Lecture 4
Energy Equation for An Ideal Fluid

Basic Hydraulic Principles Course

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Welcome

- Please put your phone on **mute**
- Please **do not place us on hold** as sometimes your "hold music" plays while you are on another call

Please Ask Questions Throughout Lecture

- Raise your hand on the WEBEX
- I will recognize you and ask you to take your phone off mute to ask

Main Points for this Lecture

- Continuity Equation

$$Q = AV$$

$$A_A V_A = A_B V_B$$

- Energy Equation

$$\frac{V_A^2}{2g} + Y_A + Z_A = \frac{V_B^2}{2g} + Y_B + Z_B = C$$

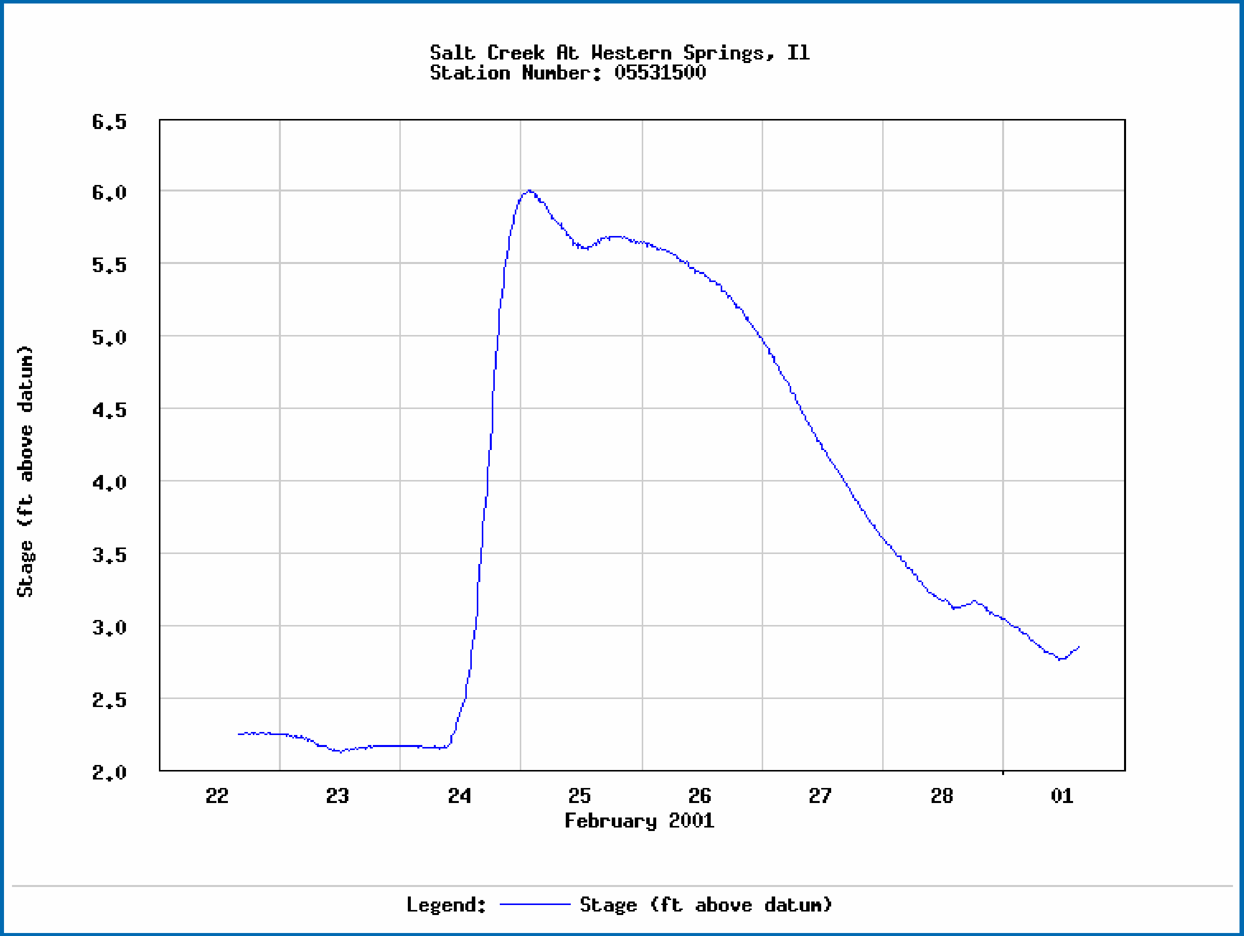
Understanding the Energy Equation and its use is CRUCIAL to success in this course

Overview of Lecture

- Flow characterization by time dependency
- Streamlines
- Continuity equation
- Energy equation
- Work a problem example
- Questions on concepts or homework problems in chapter 4
- Other questions

Flow Characterization

- Fluid flow may be either steady or unsteady.
- Steady flow exists when none of the variables in the flow problem change with time.
- If any of the variables change with time, the condition of unsteady flow exists.
- This discussion and most of this course deals with steady-flow only.



Is the above steady flow?

Steady Flow Implies:

- $Stage_{t=\text{noon on Feb 24}} = Stage_{t=\text{ midnight on Feb 24}}$
- $Q_{t=\text{noon on Feb 24}} = Q_{t=\text{ midnight on Feb 24}}$
- $V_{t=\text{noon on Feb 24}} = V_{t=\text{ midnight on Feb 24}}$

To apply energy concepts, we often have to make some steady flow approximations

Example: We assume a steady flow at the peak of a flow in order to use indirect computation methods.

Streamlines

- **Path lines**—trace made by a single particle over a period of time
- **Streamlines**—Curve that is tangent to the direction of velocity at every point on the curve
- **Streamtubes**—In 2-dimensional space, this is the area between two streamlines. It resembles a tube or passageway.

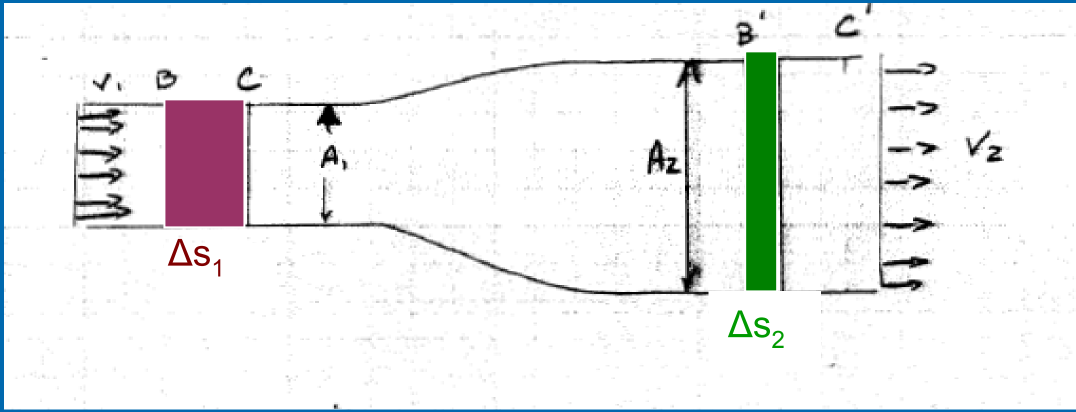
For Steady Flow, a path line and a streamline are identical

Let's put aside the concept of streamlines for a bit while we develop some other concepts. We will come back to it.

Continuity Equation

Matter can neither be created nor destroyed is the principle of **<u>conservation of mass</u>**

Application of the conservation of mass to steady flow in the streamtube results in the **Equation of Continuity**



Note: At all points in this pipe, the particles of the fluid move tangential to the streamline, which would be expected in steady flow

•In a small interval of time, Δt, fluid at the beginning of the pipe moves Δs_1, which $\Delta s_1 = v_1 \Delta t$, where v_1 is velocity in the area of pipe that has a cross sectional area = A_1

•If A_1 is the cross sectional area of the beginning of the pipe. The mass contained in the maroon area is $M_1=\rho_1 A_1 \Delta S_1= \rho_1 A_1 v_1 \Delta t$ where ρ is the density of the fluid

•Similarly, the fluid moving through the enlarged section of pipe in Δt time has a mass equal to $M_2=\rho_2 A_2 \Delta S_2= \rho_2 A_2 v_2 \Delta t$.

