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Optimal Portfolio Allocation in a World Without Treasury Securities

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Abstract

If current projections of future budget surpluses materialize, investing in Treasury securities–an asset class with which investors have long been familiar–could eventually become a thing of the past. In this paper, I examine the extent to which investors' portfolio allocation decisions are likely to be affected by the retirement of all federal government debt. The analysis suggests only small effects for most investors, especially, as is effectively the case for many institutional investors, when a no short sale constraint is in place. Under such circumstances, highly conservative investors–whose portfolios have risk-return characteristics akin to money market instruments–and very aggressive investors–who hold mostly equities–stand to be the least affected by the removal of Treasuries from the pool of investable assets. The analysis abstracts from indirect beneficial effects on investors from a Treasury debt payoff, such as the potential for greater productivity growth (and faster wealth accumulation) as more resources are freed up for investment in the private sector.

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1. Introduction

Since 1998, the U.S. government has been running a string of growing budget surpluses, reversing an era of back-to-back deficits that had its origin in the late 1960s. As a result, the stock of outstanding Treasury debt held by the public has declined from a peak of $3.4 trillion in 1996 to $3.2 trillion in early 2000. More to the point of this paper, current budget projections point to the elimination of virtually all federal government debt by the early 2010s.[1]

Although the improving budgetary outlook of the U.S. government is certainly good news–reduced federal borrowing frees up resources to the private sector, lowering market interest rates and "crowding in" private sector investment–the eventual retirement of government debt would make Treasury securities, an investment class with which investors have long been familiar, a thing of the past. In recent papers, Wojnilower (2000) and Reinhart and Sack (2000) have examined the effects of disappearing government debt primarily from a macroeconomic perspective. In this paper, I examine the extent to which investors' portfolio allocation decisions are likely to be affected by the elimination of government debt as an investable asset class. Are non-government securities, such as corporate bonds, close enough substitutes to Treasuries that investors could easily and costlessly move out of Treasuries and maintain the current risk-return profile of their portfolios?

The answer that this paper provides to the above question comes in two parts: First I

[1] Reinhart and Sack (2000) discuss the main factors behind these budgetary developments.

estimate the extent to which the risk-return tradeoffs facing investors in the market place are likely to be affected by the removal of Treasury securities from the pool of investable assets. The model discussed in this paper suggests that, despite the close correlation between Treasury and corporate debt securities, eliminating the former from the set of available assets could, under some circumstances, potentially leave some investors with inferior risk-return trade-offs. I then rely on a stylized model of investors' preferences over risk and return to attempt to quantify the extent to which investors valued the old investment opportunity set, which included Treasury debt, relative to the new one, which does not. Calibration of the model suggests that most investors would be little affected by the reduced pool of investment opportunities, especially, as is the case for many institutional investors, when short sales are disallowed. Under such circumstances, highly conservative investors–whose portfolios have risk-return characteristics akin to money market instruments–and very aggressive investors–who hold mostly equities–stand to be the least affected by the removal of Treasuries from the pool of investable assets. In contrast, in the polar case where unlimited short sales are allowed, investors with a high tolerance for risk would be among the most affected by the disappearance of Treasury securities, as the model would suggest that these investors make greater use of short positions in Treasuries to enhance their portfolio performance.

The vantage point of the analysis is the well-known capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). Such a simple, stylized model provides some valuable insights into this mostly overlooked consequence of the good news on the federal

budget. I provide a brief overview of the CAPM in section 2. In section 3, I use data on returns on a variety of financial assets to attempt to quantify the effects of an eventual government debt payoff both on the set of available investment opportunities and on the value that investors place on having Treasury securities as part of that set. Section 4 summarizes the main results and concludes.

2. Analytical Framework

I rely on the capital asset pricing model pioneered by the early work of Sharpe (1964) and Lintner (1965). The model is well-known both in the finance and economics literatures and is described in most finance textbooks (see, e.g., Huang and Litzenberger, 1997). In this short paper, I do not rederive standard results in the CAPM literature, but only highlight those features of the standard model that are most relevant for the analysis at hand.

2.1 Overview of the Model

Basic assumptions. In order to fix key concepts, I shall start with a highly stylized description of the investment environment. Underlying such an environment is an economy populated by investors who make portfolio allocation decisions in order to maximize their financial wealth. The model assumes that wealth can be allocated between the economy's only riskless asset, which I shall call Treasury debt, and a large number of risky assets. Investors are allowed to take short positions in all assets.

