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Modeling Labor Markets with Heterogeneous Agents and Matches

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Modeling Labor Markets with Heterogeneous Agents and Matches

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Abstract

I present a matching model with heterogeneous workers, firms, and worker-fim matches. The model generalizes the seminal Jovanovic (1979) model to the case of heterogeneous agents. The equilibrium wage is linear in a person-specific component, a firm-specific component, and a match specific component that varies with tenure. Under certain conditions, the equilibrium wage takes a simpler structure where the match specific component does not vary with tenure. I discuss fixed- and mixed-effect methods for estimating wage models with this structure on longitudinal linked employer-employee data. The fixed effect specification relies on restrictive identification conditions, but is feasible for very large databases. The mixed model requires less restrictive identification conditions, but is feasible only on relatively small databases. Both the fixed and mixed models generate empirical person, firm, and match effects with characteristics that are consistent with predictions from the matching model; the mixed model moreso than the fixed model. Shortcomings of the fixed model appear to be artifacts of the identification conditions.

1 Introduction

Recent developments in the creation and analysis of longitudinal linked employer-employee data have created new opportunities for modeling labor markets. Chief among these, linked employer-employee data provide an environment to formally model wages and employment as equilibrium outcomes. Arguably of equal importance, the longitudinal nature of these data provide a means to accomodate heterogeneity — both observed and unobserved — on both sides of the market. In combination, empirical equilibrium models with heterogeneous agents promise to give fresh insight into almost all aspects of the labor market. To date, however, empirical analyses on longitudinal linked data have focused on either equilibrium aspects or heterogeneity aspects, but rarely the two together.

This purpose of this paper is twofold. First, I present a matching model with heterogeneous workers, firms, and worker-firm matches. With a simple production technology and Nash bargained wage, equilibrium wages are linear in a worker-specific component, a firm-specific component, and a match-specific component. In the second part of the paper, I present methods for estimating a wage model with this structure and some preliminary empirical results.

The behavioral model generalizes the seminal Jovanovic (1979) matching model to the case of heterogeneous workers and firms. Worker and firm types are observable when the worker and firm meet. However, agents learn the value of match quality slowly by observing production outcomes. Firms employ many workers. All separations are endogenous, occurring when agents learn that match quality is low.

The model is related to the literature on heterogeneous search and matching. A recent survey of this literature is Burdett and Coles (1999). Two themes emerge from work in this area: there are externalities due to heterogeneous matching, and assortative matching arises only under certain conditions. Gautier (2000), Sattinger (1995), and Shimer and Smith (2001) focus on the externality which arises when heterogeneous agents enter into matches. When a worker of some type x matches with a firm of some type y, there is one less type x worker and one less type y firm with whom other agents can match. When search frictions are present, this externality can result in inefficient assignment of workers to jobs. Albrecht and Vroman (forthcoming) and Shimer and Smith (2000) consider the conditions under which two-sided search with heterogeneous agents generates assortative matching.[1] These conditions are generally more restrictive than those proposed by Becker (1973) for the full information (no search friction) case.

There is considerable variation in the heterogeneity structures considered by other authors. In no instance is there a distinction between firm heterogeneity and worker-firm match heterogeneity – roughly speaking, a firm is a job or collection of independent jobs. In this case, there is little to be gained by modeling firms who employ many workers. Consequently, most model firms who employ a single worker. Albrecht and Vroman (forthcoming), Gau-

[1] Assortative matching is match formation between agents of the same type (appropriately defined).

tier (2000), and Kohns (2000) develop models with exogenous heterogeneity on one side of the market, and endogenous heterogeneity on the other. Workers are characterized by an exogenous skill level (high or low). Firms choose the "complexity" or skill level of a job when opening a vacancy (also high or low). High skill workers are more productive in high skill jobs than are low skill workers, or than in low skill jobs. In contrast, Sattinger (1995), Shimer and Smith (2000), Shimer and Smith (2001), and Stern (1990) present models where agents on both sides of the market are characterized by exogenous heterogeneity. All but Shimer and Smith (2000) rely on a discrete number of agent types.

To put my matching model in context, it is useful to comment on several other aspects of the models discussed so far. In particular, it is useful to consider the mechanisms by which matches terminate, and the assumed information structure. These are intimately related in my model. Only Gautier (2000) allows endogenous separations, which arise from on the job search. Other authors either treat matches as lifetime matches (no separations) or only allow matches to dissolve at an exogenous rate. Correspondingly, there is no room for learning about agents' types or about match quality— in all but Stern (1990), agent types are observable. However in Stern's (1990) model no new information is ever acquired about agent types after a lifetime contract is signed. Thus there is no learning or endogenous dissolution.

The matching model that I present in Section 2 differs from those discussed above in some important respects, some of which I have already mentioned in passing. The foremost of these is the heterogeneity structure. I assume that workers vary in their average productivity, which I refer to as worker quality. Worker quality determines in part the worker's productivity at any firm. Firms vary in their technology, which affects the average productivity of all workers they employ. I refer to this heterogeneity component as firm quality. Firm quality is common to all workers employed by the firm. Finally, worker-firm matches also vary in their average productivity, which I refer to as match quality. Worker quality, firm quality, and match quality are all exogenous and take values on a continuum. The output produced by a worker-firm match is a function of all three heterogeneity components, and is subject to match-specific shocks. When a worker and firm meet, they can determine the other's quality by inspection. They both observe a noisy signal of match quality. On the basis of each party's quality and the signal of match quality, they determine whether or not to pursue the match. If they do, wages are the outcome of a bilateral bargain. Subsequent production outcomes are signals of match quality and prompt renegotiation of the wage. I show there is a reservation level of beliefs about match quality below which matches dissolve. Thus separations are endogenized as the result of learning about match quality.

Under certain conditions, which I discuss in Section 2.3, the equilibrium wage is linear in a worker-specific component, a firm-specific component, and a match-specific component. Such a wage model is estimable on longitudinal linked employer-employee data. The various specification discussed in Section 3 generalize the linear models with fixed person and firm effects developed in Abowd et al. (1999) and Abowd et al. (2002).

The remainder of this paper is organized as follows. In Section 2 I briefly present the matching model and some basic results. The model is a work in progress and not all aspects of equilibrium are completely characterized in this draft. Since the focus in the remainder of the paper is to develop an empirical specification for the equilibrium wage, my focus in Section 2 is to derive this. In Section 3 I present several methods based on linear and linear mixed models for estimating the equilibrium wage function. Section 4 describes the Longitudinal Employer Household Dynamics (LEHD) Program database on which I estimate the empirical model. Section 5 presents some *very* preliminary empirical results, and Section 6 concludes.

2 A Matching Model with Heterogeneous Workers, Firms, and Worker-Firm Matches

The economy is populated by a continuum of infinitely-lived workers of measure one. There is a continuum of firms of measure ϕ. All agents are risk neutral and share the common discount factor $0 < \beta < 1$. Time is discrete.

Workers are identified by the continuous index i. They are heterogeneous in their average productivity when employed, denoted θ_i. Assume

$$\theta_i \sim N(0, \sigma_\theta) \text{ iid across workers.} \tag{1}$$

I will refer to θ_i as worker quality and use F_θ to denote the distribution function in (1). Workers are also heterogeneous in the value of home production when unemployed, denoted h_i. Assume θ_i and h_i are exogenous, known to the worker, and observable to the firm when worker and firm meet. Workers seek to maximize the expected present value of wages.

Firms are identified by the continuous index j. They employ many workers. Firms operate in a competitive output market and produce a homogeneous good. The price of output is normalized to 1. Output can only be produced by a worker matched to a firm. Firms seek to maximize the expected net revenues of a match: the value of output minus a wage payment to the worker.

Firms are heterogeneous in their technology, denoted ψ_j, which affects the average productivity of all their employees. Assume

$$\psi_j \sim N(0, \sigma_\psi) \text{ iid across firms.} \tag{2}$$

I will refer to ψ_j as firm quality and use F_ψ to denote the distribution function in in (2). Firms are also heterogeneous in their cost of maintaining a vacancy, denoted k_j. Assume that firms know their own values of ψ_j and k_j, and that these parameters are observable by the worker when worker and firm meet. Both ψ_i and k_j are exogenous. Firms incur cost $c(e_j)$ to hire e_j workers. Assume c is continuous, increasing, and convex.

Unemployed workers are matched to firms with open vacancies. Search is undirected. The total number of matches formed in a period is given by $m(u, v)$ where u is the number of unemployed workers in the economy, and v is the number of open vacancies. Both u and v are determined endogenously. Assume m is non-decreasing in both u and v. The probability that a randomly selected unemployed worker will be matched to a firm in the current period is $\pi \equiv \frac{1}{u} m(u, v)$. Similarly, the probability that a randomly selected vacancy will be filled is $\lambda \equiv \frac{1}{v} m(u, v)$. With a large number of workers and firms, all agents take u and v as given.

Worker-firm matches are heterogeneous in their average productivity, denoted γ_{ij}. Assume

$$\gamma_{ij} \sim N(0, \sigma_\gamma) \text{ iid across matches.} \tag{3}$$

I will refer to γ_{ij} as match quality and use F_γ to denote the distribution function in (3). Match quality γ_{ij} is unobserved. When a worker and firm first meet, they observe a noisy signal of match quality $x_{ij} = \gamma_{ij} + \eta_{ij}$ where

$$\eta_{ij} \sim N(0, 1) \text{ iid across matches.} \tag{4}$$

I will use F_η to denote the distribution function in (4). The worker and firm form beliefs about the value of γ_{ij} on the basis of a prior and the signal x_{ij}. Workers and firms subsequently update their beliefs over γ_{ij} on the basis of output realizations. Prior beliefs and the updating process are discussed in Section 2.1.

Output is produced according to the production function:

$$q_{ij\tau} = \mu + \theta_i + \psi_j + \gamma_{ij} + \varepsilon_{ij\tau} \tag{5}$$

where τ indexes tenure (the duration of the match), μ is the grand mean of productivity, and $\varepsilon_{ij\tau}$ is a random disturbance. Assume

$$\varepsilon_{ij\tau} \sim N(0, \sigma_\varepsilon) \text{ iid across matches and tenure.} \tag{6}$$

The linear production technology (5) generalizes that of Jovanovic (1979) to the case of heterogeneous workers and firms in a discrete time setting. As a normalization, I use tenure $\tau = 1$ to refer to the period in which the match forms, i.e., before any production has taken place.

Within-period timing is as follows:

1. Unemployed workers are randomly matched to a firm with an open vacancy. Upon matching, agents observe θ_i and ψ_j, and obtain the signal x_{ij}.

2. Given all tenure-τ information about the match (θ_i, ψ_j and current beliefs over γ_{ij}) the worker and firm decide whether or not continue the match. The current period wage $w_{ij\tau}$ is simultaneously determined by a Nash bargain.

3. If agents decide to continue the match, output $q_{ij\tau}$ is produced and observed by both parties. Agents update their beliefs about γ_{ij}.

4. The negotiated wage is paid to the worker.

5. Firms open vacancies v_j.

Assume that reputational considerations preclude agents from reneging on the agreed-upon wage payment.

2.1 Beliefs About Match Quality

Assume agents' prior beliefs about θ_i, ψ_j, γ_{ij}, η_{ij}, and $\varepsilon_{ij\tau}$ are governed by equations (1), (2), (3), (4), and (6). Recall θ_i and ψ_j are learned when the match is formed. Agents update their beliefs over γ_{ij} using Bayes' rule when they acquire new information, i.e., upon observing the signal x_{ij} and production outcomes $q_{ij\tau}$.

