Privatizing Social Security While Limiting Adverse Selection in Annuities Markets

by

Jan Walliser

Congressional Budget Office

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Mailing Address: Congressional Budget Office, Macroeconomic Analysis Division, 2nd and D Streets, S.W., Washington, D.C. 20515. E-mail: JanW@cbo.gov.

Abstract

This paper employs the heterogenous-agent life cycle model developed in Walliser (1997), which successfully generates the observable amount of loading in annuities markets under current policy, to explore the effect of two funded retirement systems with mandatory annuitization. The first system requires the annuitization of a retirement account accumulated with 5 percent of payroll and provides the same rate of return to all agents (income-proportional annuity). The second system converts the same retirement accounts into a flat annuity (demogrant) and thus redistributes from those with above average earnings to those with below average earnings.

The paper shows that the redistributive system may come at the cost of increasing prices in the private annuities market. On the other hand, an income-proportional annuity that causes a smaller increase in private annuity prices than a demogrant cannot offer an actuarially fair rate of return due to the correlation between income and mortality.

Without compensatory payments an income-proportional annuity is welfare enhancing for almost everybody, whereas a demogrant increases welfare for the poor but decreases the welfare of the rich. A demogrant may offset the redistributive effects of pooling caused by the positive correlation between income and longevity. Both systems improve economic efficiency over a purely private system without government regulation.

1. Introduction

With the growing understanding that Social Security's unfunded liabilities will be difficult to cover once the baby boomers retire, Social Security reform is moving to the center of the political discussion. The potential gains from a transition to a funded system with privately managed individual accounts has received much attention recently.[1] Funding social security may substantially increase economic output and economic efficiency and could thus be beneficial beyond solving the medium term problems of the current system.

What is missing from these studies is that Social Security's role comprises more than providing retirement income. The annuity provided by Social Security insures against longevity risk and inflation risk. If private annuities markets are not actuarially fair due to adverse selection, insurance against longevity risk may be very costly or unavailable. Any proposal to partly or fully privatize Social Security must therefore consider the regulatory question whether and how to mandate annuitization in a privatized social security system.

Recent proposals by the Quadrennial Social Security Advisory Council did not agree on the method of annuitization. The "Individual Account" Plan would mandate the annuitization of the entire retirement account. In contrast, the "Personal Security Account" Plan provides a mandatory flat annuity financed with a 5 percent payroll tax.[2]

This paper analyzes how mandatory annuitization affects annuity prices and welfare in a life cycle model with agents differing by life expectancy and income. The model -- developed in Walliser (1997) -- reflects the empirically observable correlation between income and mortality and matches some characteristics of observable annuity prices.

The paper investigates two systems that eliminate social security but mandate the accumulation of funds in a mandatory retirement account with five percent of payroll. The first system requires the annuitization of the entire account upon retirement and is non-redistributive. The retirement annuity is proportional to the funds accumulated in the retirement account. This set-up

1. See Feldstein (1996), Feldstein and Samwick (1997), Kotlikoff (1996a,b), Kotlikoff, Smetters, and Walliser (1997), Kotlikoff (1997).

2. The overall payroll tax would be 7.4 percent with 2.4 percent financing disability and survivor benefits.

is therefore labeled as an income-proportional annuity below.

The second system adds redistribution by converting the retirement account into the same annuity for everybody irrespective of the actual funds in the account.[3] Hence, those with above average income receive a below average return and vice versa. In the following discussion, that system is identified as a flat annuity or a demogrant.

This paper contrasts annuity prices and welfare in a fully privatized system with those in the two regimes outlined above. I show that a flat annuity may have a more detrimental effect on private annuity prices than an income-proportional annuity because income and mortality are correlated. However, a flat annuity always offers the highest return and can offset the negative welfare effect of pooling for the low earners. An income-proportional annuity on the other hand can remove only about two-thirds of the loading attributed to adverse selection.

Without compensatory payments an income-proportional annuity financed with 5 percent of payroll is welfare enhancing for almost all model agents as compared to an economy without mandates. The redistributive demogrant, in contrast, increases welfare only for the poor but decreases the welfare of the rich. On the other hand, a demogrant offsets the redistributive effects of pooling caused by the positive correlation between income and longevity. Both types of mandates improve economic efficiency compared to regimes without mandates.

The paper is organized as follows. Section 2 discusses previous theoretical findings concerning the welfare effects of mandatory insurance. Section 3 introduces a two period model of annuity demand and discusses the return of social annuities as well as the derivation of insurance load factors. Section 4 extends the simulation model to 75 periods (ages 25-100) and simulates the effects of the two regimes outlined above. Section 5 discusses different policy options in light of the evidence of the simulation model and summarizes and concludes the paper.

2. Annuity Insurance, Adverse Selection and the Pareto Efficiency of Mandates

3. Note that a demogrant that is not redistributive would have to be very small and therefore provide only minimal protection against outliving one's resources.

Annuities protect consumers against outliving their resources in retirement. By implicitly redistributing the estate of deceased annuitants to surviving annuitants, annuities can also generate a higher rate of return than alternate bequeathable investments. Outside of a bequest motive, the standard life cycle model with lifespan uncertainty therefore predicts that consumers annuitize all their assets.

Individuals who buy annuities generally consider their own survival prospects in choosing the size of the annuity. Thus, economic agents who expect to live longer, buy larger annuities. They are also more likely to demand private annuities in the first place. Generally speaking, insurance companies are unable to observe individual survival prospects -- because the information can be successfully disguised by prospective annuitants or because legislation prevents discrimination based on race and sex.

Consequently, insurance companies will not break even if they price annuities based on the average survival probabilities of the population. Those who buy annuities live longer than average and among those who buy annuities the consumers with the highest life expectancy buy the largest annuity. This phenomenon is called adverse selection.

Mandatory insurance may remedy the adverse selection problem. A mandatory insurance forces everybody to purchase an annuity, irrespective of his or her own survival prospects. Therefore, the pool of annuitants reflects the population as a whole and a mandatory insurance can offer a return based on the characteristics of the average annuitant. This results only holds, however, if the mandatory annuity does not vary with any characteristic correlated with longevity.