Embedded in the above setup are several key additional assumptions. First, I abstract from the theoretical debate of whether government debt constitutes wealth, by assuming that

tax payers do not fully discount the tax burden on future generations that in principle is linked to today's stock of government debt.[2] Second, I ignore inflation and interest rate risk by assuming that Treasury securities are truly riskless assets. I maintain this assumption here for ease of exposition only; I shall relax it in the empirical analysis in section 3.

Opportunity set of risky assets. By combining two or more existing assets (or portfolio of assets), investors can effectively create or synthesize new assets with different risk-return characteristics. The set of all feasible risky assets can be shown to be convex in the expected return-risk space. This set is shown as the shaded area in Figure 1. Also shown in the figure is the efficient portfolio frontier, the thick solid line, which shows the maximum attainable expected returns for given levels of risk.

The Role of the Risk-Free Asset. The riskless asset provides the rate of return r_f with certainty. By adding it to the mix of available assets, it can be shown that the efficient portfolio frontier becomes the tangent line to the opportunity set of risky assets that intercepts the vertical axis at r_f (see Figure 1). Thus, an important role of the riskless asset is to expand the pool of feasible investment opportunities when it is used in combination of the risky assets.

Attitudes Towards Risk. The efficient portfolio frontier tells us which investment opportunities are available but is silent about which of these will be chosen by a given investor. To examine how investors value different risk-return combinations, I make the usual assumption that investors are risk averse, and that their sense of well-being–as

[2] Bernheim(1987) discusses the main arguments behind this debate.

measured by theoretical utility functions–increases with wealth.

By assuming a model for utility that embodies risk aversion and is increasing in wealth, it can be shown that an individual's indifference curve–which in the context of this paper correspond to the combinations of risk and expected return that are equally valued by an investor–are positively sloped in the expected return-standard deviation plane (see the dotted lines in Figure 2). Moreover, it is straightforward to see that indifference curves in the northwest corner of the expected return-standard deviation plane correspond to higher levels of expected utility that do those curves below them–for a given level of risk, individuals prefer higher levels of expected return.

2.2 Optimal Portfolio Allocations

A standard result of the CAPM is that, with positively sloped indifference curves, investors will hold an efficient portfolio in equilibrium. As shown in Figure 2, when there exists a riskless asset, investors will hold portfolio e_1, which is a linear combination of the market portfolio of risky assets, m_1, and the riskless asset. In plainer words, in equilibrium investors will choose to allocate their wealth between the riskless asset and the market portfolio of risky assets so as to produce the expected return $:_1$ and standard deviation F_1. With a portfolio with such a risk-return profile, investors achieve the highest possible utility level, i.e., the highest attainable indifference curve.

In the absence of the riskless asset, the optimal allocation of aggregate wealth would be at the new equilibrium point e_2, where, by construction, all aggregate wealth is allocated to risky assets. Relative to the initial equilibrium at e_1, we can see in Figure 2 that the

disappearance of the riskless assets forces investors to a lower indifference curve. Thus, the model suggests that a government debt payoff would potentially force investors into a less favorable trade-off between risk and expected return and to lower levels of expected utility.

3. Empirical Analysis

The message from the standard capital asset pricing model is clear: The pool of available investment opportunities may be significantly affected by the retirement of government debt, and there is a potential welfare loss associated with such an effect. From a practical standpoint, however, the analysis so far has not been able to say anything about the likely sizes of either the shift in the portfolio frontier or of the loss in investors' welfare. I turn to these empirical issues now.

3.1 The Data

The main data used in this paper are the series on holding period returns compiled by Ibbotson Associates (2000) for major asset classes in the United States. The asset classes in the Ibbotson database are equities (large and small capitalization stocks), long- and medium-term fixed-income securities (investment-grade corporate and Treasury bonds with average maturity of about 20 years and Treasury notes with average maturity of approximately five years), and money market securities (short-term Treasury bills). The data span the period between 1926 and 1999. Alternative asset classes were also considered and described below.

Table 1 summarizes the main characteristics of the data, based on compound annual

returns computed over the entire sample.[3] Small company stocks have had the highest mean return, 17-1/2 percent, but also the highest risk, a standard deviation of 33-1/2 percent. At the other end of the spectrum, short-term Treasury bills have yielded 3.8 percent on average, with a standard deviation of 3.2 percent. The table also shows the correlation coefficients between the various asset classes: While equities are weakly correlated with bonds and notes, corporate and government debt securities are strongly correlated with each other, with correlation coefficients above 0.9.