After observing the signal x_{ij}, worker and firm posterior beliefs about γ_{ij} are normally distributed with mean m_{ij1} and variance s_1 where

$$m_{ij1} = x_{ij}\left(\frac{\sigma_\gamma}{\sigma_\gamma + 1}\right) \tag{7}$$

$$s_1 = \frac{\sigma_\gamma}{\sigma_\gamma + 1}. \tag{8}$$

In each subsequent period that the match persists, worker and firm extract the signal $\gamma_{ij}+\varepsilon_{ij\tau}$ from observed output $q_{ij\tau}$. Hence at the beginning of the τ^{th} period of the match (that is, after observing $\tau - 1$ production outcomes), worker and firm posterior beliefs over match quality are normally distributed with mean $m_{ij\tau}$ and variance s_τ, where

$$\begin{aligned} m_{ij\tau} &= \left(\frac{m_{ij\tau-1}}{s_{\tau-1}} + \frac{\gamma_{ij} + \varepsilon_{ij\tau-1}}{\sigma_\varepsilon}\right) / \left(\frac{1}{s_{\tau-1}} + \frac{1}{\sigma_\varepsilon}\right) \\ &= \left(x_{ij} + \sum_{s=1}^{\tau-1} \frac{\gamma_{ij} + \varepsilon_{ijs}}{\sigma_\varepsilon}\right) / \left(\frac{\sigma_\gamma + 1}{\sigma_\gamma} + \frac{\tau - 1}{\sigma_\varepsilon}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{1}{s_\tau} &= \frac{1}{s_{\tau-1}} + \frac{1}{\sigma_\varepsilon} \\ &= \frac{\sigma_\gamma + 1}{\sigma_\gamma} + \frac{\tau - 1}{\sigma_\varepsilon}. \end{aligned} \tag{10}$$

Clearly s_τ is deterministic and does not depend on the value of the signals received. It is fairly straightforward to show that

$$\lim_{\tau\to\infty} m_{ij\tau} = \gamma_{ij} \tag{11}$$

$$\lim_{\tau\to\infty} s_\tau = 0 \tag{12}$$

which is a standard result for Bayesian updating with "correct" priors (see e.g., Blume and Easley (1998)).

Denote the complete set of tenure-τ information about the productivity of a match by $\Omega_{ij\tau} = \left(\theta_i, \psi_j, m_{ij\tau}, s_\tau\right)$. It is notationally convenient to describe the evolution of beliefs using a transition distribution $F\left(\Omega_{ij,\tau+1}|\Omega_{ij\tau}\right)$. Both θ_i and ψ_j are fixed, so the transition distribution describes the probabilistic evolution of $m_{ij,\tau+1}$ given $m_{ij\tau}$, and the deterministic evolution of $s_{\tau+1}$ given s_τ. The distribution of $m_{ij,\tau+1}|m_{ij\tau}$ is normal with mean and variance

$$E\left(m_{ij,\tau+1}|m_{ij\tau}\right) = m_{ij\tau} \tag{13}$$

$$Var\left(m_{ij,\tau+1}|m_{ij\tau}\right) = \frac{s_\tau^2}{s_\tau + \sigma_\varepsilon}. \tag{14}$$

More generally, for any $r > \tau$ the distribution of $m_{ijr}|m_{ij\tau}$ is normal with mean and variance

$$E\left(m_{ijr}|m_{ij\tau}\right) = m_{ij\tau} \tag{15}$$

$$Var\left(m_{ijr}|m_{ij\tau}\right) = \frac{s_\tau^r\left(r-\tau\right)}{s_\tau\left(r-\tau\right)+\sigma_\varepsilon}. \tag{16}$$

2.2 Match Formation, Duration, and Wages

In the Nash bargain framework, after an unemployed worker and a firm with an open vacancy meet, the match is consummated if the expected joint surplus of the match is nonnegative. The same condition determines whether or not the match persists at any subsequent tenure $\tau > 1$. At each such juncture, expectations are taken with respect to tenure-τ information about the match, $\Omega_{ij\tau}$. Thus we shall see that equilibrium wages map tenure-τ information about the match into payments from firm to worker. To reflect this, I write the tenure-τ wage as $w_{ij\tau} = w\left(\Omega_{ij\tau}\right)$ for each $\tau > 0$. I assume the function w is known to all agents, and that w is an equilibrium function in the following sense: it is chosen so that workers and firms agree about the set of acceptable job matches. These assumptions imply that for a match between worker i and firm j which has lasted τ periods, there is a reservation level of beliefs over match quality, $\bar{m}_{ij\tau}$, below which matches dissolve and above which matches continue.

In deriving the equilibrium wage and conditions under which matches form/persist, I use the following notation: $J\left[w\left(\Omega_{ij\tau}\right)\right]$ is the expected value to worker i of employment at firm j at wage $w_{ij\tau}$; Q_i is the value of worker i's outside option (unemployment); $\Pi\left[w\left(\Omega_{ij\tau}\right)\right]$ is the expected value to firm j of net revenues from a match with worker i at wage $w_{ij\tau}$; and V_j is the value of firm j's outside option (a vacancy). At tenure τ, the match continues if and only if

$$J\left[w\left(\Omega_{ij\tau}\right)\right] + \Pi\left[w\left(\Omega_{ij\tau}\right)\right] \geq Q_i + V_j. \tag{17}$$

When (17) is satisfied, the wage $w_{ij\tau} = w\left(\Omega_{ij\tau}\right)$ is given by the Nash bargaining wage condition

$$J\left[w\left(\Omega_{ij\tau}\right)\right] - Q_i = \delta\left(J\left[w\left(\Omega_{ij\tau}\right)\right] + \Pi\left[w\left(\Omega_{ij\tau}\right)\right] - Q_i - V_j\right) \tag{18}$$

or equivalently,

$$(1-\delta)\left(J\left[w\left(\Omega_{ij\tau}\right)\right]-Q_i\right)=\delta\left(\Pi\left[w\left(\Omega_{ij\tau}\right)\right]-V_j\right) \tag{19}$$

where δ is the exogenously given worker's share of the joint surplus.

In solving for the equilibrium wage and employment condition, I use the following strategy. First, I make two conjectures regarding the structure of equilibrium wages and employment. Then I characterize the various value functions under the assumption that the conjectures are true. Finally, I show that equilibrium wages and employment satisfy the two conjectures. Conjecture 1 has powerful implications for simplifying expectations over future values of $w\left(\Omega_{ij\tau}\right)$. Recall that $m_{ij\tau}$ and s_τ are the mean and variance of tenure-τ beliefs over match quality.

Conjecture 1 *The equilibrium wage offer function w is linear in $m_{ij\tau}$ and independent of s_τ.*

Conjecture 2 concerns the reservation level of beliefs over match quality, which I denote $\bar{m}_{ij\tau}$. That w is an equilibrium function in the sense that workers and firms agree about the set of acceptable matches implies that such a reservation level of beliefs exists: when the mean of beliefs over match quality falls below $\bar{m}_{ij\tau}$, the match terminates.

Conjecture 2 *The reservation level of beliefs about match quality, $\bar{m}_{ij\tau}$, is independent of tenure. That is, $\bar{m}_{ij\tau}=\bar{m}_{ij}$ for all $\tau>0$.*

I prove that Conjectures 1 and 2 are true in Propositions 4 and 5.

Before deriving the various value functions, I need to introduce some final notation. Let $G_r(x)=\Pr\left(m_{ijr}<x|\Omega_{ij\tau}\right)$. Then $G_r\left(\bar{m}_{ijr}\right)$ is the subjective probability that the match will terminate at tenure r, given tenure-τ beliefs. Note G_r is just the normal distribution with mean and variance given by (15) and (16). I will use E_τ to denote an expectation taken with respect to tenure-τ information. Tenure-τ expectations of tenure $\tau+1$ quantities can be taken with respect to the transition density of beliefs, denoted $F\left(\Omega_{ij\tau+1}|\Omega_{ij\tau}\right)$, described in Section 2.1.

2.2.1 The Worker's Value of Employment and Unemployment

The expected value to worker i of employment with firm j at wage $w_{ij\tau}=w\left(\Omega_{ij\tau}\right)$ is today's wage payment plus the discounted expected value of employment next period, adjusted for the possibility that the match terminates. That is,

$$\begin{aligned}
J\left[w\left(\Omega_{ij\tau}\right)\right] &= w_{ij\tau}+\beta\left[1-G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right)\right]E_\tau J\left[w\left(\Omega_{ij,\tau+1}\right)\right]+\beta G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right)Q_i \\
&= w_{ij\tau}+\beta\left[1-G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right)\right]\int J\left[w\left(\Omega_{ij,\tau+1}\right)\right]dF\left(\Omega_{ij\tau+1}|\Omega_{ij\tau}\right) \\
&\quad +\beta G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right)Q_i
\end{aligned} \tag{20}$$

When Conjecture 1 is true, $E_\tau w_{ijr} = w_{ijr}$ for all $r > \tau$. When Conjecture 2 is also true,

$$\begin{aligned} E_\tau J\left[w\left(\Omega_{ij,\tau+1}\right)\right] &= E_\tau w_{ij,\tau+1} + \beta\left[1 - G_{\tau+2}\left(\bar{m}_{ij}\right)\right] E_\tau E_{\tau+1} J\left[w\left(\Omega_{ij,\tau+2}\right)\right] + \beta G_{\tau+2}\left(\bar{m}_{ij}\right) Q_i \\ &= w_{ij\tau} + \beta\left[1 - G_{\tau+2}\left(\bar{m}_{ij}\right)\right] E_\tau J\left[w\left(\Omega_{ij,\tau+2}\right)\right] + \beta G_{\tau+2}\left(\bar{m}_{ij}\right) Q_i \qquad (21) \end{aligned}$$

and forward recursion gives

$$\begin{aligned} J\left[w\left(\Omega_{ij\tau}\right)\right] &= w_{ij\tau}\left(1 + \sum_{s=\tau+1}^{\infty} \beta^{s-\tau} \prod_{r=\tau+1}^{s} \left[1 - G_r\left(\bar{m}_{ij}\right)\right]\right) \\ &\quad + Q_i \sum_{s=\tau+1}^{\infty} \beta^{s-\tau} G_s\left(\bar{m}_{ij}\right) \prod_{r=\tau+1}^{s-1} \left[1 - G_r\left(\bar{m}_{ij}\right)\right]. \qquad (22) \end{aligned}$$

Equation (22) says that the expected value of employment is a weighted average of the current wage and the value of unemployment. The weights are discounted sums of probabilities of the match terminating at each future tenure τ, given current beliefs over match quality.

Deriving the value of unemployment is rather tedious and not particularly instructive. Thus I relegate it to Appendix A. When Conjectures 1 and 2 are true, the value of being unemployed today and behaving optimally thereafter is

$$Q_i = \frac{h_i + \beta\pi\left(\sum_{\tau=1}^{\infty} \beta^{\tau-1} \int w\left(\Omega_{ij1}^0\right) \prod_{s=1}^{\tau} \left[1 - G_s\left(\bar{m}_{ij}\right)\right] dF_\psi\right)}{1 - \beta(1-\pi) - \pi \sum_{\tau=1}^{\infty} \beta^{\tau} \int G_\tau\left(\bar{m}_{ij}\right) \prod_{s=1}^{\tau-1} \left[1 - G_s\left(\bar{m}_{ij}\right)\right] dF_\psi} \qquad (23)$$

where $\Omega_{ij1}^0 = \left(\theta_i, \psi_j, 0, s_1\right)$. The numerator in Equation (23) is the sum of the value of home production today and the discounted expected value of employment in subsequent periods when the identity of the matching firm is unknown. That is, before firm quality and the signal x_{ij} are known. The denominator normalizes to account for the possibility of re-entering unemployment at each future tenure.

