Price Level Determinacy and Monetary Policy under a Balanced-Budget Requirement

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Abstract

This paper analyzes the implications of a balanced budget fiscal policy rule for the determinacy of the price level in a cash–in–advance economy under three alternative monetary policy regimes. It shows that in such stylized models with flexible prices and a period–by–period balanced budget requirement the price level is determinate under a money growth rate peg and is indeterminate under a pure nominal interest rate peg. Under a feedback rule whereby the nominal interest rate is set as an increasing function of the inflation rate the price level is determinate for intermediate values of the inflation elasticity of the feedback rule and is indeterminate for both very low and very high values of the inflation elasticity. Finally, regardless of the particular monetary policy specification, a rational expectations equilibrium consistent with the optimal quantity of money may not exist.

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1 Introduction

In the past decade, the idea of imposing fiscal discipline through a balanced-budget rule has gained considerable importance in the economic policy debate. This is reflected perhaps most clearly in the proposed balanced budget amendment that was passed by the United States House of Representatives on January 26, 1995 but failed passage in the Senate. Yet, little light has been shed on the consequences of balanced budget rules for business cycle fluctuations beyond the basic Keynesian insight that balanced budget rules amplify business cycles by requiring tax increases and expenditure cuts during recessions and the reverse during booms. Even less theoretical work has been devoted to understanding the implications of balanced budget rules for nominal stability, and in particular, to understanding the restrictions that such a fiscal policy rule may impose on monetary policy, if nominal stability is to be preserved.

This paper fills this gap by providing an analysis of the implications of a balanced budget requirement for the determinacy of the price level under alternative monetary policy regimes. The type of balanced-budget rule we focus on is one in which each period the primary surplus—the difference between taxes and government expenditures—is required to be equal to the interest payments on the outstanding public debt. We combine this balanced budget rule with three simple monetary policy specifications: a nominal interest rate peg, a money growth rate peg, and a feedback rule whereby the nominal interest rate is set as an increasing function of the inflation rate. We conduct the analysis within the cash-in-advance model with cash and credit goods developed by Lucas and Stokey (1987).

We find that under the balanced-budget rule the price level is indeterminate when the monetary authority follows an interest rate peg and is determinate when the monetary authority follows a money growth rate peg. These results are not necessary consequences of the monetary policy specifications alone. For example, Auernheimer and Contreras (1990), Sims (1994), and Woodford (1994), find that if the primary surplus is set exogenously, then an interest rate peg delivers a unique price level. This comparison highlights that given the monetary policy regime the adoption of a balanced-budget rule may have important consequences for nominal stability.

If the balanced budget rule is combined with the feedback rule, the price level is determinate when the nominal interest rate is moderately sensitive to the inflation rate, and is indeterminate when the nominal interest rate is either very responsive or little responsive to the inflation rate. Again, this result is driven by the particular fiscal regime we analyze; for the same feedback rule we consider Leeper (1991) shows that when the primary surplus is exogenous — a fiscal policy to which he refers as active — the price level is not indeterminate regardless of how sensitive the feedback rule is.[1]

Leeper also shows that if the primary surplus is increasing in and sensitive enough to the stock of public debt — a fiscal policy to which he refers as passive — the price level is indeterminate for relatively insensitive feedback rules and is determinate otherwise.[2] Leeper's passive fiscal policy is similar to our balanced-budget rule because under both policies taxes are an increasing function of the stock of public

[1]Leeper studies local equilibria by characterizing solutions to a linear approximation of the equilibrium conditions near a steady state. By contrast, we perform a global analysis characterizing solutions to the exact equilibrium conditions.

[2]See Woodford (1995) and Canzoneri and Diba (1996) for further analysis of the relationship between the monetary-fiscal regime and price level determination.

debt. The reason why in our model, unlike in Leeper's, highly sensitive monetary feedback rules render the equilibrium price level indeterminate is that in our model the nominal interest rate affects the consumption/leisure, or cash/credit, margin. In Leeper's model this effect is not present because in his endowment money-in-the-utility-function model with a separable single-period utility function, the marginal utility of consumption is independent of the nominal interest rate in equilibrium.

In practice, balanced-budget proposals typically allow fiscal authorities to run secondary surpluses. Therefore, our benchmark definition of the balanced-budget rule, although analytically convenient, is clearly unrealistic since it forces the government to run a zero secondary surplus on a period-by-period basis. However, it turns out that our main results are not driven by this particular specification of the balanced-budget rule. Specifically, we show that in the case in which either the real or the nominal secondary surplus is positive and exogenous the price level remains indeterminate under an interest rate peg.

The balanced-budget requirement has implications for optimal monetary policy. We find that under a strict balanced-budget rule that eliminates budget surpluses as well as budget deficits the optimal quantity of money advocated by Milton Friedman — a monetary policy consistent with a zero nominal interest rate — cannot be attained under any of the three monetary regimes considered because in this case no rational expectations equilibrium exists. However, if the balanced budget requirement allows for positive secondary surpluses an equilibrium consistent with the optimal quantity of money may or may not exist. For example, if the balanced budget rule is combined with a money growth rate peg and is implemented in such a way that the public debt converges to zero, or if the balanced budget rule is combined with an interest rate peg and is implemented in such a way that the real secondary surplus is at least in one period strictly positive, an equilibrium consistent with the optimal quantity of money exists.

In the next section we describe the formal model and the fiscal policy regime. In sections 3, 4, and 5 we analyze the implications of the balanced-budget rule for price level determinacy when the monetary authority follows, respectively, an interest rate peg, a money growth rate peg, and a feedback rule linking the nominal interest rate to inflation. Section 6 concludes.

2 A Cash-in-Advance Economy

Households

In this section, we present a model of a cash-in-advance economy in which public and private consumption are cash goods and leisure is a credit good.[3] The economy is assumed to be populated by an infinite number of identical households with log-linear single-period utility functions defined over consumption, c_t, and leisure, $1 - h_t$, who seek to maximize their lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \theta \ln(1 - h_t)], \qquad \theta > 0 \tag{1}$$

[3] The presentation of the model follows Woodford (1994).

where $\beta \in (0,1)$ denotes the subjective discount factor and E_t denotes the expectation operator conditional on information available in period t. Each period $t \geq 0$ is divided into two non-overlapping markets. In the first market households use their nominal wealth at the beginning of the period, W_t, to pay lump–sum taxes, T_t, to acquire money, M_t^c, and to purchase state–contingent claims, D_{t+1}, which cost $E_t r_{t+1} D_{t+1}$ dollars and pay D_{t+1} dollars in period $t+1$ (i.e., r_{t+1} is the price of a one–period contingent claim divided by the probability of occurrence of that state). The household's budget constraint in the first market is then given by

$$T_t + M_t^c + E_t\{r_{t+1}D_{t+1}\} \leq W_t \tag{2.a}$$

In the second market, goods and labor services are traded. The household purchases consumption goods at a price of P_t dollars per unit using the money balances it held at the beginning of the goods market. Further, the household has access to a linear technology that enables it to produce one unit of the consumption good per unit of labor input. The household sells these consumption goods at a price of P_t dollars per unit and since it cannot use sales revenues to purchase consumption goods in the current goods market its nominal asset holdings at the beginning of period $t+1$ are

$$W_{t+1} = D_{t+1} + M_t^c - P_t c_t + P_t h_t \tag{2.b}$$

Purchases of goods are subject to a cash-in-advance constraint of the form

$$M_t^c \geq P_t c_t. \tag{2.c}$$

The household chooses sequences for W_{t+1}, D_{t+1}, M_t^c, h_t, and c_t, given $W_0 > 0$, so as to maximize (1) subject to c_t, $M_t^c \geq 0$, $0 \leq h_t \leq 1$, (2.a), (2.b), (2.c) and the following borrowing constraint that prevents it from engaging in Ponzi schemes

$$W_{t+1} \geq -q_{t+1}^{-1} \sum_{j=1}^{\infty} E_{t+1}\{q_{t+j+1}P_{t+j} - q_{t+j}T_{t+j}\} \tag{2.d}$$

the price in period 0 of one dollar in period t in a particular state of the world, which is defined as

$$q_t \equiv r_1 r_2 \cdots r_t \qquad \text{with} \qquad q_0 \equiv 1.$$

The borrowing limit (2.d) ensures that in every state of the world private debt is not greater than the amount an agent would be able to repay, which is equal to the present discounted value of the time endowment net of taxes.