3.2 Estimating the Shift in the Portfolio Frontier

In deriving the efficient portfolio frontier implied by the data, I start out by making an important departure from the theoretical framework described in section 2. I make the realistic assumption that, although Treasury debt securities are free from default risk, holders of such securities are still subject to interest rate and inflation risk. Thus, strictly speaking, the empirical analysis will entail comparing two scenarios that involve only risky assets, one that allows for the existence of Treasury debt and another that does not.

Portfolio frontier with Treasury securities. The solid line in Figure 3 shows the efficient portfolio frontier implied by the data when unlimited short sales are allowed. To derive such a line, I used the statistics listed in Table 1 as estimates of the first and second moments of compound annual returns for all asset classes. Figure 3 also shows the risk-return characteristics associated with each of the individual asset classes listed in Table 1. As shown in the figure, by appropriately combining the six original asset classes, investors can achieve a

[3] Ibbotson Associates (2000) provide a detailed discussion of the data.

wide range of risk-return pairs. For instance, an investor aiming for an expected annual return of, say, 10 percent, would have to be willing to accept a standard deviation of returns of about 9-1/2 percent.

Portfolio frontier without Treasury securities. The dashed line in Figure 3 shows the efficient portfolio frontier after Treasury bonds and notes are excluded from the asset mix. As shown in the figure, the portfolio frontier shifts clockwise by an appreciable amount after the removal of Treasury debt, suggesting that investors seeking higher levels of expected return must now be willing to bear significantly greater risk. For instance, the level of risk associated with an expected annual return of 10 percent increases from about 9-1/2 percent to approximately 12-1/2 percent.[4]

The computation of the new portfolio frontier implicitly assumes that the risk-return characteristics of Treasury bills can be perfectly replicated by other money market instruments such as federally-insured CDs and deposits and high-grade commercial paper. (Hence, Treasury bills were not removed from the portfolio mix.) Obviously, such an assumption is only an approximation: Federal deposit insurance is limited to $100,000 per account and, as the early 1970s have taught us, even high-grade commercial paper is subject to default risk. Still, because a comparable time series of returns on private money market instruments is not readily available, my analysis abstracts from imperfect substitutability between Treasury bills and other short-term investments, and thus there is a risk that the

[4] These results assume that the first and second moments of the returns on other assets are unchanged after the disappearance of Treasury securities. In principle, such moments could change, but the analysis of Reinhart and Sack (2000) suggests that the changes would be small.

model underestimates the shift in investment opportunities following the retirement of Treasury debt.

Another important assumption that underlies the methodology for estimating the effect of disappearing government debt on the efficient portfolio frontier relates to the choice of assets included in the analysis. In particular, I assume that the relatively small number of asset classes listed in Table 1 can adequately represent the set of all investment opportunities facing investors in the financial market place. This is an admittedly restrictive assumption, but one that I will partly relax below by adding agency securities and foreign assets to the investment mix.

Lastly, the sizable shift in the portfolio frontier discussed thus far corresponds to a version of the model that allows for unlimited short sales by all investors. From a practical standpoint, however, such a scenario does not correspond to the situation facing many individual and institutional investors. I discuss the implications and importance of the short selling assumption below.

3.3 Estimating the Value of Greater Diversification

Eliminating Treasuries from the asset pool potentially reduces the opportunities for investment diversification, and indeed, the data suggest that the model-implied shift in the portfolio frontier after Treasuries are removed from the original asset mix can be sizable. In principle, the extent to which investors might feel worse off as a result of such a shift depends on their specific attitudes towards risk. To address this issue, I assume that investors' preferences over risk and return are described by a utility function, U(W), of the

form

$$U(W_t) = W_t^{(1-\delta)}/(1-\delta)$$

where wealth, W, is lognormally distributed, and the expected value and standard deviation of the return on one's wealth are functions of the how much one chooses to invest in the various asset classes. It can be shown that U(W) is such that investors prefer higher returns, but dislike risk.[5] The parameter δ denotes the investors' coefficient of relative risk aversion, which I will use as a metric to characterize how investors view different trade-offs between risk and expected return.

