Mathematics Curriculum Content Standards

Mathematics: Pre-Kindergarten

The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

Strand: **M1 Numbers and Operations**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Pre-Kindergarten, all students should:

Standards:

M1a: recognize and differentiate written numbers from other symbols;

M1b: count in a sequence forward from one;

M1c: identify and name numerals from 0 to 9;

M1d: construct sets with more, fewer, or the same number of objects than a given set;

M1e: compare the number of things in two sets using comparative language, i.e., more, fewer, same number.

Essential To Know: Students use numbers to represent quantity.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Pre-Kindergarten, all students should:

Standards:

M2a: sort, classify, and order objects by one attribute;

M2b: identify, copy, extend, and create simple patterns or patterns of sounds, shapes, and motions;

M2c: recognize simple patterns in sets of objects;

M2d: sort and compare groups of objects having equal or different numbers of objects, i.e. more than, less than, or equal;

M2e: recognize and identify a change in common objects, sounds, or movements.

Essential To Know: Students sort objects by an attribute.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;

- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

In Pre-Kindergarten, all students should:

Standards:

M3a: identify, name, describe, and create common two-dimensional shapes;

M3b: identify, name, and describe three-dimensional shapes;

M3c: describe and demonstrate location and physical proximity, i.e., above, below, etc.

Essential To Know: Students name and describe two-dimensional shapes.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Pre-Kindergarten, all students should:

Standards:

M4a: describe everyday events in logical order;

M4b: recognize the passage of time and identify devices that measure time, i.e., clocks, timers, etc.;

M4c: describe people and objects using measurement terms, i.e., taller than, biggest, longest, etc.;

M4d: identify differences in temperature by using descriptors, i.e., warm, cold, hot, etc.;

M4e: recognize and name measurable attributes of objects, i.e., long, short, and heavy, etc.;

M4f: explore nonstandard measurements to measure attributes of length, height and weight, e.g., a paper clip as a unit measure of length;

M4g: order a like set of objects according to a measurable attribute, i.e. length, thickness of crayons, etc.

Essential To Know: Students identify measurable attributes and use these to make comparisons among objects, events, etc.

Strand **Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Pre-Kindergarten, all students should:

Standards:

M5a: sort and organize concrete data by similarities and differences

M5b: answer and ask questions using data displayed with objects, pictographs, and/or tables.

Essential To Know: Students recognize data by sorting objects according to one attribute.

Strand: **M6 Problem Solving**

Standard: **M6a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;

apply and adapt a variety of appropriate strategies to solve problems;

- monitor and reflect on the process of mathematical problem solving.

Strand: **M7 Reasoning and Proof**

Standard: **M7a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Strand: **M8 Communication**

Standard: **M8a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Strand: **M9 Connections**

Standard: **M9a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Strand **M10 Representation**

Standard: **M10a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

Strand: **M1 Numbers and Operations**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Kindergarten, all students should:

Standards:

M1a: recognize, write and name cardinal numbers up to 20;

M1b: count and recognize "how many" are in sets of objects;

M1c: compare and order objects using ordinal numbers;

M1d: represent and use sets of objects in multiple ways, including separating (decompose), joining (compose), and ordering sets;

M1e: model and represent addition as combining sets and subtraction as taking away sets;

M1f: demonstrate one-one correspondence using manipulatives or objects from their environment;

M1g: estimate quantities of objects within multiple sets using comparative language, i.e., more than, less than, or about the same.

Essential To Know: Students recognize the relationship between numbers and quantities.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Kindergarten, all students should:

Standards:

M2a: describe how objects are alike and different using one or two properties;

M2b: sort, classify, and order objects in more than one way;

M2c: identify, create, copy, and describe sequences of sounds, shapes, motions, and numbers;

M2d: describe, model, and extend AB and ABC patterns;

M2e: model a problem situation using actual objects;

M2f: recognize changes that are measurable.

Essential To Know: Students sort and order objects according to attributes.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- uses visualization, spatial reasoning, and geometric modeling to solve problems.

In Kindergarten, all students should:

Standards:

M3a: identify, compare, and sort two- and three-dimensional shapes;

M3b: describe locations to include direction and distance using the language of relative position;

M3c: recognize that two objects having the same shape but oriented differently in space are congruent;

M3d: draw common two-dimensional shapes from memory.

Essential To Know: Students name and describe objects and two- and three-dimensional shapes by their position, direction, and distance.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Kindergarten, all students should:

Standards:

M4a: compare and order objects according length, height, capacity, and weight by using descriptors, i.e., longer, shorter, and heavier;

M4b: order events based on time;

M4c: explore ways to measure different attributes of objects;

M4d: explore common instruments for measuring, i.e., scales, rulers, cups, etc., and identify the unit measure of each instrument;

M4e: use measuring instruments or non-standard measurement tools to compare objects, liquids, spaces, and people.

Essential To Know: Students identify and measure attributes of objects.
Students use nonstandard units to measure.

Strand: **M5 Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Kindergarten, all students should:

Standards:

M5a: gather, sort, and interpret data in response to questions posed, e.g., by class surveys, or teacher/student questions;

M5b: organize and represent data using concrete objects, pictures, and graphs;

M5c: ask and answer questions and make predictions based on data collected.

Essential To Know: Students organize and represent data to formulate a response to a question.

Strand: **M6 Problem Solving**

Standard: **M6a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;

apply and adapt a variety of appropriate strategies to solve problems;

- monitor and reflect on the process of mathematical problem solving.