2.2.2 The Firm's Value of Employment and of a Vacancy

I now turn to the firm's value of employment. The value to firm j of employing worker i at wage $w_{ij\tau} = w\left(\Omega_{ij\tau}\right)$ is today's expected net revenues plus the discounted expected value of employment next period, adjusted for the possibility that the match terminates. Thus,

$$\begin{aligned} \Pi\left[w\left(\Omega_{ij\tau}\right)\right] &= E_\tau q_{ij\tau} - w_{ij\tau} + \beta\left[1 - G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right)\right] E_\tau \Pi\left[w\left(\Omega_{ij,\tau+1}\right)\right] + \beta G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right) V_j \\ &= \mu + \theta_i + \psi_j + m_{ij\tau} - w_{ij\tau} + \beta\left[1 - G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right)\right] \int \Pi\left[w\left(\Omega_{ij,\tau+1}\right)\right] dF\left(\Omega_{ij\tau+1} | \Omega_{ij\tau}\right) \\ &\quad + \beta G_{\tau+1}\left(\bar{m}_{ij,\tau+1}\right) V_j. \qquad (24) \end{aligned}$$

Applying Conjectures 1 and 2, forward recursion analogous to (21) and (22) gives

$$\begin{aligned}\Pi\left[w\left(\Omega_{ij\tau}\right)\right] &= \left(\mu+\theta_i+\psi_j+m_{ij\tau}-w_{ij\tau}\right)\left(1+\sum_{s=\tau+1}^{\infty}\beta^{s-\tau}\prod_{r=\tau+1}^{s}\left[1-G_r\left(\bar{m}_{ij}\right)\right]\right) \\ &\quad +V_j\sum_{s=\tau+1}^{\infty}\beta^{s-\tau}G_s\left(\bar{m}_{ij}\right)\prod_{r=\tau+1}^{s-1}\left[1-G_r\left(\bar{m}_{ij}\right)\right]. \qquad (25)\end{aligned}$$

Equation (25) says that the value of employing worker i is a weighted average of the expected net revenues accruing to the match (expected output minus the current wage) and the value of a vacancy. Like the worker's value of employment, the weights are discounted sums of probabilities that the match terminates at each future tenure τ, given current beliefs over match quality.

I derive the value of a vacancy in Appendix A. When Conjectures 1 and 2 are true, this is

$$V_j=\frac{\beta\lambda\left(\sum_{\tau=1}^{\infty}\beta^{\tau-1}\int\left[\mu+\theta_i+\psi_j-w\left(\Omega_{ij1}^0\right)\right]\prod_{s=1}^{\tau}\left[1-G_s\left(\bar{m}_{ij}\right)\right]dF_\theta\right)-k_j}{1-\beta\left(1-\lambda\right)-\lambda\sum_{\tau=1}^{\infty}\beta^{\tau}\int G_\tau\left(\bar{m}_{ij}\right)\prod_{s=1}^{\tau-1}\left[1-G_s\left(\bar{m}_{ij}\right)\right]dF_\theta}. \qquad (26)$$

The numerator in (26) is the discounted expected value of employing a worker next period before the identity of the matching worker i is known (that is, before worker quality and the signal x_{ij} are known) minus the cost of maintaining the vacancy. The denominator normalizes the value to reflect the possibility of the match terminating at each future tenure.

2.2.3 The Firm's Decision to Open Vacancies

The production technology (5) implies that employees of firm j produce independently of one another. As a consequence, in each period the firm's decision to open vacancies is a static one. The number of hires today has no dynamic consequences for future hiring or productivity. When firm j opens v_j vacancies, we can model the number that are filled, e_j, as a binomial process. It follows that the number of vacancies opened by firm j in a given period solves[2]

$$\max_{v_j\in\mathbb{N}}\sum_{e_j=0}^{v_j}\binom{v_j}{e_j}\lambda^{e_j}\left(1-\lambda\right)^{v_j-e_j}\left[e_j\Pi_0\left(\psi_j\right)-c\left(e_j\right)\right]-k_jv_j \qquad (27)$$

where $\Pi_0\left(\psi_j\right)$ is the expected present value of net revenues from a match for a firm of quality ψ_j, before the identity of the matching worker i is known. I derive $\Pi_0\left(\psi_j\right)$ in Appendix A.

Note that firm size (employment) is indeterminate. However, convex hiring costs c guarantee the solution to (27) is well defined and the number of vacancies opened in any period by firm j is finite. At any point in time, the total number of vacancies in the economy is just $v=\int_0^\phi v_j dj$.

[2] The approach to modeling vacancies is based on Nagypal (2000).

2.2.4 The Equilibrium Wage

With expressions for the value functions in hand, I can now prove the two Conjectures and derive the equilibrium wage and employment conditions. Define the following terms:

$$A_{ij\tau} = 1 + \sum_{s=\tau+1}^{\infty} \beta^{s-\tau} \prod_{r=\tau+1}^{s} \left[1 - G_r\left(\bar{m}_{ij}\right)\right] \tag{28}$$

$$B_{ij\tau} = 1 - \sum_{s=\tau+1}^{\infty} \beta^{s-\tau} G\left(\bar{m}_{ij}\right) \prod_{r=\tau+1}^{s-1} \left[1 - G\left(\bar{m}_{ij}\right)\right]. \tag{29}$$

Substituting the definitions of $A_{ij\tau}$ and $B_{ij\tau}$ into (22), we see the worker's net surplus from the match is:

$$J\left[w\left(\Omega_{ij\tau}\right)\right] - Q_i = w_{ij\tau} A_{ij\tau} - Q_i B_{ij\tau}. \tag{30}$$

Similarly, the firm's net surplus from the match is:

$$\Pi\left[w\left(\Omega_{ij\tau}\right)\right] - V_j = \left(\mu + \theta_i + \psi_j + m_{ij\tau} - w_{ij\tau}\right) A_{ij\tau} - V_j B_{ij\tau}. \tag{31}$$

Substituting (30) and (31) into the Nash bargaining wage condition (19) gives the equilibrium wage:

$$w_{ij\tau} = \delta\left(\mu + \theta_i + \psi_j + m_{ij\tau}\right) + \left(\left(1-\delta\right) Q_i - \delta V_j\right) \frac{B_{ij\tau}}{A_{ij\tau}}. \tag{32}$$

Lemma 3 *Define $A_{ij\tau}$ and $B_{ij\tau}$ as in (28) and (29). Then*

$$\frac{B_{ij\tau}}{A_{ij\tau}} = 1 - \beta$$

for all $i, j,$ and $\tau > 0$.

The proof of Lemma 3 is in Appendix B. We can now prove Conjectures 1 and 2, which are restated as Propositions 4 and 5.

Proposition 4 *The equilibrium wage offer function w is linear in $m_{ij\tau}$ and independent of s_τ.*

Proof. Follows directly from equation (32) and Lemma 3. In particular, the equilibrium wage is

$$w_{ij\tau} = \delta\left(\mu + \theta_i + \psi_j + m_{ij\tau}\right) + \left(\left(1-\delta\right) Q_i - \delta V_j\right)\left(1-\beta\right). \tag{33}$$

■

As conjectured, (33) verifies that the equilibrium wage is linear in the posterior mean of beliefs about match quality. It is worthwhile relating this result to the Jovanovic (1979) equilibrium wage. In his model, workers and firms are *ex-ante* identical but matches are heterogeneous, and production occurs according to the continuous time analog of (5) with $\theta_i = \psi_j = 0$ for all i, j. The Jovanovic (1979) equilibrium wage is equal to expected marginal product, which in his case is also the posterior mean of beliefs about match quality. His result relies on the assumption that firms earn zero expected profit. Similar to Jovanovic's model, the equilibrium wage (33) is linear in expected marginal product, $\mu + \theta_i + \psi_j + m_{ij\tau}$, and in the posterior mean of beliefs about match quality, $m_{ij\tau}$. A stronger result is that when workers capture all the quasi–rents associated with the match, that is as $\delta \to 1$, so that firms earn zero expected profit; and when the cost of maintaining a vacancy is zero (as in Jovanovic (1979)); then the equilibrium wage converges to $w'_{ij\tau} = \mu + \theta_i + \psi_j + m_{ij\tau}$. That is, the equilibrium wage converges to the expected marginal product of the match. In this sense, the Jovanovic (1979) equilibrium wage is a special case of (33).

Note we can rewrite the Nash bargained wage in (33) as

$$\begin{aligned} w_{ij\tau} &= \delta\mu + [\delta\theta_i + (1-\beta)(1-\delta) Q_i] + \left[\delta\psi_j - (1-\beta)\delta V_j\right] + \delta m_{ij\tau} \\ &= \tilde{\mu} + \tilde{\theta}_i + \tilde{\psi}_j + \tilde{m}_{ij\tau} \end{aligned} \tag{34}$$

where $\tilde{\mu} = \delta\mu$, $\tilde{\theta}_i = \delta\theta_i + (1-\beta)(1-\delta) Q_i$, $\tilde{\psi}_j = \left[\delta\psi_j - (1-\beta)\delta V_j\right]$, and $\tilde{m}_{ij\tau} = \delta m_{ij\tau}$. The expression (34) shows that equilibrium wages are a function of a worker-specific component $\tilde{\theta}_i$, a firm-specific component $\tilde{\psi}_j$, and a match-specific component that varies with tenure $\tilde{m}_{ij\tau}$. I will make use of this fact in developing an empirical strategy in what follows. In Section 2.3, I develop conditions under which $\tilde{m}_{ij\tau}$ does not vary with tenure.

In what follows, I refer to $\tilde{\theta}_i$ and $\tilde{\psi}_j$ as empirical person and firm effects. We can interpret the empirical person effect as a bargaining-strength weighted average of the worker's quality and outside option, adjusted for waiting. Similarly, the empirical firm effect is just the worker's share of the firm's surplus from the match, adjusted for waiting.

Proposition 5 *The reservation level of beliefs about match quality, $\bar{m}_{ij\tau}$, is independent of tenure. That is, $\bar{m}_{ij\tau} = \bar{m}_{ij}$ for all $\tau > 0$.*

Proof. The reservation level of beliefs about match quality, $\bar{m}_{ij\tau}$, is the level at which parties to the match are indifferent between continuing the match and allowing it to dissolve. That is, the value at which the joint surplus from the match is zero. Thus $\bar{m}_{ij\tau}$ is defined by

$$J\left[w\left(\bar{\Omega}_{ij\tau}\right)\right] + \Pi\left[w\left(\bar{\Omega}_{ij\tau}\right)\right] = Q_i + V_j \tag{35}$$

where $\bar{\Omega}_{ij\tau} = (\theta_i, \psi_i, \bar{m}_{ij\tau}, s_\tau)$. Substituting (30) and (31) into (35) yields

$$\left(\mu + \theta_i + \psi_j + \bar{m}_{ij\tau}\right) A_{ij\tau} = (Q_i + V_j) B_{ij\tau}$$

and applying Lemma 3,

$$\begin{aligned} \bar{m}_{ij\tau} &= (Q_i + V_j)(1-\beta) - \mu - \theta_i - \psi_j \qquad (36) \\ &\equiv \bar{m}_{ij} \end{aligned}$$

for all $\tau > 0$. ■

In conjunction with the definitions of Q_i and V_j, equation (36) confirms that the reservation level of beliefs about match quality is a function only of worker and firm characteristics: θ_i, h_i, ψ_j, and k_j. Comparative statics on (36) can be used to characterize how employment duration and match quality vary with θ_i and ψ_j. Unfortunately Q_i and V_j are themselves complex functions of θ_i and ψ_j, which complicates signing the derivatives. Nevertheless, (36) makes clear that the effects of θ_i and ψ_j on expected duration should be symmetric.