•Because mass is conserved and the flow is steady, the mass that crosses A_1 in Δt is the same as the mass that crosses A_2 in Δt or $M_1=M_2$. Or:

$$\rho_1 A_1 v_1 \Delta t= \rho_2 A_2 v_2 \Delta t$$

Equation of Continuity

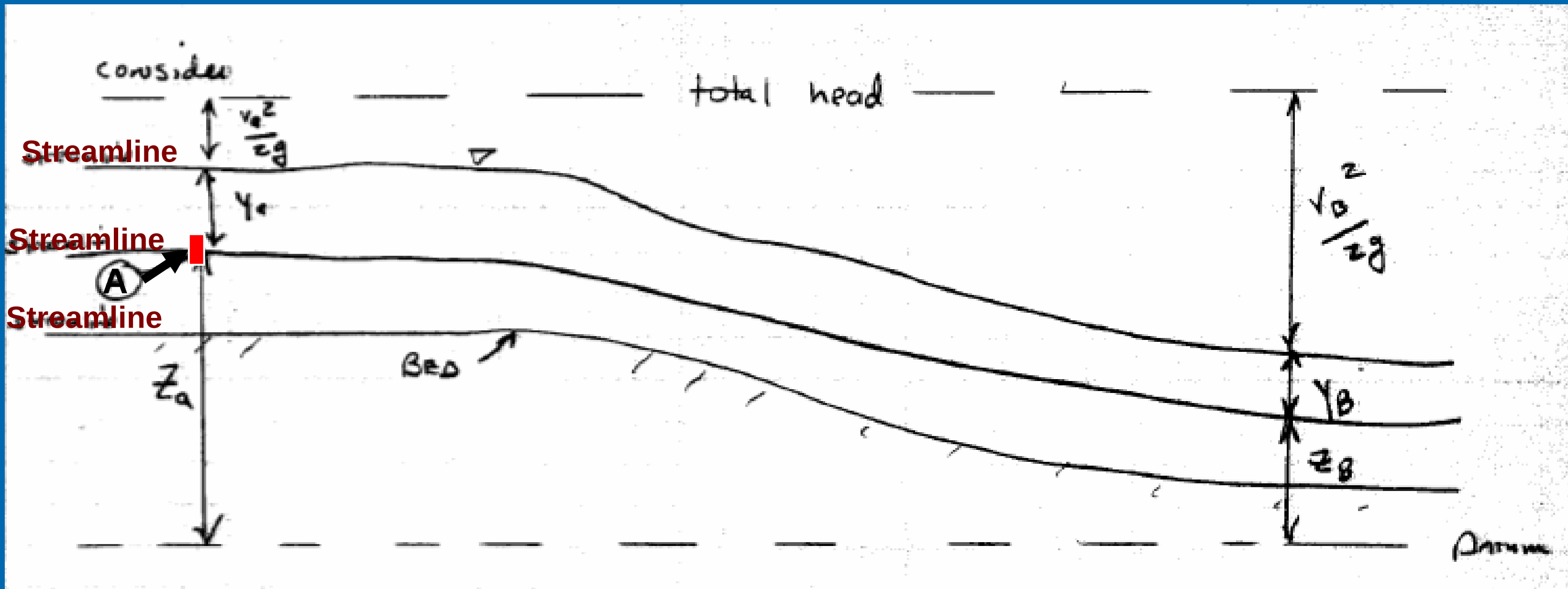
For a steady flow in an incompressible fluid, ρ is constant.

$$A_1 v_1 = A_2 v_2 = Q$$

Q=Flow rate

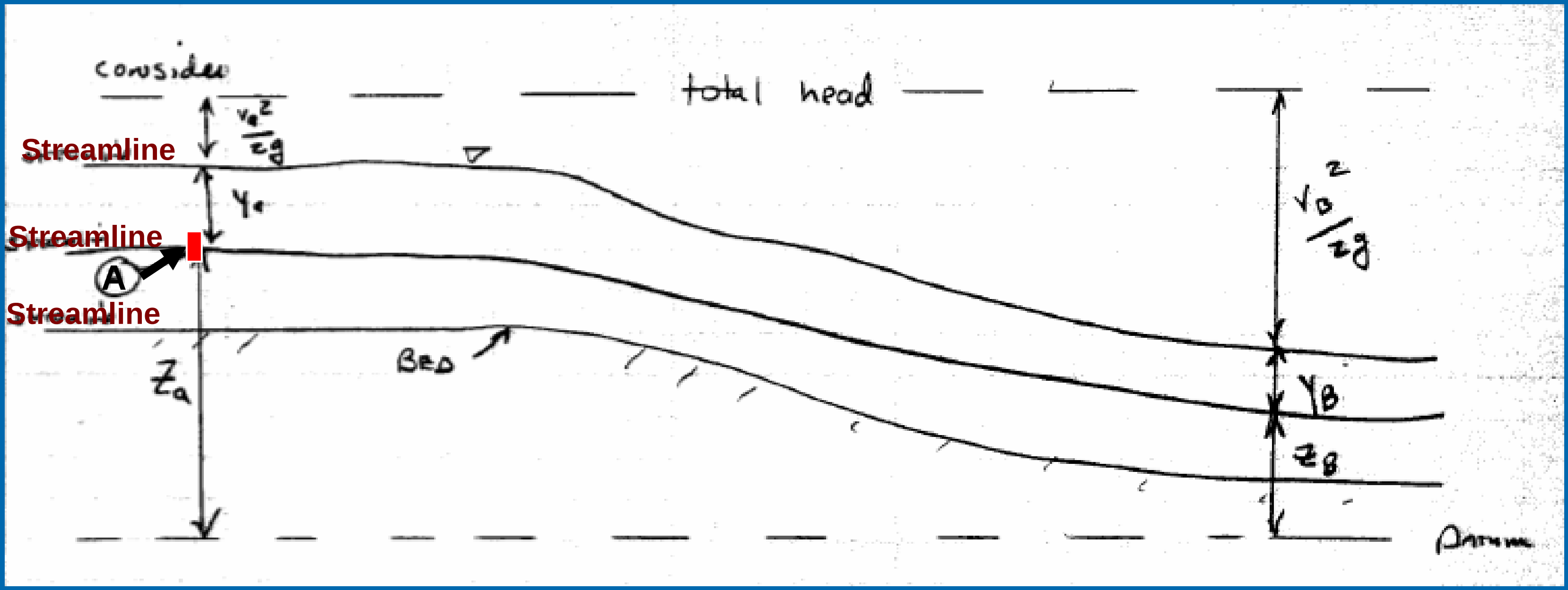
Energy Equation

- Apply the conservation of energy
- Assume an ideal fluid→ there is no shearing stress→there is no energy loss from friction
- Consider……..

