US Department
of Transportation

**Federal Railroad
Administration**

# Research Results

**RR00-06**
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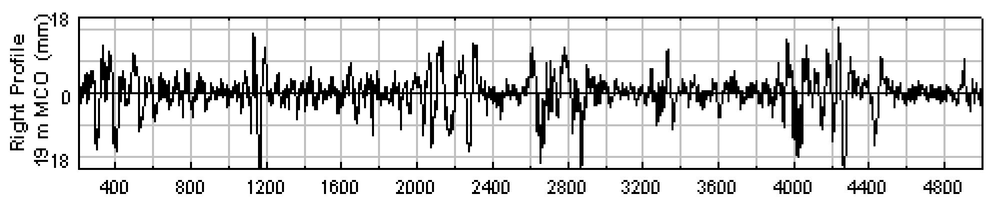
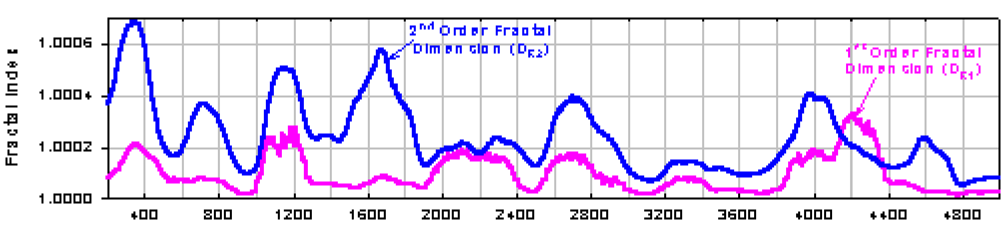
# Fractal Analysis of Geometry Data for Railroad Track Condition Assessment


## SUMMARY

The Federal Railroad Administration sponsored a research project to investigate the application of Fractal Analysis to railway track geometry data and to develop numerical indices based on this analysis for use in track condition assessment, improved safety and efficiency of operations, and diagnosis of the cause of poor track condition. Fractal Analysis is an analytical technique that can be applied to characterize and to quantify irregular patterns that are chaotic and random; as track geometry data are classified. Fractal analysis of geometry data revealed that the mid-chord offsets (MCOs) of the vertical profile data have typically two orders of roughness. This bi-fractal condition results in three fractal parameters to describe the geometry pattern: two independent fractal dimensions ($D_{R1}$ and $_{DR2}$) based on the slope of the log-log plots and the location of the breakpoint. Figure 1 is a plot of the basic geometry profile data and the derived fractal parameters ($D_{R1}$ and $D_{R2}$). Two fractal analysis software programs were developed during this research: (1) Fractal Track Evaluation (FTEval) used to evaluate the applicability of fractal analysis to track geometry, and (2) Fractal Track Automatic (FTAuto), evolved from the FTEval, which performs fractal analysis on the geometry data for any length of track. The study concluded that the geometric patterns of the track conditions could be studied through fractal analysis.


Track Geometry, 19 m (62 ft) Mid-Chord Offset Data




1st and 2nd Order Fractal Dimension

*Figure 1: Example Track Geometry and Fractal Analysis Results*

## BACKGROUND

Researchers studied the use of fractal analysis to describe railway track geometry data for improved indication of track geometry condition, better maintenance planning and problem cause evaluation. Railway track geometry consists of a vertical profile, horizontal alignment, crosslevel and gage. Specific safe and efficient train operational speeds are associated with the original design values of these geometry parameters. Deviation from these design values results in "rough" track geometry which in turn results in delays from slow orders or maintenance related interruptions. Rough track geometry can also cause degradation of track components and rolling stock, freight damage, passenger discomfort, and in extreme cases, derailments. To minimize the adverse impact of rough track the measured geometry data needs to be better utilized in the railroad decision making process. Precise and meaningful study of geometry data will lead to its improved utilization.

The focus of this research was on vertical profile measurements since these are the parameters most influenced by the track substructure conditions. It showed that fractal analysis could be used to precisely quantify track geometry measurements by providing unique numerical values that characterize the geometry pattern.

## FRACTAL ANALYSIS

Fractal analysis is used to characterize irregular geometry patterns and to quantify patterns that are seemingly chaotic and random (Mandelbrot, 1983). The fractal dimension of a pattern varies depending on the degree of "roughness" of the pattern, and will have a different value for each pattern type, with the fractal dimension being specific for that pattern (Kaye, 1989).