The mandatory annuity may provide only a portion of the total desired annuity income. Some individuals will therefore engage in the private annuities market even if the government provides a mandatory annuity to them. Clearly, prices in the private market will not be independent of the chracteristics of the mandatory annuity. This paper therefore investigates annuity prices in the private market under different mandatory regimes and compares them to a private annuity prices in an economy without mandates.

Previous papers on the welfare effects of annuity insurance find that for certain equilibria mandatory insurance may be Pareto improving. Eckstein, Eichenbaum, and Peled (1985) study the effects of mandatory insurance on welfare and annuity prices in a two period model with two types of agents. They find that mandated annuitization is always Pareto improving if the initial equilibrium without mandates takes the form of a Wilson (1977) pooling equilibrium. In this case mandatory insurance works as a screening device that enables private insurance companies to identify each type. In response, a separating equilibrium arises in the remaining private annuity market after the mandate is imposed. If the initial equilibrium is of the separating Rothschild and Stiglitz (1976) type, mandatory annuitization may increase or decrease welfare for the following reason: Pooling through a mandate always decreases the insurance premium for the low risk type but it may or may not relax the high risk type's quantity constraint on insurance. The second effect depends on the shape of the indifference curves. The mandate is only Pareto improving if it also relaxes the quantity constraint.

Eckstein, Eichenbaum, and Peled's results about the efficiency of mandated regimes rely on a number of implicit assumptions that support the existence of a Wilson or Rothschild-Stiglitz equilibrium.[4] Most importantly, the existence of these equilibria requires that each insurance company can perfectly monitor the insurance purchases of all customers. Insurers can then preclude customers from buying more than one contract.[5]

As I argue in Walliser (1997), this assumption seems less applicable to the annuity insurance market. First, contracts can be easily split between different insurers and therefore disguise financial status and effective demand. Second, contract clauses or copays that may support market separation and are common to health insurance or fire insurance, are of little practical value for annuity insurance. After all, it is unheard of that insurance companies threaten to reduce annuity payments after the annuitant improves his life style.

Instead of the Rothschild-Stiglitz equilibrium, this paper employs an equilibrium concept in which the insurer can only choose the premium of the annuity and the annuitant chooses the quantity. In effect, everyone is therefore placed in the same pool. Abel (1986) and Pauly (1974) also use this type of equilibrium. The lack of market separation coincides with the empirical finding that annuity

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4. Eckstein, Eichenbaum, and Peled (1985) do not discuss to what extent their results depend on their choice of equilibrium.

5. Aside from this important assumption, the characteristics of insurance market equilibria have been found to be dependent on their specific assumptions about the order of 'moves' in the contracting game, as Hellwig (1987) points out.

insurance companies charge a premium per dollar of monthly annuity payments and allow the customer to choose the size of the annuity.[6]

A price equilibrium implies that mandates improve welfare for those with shorter life expectancy and lower income but reduce the welfare of agents with longer lives and higher income. The reason for this result is that mandates increase prices for private annuities. The more agents continue to rely on private annuities in the presence of a social annuity the smaller are the possible welfare gains from mandatory insurance. In general, mandatory insurance may not be Pareto improving in a price equilibrium. Therefore, it is important to investigate possible welfare gains and losses as well as the impact of mandatory insurance on efficiency.

3. A Two Period Model of Annuity Demand

This section introduces the basic model used for the evaluation of adverse selection on annuity prices, annuity demand and welfare. See also Walliser (1997) for a more detailed discussion of assumptions.

3.1 The Basic Model

Consider an economy with N types of consumers, $j \in \{1,N\}$, who live for a maximum of two periods. Survival at the end of the first period of life is uncertain and occurs with probability π^j, $0 < \pi^j < 1$. Agents exogenously supply one unit of labor during their first period of life and receive labor income w^j. Survivors retire in the second period of their life. Note that each type j can therefore be perfectly characterized by the pair $\{\pi^j, w^j\}$. Income and survival probabilities may be correlated.

Consumers choose consumption in both periods $\{c_1, c_2\}$ to maximize expected utility from a time-separable utility function. Future utility is discounted with the pure rate of time preference δ,

6. However, this observation does not rule out the coalition proof Nash equilibrium.

$$(1) \qquad \max \; U(c_1^j) + \frac{\pi^j}{1+\delta} \; U(c_2^j).$$

Individuals can buy an annuity a^j which is paid in year *2* conditional on the agent's survival. An annuity a^j can be purchased in period *1* for a price of Z per dollar. Bonds in this economy offer a rate of return r. As mentioned above, if consumers do not value bequests, annuities dominate bonds and nobody holds bequeathable assets (Yaari, 1965).[7]

Government intervention may affect the intertemporal allocation of consumption. In particular, I consider three regimes: The first regime is purely private and has no mandates (NM), the second regime provides an income proportional annuity *PS* financed with a percentage *s* of labor income (SM), and the third regime provides a flat annuity also financed with a payroll tax (FM).[8] Pensions are paid in equal installments after retirement.

To evaluate the efficiency of different regimes, I employ a "lump-sum redistribution authority", which may compensate the losers of a regime change through lump sum transfers L from the winners.[9] All transfers occur in the first period of life and sum up to zero across all types of agents. Accordingly, the budget constraints under the three regimes can be characterized as follows:

$$(NM1) \qquad c_1^j \;=\; w^j - a^j Z^j + L^j$$

$$(NM2) \qquad c_2^j \;=\; a^j$$

7. This result depends on the assumption that agents do not have a bequest motive. Fischer (1973), Abel (1986) and Friedman and Warshawsky (1990) include a bequest motive in their respective models. On the other hand, Hurd (1989) does not find that a bequest motive is empirically relevant and asserts that people are in fact overannuitized by social annuities.

