paper 204 (Bureau of Labor Statistics, 1990), and "Differences in Food Prices Across U.S. Cities: Evidence From CPI Data," working paper 209 (Bureau of Labor Statistics, 1991).

[14] See *BLS Handbook of Methods*, Bulletin 2414 (Bureau of Labor Statistics, 1992), chapter 19.

[15] See Jack E. Triplett, "Price Index Research and Its Influence on Data: A Historical Review." Paper presented at the 50th Anniversary of the Conference on Income and Wealth, Washington, 1988.

[16] Braithwait, "Substitution Bias."

[17] Manser and McDonald, "An Analysis of Substitution Bias."

[18] See W. Erwin Diewert, "Superlative Index Numbers and Consistency in Aggregation," *Econometrica*, 1978, vol. 46, pp. 883–900. Diewert showed that all choices of a superlative index give the same answer to the second order and that the choice is immaterial. We have used the Tornqvist in keeping with the Manser–McDonald study.

[19] See F.G. Forsyth, "The Practical Construction of a Chain Price Index Number," *The Journal of the Royal Statistical Society*, series A, vol. 144, part 2, pp. 348–58; and Bodan J. Szulc, "Linking Price Index Numbers," in W. Erwin Diewert and C. Montmarquette, eds. *Price Level Management*, (Ottawa Canada, Statistics Canada, 1983).

[20] See, Manser and McDonald, "Substitution Bias." 1988.

[21] Jack E. Triplett, "Reconciling the CPI and the PCE Deflator," *Monthly Labor Review*, September 1981, pp. 3–15.

APPENDIX: Data used in the calculations

The two types of data necessary to construct the indexes described in this article are expenditure data to calculate relative importances and price data from which to calculate price relatives. The data, briefly described, allow the calculation of superlative indexes over the December 1982-December 1991 period.

Average monthly relative importances for each year are obtained using data from the Consumer Expenditure Survey conducted by BLS. Data are available beginning in 1980 for expenditures in a relatively high level of disaggregation both with regard to commodity items as well as geographic detail. The commodity identifiers in these files are consistent over the time period, and provide, at the most disaggregate level, expenditures for over 200 commodities. Unfortunately, a break in the data occurs in 1986, where the geographic identifiers changed slightly. A concordance was constructed to form 44 geographic areas which are consistent over the period.

The price relatives are constructed using unpublished indexes from the CPI program. Disaggregate indexes are available for geographic areas and commodity items roughly comparable to those used in the Consumer Expenditure data. A major break in the data occurred with the 1987 revision, where several commodity classes as well as areas underwent changes in classification. Several concordances were required to form geographic/commodity cells which were both consistent over time and with the Consumer Expenditure data cells. The highest level of disaggregation possible provides indexes and expenditure levels for roughly 200 commodity classes and 44 geographic areas (over 8,000 cells) from which to form the indexes described earlier.

Within a particular area and commodity item, the available indexes are used to construct month-to-month price relatives as follows:

$$P^{t}_{i} / P^{t-1}_{i} = I_{t,i} / I_{t-1,i},$$

that is, the indexes for each cell are treated as basic prices.

Although expenditure data are available beginning in 1980, the first period of analysis in this article is December 1982. A conceptual change in the treatment of shelter (the largest component in the index) which reduced the weight of the component and altered the measure of price movement, was made in December 1982. Although it may be possible to extend the new measure of shelter back to 1980, that is not attempted here, and instead December 1982 is chosen as the initial period for analysis.

Table A-1. **Changes in fixed-base and chained indexes, as measured by four formulas over the December 1982–91 period**