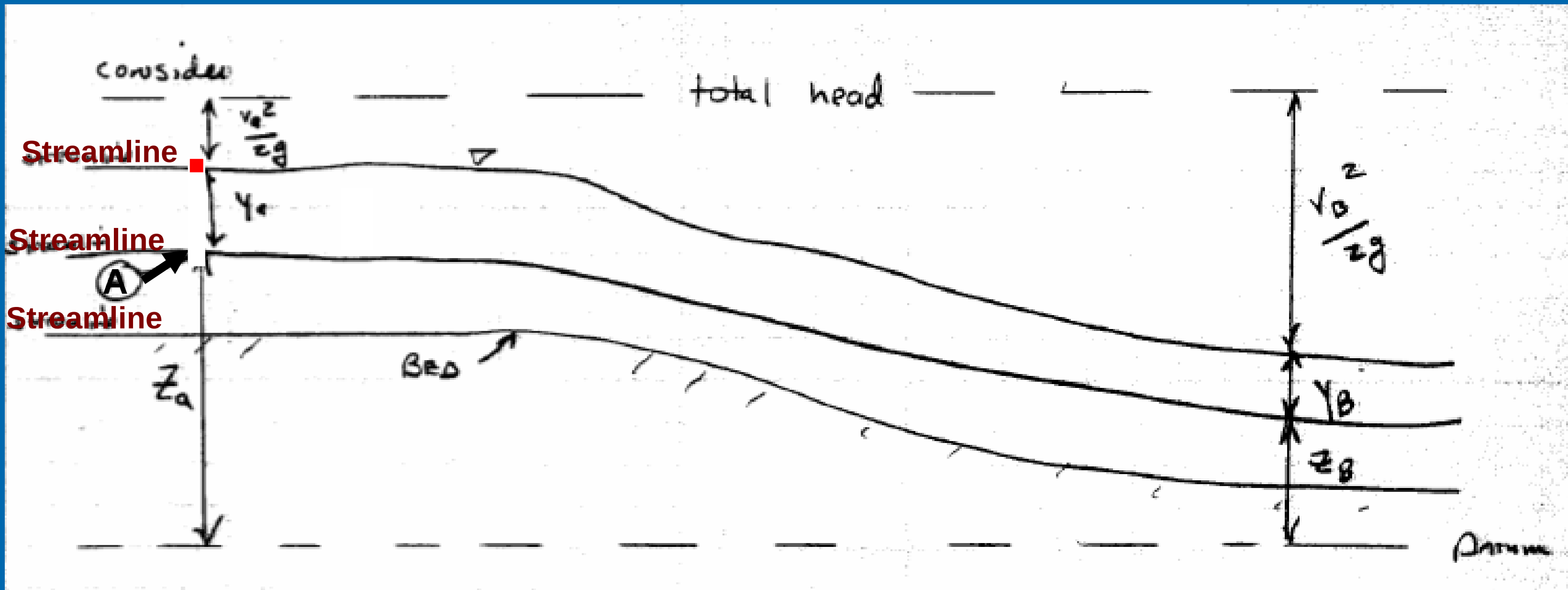


- Consider a 1 pound parcel of water at Point A on the streamline

- Parcel contains 3 types of energy:
    - Kinetic
    - Potential
    - Pressure Potential



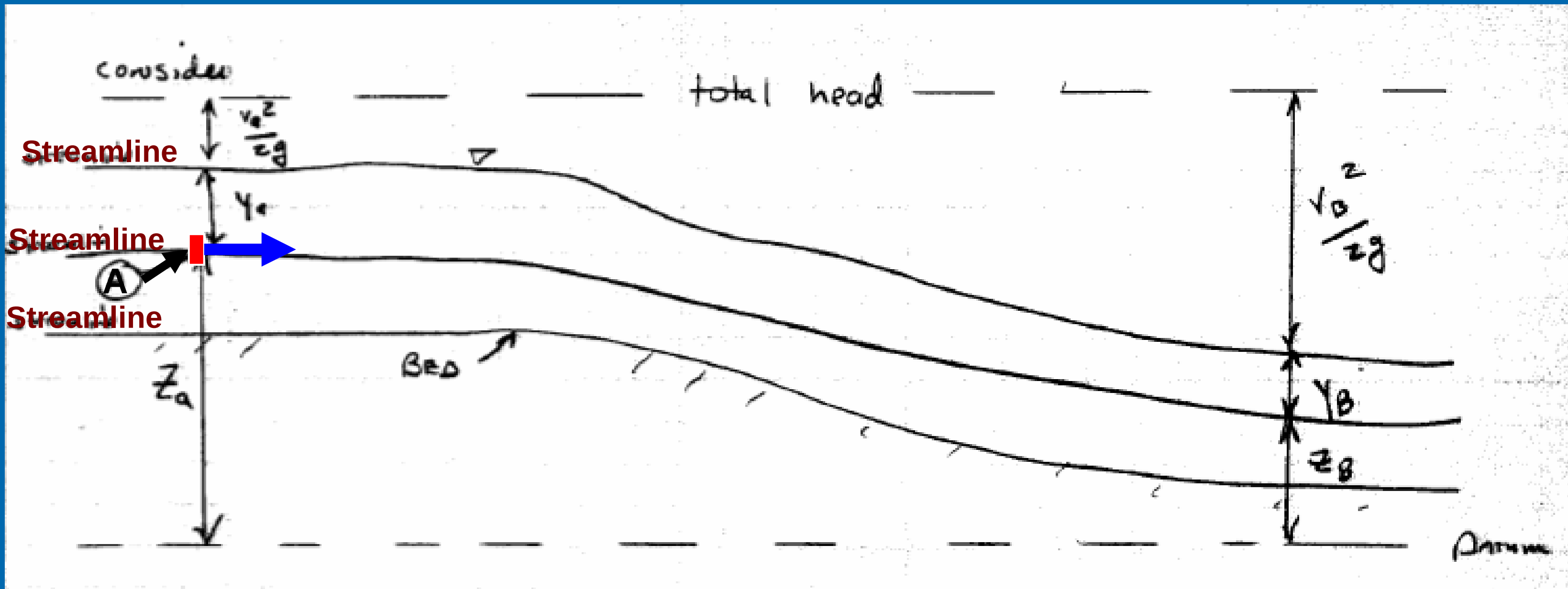
•**Potential Energy** —Weight times the distance above the datum. In our discussion in this class, we will use units for energy as energy per pound of flowing water. Therefore, for section A in the diagram, the potential energy of the one pound parcel of water at A is Z_A foot-pound per pound. We typically allow the units of weight to cancel and express the units only in length terms. In English units, that will be feet. In hydraulics, we call the potential energy **head**. So, we would say that at point A, we have **Z_A feet of head**.



•**<u>Pressure Potential Energy</u>** — A parcel of water, which is neutrally buoyant, could rise to the surface without expenditure of energy. Its effective potential energy per pound of fluid is $Z_A + Y_A$. Y_A is the pressure potential energy and is equal to the pressure at point A divided by the unit weight (p/ρg). Y_A is known as the **<u>pressure head</u>**.

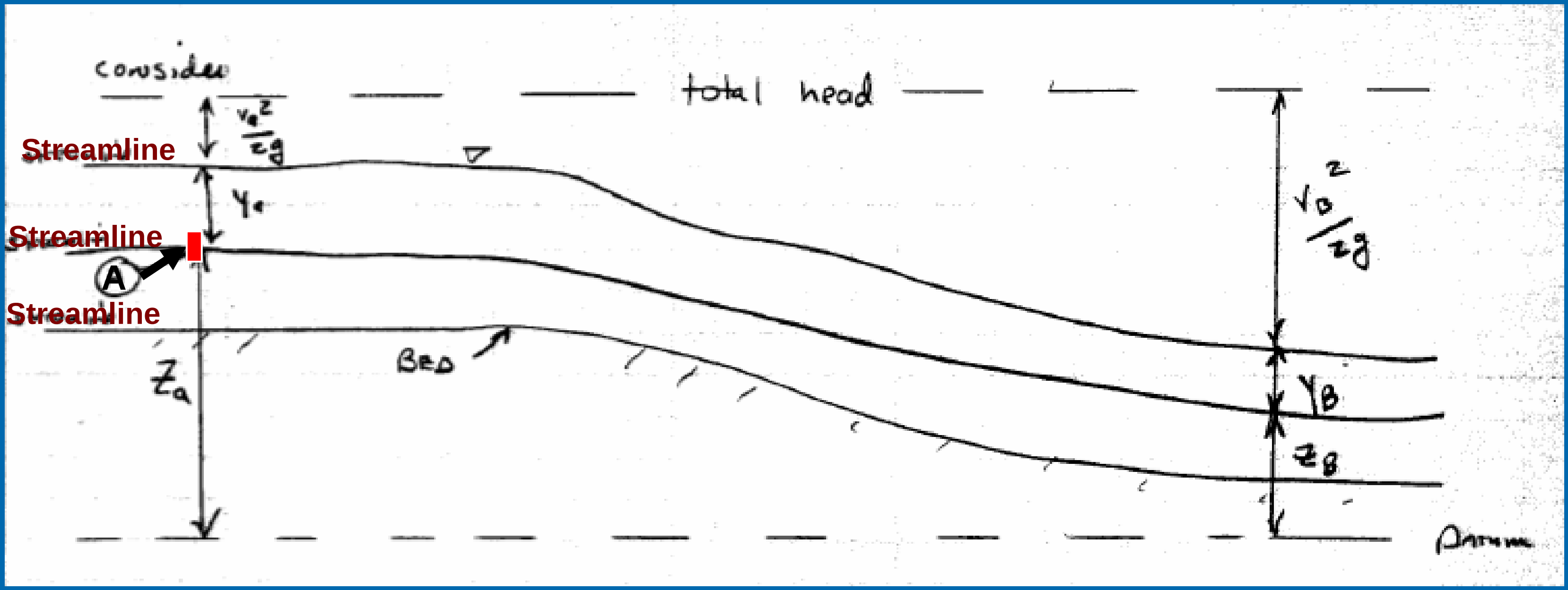
•Notice that the **effective potential energy** for any parcel of fluid at section A is the same and equal to the sum of the water depth plus the elevation of the bed above the datum.

