Activist vs. Non-Activist Monetary Policy: Optimal Rules Under Extreme Uncertainty (A Primer on Robust Control)

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EDITORIAL NOTE

This working paper is a re-issue of an unpublished manuscript written eighteen years ago.[1] The reason for resurrecting it now is that it has been referenced recently in the new literature on `robust control`. In 1982, this topic had little currency in the economics literature; and even in engineering, where this field has been developed, the topic had just begun to take hold with the publication of G. Zames' seminal 1981 article on H^∞ control.[2] Since I wrote "Activist vs. Non-Activist Monetary Policy," the technology of robust control has leap-frogged and so has understanding of the subject, thanks largely to the many contributions by Lars Hansen and Thomas Sargent.[3] Even so, many basic insights and intuitions remain the same. Because this paper is framed in simple mathematical terms, the reader may view it as a primer on robust control, useful even today, when much of the literature on the subject tends to be very technical.

*Federal Reserve Board, Washington, DC 20551. The opinions expressed herein are solely those of the author and do not necessarily represent the views of the Board of Governors or the staff of the Federal Reserve System.

[1] For technical reasons, the paper had to be be retyped (re-wordprocessed is the correct word now); and I thank Karen Blackwell for doing it so well for me. I have taken the liberty to make some editorial improvements, which, however, have not affected any of the original conclusions.

[2] "Feedback and optimal sensitivity: model reference transformations, multiplicative norms, and approximate inverses," in *IEEE Transactions on Automatic Control AC-26*.

[3] See, for example, their most recent paper, "Robust control and filtering of forward-looking models," unpublished manuscript, University of Chicago and Stanford University, October 2000.

The paper was motivated by Milton Friedman's remark that economists and policy makers know too little about their models to make them useful for setting monetary policy. Therefore, instead of varying money supply (the policy instrument of the day) in response to observed changes in economic aggregates, central banks should determine a constant rate of money growth, a CMG, and stick to it. I viewed this conclusion as too extreme, since it was well known that classical characterizations of model uncertainty would not, except in an extreme version of policy multiplier uncertainty analyzed by Brainard (1967), produce non-reactive policy. I considered, therefore, the possibility that Friedman had something far more extreme than Bayesian risk in mind, a kind of uncertainty that could not be described in terms of subjective or objective probability distributions. This was the idea of uncertainty made famous by Frank H. Knight, which required an entirely different approach to optimization. Since one cannot formalize Knightian uncertainty with well defined probability distributions in a Bayesian sense, it is impossible to formulate policy based on mathematical expectations, obliging the decision maker to resort to minimax strategies that seek to avoid worst-case outcomes.

For pedagogical reasons, I confined the analysis to a one-equation model of nominal income growth subject to uncertainty about its parameters as well as the underlying data. The paper shows how, even under extreme uncertainty, a policy maker determines a money growth rule that is usually continuously responsive to observations, where particularities of the model determine whether such a rule is more or less attenuated than one derived in a Bayesian framework. The last half of the paper compares policies when the decision maker uses Bayesian or robust filtering techniques or, alternatively, Bayesian or robust signal extraction methods for an imperfectly observed state variable, such as trend output growth.

The model is written in static form and uses a simple quadratic social loss function. The results are equivalent if one introduces dynamics and works with the implied asymptotic expected loss. The archaic seeming terms used in the paper, such as "activist" and "non-activist" may be read as "amplified" and "attenuated," respectively. Also, the paper can easily be reframed to derive interest rate rules, the focus of much modern policy analysis.

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ACTIVIST VS. NON-ACTIVIST MONETARY POLICY:
OPTIMAL RULES UNDER EXTREME UNCERTAINTY

Abstract

This paper analyzes the optimality of reactive feedback rules advocated by neo-Keynesians, and constant money growth rules proposed by monetarists. The basis for this controversy is not merely a disagreement concerning sources and impacts of uncertainty in the economy, but also an apparent fundamental difference in the attitude toward uncertainty about models. To address these differences, this paper compares the relative reactiveness of a monetary policy instrument to conditioning information for two starkly differing versions of model uncertainty about the model and the data driving it: Bayesian uncertainty that assumes known probability distributions for a model's parameters and the data and Knightian uncertainty that does not. In the latter case, the policy maker copes with extreme uncertainty by playing a mental game against ''nature,'' using minmax strategies. Contrary to common intuition, extreme uncertainty about a model's parameters does not necessarily imply less responsiveness to conditioning information--here represented by the lagged gap between nominal income growth and its trend--and it certainly does not justify constancy of money growth except in an extreme version of Brainard's (1967) result. A partial constant money growth rule can be derived in only one special case: if the conditioning variable in the feedback rule is also uncertain in either Bayesian or Knightian senses and the authority uses Neyman-Pearson likelihood ratio tests to distinguish noise from information with each new observation.