Given the utility function, it is relatively straightforward to derive optimal portfolio allocations in worlds with and without Treasury securities, along with their corresponding levels of expected utility. I then used empirical estimates of δ to parameterize the utility function and quantify the extent to which investors are worse off after the removal of Treasury debt from the set of available assets. To put a dollar amount on the resulting loss of investor welfare, I computed the additional wealth that would be required to bring investors in a world without Treasuries back to the expected utility levels experienced by investors in the world with Treasuries. Accordingly, the larger the difference between investors' initial wealth and the compensating wealth level, the higher the value that investors

[5] Such a specification of the investors' preferences, commonly referred to as power utility, is widely used in the finance and economics literatures–see, e.g., Grossman and Shiller (1982). In addition to its analytical tractability, power utility functions have several desirable properties, such as the fact that risk premia are invariant to the scale of the economy. Nonetheless, the power utility specification has its own limitations, which are especially relevant for studies that, unlike this paper, focus on the separate roles of risk aversion and individuals' decisions to consume and save over time (Hall, 1988).

attach to having Treasuries in the set of investable assets and the worse off they would be after the disappearance of federal government debt.

Results. Because there is considerable uncertainty about the available empirical values of δ, I will present results for a range of values reported in the literature. Alternatively, one can think of the different values of δ I used as pertaining to different types of investors, and, indeed, this is the interpretation that best characterizes the analysis conducted in this paper. I start with δ set to 10, at the high end of the range of values considered plausible by the stylized model of Mehra and Prescott (1985). Still, such a value of δ is below the empirical estimate reported by Reinhart and Sack (2000), who find a coefficient of risk aversion of about 20, close to the number implied by the work of Hansen and Jagannathan (1991)–see also Cambell, Lo, and MacKinlay (1997). In addition to experimenting with δ=20, I also report results based on a coefficient of relative risk aversion of 2, which is suggested by the earlier work of Friend and Blume (1975) and used by Frankel (1985).

Figure 4 shows the optimal portfolio allocations that correspond to the case where δ is set to 10. As government debt securities disappear, investors move from the indifference curve labeled I to that labeled II. In terms of welfare loss, a nearly 1 percent rise in wealth would be required to compensate these investors for removing Treasuries from the investment pool. Such a compensating wealth differential seems large enough to suggest that, despite their high correlation with Treasuries, corporate debt securities are an imperfect substitute to Treasuries, and the investment opportunities made available by the existence of Treasury securities are potentially considerably valued by at least some investors.

Which asset classes take the place of Treasuries as government debt is retired? It is noteworthy that investors actually choose a less risky portfolio as Treasury debt disappears. The standard deviation of the optimal portfolio's return falls from about 7-1/2 percent to 5-3/4 percent in the new equilibrium–the expected return on wealth declines from 8-1/2 percent to 6-1/2 percent. This happens as investors shift the funds previously invested into Treasuries primarily into money market instruments and, to a lesser degree, corporate bonds–the allocations into equities remains little changed after the removal of Treasuries.

The results summarized in Figure 4 allow for short sales of individual assets, a setup that we could interpret as corresponding to the analysis of individual classes of investors or of investors within a particular country. Yet, from a practical standpoint, many institutional investors, such as mutual funds, either do not engage in short selling or do so under various legal and self-imposed restrictions. Moreover, from a theoretical perspective, to examine the welfare implications of an eventual government debt payoff on the representative global investor, while making the strong assumption that all investors have identical attitudes towards risk, would require that short sales be disallowed. Without short sales, the required rise in wealth to compensate for disappearing Treasuries amounts 0.2 percent, appreciably smaller than in the scenario that allowed for unlimited short sales. As with the case where short sales were allowed, money market instruments receive most of the funds previously invested in Treasuries, followed by corporate bonds. I should note, however that the model without short sales has the counterfactual implication that neither long-term Treasury bonds nor corporate debt would be held in the initial equilibrium. In reality, of course, truth lies in

between the two polar cases of unlimited short sales and no short sales at all, although as I have argued above, from a practical standpoint, many investors would likely be closer to the latter case.

The last two rows of Table 2 summarize the results for alternative values of the coefficient of risk aversion (δ). For investors with a high tolerance for risk (δ=2), the compensating rise in wealth is a sizable 5-1/2 percent when unlimited short sales are allowed. By contrast, in the case where short sales are disallowed and δ=2, the removal of Treasury securities has virtually no effect on investors' expected utility levels. Taken together, these findings explain why aggressive investors are more affected by the disappearance of Treasuries in the unlimited short sales version of the model: Such investors seek higher expected returns in their portfolios in part by establishing short positions in Treasury securities in order to invest in other assets; paying off the federal debt makes such a strategy impractical.