Strand: **M7 Reasoning and Proof**

Standard: **M7a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Strand: **M8 Communication**

Standard: **M8a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

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- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Strand: **M9 Connections**

Standard: **M9a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

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- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
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Strand **M10 Representation**

Standard: **M10a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas;

- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

Strand: **M1 Numbers and Operations**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Grade 1, all students should:

Standards:

M1a: recognize place values of numbers (ones, tens, and hundreds) and identify groups of each quantity;

M1b: identify and generate equivalent forms of the same number using concrete objects and number statements;

M1c: recognize wholes and parts of wholes, i.e., ½, , and, ¼;

M1d: express the concepts of addition and subtraction through drawings, number statements and/or verbal explanation, as well as using plus (+) and minus (-) symbols;

M1e: explain the relationship between addition and subtraction as inverse operations;

M1f: explain and perform addition and subtraction of one-digit whole numbers;

M1g: use estimation based on a benchmark and recognize reasonable answers;

M1h: select, explain, and use appropriate computational procedures to solve real-world problems.

Essential To Know: Students use the concept of place value to decompose and compose whole numbers up to 100.
Students explain, model, and demonstrate the meaning of addition and subtraction with whole numbers.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 1, all students should:

Standards:

M2a: sort, classify, and order objects by two or more attributes and explain how objects were sorted;

M2b: identify, describe, extend and create repeating patterns and number sequences;

M2c: solve open sentences using the commutative property of addition by representing an expression in more than one way;

M2d: write mathematical equations using symbols;

M2e: model and describe a problem situation using representations, i.e. words, objects, number phrase or sentence;

M2f: experiment with equivalency using concrete materials;

M2g: identify measurable changes that are predictable, e.g., students grow taller, not shorter, as they get older.

Essential To Know: Students recognize, extend, and create patterns.
Students recognize and describe and describe changes using words and numbers.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- uses visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 1, all students should:

Standard:

M3a: identify two-dimensional shapes in three-dimensional shapes;

M3b: create new shapes by combining or cutting or taking apart existing shapes;

M3c: describe and name the direction and distance in navigating space, e.g., which way, how far, etc.;

M3d: identify and determine whether two-dimensional shapes are congruent (same shape and size) or similar (same shape different size);

M3e: recognize and explore symmetry;

M3f: recognize geometric shapes and structures in the environment and specify their location.

Essential To Know: Students describe the attributes and parts of two- and three-dimensional shapes.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grade 1, all students should:

Standard

M4a: measure and differentiate objects using both comparative terms and standard units of measure, i.e., inches, centimeters, etc.;

M4b: recognize repeating pattern of time;

M4c: tell time to the hour and half hour using digital and analog timepieces;

M4d: order a sequence of events that occur over time;

M4e: estimate and measure a variety of attributes of objects using standard and nonstandard units;

M4f: make reasonable estimates about the passage of time in commonplace events, e.g., tasks being completed, living things growing, etc.

Essential To Know: Students use standard units of measurement.

Strand: **M5 Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 1, all students should:

Standard:

M5a: identify multiple categories for sorting data;

M5b: collect, organize, represent and interpret data using concrete objects, pictures, tallies, and graphs;

M5c: compare and contrast similar data sets;

M5d: construct questions that can be answered by using information from a graph or table;

M5e: describe events related to student's experiences as more likely or less likely to happen;

M5f: read and interpret graphs and tables to make comparisons and predictions.

Essential To Know: Students collect, sort, represent, and analyze data.

Strand: **M6 Problem Solving**

Standard: **M6a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- build new mathematical knowledge through problem solving;
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- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Grade 2, all students should:

Standards:

M1a: use place values to represent whole numbers through a hundred using numerals, words, and physical models;

M1b: use words, numerals, and physical models to show an understanding of fractions and their relationship to a whole;

M1c: identify numbers as even or odd whole numbers;

M1d: show equivalent representations for whole numbers by using addition and subtraction facts;

M1e: explain multiplication as repeated addition and equal groupings of objects and division as repeated subtraction and equal sharing;

M1f: explain and perform addition and subtraction for two-digit numbers;

M1g: use various estimating techniques and rounding of whole numbers;

M1h: solve a variety of non-routine multi-step problems involving addition and subtraction.

Essential To Know: Students estimate, calculate, and develop strategies for solving addition and subtraction problems based on number relationships.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 2, all students should:

Standards:

M2a: create and describe patterns with multiple attributes;

M2b: use patterns to make generalizations and predictions by determining the rule and/or identifying missing elements in a pattern and justifying their inclusion;

M2c: use symbols to represent unknown quantities and identify values for symbols;

M2d: represent equivalence and extend the concept to situations involving symbols, i.e., + = 10;

M2e: solve open sentences by representing an expression in more than one way using the associative property of addition;

M2f: model and describe a problem situation using symbols and operations;

M2g: describe qualitative changes;

M2h: describe quantitative changes, especially those involving addition and subtraction.

Essential To Know: Students generalize a pattern to determine a rule.
Students represent information using words, numbers, and symbols.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- uses visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 2, all students should:

Standards:

M3a: predict the results of putting together and taking apart two- and three-dimensional shapes.

M3b: find and name locations using simple relationships and in coordinate systems, i.e., grids, maps, etc.;

M3c: use models to demonstrate slides, flips, and turns of shapes.

M3d: recognize and create shapes with symmetry;

M3e: predict what new shapes will be formed by combining or cutting apart existing shapes.