One can show that the set of sequences $\{c_t, h_t, M_t^c\}$ satisfying the budget constraints (2.a)-(2.d) are equivalent to the set of sequences $\{c_t, h_t, M_t^c\}$ satisfying the cash-in-advance constraint (2.c) and the following present value budget constraint[4]

$$W_0 + E_0 \sum_{t=0}^{\infty} [q_{t+1} h_t P_t - q_t T_t] \geq$$
$$E_0 \sum_{t=0}^{\infty} [(q_t - q_{t+1})(M_t^c - P_t c_t) + q_t p_t c_t]. \tag{3}$$

From the first–order conditions of the household's optimization problem, consumption and hours must satisfy

$$\frac{1}{P_t c_t} r_{t+1} = \beta \frac{1}{P_{t+1} c_{t+1}} \tag{4}$$
$$\frac{\theta}{1 - h_t} = \beta E_t \frac{P_t}{P_{t+1} c_{t+1}}. \tag{5}$$

The first equation is a standard pricing equation for a one-step-ahead contingent claim, and equates the loss in utility from buying a contingent claim today with the expected gain in utility realized from consuming its pays off. The second equation is a labor supply equation and says that the disutility of working an extra hour in period t has to equal the utility derived from spending the wage on consumption goods in period $t + 1$. A further requirement for optimality of the household's consumption, money demand, and labor plans is that they satisfy the present value budget constraint, (3), with equality.

The government

We assume that the government issues only a riskless pure discount bond (i.e., a bond that pays off one dollar in the following period regardless of the state realized) which is denoted by B_t. The government starts in period zero with some outstanding stock of debt, B_{-1}. Its period–by–period budget constraint is

$$M_t + \frac{B_t}{R_t} = B_{t-1} + M_{t-1} + P_t g - T_t \tag{6}$$

where M_t denotes the money supply, g denotes constant real government purchases, and R_t denotes the gross nominal interest rate paid on the riskless bond. The interest rate must satisfy

$$R_t = \frac{1}{E_t r_{t+1}}. \tag{7}$$

The government, like households, is subject to a cash-in-advance constraint on its purchases of goods

$$M_t^g \geq P_t g.$$

The central element that distinguishes this paper from other studies of price level determination under alternative monetary policy regimes is the specification of fiscal policy. We assume that the government is subject to a balanced-budget requirement whereby the primary surplus must be equal to the interest payments

[4] See Woodford (1994).

on the outstanding public debt

$$T_t - P_t g = B_{t-1}/R_{t-1}(R_{t-1} - 1). \tag{8}$$

This specification of the balanced budget rule implies that seignorage income cannot be used to finance current spending or to pay interest on the debt. Combining (6) and (8) yields

$$M_t + \frac{B_t}{R_t} = M_{t-1} + \frac{B_{t-1}}{R_{t-1}}, \tag{9}$$

which says that total nominal government liabilities are constant over time; that is, under the balanced-budget rule, changes in the stock of money are implemented exclusively through open market operations.

Throughout the paper, we maintain the same fiscal policy regime (i.e., a balanced budget requirement) but consider three alternative monetary policy regimes: (i) a pure interest rate peg, (ii) a money supply growth rate peg, and (iii) a feedback rule whereby the nominal interest rate is set as an increasing function of the inflation rate. Under policy regimes (i) and (iii), the central bank sets the nominal interest rate by fixing the price of the riskless one-period nominal bond and stands ready to exchange money for bonds in any quantities demanded. This means that M_t and B_t are endogenous. Under policy regime (ii), the government specifies a deterministic path for the money supply, so that B_t and R_t are endogenous.

Equilibrium

In equilibrium the product and money market clear,

$$h_t = c_t + g \tag{10}$$

and

$$M_t = M_t^c + M_t^g = M_t^c + P_t g,$$

where the last equality follows from the assumption that the government's cash–in–advance constraint is always binding. Because all agents are identical and government bonds are the only financial assets in positive aggregate net supply, it must be the case that

$$D_{t+1} = B_t.$$

In any equilibrium, the nominal interest rate must be non-negative. When the nominal interest rate is positive, the household's cash-in-advance constraint holds with equality, $M_t^c = P_t c_t$, and when the nominal interest rate is zero ($R_t = 1$), consumption is equal to the unconstrained optimum,

$$c_t = \hat{c} \equiv \text{argmax}[\ln c_t + \theta \ln(1 - c_t - g)].$$

Therefore, in any equilibrium

$$c_t = \min(\hat{c}, m_t - g) \tag{11}$$

where

$$m_t = M_t/P_t. \tag{12}$$

Using the definitions

$$F(m) \equiv \theta\, m/[1 - \min(m, \hat{c} + g)],$$

and

$$G(m) \equiv m/[\min(m, \hat{c} + g) - g],$$

and combining the market clearing conditions with the first order conditions (4) and (5) yields,

$$G(m_t)\, r_{t+1} = \beta G(m_{t+1})\, M_t/M_{t+1} \tag{13}$$

and

$$F(m_t) = \beta E_t \left[G(m_{t+1})\, M_t/M_{t+1}\right]. \tag{14}$$

Taking expected values of both sides of (13) and substituting (7) and (14) implies a demand for real balances of the form

$$R_t = \frac{G(m_t)}{F(m_t)}. \tag{15}$$

Since $G(.)$ is strictly decreasing and $F(.)$ is strictly increasing in m for $m \le \hat{c} + g$, R_t is strictly decreasing in m_t for $m \le \hat{c} + g$. On the other hand, since $G(m) = F(m)$ for $m \ge \hat{c} + g$, $m \ge \hat{c} + g$ for $R = 1$.

From (13) we can express the present value deflator as

$$q_t = \beta^t M_0/M_t \frac{G(m_t)}{G(m_0)}. \tag{16}$$

Substituting the market clearing conditions into (3), which in equilibrium must hold with equality, yields

$$\sum_{t=0}^{\infty} E_0(q_t - q_{t+1})M_t = (M_{-1} + B_{-1}) + \sum_{t=0}^{\infty} E_0 q_t(P_t g - T_t), \tag{17}$$

Given (6), (17) is equivalent to

$$\lim_{t\to\infty} E_0 \left[q_t(M_t + B_t/R_t) + (q_{t+1} - q_t)M_t\right] = 0. \tag{18}$$

Using (9) and (16) to eliminate $M_t + B_t/R_t$ and q_t from this expression yields

$$\lim_{t\to\infty} E_0 \beta^t \left[G(m_t)A_0/M_t + F(m_t) - G(m_t)\right] = 0. \tag{19}$$

where A_0 ($\equiv M_{-1} + B_{-1}/R_{-1}$) denotes initial nominal government liabilities and is an initial condition in period 0. We assume that $A_0 > 0$.

A rational expectations equilibrium is a set of processes $m_t > 0$, $M_t > 0$, and $R_t \ge 1$ satisfying (14), (15), (19), and one additional equation specifying the monetary policy regime. One can then uniquely determine consumption, c_t, from (11); labor, h_t, from (10); the price level, P_t, from (12); the pricing kernel, r_{t+1}, from (13); the present value deflator, q_t, from (16); the stock of public debt, B_t/R_t, from (9); and lump sum taxes, T_t, from (6).