Although less interesting from a theoretical perspective, comparative statics on (36) with respect to the empirical person and firm effects $\tilde{\theta}_i$ and $\tilde{\psi}_j$ are readily obtained. In particular, it is easy to show that

$$\frac{\partial \bar{m}_{ij}}{\partial \tilde{\theta}_i} = \frac{\partial \bar{m}_{ij}}{\partial \tilde{\psi}_j} = -\frac{1}{\delta} < 0. \tag{37}$$

Thus the empirical person and firm effects have a symmetric negative effect on reservation match quality. Based on (37), on average we should expect higher values of $\tilde{\theta}_i$ and $\tilde{\psi}_j$ to be associated with longer job duration. Furthermore, the duration-weighted correlation between $\tilde{\theta}_i$ and $\tilde{\psi}_j$ should be positive.[3]

2.3 Special Cases Where Wages are Linear in a Match-Specific Component

For the empirical application that follows, I need to derive conditions under which the equilibrium wage is linear in person-, firm-, and match-specific components. There are two such conditions. The first is an asymptotic result. The second is when match quality is observable.

2.3.1 An Asymptotic Result

It is a standard result that Bayesian learning with "correct" priors is *consistent* (see Blume and Easley (1998) for a formal definition). In the context of this model, consistency is summarized by (11) and (12). Applying these to the equilibrium wage (34) it is easy to verify that

$$\begin{aligned}\lim_{\tau\to\infty} w_{ij\tau} &= \delta\mu + [\delta\theta_i + (1-\beta)(1-\delta)Q_i] + \left[\delta\psi_j - (1-\beta)\delta V_j\right] + \delta\gamma_{ij} \\ &= \tilde{\mu} + \tilde{\theta}_i + \tilde{\psi}_j + \tilde{\gamma}_{ij}\end{aligned} \tag{38}$$

where $\tilde{\mu}$, $\tilde{\theta}_i$, $\tilde{\psi}_j$ were defined previously and $\tilde{\gamma}_{ij} = \delta\gamma_{ij}$. The key feature of (38) is that it is linear in person-, firm-, and match-specific components.

[3] To the extent that an empirical model with person, firm, and match effects approximates (34), we should also expect the duration-weighted correlation between the match effect and person/firm effects to be positive.

2.3.2 Observable Match Quality

A second case that generates equilibrium wages with the same structure as (38) is when match quality is observable. Retaining all other aspects of the model, it is easy to verify that when γ_{ij} is observable

$$
\begin{aligned}
J\left[w\left(\Omega_{ij\tau}\right)\right] &= \frac{w_{ij\tau}}{1-\beta} &(39)\\
\Pi\left[w\left(\Omega_{ij\tau}\right)\right] &= \frac{\mu+\theta_i+\psi_j+\gamma_{ij}-w_{ij\tau}}{1-\beta} &(40)
\end{aligned}
$$

and Q_i and V_j are special cases of (23) and (26), respectively. Applying the Nash bargaining wage condition (19) yields the equilibrium wage

$$
\begin{aligned}
w_{ij\tau} &= \delta\left(\mu+\theta_i+\psi_j+\gamma_{ij}\right)+(1-\beta)\left[(1-\delta)Q_i-\delta V_j\right]\\
&= \delta\mu+\left[\delta\theta_i+(1-\beta)(1-\delta)Q_i\right]+\left[\delta\psi_j-(1-\beta)\delta V_j\right]+\delta\gamma_{ij}\\
&= \tilde{\mu}+\tilde{\theta}_i+\tilde{\psi}_j+\tilde{\gamma}_{ij}. &(41)
\end{aligned}
$$

The separation condition is unchanged from equation (36). Of course in this context there are no endogenous separations. In fact, there are no separations at all. When $\gamma_{ij} \geq \bar{m}_{ij}$, the match forms and never ends. If $\gamma_{ij} < \bar{m}_{ij}$ then agents separate and no production takes place. Though it is a little unsatisfying, it is a trivial modification to accommodate exogenous separations. This is the route most of the literature has taken.

3 Empirical Model

I now develop an empirical strategy based on the matching model in Section 2. Though an empirical specification based on the general form of the equilibrium wage function (34) is feasible, it is complicated by the presence of $m_{ij\tau}$. This term — the mean of beliefs about match quality — implies a complex within-match covariance structure. Such a model is related to that of Farber and Gibbons (1996).[4] As a first step towards estimating a more general specification, in this version I focus on the special cases presented in Section 2.3. That is, a wage function that is linear in person-, firm-, and match-specific components like (38) and (41). As discussed in the previous Section, this model can be interpreted as an asymptotic approximation to (34), or the exact equilibrium wage when match quality is observable.

[4] Farber and Gibbons (1996) consider the case with homogeneous firms and jobs, and unobservable person heterogeneity. Person heterogeneity in their model plays a role similar to match quality in mine: firms learn about the worker's unobservable productivity by observing production outcomes.

Predictably, I model wages in logs rather than levels. Applying an often used transformation to (38) or (41):

$$w_{ij\tau} = \tilde{\mu}\left(1 + \frac{\tilde{\theta}_i}{\tilde{\mu}} + \frac{\tilde{\psi}_j}{\tilde{\mu}} + \frac{\tilde{\gamma}_{ij}}{\tilde{\mu}}\right) \tag{42}$$

which implies

$$\begin{aligned}\ln w_{ij\tau} &\approx \ln \tilde{\mu} + \frac{\tilde{\theta}_i}{\tilde{\mu}} + \frac{\tilde{\psi}_j}{\tilde{\mu}} + \frac{\tilde{\gamma}_{ij}}{\tilde{\mu}} \\ &= \check{\mu} + \check{\theta}_i + \check{\psi}_j + \check{\gamma}_{ij}\end{aligned} \tag{43}$$

where the first line of (43) uses $\ln(1+x) \approx x$, and where $\check{\mu} = \ln \tilde{\mu}$, $\check{\theta}_i = \frac{\tilde{\theta}_i}{\tilde{\mu}}$, $\check{\psi}_j = \frac{\tilde{\psi}_j}{\tilde{\mu}}$, and $\check{\gamma}_{ij} = \frac{\tilde{\gamma}_{ij}}{\tilde{\mu}}$.

In the model of Section 2, there were no aggregate shocks to the economy, nor did the average productivity of a given worker vary with labor force experience. I relax these assumptions for the empirical specification, and in the sequel I include a set of time-varying covariates $x_{ij\tau}$ in the wage model.[5] The basic estimating equation is

$$\ln w_{ij\tau} = \check{\mu} + x_{ij\tau}\beta + \check{\theta}_i + \check{\psi}_j + \check{\gamma}_{ij} + \check{\varepsilon}_{ij\tau}. \tag{44}$$

where $\check{\varepsilon}_{ij\tau}$ is a statistical residual.

The empirical wage model (44) is estimable on longitudinal linked employer-employee data. Identifying the person, firm, and match effects requires repeated observations on workers at multiple firms, and repeated observations on multiple workers in a given firm. One can model the person, firm, and match effects as either fixed or random. Each approach has both desirable and undesirable characteristics.

3.1 Modeling Wages With Fixed Worker, Firm, and Match Effects

Before proceeding, I restate the basic estimating equation (44) in notation more consistent with the linear models literature and the previous work of Abowd et al. (1999) and Abowd et al. (2002). In particular, I drop the inverse-hats from parameters in (44). With considerable abuse of earlier notation, I estimate the wage equation:

$$y_{ijt} = \mu_y + (x_{ijt} - \mu_x)\beta + \theta_i + \psi_j + \gamma_{ij} + \varepsilon_{ijt} \tag{45}$$

where y_{ijt} is the natural logarithm of quarterly earnings of individual $i = 1, ..., N$ at firm $j = 1, ..., J$ in quarter $t = n_{i1}, ..., n_{iT_i}$; x_{ijt} is a vector of P characteristics of individual i which

[5] In the application that follows, these include fixed year and quarter effects, and a fixed experience effect. Note the model as already specified accomodates time-invariant characteristics that are observable to the econometrician, e.g., education or industry. These are just components of the person, firm, and match effects.

vary over time and within job;[6] β measures returns to characteristics in x_{ijt}; θ_i is the pure person effect; ψ_j is the pure firm effect for the firm j at which worker i is employed at date t (formalized by the link function $j = J(i,t)$); γ_{ij} is the pure person-firm match effect; μ_y is the grand mean of y_{ijt} and μ_x is the grand mean of x_{ijt}; and ε_{ijt} is a statistical residual. Note that whereas τ was used to denote tenure in the previous sections, here t denotes calendar time.[7] I reiterate that θ_i, ψ_j, and γ_{ij} are the empirical effects of (44) and not the theoretical person, firm, and match quality of Section 2. Assume ε_{ijt} has the following properties:

$$\begin{aligned} E\left[\varepsilon_{ijt}|i,j,t,x_{ijt}\right] &= 0 \\ Cov\left[\varepsilon_{ijt},\varepsilon_{kls}|i,j,t,k,l,s,x_{ijt},x_{kls}\right] &= \begin{cases} \sigma_\varepsilon^2 & \text{if } i=k,\ j=l, \text{ and } t=s, \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

The pure person effect is further decomposed into components observed and unobserved by the econometrician as follows:[8]

$$\theta_i = \alpha_i + u_i\eta$$

where u_i are observable time-invariant person characteristics; α_i is a person-specific intercept; and η measures returns to characteristics. Rewriting equation (45) in matrix notation,

$$y = X\beta + D\theta + F\psi + G\gamma + \varepsilon \tag{46}$$

where X is the $N^* \times P$ matrix of observable time-varying characteristics (in deviations from grand means), D is the $N^* \times N$ design matrix for the person effects, F is the $N^* \times J$ design matrix for the firm effects, G is the $N^* \times M$ design matrix for the match effects, y is the $N^* \times 1$ vector of annual earnings (also in deviations from the grand mean), ε is the conformable vector of residuals, $M \leq NJ$ is the number of worker-firm matches, and $N^* = \sum_{i=1}^{N}(n_{iT_i} - n_{i1} + 1)$ is the total number of observations.[9] The parameter vectors β, θ, ψ, and γ are of conformable dimension. Note that the wage models of Abowd et al. (1999) and Abowd et al. (2002) are special cases of (46) where $\gamma = 0$.

For what follows, the rows of y, X, D, F, and G are arranged in lexicon order, that is by

[6] In general, x_{ijt} could also contain time-varying firm and job characteristics.

[7] The distinction only matters because the model includes time effects in x_{ijt}.

[8] Similar decompositions of the pure firm and match effects are also possible. For example,

$$\begin{aligned} \psi_j &= \phi_j + z_j\zeta \\ \gamma_{ij} &= \varsigma_{ij} + w_{ij}\xi. \end{aligned}$$

In fact, even more general decompositions of the pure person, firm, and match effects are possible with sufficient data. For example, the pure firm effect could include a firm-specific tenure effect. In this application, I only consider the simple decomposition of θ_i into observed and unobserved compnents.

[9] For simplicity, I assume one record per person per period.

t within j within i. Hence,

$$y = \begin{bmatrix} y_{1,J(1,n_{11}),n_{11}} \\ \dots \\ y_{1,J\left(1,n_{1T_1}\right),n_{1T_1}} \\ \dots \\ y_{N,J(N,n_{N1}),n_{N1}} \\ \dots \\ y_{N,J\left(N,n_{NT_N}\right),n_{NT_N}} \end{bmatrix} \tag{47}$$

with $J(i, n_{i,t-1}) \leq J(i, n_{it})$; X, D, F, G, and ε are arranged conformably.