Turning now to investors with relatively high levels of risk aversion (δ=20), the compensating wealth differential is estimated to be 0.4 percent when short sales are allowed and only 0.1 percent in the no short sales specification of the model. The intuition behind such small effects is relatively straightforward: These highly risk averse investors started out by holding a portfolio with risk and return characteristics that are closer to those of money market instruments than did the other two investor classes analyzed above. Therefore, the disappearance of Treasury debt, and the resulting reallocation into money market securities, generally leads to small changes in the risk-return profile of their portfolios.

To sum up, when short sales are allowed, the higher one's tolerance for risk, the more one might be adversely affected by a complete payoff of marketable Treasury debt. In contrast, when no short sales are allowed, moderately risk averse investors might stand to be the most affected by a retirement of Treasury debt: They are more likely to be holding a larger share of their wealth in medium- and long-term fixed income products than either the more aggressive investors–who hold a lot of equities–or their more conservative counterparts–who have a larger share of their portfolios in money market instruments.

3.5 Further Sensitivity Analysis

In addition to allowing for alternative values of D, I checked the robustness of the main findings reported above along two dimensions. First I recomputed the efficient portfolio frontiers using different time periods. The main conclusions were generally the same: By either restricting the analysis to the period starting in the 1980s or to the post-war period, it was still the case that some investors were generally appreciably worse off after Treasury securities were removed from the portfolio mix.

Second, I also included additional asset classes in the analysis, supplementing the Ibbotson database with returns based on market indexes computed by Salomon Smith Barney (SSB) and Morgan Stanley Capital International (MSCI). These indexes were not included in the original analysis because their time series are much shorter, generally going back only to the early 1980s. I started by adding SSB's U.S. agency bond and U.S. mortgage-backed securities return indexes to the analysis. Returns on agency debt securities are highly correlated with returns on Treasuries–as high as 0.9896 over the 1981-1999 time period–and

indeed are often mentioned as a potentially leading substitutes to the liquidity and safety of government debt. Nonetheless, the model suggests that the risk-return characteristics of Treasuries are such that the main results discussed above are little changed: For a coefficient of risk aversion of 10, and allowing for short sales, investors would require a rise in initial wealth of roughly 1.5 percent to fully compensate them for the reduced investment opportunities resulting from paying off the federal debt. The fact that returns on agency and Treasury debt are so strongly correlated might make this result surprising to some, but here again it should be noted that the existence of both Treasuries and agencies gives investors opportunities to use short and long positions in both markets to enhance the performance of their portfolios, and such opportunities disappear with the removal of Treasuries from the pool of available assets. Indeed, the welfare effect is negligible when short sales are disallowed. Thus the existence of agency debt securities per se does not allow investors to costlessly shift out of Treasuries, and the main thrust of the results stands.

Lastly, I experimented with adding foreign assets to the portfolio mix, namely MSCI's Japanese and European equity indexes.[6] Although it was the case that the efficient portfolio frontier shifted up significantly after the introduction of such assets, the model results were still such that removing Treasuries from the opportunity set of investable assets had a significant effect on the expected utility of investors.

4. Concluding Remarks

[6] Ideally, one would want to also include foreign fixed-income assets into the asset mix, but I have not been able to find a long enough time series to use in the analysis.

Examining investment returns for various asset classes for the 1926-1999 period, I found that, an eventual payoff of U.S. Treasury debt has the potential to force some investors to less favorable trade-offs between the risk and expected return of their portfolios, although in most cases such effects are small. Based on a well-known model of investors' preferences over risk and return, the findings suggest that, from a pure portfolio allocation perspective, some investors might be worse off as they move from a world with Treasury securities to one without them, but the most significant effects are limited to the case where investors are willing and able to engage in unlimited short sales. When the model is modified to more closely proxy the investment strategies used by many institutional and individual investors, who do not engage in short selling or do so under very limited conditions, the results suggest that, while small, portfolio effects are more evident for investors with intermediate tolerance for risk: Highly risk averse investors–who hold mostly money-market type instruments–and aggressive investors–whose holdings are dominated by equities–stand to be the least affected by an eventual Treasury debt payoff.