3 Equilibrium under an Interest Rate Peg

The class of monetary policy regimes to be considered in this section are interest-rate pegs defined by the choice of a constant nominal interest rate

$$R_t = R \geq 1.$$

We first analyze the case in which the nominal interest rate is set at zero ($R = 1$). This policy corresponds to the optimal quantity of money because it eliminates any inefficiencies associated with holding money. Since for $m \geq \hat{c} + g$, $F(m) = G(m)$, it follows that when $R_t = 1$, any $m_t \geq \hat{c} + g$ solves (15). However, (19) will not be satisfied by any of these solutions. Since in this case $F(m) = G(m)$, (19) simply requires that $\lim_{t\to\infty} E_0 \beta^t G(m_t)/M_t A_0 = 0$. Using (14) and (15) one can express $E_0 G(m_t)/M_t$ as $(\beta R)^{-t} G(m_0)/M_0$ so that for $R = 1$ $\lim_{t\to\infty} E_0 \beta^t E_0 G(m_t)/M_t A_0 = G(m_0)/M_0 A_0 \neq 0$. Consequently, under a balanced budget rule no rational expectations equilibrium exists when the nominal interest rate is pegged at zero, and therefore the optimal quantity of money cannot be brought about.

Next, we show that when the nominal interest rate is pegged at a positive value, then a rational expectations equilibrium exists and the associated resource allocation is unique while the price level is indeterminate. Let the nominal interest rate be pegged at some positive value $R > 1$. Since $G(m)/F(m)$ is monotonically decreasing in m, strictly greater than one for all $m < \hat{c} + g$, and equal to one for $m = \hat{c} + g$, it follows from (15) that real balances are uniquely determined, that is,

$$m_t = m \quad \forall t \geq 0 \quad \text{and} \quad g < m < \hat{c} + g.$$

If $R > 1$, the household's cash-in-advance constraint is always binding and therefore $c_t m - g$ for all t. From market clearing in the product market it follows that $h_t = m$. That is, under a pure interest rate peg with $R > 1$ the real resource allocation is unique and consumption and hours are constant.

Using $E_0[G(m_t)A_0/M_t] = (\beta R)^{-t} G(m_0) A_0/M_0$ and $m_t = m$ in (19) yields

$$\lim_{t\to\infty} R^{-t} A_0 + \beta^t (R^{-1} - 1) M_0 = 0 \tag{20}$$

This equation is satisfied for any M_0. Therefore, in equilibrium the price level in period 0 is indeterminate, $P_0 = M_0/m$. This represents the main result of this section, namely that a balanced budget rule combined with a pure interest rate peg leads to nominal indeterminacy.

Perfect foresight equilibria

It is useful to characterize the evolution of real debt and real taxes under the monetary-fiscal regime considered in this section. To simplify matters, we will restrict the analysis to perfect foresight paths. Given M_0, the perfect foresight path of M_t is uniquely determined by (14) as

$$M_t = (\beta R)^t M_0.$$

Using this expression and the fact that both total nominal government liabilities and real balances are constant, real debt can be expressed as

$$b_t \equiv B_t/P_t = Rm(\beta R)^{-t} A_0/M_0 - Rm.$$

According to this expression, the stock of real debt depends on the initial money supply M_0 and is therefore not unique. The long–run level of real debt depends on the level of the nominal interest rate. If the monetary authority pegs the nominal interest rate at a value that exceeds the real interest rate ($R > \beta^{-1}$), then $\lim_{t\to\infty} b_t = -Rm$, so the real stock of debt converges to a finite (negative) value. That is, the government ends up being a net lender to the public. Alternatively, if the monetary authority pegs the nominal interest rate at a value below the real interest rate ($1 < R < \beta^{-1}$), then $\lim_{t\to\infty} b_t = \infty$, so the real stock of debt grows without bound.[5] The reason for this explosive behavior of real debt is that when $\beta R < 1$, the money supply falls at the rate $1 - \beta R$ and therefore seignorage income is negative. In turn, these losses must be financed with new debt because, according to the balanced budget rule, the government is only allowed to raise taxes to cover government spending and interest on the outstanding debt. However, private agents are willing to hold the ever increasing government debt because they face a path of real lump-sum taxes, τ_t, which is also increasing over time

$$\tau_t = T_t/P_t = g + b_{t-1}/R(R-1)/(\beta R).$$

Allowing for fiscal surpluses

In this subsection we consider balanced-budget rules that allow for positive secondary surpluses. This type of rules are clearly more realistic than our baseline specification of a zero secondary surplus.[6] Suppose that the fiscal authority were to set exogenously the time path of the (non-negative) secondary surplus.[7] For simplicity we assume that either the real secondary surplus, s_t, is exogenous and bounded above by $\overline{s}$ or that the nominal secondary surplus,$S_t = P_t s_t$, is exogenous and bounded above by $\overline{S}$. Under either assumption the fiscal policy rule takes the form

$$T_t - P_t g - (R_{t-1} - 1)B_{t-1}/R_{t-1} = S_t.$$

[5]Note that although the real stock of debt is increasing over time, its present discounted value ($\lim_{t\to\infty} \beta^t b_t$) is zero.

[6]For example, the proposed balanced budget amendment that was passed in January 1995 by the U.S. House of Representatives allows for positive secondary surpluses.

[7]The requirement of an, at least sometimes, strictly positive secondary surplus by itself is not a complete description of the fiscal policy regime. Further restrictions on the way in which the secondary surplus is brought about are necessary.

Substituting this specification of the balanced budget rule in the government's sequential budget constraint (6), the evolution of total nominal government liabilities, $M_t + B_t/R_t$, becomes

$$M_t + \frac{B_t}{R_t} = M_{t-1} + \frac{B_{t-1}}{R_{t-1}} - S_t = \ldots = A_0 - \sum_{j=0}^{t} S_j.$$

Using this expression to eliminate $M_t + B_t/R_t$ in the transversality condition (18) yields

$$\lim_{t\to\infty} E_0 \left[q_t(A_0 - \sum_{j=0}^{t} S_j) + (q_{t+1} - q_t)M_t \right] = 0, \tag{21}$$

which together with (16) replaces equilibrium condition (19).

Suppose that $R = 1$. In this case $E_t q_{t+j}/q_t = R^{-j} = 1$, thus $E_0(q_{t+1} - q_t)M_t = 0$ and $E_0 q_t = 1$. Consequently (21) simplifies to

$$A_0 = \lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j) = 0 \tag{22}$$

We first analyze the case that the real secondary surplus is exogenous and deterministic. From (15) it follows that in any equilibrium $m_t \geq \hat{c} + g$, which implies, by (16), that $G(m_t) = m_t/\hat{c}$ and $q_t = \beta^t P_0/P_t$ so that $E_0 q_t \sum_{j=0}^{t} P_j s_j = \beta^t P_0 \sum_{j=0}^{t} E_0 P_j/P_t s_j = \beta^t P_0 \sum_{j=0}^{t} \beta^{j-t} s_j$, where the last equality follows from $P_j/P_t = \beta^{j-t} q_t/q_j$ and from the fact that if $R = 1$, $E_0 q_t/q_j = 1$ for any $t \geq j \geq 0$. Thus (22) becomes

$$A_0 = P_0 \lim_{t\to\infty} \sum_{j=0}^{t} \beta^j s_j.$$

Since $\sum_{j=0}^{t} \beta^j s_j$ is monotonically increasing and bounded above by $\overline{s}/(1-\beta)$, it converges to some finite positive value s. Therefore the transversality condition will be satisfied only if

$$P_0 = (1-\beta)A_0/s,$$

that is, the transversality condition uniquely determines the equilibrium price level. However, the initial money supply is indeterminate, any M_0 that satisfies $M_0 \geq (\hat{c}+g)A_0 P_0$ is an equilibrium. From this result it follows that under a balanced budget requirement in which the secondary real surplus is deterministic and strictly positive in at least one period, (i) an equilibrium consistent with the optimal quantity of money exists and (ii) the associated price level is unique. The reason why an equilibrium consistent with the optimal quantity of money exists when the real secondary surplus is positive and does not exist when the secondary surplus is exactly zero period by period is the following. In equilibrium the ratio of the government's initial nominal liabilities to the initial price level must be equal to the present discounted value of real primary surpluses.

