DISCREPANCIES BETWEEN ANALYTICAL SOLUTIONS OF TWO BOREHOLE PERMEAMETERS FOR ESTIMATING FIELD-SATURATED HYDRAULIC CONDUCTIVITY

J. D. Jabro, R. G. Evans

ABSTRACT. Field-saturated hydraulic conductivity (K_{fs}) is considered the most important parameter for water flow and chemical transport phenomena in soils. The Richards' and Laplace's solutions of the Guelph Permeameter (GP) and the Glover's solution of the Compact Constant Head Permeameter (CCHP) for calculating K_{fs} were compared. Steady-state flow rates of water into soil at a single constant head infiltration (H = 20 cm) from a borehole measured with the Guelph permeameter method were used to estimate K_{fs} values using these solutions. The geometric mean values of K_{fs} calculated using Richards', Laplace's, and Glover's solutions were 0.112, 0.185, and 0.224 cm h⁻¹, respectively, for a Duffield silt loam soil.

The Glover's and Laplace's solutions, neither of which takes into account the effect of unsaturated capillary flow, produce K_{fs} values approximately 1.5 to 2 times larger than the K_{fs} values calculated using Richards' solution. While the Glover's solution gives K_{fs} values nearly 1.4 times larger than those estimated by the Laplace's solution. The student t-test showed that the mean difference (M_d) among Richards', Laplace's, and Glover's solutions were significantly different from zero at p < 0.01. Thus, statistical analyses indicated that the three analytical methods result in dissimilar estimates of K_{fs} values. Negative K_{fs} values are often obtained using simultaneous equations of Richards' solution approach in heterogeneous soils and the Richards' solution can only be used at one constant water depth when an α value must be estimated or assigned based on the soil texture and structure. Both Glover's and Laplace's solutions can be used in coarse textured soils where the capillarity effect is minimal and initial water content in the soil is near the field capacity level.

This indicates that the variability in K_{fs} estimates depend not only on soil structure, texture, and other soil characteristics, but also on the method of estimation imposed by the borehole analytical solution.

Keywords. Hydraulic conductivity, Permeameter, Constant head, Solution, Water flow.

aturated hydraulic conductivity (K_s) is an important soil property that measures the ability of soil to transmit water under saturated conditions (Klute and Dirksen, 1986). Further, K_s plays an important role in many agronomic, engineering, environmental, and hydrological investigations. Measurement of K_s is often done using borehole permeameters (Amoozegar and Warrick, 1986; Elrick and Reynolds, 1992a).

In-situ measurements of K_s are essential for describing water movement and chemical transport in the soil. When K_s is measured in the unsaturated zone, it is often referred to as the field-saturated hydraulic conductivity, K_{fs} (Reynolds, 1993; Rodgers and Mulqueen, 2004; Bagarello et al., 2004).

Numerous field techniques have been developed for measuring K_{fs} of soils in the unsaturated zone (Stephens and Newman, 1982; Reynolds et al., 1983; Reynolds and Elrick, 1985; Amoozegar and Warrick, 1986; Perroux and White,

1988; Amoozegar, 1989a, b). Several of these techniques have been compared to each other and the results of these comparisons showed high inconsistency and different trends under various soil types and field conditions (Mohanty et al., 1994). Further, the operating ranges, assumptions, methods of calculation and limitations of some of these methods did not appear to be well understood in some of these comparisons (Salverda and Dane, 1993).

Several water flow models or solutions (e.g., Richards, Laplace, Gardner, Glover, Philip, and others) have been used to describe infiltration into soil surrounding a borehole at a constant water head (Philip, 1985; Reynolds and Elrick, 1985; Elrick et al., 1989; Amoozegar, 1989b; Elrick and Reynolds, 1992a; Wu et al., 1993). Steady water flow rate measurements into soil from a borehole under a constant head have been used to estimate K_{fs} values using the aforementioned analytical solutions. Wu et al. (1993) tested and compared several water flow solutions, however, they did not include the Glover's solution in their study because their water flow rate measurements were made at 5- and 10-cm constant water heads and the Glover's solution requires a ratio of water head (H) in a hole to borehole radius (r) larger than 5, H/r >5 (Amoozegar, 1989a, b).

