# Comparison of Two Methods for Calculating Mean Solar Radiative Fluxes in Two-Layer Broken Clouds (Visible Range)

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## Introduction

Cloud layers may simultaneously occur at different atmospheric levels, and two or more cloud layers may not be uncommon during passage of atmospheric fronts or disintegration of massive cumulus or cumulonimbus clouds. Frontal cloud layering throughout the troposphere over the European part of the former USSR was studied by Baranov (1983): on an annually mean basis, the clouds occur in a single layer in 40% to 50% of the cases, in two layers in about 30% to 40% of the cases, and in three or four layers in 10% to 30% of the cases.

Presently, well-developed methods exist for calculating radiation effects of *horizontally homogeneous* multilayer clouds (Lenoble 1990). However, when clouds in at least one layer are broken, the radiation calculations use approximate methods developed within the framework of deterministic radiative transfer theory. This circumstance makes them especially attractive for practical usage, such as for calculating shortwave influxes in atmospheric general circulation model (GCM) radiation codes. The approach is seriously deficient in that the methods have never been subjected to rigorous tests.

Using the Poisson model of broken clouds, we obtained equations for mean intensity and developed a Monte Carlo algorithm for calculating the mean fluxes of solar radiation in two-layer clouds. The purpose of this work is to test, against the algorithm proposed here, the accuracy of the approximate method for calculating the mean fluxes in two-layer broken clouds. This will ascertain the applicability range of a horizontally homogeneous cloud model for the class of problems dealing with calculations of the energy characteristics of two-layer clouds.

# Model

The cloudy-aerosol atmospheric model is defined in the height interval $0 \le z \le H^{t}_{atm}$ as K separate atmospheric layers (Figure 1). A unit solar flux is incident on the atmospheric upper boundary $z = H^{t}_{atm}$ in the direction $\vec{\omega}_{\oplus} = (\xi_{\oplus}, \varphi_{\oplus})$, where $\xi_{\oplus}$ and $\varphi_{\oplus} = 0$ are zenith and azimuthal solar angles.

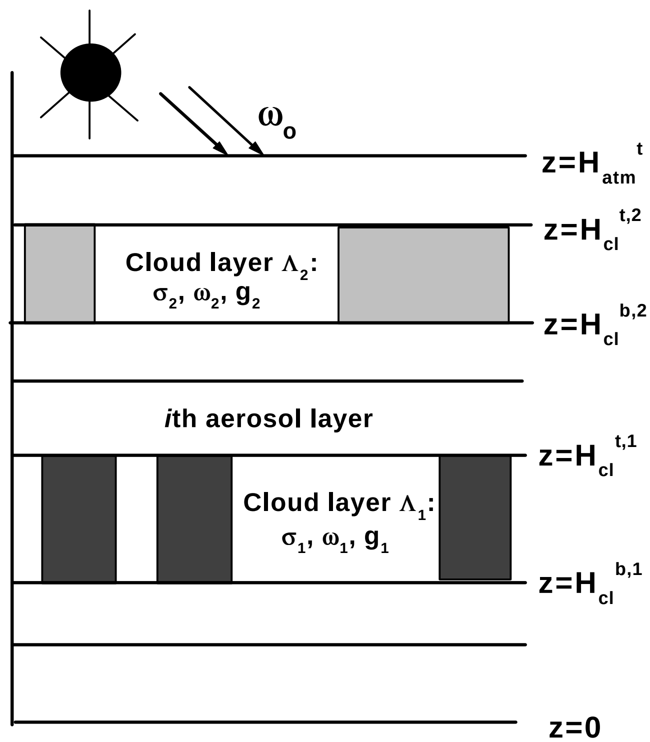


**Figure 1**. Schematic illustration of the cloudy aerosol atmosphere containing two layers of broken clouds.

*Cloud model.* Clouds occupy two separate layers $\Lambda_i$, i = 1,2, with subscript i = 1 corresponding to the lower, and $i = 2$ to the upper layer. Each cloud layer is characterized by the heights $H^{b,i}_{cl}$ and $H^{t,i}_{cl}$ of the bottom and top boundaries: $H^{b,i}_{cl} \le z \le H^{t,i}_{cl}$, i = 1,2; $H^{t,1}_{cl} \le H^{b,2}_{cl}$.