8. Note that the latter two regimes differ if households differ in their labor income.

9. See Auerbach and Kotlikoff (1987).

$$(SM1) \quad c_1^j = w^j(1-s) - a^j Z^j + L^j$$

$$(SM2) \quad c_2^j = a^j + PS^j$$

$$(FM1) \quad c_1^j = w^j(1-s) - a^j Z^j + L^j$$

$$(FM2) \quad c_2^j = a^j + PF$$

The size of each cohort's social annuities PS^j and PF is solely determined by the tax payments of the same cohort. The pension PS^j pays a multiple PSM of wage income w^j and is not redistributing income since everybody receives the same rate of return. In contrast, since the pension PF has the same size for everybody, it redistributes from richer to poorer members of the same cohort.[10] Let the pension PF be defined as multiple PFM of mean income.[11]

Calculating the implicit price of the social annuities for the government provides convenient measures for the cost advantage of mandatory insurance. The implicit premia -- Z^S for the income proportional social annuity and Z^F for the fixed social annuity -- are defined as the ratio of tax payments divided by the discounted expected annuity payments.[12]

If income and survival are positively correlated, the price Z^S per dollar of expected income-proportional social annuity payments exceeds the price Z^F per dollar of expected fixed annuity payments. The return Z^S on an income proportional social annuity is equal to:

10. Alternatively one could also devise a non-redistributive mandatory annuity of fixed value. However, since it would be based on the wages of the poorest members of each cohort, it would be unrealistically small.

11. Note that the rate of return for the income proportional annuity is identical for all participants whereas for the fixed annuity only the return at mean income represents the true return for the government since the system is redistributive.

12. This premium is not actually paid. Instead, it just reflects the overall internal rate of return of social annuities.

$$(2) \qquad Z^S = \frac{s}{PSM} = \frac{1}{R} \frac{\sum_{j=1}^{N} \epsilon^j \pi^j w^j}{\sum_{j=1}^{N} \epsilon^j w^j}$$

The equivalent expression for the fixed pension Z^F equals

$$(3) \qquad Z^F = \frac{s}{PFM} = \frac{1}{R} \sum_{j=1}^{N} \epsilon^j \pi^j = \frac{\bar{\pi}}{R}.$$

If $w^j = w$ for all types j both pensions have the same price since the wage drops out in the equation for Z^S. However if $\partial \pi^j/\partial w^j > 0$, then $\sum_j \epsilon^j \pi^j w^j > \sum_j \epsilon^j w^j \sum_j \epsilon^j \pi^j$ and it holds that $Z^S > Z^F$. Note that Z^F is the price based on average population mortality as also discussed in Walliser (1997).

If income and survival probabilities move together, even a mandatory annuity proportional to income cannot offer a rate of return based on average mortality. Clearly, the result follows because the government or other institution that provides the mandatory annuity needs to pay larger benefits to individuals who have a higher chance of survival.

Only a fixed annuity provides an overall return based on average survival probabilities. However, if the flat annuity is financed with a proportional payroll tax, this rate of return does not accrue to everybody. Individuals with above average income receive a lower return than implied by average mortality rates. On the other hand, individuals with below average income receive a higher rate of return than implied by average mortality rates. Nonetheless, measured as ratio of payouts to contribution, the flat annuity has a lower cost to the provider of the mandatory insurance since $Z^F < Z^S$.

Solving the intertemporal maximization problem as specified by the utility functions and the budget constraints yields an annuity demand function $a^j = a^j(Z^j, \pi^j, w^j, s)$ for each type j. I make the following assumptions:

A1. *The utility function is twice continuously differentiable with $U' > 0$ and $U'' < 0$.*

A2. *Annuity demand is non-negative, i.e. $a \geq 0$.*

Assumption 1 is standard and ensures that annuity insurance demand is continuous in Z.[13] Assumption 2 precludes the possibility that agents borrow and simultaneously buy life insurance.

Given the first two assumptions, annuity demand has to satisfy the following first order condition:

$$(4) \quad U'(c_1^j)\; Z^j \;\geq\; \frac{\pi^j}{1+\delta}\; U'(c_2^j)$$

Due to assumption 2, equation (4) may hold with inequality if agents receive larger social annuities than desired in regimes (SM) and (FM).

Lifetime resources are divided between the two periods of life according to the personal discount factor. The latter is a combination of the pure rate of time preference and the survival probability. With increasing survival probability, future consumption is discounted less and annuity demand increases. Moreover, annuity demand increases with lifetime income since higher wages imply higher consumption in both periods of life.

In addition to A1 and A2, I make the following assumptions:

A3. *Each type j comprises a share ϵ^j of the population with $\sum_j \epsilon^j = 1$.*
A4. *The rate of return r on riskless assets is fixed.*
A5. *Insurance companies have no information about the type of an annuity purchaser. They cannot efficiently monitor the overall amount of annuities purchased by their customers and can therefore not restrict the quantity purchased. Insurance companies can observe age and sex but not income.*

The fourth assumption excludes general equilibrium feedback effects. Assumption 5 generates the adverse selection problem by eliminating the first best outcome in which annuities can be purchased based on individual survival prospects.

By virtue of assumption 5 insurance companies offer annuity insurance and charge a

13. This is a standard textbook result of consumption theory.

premium Z per dollar of annuity payments in period 2. An annuity a^j sold to a type j customer thus generates expected profits $(Z-\pi^j/R)\, a^j$, the premium received less expected payouts.

In a competitive insurance market, the premium will be sufficient for companies to break even across all types of customers in equilibrium. Accordingly, Z is implicitly defined by the zero profit condition,

$$(5) \qquad P(Z) = \sum_{j=1}^{N} \epsilon^j \; (Z-\pi^j/R) \;\; a^j(Z,\pi^j,w^j,s) = 0.$$

As discussed in Walliser (1997), there exists always at least one annuity premium for which insurance companies break even. If social annuities are sufficiently large, the solution to (5) may be the trivial solution with no annuities demanded. The equilibrium may, in general, not be unique and depends on the specifics of the utility function and the corresponding behavior of annuity demand functions.

The equilibrium premium for an annuity as derived from equation (5) will be higher than a premium based on average survival probabilities.[14] This result is clearly related to adverse selection: individuals with higher life expectancy buy larger annuities. In addition, due to the positive correlation between income and longevity, agents with longer than average life expectancy occupy a larger share of the market. This outcome contributes to higher prices.

How can we measure adverse selection in the described modeling framework? The previous discussion about the price of mandatory annuity insurance provides two simple approaches to calculate load factors.[15] The deviation of the annuity premium Z^* from the 'fair' premium equals simply $Q^F = Z^*/Z^F$ since, as established by equation (3), the return of a fixed mandated annuity coincides with the return based on average mortality. This figure can then be directly compared to empirical load factors.