Also note….the sum of the head and the pressure head, which above we call the effective potential energy, is also known as the **<u>piezometric head</u>**



Kinetic Energy: From Physics, we know that the kinetic energy of a particle of mass m and a speed of v is defined as $KE=1/2\ mv^2$. For a pound of water, it has a mass $m=1/g$ (from Newton's 2nd law where weight is a Force (F) and $F=ma$, where in this case a is the acceleration of gravity (g)) Therefore, Kinetic energy of a one pound parcel of water → $KE=1/2(1/g)v^2=v^2/2g$

Also known as **velocity head**



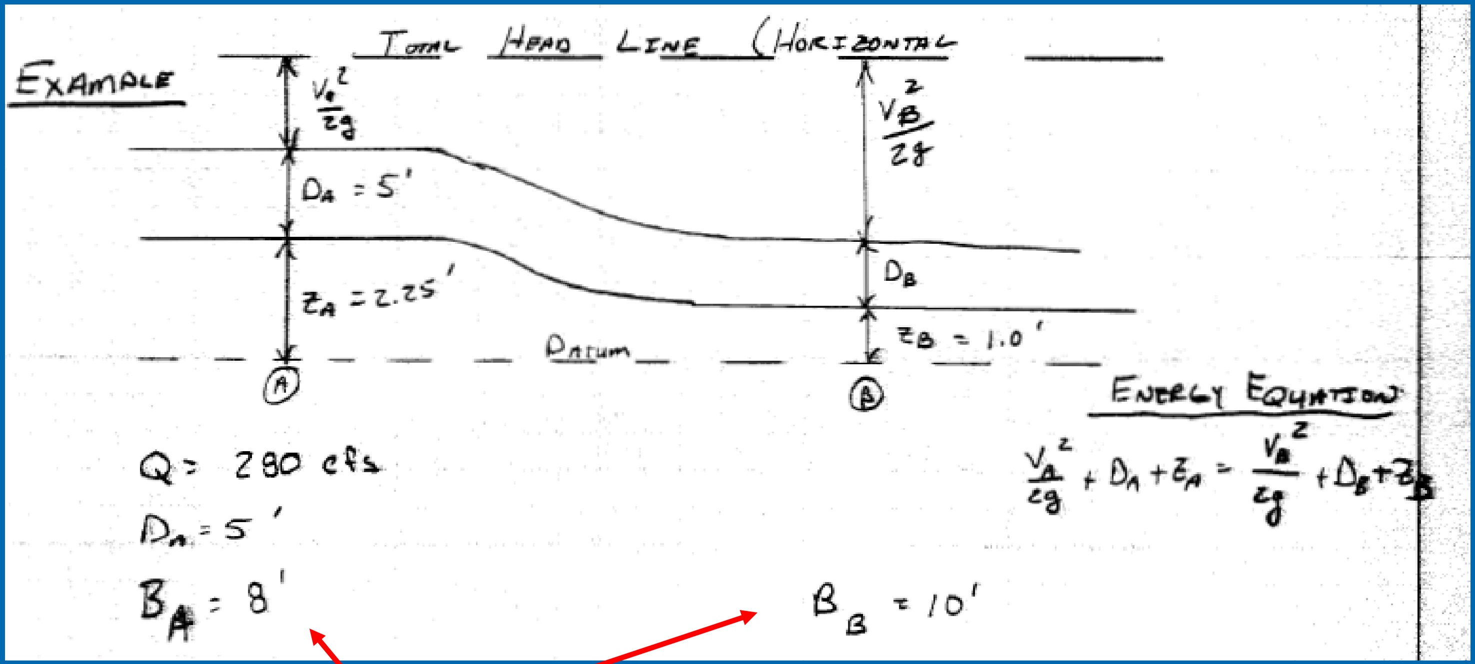
The sum of all 3 energies is called the total head or total energy

For an **IDEAL FLUID** (no frictional resistance in the fluid), the total energy along a streamline is constant

$$\frac{V_A^2}{2g} + Y_A + Z_A = \frac{V_B^2}{2g} + Y_B + Z_B = C$$

Numerous engineering problems can be solved by this simplified situation, however, if friction losses are large, the results will be poor.

Example



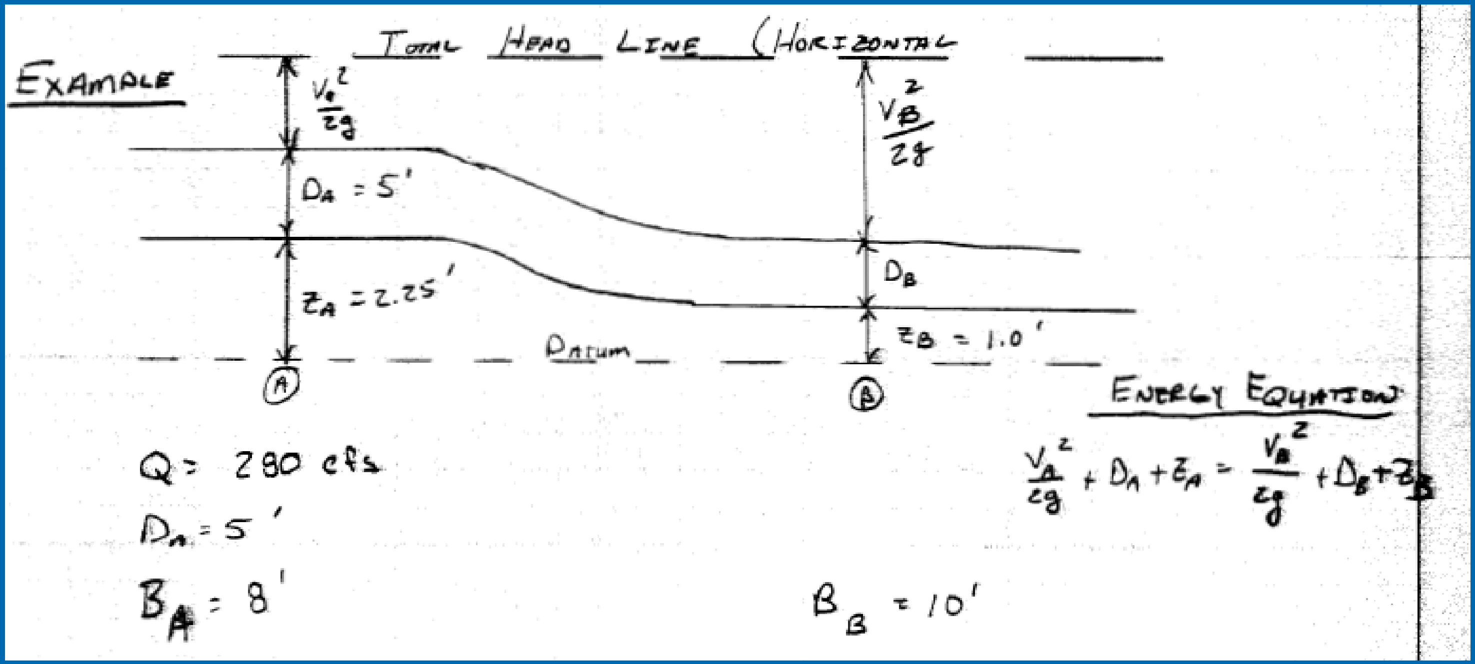
Write down what you know and your unknowns: → V_A, V_B, D_B

Write down the equations at your disposal to help you solve things:

Continuity → $A_A v_A = A_B v_B = Q$

Energy Equation →
$$\frac{V_A^2}{2g} + Y_A + Z_A = \frac{V_B^2}{2g} + Y_B + Z_B = C$$

Example



We can solve for V_A just by using the continuity equation.