Within $\Lambda_i$, the optical model is specified in terms of the random scalar fields of the extinction coefficient $\sigma_{\lambda,I}(r)\kappa_i(r)$, single scattering albedo $\omega_{\lambda,i}(r)\kappa_i(r)$, and scattering phase function $g_{\lambda,i}(\omega, \omega',r)\kappa_i(r)$, i = 1,2; the subscript "$\lambda$" stands for a wavelength. The random fields $\kappa_1(r)$ and $\kappa_2(r)$ are assumed independent of each other; and the mathematical model of $\kappa_1(r)$, i = 1,2, is constructed based on the Poisson point fluxes on the straight lines (Titov 1990). Optical characteristics inside an individual cloud are assumed constant.

Each i[th] *aerosol layer* is assumed horizontally homogeneous, and it is characterized by the extinction coefficient $\sigma^{a}_{\lambda,i}$, single-scattering albedo $\omega^{a}_{\lambda,i}$, and scattering phase function $g^{a}_{\lambda,i}(\vec{\omega},\vec{\omega}')$.

The *underlying surface* reflects the incident radiation according to Lambert law and has the albedo $A_s$.

## Calculation Methods

**Method of closed equations.** In an earlier work, based on the Poisson model of ***one-layer*** broken clouds, Titov (1990) derived a system of closed equations for mean intensity in the case of statistically homogeneous cloud fields, and developed an algorithm of its solution by the Monte Carlo method (MCE). Here those results are generalized to the case for ***two-layer*** broken clouds under the assumption that the random fields $\kappa_i(r)$, i = 1,2, are independent and statistically homogeneous.

**Approximate calculation technique.** The approximate method for calculating radiative fluxes under conditions of two-layer broken clouds is defined by the formula:

$$F = K_{clr=1,clr=2} \times F_{clr=1,clr=2} + K_{pp=1,clr=2} \times F_{pp=1,clr=2} + K_{clr=1,pp=2} \times F_{clr=1,pp=2} + K_{pp=1,pp=2} \times F_{pp=1,pp=2}\,.$$

Here $F_{clr=1,clr=2}$, $F_{pp=1,clr=2}$, $F_{clr=1,pp=2}$, $F_{pp=1,pp=2}$ are clear-sky ("clr") and overcast ("pp") values of F in one- and two-layer clouds, calculated from a deterministic radiative transfer equation. They are used with weights, chosen in accordance with an employed hypothesis of cloud overlap. Of three known hypotheses, i.e., of minimum, maximum, and random overlap, GCMs generally use the latter two or their combination. A combined use assumes maximum overlap for contiguous cloud layers (e.g., those located at the same atmospheric level) and random overlap for widely separated layers (e.g., those located at different atmospheric levels) (Mokhov et al. 1994).

Let $N_1$ and $N_2$ be the cloud fractions in the lower and upper cloud layers, respectively. Then, the upward/downward solar radiative fluxes $F^{\uparrow(\downarrow)}$ at an altitude level z for the hypothesis of *random* overlap are given by formula:

$$\begin{aligned} F^{\uparrow(\downarrow)}_{rand}(z) = {} & (1 - N_1) \times (1 - N_2) \times F^{\uparrow(\downarrow)}_{clr=1,clr=2}(z) \\ & + N_1 \times (1 - N_2) \times F^{\uparrow(\downarrow)}_{pp=1,clr=2}(z) \\ & + N_2 \times (1 - N_1) \times F^{\uparrow(\downarrow)}_{clr=1,pp=2}(z) \\ & + N_1 \times N_2 \times F^{\uparrow(\downarrow)}_{pp=1,pp=2}(z) \end{aligned}$$

# Calculation Results

The mean fluxes in two-layer broken clouds are calculated by the method of closed equations, $F_{MCE}^{\uparrow(\downarrow)}$, and by the approximate method using the hypothesis on random cloud overlap, $F_{rand}^{\uparrow(\downarrow)}$. The relative differences between radiative fluxes $F_{MCE}^{\uparrow(\downarrow)}$ and $F_{rand}^{\uparrow(\downarrow)}$ will be characterized by the quantity

$$\delta F^{\uparrow(\downarrow)}(z) = 100\% \times \left(F_{rand}^{\uparrow(\downarrow)}(z) - F_{MCE}^{\uparrow(\downarrow)}(z)\right) / F_{MCE}^{\uparrow(\downarrow)}(z).$$