However, the load factor Q^F reflects two reasons why annuity prices deviate from 'fair' prices based on average mortality: 1) Higher life expectancy increases annuity demand, and 2) annuity

14. See Abel (1986) for a formal proof.

15. See also Friedman and Warshawsky (1988) and Walliser (1997).

demand rises proportionally with income. The second effect alone will cause loading if income and longevity are positively correlated. A measure for the cost of an income-proportional annuity is conveniently provided by Z^S. Consequently, the load factor that corrects for the effect of higher income on demand can be defined as $Q^S = Z^*/Z^S$.

By comparing Q^S and Q^F one can assess to which extent the income mortality correlation matters. If Q^S substantially exceeds Q^F then the income mortality correlation plays an important role in explaining the observable loading of annuities. As a consequence, simply mandating private accounts that are proportional to income would not remove a significant portion of the measured adverse selection. Walliser (1997) shows that Q^S may reach almost 40 percent of Q^F in a calibrated life cycle model.

Clearly, many question related to mandatory annuitization involve changes in welfare. The next two equations derive the marginal change in welfare for the two mandatory regimes. Using the first order condition (4), it can be shown that independent of the form of the utility function the marginal welfare change at $s = 0$ depends on expressions

$$(6) \qquad [(Q^S - 1)\ w^j - a^j\ \frac{\partial Z^*}{\partial s}]$$

and

$$(7) \qquad [Q^F\ \bar{w}\ -\ w^j\ -\ a^j\ \frac{\partial Z^*}{\partial s}]$$

for the two types of social annuities, respectively. $\bar{w}$ corresponds to the average wage, which determines the rate of return if the mandatory annuity is flat. Annuity demand is measured before the introduction of social annuities.

Two opposing effects influence the sign of the marginal change in utility. Equation (6) shows that an income-proportional annuity expands available resources by providing a higher rate of return than the market annuity. This fact is captured by the load factor $Q^S > 1$. For the fixed annuity, this result only holds for individuals whose income does not exceed the product of mean income and the load factor Q^F. The second effect on welfare stems from the reaction of private annuity prices to the introduction of mandatory annuities and is negative in both cases.

Quite clearly, the welfare changes are determined by the weight of private annuities. An agent with high demand for private annuities is more likely to suffer from a mandate at the margin since he complements the social annuity with a private annuity. The price of the private annuity, in turn, rises with the mandate. In case of the income proportional social annuity welfare losses are likely for individuals with high survival probabilities who purchase more private annuities. In case of the fixed annuity the redistribution leads to further welfare losses although the rate of return on the mandatory annuity is higher.

To conclude, in an economy with a positive correlation between income and mortality, welfare losses after mandating annuitization are likely for individuals with high income and higher survival probabilities. On the other hand, welfare gains accrue for those with smaller survival probabilities for whom private markets matter less. Redistribution renders these effects more pronounced if the annuity is flat.

3.2. Some Results for the Log Utility Case

To solve analytically for the annuity premium in a private annuity market, I assume in the following that utility is logarithmic. Accordingly, the lifetime utility function can be expressed as:

$$(8) \qquad U = \log(c_1^j) + \frac{\pi^j}{(1+\delta)} \log(c_2^j).$$

Annuity demand can then be derived from the first order condition and the budget constraints provided above. In absence of social annuities we obtain:

$$(9) \qquad a^j = \frac{w^j \pi^j}{(1+\pi^j+\delta)\ Z}$$

Annuity demand increases with survival probabilities and income and decreases with the price of annuities.

Solving algebraically for Z from equation (5) we obtain the following unique equilibrium annuity insurance premium for the log utility case:

$$(10) \qquad Z^* = \frac{1}{R} \frac{\sum_{j=1}^{N} \epsilon^j \pi^j w^j \left(\frac{\pi^j}{1+\pi^j+\delta} \right)}{\sum_{j=1}^{N} \epsilon^j w^j \left(\frac{\pi^j}{1+\pi^j+\delta} \right)}$$

As Appendix 1 shows formally, the premium for a private annuity without government intervention will be less favorable than both kinds of mandatory insurance. It holds that $Z^* > Z^S \geq Z^F$.

How will social annuities affect the prices of private annuities? As shown by Abel (1986), it is a general property of the price equilibrium that annuity prices react to government intervention because the change of private annuity demand differs by survival probabilities. This property is in sharp contrast to the Rothschild-Stiglitz separating equilibrium. More specifically, the annuity demand functions in the regime with an income-proportional mandate can be derived as follows:

$$(11) \qquad a^j = \frac{w^j \pi^j \ (1-s) - w^j s Q^S (1+\delta)}{(1+\delta+\pi^j) \ Z}$$

as long as this expression is non-negative. Note that the second term includes the load factor characterizing the percentage difference between the cost of an income proportional mandatory annuity and private annuities. Because of the lower costs of the mandatory annuity, private annuity demand decreases more than one for one if annuitization is mandatory.

A similar result holds for the private annuity demand if the mandatory annuity is fixed. The demand can be derived as follows:

$$(12) \qquad a^j = \frac{w^j (1-s) \ \pi^j - Q^F \ \bar{w} s \ (1+\delta)}{(1+\delta+\pi^j) \ Z}$$

as long as the expression is non-negative. In case of the flat annuity the reduction in demand does not depend on type *j*'s own wage rate but on the average wage rate in the economy. Therefore, under a demogrant, the decrease in private annuity demand will be stronger for types with below average income. However, it will be weaker for types with above average income than for the income-proportional annuity.[16]

How annuity demand changes at the margin if annuitization is mandatory follows directly from the demand functions . Equation (13) shows the marginal change in demand for a mandatory income proportional annuity. Equation (14) displays the same expression for the flat mandatory annuity.

$$(13) \qquad \frac{\partial a^j}{\partial s}\Big|_{s=0} = -w^j \frac{(\pi^j + Q^S(1+\delta))}{(1+\delta+\pi^j)\ Z^*}$$

$$(14) \qquad \frac{\partial a^j}{\partial s}\Big|_{s=0} = -\bar{w}\ \frac{(\frac{w^j}{\bar{w}}\pi^j + Q^F(1+\delta))}{(1+\delta+\pi^j)\ Z^*}$$

In both cases the reduction in demand is higher for individuals with lower survival probabilities: the derivative of (13) and (14) with respect to π is negative. Thus, in both cases the prices for private annuities must increase after introducing a mandatory annuity since the insurance firm will incur losses at the previous equilibrium price. A mandatory annuity therefore drives disproportionately more demand of types with shorter live expectancy out of the private annuities market.