Continuity ⟶

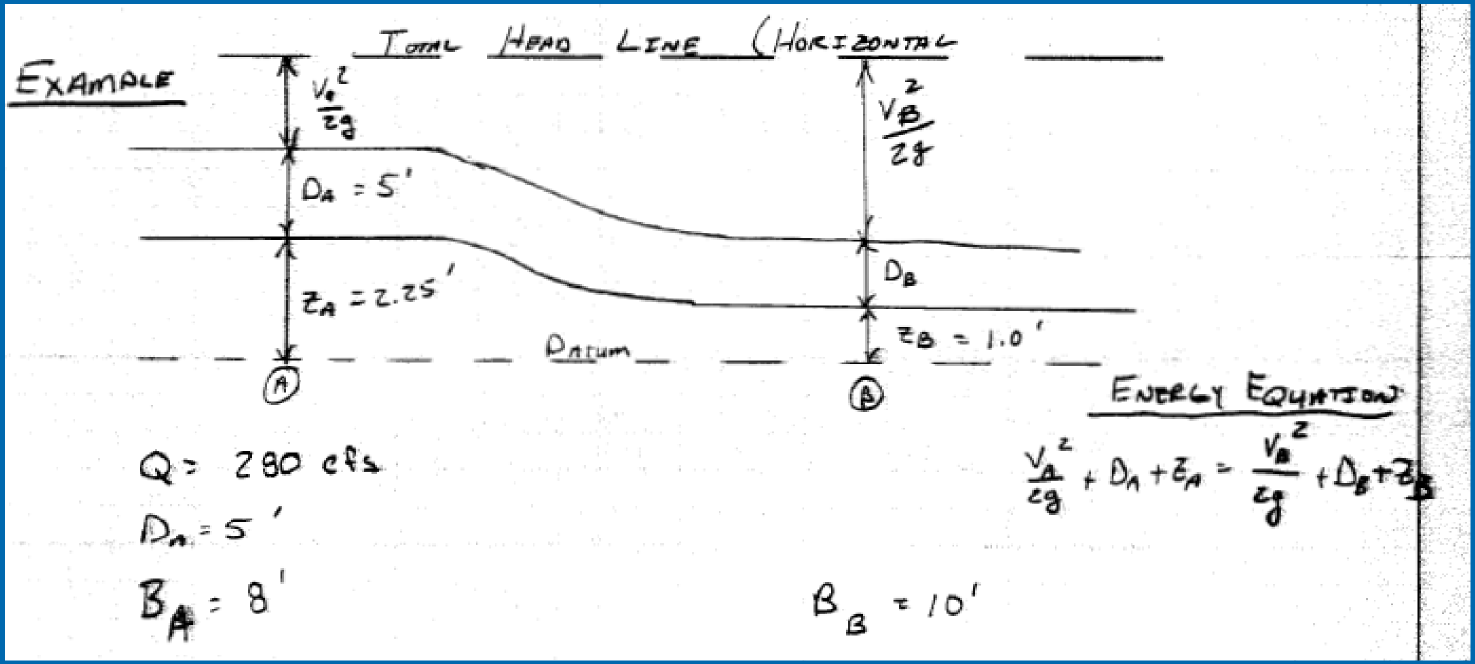
$A_A v_A = Q$

$(5*8)\, v_A = 280$

$v_A = 280/(5*8)$

$v_A = 7$ ft/sec

Example



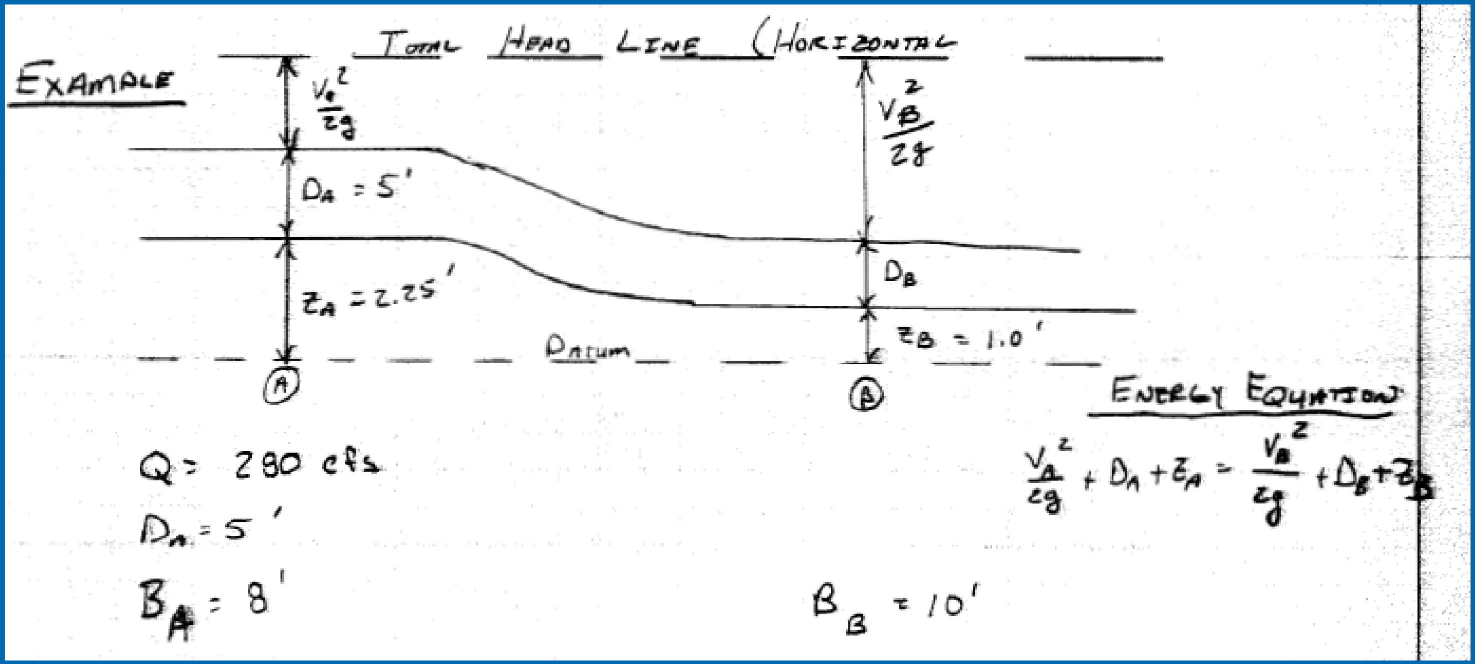
Utilizing the Energy Equation, I can solve for the total energy at Section A.