**Cloud Parameters**. Fluxes of visible solar radiation are calculated for characteristic optical and geometrical parameters of typical cloud systems (St) – (As), (St) – (Ci), (Cu) – (As), (Cu) – (Ci) at midlatitudes of the Northern Hemisphere (Handbook of clouds and the cloudy atmosphere, 1989), wavelength $\lambda = 0.69$ $\mu$m. The scattering phase function of water clouds was calculated from Mie theory for cloud $C_1$, and the flux calculations for ice clouds use a scattering phase function of randomly and horizontally oriented hexagonal ice crystals (Takano and Liou 1989). Single scattering albedo $\omega_i = 1$, $i = 1,2$. Aspect ratio $\gamma_i$ $H_i$ / $D_i$ (where $H_i$ and $D_i$ are the thickness and characteristic horizontal size of the cloud elements of the $i^{th}$ cloudy layer) in most calculations varies in the range $0 \le \gamma_i \le 2$, $i = 1,2$. Aspect ratios $\gamma << 1$ correspond to stratus, and $0.5 \le \gamma \le 2$ to convective clouds. Surface albedo $A_s = 0$.

**Comparison of Calculated Results**. To better understand the radiative transfer in two-layer clouds, we will consider two extreme cases, when one layer is overcast while the other is broken.

*Case 1*. Suppose that $N_1 = 1$ and $N_2 < 1$, which means that the broken cloud layer $\Lambda_2$ is located over a horizontally homogeneous reflecting surface $\Lambda_1$. We will consider cloud cases corresponding to intermediate cloud fractions $N_2$. As calculations showed, at $\xi_\oplus \le 75°$ and for optical depths $10 \le \tau_1 \le 40$, $1 \le \tau_2 \le 25$ ($\tau = 1$ for ice clouds and $\tau \ge 4$ for water clouds) the values of $\left|\delta F^{\uparrow}\left(H_{cl}^{t,2}\right)\right|$ and $\left|\delta F^{\downarrow}\left(H_{cl}^{b,1}\right)\right|$ do not exceed 5% to 8% when $0 \le \gamma_2 \le 2$ (Figure 2a). The difference between mean upward fluxes $F_{MCE}^{\uparrow}$ and $F_{rand}^{\uparrow}$ at the level $z = H_{cl}^{t,2}$ translates into the difference of up to ≈-20% between $F_{MCE}^{\uparrow}$ and $F_{rand}^{\uparrow}$ at the level of the bottom boundary of the layer $\Lambda_1$ (Figure 2b).

*Case 2*. Suppose that $N_2 = 1$, $N_1 < 1$ (placement of overcast cloud layer $\Lambda_2$ over the broken layer $\Lambda_1$ is equivalent to redefining boundary conditions for underlying cloud layer and switching from a monodirectional source of radiation to a diffuse one).

When the upper layer $\Lambda_2$ is optically thin, it is expected that part of the comparison between the two methods of mean flux calculations can be made using earlier results of Skorinov and Titov (1984).