If $R = 1$, the real primary surplus is equal to the real secondary surplus because interest payments on the debt are zero. Therefore, if the real secondary deficit is positive in at least one period, the present discounted value of real primary surpluses is also positive. Consequently, there is a unique price level which makes the real value of the initial nominal government liabilities equal to this present discounted value. If, however, the secondary surplus is equal to zero period by period, the real primary surplus is also equal to zero. This implies that unless the initial government liabilities are equal to zero, there exists no price level which makes the real value of the government's initial nominal liabilities equal to the present discounted value of real primary surpluses.

Assume now that the nominal secondary surplus is exogenous and deterministic, then (22) becomes

$$A_0 = \lim_{t\to\infty} \sum_{j=0}^{t} S_j$$

But this will certainly not be true for any arbitrary path of the nominal secondary surplus. Further, even if the sequence of nominal secondary surpluses satisfied the above condition, the price level would be indeterminate. Therefore we conclude that under a balanced budget requirement a rational expectations equilibrium consistent with the optimal quantity of money may or may not exist, and even if it does exist the price level may or may not be determinate depending on the particular implementation of the balanced budget requirement.

Now suppose that $R > 1$. In this case, (12) implies that real balances are uniquely determined and constant over time, $m_t = m^*$. With real balances uniquely determined, the real allocation is also uniquely determined. Using $m_t = m^*$, $E_t q_{t+1}/q_t = E_t r_{t+1} = R^{-1}$, and (16), one can express the last term on the left side of (21), $E_0(q_{t+1} - q_t)M_t$, as $(R^{-1} - 1)\beta^t M_0$ which converges to zero as t converges to infinity. Replacing $E_0 q_t$ by R^{-t} yields $E_0 q_t A_0 = R^{-t} A_0$, which implies that the first term on the left side of (21) converges to zero as t converges to infinity. Thus (21) becomes

$$\lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j = 0. \tag{23}$$

Assuming that the real secondary surplus is exogenous and using the facts that $q_t = \beta^t M_0/M_t$ and $E_0 M_j/M_t = (\beta R)^{j-t}$ the left side of (23) can be written as

$$\begin{aligned}
E_0 q_t \sum_{j=0}^{t} S_j &= \lim_{t\to\infty} E_0 \beta^t \frac{M_0}{M_t} \sum_{j=0}^{t} P_j s_j \\
&= \lim_{t\to\infty} \beta^t \frac{M_0}{m^*} \sum_{j=0}^{t} E_0 \frac{M_j}{M_t} s_j \\
&\leq \lim_{t\to\infty} \beta^t \frac{M_0}{m^*} \sum_{j=0}^{t} E_0 \frac{M_j}{M_t} \overline{s} \\
&= \lim_{t\to\infty} \beta^t \frac{M_0 \overline{s}}{m^*} \sum_{j=0}^{t} (\beta R)^{j-t} \\
&= \lim_{t\to\infty} R^{-t} \frac{M_0 \overline{s}}{m^*} \quad \frac{1-(\beta R)^{t+1}}{1-\beta R} \\
&= 0,
\end{aligned}$$

This shows that the transversality condition is satisfied for any value of M_0 and hence the initial money supply is indeterminate. The initial price level, being equal to M_0/m^*, is thus also indeterminate.

Alternatively, if the nominal secondary surplus is exogenous, the left side of (23) can be written as

$$\begin{aligned} \lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} S_j &\leq \lim_{t\to\infty} E_0 q_t \sum_{j=0}^{t} \overline{S} \\ &= \lim_{t\to\infty} \overline{S}\, t\, E_0 q_t \\ &= \lim_{t\to\infty} \overline{S}\, t\, R^{-t} \\ &= 0, \end{aligned}$$

where we used the facts that S_j is bounded and that $E_0 q_t = R^{-t}$. This implies that the transversality condition will be satisfied regardless of the value of M_0 and hence the initial price level is again indeterminate. It follows that our earlier finding that if $R > 1$ the price level is indeterminate, also holds when the balanced budget requirement is implemented through a fiscal policy in which the secondary surplus is exogenous and positive.

Related literature

We close this section by comparing our results to those obtained under a fiscal policy in which the primary surplus is exogenous. Specifically, as mentioned earlier, Auernheimer and Contreras (1994), Sims (1994), and Woodford (1994) find that if the paths of government purchases and taxes are exogenous, then the price level is uniquely determined under an interest rate peg. To see this, use equations (7) and (16) and the fact that for $R > 1$ real balances are constant (by the liquidity preference equation (15)), to express the transversality condition (17) as

$$\sum_{t=0}^{\infty} \beta^t M_0 (1 - R^{-1} + E_0(\tau_t - g_t)/m) = M_{-1} + B_{-1}.$$

When the process for the primary surplus, $\tau_t - g_t$, is exogenous, this equation implies that M_0 is only a function of the predetermined variables M_{-1} and B_{-1} and of the present discounted value of current and expected future primary surpluses. Consequently, M_0 is uniquely determined and thus so is the price level $P_0 = M_0/m$. Comparing this result with the one obtained under a balanced budget rule, it follows that given the monetary policy regime, the adoption of a balanced budget rule has important consequences for price stability. Under the monetary policy considered here, a fiscal policy shift whereby a policy in which the real primary surplus is set exogenously is replaced by a balanced budget policy in which the secondary surplus is set exogenously would result in the loss of the nominal anchor.

4 Equilibrium under a Money Growth Rate Peg

Under the monetary policy regime to be considered in this section the government sets a constant growth rate $\mu > 0$ for the money supply

$$M_t = \mu^t M_0. \tag{24}$$

Given the money growth rate peg, (14) reduces to

$$F(m_t) = \frac{\beta}{\mu} E_t G(m_{t+1}) \tag{25}$$

and the transversality condition (19) becomes

$$\lim_{t\to\infty} \beta^t E_0 \left[G(m_t)\mu^{-t} A_0 + (F(m_t) - G(m_t))M_0 \right] = 0. \tag{26}$$

Given A_0 and M_0, a rational expectations equilibrium is a stochastic process $\{m_t\}$ satisfying (25) and (26).

Steady state equilibria

We first analyze the existence of steady state equilibria, that is, non-stochastic equilibria with constant real balances. Clearly, a steady state equilibrium does not exist if $\mu \leq \beta$ because in that case no constant value of real balances satisfies (26). This means that the policy maker cannot bring about the efficient allocation, or optimal quantity of money, by setting the money growth rate to $\beta - 1$.[8] Combining this result with the one obtained in the previous section, we conclude that under a balanced budget requirement an interest rate peg as well as a money growth rate peg equilibrium may fail to be consistent with the optimal quantity of money advocated by Milton Friedman. This result is a consequence of the particular fiscal policy we are analyzing. For example, Woodford (1994) shows that for a fiscal policy in which the real primary surplus is positive and exogenous a steady state equilibrium consistent with the optimal quantity of money exists under a pure interest rate peg. He also shows that for a fiscal policy in which the stock of public debt is zero and government purchases are exogenous a steady state equilibrium consistent with the optimal quantity of money exists under a money growth rate peg.