In assessing the effect on investors of Treasury debt paydowns, this paper has focused narrowly on the risk-return characteristics of optimal investment portfolios and on how different types of investors might value such characteristics. In particular, the paper has abstracted from consideration of other channels through which an eventual elimination of the U.S. government's debt would have beneficial effects on those same investors. For instance, paying off the federal debt would further reduce the government's claim on available savings, potentially helping to facilitate increased investment in new technologies

and faster productivity growth in the United States. These in turn could well generate the additional wealth needed to compensate individual classes of investors for their reduced investment opportunities in a world without Treasuries.

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Table 1

Summary Statistics of the Returns Data
(Compound annual returns, 1926-1999)

Asset class	Mean	Std dev	Correlation matrix					
Large cap stocks	13.3	20.1	1.00					
Small cap stocks	17.6	33.6	0.79	1.00				
Corporate bonds	5.9	8.7	0.25	0.10	1.00			
Long-term gov bonds	5.5	9.3	0.19	0.02	0.94	1.00		
Medium-term gov notes	5.4	5.8	0.11	-0.04	0.91	0.91	1.00	
Money market instruments	3.8	3.2	-0.02	-0.09	0.21	0.23	0.49	1.00

Source: Ibbotson Associates (2000)

Table 2

Percentage Increase in Wealth Required to Compensate Investors for the Removal of Treasuries from the Investment Opportunity Set

Coefficient of Risk Aversion	Compensating Rise in Wealth	
	With Short Sales	Short Sales not Allowed
$D=10$	0.9	0.2
$D=20$	0.4	0.1
$D= 2$	5.5	0.0