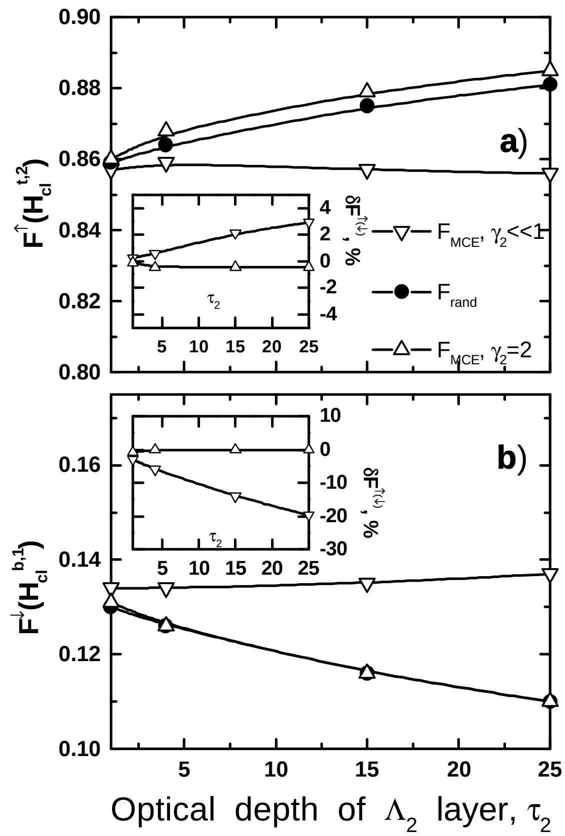


**Figure 2.** Mean upward and downward radiative fluxes and relative difference $\Delta F^{\uparrow(\downarrow)}$ (in %) at the boundaries of (a) top cloud layer $\Lambda_2$ and (b) bottom cloud layer $\Lambda_1$ for $N_1 = 1$, $\tau_1 = 40$, $N_2 = 0.5$, $\xi_{\oplus} = 75°$, and different aspect ratios $\gamma_2$.

Indeed, consider two typical cloud systems, (St) – (Ci) and (Cu) – (Ci). When $\gamma_1 << 1$, over almost entire ranges of input model parameters ($10 \le \tau_1 \le 40$ and $\xi_{\oplus} \le 75°$) $\left|\delta F^{\uparrow}\left(H_{cl}^{t,2}\right)\right|$ and $\left|\delta F^{\downarrow}\left(H_{cl}^{b,1}\right)\right|$ do not exceed 5%. If the lower layer is occupied by cumulus clouds, at small and intermediate cloud fractions $F^{\uparrow}\left(H_{cl}^{t,2}\right)$ and $F^{\downarrow}\left(H_{cl}^{b,1}\right)$ differ stronger between the models: at $\gamma_1 = 2$, $\delta F^{\uparrow}\left(H_{cl}^{t,2}\right) \approx -(10\% \text{ to } 15\%)$ and $\delta F^{\downarrow}\left(H_{cl}^{b,1}\right) \approx 10\% \text{ to } 30\%$.

Now we assume that the upper cloud layer has large optical depth: $\tau_2 = 25$ (middle-level *As* clouds). The upward flux at the level of the top boundary of two-layer clouds $z = H_{cl}^{t,2}$ consists both of photons scattered only within $\Lambda_2$ and photons participating in radiative exchange between the cloud layers. As calculations show, the layer $\Lambda_2$ by itself contributes to $F^{\uparrow}\left(H_{cl}^{t,2}\right)$ more than 70% to 90%. As a consequence, $F^{\uparrow}\left(H_{cl}^{t,2}\right)$ values, calculated by different methods, agree well over almost the entire range of input model parameters: $\left|\delta F^{\uparrow}\left(H_{cl}^{t,2}\right)\right| \leq 5\%$. For intermediate values of optical depth $1 < \tau_2 < 25$, the difference $\left|\delta F^{\uparrow}\left(H_{cl}^{t,2}\right)\right|$ may reach 10%, and the largest $\delta F^{\downarrow}\left(H_{cl}^{b,1}\right)$ value may be ≈-(10÷15)%.

We now assume that the upper and lower layers are filled *with partial cloudiness*: $N_1 < 1$, $N_2 < 1$. Calculations are made for cloud fractions $N_i$, $i = 1,2$, most typical for low-, middle-, and high-level clouds in midlatitude summer of the northern hemisphere (Meleshko 1980): 0.3÷0.5 for low clouds, 0.2÷0.3 for midlevel and high clouds.

For cloud system (St) – (Ci) the relative difference $\delta F^{\uparrow(\downarrow)}$ does not exceed 10%. Increasing aspect ratio $\gamma_1$, i.e., switching to the system (Cu) – (Ci), has diverse effects on $\delta F^{\uparrow(\downarrow)}$: $\delta F^{\uparrow(\downarrow)} \leq 10\%$ for $\tau_1 = 10$ and $\xi_{\oplus} = 30°$; however $\delta F^{\uparrow(\downarrow)}$ increases to tens of a percent when $\tau_1 = 40$ and $\xi_{\oplus} = 75°$. For instance, at $\gamma_1 = 2$, $N_1 = 0.5$ and $N_2 = 0.3$, $\delta F^{\uparrow}\left(H_{cl}^{t,2}\right) \approx -25\%$, while $F_{rand}^{\downarrow}\left(H_{cl}^{b,1}\right)$ greatly exceeds $F_{MCE}^{\downarrow}\left(H_{cl}^{b,1}\right)$: $\delta F^{\downarrow}\left(H_{cl}^{b,1}\right) \approx 60\%$ (Figure 3).