In what follows, we assume that the growth rate of the money supply exceeds the discount factor ($\mu > \beta$). Given this assumption, any constant value of m satisfies (26). So the question of whether a steady state equilibrium exists and whether it is unique, reduces to studying solutions to (25) in which $m_t = m_{t+1} = m^*$ for all t

$$F(m^*) = \frac{\beta}{\mu} G(m^*). \tag{27}$$

For $g < m < g + \hat{c}$, $G(m)$ is monotonically decreasing, and $F(m)$ is monotonically increasing. For

[8] If the balanced budget rule allows for surpluses, a steady–state equilibrium consistent with the optimal quantity of money may exist. To see this, note that if $\mu = \beta$, then (16) implies that in steady state $q_t = q_{t+1} = 1$ and (15) together with (25) implies that $R_t = 1$. From (18) it then follows that $\lim_{t\to\infty}(M_t + B_t)$ must be equal to zero in a steady–state equilibrium. As $\lim_{t\to\infty} M_t = \lim_{t\to\infty} \beta^t M_0 = 0$, this requires that $\lim_{t\to\infty} B_t = 0$ as well. This could certainly be the case but it does not need to be: a balanced budget rule does not necessarily imply that the stock of public debt converges to zero.

$m \geq \hat{c} + g$, $F(m) = G(m)$, and since $\beta/\mu < 1$ the left-hand side of (27) is greater than the right-hand side of (27) (figure 1). As m approaches g from above, the right-hand side of (27) becomes larger than the left-hand side because $G(m)$ converges to ∞ while $F(m)$ converges to $\theta g/(1-g)$; therefore, there exists a unique solution m^* to (27) satisfying $g < m^* < g + \hat{c}$. Since $m^* < g + \hat{c}$, the consumer's cash-in-advance constraint is binding in any steady state equilibrium and from (15) the nominal interest rate, $R^* - 1$, is positive and equal to $\mu/\beta - 1 > 0$. Because real balances are constant in steady state, the inflation rate is also constant and equal to the growth rate of the money supply, μ. Using the fact that total nominal government liabilities ($M_t + B_t/R_t = A_0$), real balances, and the nominal interest rate are constant, real debt can be expressed as

$$b_t = \mu^{-t} R^* m^* A_0/M_0 - R^* m^*.$$

Hence, if the money growth rate is negative ($\mu < 1$), then the real stock of debt associated with the steady-state equilibrium grows without bounds. If, on the other hand, the money growth rate is positive ($\mu \geq 1$), then the long-run real stock of debt is finite (and negative for $\mu > 1$).

Non–steady–state equilibria

We make the following assumption regarding preferences and the size of government purchases

$$F(g) < \beta/\mu \inf_{m>g} G(m). \tag{A1}$$

This assumption is satisfied for any value of μ for which private consumption exceeds government consumption in the steady-state equilibrium. The following proposition shows that if (A1) is satisfied, then m_t is bounded above and below (away from g) in any equilibrium. In particular, equilibria in which the private sector ends up completely demonetized are ruled out.

Proposition 4.1 *Suppose that preferences satisfy (A1) and that $\mu > \beta$, then there exists a lower bound $\underline{m}$, $g < \underline{m} < \hat{c} + g$, such that $m_t \geq \underline{m}$ at all times in any equilibrium, and there exists an upper bound $\overline{m} < \infty$ such that $m_t \leq \overline{m}$ at all times in any equilibrium.*

Proof: The proof of this proposition, which draws heavily on the proof of proposition 5 in Woodford (1994), is presented in the appendix ■

Proposition 4.1 establishes that in any equilibrium there are at most bounded fluctuations. This result depends crucially on the balanced-budget requirement and the assumption of a positive initial stock of government debt. For an alternative fiscal policy, one in which the stock of debt and government purchases are zero, Woodford (1994) proves that Proposition 4.1 holds for non-negative money growth rates ($\mu \geq 1$) but not for negative money growth rates ($\beta < \mu < 1$). In particular, he shows that for negative money growth rates speculative hyperdeflations are possible. The existence of the lower bound for real balances $\underline{m}$, and hence the impossibility of speculative hyperinflations, in our setup is due to the fact that under our fiscal policy government purchases are positive, exogenous, and subject to a cash-in-advance constraint which prevents the price level from growing (in the long run) at a faster rate than the money supply. However, the reason why we can rule out speculative hyperinflations is not simply a consequence of this aspect of our fiscal policy specification. Even if government purchases and the initial stock of debt were zero, as in Woodford, our log-linear preference specification would rule out the existence of speculative hyerinflations.

If (A1) is replaced by the slightly stronger assumption that

$$g < \frac{\beta}{\mu\theta} \equiv \gamma, \tag{A2}$$

then one can show that the steady-state equilibrium is the unique rational expectations equilibrium. Assumption (A2) is still satisfied whenever private consumption exceeds government consumption in steady state.[9] Some simple algebra shows that if (A2) holds, then $\overline{m} = \hat{c} + g$. Consider first the existence of perfect-foresight non-steady state equilibria. Assuming that $m_t \leq \hat{c} + g$ for all t, (25) can be solved for m_{t+1} as a function of m_t

$$m_{t+1} = \frac{g m_t}{(1+\gamma)m_t - \gamma} \equiv f(m_t). \tag{28}$$

If $\mu > \beta$ and if m_t is bounded, the transversality condition (26) is always satisfied. Therefore, if (A2) holds, any sequence $\{m_t\}$ that satisfies (28) and $\underline{m} \leq m_t \leq \overline{m}$ constitutes a perfect foresight equilibrium.

Proposition 4.2 *If $\mu > \beta$ and assumption (A2) is satisfied, then there exists a unique perfect foresight equilibrium $m_t = m^*$ for all t.*

Proof: Since the g, β, and μ that satisfy (A2) also satisfy (A1) it follows from our analysis of steady state equilibria that a steady state equilibrium m^* exists, that it is unique and that it satisfies $g < m^* < \hat{c} + g$. Suppose there exists another perfect foresight equilibrium $\{m_t\}$ with $m_0 < m^*$. Let $\tilde{m}_t = m_{2t}$, $t \geq 0$; From (28) it follows that $\tilde{m}_{t+1} = (g^2\tilde{m}_t)/((1+\gamma)(g-\gamma)\tilde{m}_t + \gamma^2)$. (A2) and $m_0 < m^*$ imply that $\{\tilde{m}_t\}$ is a monotonically decreasing sequence, and Proposition 4.1 implies that it is bounded below by $\underline{m}$. Therefore, the sequence $\{\tilde{m}_t\}$ must converge to some $\tilde{m}$, with $\underline{m} \leq \tilde{m} < m^*$. Being a limit, $\tilde{m}$ must solve $\tilde{m} = (g^2\tilde{m})/((1+\gamma)(g-\gamma)\tilde{m} + \gamma^2)$. The only non-zero solution to this equation is $\tilde{m} = (\gamma + g)/(1+\gamma) = m^*$, which is a contradiction. By a similar argument one can show that there does not exist a perfect foresight equilibrium with $m_0 > m^*$. Hence the unique perfect foresight equilibrium is $m_t = m^*$ for all t ■

This proposition can be used to show that the only rational expectations equilibrium is the steady state.