Energy Equation

$$\frac{V_A^2}{2g} + Y_A + Z_A = \frac{V_B^2}{2g} + Y_B + Z_B = C$$

$$\frac{V_A^2}{2g} + Y_A + Z_A = \frac{7^2}{2(32.2)} + 5 + 2.25 = 0.761 + 5 + 2.25 = 8.011\,ft$$

Example

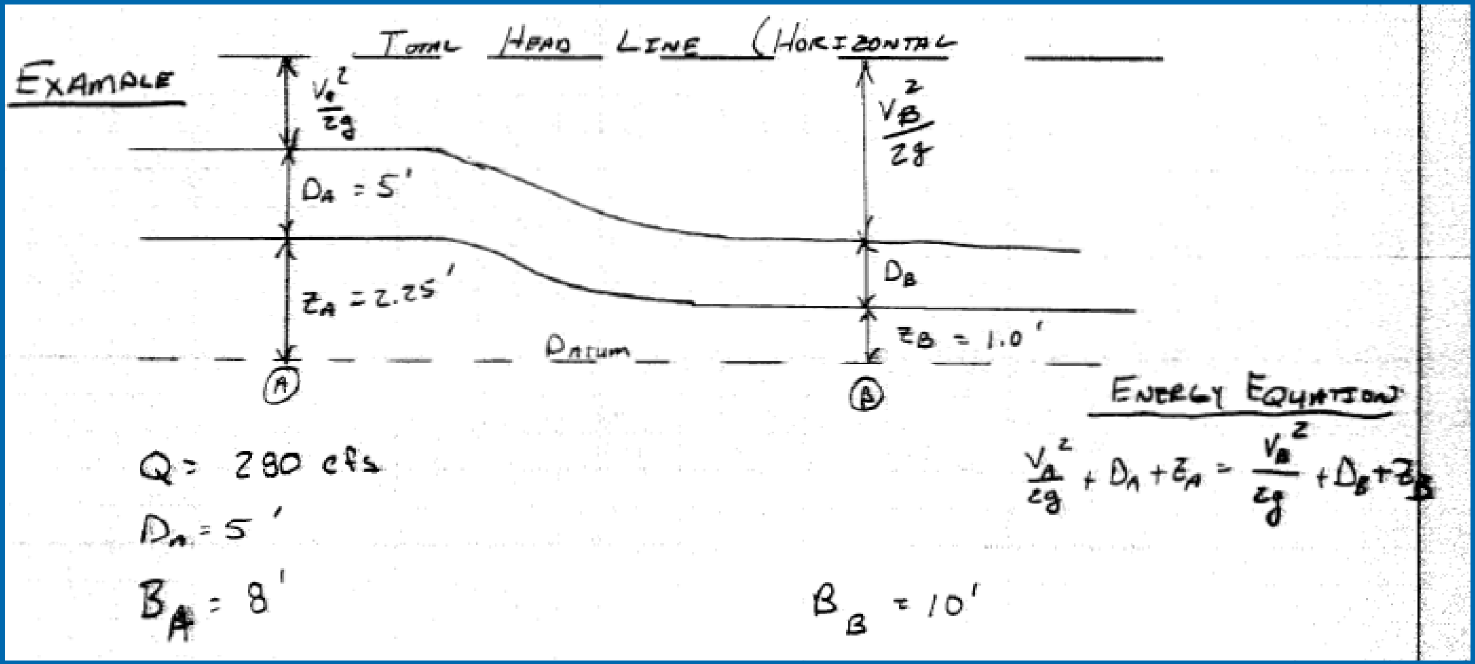


Still Utilizing the Energy Equation, I can solve items at section B because since this is an IDEAL fluid, the total energy at section A is equal to the total energy at section B

$$\frac{V_A^2}{2g} + Y_A + Z_A = \frac{V_B^2}{2g} + Y_B + Z_B = C$$

$$8.011\,ft = \frac{V_B^2}{2g} + Y_B + Z_B = \frac{V_B^2}{2(32.2)} + D_B + 1$$

Example



We don't know V_B or D_B yet…..but we can once again use the continuity equation

$A_B\, v_B = (D_B*10)*V_B = Q$ → $v_B = Q/(D_B*10) = 280/(D_B*10) = 2.8/D_B$

$$8.011\,ft = \frac{2.8^2}{2(32.2)D_B^2} + D_B + 1 \quad \rightarrow \quad 7.011\,ft = \frac{12.174}{D_B^2} + D_B$$

This is a cubic equation which can be solved by trial and error

See Text Book on Page 31

Different values of DB are assumed until the right hand side of the equation is equal to 7.011

$$7.011\,ft = \frac{12.174}{D_B^2} + D_B$$

D_B(ft)	$\frac{12.174}{D_2^2} + D_2$	D_B(ft)	$\frac{12.174}{D_2^2} + D_2$
10.0	10.12	2.0	5.043
8.0	8.19	1.8	5.557
7.0	7.248	1.5	6.911
6.75	7.017	1.49	6.973
	>*		>*
6.74	7.008	1.48	7.038
6.0	6.338	1.30	8.503
4.0	4.761	1.00	13.174

Notice that a depth of either 6.743 or 1.484 feet satisfies the equation and is possible for an ideal fluid. These are called alternate depths. Unless some constriction downstream caused the water to back up, the flow would accelerate as shown on the figure and the smaller depth will occur. In this case, the velocity at the section would be

Using the computed value of D_B of 1.284, compute the velocity in section B

$$v_B = \frac{Q}{A_B} = \frac{280}{10\ (1.484)} = 18.87\ \text{ft/s},$$

Homework Problems Chapter 4

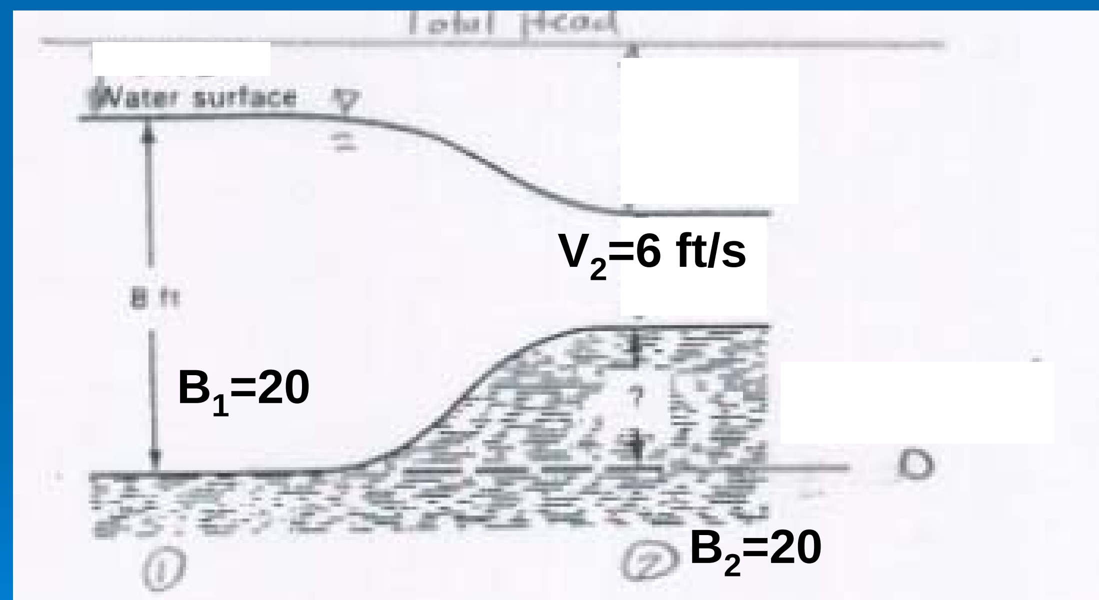
Problem 4.1

PROBLEMS

1. 500 ft^3/s of water flow in a rectangular open channel that is 20 feet wide and 8 feet deep. After passing through a transition structure, the width of the rectangular channel narrows to 15 feet and the bed raises as shown. The velocity in the contracted section is found to be 6 ft/s.
   (a) What is the water depth in the narrow channel?
   (b) What are the velocity heads in each section?
   (c) Draw and label the total and piezometric (water surface) head lines.
   (d) How much does the bed elevation increase in the contracted section?

Remember:

1. **Write down all you know and what you don't know**
2. **Write down equations**



Part A. Use Continuity Equation

A) $Q = A_2 V_2 = 500\ ft^3/s = (15\ ft)(6\ ft/s)\ D_2$

$D_2 = \frac{500}{(15)(6)} = 5.56\ ft/s$

Problem 4.1

PROBLEMS

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Part B.