Figure 1

Efficient Portfolio Frontier

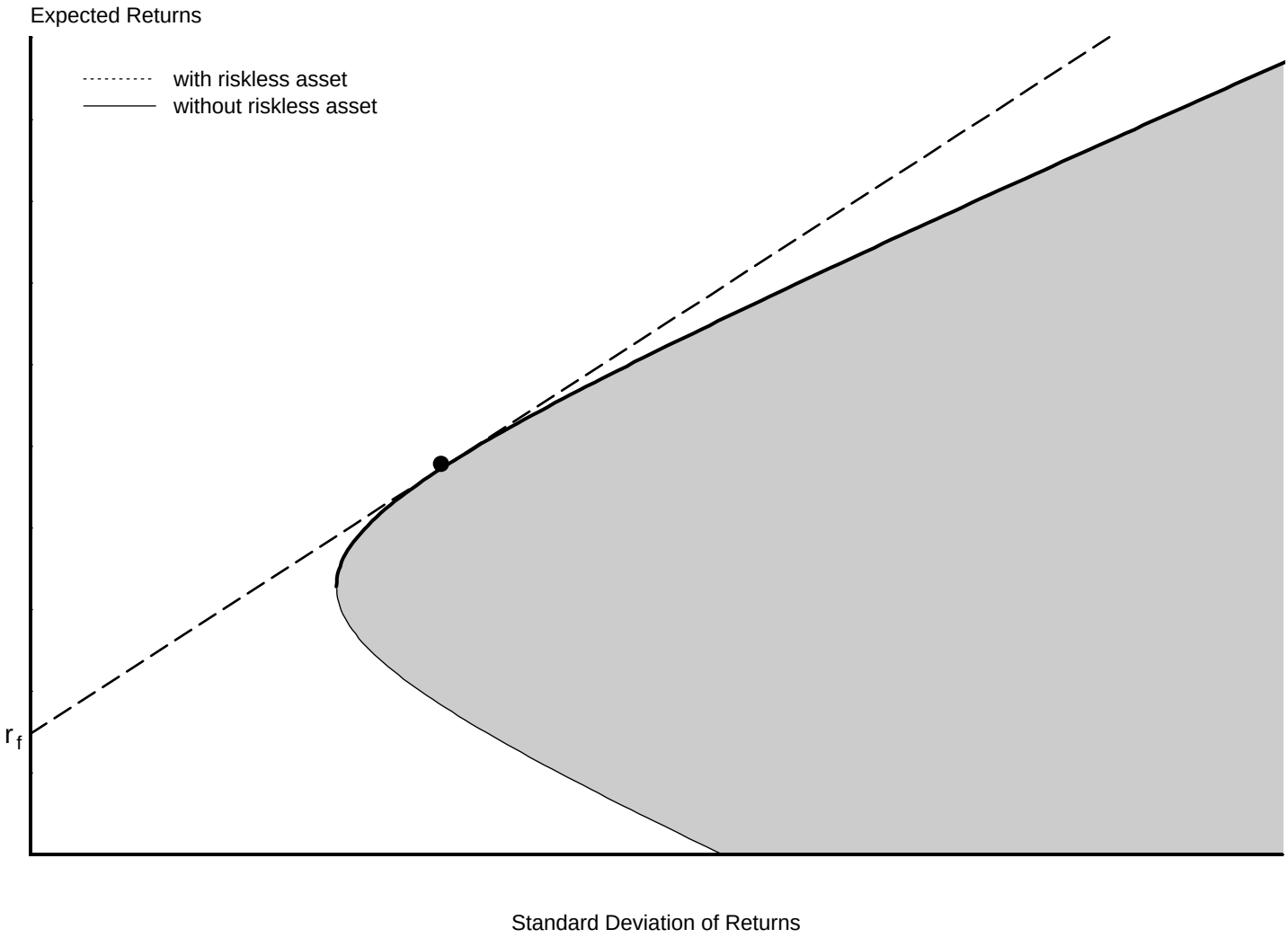

Figure 2

Optimal Portfolio Allocations