Proposition 4.3 *If $\mu > \beta$ and assumption (A2) is satisfied, then there exists a unique rational expectations equilibrium $m_t = m^*$ for all t.*

Proof: Suppose there exists another rational expectations equilibrium with $m_t > m^*$ for some t. Let $m^P_{t+j} \equiv f^j(m_t)$, where $f(.)$ is defined by (28). It follows that there exists an even integer J such that for all $s \leq J$ $m^P_{t+s} \in [\underline{m}, \overline{m}]$ and $g < m^P_{t+J+1} < \underline{m}$. Note that $m^* > m^P_{t+1}$ and that $P(m_{t+1} \leq m^P_{t+1}) > 0$ because $E_t G(m_{t+1}) = G(m^P_{t+1})$ and $G(.)$ is strictly decreasing. Further, let $\epsilon_{t+s} = P(m_{t+s} \geq m^P_{t+s} | m_{t+s-1} \leq m^P_{t+s-1})$, if s is even, and let $\epsilon_{t+s} = P(m_{t+s} \leq m^P_{t+s} | m_{t+s-1} \geq m^P_{t+s-1})$, if s is odd. The probabilities ϵ_{t+s} are strictly positive for all $s \leq J$. To see this, assume first that $m_{t+s-1} \leq m^P_{t+s-1}$, then since $F(m)$ is

[9] (A1) is satisfied whenever $\theta g < \frac{\beta/\mu}{1-\beta/\mu}$. (A2) is stronger than (A1) because $\frac{\beta/\mu}{1-\beta/\mu} > \beta/\mu$.

strictly increasing, $F(m_{t+s-1}) \leq F(m^P_{t+s-1})$; using (25) this implies that $E_{t+s-1}G(m_t + s) \leq G(m^P_{t+s})$ and, since G(.) is strictly decreasing, this implies that $\epsilon_{t+s} > 0$. Similarly if $m_{t+s-1} \geq m^P_{t+s-1}$, then $E_{t+s-1}G(m_{t+s}) \geq G(m^P_{t+s})$ and hence $\epsilon_{t+s} > 0$. Next, note that

$$
\begin{aligned}
P(m_{t+J} \geq m^P_{t+J}) &= \epsilon_{t+J} P(m_{t+J-1} \leq m^P_{t+J-1}) + \\
&\quad (1-\epsilon_{t+J}) P(m_{t+J-1} > m^P_{t+J-1}) \\
&\geq \epsilon_{t+J} P(m_{t+J-1} \leq m^P_{t+J-1}) \\
&\geq \dots \\
&\geq \prod_{s=2}^{J} \epsilon_{t+s} P(m_{t+1} \leq m^P_{t+1}) \\
&> 0.
\end{aligned}
$$

Suppose that $m_{t+J} \geq m^P_{t+J}$, then

$$
\begin{aligned}
\frac{\beta}{\mu} E_{t+J} G(m_{t+J+1}) &= F(m_{t+J}) \\
&\geq F(m^P_{t+J}) \\
&= \frac{\beta}{\mu} G(m^P_{t+J+1})
\end{aligned}
$$

Since $g < m^P_{t+J+1} < \underline{m}$ and $G(.)$ is monotone decreasing for $m \in (g, \overline{m}]$, this implies that

$$P(m_{t+J+1} < \underline{m} | m_{t+J} \geq m^P_{t+J}) > 0.$$

But by Proposition 4.1 this can never be true in any equilibrium. Therefore, in any equilibrium $m_{t+J} < m^P_{t+J}$ with probability one. But this is a contradiction because we just showed that if $m_t > m^*$, then the probability that $m_{t+J} \geq m^P_{t+J}$ is strictly positive. Hence, there does not exist a rational expectations equilibrium with $m_t > m^*$ at any t. Similarly, one can show that no rational expectations equilibrium exists with $m_t < m^*$ at some t, and therefore the unique rational expectations equilibrium is $m_t = m^*$ for all t ∎

5 Equilibrium under a Feedback Rule

The monetary policy regimes we have analyzed thus far — a pure interest rate peg and a pure money growth rate peg — share the characteristic of being insensitive to current economic conditions. By contrast, the monetary policy we consider in this section allows for feedback from endogenous variables. We assume that the monetary authority responds to increases in inflation by raising the nominal interest rate. Specifically, the monetary authority is assumed to set the nominal interest rate according to the following feedback rule

$$R_t = R + \alpha(\pi_t - \beta R) \qquad \alpha > 0, R \geq 1, \tag{29}$$

where π_t ($\equiv P_t/P_{t-1}$) denotes the gross rate of inflation. Under this monetary policy regime the quantity of money and bonds is endogenous, as is the case under a pure interest rate peg discussed in section three.

In what follows we restrict the analysis to perfect foresight equilibria. Expressing π_t as m_{t-1}/m_t M_t/M_{t-1} and using (15), the feedback rule can be written as

$$\frac{G(m_t)}{F(m_t)} = R(1-\alpha\beta) + \alpha\frac{m_{t-1}}{m_t}\frac{M_t}{M_{t-1}}. \tag{30}$$

Using (14) to eliminate M_{t+1}/M_t, the above equation can be written as

$$\frac{G(m_0)}{F(m_0)} = R(1-\alpha\beta) + \alpha\frac{m_{-1}}{m_0}\frac{M_0}{M_{-1}} \quad \text{and} \tag{31}$$

$$\frac{G(m_{t+1})}{F(m_{t+1})} = R(1-\alpha\beta) + \alpha\beta\frac{m_t}{m_{t+1}}\frac{G(m_{t+1})}{F(m_t)}, \quad \text{all} t \geq 0. \tag{32}$$

In perfect foresight, the transversality condition (19) becomes

$$\lim_{t\to\infty} \beta^t \left[G(m_t)(A_0/M_t - 1) + F(m_t)\right] = 0. \tag{33}$$

A perfect foresight equilibrium consists of a set of sequences $\{m_t, M_t\}$ satisfying $m_t, M_t > 0$, (14), (31)-(33) given A_0, m_{-1}, M_{-1}.

Using the definitions of $F(.)$ and $G(.)$ equation (32) is a simple first order linear difference equation

$$\min[m_{t+1}, \hat{c}+g] = a + b\min[m_t, \hat{c}+g] \tag{34}$$

with

$$a = \frac{(1-\alpha\beta)(1+\theta Rg)}{1+\theta R(1-\alpha\beta)}$$

and

$$b = \frac{\alpha\beta}{1+\theta R(1-\alpha\beta)}.$$

For $R > 1$, this difference equation has a unique constant solution

$$m^* = \frac{1+\theta Rg}{1+\theta R} \in (g, \hat{c}+g).$$

and for $R = 1$ it has a continuum of constant solutions $m^* \geq \hat{c}+g$.

Evaluating (30) at $m_t = m_{t+1} = m^*$, it follows that both the steady-state gross money growth rate, M_t/M_{t-1}, and the steady-state gross inflation rate are equal to βR. Evaluating the feedback rule (29) at $\pi_t = \beta R$ implies that R is the steady-state nominal interest rate. The fact that the steady-state money growth rate is βR and implies that the transversality condition (33) will be satisfied if $R > 1$ but will not be satisfied if $R = 1$; that is, no steady state equilibrium exists for $R = 1$. Accordingly, for the remainder of this section we assume that $R > 1$. Note that the steady-state level of real balances is independent of α and that it is identical to the level of real balances obtained under a pure interest rate peg.

Figure 2 plots the right and left hand sides of (34) as functions of m. The upper left panel displays the case $0 < b < 1$, which is equivalent to $\alpha\beta < 1$. For any $m_0 \in (g, \infty)$, (34) implies a sequence of real balances that converges monotonically to the steady state m^*. Given m_0, equation (31) determines the nominal money supply in the initial period, M_0. Clearly, M_0 will be strictly positive for any $m_0 \in (g, \infty)$, if, in addition to $\alpha\beta < 1$, the condition $R(1 - \alpha\beta) < 1$ is satisfied. So among the many possible perfect foresight equilibria there are some in which initial real balances are arbitrarily large and the initial nominal interest rate is zero. If, on the other hand, $R \geq 1/(1 - \alpha\beta)$, then M_0 will be strictly positive only if $m_0 < \frac{1+g\theta R(1-\alpha\beta)}{1+\theta R(1-\alpha\beta)} \in (m^*, \hat{c} + g)$. This implies that in this case, the nominal interest rate is always positive along the perfect foresight path and real balances are always below the minimum level consistent with the efficient allocation ($m_t < \hat{c} + g$). To summarize, when the monetary authority follows a feedback rule for the nominal interest rate that is not very sensitive to current inflation ($\alpha\beta < 1$), both the price level and the real allocation are indeterminate. By comparison under a pure interest rate peg the price level is also indeterminate but the real allocation is unique. In this sense the feedback rule can make the indeterminacy problem worse.