It follows as a second result that prices for private annuities will increase more if the mandatory annuity is flat and income and longevity are positively correlated. To see that note that the reduction in demand depends on mean income if the mandatory annuity is flat. Therefore, low income and low survival types will decrease their demand more as compared to an income-proportional annuity. The opposite is true for the high income types with higher life expectancy. As a consequence, the original equilibrium premium for private annuities will cause higher losses if the

16. Private annuity prices in presence of social annuities can be derived from a quadratic equation (see Appendix 2) using equations (11) and (12).

mandatory annuity takes the form of a demogrant. Expressed differently, private annuity prices must rise more if the mandatory annuity is flat.

This result is of practical importance for the design of a private retirement system replacing Social Security. For example, the "Private Security Account" Plan suggested by Advisory Council members Schieber and Weaver, would not require annuitization of the private retirement accounts. However, it provides a flat annuity for everyone participating in the system. The Schieber-Weaver plan may have the potential to increase the price of private annuities compared to an economy without a demogrant. However, the flat annuity may offer better protection to low income households than an equally financed income-proportional plan since the flat annuity increases the implicit rate of return for types with low income.

Table 1 illustrates the previous results with a few examples. The pure rate of time preference is set to 0.35, and the interest factor to 2.5, consistent with a 1 percent pure rate of time preference and a 3 percent interest rate over a 30-year-period. The mean wage rate is set to 1. There are nine types of economic agents with survival probabilities 0.1 to 0.9, the mean survival probability is 0.5. Survival probabilities are assumed to be either distributed uniform or Normal. The Normal distribution is considered with standard deviations of 0.1 and 0.2. Finally, if income and longevity are correlated I assume that for each increase of income by 0.1, survival probabilities increase by 0.05.[17] The type with mean income thus also has the mean survival probability.

The table shows the 'conventional' load factor Q^F and also the income corrected load factor Q^S. Recall that Q^F represents the percentage increase of private annuity prices over the rate of return the government could offer on average on the flat mandatory annuity. Q^S shows how much more expensive the private annuity is compared to an income-proportional mandatory annuity.

If income and longevity are not correlated, both types of load factors coincide and there is also no difference between the fixed and the income proportional social annuity. For this reason, the respective values are omitted. As the first data column shows, load factors increase sharply with the introduction of social annuities. If income and longevity are positively correlated the flat annuity clearly has a much more detrimental effect on private annuity prices than the income-proportional

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17. Since empirically other factors than income influence mortality the assumed linear relationship will overstate the true income mortality correlation.

annuity.

The percent increase of annuity prices caused by the correlation between income and mortality can be substantial. Without any mandate, between 25 and 40 percent of the loading is attributable to the income - mortality correlation in this example. With mandatory insurance, the difference between Q^S and Q^F shrinks. This implies that a larger portion of the remaining adverse selection stems from the income mortality correlation.

The increase in load factors for the flat annuity is much more pronounced than for the income-proportional annuity because the flat annuity drives the low income - high mortality types entirely out of the annuity market. Consequently the measured loading increases more sharply since low mortality types begin to dominate the market.

4. Mandatory Insurance, Annuity Prices and Welfare

4.1 Extending the Model to 75 Years

<u>The Model</u>

The model of annuity demand can now be extended to 75 years (ages 25 to 100). A type *j* consumer maximizes the following utility function:

$$(15) \qquad \sum_{t=1}^{75} U(c_t^j) \prod_{s=1}^{t-1} \frac{\pi_s^j}{(1+\delta)}$$

where π_t^j stand for the probability that type *j* lives to from age *t* to age *t+1.* The maximization is subject to a sequence of budget constraints evolving as follows:

$$(16) \qquad c_t^j = w_t^j(1-s) + a_t^j - a_{t+1}^j Z^* + PS_t^j + L_t^j$$

if the mandatory annuity is proportional to income. A similar constraint holds for the demogrant by replacing PS^j with PF. Annuity demand is zero in the last period of life.

The size of the mandatory annuity is determined such that expected contributions are sufficient to finance expected payments in present values. For the demogrant, it must hold that:

$$(17) \qquad \sum_{t=1}^{44} \sum_{j=1}^{N} \epsilon_1^j \, sw_t^j \prod_{s=1}^{t-1} \frac{\pi_s^j}{(1+r)} = \sum_{t=45}^{75} \sum_{j=1}^{N} \epsilon_1^j \, PF \prod_{s=1}^{t-1} \frac{\pi_s^j}{(1+r)}$$

which can be easily solved for PF. A similar relationship holds for the mandatory annuity PS^j by replacing PF with PS^1. In the latter case (17) can be solved by recognizing that PS^1 is a multiple of PS^1. The size of the payment PS^1 simply depends on the relative tax payments made by type j compared to type 1. Note also that the calibrated model below maintains the social security payroll tax ceiling.

In extending the two period model to 75 years of life, an additional assumption is required:

A6. *Survival probabilities are strictly ranked across types.*

Assumption 6 ensures that a type j with higher chances to live from age 50 to age 51 than type k, say, also has a higher probability to live from age 51 to 52 than type k and so on. This assumption is satisfied if socioeconomic characteristics shift survival probabilities proportionally. This assumption ensures that individuals cannot be made better off by designing a multi-period contract among a subgroup of types. The type with the highest survival probability has the highest survival probability throughout all periods of life. He would therefore always prefer to not reveal his longevity characteristics to reduce the cost of his annuity premium. The same holds for the type with the next highest survival probability and so on.

Under assumptions A1 to A6 the multi-period equilibrium for annuities can be defined as a sequence of single period contracts satisfying equation (5). Hence, the existence of the multi-period equilibrium follows from the existence of a zero profit equilibrium for each period.