## Conclusion

Mean fluxes of solar radiation in two-layer broken clouds are calculated using two methods: 1) an approximate method (based on the assumption of random cloud overlap) and 2) the method of closed equations, based on the Monte Carlo solution of the system of equations for the mean intensity. The MCE not only accounts for the stochastic cloud structure, but also, in comparison with the approximate method, provides a more exact description of radiative interaction between broken cloud layers.

It is shown that the relative differences between upward fluxes at the top boundary of layer $\Lambda_2$ and between downward fluxes at the bottom boundary of layer $\Lambda_1$ in the cloud system (St) – (Ci) generally do not exceed 5% to 10%. When the upper layer $\Lambda_2$ is occupied by the water clouds of moderate optical depth $\left((St)-(As), 4 \leq \tau_2 \leq 25\right)$, increases in $\tau_1$ from 10 to 40 and in $\xi_{\oplus}$ from 30° to 75° may cause $\left|\delta F^{\uparrow}\left(H_{cl}^{t,2}\right)\right|$ and $\left|\delta F^{\downarrow}\left(H_{cl}^{b,1}\right)\right|$ to increase up to ≈20%. In the cloud systems (Cu) – (Ci), (Cu) – (As), the mean fluxes $F^{\uparrow}\left(H_{cl}^{t,2}\right)$ and $F^{\downarrow}\left(H_{cl}^{b,1}\right)$ depend fairly weakly on the calculation technique when optical thickness of the cloud layer $\Lambda_1$ is relatively small ($\tau_1 = 10$) and when $\xi_{\oplus} \leq 30°$: $\left|\delta F^{\uparrow}\left(H_{cl}^{t,2}\right)\right|$ and $\left|\delta F^{\downarrow}\left(H_{cl}^{b,1}\right)\right|$ typically are within 5% (rarely amounting to 10%). The larger that either the optical depth of the cloud layers or the zenith solar angle is, the greater the difference between mean fluxes. For instance, the approximate method may underestimate $F^{\uparrow}\left(H_{cl}^{t,2}\right)$ and overestimate $F^{\downarrow}\left(H_{cl}^{b,1}\right)$ relative to MCE values by 25% to 30% and 50% to 60%, respectively.

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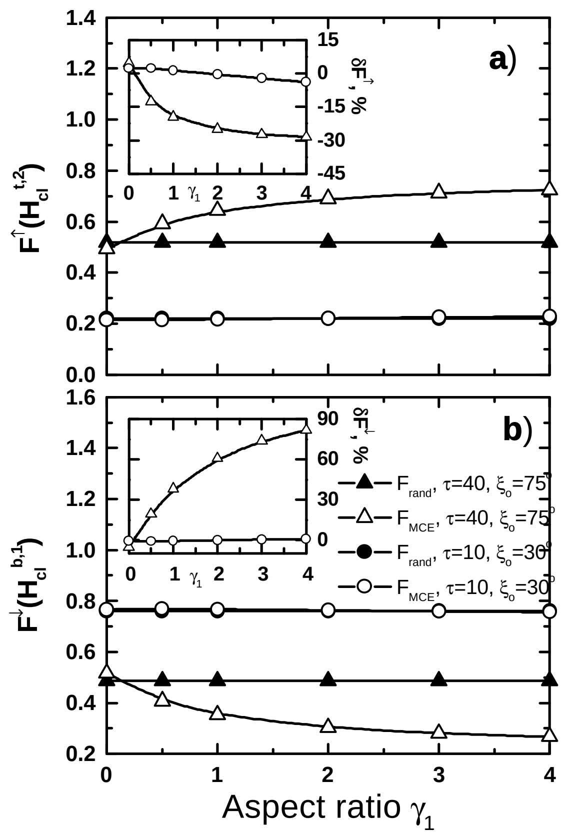


**Figure 3.** Mean upward and downward radiative fluxes and relative difference $\Delta F^{\uparrow(\downarrow)}$ (in %) at the boundaries of (a) top cloud layer $\Lambda_2$ and (b) bottom cloud layer $\Lambda_1$ for N1 = 0.5, $\tau_2$ = 1, $N_2$ = 0.3 $\gamma_2$ << 1, and different parameters of the lower cloud layer and illumination conditions.

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