The lower left panel of figure 2 displays the case $b \in (-1, 0)$, that is, $\alpha\beta > (1 + \theta R)/(\theta R - 1)$ and $\theta R > 1$. In this case there is a continuum of perfect foresight equilibria converging to the steady state m^* in an oscillating fashion.[10] Again, the price level and the real allocation are indeterminate. Interestingly, in this case the indeterminacy occurs for a very `active' monetary policy. In models in which in equilibrium the marginal utility of consumption is constant or exogenous —as in the money-in-the-utility-function model with separable single-period preferences studied by Leeper (1991)—the standard result is that the higher the elasticity of the feedback rule the less likely it is that the equilibrium is indeterminate.

The upper right panel of figure 2 shows the case $b > 1$, which occurs when $1 < \alpha\beta < 1 + 1/(\theta R)$, and the lower right panel shows the case $b < -1$ which occurs when either $\alpha\beta > 1 + 1/(\theta R)$ and $\theta R < 1$ or $1 + 1/(\theta R) < \alpha\beta < (1 + 1/(\theta R))/(1 - 1/(\theta R))$ and $\theta R > 1$. In these cases the only perfect foresight equilibrium is the steady state, $m_t = m^*$ for all t, because for any $m_0 \neq m^*$ the sequence of real balances implied by (34) either violates the lower bound g or is such that at some point the right hand side of (34) exceeds $\hat{c} + g$. The determinacy of equilibrium when $|b| > 1$ demonstrates that under a balanced-budget requirement the monetary authority can bring about nominal stability without directly controlling the monetary aggregate.

Figure 3 summarizes the relation between price level determinacy and the elasticity of the feedback rule α. It shows with open dots the pairs $(R, \alpha\beta)$ for which the price level is determinate and with solid dots the pairs for which the price level is indeterminate. Given the steady-state nominal interest rate, both relatively insensitive and very sensitive feedback rules generate indeterminacy of the perfect foresight equilibrium, and for intermediate degrees of sensitivity the equilibrium is unique. At the same time, given the sensitivity of the feedback rule, the higher the nominal interest rate the more likely it is that the perfect foresight equilibrium is indeterminate.

[10] The initial real balances m_0 must satisfy $m_0 \geq \max[g, (\hat{c} + g - a)/b]$ and $m_0 < (g - a)/b$ if $(g - a)/b < \hat{c} + g$.

6 Conclusion

In this paper we argue that a balanced-budget rule has important implications for the determinacy of the price level. Using a standard cash-in-advance framework we find that if the balanced budget rule is coupled with an interest rate peg, the price level is indeterminate, whereas if it is coupled with a money growth rate, the price level is determinate. We also find that if the balanced budget rule is coupled with a feedback rule whereby the nominal interest rate is an increasing function of the inflation rate, the determinacy of the price level depends on the responsiveness of the feedback rule. For both little and highly sensitive feedback rules, the price level is indeterminate, whereas for moderately responsive feedback rules the price level is unique.

The results of this paper suggest that one largely ignored aspect of balanced-budget fiscal policy rules is their implications for price level determinacy. For a given monetary policy regime the price level can be determinate under some fiscal policy regime but indeterminate under a balanced budget requirement and vice versa. Therefore, any discussion on the macroeconomic consequences of balanced budget rules should take into account its implications for nominal stability.

Our finding that for certain monetary policy specifications a balanced budget rule leads to nominal instability complements our earlier work on the macroeconomic consequences of balanced-budget rules, Schmitt-Grohé and Uribe (1996), where we show that in a standard neoclassical growth model without money such a fiscal policy may lead to real instability by allowing for equilibria in which expectations of future tax increases can be self-fulfilling.

The study of the implications of balanced budget rules for nominal stability presented in this paper could be extended in several directions. First, it would be worth studying how the results are modified in a model augmented with nominal frictions such as sticky prices. Second, expanding the set of monetary policies might provide additional insights into the restrictions that the particular fiscal policy studied in this paper imposes on the conduct of monetary policy. In the context of a model with nominal rigidities the family of Taylor rules, that is, feedback rules whereby the nominal interest rate depends not only on current inflation but also on the output gap, are especially interesting from an empirical point of view. Lastly, the period-by-period balanced-budget rule we consider is an obvious starting point, but there are certainly more realistic specifications, such as rules that require the budget to be balanced over non-overlapping intervals of more than one period.

Appendix

Proof of Proposition 4.1

We first prove the existence of the lower bound $\underline{m}$. Define

$$\underline{m} \equiv \inf \left\{ m \geq g \,\middle|\, F(m) \geq \beta/\mu \inf_{m' \geq g} G(m') \right\}$$

Note that $\underline{m} > g$ by (A1). Since $m \geq \hat{c} + g$, implies $F(m) = G(m) > \beta/\mu\, G(m)$, one must have $\underline{m} < \hat{c} + g$. Suppose $m_t < \underline{m}$, this would require that

$$F(m_t) < \beta/\mu \inf_{m' > g} G(m') \leq \beta/\mu E_t[G(m_{t+1})]$$

contradicting (25). Therefore, $m_t \geq \underline{m}$ in any equilibrium. Next we show the existence of the upper bound $\overline{m}$. Define

$$\overline{m} \equiv \max \left\{ \beta/\mu\, \hat{c} \sup_{\underline{m} \leq m \leq \hat{c}+g} G(m), \hat{c} + g \right\}.$$

The fact that $G(m)$ is continuous on the compact interval $[\underline{m}, \hat{c}+g]$ implies that $\overline{m} < \infty$. The definition of $\overline{m}$ furthermore implies that for all m in the interval $[\underline{m}, \hat{c} + g]$,

$$G(m) \leq \mu/\beta \frac{\overline{m}}{\hat{c}} \tag{35}$$

Observe also that for all $\hat{c} + g < m \leq \overline{m}$,

$$G(m) = m\frac{1}{\hat{c}} \leq \overline{m}\frac{1}{\hat{c}} \leq \mu/\beta\overline{m}\frac{1}{\hat{c}}$$

Thus (35) holds for all m in the interval $\underline{m} \leq m \leq \overline{m}$. Now suppose that at some date $m_t \geq \overline{m}$. It follows that

$$\begin{aligned}
\frac{m_t}{\hat{c}} &= F(m_t) = \frac{\beta}{\mu} E_t[G(m_{t+1})] \\
&= \frac{\beta}{\mu} P_t(m_{t+1} \leq \overline{m}) E_t[G(m_{t+1}) | m_{t+1} \leq \overline{m}] + \\
&\quad \frac{\beta}{\mu} P_t(m_{t+1} > \overline{m}) E_t[G(m_{t+1}) | m_{t+1} > \overline{m}] \\
&\leq P_t(m_{t+1} \leq \overline{m})\overline{m}\frac{1}{\hat{c}} + \frac{\beta}{\mu} P_t(m_{t+1} > \overline{m}) E_t[m_{t+1} | m_{t+1} > \overline{m}]\frac{1}{\hat{c}} \\
&= \frac{\overline{m}}{\hat{c}} - P_t(m_{t+1} > \overline{m})\frac{\overline{m}}{\hat{c}} + \frac{\beta}{\mu} P_t(m_{t+1} > \overline{m}) E_t[m_{t+1} | m_{t+1} > \overline{m}]\frac{1}{\hat{c}} \\
&\leq \frac{\overline{m}}{\hat{c}} + \frac{\beta}{\mu} P_t(m_{t+1} > \overline{m})\frac{1}{\hat{c}}[E_t(m_{t+1} | m_{t+1} > \overline{m}) - \overline{m}] \\
&= \frac{\overline{m}}{\hat{c}} + \frac{\beta}{\mu} E_t[\max(m_{t+1} - \overline{m}, 0)]\frac{1}{\hat{c}}
\end{aligned}$$

where $P_t(\ldots)$ denotes the probability of the event in question conditional upon information available at time

t. In deriving these equalities and inequalities we have used (25), (35), and the fact that $m_{t+1} \geq \underline{m}$ with probability one. Then for any value of m_t, it follows that

$$E_t[\max(m_{t+1} - \overline{m}, 0)] \geq \mu/\beta \max(m_t - \overline{m}, 0).$$

Since $G(m_{t+T}) \geq (1/\hat{c}) \max(m_{t+T} - \overline{m}, 0)$ it follows that $E_t G(m_{t+T}) \geq (1/\hat{c})(\mu/\beta)^T \max[m_t - \overline{m}, 0]$. As a result (26) can be written as

$$\lim_{T\to\infty} \beta^T E_t \left[G(m_{t+T})(\frac{A_t}{\mu^T} - M_t) + \frac{\beta}{\mu} G(m_{t+T+1}) M_t \right] \geq \max[m_t - \overline{m}, 0] \frac{A_t}{\hat{c}}.$$

which is violated whenever $m_t > \overline{m}$. Hence, there can be no equilibrium in which $m_t > \overline{m}$ ■

References

Auernheimer, Leonardo and Benjamin Contreras, Control of the Interest Rate with a Government Budget Constraint: Determinacy of the Price Level and Other Results, mimeo, Texas A&M University, 1990.