Price change from December	To December—									
	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Fixed-base indexes										
Laspeyres:										
1982	100.000	103.848	108.073	112.496	114.436	119.634	124.985	130.908	139.214	143.783
1983		100.000	104.123	108.425	110.471	115.336	120.395	126.098	133.873	138.503
1984			100.000	104.137	106.127	110.723	115.558	120.903	128.233	132.664
1985				100.000	101.925	106.355	111.014	116.090	123.038	126.924
1986					100.000	104.595	109.082	114.101	121.154	124.966
1987						100.000	104.443	109.170	115.770	119.391
1988							100.000	104.653	111.065	114.619
1989								100.000	106.178	109.560
1990									100.000	103.556
1991										100.000
Paasche:										
1982	100.000	103.493	107.636	111.650	112.855	117.839	122.483	127.707	135.498	138.937
1983		100.000	103.860	107.810	109.048	113.834	118.284	123.286	130.801	134.152
1984			100.000	103.794	105.062	109.658	114.002	118.741	125.960	129.175
1985				100.000	101.253	105.666	109.853	114.522	121.454	124.558
1986					100.000	104.003	108.146	112.876	119.379	122.514
1987						100.000	103.848	108.278	114.754	117.728
1988							100.000	104.212	110.260	113.086
1989								100.000	105.863	108.526
1990									100.000	102.885
1991										100.000
Fisher Ideal:										
1982	100.000	103.670	107.854	112.072	113.643	118.733	123.728	129.298	137.343	141.339
1983		100.000	103.991	108.117	109.757	114.583	119.335	124.684	132.328	136.310
1984			100.000	103.965	105.593	110.189	114.777	119.817	127.091	130.908
1985				100.000	101.588	106.010	110.432	115.303	122.243	125.735
1986					100.000	104.299	108.613	113.487	120.263	123.734
1987						100.000	104.145	108.723	115.261	118.557
1988							100.000	104.432	110.662	113.850
1989								100.000	106.020	109.042
1990									100.000	103.220
1991										100.000
Tornqvist:										
1982	100.000	103.664	107.842	112.066	113.626	118.720	123.726	129.352	137.005	141.226
1983		100.000	103.983	108.110	109.764	114.577	119.324	124.733	131.980	136.203
1984			100.000	103.966	105.626	110.185	114.790	119.879	126.750	130.809
1985				100.000	101.615	106.014	110.434	115.353	121.910	125.641
1986					100.000	104.290	108.604	113.500	119.886	123.610
1987						100.000	104.142	108.735	114.943	118.456
1988							100.000	104.443	110.357	113.745
1989								100.000	105.744	108.950
1990									100.000	103.134
1991										100.000

Table A-1. **Continued—Changes in fixed-base and chained indexes, as measured by four formulas over the December 1982–91 period**

Price change from December	To December— 1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Chained indexes										
Laspeyres:										
1982	100.000	103.848	108.130	112.603	114.771	120.044	125.378	131.212	139.318	144.272
1983		100.000	104.123	108.431	110.518	115.596	120.732	126.350	134.156	138.926
1984			100.000	104.137	106.142	111.019	115.951	121.347	128.843	133.425
1985				100.000	101.925	106.608	111.345	116.526	123.725	128.125
1986					100.000	104.595	109.242	114.325	121.388	125.705
1987						100.000	104.443	109.303	116.055	120.182
1988							100.000	104.653	111.118	115.070
1989								100.000	106.178	109.954
1990									100.000	103.556
1991										100.000
Paasche:										
1982	100.000	103.493	107.488	111.566	112.964	117.486	122.007	127.146	134.600	138.483
1983		100.000	103.860	107.800	109.151	113.521	117.889	122.854	130.057	133.809
1984			100.000	103.794	105.095	109.301	113.507	118.288	125.224	128.836
1985				100.000	101.253	105.306	109.358	113.965	120.646	124.127
1986					100.000	104.003	108.005	112.554	119.153	122.591
1987						100.000	103.848	108.222	114.567	117.872
1988							100.000	104.212	110.322	113.505
1989								100.000	105.863	108.917
1990									100.000	102.885
1991										100.000
Fisher Ideal:										
1982	100.000	103.670	107.808	112.083	113.864	118.758	123.681	129.163	136.939	141.348
1983		100.000	103.991	108.115	109.832	114.554	119.302	124.590	132.091	136.344
1984			100.000	103.965	105.617	110.157	114.723	119.808	127.021	131.111
1985				100.000	101.588	105.955	110.347	115.238	122.176	126.110
1986					100.000	104.299	108.622	113.436	120.266	124.138
1987						100.000	104.145	108.761	115.309	119.022
1988							100.000	104.432	110.719	114.285
1989								100.000	106.020	109.434
1990									100.000	103.220
1991										100.000
Tornqvist:										
1982	100.000	103.664	107.793	112.068	113.878	118.763	123.682	129.178	136.598	140.879
1983		100.000	103.983	108.107	109.853	114.566	119.311	124.612	131.770	135.899
1984			100.000	103.966	105.645	110.177	114.741	119.839	126.722	130.694
1985				100.000	101.615	105.974	110.364	115.267	121.888	125.708
1986					100.000	104.290	108.610	113.435	119.951	123.710
1987						100.000	104.142	108.769	115.017	118.621
1988							100.000	104.443	110.442	113.903
1989								100.000	105.744	109.058
1990									100.000	103.134
1991										100.000