This study focuses on the Richards' and Laplace's solutions of the Guelph Permeameter (GP) and the Glover's solution of the Compact Constant Head Permeameter (CCHP) for calculating K_{fs} . Both GP and CCHP (also named Amoozemeter) are constant-head well permeameters or shallow well pump-in techniques that are commonly used for

549

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measuring K_{fs} in soil. The two devices are basically in-hole permeameters operating on the marriote bottle principle and are used to measure flow rate of water from a cylindrical borehole at a constant water depth in a small diameter hole (approximately 6 cm). Both GP and CCHP devices are compact, portable, and easy to use. Further, both devices only need a small amount of water and time and can be easily operated by one person (Reynolds and Elrick, 1985; Amoozegar, 1989b, respectively). However, the two permeameter techniques use different solutions or analyses for calculating K_{fs}. Several previous attempts (Reynolds and Elrick, 1985; Amoozegar, 1989a; Wu et al., 1993) were made to compare various analytical solutions for estimating soil hydraulic conductivity, but no distinct conclusion for calculating K_{fs} was reached from those previous studies regarding which calculation method to use and under what soil or field conditions the method should be applied.

This article also explores different scenarios for calculating K_{fs}: Two simultaneous equations of Richards' solution for estimating an α parameter; the Richards' solution with a known or an estimated value of soil α parameter; the Laplace's solution; and finally the Glover's solution. Therefore, the general objective of this study is to compare the Richards' and Laplace's solutions of GP with the Glover's solution of CCHP for calculating K_{fs} using steady-state water flow rate measurements obtained at a single constant-head (20-cm) infiltration from a borehole measured with the GP technique.

THEORETICAL CONSIDERATIONS

RICHARDS' AND LAPLACE'S SOLUTIONS OF THE GUELPH PERMEAMETER (GP)

The steady state flow of water from a cylindrical borehole augured to a given depth below the soil surface is based on Richards' analysis or analytical solution presented by Reynolds et al. (1985) as:

$$2\pi H^2 K_{fs} + C\pi r^2 K_{fs} + 2\pi H \phi_m = CQ \qquad (1)$$

where H is the steady depth of water in the hole (cm), K_{fs} is the field-saturated hydraulic conductivity of the soil (cm h⁻¹), r is the radius of the hole (cm), C is a dimensionless shape factor that depends primarily on the H/r ratio and on soil texture/structure properties and is a function of both H and r (C = 0.8-2.5); ϕ_m is the matrix flux potential (cm² h⁻¹), and Q is the steady-state flow rate out of the borehole (cm³ h⁻¹). More information regarding calculation of K_{fs} , definition and values of equation 1 components are given in Reynolds and Elrick (1985, 1987).

Equation 1 was re-arranged using $\phi_m = K_{fs}/\alpha$ when the water content in the soil was at or below the field capacity (Elrick et al., 1989) to derive the following equation:

$$K_{\rm fs} = CQ / [2 \pi H^2 + C\pi r^2 + (2 \pi H / \alpha)]$$
(2)

where α is a soil texture/structure parameter (cm⁻¹), which represents the effect of soil capillarity under steady flow conditions. Further details regarding the α parameter and its estimates for different types of soils are given in Reynolds and Elrick (1985), Elrick et al. (1989), and Reynolds (1993).

The first term in the denominator on the right side of equation 2 denotes the pressure component of flow, the second term denotes the gravitational component of the flow, and the third term is the capillarity component which denotes the flow due to the capillary effect of the unsaturated soil conditions surrounding the borehole.