The price of an annuity spanning several time periods can then be calculated from the sequence of prices prevailing in each period. In particular, the premium PR_t of a \$1 life annuity in time t generated by the model can be derived as:

$$(18) \quad PR_t = Z_t^* + Z_t^* Z_{t+1}^* + Z_t^* Z_{t+1}^* Z_{t+2}^* + \ldots$$

Accordingly, the actuarial fair premium PR_t^F can be calculated as:

$$(19) \quad PR_t^F = Z_t^F + Z_t^F Z_{t+1}^F + Z_t^F Z_{t+1}^F Z_{t+2}^F + \ldots$$

and the percentage deviation of annuity prices from actuarial fairness ("load factor") is just the ratio of of the expression in equations (18) and (19). A similar derivation holds for the annuity price that uses an income weighted average population survival probability.

Calibration

The extended model contains roughly 1450 types, corresponding to 50-year-old individuals for whom survival probabilities are predicted with coefficients from an estimation by Lillard and Panis (1996). Survival probabilities depend on family permanent income, marital status, educational attainment, and race. In addition, survival probabilities take the mortality time trend and also the probability of spousal death into account. Survival probabilities derived from this calibration correspond closely to those life tables. Labor income profiles have the same concavity as profiles estimated by Altig et al. (1997). However, income profiles pass through the observable CPS labor income, and are corrected for hours worked and taxes. A couple's labor income is divided equally between both, reflecting the assumption that both spouses have access to half of the couple's resources. Population weights for age 50 individuals are from the CPS and evolve according to survival probabilities over time. For a more detailed description of the calibration see Walliser (1997).

4.2. Results

<u>Annuity Prices</u>

How will government intervention affect annuity prices? As discussed in the previous section, theory predicts that mandatory insurance exacerbates adverse selection as measured with the load factor. This section employs the extended 75-period life cycle model to compare different regimes of mandatory insurance with purely private markets.

An important and previously ignored question is whether a mandatory annuity would pool males and females. Currently, the annuity provided by Social Security does not differentiate the benefits by sex of the beneficiary. Social Security thus implicitly offers a higher rate of return to women because women live longer on average than men. Private annuities markets, in the contrary, tend to charge different premia to male and female annuitants. On the other hand, group annuities apply unisex premia following a Supreme Court decision which considered separate life tables as sex discrimination.

Since the regulatory framework in a system with mandatory annuities is not obvious, this paper compares three different regimes. As displayed in Table 2, the first regime assigns separate mandatory and private annuities to males and females. The second regime assigns a mandatory annuity that pools males and females but allows the private market to stay separated. Table 2 shows the outcome of the second regime for males. The third regime pools males and females for the mandatory annuity and also enforces pooling in the private market.

As outlined before, the mandates can take the form of a demogrant -- a flat annuity of equal value for all -- or a mandatory annuity that is proportional to income. Table 2 exhibits the load factors for these two cases in the last two columns. The other three data columns show the load factors under a purely private system, the "corrected" load factor under a private system, and the load factor under the current Social Security system, respectively.[18] The "corrected" load factor in column 2 shows how much of the load factor in column 1 is due to the positive correlation between income and longevity.

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18. See Walliser (1997) for the calculation of load factors under the current system.

Load factors increase with the age of the annuitant and are smaller for females than for males. Load factors decrease with the rate of risk aversion since higher risk aversion puts a larger segment of the population into the annuities market. See also Walliser (1997).

The increase in private annuity prices caused by a demogrant financed with 5 percent of payroll is similar to price increases caused by the existence of Social Security. Take, for example, the load factor for the separated market. If the risk aversion parameter is set to 2, the model predicts a load factor of 7.6 percent for a 65- year-old male if he receives payments from a demogrant. In comparison, it predicts a load factor of 7.2 percent if benefits correspond to current law. For a 65 year old female the load factors are identical under current law and with a demogrant.

This result follows from the progressive nature of the demogrant which drives low earners with shorter life expectancy out of the private annuities market. The benefits paid by the demogrant -- about $900 per month for men and $690 for women if benefits differ by sex or $790 per month for a unisex mandatory annuity -- are lower than social security benefits for many.[19] However, benefits are higher for the very poor who tend to have the worst longevity prospects. Consequently, consumers at the bottom of the income scale withdraw in even larger numbers from the private annuities market. On the other hand, the relatively small size of the demogrant leads to higher annuity demand of low to middle income earners who receive more generous benefits in the current system. On net, this change in demand patterns causes a similar size of the load factor under a demogrant as compared to social security.

The income-mortality correlation tends to impact annuity prices more under a demogrant than under social security. This result can be inferred by comparing load factors for males and females. Take the 65-year-old male and female with risk aversion parameter 2, for example. A comparison of columns 1 and 2 reveals why the positive correlation between income and longevity is important. For males, about 28 percent of the loading in an entirely private market is due to the correlation, for females the same figure is just 16 percent. Turning to columns 3 and 4, the results show that the load

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19. Note that the monthly benefits generated by the model are higher than the demogrant financed with 5 percent of payroll under the Schieber-Weaver plan. Two reasons are behind these differences: First, the Schieber-Weaver Plan finances the demogrant on a pay-as-you-go basis with an implicit rate of return between 1 and 2 percent whereas the flat benefits in my model is paid for by accumulated contributions which receive a rate of return of 3 percent. Second, my model does not reflect interruptions in earnings histories. At least one family member is continuously employed between ages 25 and 65.

factors for males under a demogrant are higher than with social security. In the contrary, the load factors for women are less or equal to load factors in a system with social security. The demogrant drives more low income and high mortality types out of the market for males than for females and correspondingly increases the load factor more.

The load factor of annuities sold to males is smaller once the demogrant pays the same benefit to males and females. This result is shown in the third panel of Table 2. As mentioned above, the unisex demogrant is smaller than the demogrant for males but higher than the demogrant for females. The difference follows largely from differences in longevity and partly because single women tend to have lower income than single men. Because of the smaller demogrant, less low income males drop out of the private annuities market and the pool of annuity purchasers contains more low risk types for the annuity provider.

The opposite holds for the market for females (not displayed in Table 2). The larger unisex demogrant drives more women with shorter life expectancy out of the market and hence increases the load factor.

