

The problem: A certain ground motion has an x percent probability of being exceeded in Y years. What is the probability, w , that that same ground motion is exceeded in Z years?

The solution: In the algorithm used to create the maps, one specifies a probability of exceedance for a given number of years. The computer finds the ground motion that has an annual rate that satisfies the following equation:

$$1 - r(a) = F(a) = e^{-T\varphi(a)}$$

where $r(a)$ is the exceedance probability of the ground motion a , $F(a)$ is the corresponding probability of non-exceedance, e is the base of the natural logarithm scale, T is the number of years for which we want to know the corresponding probability, and $\varphi(a)$ is the annual rate of exceedance of ground motion a . This equation is nothing more than the Poisson probability that given an expected number n of events, e^{-n} is the probability of getting none.

Given a ground motion, a , the annual rate at a site $\varphi(a)$ is constant. Taking logs and solving for $\varphi(a)$, we can make the equation,

$$\frac{\ln F(a)}{T} = -\varphi(a)$$

and equate for the two cases, number of years = Y and Z

$$\frac{\ln(1 - x)}{Y} = \frac{\ln(1 - w)}{Z}$$

Solving for $1 - w$,

$$\ln(1 - w) = \frac{Z}{Y} \ln(1 - x)$$

$$(1 - w) = e^{\frac{Z}{Y} \ln(1 - x)} = (1 - x)^{\frac{Z}{Y}}$$

So, for instance, for $r = 0.10$, $Y = 50$, and $Z = 500$; $F = 0.9$ and

$$(1 - w) = (0.90)^{10} = 0.35, \text{ hence } w = 0.65$$

In a similar way, for $r=0.05$, $w = 0.40$, and for $r = 0.02$, $w = 0.18$

If a ground motion has this probability of being exceeded in 50 years,	the same ground motion has the following probability of being exceeded in 500 years.	Prob of exceedance in 500 ----- Prob of exceedance in 50
0.02	0.18	9.0
0.05	0.40	8.0
0.10	0.65	6.5