As we noticed, equation 2 or Richards' analysis has two unknown values (K_{fs} and α parameter), thus two equations are required to solve this equation simultaneously for K_{fs} and α using two constant water depths.

Eliminating the third term (capillarity component) of equation 2 yields Laplace's solution (Reynolds et al., 1983):

$$K_{\rm fs} = CQ / [2 \pi H^2 + C\pi r^2]$$
 (3)

Equations 2 and 3 can be solved for K_{fs} using a single depth of water in an auger hole without any possibility of obtaining negative values of K_{fs} .

GLOVER'S SOLUTION OF COMPACT CONSTANT HEAD PERMEAMETER (CCHP)

The calculation of K_{fs} started with the Glover solution (Zangar, 1953) suggested by Amoozegar and Warrick (1986), Amoozegar (1989b), and Rodgers and Mulqueen (2004) using the following equation:

$$K_{\rm fs} = Q[\sinh^{-1}(H/r) - \{(r/H)^2 + 1\}^{0.5} + r/H]/2\pi H^2 \quad (4)$$

where Q is a steady-state infiltration rate from the borehole $(cm^3 h^{-1})$, H is the steady depth of water in the hole (cm), r is the radius of the hole (cm), and $sinh^{-1}$ is the inverse hyperbolic sine function calculated as:

$$\sinh^{-1}(H/r) = \log_{e} \left[(H/r) + ((H/r)^{2} + 1)^{0.5} \right]$$
 (5)

Equation 4 only takes into account a pressure head in a cylindrical borehole and neglects both gravitational and matric potential gradients in the soil. Nonetheless, the Glover solution can provide good estimates of K_{fs} using a larger H/r ratio (H/r >10), where hydrostatic pressure dominates the flow (Elrick and Reynolds, 1992b). Equation 4 is used for calculating K_{fs} in the absence of an impermeable layer or when the distance between the impermeable layer and the bottom of the borehole is at least twice as large as the height of the water in the hole (Amoozegar, 1989b). When the H value is much bigger than r (H> r), equation 4 can be written in a more simple form as:

$$K_{\rm fs} = Q[\sinh^{-1}(H/r) - 1]/2\pi H^2$$
 (6)

More discussion regarding the CCHP, its theoretical aspects, equations and calculations are given in Amoozegar (1989a, b).

MATERIALS AND METHODS

FIELD SITE AND SOIL

The field site is locatedon the Weaver Homestead farm in Lancaster County, Pennsylvania. The soil is classified as Duffield silt loam (fine-silty, mixed, mesic Ultic Hapludalf) developed from limestone parent material. The soil is deep, well drained, and moderately structured. The slope of the field site was between 2-3%. Selected measured soil physical and hydraulic characteristics for the Duffield silt loam are given in table 1.

Table 1. Selected soil physical and hydraulic characteristics for a Duffield silt loam

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Value ^[a]					
0.40-0.60					
1.53					
178					
571					
251					
0.384					
0.365					
0.227					

^[a] Each value is a mean of nine measurements.

WATER FLOW RATE MEASUREMENTS

A total of 60 in-situ measurements of steady-state flow rate were taken on a 10- \times 10-m sampling grid scheme. Steady-state water flow rates were measured using the Guelph permeameter technique (Reynolds and Elrick, 1985). Flow rates were measured when the initial water contents in the soil were below the field capacity ranging between 0.238 and 0.304 m³ m⁻³.

For each measurement, a 6-cm diameter cylindrical hole was augured to a depth ranging from 40 to 60 cm. A rigorous wire brush was used to prepare a clean borehole and to minimize wall smearing, which can cause erroneous unrepresentative K_{fs} values (Reynolds, 1993). One set of steady flow rate measurements was made at a constant pressure head of 20-cm water for each hole (Elrick et al., 1989; Salverda and Dane, 1993). Steady-state flow rates were assumed when the last three consecutive readings were approximately the same ($\pm 5\%$). Further details about the Guelph permeameter technique are given in Reynolds (1993).