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I. INTRODUCTION: ON THE APPROPRIATENESS OF FEEDBACK POLICY

According to monetarist orthodoxy, the supply of money should be made to grow at a constant rate. This constant money growth (CMG) proposition has been defended on the grounds that policy should not be left in generally incompetent hands, and that even if applied rationally, monetary policy has unknown current or lagged effects that may be destabilizing. Such uncertainty is said to result from the diffusion of information in markets to which policy makers typically have no access, and from confusion caused by policy concerning short-term vs.long-term monetary phenomena. Exponents of this policy perverseness proposition include Friedman (1970, 1971), Brunner and Meltzer (1972), and Brunner (1980).

An opposing group of economists, whom I shall label ''neo-Keynesians,'' holds to a theory of structural instability requiring active, contingent intervention via optimal control procedures to stabilize the course of the economy. Exponents of this policy effectiveness proposition include Gordon (1977), Fischer (1977), Leijonhufoud (1968), Modigliani (1977), Modigliani and Papademus (1980), and Okun (1972).

A thought provoking case against activist policies, including policies based on optimal control procedures, has recently been made by Brunner (1981). In essence, Brunner notes that frequently advocated control procedures are model specific and require the possibly arrogant assumption that the policy maker's information is sufficient and superior to the information held by agents in the economy. However, if information sets are identical, then the rational expectations literature [e.g.,Barro (1976), Lucas (1976), and Sargent and Wallace (1973)] points to an irrelevance proposition for policy.[4]

[4]Even in rational expectations models that have been the paradigm for the ineffectiveness propositions, activist policies can be optimal if (1) there exist contractual precommitments among agents in the economy [Fischer (1977)], (2) private agents obtain information with lags [Taylor (1975)], or (3) behavioral equations describing the economy contain unknown parameters,

Neither paradigm in its extreme version seems to have much empirical plausibility. Indeed, the strictest interpretation of the rational expectations hypothesis, equating subjective and objective probability distributions (and/or expectations) has been shown by Swamy, Tinsley, and Barth (1981) to be problematical. However, one may speak of <u>endogenous</u> expectations generated by the coherent behavior of all participants in an economy where subjective opinions guide all decisions, subject to an information state that (1) evolves over time and (2) is possibly endogenous. In such an environment, endogenously generated expectations may have neutralizing effects, but policy may not be altogether irrevelant.

A further, challenging criticism of optimal control techniques has been offered in the context of non-causal or forward looking models. Non-causal models describe economies in which current values of state variables depend on expected future values. Generally, non-causal models lead to situations in which the authority is engaged in dynamic games with the private sector because participants in the economy make decisions contingent on expectations of <u>future</u> policy decisions. As a result, there arises the question of optimality vs.''time-consistency'' in macro economic policy, an issue that is currently being debated in the literature [vid.Buiter (1981), Calvo (1978) Kydland and Prescott (1977) and Lee (1981)].

In forward looking models, i.e., in models in which agents in the economy base decisions upon expected <u>future</u> policy decisions (and in which as a result the current state of the economy is in part determined by the future expected state of the economy), situations may arise in which at the time the known policy is to be enacted, there emerges a better policy, even though the initial policy was optimal when computed. In an environment in which agents did not form forward expectations, the original policy would remain optimal, and deviations from that policy would be sub-optimal.

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and information costs are positive [Howitt (1981)].

However, in economies containing forward expectations of policy, an optimal policy may be different from the policy as originally announced and is called <u>time-inconsistent</u>. The authority may well be tempted to revise its originally announced policy, or, as suggested by Buiter (1981) and Lee (1981), it may plan to revise policy in a manner not predicted by the private sector but optimal from a social welfare point of view. Such time-inconsistency may eventually give rise to credibility problems and render policy self-defeating. Unfortunately, consistency of action implied by adherence to announced policies (including Keynesian feedback and monetarist CMG rules) can also lead to credibility problems, since by construction, consistent policies are non-optimal in non-causal environments: hence they fail to promote society's goals. Various solutions have been offered, including proposals to construct optimal policies under the <u>constraint</u> that the authority must follow through with the plan, or to include a penalty for unanticipated policy changes in the central bank's loss function. The imposition of constraints on the volatility of the federal funds rate in controlling money supply aggregates suggests the usefulness of such an approach. [vid.Tinsley and von zur Muehlen (1981)]. An important feature of these approaches to controlling non-causal environments is that such time-inconsistent rules are, indeed, contingent, i.e., reactive feedback rules.

Although the above comment points to interesting and important issues relating to the usefulness of reactive policy, this paper abstracts from problems raised by endogenous expectations, the focus being on the nature of optimal reactions to extreme uncertainty. Nevertheless, as indicated later, the model, as specified, may encompass provisions for endogenous expectations.