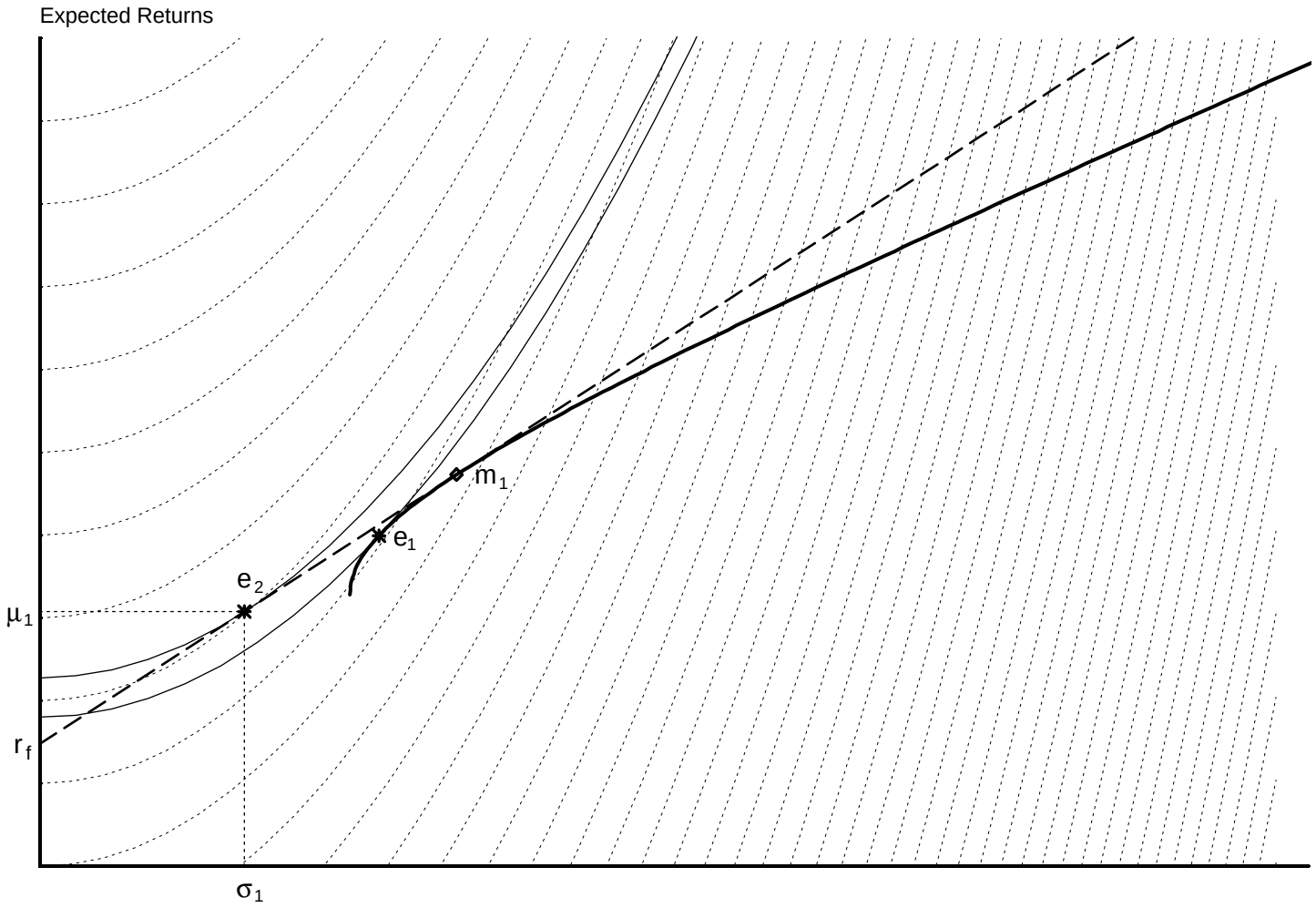

Figure 3

Portfolio Frontiers

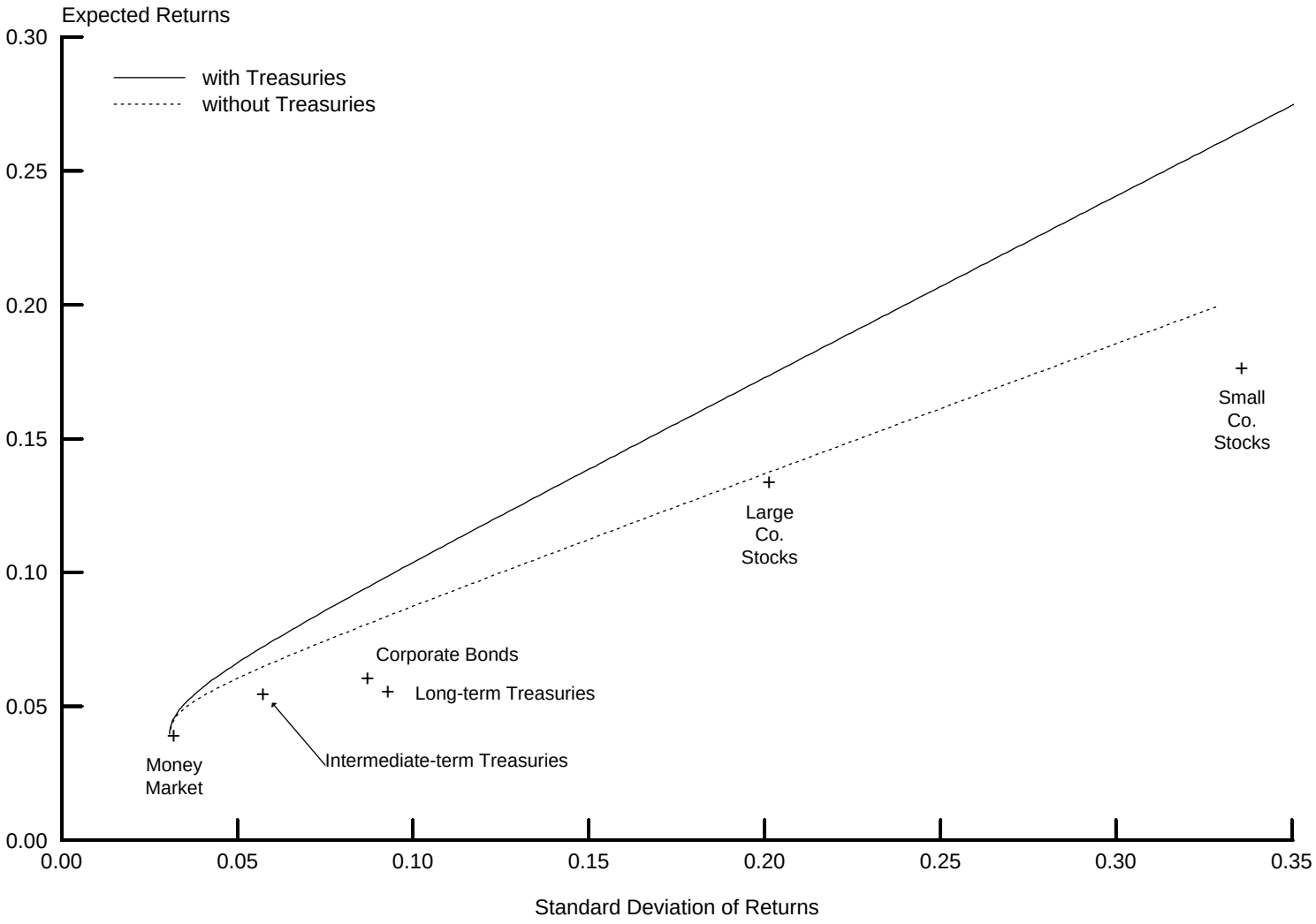

Figure 4

Optimal Portfolio Allocations