3.1.1 Estimation

The least squares estimator of (46) solves the normal equations:

$$\begin{bmatrix} X'X & X'D & X'F & X'G \\ D'X & D'D & D'F & D'G \\ F'X & F'D & F'F & F'G \\ G'X & G'D & G'F & G'G \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{\theta} \\ \hat{\psi} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} X'y \\ D'y \\ F'y \\ G'y \end{bmatrix}. \tag{48}$$

In principle, (48) could be solved directly for the least squares estimators $\hat{\beta}$, $\hat{\theta}$, $\hat{\psi}$, and $\hat{\gamma}$. In practice, however, for large N and J, computing the (generalized) inverse of the cross-products matrix in (48) is infeasible due to its extreme dimension — $(P + N + J + M) \times (P + N + J + M)$. The following method solves for the least squares estimators without inverting the cross-products matrix.

Applying standard results for partitioned regression, the least squares estimator of β is

$$\hat{\beta} = \left(X'M_{[D\ F\ G]}X\right)^{-} X'M_{[D\ F\ G]}y \tag{49}$$

where A^{-} denotes the generalized inverse of matrix A and $M_{[D\ F\ G]}$ is the column null space of $\begin{bmatrix} D & F & G \end{bmatrix}$:

$$M_{[D\ F\ G]} = \begin{bmatrix} M^{11}_{[D\ F\ G]} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & M^{12}_{[D\ F\ G]} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & & & \dots \\ \dots & \dots & & & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & M^{NJ}_{[D\ F\ G]} \end{bmatrix} \tag{50}$$

$$M^{ij}_{[D\ F\ G]} = \mathbf{1}(T_{ij} > 0) \left(I_{T_{ij}} - \frac{1}{T_{ij}} i_{T_{ij}} i'_{T_{ij}} \right) \tag{51}$$

$$T_{ij} = \sum_{t=n_{i1}}^{n_{iT_i}} \mathbf{1}\left(J\left(i,t\right)=j\right) \tag{52}$$

where the function $\mathbf{1}\left(A\right)$ takes the value one if A is true, and zero otherwise; I_A is the $A \times A$ identity matrix and i_A is an $A \times 1$ vector of ones. Equations (49) - (52) imply that the least squares estimator of β can be obtained from a regression of y on X, both in deviations from match-specific means. That is, from the regression

$$y_{ijt} - \mu_y - \bar{y}_{ij\cdot} = \left(x_{ijt} - \mu_x - \bar{x}_{ij\cdot}\right)\beta + \nu_{ijt} \tag{53}$$

where $\bar{y}_{ij\cdot} = \frac{1}{T_{ij}} \sum_{t=n_{i1}}^{n_{iT_i}} \mathbf{1}\left(J\left(i,t\right)=j\right) y_{ijt}$, $\bar{x}_{ij\cdot} = \frac{1}{T_{ij}} \sum_{t=n_{i1}}^{n_{iT_i}} \mathbf{1}\left(J\left(i,t\right)=j\right) x_{ijt}$, and ν_{ijt} is a statistical residual.

The least squares estimators of the person, firm, and match effects solve the remaining normal equations from the partitioned regression:

$$\begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} X\hat{\beta} + \begin{bmatrix} D'D & D'F & D'G \\ F'D & F'F & F'G \\ G'D & G'F & G'G \end{bmatrix} \begin{bmatrix} \hat{\theta} \\ \hat{\psi} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} y \tag{54}$$

or,

$$\begin{bmatrix} \hat{\theta} \\ \hat{\psi} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} D'D & D'F & D'G \\ F'D & F'F & F'G \\ G'D & G'F & G'G \end{bmatrix}^{-} \begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} \left(y - X\hat{\beta}\right) \tag{55}$$

which implies that for each worker-firm match,

$$\begin{aligned} \hat{\lambda}_{ij} &\equiv \hat{\theta}_i + \hat{\psi}_j + \hat{\gamma}_{ij} \\ &= \bar{y}_{ij\cdot} - \bar{x}_{ij\cdot}\hat{\beta}\ . \end{aligned} \tag{56}$$

3.1.2 Identification

There are several approaches to decomposing $\hat{\lambda}_{ij}$ into person, firm, and match effects. All require additional identifying assumptions. To see why, notice there are only M cell means (the $\hat{\lambda}_{ij}$) on which to estimate $N + J + M$ person, firm, and match effects. This is frequently called an overparameterized model.

A common approach is to impose linear restrictions on the estimated effects. An often applied set of restrictions, frequently called Σ^*-restrictions (see e.g., Searle (1987, p. 328-

330)) is

$$\sum_{i=1}^{N}\hat{\theta}_i = 0 \text{ and } \sum_{j=1}^{J}\hat{\psi}_j = 0 \tag{57}$$

$$\sum_{i=1}^{I}\hat{\gamma}_{ij} = 0\ \forall j \text{ and } \sum_{j=1}^{J}\hat{\gamma}_{ij} = 0\ \forall i \tag{58}$$

within a connected group of data.[10] The Σ^*-restriction approach is not without drawbacks. An example of one such drawback is that the set of i appearing in the sum $\sum_{i=1}^{I}\hat{\gamma}_{ij} = 0\ \forall j$ is different for each j. Thus, for example, adding one more worker or firm to the sample can change the value of the estimated effects for all other connected workers and firms. This complicates any interpretation of the estimated effects. For this reason, I opt for an alternate identification strategy.

The approach I adopt is to assume the match effects are orthogonal to the person and firm effects. In this case, the match effects are identified whenever both $\hat{\theta}_i$ and $\hat{\psi}_j$ are identified. Identification conditions for $\hat{\theta}_i$ and $\hat{\psi}_j$ are given in Abowd et al. (2002). Briefly, these rely on a notion of connectedness in the data (see footnote 10). Graph theoretic methods are used to determine groups of connected persons and firms. When there are G mutually exclusive connected groups in the data, there are exactly $N + J - G$ identified person and firm effects.

Under the orthogonality assumption, it is possible to decompose $\hat{\lambda}_{ij}$ into person, firm, and match effects without computing the generalized inverse of the cross-products matrix in (55) by methods such as least squares conjugate gradient (LSCG) optimization. An algorithm for solving large two-factor analyses of covariance (with main effects only) via LSCG has been developed at the U.S. Census Bureau (see Abowd et al. (2002) for details). In principle, the complete set of effects $(\beta, \theta, \psi, \gamma)$ could be estimated via LSCG. However its current implementation at the Census Bureau is designed to solve for covariate, worker, and firm effects only. This necessitates a two-step procedure. I first estimate $\hat{\beta}$ from the regression (53). With $\hat{\beta}$ in hand, I compute $\hat{\lambda}_{ij}$ as in (56). To estimate the worker, firm, and match effects, I use LSCG to decompose $\hat{\lambda}_{ij}$ into worker and firm effects $\left(\hat{\theta}_i, \hat{\psi}_j\right)$, and a residual component $\hat{\gamma}_{ij}$ — the estimated match effect. By definition, $\hat{\gamma}_{ij}$ is orthogonal to $\hat{\theta}_i$ and $\hat{\psi}_j$.

[10] See Searle (1987) for a general discussion of connectedness. In labor market data, firms are connected by common employees; workers are connected by common employers. Abowd et al. (2002) develop a graph theoretic algorithm for finding connected groups of workers and firms in longitudinal linked employer-employee data.

3.1.3 Advantages and Disadvantages of Modeling the Person, Firm, and Match Effects as Fixed

The linear model approach to estimating fixed person, firm, and match effects has several advantages and disadvantages as compared to competing estimators. Under the orthogonality assumption it is feasible for very large databases, such as the one employed here. Conversely, on small samples or survey data which are not highly connected (that is, when G is large), there are few identified effects. A mixed model approach relies less heavily on connectedness, and in general requires less restrictive identification assumptions. Finally, though the estimates $\hat{\theta}_i, \hat{\psi}_j$, and $\hat{\gamma}_{ij}$ are inconsistent, they are unbiased.

3.2 Modeling Wages With Random Worker, Firm, and Match Effects

An alternative to the linear model methods discussed in the previous section is to adopt a mixed models approach. That is, to treat the person, firm, and match effects as random. Maintaining notation from the previous section, I estimate the linear mixed model:

$$y_{ijt} = \mu_y + (x_{ijt} - \mu_x)\,\beta + u_i\eta + \alpha_i + \psi_j + \gamma_{ij} + \varepsilon_{ijt} \tag{59}$$

where

$$\begin{aligned} \alpha_i &\sim N\left(0, \sigma_\alpha^2\right) \text{ iid} \\ \psi_j &\sim N\left(0, \sigma_\psi^2\right) \text{ iid} \\ \gamma_{ij} &\sim N\left(0, \sigma_\gamma^2\right) \text{ iid} \\ \varepsilon_{ijt} &\sim N\left(0, \sigma_\varepsilon^2\right) \text{ iid} \end{aligned}$$

and β and η are treated as fixed. The model can be estimated by standard techniques as described in Searle et al. (1992) or McCulloch and Searle (2001). Such methods include maximum likelihood (ML) or restricted maximum likelihood (REML). I use the latter.

The REML approach is to apply maximum likelihood to linear functions of y, say $K'y$, where K' is chosen so that $K'y$ contains none of the fixed effects (β and η). This yields REML estimates of the variance components (σ_α^2, σ_ψ^2, σ_γ^2, σ_ε^2). It is a standard result that the estimated variance components are invariant to the values of the fixed effects. Unlike ML, REML has the further advantage of explicitly taking into account degrees of freedom for the fixed effects.

With REML estimates of the variance components in hand one can compute estimates of the fixed effects and Best Linear Unbiased Predictors (BLUPs) of the random effects. Let

$$\Lambda = \sigma_\varepsilon^2 I_{N^*} \tag{60}$$

$$\Omega = \begin{bmatrix} \sigma_\alpha^2 I_N & 0 & 0 \\ 0 & \sigma_\psi^2 I_J & 0 \\ 0 & 0 & \sigma_\gamma^2 I_M \end{bmatrix} \tag{61}$$

and let $\hat{\Lambda}$ and $\hat{\Omega}$ denote the REML estimates of Λ and Ω. Rewrite (59) in matrix notation

$$y = X\beta + U\eta + D\alpha + F\psi + G\gamma + \varepsilon. \tag{62}$$

The BLUPs and estimates of the fixed effects solve the mixed model equations

$$\begin{bmatrix} \begin{bmatrix} X' \\ U' \end{bmatrix} \hat{\Lambda}^{-1} \begin{bmatrix} X & U \end{bmatrix} & \begin{bmatrix} X' \\ U' \end{bmatrix} \hat{\Lambda}^{-1} \begin{bmatrix} D & F & G \end{bmatrix} \\ \begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} \hat{\Lambda}^{-1} \begin{bmatrix} X & U \end{bmatrix} & \begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} \hat{\Lambda}^{-1} \begin{bmatrix} D & F & G \end{bmatrix} + \hat{\Omega}^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{\eta} \\ \hat{\alpha} \\ \hat{\psi} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} X' \\ U' \end{bmatrix} \hat{\Lambda}^{-1} y \\ \begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} \hat{\Lambda}^{-1} y \end{bmatrix} \tag{63}$$

It is worth noting I do *not* assume that the design of the random effects (i.e., $\begin{bmatrix} D & F & G \end{bmatrix}$) is orthogonal to the design of the fixed effects $\left(\begin{bmatrix} X & U \end{bmatrix}\right)$. Most econometric specifications of mixed models assume an orthogonal design. Although the orthogonal design assumption considerably simplifies solving the mixed model equations (63),[11] it is by no means necessary. Furthermore, the orthogonal design assumption is generally violated in economic data. Finally, note the Abowd et al. (1999) and Abowd et al. (2002) wage models with fixed person and firm effects are special cases of (62) with $\gamma = 0$ as $\Omega \to \infty$.