Finally, if both the private market and the demogrant pool women and men -- a case shown in the fourth panel of Table 2 -- loading falls somewhere in between the loading for males and females in separate markets. Again, the load factors for the demogrant are very close to load factors under the current Social Security system.

Consider next how load factors develop if the mandatory annuity is proportional to income and financed with 5 percent of payroll. The results are displayed in the last column of Table 2. The income proportional annuity increases loading as compared to a purely private market, however the increase is smaller than the increase caused by a demogrant. This finding is in line with the theoretical derivations in the previous section.

The increase in the load factor induced by the income-proportional annuity is in most cases less than half of the increase caused by a demogrant. For example, for 65-year-old males with a risk aversion parameter of 2, the load factor increases from 5.7 percent to 6.3 percent if the mandatory annuity rises proportionally with income. If the mandatory annuity is flat, the load factor rises from 5.7 to 7.6 percent. If males and females are pooled in the private market, the increase in load factors under the income-proportional mandate accounts for more than half of the increase under the demogrant.

Again, the income longevity correlation is important in understanding the size of the load factor increase. The price increase in the annuities market for females is much less pronounced than in the annuities market for men. This result is related to the income mortality correlation. If lower survival probabilities are less strongly connected with income the annuity demand of shorter lived agents falls to a lesser extent. Put differently, the annuity demand reduction is less concentrated among low earners, which in turn keeps the composition of annuitants closer to the average.

To conclude, any regulatory framework that includes mandatory annuitization has strong repercussion on prices of private annuities. A demogrant financed with 5 percent of payroll would likely not reduce the loading of annuities below currently observed levels. It increases the cost of insurance against lifespan uncertainty for middle and high earners, since the benefits provided by the demogrant are below current social security benefits. An income-proportional mandatory annuity can mitigate the increases in annuity prices. However, benefits for low earners may be substantially smaller than under social security. If the mandatory annuity pools men and women but does not enforce pooling in private markets, the increase in load factors is reduced in the annuities market for males and exacerbated in the annuities market for females. If private markets are also forced to treat men and women identically, load factors settle somewhere in between the load factors for separate markets.

Welfare

The previous findings -- while important for assessing the impact of mandates on private annuity prices -- provide little information on who wins and who loses compared to an entirely private system. Table 3 exhibits changes in lifetime utilities for married agents differentiated by income level. Note that all comparisons abstract from any transition.

A pooled annuities market redistributes from the poor to the rich. This result can be inferred from the difference in lifetime utility between the first best where everybody can buy annuities based on individual survival prospects and a pooled market as outlined above. The change in welfare is displayed in column 1 of Table 3. Low income males and females both lose almost 4 percent of their lifetime income by being forced to pay the same annuity premium as everybody else in a pooled market. Middle and high earners, on the other hand, gain in a private market. This result again illustrates the strong impact of the income mortality correlation. As a consequence of this correlation, a pooled private annuities market implicitly redistributes from the poor to the rich.

Married men with income of around $30,000 already profit from being pooled in a private market. Due to the strong effect of marital status their survival probabilities are already above the average for men. For women, gains kick in at somewhat higher incomes since a married woman with income of $30,000 still has survival prospects slightly below average.

An income-proportional annuity is welfare improving for all agents displayed in Table 3 as long as mandatory and private annuity either both differentiate by sex or both don't differentiate by sex. As columns 2 and 4 show, replacing a sex-specific income-proportional annuity with a private market separated by sex, would reduce lifetime utility by about 0.3 percent for males and 0.2 percent for females. A very similar result holds if a unisex mandatory annuity is replaced with a private market charging the same premium to males and females.

The welfare gain stems from the higher return offered by the mandatory annuity. As outlined in section 3, welfare changes follow from two opposing effects: On the one hand, a mandatory annuity is cheaper. On the other hand mandates increase the prices for private annuities. Apparently, for all types represented in Table 3 the higher return of the income proportional mandatory annuity more than outweighs the higher prices of private annuities.

If a unisex mandatory annuity is replaced by a separated private market, men would prefer the private market. This finding -- shown in column 6 -- clearly follows from the implicit redistribution from males to females by a unisex mandatory annuity. The redistribution amounts to around 0.6 percent of lifetime income. Males life shorter lives than females on average and therefore get below average payouts if pooled with women. Consequently, if men are able to purchase an annuity in the private market based on their own survival prospects, they prefer the private annuity over the mandatory annuity. Obviously, women who gain from the unisex treatment prefer the mandatory annuity.

An income-proportional annuity is not Pareto improving. Although all agents who are displayed in columns 2 and 4 of Table 3 gain from a mandatory income proportional annuity there are still a few consumers who lose -- mostly those with very high incomes who hold the largest annuity wealth. This group profits most from lower load factors in private annuities markets.[20]

However, this does not rule out an increase in efficiency. In order to investigate efficiency properties of mandates, the "lump sum redistribution authority" is employed: All the losers of the regime change are compensated such that all the winners end up with an identical change in utility. If the utility still increases the regime change allows to compensate the losers and can still make everybody else better off. If after compensation the former winners are worse off, then the reform decreases economic efficiency. The efficiency calculations also account for the feedback effect of the redistribution on annuity prices.

An income-proportional annuity is efficient. The gains of the very rich are not large enough to compensate all the losers when abandoning the mandate. Hence, the income proportional mandate still improves economic efficiency. This result also holds if the mandate is unisex and private markets are separated for males and females since the welfare gains of males are not sufficient to offset the welfare losses of females.

A demogrant improves welfare for the poor but reduces the welfare of the middle and high earners. Columns 3, 5, and 7 of Table 3 exhibit the respective welfare changes. Due to the redistribution involved in the payroll tax finance of the demogrant, the poorest can lose up to 11 percent of lifetime income when the demogrant is abolished. Individuals with mean income however would already gain from removing the demogrant. Welfare gains are even higher for high earners. They do not rise monotonically with income, however, due to the payroll tax ceiling. If the demogrant pools men and women but the private market is separated, the welfare changes also reflect the change in redistribution between sexes.

The higher overall return a demogrant can offer due to the pooling of the population is for most individuals more than offset by the redistributive effect of the demogrant. Thus, most individuals would prefer the private market over the demogrant. Note, though, that the demogrant can offset the implicit redistribution from poor to rich that is induced by pooling in the private market. In the current example the demogrant more than offsets the redistribution.