Canzoneri, Matthew B. and Behzad Diba, Fiscal Constraints on Central Bank Independence and Price Stability, mimeo, Georgetown University, July 1996.

Leeper, Eric, 1991, Equilibria under `Active' and `Passive' Monetary and Fiscal Policies, Journal of Monetary Economics 27, 129-147.

Lucas, Robert E. Jr. and Nancy Stokey, 1987, Money and interest in a cash-in-advance economy, Econometrica 55, 491-514.

Schmitt-Grohé, Stephanie and Mart´n Uribe, Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability, working paper, The Board of Governors of the Federal Reserve System, Washington D.C., July 1996, forthcoming Journal of Political Economy.

Sims, Christopher, 1994, A simple model for the study of the determination of the price level and the interaction of monetary and fiscal policy, Economic Theory 4, 381-399.

Woodford, Michael, 1994, Monetary policy and price level determinacy in a cash-in-advance economy, Economic Theory 4, 345-380.

Woodford, Michael, 1995, Price-level determinacy without control of the monetary aggregate, Carnegie-Rochester Conference Series on Public Policy 43, 1-46.

Figure 1
Money Growth Rate Peg
($\mu > \beta$ and (A2))

$$G(m_t) = \frac{m_t}{\min(m_t, \hat{c} + g) - g}$$
$$F(m_t) = \frac{\theta m_t}{1 - \min(m_t, \hat{c} + g)}$$

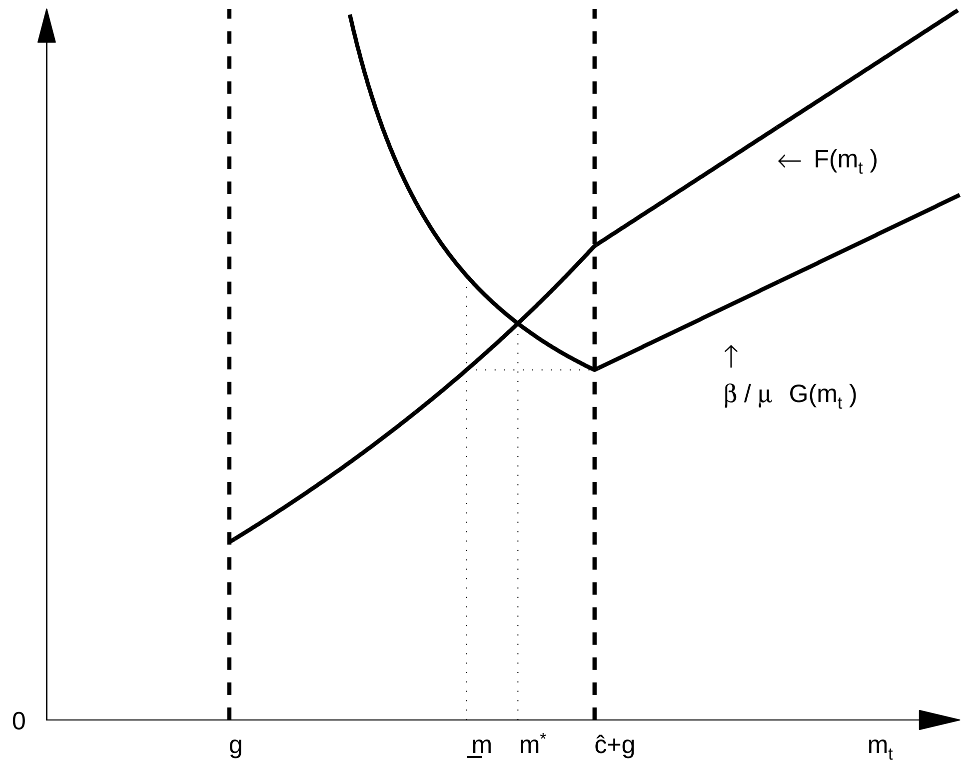

Figure 2

Feedback Rule: $R_t = R + \alpha(\pi_t - \beta R)$

$$\min[m_{t+1}, \hat{c} + g] = a + b \min[m_t, \hat{c} + g]$$

—— = $\min[m, \hat{c} + g]$

-o-o-o = $a + b \min[m, \hat{c} + g]$

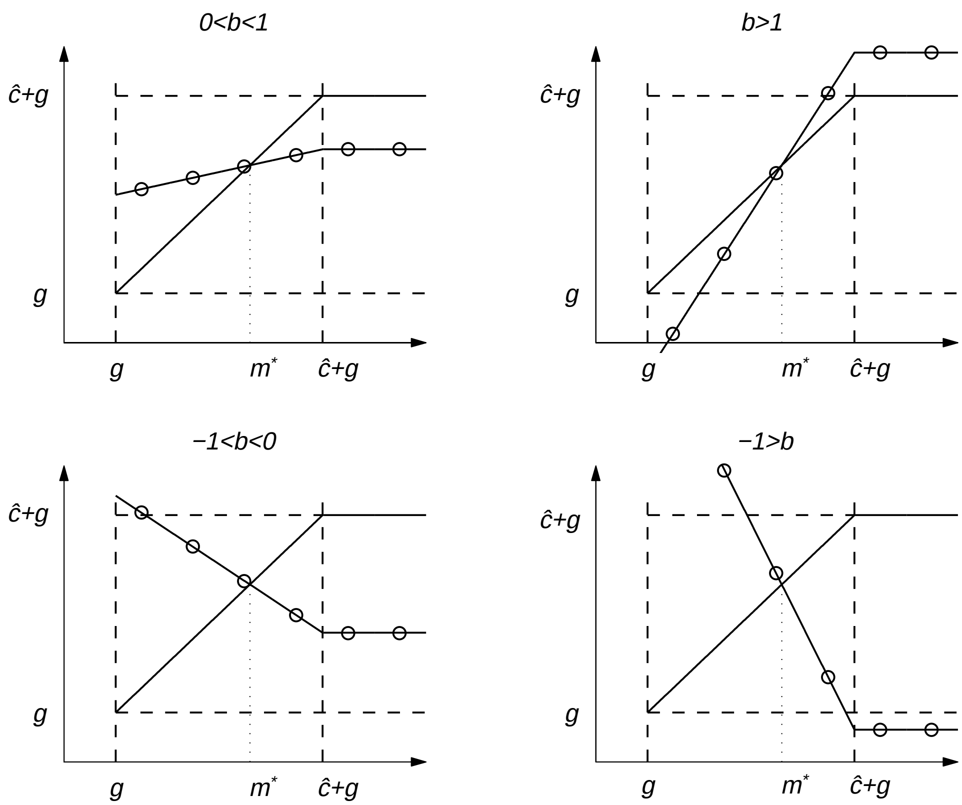

Figure 3

Relationship between Price Level Determinacy and values of α and R

o price level is determinate

· price level is indeterminate