3.2.1 Advantages and Disadvantages of the Mixed Model Approach

Unlike the linear models of Section 3.1, mixed model estimation is generally limited to rather small samples. In the animal and plant breeding literature, where mixed models are commonplace, the solution has been to estimate variance components on a subset of the data, and use these to compute BLUPs and fixed effects for the entire sample. To its credit, however, the mixed model approach does not require restrictive identification assumptions such as those discussed in Section 3.1.2. Instead, prior information embedded in the distributional assumptions of (59) identifies the effects. Finally, although some connectedness is required to precisely estimate Ω, the degree of connectedness required is considerably less than in the linear model case.

A decided advantage of a mixed models approach is that one can accommodate much more general covariance structures than Λ and Ω. In particular, it is feasible to estimate the

[11] With an orthogonal design, $\begin{bmatrix} X' \\ U' \end{bmatrix} \begin{bmatrix} D & F & G \end{bmatrix} = 0$. Thus the off-diagonal blocks in (63) are zero, and one can solve for the BLUPs and fixed effects separately:

$$\begin{bmatrix} \begin{bmatrix} X' \\ U' \end{bmatrix} \hat{\Lambda}^{-1} \begin{bmatrix} X & U \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} \hat{\Lambda}^{-1} \begin{bmatrix} D & F & G \end{bmatrix} + \hat{\Omega}^{-1} \end{bmatrix} \begin{bmatrix} \beta \\ \eta \\ \alpha \\ \psi \\ \gamma \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} X' \\ U' \end{bmatrix} \hat{\Lambda}^{-1} y \\ \begin{bmatrix} D' \\ F' \\ G' \end{bmatrix} \hat{\Lambda}^{-1} y \end{bmatrix}. \tag{64}$$

complex within-match covariance structure predicted by the general form of the equilibrium wage function (34). I leave this for future work.

4 Data

Data in this study are from the Longitudinal Employer-Household Dynamics (LEHD) Program database, under development at the U.S. Census Bureau. The LEHD database includes seven states: California, Florida, Illinois, Maryland, Minnesota, North Carolina, and Texas. Together, these states represent about 60 percent of U.S. employment. In this paper, I use data from two of the seven LEHD states. The identity of the two states cannot be revealed for confidentiality reasons.

The LEHD data are administrative, constructed from quarterly Unemployment Insurance (UI) system wage reports. Every state in the U.S., through its Employment Security Agency, collects quarterly earnings and employment information to manage its unemployment compensation program. The characteristics of these UI wage data are detailed elsewhere, for example Stevens (2002) and Burgess et al. (2000). With the UI wage records as its frame, the LEHD data comprise the universe of employers required to file UI system wage reports — that is, all employment covered by the UI system in the seven participating states. The data span the first quarter 1990 through the fourth quarter 1999 (40 quarters).

Individuals are uniquely identified in the data by a Protected Identity Key (PIK). Employers are identified by a state unemployment insurance account number (SEIN). Unfortunately, the UI wage records contain only very limited information: PIK, SEIN, and quarterly earnings. In the LEHD database, additional demographic characteristics are integrated with these data from various internal Census Bureau sources. Such characteristics include sex, race, date of birth, and education.[12]

The dependent variable for the wage regressions is real full-quarter earnings. Individual i is identified as having worked a full quarter at SEIN j in quarter t if there are UI wage records for which $J(i,t-1) = J(i,t) = J(i,t+1)$. If the individual worked a full quarter at firm j in t, then the full-quarter earnings measure w_{ijt} is simply reported earnings (about 80 percent of the analysis sample). If the individual did not work a full quarter in t, and worked no full quarters at this employer in the previous or subsequent 4 quarters (a nine quarter moving window), the record was dropped (about 15 percent of all wage records). If the individual did not work a full quarter in t but did work at least one full quarter in the nine quarter window, one of two earnings measures was used. First, if reported earnings in quarter t were at least 80 percent of earnings in the full quarter, then reported earnings were treated as full-quarter earnings (about 12 percent of the analysis sample). If on the other hand reported earnings were less than 80 percent of those in the full quarter, earnings were

[12] Sex, race, and date of birth are based on an exact match to administrative data sources. Education is based on a statistical match.

imputed to the full-quarter level (about 7 percent of the analysis sample).

Missing data items are multiply-imputed using the Sequential Regression Multivariate Imputation (SRMI) method. See Rubin (1987) for a general treatment of multiple-imputation; the SRMI technique is due to Raghunathan et al. (1998); Abowd and Woodcock (2001) generalize SRMI to the case of longitudinal linked data. Missing data items include full-quarter earnings (in the case described above), full-time status, and education. I generate three implicates of the missing data items. The results presented in this draft are based on a single implicate only.

A dominant employer is identified for each individual in each quarter. Individual i's dominant employer in quarter t is the employer at which i's earnings were largest in t. The analysis sample is restricted to full-time private sector employees at their dominant employer, who worked at least one full quarter in the nine quarter window, between 16 and 70 years of age, with real quarterly earnings of at least \$250 (1982 dollars), employed in non-agricultural jobs which lasted at least 2 quarters, at SEINs with at least five employees. For the two states on which the analysis is performed, the resulting analysis sample consists of 190 million quarterly earnings observations on 10 million individuals employed at approximately 275,000 firms, with a total of over 18 million unique worker-firm matches.

Covariates X in the earnings regressions include year and quarter dummies, a dummy to indicate whether earnings were imputed to the full-quarter level (interacted with sex), and a quartic in labor force experience (interacted with sex). The experience measure is defined as follows. At each individual's first appearance in the data, labor force experience is defined as potential experience (age-education-6). The individual accrues 0.25 years of experience for each subsequent quarter of full-time employment. Covariates in U are education (five categories) interacted with sex.

Table 1 presents basic summary statistics. There are few surprises. Men comprise 55 percent of the sample. The average real full quarter earnings of women, \$8853, are approximately 61 percent of men's average earnings. On average, women have 6 months more labor force experience than men, and one year less education. Additional demographic information will be available in a future version of this paper.

5 Results

In this Section I report some very preliminary results of estimating the fixed wage model (45) and mixed wage model (59) on the LEHD database. Each model is estimated separately on each state. The resulting estimates are pooled to be representative of 1997 employment. As mentioned, the fixed model is particularly well suited to estimation on large databases. Consequently, the estimation is performed on the entire analysis sample. In comparison, the mixed model is very computationally intensive. For this reason, the mixed model estimation is performed on a 3 percent simple random subsample of individuals. The mixed model subsample consists of 4.6 million observations on 330,000 individuals employed at 97,000 firms, with a total of approximately 532,000 unique worker-firm matches.[13]

The data are highly connected. Considering these are universe data, this is not terribly surprising. For the complete analysis sample, the largest connected group in each state contains approximately 99.9% of observed jobs. Even in the mixed model subsample, the largest connected group in each state contains more than 90% of jobs.[14]

Figure 1 plots the estimated returns to experience under the two models. As is typically found, the returns to experience are lower for women than men and accrue more slowly to women than to men. That is, the women's profile is flatter and lies everywhere below that of men. At low levels of experience the fixed and mixed model profiles are very similar. However, above 10 years of labor force experience the profiles diverge markedly. At higher levels of experience, the mixed model attributes considerably less wage growth to experience than does the fixed model.

Lacking much else in the way of detailed demographic data, it is the person, firm, and match effects themselves which are of primary interest. However, for the fixed models I estimate nearly 30 million effects; nearly 1 million for the mixed model. Summarizing these in an informative fashion remains a challenge.

Table 2 presents correlations among the estimated wage components from both wage models. The data are duration weighted. All wage components exhibit a strong positive correlation with log earnings under both models. Among wage components, the person effect θ_i is most strongly correlated with log earnings: 0.64 in the fixed model, 0.69 in the mixed model. Match effects γ_{ij} exhibit a dramatically higher correlation with log earnings under the mixed model (0.59) than under the fixed model (0.23). This is presumably an artifact of the orthogonality condition that identifies the match effect in the fixed model. Both models yield person effects which are negatively correlated with time-varying observables $x_{ijt}\beta$. The negative correlation is considerably stronger in the fixed model (-0.30) than in the mixed model (-0.03). In contrast, in both models firm effects ψ_j are positively correlated with time-varying observables.

Recall that the matching model of Section 2 predicted that the estimated person and firm effects should be positively correlated. This is true under both the fixed and mixed specifications. The matching model also predicted a positive correlation between θ_i and γ_{ij}, and between ψ_j and γ_{ij}. In the fixed model, the match effects are orthogonal to the person and firm effects by design. In contrast, under the mixed model the match effects are positively correlated with person effects (0.52) and firm effects (0.05), as predicted. This suggests that the orthogonality assumption used to identify the match effect in the fixed model may be too restrictive. Certainly, it appears inconsistent with the matching model.

Table 3 presents the estimated variance components from the mixed model. Person effects exhibit the greatest variation ($\sigma^2_\alpha = 0.197$), followed by firm effects ($\sigma^2_\psi = 0.151$) and match

[13]BLUPs are only computed for the subsample.

[14]Exact counts of workers, firms, and jobs in each group are excluded for confidentiality reasons.

effects ($\sigma_\gamma^2 = 0.135$).[15] Coupled with the strong correlation between person effects and log earnings, the large variance component σ_α^2 suggests that personal heterogeneity is a more important contributor to earnings variation than are firm and match heterogeneity.

Figures 2 and 3 plot the means of the estimated wage components by observed job duration. As one would expect, longer lasting employment relations are on average associated with higher earnings. This is consistent with the matching model in Section 2. The matching model also predicts that on average higher values of person, firm, and match effects should be associated with longer job duration. This is true of effects estimated under the mixed model (Figure 3). However, under the fixed model the mean match effect is increasing in job duration only up to about 5 years, beyond which it declines.

Figures 2 and 3 both indicate that time-varying observables are important earnings determinants among low duration jobs, but less important in employment relations that last a long time. This is true for the mixed model moreso than for the fixed model. In contrast, the person effect is on average a relatively small component of earnings for low duration jobs, and the largest component among long lasting jobs. Under the mixed model, the match component is also a very important determinant of earnings among long lasting jobs. Finally, the firm effect appears to be a more important component of earnings among low duration jobs than among long lasting jobs.

6 Conclusion

I presented a matching model with heterogeneous workers, firms, and worker-fim matches. The model generalizes the seminal Jovanovic (1979) model to the case of heterogeneous workers and firms. The equilibrium wage is linear in a person-specific component, a firm-specific component, and a match specific component that varies with tenure. Under certain conditions, the equilibrium wage takes a simpler structure where the match specific component does not vary with tenure. I discussed fixed- and mixed-effect methods for estimating wage models with this structure on longitudinal linked employer-employee data. The fixed effect specification relies on restrictive identification conditions, but is feasible for very large databases. The mixed model requires less restrictive identification conditions, but is feasible only on relatively small databases. Both the fixed and mixed models generate empirical person, firm, and match effects with characteristics that are consistent with predictions from the matching model; the mixed model moreso than the fixed model. Shortcomings of the fixed model appear to be artifacts of the identification conditions.

The paper suggests several directions for future work. The first is to develop an estimator for the general case of the equilibrium wage function in the matching model. This is feasible but difficult due to a complicated within-match covariance structure. The mixed model

[15] I stress that these are estimates of the variance components from the mixed model (59), and not the structural variances defined in (2) and (3). Recovering structural parameters is left for future work.

can accommodate such a covariance structure, but not without a considerable increase in computational requirements. An alternative is to fit residuals from a simple regression of earnings on observables to this covariance structure, using some minimum distance estimator.

As mentioned at the outset, empirical equilibrium models with heterogeneous agents promise to shed new light on many labor market phenomena. This paper is primarily a methods piece and has not addressed specific labor market phenomena in any detail, though this promises to be a fruitful area for future research. Consider, for example, the source and determinants of inter-industry wage differentials. This remains one of the most pervasive and difficult to explain phenomena in the discipline. Along the lines of Abowd and Kramarz (1999), one could decompose observed inter-industry wage differentials into components due to workers, components due to firms, and components due to worker-firm matches. An application along these lines will be available in a future version of this paper.