You know that Velocity Head →

$$\frac{V^2}{2g}$$

B) $$\frac{V_1^2}{2g} = \frac{Q^2}{A_1^2\, 2g} = \frac{500^2}{(8^2)(20^2)\,64.4} = 0.15\ \text{ft}$$

$$\frac{V_2^2}{2g} = \frac{(6)^2}{64.4} = 0.56'$$

Problem 4.1

PROBLEMS

1. 500 ft^3/s of water flow in a rectangular open channel that is 20 feet wide and 8 feet deep. After passing through a transition structure, the width of the rectangular channel narrows to 15 feet and the bed raises as shown. The velocity in the contracted section is found to be 6 ft/s.
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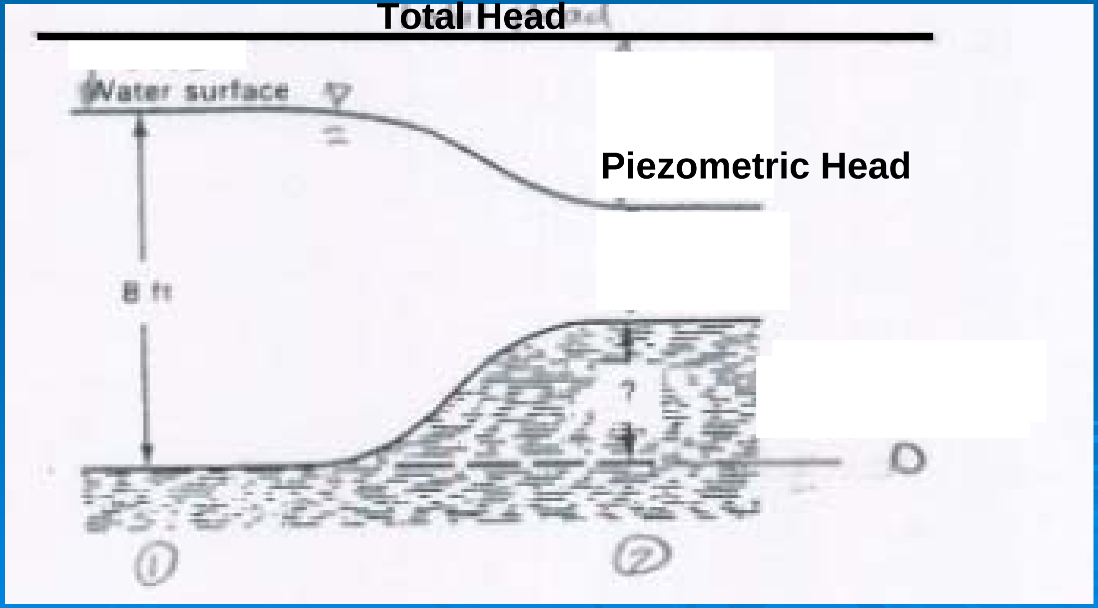
Part C.



Problem 4.1

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Part D.

Write the energy equation from Section 1 to 2

$$\frac{V_1^2}{2g} + Y_1 + Z_1 = \frac{V_2^2}{2g} + Y_2 + Z_2$$

D) Energy Equation ① → ②

$$\frac{V_1^2}{2g} + D_1 + Z_1 = \frac{V_2^2}{2g} + D_2 + Z_2$$

$$0.15 + 8 + 0 = 0.56 + 5.56 + Z_2$$

$$Z_2 = \underline{\underline{2.03 \text{ ft}}}$$

Problem 4.2

2. Water stands 9 feet deep in a large tank. A hole with an area of 0.1 ft^2 is punched in the side of the tank 5 feet above the bottom.
   (a) Compute the discharge from the hole.
   (b) Draw and label the total and piezometric head lines.
   (c) What is the velocity of the water as it hits the ground?

   Note: If the hole is rounded as shown, the answers you compute for an ideal fluid will be correct to within about 1 percent.

Remember:

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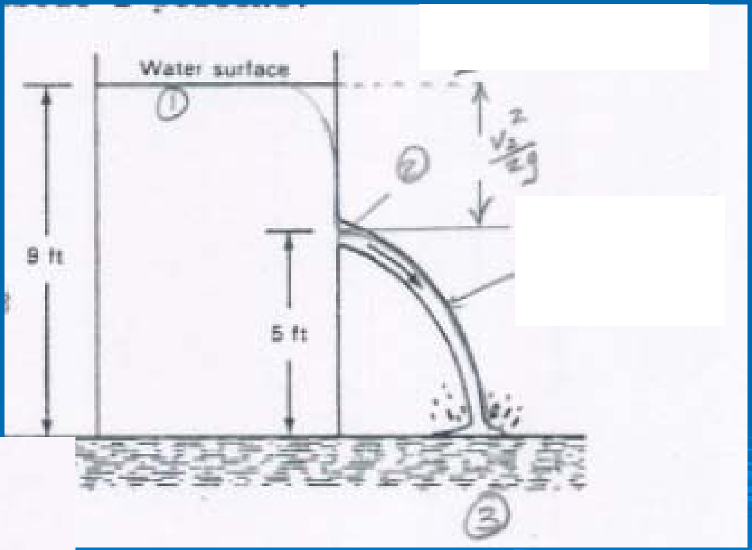
2. **Write down equations**



Part A.

A) Apply energy equation ① → ②

$V_1 = 0$; $D_1 + Z_1 = 9'$; $D_2 + Z_2 = 5'$

$$\frac{V_1^2}{2g} + D_1 + Z_1 = \frac{V_2^2}{2g} + D_2 + Z_2$$

$$0 + 9 = \frac{V_2^2}{2g} + 5 \quad \therefore \quad V_2 = \underline{16.05 \text{ ft/sec}}$$

$$Q = A_2 V_2 = (0.1 \text{ ft}^2)(16.05) = \underline{1.605 \ \frac{\text{ft}^3}{\text{s}}}$$

Problem 4.2

2. Water stands 9 feet deep in a large tank. A hole with an area of 0.1 ft^2 is punched in the side of the tank 5 feet above the bottom.
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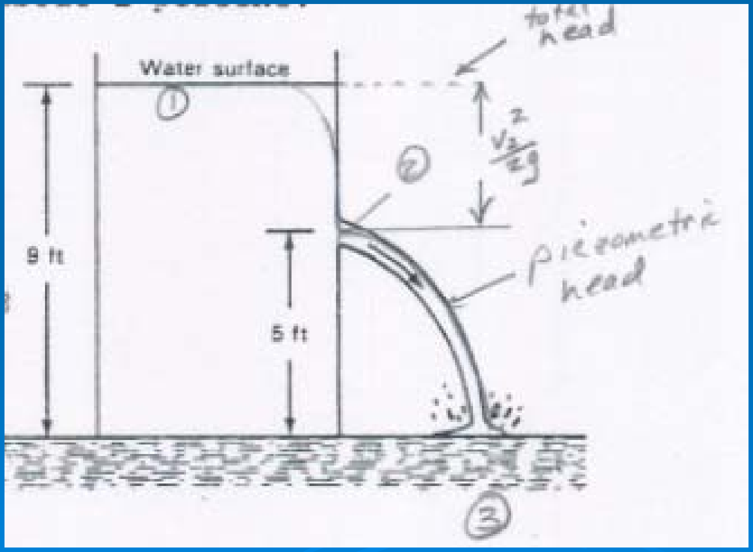
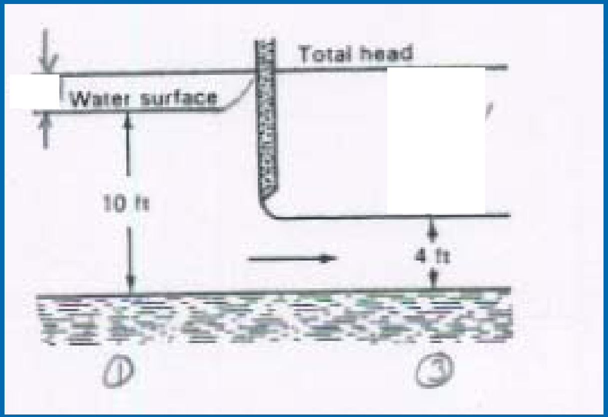
Part B



Problem 4.3

3.Compute the discharge in the 20-foot wide rectangular channel shown below. Draw and label the total head line and the water surface near the gate.