Local tap water was used in the tests. To eliminate temperature effects, the temperature of the water was recorded prior to taking the measurements and the water flow readings were then corrected to a water viscosity at 20°C.

RESULTS AND DISCUSSION

SIMULTANEOUS EQUATIONS APPROACH OF RICHARD'S SOLUTION

The simultaneous equations approach of Richard's solution (eq. 2) was used at 10- and 20-cm water depths to obtain α parameter values. Out of 60 estimations, 48 resulted in negative α and K_{fs} values. The arithmetic mean of the positive α values of 12 calculations was approximately 0.1114 cm⁻¹. Duffield silt loam soil has similar physical and hydraulic characteristics to those of Hagerstown soil. Therefore, equation 2 with $\alpha = 0.12$ cm⁻¹ and H = 20 cm was used to calculate K_{fs} values for all 60 boreholes rather than using the two constant water depths of Richards' analysis approach (Reynolds and Elrick, 1985).

The simultaneous equation approach of Richards' solution seems problematic and frequently produces negative unrealistic values for K_{fs} mainly in heterogeneous layered soil profile conditions. Therefore, Elrick et al. (1989) suggested α values for various classes of soil types in order to avoid negative values of K_{fs} . They also suggested an α parameter value of 0.12 cm⁻¹ for most structured and clay textured soils. Further, Campbell and Fritton (1994) calculated an α value of 0.12 cm⁻¹ for a Hagerstown silt loam soil which was used in K_{fs} calculation at one depth of water (H = 20 cm).

COMPARISON OF RICHARDS', LAPLACE'S, AND GLOVER'S SOLUTIONS

The steady-state flow rates of water at a single constanthead infiltration from a borehole measured by a Guelph permeameter apparatus were used to calculate K_{fs} values using the Richards' and Laplace's solutions of GP and the Glover's solution of CCHP as indicated in equations 2, 3, and 6, respectively.

Table 2 summarizes K_{fs} statistical results obtained from each solution and the comparison of their arithmetic and geometric means.

The geometric mean values of K_{fs} calculated using Richards', Laplace's, and Glover's solutions were 0.112, 0.158, and 0.224 cm h⁻¹, respectively, for the Duffield silt loam soil (table 2). The geometric means of K_{fs} were used for comparison because K_{fs} data are better described by a log-normal probability frequency distribution.

The Glover's and Laplace's solutions, neither of which include the soil capillary effect (the flow due to the capillary suction of the unsaturated soil surrounding the borehole), produce K_{fs} values approximately 1.5 to 2 times larger than the K_{fs} values calculated using Richards' solution. The results presented in this paper are in agreement with those found by Amoozegar (1989a) which indicated that the K_{fs} values obtained by the Glover's solution were larger than those obtained by the simultaneous equations of Richards' approach.

Both the Glover's and Laplace's solutions (eqs. 3 and 4) overestimate the K_{fs} in soils compared to a single-head Richards' solution of GP with a given value of α parameter because the latter takes into account the unsaturated flow as indicated in equation 1. The unsaturated flow, or the capillarity effect, is a very important component when the initial water content in the soil is below the field capacity, particularly in soils with high clay content.

Further, the mean difference $(M_d = \Sigma(\text{solution1} - \text{solution2})/\text{number of observations})$ was used to measure the average difference between K_{fs} values estimated by any two solutions. An M_d value equal to zero (null hypothesis, H_0 : $M_d = 0$) indicates no overall difference in K_{fs} even though the two solutions values can differ in individual measurements. The sign implies whether the solution tends to overestimate (+) or underestimate (-). The M_d was also tested for significant difference from zero using a t-test (SAS Institute, 2003).

Table 2. Statistical summary of three analytical solutions.