Essential To Know: Students identify and describe a single transformation of a simple shape.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grade 2, all students should:

Standards:

M4a: tell time to the nearest one-minute interval on a digital clock and the nearest five-minute interval on an analog clock;

M4b: describe and compare the relationships among units of measure, e.g., pints and quarts, hours and half hours, etc.;

M4c: select the appropriate unit of measure for the attribute being measured, i.e., area, capacity, length, etc.;

M4d: make and test predictions about measurements, using different units to measure the same length or volume;

M4e: use repetition of a single unit to measure something larger than the unit;

M4f: estimate and measure the length and weight of common objects to the nearest unit.

Essential To Know: Students make and use estimates of measurement.

Students select and correctly use the appropriate measurement tool and unit.

Strand: **M5 Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

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- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 2, all students should:

Standards:

M5a: develop categories for sorting a collection of materials;

M5b: collect, organize, represent and interpret data using pictographs, bar graphs, and tables;

M5c: generate questions, collect, and organize data to address the questions and draw conclusions;

M5d: read and interpret graphs and tables to identify main ideas, draw conclusions, and make predictions;

M5e: describe events that are more likely, least likely, or equally likely to happen;

M5f: use physical models and pictures to represent possible arrangements of two or three objects;

M5g: identify events that can have more than one outcome, e.g., predicting weather, tossing coins, etc.

Essential To Know: Students read, interpret and create graphs and tables.

Strand: **M6 Problem Solving**

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- understand how to compute fluently and make reasonable estimates.

In Grade 3, all students should:

Standards:

M1a: use place values to read, model, and interpret whole numbers through thousands and decimals through hundredths; explain the values of the digits;

M1b: explain the relationship of commonly used fractions to their equivalent forms, and explain their relationship to a whole;

M1c: identify and describe numbers according to their characteristics such as even, odd, multiples, and/or factors;

M1d: use mathematical language and symbols to compare and order numbers and objects;

M1e: explain and use addition, subtraction, multiplication, and division to show equivalent whole numbers;

M1f: explain the relationship between multiplication and division as inverse operations.

M1g: use properties of operations on whole numbers, i.e., commutativity and associativity;

M1h: explain and perform addition and subtraction for two- and three-digit numbers and multiplication of one- and two-digit numbers;

M1i: model and explain multiplication and division using appropriate symbols and strategies;

M1j: develop and use strategies to estimate the results of whole number computations and judge the reasonableness of the computed results;

M1k: solve non-routine multi-step problems using appropriate tools and strategies involving addition, subtraction, and multiplication.

Essential To Know:

Students select, explain the meaning of, and use a variety of models to demonstrate multiplication and division of whole numbers.
Students explain and represent with models that fractions are parts of a whole or parts of set.

Strand: **M2 Algebra**

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- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 3, all students should:

Standards:

M2a: identify, describe, and extend the rules of multiplicative and growing patterns;

M2b: make predictions, identify relationships, and solve problems by using the concept of patterns;

M2c: express mathematical relationships as equations or inequalities with appropriate symbols;

M2d: solve open sentences by representing an expression in more than one way using the commutative and associative properties for multiplication;

M2e: organize and order data in labeled tables to discover patterns and rules;

M2f: represent mathematical situations to solve problems using equations or inequalities;

M2g: recognize patterns and make predictions based on collected data;

M2h: describe the difference between qualitative and quantitative changes.

Essential To Know: Students describe, extend, and make generalizations about patterns involving multiplicative and growing patterns.
Students use algebraic properties to identify numeric relationships.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- uses visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 3, all students should:

Standards:

M3a: describe properties of two- and three-dimensional shapes using mathematical terminology;

M3b: identify and describe the relative size of angles with right angles as a reference;

M3c: use coordinate systems to specify locations and describe paths;

M3d: verify symmetry by drawing lines of symmetry in shapes and objects;

M3e: build and draw geometric shapes.

Essential To Know: Students identify and compare the structure of two- and three- dimensional shapes.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grade 3, all students should:

Standards:

M4a: explain the need for using standard units when making measurements;

M4b: explore standard units within the customary and metric systems and describe the relationship of units within each system;

M4c: use accurate vocabulary to describe measurement, i.e., meters for length, degrees for temperature, minutes to hours, etc.;

M4d: use counting techniques to explain how to find the area and perimeter of regular shapes;

M4e: estimate measurements using a personal reference;

M4f: uses appropriate measurement tools and techniques to construct a figure;

M4g: select and apply appropriate standard units and tools to compare the measurable attributes of a variety of objects;

M4h: develop strategies for estimating the perimeter of irregular shapes;

M4i: read thermometers accurately;

M4j: use models to estimate perimeter and area;

M4k: calculate the area and perimeter of regular shapes;

M4l: tell time to the nearest minute and measure elapsed time using a clock or calendar.

Essential to Know: Students estimate and find area and perimeter using diagrams, models, grids or by standard-unit measuring.

Strand: **M5 Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 3, all students should:

Standards:

M5a: develop and implement a plan to collect and organize data to address a given question;

M5b: translate information from one data representation to another, i.e., graph to table;

M5c: support a conclusion or a prediction with evidence from data;

M5d: organize and graphically display data using a variety of categories and intervals;

M5e: describe the characteristics of graphically represented data, i.e., identify the mode;

M5f: examine graphs and tables that display the same set of data to identify what each representation contributes to the interpretation of data and conclusions drawn;

M5g: select a question for study, predict possible outcomes, conduct simple experiments, and compare results to predictions.