Equations to Use

$A_1 v_1 = A_2 v_2 = Q$

$$\frac{V_1^2}{2g} + Y_1 + Z_1 = \frac{V_2^2}{2g} + Y_2 + Z_2$$

Problem 4.3

3.Compute the discharge in the 20-foot wide rectangular channel shown below. Draw and label the total head line and the water surface near the gate.

Apply Continuity ① → ③

$$Q = (10)(20)V_1 = 4(20)V_2$$

$$V_1 = 0.4 V_2 \;;\; V_1^2 = 0.16 V_2^2$$

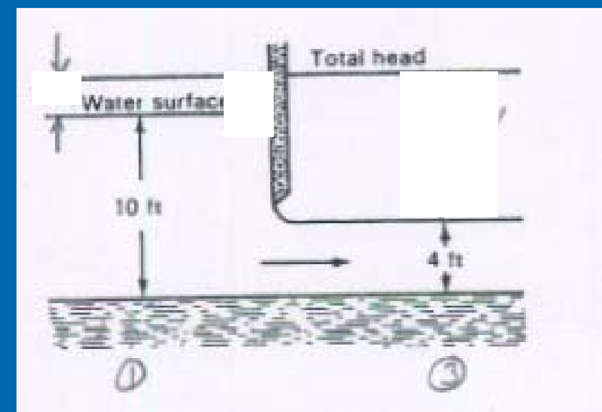
Apply energy ① → ③

$$\frac{V_1^2}{2g} + D_1 + z_1 = \frac{V_2^2}{2g} + D_2 + z_2$$

By substitution from continuity

$$\frac{0.16 V_2^2}{2g} + 10 + 0 = \frac{V_2^2}{2g} + 4 + 0$$

$$\frac{V_2^2}{2g}(1 - 0.16) = 10 - 4$$



$$\frac{V_2^2}{2g} = \frac{6}{0.84} = 7.14' \rightarrow V_2 = 21.45 \text{ ft/s}$$

$$\therefore \frac{V_1^2}{2g} = 1.14'$$

$$Q = A_2 V_2$$

$$= (4)(20)(21.45)$$

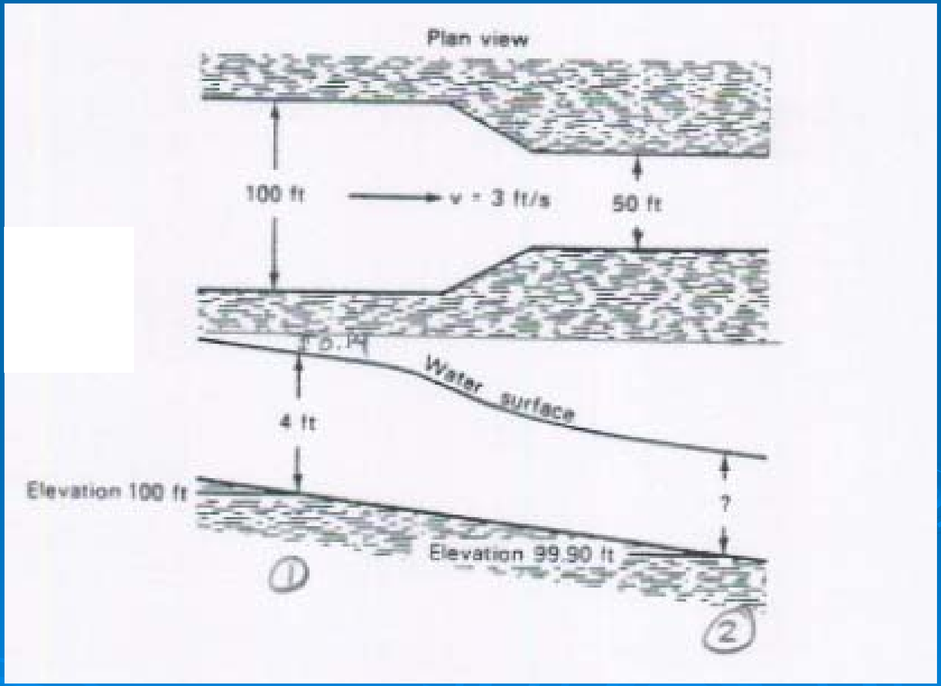
$$= 1716 \text{ cfs}$$

Problem 4.4

4.Compute the discharge and depth in the contracted section for the indicated rectangular channel.

We have enough data to compute velocity head at section 1

$$\frac{V_1^2}{2g} = \frac{3^2}{64.4} = 0.14'$$



Apply Continuity to get velocity head at Section 2

Apply Continuity

$$Q = A_1 V_1 = (4)(100)(3) = A_2 V_2 = (50)(D_2)(V_2)$$

$$\therefore V_2 = \frac{24}{D_2}$$

Problem 4.4

4.Compute the discharge and depth in the contracted section for the indicated rectangular channel.

Apply the Energy Equation

Apply energy

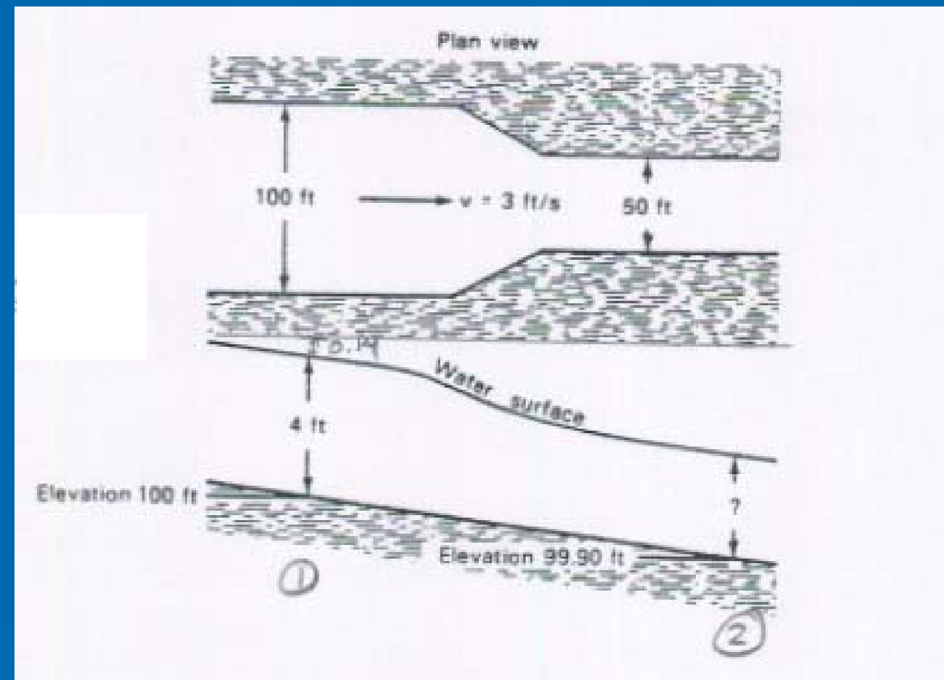
$$\frac{V_1^2}{2g} + D_1 + Z_1 = \frac{V_2^2}{2g} + D_2 + Z_2$$

$$0.14 + 4 + 100 = \frac{24^2}{D_2^2\,64.4} + D_2 + 99.9$$

Simplify

$$4.24 = D_2 + \frac{8.9441}{D_2^2}$$

Solve By Trial + Error



Assume various values of D2 and solve for Left Hand Side (LHS)

D_2	LHS
4.0	4.559
3.9	4.488
3.8	4.42
3.5	4.23
3.0	3.99
2.0	4.24
1.99	4.25

Both of these depths satisfy the equation. These are known as alternate depths. Alternate depths will be discussed in Lesson 11 on Specific Energy