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Solution	Mean (cm h ⁻¹)	Standard Deviation (cm h ⁻¹)	Maximum (cm h ⁻¹)	Minimum (cm h ⁻¹)	Geometric ^[a] Mean (cm h ⁻¹)
Glover	0.385	0.400	1.742	0.029	0.224
Richards	0.192	0.199	0.870	0.015	0.112
Laplace	0.271	0.281	1.228	0.021	0.158

 [a] Geometric mean values were calculated because K_{fs} is better described by a log-normal frequency distribution. The mean difference (M_d) in K_{fs} values between the Glover's and Richards' solutions was significantly different from zero $(M_d = 0.197 \text{ cm h}^{-1}, t = 7.22, p < 0.01)$, which validated the discrepancy in K_{fs} estimated using these two solutions (table 3). Significant differences were also found between both Glover's and Laplace's solutions $(M_d = 0.114 \text{ cm h}^{-1}, t = 7.23, p < 0.01)$ on one hand and between Laplace's and Richards' solutions $(M_d = 0.079 \text{ cm h}^{-1}, t = 7.22, p < 0.01)$ on the other hand.

From the results presented in this article, we recommend that K_{fs} not be calculated using the two simultaneous equations of Richards' solution generated from two measurements at two constant water depths. This solution often produces negative unrealistic values for K_{fs} particularly in heterogeneous soil profile conditions since soils are likely to be non-homogeneous, anisotropic, and consist of variable soil horizons or layers. The Richards' solution can only be used at a single constant head (H) and with an α parameter value estimated or assigned based on the soil texture and structure characteristics (Elrick et al., 1989). Further, this solution can be used in various ranges of soil texture where initial soil water content is at field capacity level or lower.

Both the Glover's and Laplace's solutions, which ignore the unsaturated flow, can be used in coarse textured soils where the capillarity effect is minimal and initial water contents in the soil are near or at the field capacity levels. The unsaturated flow or the capillarity effect is a very important factor when the initial water content in the soil is below the field capacity, particularly in soils with high clay content. Further, the Glover's solution has been recommended for calculating K_{fs} when distance between the bottom of the borehole and impermeable layer is 2H (Amoozegar and Warrick, 1986).

The results of this study confirm that discrepancies and lack of agreement exist between $K_{\rm fs}$ estimates using borehole permeameter techniques. This also indicates that the variability and magnitude of $K_{\rm fs}$ estimates depend not only on soil structure, texture, flow geometry, soil disturbance and other soil factors, but also on analytical solutions and methods of estimation.

SUMMARY AND CONCLUSIONS

The Richards' and Laplace's solutions of the Guelph permeameter (GP) and the Glover's solution of the compact constant head permeameter (CCHP) were compared for their ability to estimate K_{fs} in a Duffield silt loam soil. Steady-state flow rates at a single constant-head infiltration from a borehole measured by the Guelph permeameter were made. The following conclusions were drawn from the results of this study: 1) The Glover's and Laplace's solutions produce K_{fs} values larger (1.5 to 2) than those estimated with the Richards' solution for a Duffield silt loam soil; These discrepancies between the estimates of these solutions exist because both Glover's and Laplace's solutions exclude the effect of the water flow due to capillarity of the unsaturated soil surrounding the borehole; 2) The simultaneous equations of Richards' solution often produces negative values for K_{fs}, particularly in heterogeneous soil profile conditions; 3) The Richards' solution can only be used at a single constant head (H) and with an α parameter value estimated based on the soil texture and structure characteristics in various textured soils

where initial water content is at field capacity level or lower, and 4) The Glover's and Laplace's solutions can be used in coarse textured soils where the capillarity effect is minimal and initial water contents in the soil are near or at field capacity limit.

In addition, the K_{fs} results are dependent on method of calculation and further work appears to be warranted for developing a suitable and universal K_{fs} estimation method for borehole permeameters. Despite the uncertainty of selecting an appropriate solution for calculating K_{fs} and discrepancies between the analytical solutions' results, both GP and CCHP methods are currently considered common, simple and convenient methods for measuring K_{fs} .

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