Essential To Know: Students translate one form of data representation to another and evaluate the different aspects of information offered by each form.

Strand: **M6 Problem Solving**

Standard: **M6a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;

apply and adapt a variety of appropriate strategies to solve problems;

- monitor and reflect on the process of mathematical problem solving.

Strand: **M7 Reasoning and Proof**

Standard: **M7a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Strand: **M8 Communication**

Standard: **M8a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Strand: **M9 Connections**

Standard: **M9a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Strand **M10 Representation**

Standard: **M10a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

Strand: **M1 Numbers and Operations**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Grade 4, all students should:

Standards:

M1a: explain the importance of place value in recognizing the magnitude of whole numbers up to a million and decimals through thousandths;

M1b: identify and generate equivalent representations for the same number by decomposing and composing the number;

M1c: judge the size of fractions in relation to benchmarks, i.e., 0, ½, 1;

M1d: identify and describe numbers according to their characteristics including primes, composites, and perfect squares;

M1e: explore the relationships between fractions, mixed numbers, and decimals;

M1f: model division problems and explore the meaning of remainders;

M1g: use models, benchmarks, and equivalence to add and subtract fractions with like denominators;

M1h: use models and benchmarks to add and subtract decimals;

M1i: develop and apply strategies and methods for division of two-digit whole numbers by one-digit divisors;

M1j: use the inverse relationships of addition and subtraction, and multiplication and division to solve problems and verify solutions;

M1k: use estimation to make predictions and check the reasonableness of result;

M1l: identify, compare and order the relative position of commonly used fractions and decimals on a number line;

M1m: demonstrate proficiency in basic facts for all operations.

Essential to Know:

Students explain and represent with models the relationship between whole numbers, common fractions, and decimals.
Students select and use estimation strategies and judge the reasonableness of the answer.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 4, all students should:

Standards:

- **M2a:** use models and words to describe, extend, and generalize patterns and relationships;
- **M2b:** represent and analyze patterns and functions using words, tables, and graphs;
- **M2c:** describe mathematical relationships using expressions, equations or inequalities;
- **M2d:** apply order of operations and the commutative and associative properties to algebraic expressions, equations, and inequalities;
- **M2e:** use and interpret variables, mathematical symbols, and properties to write and simplify mathematical expressions and sentences;
- **M2f:** develop and solve equations or inequalities using variables that represent problem situations;
- **M2g:** identify and describe patterns of change to make predictions that identify the relationship represented in a table or graph.

Essential to Know: Students use relationships in patterns to make predictions by using tables, charts, physical objects, and symbols.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- uses visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 4, all students should:

Standards:

- **M3a:** describe attributes of planes, points, and lines, i.e., parallel and perpendicular line segments;
- **M3b:** identify and draw right, obtuse, and acute angles;
- **M3c:** identify and draw congruent figures;
- **M3d:** investigate the results of subdividing, combining, and transforming shapes;
- **M3e:** make and use coordinate systems to specify locations and to describe paths;
- **M3f:** find the distance between points along horizontal and vertical lines of a coordinate system;
- **M3g:** predict and describe transformations to show that two shapes are congruent;
- **M3h:** identify and describe line and rotational symmetry in two-dimensional shapes and designs;
- **M3i:** identify geometric solids which could be composed of other solids.

Essential to Know: Students describe geometric properties and relationships using appropriate vocabulary.

Students use two-dimensional coordinate grids to represent points and to graph lines and simple figures.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grade 4, all students should:

Standards:

M4a: recognize that measurements are approximations;

M4b: measure with accuracy using both customary and metric systems of measurement;

M4c: extend recognition of measurable attributes to include area and angles;

M4d: determine the possible dimensions of rectangles when the area is constant;

M4e: estimate measurements of perimeter, area, and angle size;

M4f: extend use of appropriate standard tools and units to include measure of perimeter and area;

M4g: explore strategies to determine the perimeter and area of right triangles;

M4h: develop strategies for estimating the area of irregular shapes;

M4i: determine elapsed time;

M4j: solve problems involving perimeter and areas of rectangles.

Essential to Know: Students carry simple unit conversions within a system of measurement.

Strand: **M5 Data Analysis** a**nd Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 4, all students should:

Standards:

M5a: describe how data collection methods affect the information that is gathered to address a question;

M5b: identify the median of a data set and describe what it indicates about the data set;

M5c: use the median, mode, and range to compare and contrast the characteristics of related data sets;

M5d: compare different representations of the same data to evaluate how each representation shows important aspects of the data;

M5e: select the appropriate data representation form for a diverse set of investigations and justify the choice in each case;

M5f: relate the concepts of impossible and certain events to the numerical values of 0 (impossible) and 1 (certain);

M5g: investigate experimental probability;

M5h: list and count all possible combinations using one member from each of several sets.

Essential to Know: Students appropriately represent and interpret data.

Strand: **M6 Problem Solving**

Standard: **M6a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;

apply and adapt a variety of appropriate strategies to solve problems;

- monitor and reflect on the process of mathematical problem solving.