The divider method for fractal dimensioning shows how fractal analysis can be used to quantify a rough pattern. Consider a rough pattern that is measured using a “divider” or “ruler” of length, $\lambda$, as illustrated on Figure 2. The rough pattern is measured by placing the ends of the ruler on consecutive points of intersection along the line. As the line is measured with smaller rulers (top down in Figure 2) the measured Total Length, $L(\lambda)$, increases since the smaller rulers have the ability to intersect more points on the line and hence better approximate the actual length. The

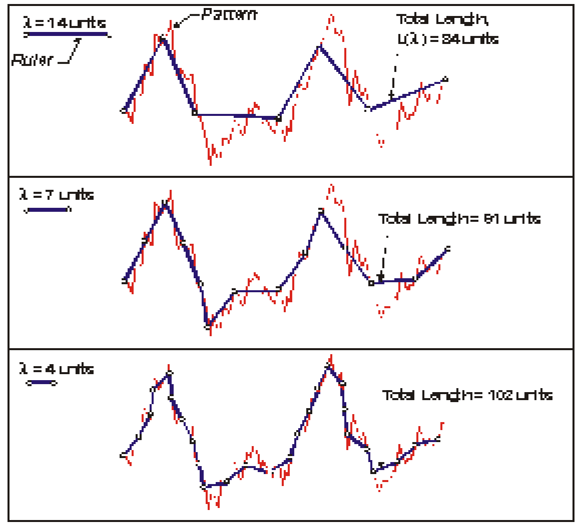



***Figure 2: Measurement of Rough Pattern with Various Ruler Sizes***

fractal dimension, $D_R$, is determined from the relationship between a series of ruler lengths and the corresponding measured Total Lengths.

If the arbitrarily rough line in Figure 2 was smoother than that shown, the difference in consecutive measured Total Lengths would not be as great, and the fractal dimension value would be less.

## APPLICATION TO TRACK GEOMETRY DATA

Two computer programs were developed for calculating the fractal dimension for large volumes of railway track data. FTEval (Fractal Track Evaluation) was developed to apply fractal analysis to discrete lengths of geometry data. This program was used to evaluate the applicability of fractal analysis to geometry data and to establish analysis protocols. The FTEval program evolved into the FTAuto (Fractal Track Automatic) program which performs fractal analysis on the track geometry data in a moving window format to provide continuous characterization of geometry roughness for any length of track.

Figure 3 presents an image of the FTAuto user interface. The user interface allows for intuitive adjustment of key variables in the fractal analysis.

FTEval and FTAuto were used to perform fractal analysis on a large amount of railway track vertical profile data (from 1994 to present) that was obtained using a high-speed inertial-based geometry car on the Northeast Corridor (NEC).

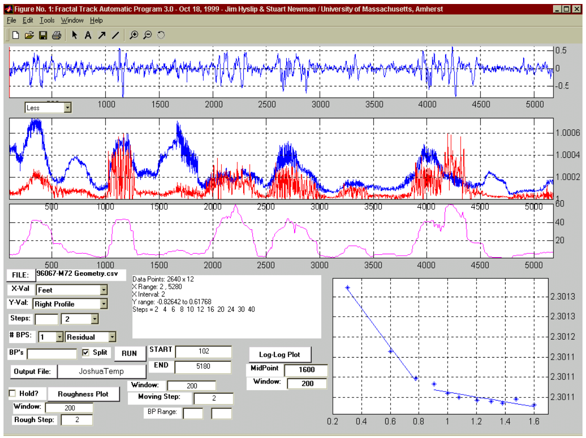


***Figure 3**: FTAuto User Interface*

The focus of this research was on vertical profile measurements since these are the parameters most influenced by the track substructure conditions and indicative of the cause of track condition deterioration.

The fractal analysis of the profile data typically indicates two orders of roughness. These orders of roughness are shown on the geometry deviation plot in Figure 4. The 1$^{st}$ order roughness is associated with the overall shape of the line and

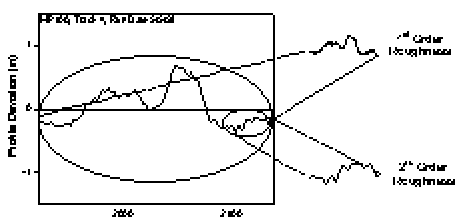


***Figure 4: Orders of Roughness of Profile Data***

the 2$^{nd}$ order roughness is associated with the texture of the line.