If it is legitimate to say that a policy maker is absolutely ignorant about the economy's structure, i.e., faces a ''black box,'' then it is true in a trivial sense that no decision can be made. This parallels the statement in logic that in the absence of a set of axioms, it is impossible to arrive at a conclusion. Constant monetary growth rules proscribing discretionary action are, of course, not derived from such a severely restricting assumption concerning ignorance. Indeed, the decision not to act in the sense suggested by a CMG rule is itself a rule to permit certain kinds of activities and -- more importantly -- to allow the effects of uncertainty to be allocated in a particular manner that would not be implicit in a more reactive agenda. For example, a constant money growth rule means that shocks to the demand for money are absorbed by movements in interest rates, while an accommodative reaction that allows the growth of money to vary reallocates the effect of the shock to the monetary aggregate [vid.Tinsley and von zur Muehlen (1981)]. Indeed, for a constant money growth policy, the volatility outcome depends on the particular operating regime adopted. Under a reserves operating procedure, supply side shocks not offset by accommodative changes in reserves must be reflected in increased interest rate volatility. If, alternatively, such shocks were accommodated by reserves changes, the interest rate volatility would be less, even within the same framework of a constant money growth path. These two regimes, while aiming at the same money growth target, would thus differ in the volatility of reserves and the interest rate.

Care must be taken to distinguish reduced activism and non-activism. To be sure, in a single-period context, increased multiplier uncertainty reduces the size of policy responses [Brainard (1967)], but it does not eliminate the contingent character in an optimal (linear-quadratic) feedback rule.[5] In a recent paper, Craine (1979) addressed this distinction and,

[5]This proposition does not necessarily extend to the multi-instrument case and depends on whether the reaction matrices before and after the change in covariances of the impact multipliers commute [Pohjola (1981)]. Further, in multi-period stochastic control problems in which the distributions of random variables are conditioned on current information, the need to evaluate expected future information for close-loop control can lead to probing, which adds to the activity of a control [Bar-Shalom and Tse (1973) and Bar-Shalom (1981)].

based on simulations of neo-Keynesian and monetarist versions of a dynamic random coefficient model, concluded that when there is uncertainty about the impact of policy alone, the optimal policy converges to a fixed money growth rate as the multiplier variance becomes large. When there is uncertainty about the transition dynamics only, the optimal policy will be reactive at all levels of uncertainty. If both types of uncertainty co-exist, policy may become more active even as multiplier uncertainty increases. Thus, multiplier uncertainty may be an insufficient reason for caution. In a historical context, Craine and Havenner (1978) found reactive policies to be less successful in terms of welfare losses than certain fixed rules. This conclusion was based on simulations of alternative policies using the MIT-Penn-SSRC model over the period 1973 III- 1975 II. That period was, of course, characterized by significant supply side shocks with regard to which the model was possibly misspecified. One possible explanation is that policy makers of that period felt they confronted a world more fitting the monetarists' conception of insurmountable uncertainty.

James Tobin once characterized constant growth strategies advocated by monetarists as essentially equivalent to minmax strategies. Minmax approaches are typical in situations of extreme uncertainty, and judging from the language used by advocates of CMG rules, this characterization may not be unwarranted. Intuition suggests that policies designed to avoid worst-case outcomes may be called for if one knows little or nothing about the consequences of policy but fears the worst. Such intuition turns out not to be well founded in all cases: this paper shows that even when facing worst-case scenarios, the policy maker formulates rules that, with minor exceptions, have all the appearance of continuous feedback rules familiar from Bayesian control methods. Further, also contrary to conventional wisdom, robust rules determined under Knightian uncertainty need not be less aggressive in their reactions to observed data than certainty equivalent rules derived under Bayesian assumptions.

Section IV derives Bayesian and robust policies under data uncertainty, i.e., when the conditioning information in a feedback rule is uncertain. In my model, the conditioning variable is the lagged spread between nominal income growth and its trend, where it is natural to assume that one of its components, real trend output growth, is not fully revealed for many periods. I show that when the distribution of disturbances to nominal income growth is unknown, a minmax filtering solution leads to a rule that is not necessarily more attenuated than one based on a Bayesian filtering rule, which assigns a known distribution to the noisy process. If the stochastic process is dichotomous, consisting of either pure noise or noise with information, then policy is affected by prior signal extraction that seeks to distinguish between these two cases. I find that both Bayesian and minmax signal extraction procedures lead to regime switching rules in which periods of constant money growth alternate with periods of reactive money growth, with lagged observed income growth serving as the conditioning variable.

To preserve simplicity, I describe only single-period, static policies. But, as indicated in several footnotes, in the present context, static minmax solutions are identical to dynamic minmax solutions anyway.[6] More generally, results in Craine and Havenner (1978) and Kalchbrenner and Tinsley (1976) indicate that in a control theoretic framework with Bayesian uncertainty, offsetting effects of current and future uncertainty can lead to increased aggressiveness. Therefore, confining the analysis to a single-period treatment of the problem will, if anything, prejudice the results in favor of fixed rules.