A Appendix: Omitted Derivations of Value Functions

A.1 The Value of Unemployment

I begin by deriving the value of unemployment. Let $J_0(\theta_i)$ denote the expected present value of wages of a worker of quality θ_i who was unemployed last period and who is about to draw a match. Clearly, $J_0(\theta_i)$ is simply the expected value of $J[w(\Omega_{ij1})]$ before the match is formed. That is, before the identity of the matching firm j is known, and before the signal x_{ij} is observed. Then

$$\begin{aligned} J_0(\theta_i) &= E_0[1-G_1(\bar{m}_{ij})]w_{ij1} + Q_i E_0 G_1(\bar{m}_{ij}) \\ &\quad +\beta E_0[1-G_1(\bar{m}_{ij})][1-G_2(\bar{m}_{ij})]J[w(\Omega_{ij2})] + \beta Q_i E_0[1-G_1(\bar{m}_{ij})]G_2(\bar{m}_{ij}) \\ &= \int [1-G_1(\bar{m}_{ij})] \int\int w_{ij1} dF_\eta dF_\gamma dF_\psi + Q_i \int G_1(\bar{m}_{ij}) dF_\psi \\ &\quad +\beta \int [1-G_1(\bar{m}_{ij})][1-G_2(\bar{m}_{ij})] \int\int J[w(\Omega_{ij2})] dF_\eta dF_\gamma dF_\psi \\ &\quad +\beta Q_i \int [1-G_1(\bar{m}_{ij})] G_2(\bar{m}_{ij}) dF_\psi \end{aligned} \tag{65}$$

which accounts for the possibility that the signal x_{ij} is too low for the match to persist. Let $\Omega^0_{ij\tau} = (\theta_i, \psi_j, 0, s_\tau)$. When w is linear in $m_{ij\tau}$ we can write

$$\begin{aligned} E_0 w_{ij1} &= \int\int\int w_{ij1} dF_\eta dF_\gamma dF_\psi \\ &= \int w(\Omega^0_{ij1}) dF_\psi. \end{aligned} \tag{66}$$

When w is also independent of s_τ,

$$\begin{aligned} E_0 w_{ij\tau} &= \int w(\Omega^0_{ij\tau}) dF_\psi \\ &= \int w(\Omega^0_{ij1}) dF_\psi \text{ for all } \tau > 0. \end{aligned}$$

Consequently when Conjectures 1 and 2 are true,

$$
\begin{aligned}
& E_0 \left[1 - G_1 (\bar{m}_{ij})\right] \left[1 - G_2 (\bar{m}_{ij})\right] J \left[w (\Omega_{ij2})\right] \\
= & \int \left[1 - G_1 (\bar{m}_{ij})\right] \left[1 - G_2 (\bar{m}_{ij})\right] \int \int J \left[w (\Omega_{ij2})\right] dF_\eta dF_\gamma dF_\psi \\
= & \int \left[1 - G_1 (\bar{m}_{ij})\right] \left[1 - G_2 (\bar{m}_{ij})\right] \int \int \left[\begin{array}{c} w (\Omega_{ij2}) \\ +\beta \left[1 - G_3 (\bar{m}_{ij})\right] E_0 J \left[w (\Omega_{ij3})\right] \\ +\beta G_3 (\bar{m}_{ij}) Q_i \end{array} \right] dF_\eta dF_\gamma dF_\psi \\
= & \int \left[1 - G_1 (\bar{m}_{ij})\right] \left[1 - G_2 (\bar{m}_{ij})\right] w \left(\Omega^0_{ij1}\right) dF_\psi \\
& +\beta \int \left[1 - G_1 (\bar{m}_{ij})\right] \left[1 - G_2 (\bar{m}_{ij})\right] \left[1 - G_3 (\bar{m}_{ij})\right] \int \int E_0 J \left[w (\Omega_{ij3})\right] dF_\eta dF_\gamma dF_\psi \\
& +\beta Q_i \int \left[1 - G_1 (\bar{m}_{ij})\right] \left[1 - G_2 (\bar{m}_{ij})\right] G_3 (\bar{m}_{ij}) dF_\psi. \qquad (67)
\end{aligned}
$$

Forward recursion on (65) gives:

$$
\begin{aligned}
J_0 (\theta_i) = & \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int w \left(\Omega^0_{ij1}\right) \prod_{s=1}^{\tau} \left[1 - G_s (\bar{m}_{ij})\right] dF_\psi \\
& +Q_i \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int G_\tau (\bar{m}_{ij}) \prod_{s=1}^{\tau-1} \left[1 - G_s (\bar{m}_{ij})\right] dF_\psi. \qquad (68)
\end{aligned}
$$

Recall that Q_i is the value of the worker's outside option – that is, the value of being unemployed today and behaving optimally thereafter. Thus $Q_i = h_i + \beta\pi J_0 (\theta_i) + \beta (1 - \pi) Q_i$ where $\pi = \frac{1}{u} m (u, v)$ is the worker's probability of drawing a match. Using (68),

$$
Q_i = \frac{h_i + \beta\pi \left(\sum_{\tau=1}^{\infty} \beta^{\tau-1} \int w \left(\Omega^0_{ij1}\right) \prod_{s=1}^{\tau} \left[1 - G_s (\bar{m}_{ij})\right] dF_\psi\right)}{1 - \beta (1 - \pi) - \pi \sum_{\tau=1}^{\infty} \beta^{\tau} \int G_\tau (\bar{m}_{ij}) \prod_{s=1}^{\tau-1} \left[1 - G_s (\bar{m}_{ij})\right] dF_\psi}. \qquad (69)
$$

A.2 The Value of a Vacancy

I now turn to the value of a vacancy. Let $\Pi_0 \left(\psi_j\right)$ denote the expected present value of net revenues from a match for a firm of quality ψ_j before the identity of the matching worker i

is known. Then

$$\begin{aligned}
\Pi_0\left(\psi_j\right) &= E_0\left[1-G_1\left(\bar{m}_{ij}\right)\right]\left(q_{ij1}-w_{ij1}\right)+E_0 V_j G_1\left(\bar{m}_{ij}\right) \\
&\quad +\beta E_0\left[1-G_1\left(\bar{m}_{ij}\right)\right]\left[1-G_2\left(\bar{m}_{ij}\right)\right]\Pi\left[w\left(\Omega_{ij2}\right)\right]+\beta V_j E_0\left[1-G_1\left(\bar{m}_{ij}\right)\right]G_2\left(\bar{m}_{ij}\right) \\
&= \int\left[1-G_1\left(\bar{m}_{ij}\right)\right]\int\int\left(q_{ij1}-w_{ij1}\right)dF_\eta dF_\gamma dF_\theta+V_j\int G_1\left(\bar{m}_{ij}\right)dF_\theta \\
&\quad +\beta\int\left[1-G_1\left(\bar{m}_{ij}\right)\right]\left[1-G_2\left(\bar{m}_{ij}\right)\right]\int\int\Pi\left[w\left(\Omega_{ij2}\right)\right]dF_\eta dF_\gamma dF_\theta \\
&\quad +\beta V_j\int\left[1-G_1\left(\bar{m}_{ij}\right)\right]G_2\left(\bar{m}_{ij}\right)dF_\theta \qquad (70)
\end{aligned}$$

which reflects the possibility that the signal x_{ij} is too low for the match to form. Following Section A.1, we can write

$$\begin{aligned}
E_0 w_{ij\tau} &= \int w\left(\Omega^0_{ij\tau}\right)dF_\theta \\
&= \int w\left(\Omega^0_{ij1}\right)dF_\theta \text{ for all } \tau>0. \qquad (71)
\end{aligned}$$

and solve $\Pi_0\left(\psi_j\right)$ recursively to obtain

$$\begin{aligned}
\Pi_0\left(\psi_j\right) &= \sum_{\tau=1}^{\infty}\beta^{\tau-1}\int\left[\mu+\theta_i+\psi_j-w\left(\Omega^0_{ij1}\right)\right]\prod_{s=1}^{\tau}\left[1-G_s\left(\bar{m}_{ij}\right)\right]dF_\theta \\
&\quad +V_j\sum_{\tau=1}^{\infty}\beta^{\tau-1}\int G_\tau\left(\bar{m}_{ij}\right)\prod_{s=1}^{\tau-1}\left[1-G_s\left(\bar{m}_{ij}\right)\right]dF_\theta. \qquad (72)
\end{aligned}$$

Recall V_j is the value of firm j's outside option – that is, the value of a vacancy. Thus $V_j=-k_j+\beta\lambda\Pi_0\left(\psi_j\right)+\beta\left(1-\lambda\right)V_j$, where k_j is firm j's cost of maintaining a vacancy and $\lambda=\frac{1}{v}m\left(u,v\right)$ is the probability of a given vacancy being filled. Hence,

$$V_j=\frac{\beta\lambda\left(\sum_{\tau=1}^{\infty}\beta^{\tau-1}\int\left[\mu+\theta_i+\psi_j-w\left(\Omega^0_{ij1}\right)\right]\prod_{s=1}^{\tau}\left[1-G_s\left(\bar{m}_{ij}\right)\right]dF_\theta\right)-k_j}{1-\beta\left(1-\lambda\right)-\lambda\sum_{\tau=1}^{\infty}\beta^{\tau}\int G_\tau\left(\bar{m}_{ij}\right)\prod_{s=1}^{\tau-1}\left[1-G_s\left(\bar{m}_{ij}\right)\right]dF_\theta}. \qquad (73)$$

B Appendix: Proof of Lemma 3

Proof. To simplify notation, let $g_p = G_{\tau+p}(\bar{m}_{ij,\tau+p})$ for $p = 1, 2, 3, ...$. From the definitions in (28) and (29) — and without using Conjecture 1 or Conjecture 2,

$$
\begin{aligned}
A_{ij\tau} &= 1 + \sum_{s=\tau+1}^{\infty} \beta^{s-\tau} \prod_{r=\tau+1}^{s} [1 - G_r(\bar{m}_{ijr})] \\
&= 1 + \beta(1-g_1) + \beta^2(1-g_1)(1-g_2) + \beta^3(1-g_1)(1-g_2)(1-g_3) + ... \\
B_{ij\tau} &= 1 - \sum_{s=\tau+1}^{\infty} \beta^{s-\tau} G(\bar{m}_{ij}) \prod_{r=\tau+1}^{s-1} [1 - G(\bar{m}_{ij})] \\
&= 1 - \beta g_1 - \beta^2(1-g_1) g_2 - \beta^3(1-g_1)(1-g_2) g_3 + ...
\end{aligned}
$$