Strand: **M7 Reasoning and Proof**

Standard: **M7a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Strand: **M8 Communication**

Standard: **M8a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Strand: **M9 Connections**

Standard: **M9a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Strand **M10 Representation**

Standard: **M10a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas;

- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

Strand: **M1 Numbers and Operations**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Grade 5, all students should:

Standards:

M1a: understand place value and explain the relationship to addition and subtraction and multiplication and division of decimals;

M1b: identify and generate equivalent forms of fractions, decimals, and percents;

M1c: explain how decimals and percents are parts of a whole;

M1d: use models to develop the concept of ratio as part to part and part to whole;

M1e: represent and compare numbers less than zero by extending the number line and using familiar applications, like temperature, to demonstrate the usefulness of negative numbers;

M1f: identify and use the distributive properties to simplify and/or perform computations;

M1g: use order of operations, including the use of parentheses, to simplify numerical expressions;

M1h: explain why fractions need common denominators to be added or subtracted;

M1i: understand the concept of multiplication and division of fractions;

M1j: understand and compute positive integer powers of nonnegative integers as repeated multiplication;

M1k: demonstrate proficiency with two-digit divisors;

M1l: use models and equivalent forms to add and subtract fractions with like and unlike denominators expressing answers in simplest form;

M1m: estimate the results of computations involving whole numbers, fractions, and decimals, using a variety of strategies;

M1n: compute and perform simple multiplication and division of fractions and decimals.

Essential To Know:

Students apply the appropriate order of operations for expressions involving addition, subtraction, multiplication, and division.
Students use, interpret, and construct multiple representations of a number and translate among equivalent relationships for integers, fractions, decimals, and percents.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 5, all students should:

Standards:

M2a: express a general rule for a pattern or a function by using visual representations, words, tables, or graphs;

M2b: explain the concept of variable;

M2c: use variables as unknown quantities in general rules when describing mathematical patterns and relationships;

M2d: apply algebraic order of operations and the commutative, associative and distributive properties to algebraic expressions, equations, and inequalities;

M2e: construct tables and graphs that accurately represent the relationship between two variables;

M2f: identify, describe, and compare situations that represent constant or varying rates of change.

Essential To Know: Students use symbolic algebra to represent and explain mathematical relationships.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- uses visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 5, all students should:

Standards:

M3a: identify faces, edges, vertices and bases of three-dimensional shapes;

M3b: identify and plot ordered pairs in the first quadrant of a coordinate system;

M3c: explore patterns that result from a combination of reflections, rotations, and translations of geometric figures, including rotational symmetry;

M3d: visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Essential To Know: Students compare and analyze attributes and other features of two- and three-dimensional geometric shapes.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grade 5, all students should:

Standards:

M4a: extend the recognition of measurable attributes to include volume (cubic units);

M4b: convert standard units of measurement within both customary and metric systems of measurement, e.g., inches to feet, centimeters to meters, etc.:

M4c: develop strategies for estimating the volume of various shapes;

M4d: extend the use of appropriate standard tools and units to include measures of volume and angle size;

M4e: develop strategies to determine the surface areas and volumes of rectangular solids;

M4f: differentiate between units of measurement for two- and three-dimensional objects and use appropriately.

Essential To Know: Students use appropriate units of measurement to measure two- and three-dimensional objects.

Strand: **M5 Data Analysis** a**nd Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 5, all students should:

Standards:

M5a: explain and conduct sampling techniques for gathering data;

M5b: select and use a graph that is appropriate for the type of data to be displayed;

M5c: read and interpret quantitative and qualitative data;

M5d: investigate the role of the mean as a balance point for the data set;

M5e: recognize samples as subsets of larger populations;

M5f: use a sample to make projections for a larger population;

M5g: use common fractions to represent the probability of events that are neither certain nor impossible;

M5h: compare theoretical and experimental outcomes in a simple experiment;

M5i: make predictions based on experimental and theoretical probabilities.

Essential To Know: Students project information for a larger population based on a sample. Students explain the relationship between experimental and theoretical probabilities.

Strand: **M6 Problem Solving**

Standard:

M6a: Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;

apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Strand: **M7 Reasoning and Proof**

Standard: **M7a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:
- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

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- use the language of mathematics to express mathematical ideas precisely.

Strand: **M9 Connections**

Standard: **M9a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:
- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Strand **M10 Representation**

Standard: **M10a:** Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:
- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

Strand: **M1 Numbers and Operations**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Grade 6, all students should:

Standards:

M1a: decompose and recompose whole numbers using factors and exponents;

M1b: find and use prime factorization of composite numbers;

M1c: use simple expressions involving integers to represent and solve problems;

M1d: compare and order positive and negative decimals and fractions and find their locations on a number line;

M1e: interpret and use ratios in different contexts to show relative sizes of two quantities, using appropriate notations, i.e., a/b, a to b, a:b;

M1f: use order of operations, including the use of exponents, decimals, rational numbers, to simplify numerical expressions;

M1g: explain the meaning and effects of arithmetic operations with positive numbers to include fractions, decimals, and percents;

M1h: perform fraction and decimal computations and justify the solutions;

M1i: estimate reasonableness of solutions to problems involving fractions and decimals;

M1j: select and use appropriate methods and tools for computing with fractions and decimals.

Essential To Know: Students select and use a combination of appropriate arithmetic operations to solve problems that use rational numbers.
Students apply and explain number theory concepts to solve problems.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 6, all students should:

Standards:

M2a: recognize and generate equivalent forms of algebraic expressions.

M2b: explain how the commutative, associative and distributive properties generate equivalent forms;

M2c: solve simple linear equations and inequalities;

M2d: use symbolic algebra to represent situations, i.e., relationships found in geometry;

M2e: evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations;

M2f: create and interpret tables and graphs to draw conclusions and make predictions;

M2g: create and compare representations that display constant and varying rates of change.