II. THE MODEL

[6]Although an extension of this discussion to dynamics is formally straightforward, the issue of long-run planning may present problems. As shown by Athans and Gershwin (1977), the existence of optimal long-range stochastic control is subject to an ‘‘uncertainty threshold’’ principle, which states that such control is possible only if a certain index of dynamic uncertainty (as quantified by a ratio of means and variances of the random parameters) is less than one.

A suitable framework for analysis is the following model of nominal income growth,

$$y_t \; = \; a_t \, y_{t-1} \; + \; b_t \, m_t \; + v_t, \tag{1}$$

where y_t is the deviation of the rate of growth of nominal GNP from its target rate; m_t is the rate of growth of money; and a_t, b_t, and v_t are variables that are alternately assumed to be known, or random with known distributions, or random with unknown distributions.[7] The additive stochastic term, v_t, may encompass the set of exogenous variables known to affect y_t.[8] Below, I discuss situations when y_t is uncertain in either the Bayesian or Knightian sense. Uncertainty about y highlights a typical problem for policy makers who very often must deal with uncertainty in the trend growth rates of output and productivity.

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[7]The President's Council of Economic Advisors provides an annual set of targets for income, employment, and inflation, as mandated by Humphrey-Hawkins legislation (the ''Full Employment and Balanced Growth Act of 1978''). Tinsley and von zur Muehlen (1981) analyzed policy rules that seek to maximize the likelihood of achieving such a target set.

[8]Equation (1) encompasses a variety of substructures relating to agents' decisions and information sets. Defining the set of random parameters $\beta_t' \; = \; [a_t, b_t, v_t]$, such that $y_t \; = \; x_t'\beta_t$, where $x_t \; = \; [y_{t-1}, m_t, 1]$. A general description of the β_t process is $\beta_t = \bar{\beta} q_t + e_t$, where $\bar{\beta} = [\bar{\beta}_1, \bar{\beta}_2, \ldots, \bar{\beta}_m]$ is a 3xm matrix of fixed parameters, $q_t \; = \; [q_{1t}, q_{2t}, \ldots, q_{mt})'$ is an mx1 vector of fixed variables (possibly including policy parameters), and $\{e_t\}$ is a purely indeterministic stationary process. The vector, q_t, may be interpreted as the set of conditioning information variables in the economy on which agents and the authority base their respective decisions. Following Swamy, Barth, and Tinsley (1981), the specification in (1) can be viewed as encompassing various degrees of interdependence of decisions and asymmetric information sets playing upon the model's parameters, β_t. Accordingly, equation (1) is within the framework of a fully rational, interdependent system in which the parameters, β_t, are not invariant to policy. The polar case, for which the prerequisites for active control are supposedly violated, as suggested by Brunner (1981), holds that certain elements of β_t are unknown.

The periodic loss is assumed to be quadratic in y and m,[9]

$$L(a_t,\ b_t,\ m_t\, v_t,)\ =\ y_t^2 + \alpha_m m_t^2, \tag{3}$$

and the policy maker is assumed to minimize the expectation $E(L)$, which may be loosely interpreted as a weighted sum of the unconditional variances of y and m.[10] Solving this problem requires knowledge of the first two moments of all random variables. For minmax strategies, the criterion is also (2), but selected parameters and variables are taken to be random with unknown distributions. To assure existence of optimal minmax policies, the random variables are assumed to have finite support, as indicated below.

Under the form of Knightian uncertainty adopted in this paper, the

[9]Although I will set $\alpha_m = 0$ in the discussion, a term for money growth in the loss function can be motivated by the following argument: For various reasons, central banks generally try to minimize excessive movements in interest rates and would therefore probably include an interest rate term in the loss function. Imagine a (background) equation describing the demand for money, modeled as a function of income and the interest rate,

$$m_t = c_t y_t + k_t i_t + u_t, \tag{2}$$

where i_t is a market interest rate, c_t and k_t are known parameters, and u_t is a stochastic error, uncorrelated with v_t. Given money and income growth, the equation determines changes in the interest rate, the square of which then depends on the squares of money and income growth (plus cross terms).

[10]The analysis in the text focuses on a single period. For our purposes, this is of little consequence, since the corresponding expected discounted multi-period loss is similar to (3),

$$\begin{aligned} E(L) &= (1-\beta)\sum_1^\infty \beta^t\,\{y_t^2 + \alpha_m m_t^2\}, \\ &= (1-\beta)\sum_1^\infty \beta^t\,\{Ey_t^2 + \alpha_m Em_t^2\}, \\ &= \sigma_y^2 + \alpha_m \sigma_m^2, \end{aligned}$$

where β is the social discount factor.

authority is assumed to know very little, other than that parameter and data uncertainty is bounded, and that the nominal model used for policy is a reasonable approximation of the true model. While this characterization of uncertainty may not reflect complete ignorance, it is probably no less than even most skeptical observers could agree on.