Now define the terms

$$
\begin{aligned}
a_0 &= 1 \\
a_1 &= \beta(1-g_1) \\
a_2 &= \beta^2(1-g_1)(1-g_2) \\
a_3 &= \beta^3(1-g_1)(1-g_2)(1-g_3)
\end{aligned}
$$

and so on, so that $A_{ij\tau} = \sum_{s=0}^{\infty} a_s$. Similarly, define

$$
\begin{aligned}
b_0 &= 1 \\
b_1 &= -\beta g_1 \\
b_2 &= -\beta^2(1-g_1) g_2 \\
b_3 &= -\beta^3(1-g_1)(1-g_2) g_3
\end{aligned}
$$

and so on, so that $B_{ij\tau} = \sum_{s=0}^{\infty} b_s$. Now notice that

$$
\begin{aligned}
a_1 &= \beta + b_1 \\
a_2 &= \beta^2 + \beta b_1 + b_2 \\
a_3 &= \beta^3 + \beta^2 b_1 + \beta b_2 + b_3
\end{aligned}
$$

and so on, so that

$$
\begin{aligned}
a_s &= \beta^s + \beta^{s-1} b_1 + \beta^{s-2} b_2 + \beta^{s-3} b_3 + ... + \beta b_{s-1} + b_s \\
&= \sum_{r=0}^{s} \beta^{s-r} b_r
\end{aligned}
$$

for $s = 0, 1, 2, 3, ...$. Thus,

$$\begin{aligned}
A_{ij\tau} &= \sum_{s=0}^{\infty}\sum_{r=0}^{s}\beta^{s-r}b_r \\
&= 1 + (\beta + b_1) + \left(\beta^2 + \beta b_1 + b_2\right) + \left(\beta^3 + \beta^2 b_1 + \beta b_2 + b_3\right) + ... \\
&= \left(1 + \beta + \beta^2 + \beta^3 + ...\right) + b_1\left(1 + \beta + \beta^2 + \beta^3 + ...\right) + b_2\left(1 + \beta + \beta^2 + \beta^3 + ...\right) + ... \\
&= \frac{1}{1-\beta} + \frac{b_1}{1-\beta} + \frac{b_2}{1-\beta} + ... \\
&= \frac{1}{1-\beta}\sum_{s=0}^{\infty} b_s \\
&= \frac{B_{ij\tau}}{1-\beta}
\end{aligned}$$

■

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Table 1: Summary Statistics

Variable	N	Mean	Std. Dev.
Men			
Full Quarter Earnings ($1982)	105,258,200	14428	40645
Labor Force Experience (Years)	105,258,200	19.3	11.43
Education (Years)	105,258,200	13.1	2.86
Earnings Imputed to the Full Quarter Level	105,258,200	0.076	0.264
Women			
Full Quarter Earnings ($1982)	85,100,389	8853	14334
Labor Force Experience (Years)	85,100,389	19.9	11.93
Education (Years)	85,100,389	12.2	4.47
Earnings Imputed to the Full Quarter Level	85,100,389	0.067	0.251
Complete Analysis Sample			
Number of Observations	190,358,589		
Number of Workers	10,337,435		
Number of Firms	274,884		
Number of Worker-Firm Matches	18,091,178		
Mixed Model Subsample			
Number of Observations	4,619,308		
Number of Workers	330,573		
Number of Firms	97,012		
Number of Worker-Firm Matches	532,104		

Table 2: Simple Correlations Among Wage Components (Duration-Weighted Data)

Fixed Model

	y	$X\beta$	θ	$U\eta$	α	ψ	γ	ε
Log(Real Full Quarter Earnings) (y)	1							
Time-Varying Observables ($X\beta$)	0.27291	1						
Pure Person Effect (θ)	0.64237	-0.29602	1					
Observable Component ($U\eta$)	0.30173	-0.09530	0.36922	1				
Unobservable Component (α)	0.57134	-0.29500	0.92934	0.00000	1			
Firm Effect (ψ)	0.45897	0.09373	0.04213	0.12355	-0.00375	1		
Match Effect (γ)	0.23495	0.00710	0.00000	0.00034	-0.00013	0.00158	1	
Residual (ε)	0.39243	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1

Mixed Model

	y	$X\beta$	θ	$U\eta$	α	ψ	γ	ε
Log(Real Full Quarter Earnings) (y)	1							
Time-Varying Observables ($X\beta$)	0.35538	1						
Pure Person Effect (θ)	0.69054	-0.03043	1					
Observable Component ($U\eta$)	0.39129	0.00134	0.49867	1				
Unobservable Component (α)	0.62475	-0.03546	0.84605	-0.04012	1			
Firm Effect (ψ)	0.53710	0.11405	0.15089	0.11806	0.10077	1		
Match Effect (γ)	0.58864	-0.00843	0.51988	-0.02028	0.61182	0.05525	1	
Residual (ε)	0.43263	0.00033	0.03770	0.00004	0.04343	0.01203	0.08102	1

Table 3: Estimated Variance Components

Source	Variance Component	Std. Err.
Person Effect (α)	0.197	0.0013
Firm Effect (ψ)	0.151	0.0018
Match Effect (γ)	0.135	0.0008
Residual (ε)	0.100	0.0001

Figure 1
Estimated Returns to Experience

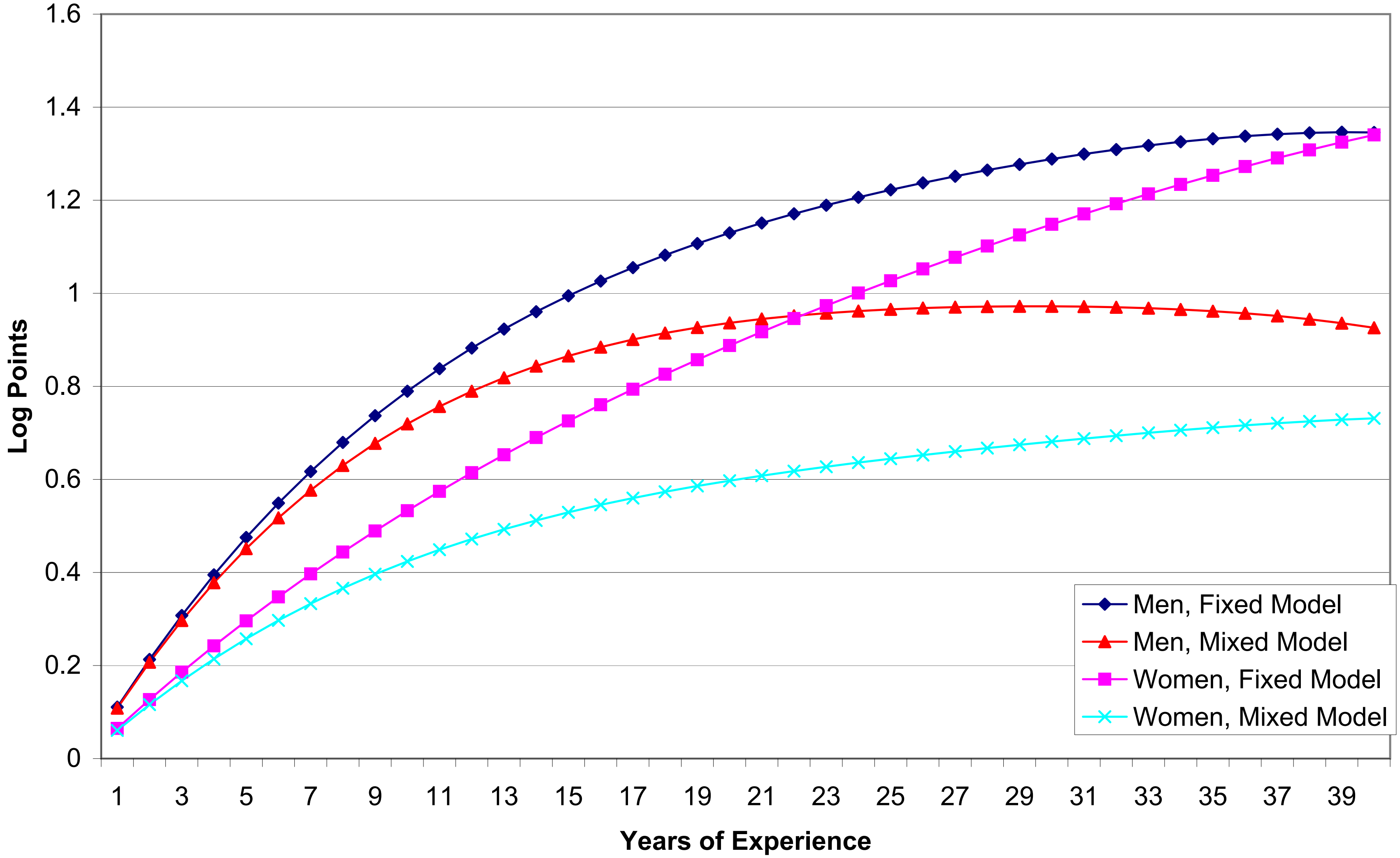

Figure 2
Mean of Estimated Effects by Duration, Fixed Model

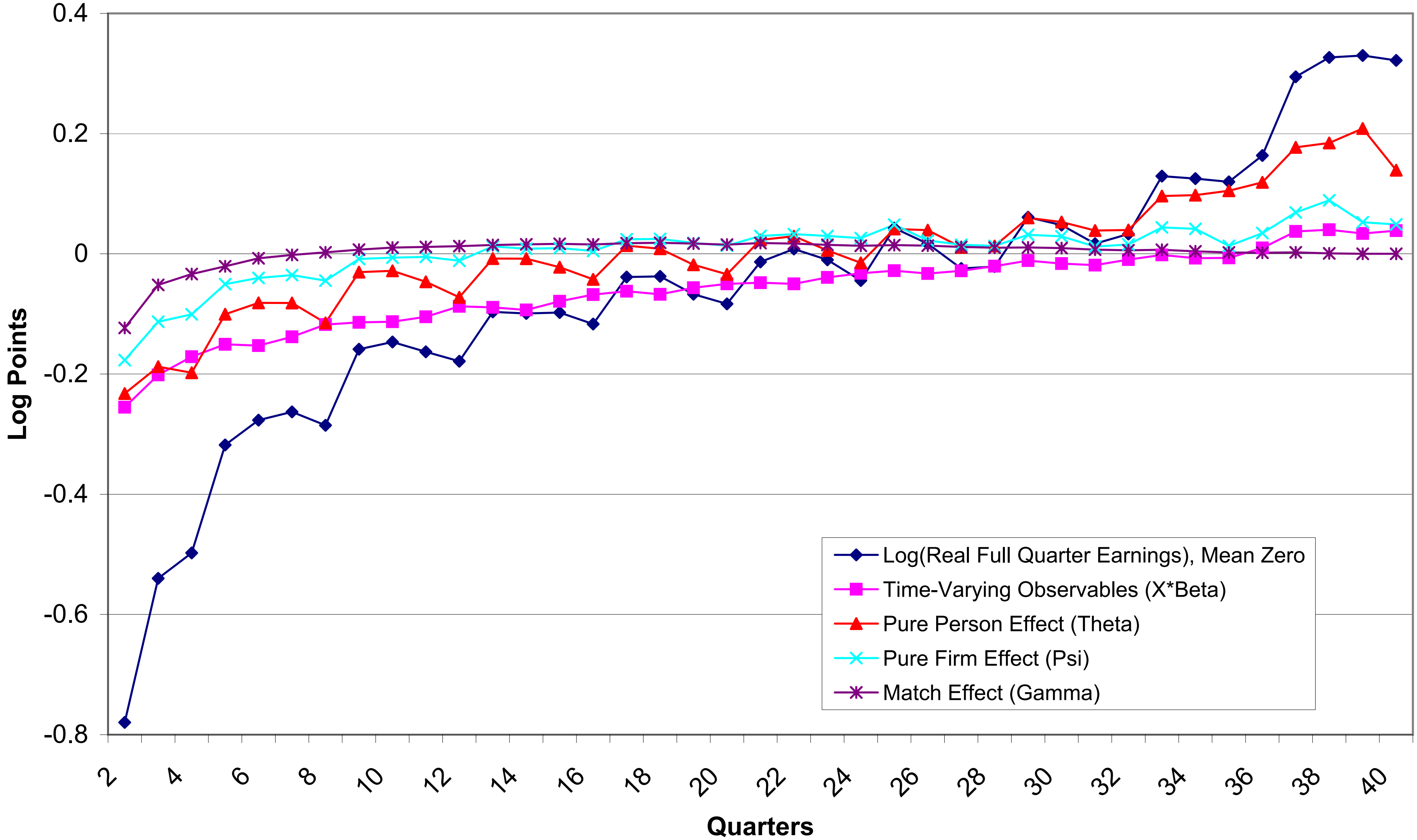

Figure 3
Mean of Estimated Effects by Duration, Mixed Model