## INDICATION OF GEOMETRY CONDITION

The three parameters that have been defined based on the fractal analysis of track geometry data provide good indicators of the roughness of the geometry data. As seen on Figure 1, $D_{R1}$ and $D_{R2}$ indicate significant track geometry trends and the parameters correlate well with track roughness ($R^{2)}$ an index value developed at UMass that is currently being used to characterize track geometry data.

Figure 5 shows an expanded view of the MCO data for two sections from Figure 1 and shows the ability of fractal analysis to indicate zones of large and small scale roughness. The pattern in the top plot has high small-scale roughness and correspondingly has a high $D_{R2}$ and a low $D_{R1}$. The pattern in the bottom plot has a "smoother" small-scale pattern and a "rougher" large-scale pattern. Correspondingly, the pattern in the lower plot has a low $D_{R2}$ and a high $D_{R1}$.

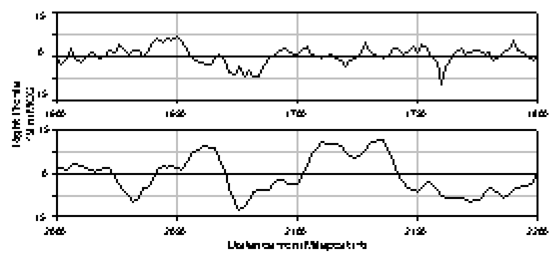


***Figure 5: Expanded View of Section 1600' - 1800' and 2000' - 2200' of Figure 1***

Fractal analysis also appears to be able to separate and characterize the wavelength components within a waveform. This is exemplified in Figure 6, which shows the results of the fractal analysis of composite sinusoidal waveforms. Composite waveforms were constructed with a short wavelength (high frequency) sinusoidal waves superimposed on

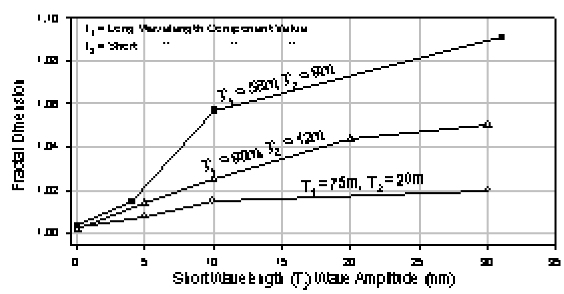



***Figure 6: Increase in $D_{R2}$ with Increase in High Frequency Wave Amplitude***

longer wavelength (low frequency) "carrier" waves. Analysis with FTEval showed that as the wave amplitude of the short wavelength component was reduced in magnitude, with the amplitude of the long wavelength held constant, $D_{R2}$ correspondingly reduced.

## MAINTENANCE PLANNING AND CONDITION EVALUATION

By quantifying track geometry data with fractal analysis and developing $D_R$ trends over time, the deterioration rate of the geometry can be assessed and maintenance/remedial input planned accordingly. Fractal analysis provides three basic parameters: the slopes of the bi-fractal plot shown ($D_{R1}$ and $D_{R2}$) and the breakpoint between the slopes. The variation in these parameters along the track can also provide an indication of the track condition, providing up to six parameters for trend analysis: 3 fundamental and 3 derived. Each of these parameters could potentially highlight different safety or maintenance concerns.

Track geometry patterns are influenced by substructure conditions. Since $D_R$ provides precise numerical values of rough patterns, it is reasoned that fractal analysis has the potential to evaluate track substructure condition. Based on this hypothesis, an empirical study has been undertaken to correlate the fractal indices to areas of track with known substructure conditions. There are indications that fractal analysis can provide information on the cause of geometry roughness and thereby aid in identifying useful remedial actions.

## FURTHER WORK

The research on the use of fractal analysis to characterize track geometry data is continuing. Further work is planned to:

- perform a detailed field study of revenue service track in order to establish correlations between track structure condition and fractal dimensions of geometry data.
- examine geometry degradation using fractal analysis values for proactive maintenance planning.
- examine fractal characteristics of other railroad data such as wheel/car accelerations and GRMS.

## ACKNOWLEDGMENTS

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**KEYWORDS:** fractal analysis, fractal dimension, divider method, track geometry data