Essential to Know: Students should represent, analyze, and generalize patterns and relations with tables, graphs, and words.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 6, all students should:

Standards:

M3a: describe and classify two- and three-dimensional shapes using their defining properties;

M3b: identify and plot points on a coordinate plane in all quadrants;

M3c: describe sizes, positions, orientations of shapes, after rotations, reflections, and translations;

M3d: recognize, explain, and perform up to two transformations on two-dimensional shapes;

M3e: draw and identify two-dimensional geometric figures with specific side length or angle measure;

M3f: describe and use properties of similarity and congruency with two-dimensional figures to solve problems.

Essential To Know: Students predict, describe, and perform transformations on two-dimensional shapes.
Students identify relationships among points, lines, and planes.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grade 6, all students should:

Standards:

M4a: explain the relationship between area and perimeter of a rectangle when one attribute is changed and the other remains constant;

M4b: investigate the precision of measurement required for tasks as well as the capability/accuracy of the instruments;

M4c: develop and use formulas to find the perimeters and areas of triangles and quadrilaterals and to find the area and circumference of circles;

M4e: find the perimeter and area of irregular polygons;

M4f: identify rate as a form of measurement based on time, i.e., mph, rpm, cc/min.

Essential to Know: Students explain the relationships between perimeter and area and circumference and area of a circle.
Students use formulas to find perimeter, circumference and area.
Students identify rate as a form of measurement.

Strand: **M5 Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data, and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 6, all students should:

Standards:

M5a: read and use graphical representations to make predictions and/or draw conclusions;

M5b: formulate questions, design a study, and evaluate the data to reach a conclusion about characteristics shared by two populations or different characteristics that exist within a population;

M5c: identify the measures of central tendency and spread of a data set to describe what it indicates about the data set;

M5d: explain the effects of scale and/or interval changes in graphs that lead to misunderstandings;

M5e: select, construct, interpret, and justify the appropriate graphical representation of data;

M5f: use 0, 1, and ratios between 0 and 1 to represent the probability of outcomes for an event;

M5g: describe and model all possible outcomes of simple events using tree diagrams, organized lists, etc.;

M5h: explain why the sum of the probabilities of all possible outcomes of a particular event is one.

Essential to Know: Students select, create, interpret, and justify the appropriate graphical representation of data.
Students understand and apply the fundamental concepts of probability.

Strand: **M6 Problem Solving**

Standard: **M6a:** Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

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- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to none another;
- understand how to compute fluently and make reasonable estimates.

In Grade 7, all students should:

Standards:

M1a: use, interpret and compare numbers in several equivalent forms such as integers, fractions, decimals, and percents;

M1b: develop meaning of percent greater than 100 or less than 1;

M1c: understand and use ratio and proportion to represent quantitative relationships;

M1d: describe the differences between rational and irrational numbers;

M1e: explain the relationship, meaning and effects of arithmetic operations with the set of integers;

M1f: use order of operations and properties to simplify numerical expressions involving integers, fractions, decimals and exponents;

M1g: simplify numerical expressions and solve real-life problems using the set of integers;

M1h: estimate and solve problems including ratios, proportions and percents, and justify reasoning.

Essential To Know: Students analyze and explain methods for solving problems involving fractions, decimals, percents, proportions and ratios.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 7, all students should:

Standards:

M2a: represent, analyze, and generalize relations and functions with tables, graphs, words, and when possible, algebraic expressions and equations;

M2b: explain relationships between graphs of lines and their equations;

M2c: generate equivalent forms of algebraic expressions by combining like terms;

M2d: use variables and appropriate operations to write an expression, equation, or inequality that represents a verbal description;

M2e: model and solve equations using inverse operations;

M2f: represent linear equations and inequalities by plotting points;

M2g: analyze functional relationships to explain how a change in one quantity results in a change in the other;

M2h: recognize a variety of uses for variables.

Essential To Know: Students represent, analyze, and generalize relations and functions with tables, graphs, words, and when possible, algebraic expressions and equations.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 7, all students should:

Standards:

M3a: demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures;

M3b: use proportional reasoning to describe and express relationships between similar and congruent figures;

M3c: classify and identify triangles by side and angle measurement and polygons as regular or irregular and/or by the number of sides;

M3d: recognize and explain the following attributes of a circle, i.e., radius, diameter, arc, chord, semicircle, and central angle;

M3d: use coordinate geometry to examine special geometric shapes, such as regular polygons and polygons with pairs of parallel or perpendicular sides;

M3e: determine the length of a side of a figure drawn on a coordinate plane with vertices having the same x or y coordinates;

M3f: examine congruence, similarity, and line or rotational symmetry of an object using transformations.

Essential To Know: Students describe and apply the properties of similarity and congruent figures and justify conjectures involving similarity and congruence.
Students graph points and identify coordinates of points in the coordinate plane.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques and tools.

In Grade 7, all students should:

Standards:

M4a: select and use appropriate tools and units of measure when measuring and calculating angles, surface areas, and volumes of rectangular prisms;

M4b: Analyze the structure and uniformity of the metric system and contrast with the customary system;

M4c: Develop strategies to determine the surface area and volume of rectangular prisms using geometric models and materials;

M4e: Understand the difference between surface area and volume, and demonstrate that two objects may have the same surface area, but different volumes—or may have the same volume, but different surface areas;

M4f: use ratios and proportions to solve problems involving scale factors.