III. POLICIES UNDER ALTERNATIVE SPECIFICATIONS OF MODEL UNCERTAINTY

III.1. Uncertain Intercept

To keep the discussion a simple as possible, I assume strict nominal income targeting and set $\alpha_m = 0$ throughout. In this section, I assume that a_t and b_t are assumed to be known constants, while v_t is the realization of a random variable, v, with unknown distribution, where $-\infty < v_1 \leq v_t \leq v_2 < \infty, v_1 < 0, v_2 > 0$. Note that since v has an unknown distribution, it is impossible to form expectations in closed form. Instead, I assume that v_t is a deterministic sequence chosen by nature to maximize the decision maker's loss.

By the minmax criterion, the authority wishes to protect the economy from disaster in the current period, as represented by the maximum of the loss function (2). Substitute (1) into (2) and note that the maximum of (2) is given by

$$L^* = \max_{v_t} L = \max_{v_t} [S(y_{t-1}, m_t) + v_t]^2, \tag{4}$$

where

$$S(y_{t-1},\ m_t) = a_t\, y_{t-1} + b_t\, m_t.$$

For any chosen value of m_t, a maximum of L requires

$$sign\ v_t = sign\ S(y_{t-1},\ m_t),$$

since $v_1 < 0,\ v_2 > 0.$ Since nature is assumed to inflict greatest harm, it will choose v_1 or v_2 accordingly. The authority would like to immunize social welfare losses to that choice, and I shall call minmax policies that achieve this objective ''neutralizing.'' A neutralizing policy that makes L the same whether v_1 or v_2 is chosen is the feedback rule,[11]

$$\hat{m}_t = -b_t^{-1}[a_t\, y_{t-1} + \frac{v_1 + v_2}{2}]. \tag{5}$$

The implied y_t process is found by substituting this rule into (1),

$$y_t = v_t - \frac{v_1 + v_2}{2}. \tag{6}$$

By comparison, if the distribution of v is known (for example, assume that $v \sim N(\bar{v}, \sigma_v^2)$), then the certainty-equivalent, linear-quadratic (LQG) rule is

$$m_t^c = -b_t^{-1}\,[a_t\, y_{t-1} \ + \bar{v}], \tag{7}$$

so that the regulated y_t process becomes

$$y_t = v_t - \bar{v}. \tag{8}$$

Under the minmax rule, the authority is indeed indifferent to nature's choice, since, whether nature choses v_1 or v_2, the loss is the

[11] Observe that $(v_1 + v_2)/2$ is the expected value of a uniformly distributed variable, v, with support v_1 and v_2. Thus, rule (5) could also have been obtained via minimization of the expected loss $E(L)$ under the uniform distribution assumption. With that interpretation, the minimized expected loss is given by

$$E(L^c) = \sigma_v^2 = (\frac{v_2 - v_1}{2})^2,$$

which is also independent of the choice of v. Therefore, in the present example, a minmax solution formally implies the same rule obtained under the assumption that v is uniformly distributed over the range $[v_1, v_2]$.

same:

$$L(a_t,\ b_t,\ m_t,\ v_1) = (\frac{v_2 - v_1}{2})^2 = L(a_t,\ b_t,\ m_t,\ v_2).$$

In comparing rules (7) and (5), note that both react identically to innovations in income growth, based on the feedback coefficient, $-a/b$. Both rules augment money growth with a constant factor, as determined by either the mean of the v process or by its boundary midpoint $(v_1 + v_2)/2$. One cannot say, then, that the minmax rule is necessarily less activist than the certainty-equivalent rule.

III.2. Uncertain System Dynamics

Assume now that b_t is constant and known, that v_t is either known or random with distribution $N(\bar{v}, \sigma_v^2)$, and that a_t is a random variable with unknown distribution and finite support, $0 < a_1 \leq a_t \leq a_2 < \infty$. Stability requires $|a_t| < 1$. Potentially, one of nature's tricks is to set $a > 1$.

The loss function, given a choice of m_t, can be re-written as

$$L = y_{t-1}^2(a_t + \frac{b_t}{y_{t-1}}m_t + \frac{v_t}{y_{t-1}})^2,$$

which, for any choice of m_t, has a maximum at a^*, such that

$$sign\ a^* = sign[\frac{b_t}{y_{t-1}}m_t + \frac{v_t}{y_{t-1}}],$$

if the term in brackets is nonzero. For any outcome, v_t, the authority wants to neutralize nature's choice which, by the maximum harm conjecture, will be a_1 or a_2. Therefore, an neutralizing policy is given by the rule,

$$\hat{m}_t = -b_t^{-1}(\frac{a_1 + a_2}{2})y_{t-1}. \tag{9}$$

Notice that $\hat{m}_t = 0$ only if $a_1 = a_2 = 0$ or $a_1 = -a_2$. Both cases are ruled out -- the latter by assumption and the former because it contradicts the premise of Knightian (or any) uncertainty. A CMG rule is implied, albeit in a formal sense, only in a situation of extreme ignorance about a, i.e., when a_1 and a_2 are also unknown, i.e., if the support of a is unknown or unbounded. But then, since no policy exists to prevent catastrophy, all policy would be pointless, in any case.[12]