A demogrant is efficient. If the demogrant is abolished, the welfare gains of the middle and

20. See equations (6) and (7) for the underlying formal arguments.

high earners are not sufficient to fully offset the gains of the losers. This holds despite the reduction in private annuity prices. Again, this finding follows from the higher economic return of a demogrant, which may not accrue to everybody in terms of higher benefits but still accrues to society as a whole. Thus, by abolishing the mandate, the fall in prices for private annuities is not large enough to compensate for the higher return of the mandatory insurance. The results holds for all cases shown in Table 3.

5. Is There a Superior Way to Overcome Adverse Selection?

Mandatory annuitization increases the cost of private annuities. The mandatory annuity replaces a larger proportion of private annuity demand for those with shorter life expectancy. As a result, the market share of consumers with longer lives increases and thus the equilibrium price rises.

Income-proportional mandatory annuities increase private annuity prices to a much lesser extent than a demogrant. This finding follows from the positive correlation between income and longevity. A redistributive demogrant reduces the private annuity demand of the poor and short lived to a larger extent than an income proportional mandatory annuity. This difference in the demand composition affords a higher price for private annuities under a demogrant than an income-proportional annuity.

Neither of the two considered mandates is Pareto improving. In contrast to findings by Eckstein, Eichenbaum, and Peled (1985), mandatory insurance does not improve welfare for everybody. However, an income proportional mandate reduces the welfare of only a few as compared to a private market without mandates. A demogrant increases the welfare of the poor and decreases the welfare of the rich.

Both forms of mandates improve economic efficiency. If the mandates were replaced by a private market the winners could not compensate the losers and still be better off. Therefore, a regulatory framework without mandatory annuitization forfeits possible efficiency gains for the economy.

Neither of the two mandates considered in this paper is superior to the other. However, both mandates raise efficiency over a purely private market without mandates. The income-proportional mandate has the advantage of mitigating the effect of mandates on private annuity premia. However, this type of mandate cannot offset the redistribution from poor to rich that is implied by pooling low and high income individuals. A demogrant's redistributive feature may be attractive because it offsets the effect of the income mortality correlation. For practical purposes, both types could also be combined in order to target the desired amount of redistribution.

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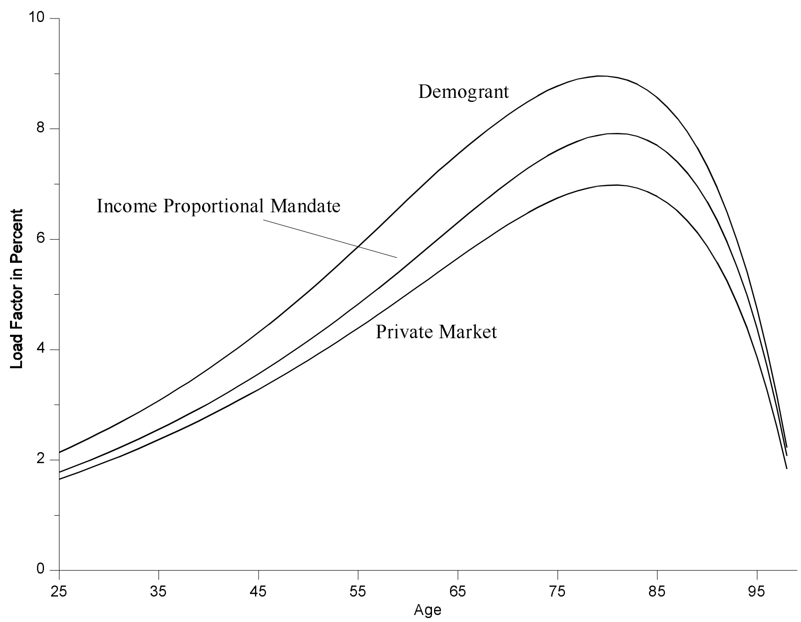
Figure 1: Load Factors for Private Annuities for Males under Different Regimes, r = 0.03, δ = 0.01, γ = 2.



Appendix 1.

Proposition: In a regime without social annuities, the premium for private annuities will always exceed the premium for social annuity and it holds that $Z^* > Z^S \geq Z^F$ if income and survival are not negatively correlated.

Proof : Recall the expressions for Z^S and Z^F from equations *(2)* and *(3)*. $Z^S \geq Z^F$ follows directly from earlier results. What remains to be shown is that $Z^* > Z^S$. Let's slightly reformulate equation (10):

$$(1A) \qquad Z^* = \frac{1}{R}\left(\frac{\sum_{j=1}^{N} \epsilon^j \pi^j w^j}{\sum_{j=1}^{N} \epsilon^j w^j} + \frac{\sum_{j=1}^{N} \epsilon^j \pi^j w^j \left(\frac{\pi^j}{1+\pi^j+\delta} - 1 \right)}{\sum_{j=1}^{N} \epsilon^j w^j \left(\frac{\pi^j}{1+\pi^j+\delta} - 1 \right)} \right)$$

Since $\frac{\pi^j}{1+\pi^j+\delta} < 1$ for all j, clearly the second fraction is always positive and $Z^* > Z^S$.

Appendix 2.

For a two period, N type model with heterogeneity in income the annuity premium for an income proportional social annuity can be calculated analytically as long as annuity demand is positive (otherwise an iterative strategy is needed).

$1/Z^* = R^A$ is the positive root of the following quadratic equation:

$$(A2) \qquad a(R^A)^2 + bR^A + c = 0$$

with:

$$(A3) \qquad a = \sum_{j=1}^{N} \frac{\epsilon^j w^j (1-s)(\pi^j)^2}{1+\delta+\pi^j}$$

$$(A4) \qquad b = -\sum_{j=1}^{N} \frac{\epsilon^j \left(Rw^j \pi^j (1-s) + \pi^j w^j s \dfrac{1+\delta}{Z^S} \right)}{1+\delta+\pi^j}$$

$$(A5) \qquad c = \sum_{j=1}^{N} \frac{R\epsilon^j w^j s \; (1+\delta)}{Z^S (1+\delta+\pi^j)}$$

(A similar derivation holds for the fixed social annuity by substituting mean income for $w^j s$).