Essential To Know: Students use investigation to determine how geometric formulas were derived. Students understand the characteristics of a system of measurements.

Strand: **M5 Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 7, all students should:

Standards:

M5a: Read, create and interpret box and whisker plots, stem and leaf plots, scatter plots, and other appropriate types of graphs;

M5b: Analyze the effect of graphing decisions on graphical representation, e.g., scaling, types of graphs, etc.;

M5c: Find, interpret, and appropriately use quartile, interquartile range, and outliers;

M5d: Explain how measures of central tendency are affected by extremes;

M5e: Find and make predictions based on the line of best fit;

M5f: Identify possible misuses of measures of central tendency;

M5g: Use proportionality and probability to make and test conjectures about the results of experiments and simulations;

M5h: Describe multiple outcomes of compound independent events, i.e., using tree diagrams and organized lists.

Essential To Know: Students understand and apply the fundamental concepts of measures of central tendency.
Students represent probabilities as ratios, proportions, decimals between 0 and 1 and percentages between 0 and 100.

Strand: **M6 Problem Solving**

Standard: **M6a:** Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;

- monitor and reflect on the process of mathematical problem solving.

Strand: **M7 Reasoning and Proof**

Standard: **M7a:** Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Strand: **M8 Communication**

Standard: **M8a:** Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Strand: **M9 Connections**

Standard: **M9a:** Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Strand **M10 Representation**

Standard: **M10a:** Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

Strand: **M1 Numbers and Operations**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates

In Grade 8, all students should:

Standards:

M1a: explain the meaning of exponents that are negative and zero;

M1b: use scientific, exponential and calculator notation to express very large or small numbers;

M1c: expand scientific notation to include negative exponents;

M1d: explain and use the additive and multiplicative identities and the additive and multiplicative inverses;

M1e: apply order of operations to simplify expressions and perform appropriate operation(s) involving numbers written in exponential notation or radical form;

M1f: make reasonable estimates and then solve problems that include rational numbers, ratios, and proportions.

Essential To Know: Students represent and compare the magnitude of numbers appropriately using exponential, scientific, and calculator notation.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grade 8, all students should:

Standards:

M2a: generalize patterns and sequences by describing the way to find the nth term.

M2b: identify functions as linear or nonlinear and contrast their properties using tables, graphs, or equations;

M2c: analyze relationships between linear equations and their graphs by connecting the meaning of intercepts and slope to the context of the situation;

M2d: use symbolic algebra to represent situations and to solve problems involving linear and nonlinear relationships;

M2e: recognize, generate, and justify equivalent forms of algebraic expressions;

M2f: solve linear equations and inequalities;

M2g: represent situations using systems of linear equations and solve graphically;

M2h: model and solve problems using various representations, i.e., graphs, tables, and equations;

M2i: connect the rate of change to the slope of a line;

M2j: analyze changes in linear relationships using graphs;

M2k: Describe and compare how changes in an equation affect the related graph.

Essential To Know: Students model and solve real world problems using various representations such as graphs, tables, and equations.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

In Grade 8, all students should:

Standards: **M3a:** understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;

M3b: verify the Pythagorean Theorem;

M3c: apply the Pythagorean Theorem to determine if a triangle is a right triangle or to find a missing side of a right triangle;

M3d: identify and describe angle relationships formed by parallel lines cut by a transversal using appropriate terminology, i.e., alternate interior, alternate exterior, supplementary, vertical angles, corresponding angles, complementary, consecutive interior;

M3e: plot ordered pairs of rational numbers on the coordinate plane in all four quadrants;

M3f: use geometric models to represent and explain numerical and algebraic relationships.

Essential To Know: Students apply the Pythagorean theorem by constructing figures that meet specific conditions.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grade 8, all students should:

Standards:: **M4a:** describe and demonstrate how perimeter, area, and volume are affected by changes of scale;

M4b: develop strategies to determine the surface area and volume of selected prisms, pyramids and cylinders;

M4c: use formulas to a specified level of precision in finding the surface area and volume of prisms, pyramids and cylinders and the volume of spheres and cones;

M4d: find the sum of the interior and exterior angles of regular convex polygons with and without the use of a protractor;

M4e: solve simple rate problems.

Essential To Know: Students use strategies to determine the surface area and volume of prisms, pyramids and cylinders.

Strand: **M5 Data Analysis and Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grade 8, all students should:

Standards:

M5a: Differentiate between discrete and continuous data and appropriate ways to represent each;

M5b: Find, interpret and appropriately use measures of center, quartile, and interquartile range to compare two sets of data;

M5c: Find the equation of a line of best fit for data represented as a scatter plot;

M5d: Describe sampling methods and analyze effects of random versus biased sampling and justify conclusions;

M5e: Construct convincing and appropriate arguments for a conclusion based on analysis of data presented;

M5f: Recognize faulty arguments or common errors in data analysis;

M5g: Compute the probability of the occurrence of independent and simple dependent events;

M5h: Distinguish between permutations and combinations.

Essential To Know: Students construct convincing and appropriate arguments based on analysis of data and interpretation of graphs.
Students explain the difference between independent and dependent events.

Strand: **M6 Problem Solving**

Standard: **M6a:** Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

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The process standards of **problem solving, reasoning and proof, connections communication, and representation** are interwoven and independent with the content standards and are necessary for the comprehensive understanding of mathematics.