Under the minmax rule, the path of the economy is

$$y_t = (a_t - \frac{a_1 + a_2}{2})y_{t-1} + v_t, \tag{12}$$

so that the welfare loss is independent of nature's worst possible choice of a:

$$L(a_1, b_t, m_t, v_t) = (\frac{a_2 - a_1}{2})^2 \, y_{t-1}^2 + v_t^2 = L(a_2, b_t, m_t, v_t).$$

When the probability distribution of a is known, the linear-quadratic

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[12]The minmax rule can be given a Bayesian interpretation by positing a_t to be uniformly distributed on $[a_1, a_2]$, and to be uncorrelated with v_t. The optimal money growth rule, obtained by minimizing $E(L)$, is

$$m^* = -b_t^{-1}(\frac{a_1 + a_2}{2})y_{t-1} - b_t^{-1}\bar{v}, \tag{10}$$

which differs from (9) by the quantity $-b^{-1}\bar{v}$. Thus, while the expected minimized loss under (9) is

$$E(\hat{L}) = (\frac{a_2 - a_1}{2})^2 y_{t-1}^2 + \sigma_v^2 + \bar{v}^2,$$

the expected loss under (10) is

$$E(L^*) = E(\hat{L}) - \bar{v}^2 < E(\hat{L}). \tag{11}$$

Therefore (10) dominates (9). In either case, the expected loss is independent of the choice of a_2 or a_1; thus m^* is also minmax.

optimal rule is

$$m_t^c = -b_t^{-1}(E(a)\ y_{t-1} + \bar{v}), \tag{13}$$

from which follows the stabilized output path,

$$y_t = (a_t - Ea)y_{t-1} + (v_t - \bar{v}).$$

In comparing the Bayesian rule (13) with the minmax rule (9), note that both react to lagged income growth using some notion of an <u>average</u> of the income persistence parameter, a. Whether one or the other rule is more activist, depends entirely on assumptions made about this parameter in these two situations. In either case, the preceding discussion demonstrates that Knightian uncertainty about system dynamics clearly does not imply a constant money growth rule.

III.3. <u>Uncertain Policy Multiplier</u>

Assume now that a and v are either known or, if random, have known (normal) distributions, $N(\bar{a}, \sigma_a^2)$ and $N(\bar{v}, \sigma_v^2)$, respectively, where $0 < a_t < 1$, and that b is random with bounded support $0 < b_1 \leq b_t \leq b_2 < \infty$.

The loss function can be written as follows:

$$L = m_t^2(\frac{a_t y_{t-1} + v_t}{m_t} + b_t)^2,$$

which has a maximum at b^* for any chosen m_t if

$$sign\ b^* = sign(\frac{a_t\ y_{t-1} + v_t}{m_t}).$$

Given a_t, this implies the minmax rule,

$$\hat{m}_t = -\frac{a_t}{(b_1 + b_2)/2} y_{t-1}, \tag{14}$$

and the regulated nominal income growth equation,

$$y_t = a_t(1 - \frac{b_t}{(b_1 + b_2)/2})y_{t-1} + v_t. \tag{15}$$

Notice that stable nominal income growth requires that $a_t|b_2 - b_1| < b_1 + b_2$, a condition that is always fulfilled if $a < 1$, as posited.

It is easily demonstrated that the implied loss is neutral to nature's choice of b, since

$$L(a_t, b_1, m_t, v_t) = a_t^2 \left(\frac{b_2 - b_1}{b_1 + b_2}\right)^2 y_{t-1}^2 = L(a_t, b_2, m_t, v_t). \tag{16}$$

The minmax solution may be contrasted with the Bayesian formula under the assumption that b is random with known distribution and a_t is known,

$$m_t^c = -\frac{a_t}{\bar{b} + \sigma_b^2/\bar{b}} y_{t-1} - \frac{\bar{v} + \sigma_{vb}}{\sigma_b^2 + \bar{b}^2}, \tag{17}$$

where $^-$ is used to indicate the mean of a variable and σ_b^2 and σ_{vb} denote the variance of b and the covariance between v and b, respectively. As this rule indicates, an increase in the variance of the policy multiplier, b, leads the policy maker to become less activist; and, as σ_b^2 becomes very large, the optimal policy even approaches a CMG rule of doing nothing.[13]

The preceding Brainard rule implies the following stabilized transition equation for nominal income growth:

$$y_t = (a_t - b_t \frac{\bar{a}}{\bar{b} + \sigma_b^2/\bar{b}})y_{t-1} + v_t - b_t(\frac{\bar{v}\bar{b} + \sigma_{vb}}{\sigma_b^2 + \bar{b}^2}). \tag{18}$$

[13]This result was first introduced by Brainard (1967) and was further explored by Craine(1979). In a somewhat different context, rules that feature episodes of action and no action, subject to triggering thresholds, were derived by von zur Muehlen (1978), who analyzed a policy model in which monetary policy steps judged to be out of the ordinary act as signals of changed policy intentions, causing agents to alter market behavior.

