EQUATIONS FOR BASIC HYDRAULIC PRINCIPLES

Lesson 1: Fluid Properties

Force = Mass (M) times acceleration (a) (Newton's Law)
or
Weight (pounds) = mass (slugs) times gravity (g in ft/sec^2),
where acceleration, g = 32.2 ft/sec^2

Density (ρ) = Mass of fluid/unit volume, where ρ_{water} = 1.94 slugs/ft^3

Unit weight (γ) = weight of fluid/unit volume = ρg
where γ_{water} = 62.4 pounds/ft^3

Dynamic viscosity of fluid (μ, in $\frac{\text{sec lb}}{\text{ft}^2}$ or $\frac{\text{slug}}{\text{sec ft}}$) is defined by $\tau = \mu\frac{\partial v}{\partial y}$, where τ is the shearing stress in lbs/ft^2, and the right-hand term is the rate of change of velocity in the fluid.

Kinematic viscosity (ν, in ft^2/sec) = μ/ρ

Pressure (P) = unit weight of water (γ_{water}) times depth (y),
which says pressure increases linearly with depth.

Lesson 2: Forces on Submerged Objects

Force on a vertical rectangular gate:

$F = \frac{\gamma W D^2}{2}$, where D = depth, W = width (triangular pressure prism)

Force is always equal to volume of pressure prism, and acts through the centroid of the pressure prism. Force is also equal to the pressure at the centroid of the gate area times the area of the gate.

To calculate centroid of a complex shape:

$F_R \cdot y_R = \Sigma(F \cdot y)$ or $y_R = \frac{\Sigma(F \cdot y)}{F_R}$, were F_R is the resultant force

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Lesson 3: Similitude and Dimensional Analysis

Reynolds Number, $Re = Vl\rho/\mu$ (Velocity, length, density/viscosity)

Froude Number, $Fr = \frac{V}{\sqrt{gD}}$ where V = velocity, D = depth, g = gravity, 32.2 ft/sec^2

Lesson 4: Ideal Energy Equation

Continuity $Q = V_1 A_1 = V_2 A_2$

Energy $$\frac{V_1^2}{2g} + y_1 + Z_1 = \frac{V_2^2}{2g} + y_2 + Z_2$$

Lesson 5: Velocity Profiles

$\tau = \gamma (D - y) \sin\theta = \gamma (D - y) S$, where τ = shear stress at any level in the fluid

Laminar Flow: $v = \frac{\gamma}{\mu}\sin\theta\left(Dy - \frac{y^2}{2}\right)$ velocity at depth y

$q = \frac{\gamma}{\mu}\sin\theta\frac{D^3}{3}$ unit discharge

$V = \frac{q}{D} = \frac{\gamma}{\mu}\sin\theta\frac{D^2}{3}$ average velocity of section

Turbulent Flow: $v = 5.75 u_* \log\left(\frac{9yu_*}{\upsilon}\right)$ (smooth bed)

$v = 5.75 u_* \log\left(\frac{30y}{k}\right)$ (rough bed)

where $u_* = \sqrt{\tau_0 / \rho}$, called the "shear velocity" or "friction velocity"

Also, from continuity eqn, Unit discharge, $q = Q/W$

Lesson 6: Energy Equation for Real Fluids

$$\frac{\alpha_1 V_1^2}{2g} + y_1 + Z_1 = \frac{\alpha_1 V_2^2}{2g} + y_2 + Z_2 + h_{l(1-2)}$$

where $\alpha = \frac{\sum (v_i^3 a_i)}{V^3 A}$ and is called "kinetic energy coefficient" or "Coriolis coefficient," where v_i = average velocity for subarea a_i, and V is average velocity for total area A, and $h_{l(1-2)}$ is the head loss between points 1 and 2

Lesson 7: Flow Resistance

Hydraulic radius, $R = A/P$, where A = flow area, P = wetted perimeter

Shear stress at bed, $\tau_0 = \gamma R S_f$ (general) where S_f = slope of energy grade line
or $\tau_0 = \gamma R S_o$ (steady-uniform flow) where S_o = bed slope

Shear Velocity, $u_* = \sqrt{\tau_0 / \rho} = \sqrt{gRS}$

Manning's equation $V = \frac{1.49}{n} R^{2/3} \sqrt{S_f}$

or $Q = \frac{1.49}{n} A R^{2/3} \sqrt{S_f} = K \sqrt{S_f}$

Chezy equation $V = C\sqrt{RS_f}$, where $C = \frac{1.49}{n} R^{1/6}$

Darcy-Weisbach eqn $V = \sqrt{\frac{8}{f}} \sqrt{gRS}$

Lesson 8: Uniform Flow

Conveyance $K = \frac{1.49}{n} A R^{2/3} = \frac{Q}{\sqrt{S_f}}$

Lesson 9: Flow in Channels with Variable Roughness

Total Discharge of channel with overbank flow (floodplain flow):

$Q_T = \sum^{N} Q_i = \sum^{N} K_i \sqrt{S_f}$, where K_i is the conveyance of subarea i

$$\alpha = \frac{\Sigma(K_i^3 / a_i^2)}{K_T^3 / A^2} = \frac{\Sigma(K_i^3 / a_i^2)}{(\Sigma K_i)^3 / (\Sigma a_i)^2} = \frac{(K_1^3/a_1^2 + K_2^3/a_2^2 + \ldots + K_N^3/a_N^2)}{\dfrac{(K_1 + K_2 + \ldots + K_N)^3}{(a_1 + a_2 + \ldots + a_N)^2}}$$

Lesson 10: Momentum

Conservation of Momentum $\Sigma \vec{F} = Q\rho(\vec{V}_2 - \vec{V}_1)$

Lesson 11: Specific Energy

Specific Energy $E = \frac{\alpha V^2}{2g} + D$

At critical depth, $Fr = 1$, or $\frac{V_c^2}{2g} = \frac{D_c}{2}$, (assuming $\alpha = 1.0$)

Lesson 14: Local Losses

Local loss $h_e = k \frac{\left|\alpha_1 V_1^2 - \alpha_2 V_2^2\right|}{2g}$, where $k_{expansion} = 0.0$ to 1.0

$k_{contraction} = 0.0$ to 0.5

Lesson 15: Water-Surface Profile Computations

Water-surface elevation at upstream section:

$$h_u = h_d + \frac{\alpha_d V_d^2}{2g} - \frac{\alpha_u V_u^2}{2g} + h_f + h_e \text{, where} \qquad h_f = \bar{S}_f \cdot L \text{, and}$$

$$\bar{S}_f = \sqrt{S_{f_d} \cdot S_{f_u}}$$

Lesson 16: Rapidly Varied Flow at Constrictions

Mattai's equation 1:

$$Q = \frac{A_3}{\sqrt{\alpha_3(1+k_e)}} \sqrt{2g\left(\Delta h + \frac{\alpha_1 V_1^2}{2g} - h_{f(1-3)}\right)} \quad \text{or} \quad Q = CA_3\sqrt{2g\left(\Delta h + \frac{\alpha_1 V_1^2}{2g} - h_{f(1-3)}\right)}$$

where $\Delta h = (D_1 + Z_1) - (D_3 + Z_3)$, and

$C = C' k_F k_\phi k_r$

where C' is a function of the channel contraction ratio, $m = (Q - q')/Q$, and the bridge length to width ratio, L/b, and all other coefficients can be determined from charts.

Lesson 18: Flow over Weirs

General equation: $Q = b \cdot C \cdot H^{3/2}$

where b = width normal to flow

C = discharge coefficient (different for each type of weir and flow conditions)

H = total upstream head