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Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- understand how to compute fluently and make reasonable estimates.

In Grades 9–12, all students should:

Standards:

M1a: connect physical, verbal and symbolic representations of irrational numbers and properties of special numbers, i.e., , ;

M1b: compare, order, and determine equivalent forms for rational and irrational numbers;

M1c: define the concept of complex numbers in the context of the square root of a negative number;

M1d: using powers and roots including rational exponents, simplify number expressions;

M1e: define the properties of matrices;

M1f: identify and explain which mathematical properties hold for a given set or operations for the real number system, i.e., density, closure, commutative, associative, distributive;

M1g: solve equations and inequalities using the inverse relationship of operations to include powers and roots;

M1h: organize and analyze data using the operations of addition, subtraction, and scalar multiplication for matrices;

M1i: estimate the approximate value of square and cube roots without the use of a calculator;

M1j: use estimation to judge the reasonableness of numerical computations and their results;

M1k: develop fluency in operations with real numbers using mental computation, paper and pencil calculations, and technology;

M1l: Use properties of the number system to judge the validity of results and justify each step of a procedure.

Strand: **M2 Algebra**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

In Grades 9–12, all students should:

Standards:

M2a: analyze, generalize, and create a variety of mathematical patterns;

M2b: analyze, interpret, and translate between relationships of patterns, functions, and relationships represented in tables, graphs, and matrices;

M2c: identify, describe, and compare the characteristics and properties of functions and relations including linear and nonlinear;

M2d: represent linear and nonlinear functions with tables, graphs, verbal rules, and symbolic rules and interpret these representations;

M2e: use algebraic representations and functions to generalize geometric properties and relationships;

M2f: write, solve, and interpret the relationship of equivalent forms for equations, inequalities, and systems of equations;

M2g: explain and demonstrate the relationship between various representations of a linear equation;

M2h: add, subtract, and multiply polynomials and divide polynomials by monomials;

M2i: translate between numeric and symbolic form of a sequence or series;

M2j: apply direct and inverse variation to both real-world and mathematical models;

M2k: solve and analyze real-world problems that can be modeled using linear, and nonlinear functions;

M2l: solve and analyze real-world problems that can be modeled using systems of equations and inequalities;

M2m: predict a reasonable conclusion for a problem being modeled, and verify the conclusion through solving the problem;

M2n: approximate and interpret rates of change from graphical and numerical data;

M2o: identify and explain how changes in parameters affect graphs of functions;

M2p: explain and graph the relationship between two variables for linear, periodic exponential, quadratic relationships and a limiting value.

Strand: **M3 Geometry**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

In Grades 9–12, all students should:

Standards:

M3a: identify undefined terms and explain the need for undefined terms;

M3b: use a variety of ways to represent geometric ideas and recognize relationships among them including coordinates, networks, transformations, and matrices;

M3c: identify and explain relationships among classes of two- and three-dimensional geometric objects, i.e., sides, angles, etc.;

M3d: make conjectures, test, and prove relationships among two- and three-dimensional geometric objects, i.e., congruent triangles;

M3e: distinguish between postulates and theorems and apply them appropriately;

M3f: identify and explain examples of induction and deductive;

M3g: analyze geometric situations using Cartesian coordinates and other appropriate coordinate systems;

M3h: use rectangular coordinates; calculate midpoints of segments, slopes of lines and segments, and distances between two points to solve problems;

M3h: use sketches, coordinates, function notation, and matrices to represent translations, reflections, rotations, and dilations of objects in the plane;

M3i: draw and construct representations for two- dimensional objects using a variety of tools;

M3j: construct vertex-edge graphs to model and solve problems;

M3k: identify and explain projections and cross sections by visualizing different perspectives of three- dimensional objects and spaces;

M3l: Solve problems by applying properties and theorems of lines, angles, polygons, and circles.

Strand: **M4 Measurement**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

In Grades 9–12, all students should:

Standards:

M4a: use the appropriate unit or dimensional analysis in measurement situations;

M4b: explain the effect of changes in the measurement of one attribute of an object relating to changes on other attributes;

M4c: recognize and apply alternative methods of measurement;

M4d: apply appropriate formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;

M4e: analyze and explain precision, accuracy, and approximate error in measurement situations.

Strand: **M5 Data Analysis** a**nd Probability**

Pre-Kindergarten through Grade 12 instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;

- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

In Grades 9–12, all students should:

Standards:

M5a: classify and describe data as single (univariate) or two variable (bivariate) and as quantitative (measurement) or qualitative (categorical) data;

M5b: design surveys and apply random sampling techniques to avoid bias in data collection;

M5c: use multiple graphical displays and statistical measures to display and interpret the relationship between two variables;

M5d: compare different sets of data by using summary statistics and select the appropriate graphical representation;

M5e: explain the ways representations can skew data or bias presentations;

M5f: describe and explain the characteristics and limitations of various sampling methods;

M5g: describe and explain how the validity of predictions from a data set are affected by the relative size of a sample and the population;

M5h: use counting techniques and/or combinations to solve explain probability problems;

M5i: describe, create, and analyze a sample space, then calculate the probability;

M5j: use the concept of conditional probability and independent events to apply and interpret the results of a set;

M5k: calculate and explain the probability of compound events;

M5l: use sampling or simulation to construct empirical probability distributions to compare and explain corresponding theoretical probabilities;

M5m: differentiate and explain the relationship between the probability of an event and the odds of an event.

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