Comparing the minmax and Bayesian rules in the case of multiplier uncertainty, we note that these rules are actually quite similar in conception: both are increasing functions of the persistence parameter, a_t, and inverse functions of either the mean of the multiplier, b_t, in the Brainard rule, or of the midpoint of its support, $(b_1 + b_2)/2$, in the minmax rule. The latter fraction is not a mean (which is unknown) but a measure of the spread between the supports of b, measuring the decision maker's view of the feasible range of alternative models. Policy, in a very real sense, is set with an eye toward the extremes of feasible models, not some average of them.[14] In the Bayesian rule, the role of this spread is played by $\sigma_b^2/\bar{b}$. How much more or less activist the minmax rule is in comparison with the Bayesian rule is a matter of the exact specification of the parameters. The Bayesian rule is more attenuated, for example, if $\sigma_b^2/\bar{b} > (b_1 + b_2)/2$, that is, if the range of Bayesian uncertainty surrounding the mean of b is greater than the spread of the upper and lower limits of the support of b under Knightian uncertainty. This seems like a natural conclusion and probably generalizes to more complex cases. There is no <u>a priori</u> clear reason that policy under Knightian uncertainty is any more or less aggressive than under Bayesian uncertainty.

III.4. <u>Combined Uncertainty in Persistence and Policy Effectiveness</u>

I now combine the previous two cases for the sake of completeness and assume that both a_t and b_t are uncertain with supports as stated in the previous sections. It is not difficult to establish that the minmax neutralizing criterion requires the money growth rule,

$$\hat{m}_t = -\frac{a_1 + a_2}{b_1 + b_2} y_{t-1}, \tag{19}$$

which is easily shown to leave welfare loss immune to nature's choice of a

[14] This should not be interpreted as saying that policy makers who prefer robustness fear gloom and doom. Rather, within a possibly narrow range of feasible models, they find themselves unable to state with any precision which one might be the most likely model. Minmax control should be viewed as dealing with small differences in models.

and b. That is, any of the combinations $(a_i, b_j,\ i,j = 1,2)$ causes the same maximum loss. Consider now the case (a_2, b_1) representing a model with the highest feasible persistence and the lowest feasible policy effectiveness. This is clearly the worst possible model a policy maker might face. Substituting these values and the above rule into (1) generates the controlled transition equation for nominal income growth,

$$y_t = (a_2 - b_1 \frac{a_1 + a_2}{b_1 + b_2}) y_{t-1} + v_t. \tag{20}$$

Stability of nominal income growth requires that $a_2 b_1 - a_1 b_2 < b_1 + b_2$, or, equivalently, that $a_2 < 1 + (1 + a_1) b_1 / b_2$. A sufficient condition is that $a_2 < 1$. But, in the mind of a policy maker, the worst-case persistence parameter, a_2, could exeed 1 by some amount that is inversely related to the worst-case policy multiplier, b_1. So, for a robust rule to work at all, the boundaries of the feasible set of models are jointly constrained: if, at its worst, policy is very limited in its effects, the uncontrolled worst-case persistence of output growth must then be correspondingly smaller.

IV. DATA UNCERTAINTY

As previously defined, y is the gap between the growth rate of nominal GNP and that of its trend or potential, where uncertainty about the latter typically lingers for a considerable number of periods. All the feedback rules derived above are therefore likely to be subject to additional errors. In the following, I explore various ways of coping with uncertainty surrounding the conditioning variable in a feedback rule. Let

$$z_t = y_{t-1} + \varepsilon_t, \tag{21}$$

where $y_{t-1} \sim N(Ey_{t-1}, \sigma_y^2)$ is the unknown true series, z_t is the observed series, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is its noise component, and y_{t-1} and ε_t are uncorrelated.

I will distinguish two related cases: (1) a model in which ε is unknown but has finite support allowing the authority to formulate a minmax strategy; and (2) a model that assumes z_t contains either a combination of signal and noise, or pure noise with a known probability distribution. Case (2) is handled using signal detection methods which precede the choice of a policy action.

IV.1. Bayesian vs. Minmax Filtering of Trend Growth

Let ε have an unknown random distribution satisfying $-\infty < \varepsilon_1 \leq \varepsilon_t \leq \varepsilon_2 < \infty, \leq \varepsilon_1 < 0, \varepsilon_2 > 0$. To simplify, assume that a_t and b_t are known constants, where $0 < a_t < 1$, and that $v \sim N(0, \sigma_v^2)$ is white noise uncorrelated with ε_t and y_{t-1}. The implied loss function is

$$L = (a_t z_t + b_t m_t + v_t - a_t \varepsilon_t)^2,$$

where (21) has been substituted for y_{t-1}. The authority chooses a feedback rule for m to minimize L under the assumption that nature selects ε_t to maximize L. Accordingly, since for any realization of a_t, b_t, and v_t, and any choice of m_t, L is a maximum at ε^* whenever

$$sign\ \varepsilon^* = -sign(\frac{a_t z_t + b_t m_t + v_t}{a_t}),$$

the optimal, neutralizing choice of m_t is

$$\hat{m}_t = -\frac{a_t}{b_t}(z_t - \frac{\varepsilon_1 + \varepsilon_2}{2}), \tag{22}$$

since v_t is unknown to the authority. This feedback rule is similar to (5), except that the authority responds to z_t rather than to y_{t-1}. Note that even under Knightian uncertainty about trend growth, it remains optimal to react with a committed rule to the available information, here represented by z_t. Upon substituting this rule into (1), the implied growth rate of nominal income is shown to be the mixed random process,

$$y_t = a_t(z_t - \frac{\varepsilon_1 + \varepsilon_2}{2}) + v_t. \tag{23}$$

By comparison, Bayesian minimization of the conditional expected loss, L, under the assumption that all relevant distributions are known, implies the transition law for y,

$$y_t = a_t(1 - \frac{\sigma_y^2}{\sigma_y^2 + \sigma_\varepsilon^2})(y_{t-1} - Ey_{t-1}) - a_t\frac{\sigma_y^2}{\sigma_y^2 + \sigma_\varepsilon^2}\varepsilon_t + v_t, \tag{24}$$

which follows from the feedback rule,

$$\begin{aligned} m_t^c &= -\frac{a_t}{b_t}\frac{\sigma_y^2}{\sigma_y^2 + \sigma_\varepsilon^2}(z_t - Ey_{t-1}) - \frac{a_t}{b_t}Ey_{t-1}, \qquad (25)\\ &= -\frac{a_t}{b_t}\frac{\sigma_y^2}{\sigma_y^2 + \sigma_\varepsilon^2}z_t - \frac{a_t}{b_t}(1 - \frac{\sigma_y^2}{\sigma_y^2 + \sigma_\varepsilon^2})Ey_{t-1}, \end{aligned}$$

where the expectation of y_{t-1} is conditional on the observation, z_{t-1}. In this rule, money growth responds to observed income, z_t, adjusted for a term representing the surprise between observed and expected growth. Comparing this rule with the minmax rule derived in (22), note that in the latter, money growth responds to the observation, z_t, with a factor, $-a_t/b_t$. In the Bayesian rule, the response parameter is attenuated by a factor which is clearly less than 1. The degree of attenuation increases as relative uncertainty about income growth rises, where, in the limit, money growth is determined solely by expected income growth, unaltered by new observations. This is clearly not what happens in the minmax rule. Under the robust rule in (22), a widening of the range of uncertainty in nominal income growth leads to an increase -- not a decrease -- in the desired growth of money!

IV.2. Bayesian vs. Minmax Signal Detection of Trend Growth

Occasionally, observed changes in the conditioning variable may be no more than noise, i.e., not a mixture of signal and noise.[15] If one could be sure of such instances, one would know not to change policy, either. In reality, one cannot distinguish with certainty among instances when observations provide true information and when they do not. To analyze this situation, assume first that the distribution of ε_t is known. The task now is to distinguish when an observation is noise and when it might be a signal. To find a rule for choosing, a decision-theoretic framework [Lehman (1959)] is used.

Let H_0 be the hypothesis that an observation is pure white noise:

$$H_0 : z_t = \varepsilon_t,$$

and let H_1 be the hypothesis that z_t also contains the signal, y_{t-1}:

$$H_1 : z_t = y_{t-1} + \varepsilon_t.$$

If ε_t is normally distributed with mean zero and variance σ_ε^2, then the two corresponding densities are defined as

$$H_0 : p(z|H_0) = N(0, \sigma_\varepsilon^2),$$

$$H_1 : p(z|H_1) = N(y_{t-1}, \sigma_\varepsilon^2).$$

Although there are various criteria for establishing tests of a signal, the Neyman-Pearson likelihood ratio test assigning observation

[15] I am not asserting that the case described here is typical or even important. I discuss this example mainly because it leads to the rare instance of policy turning constant, at least episodically.

thresholds to a variable is selected here. Let Π_0 and Π_1 denote the a priori frequency of noise, and of noise plus signal, respectively. A decision adopting H_0 is denoted d_0 and a decision adopting H_1 is denoted d_1. Either decision, d_i, may be correct or incorrect, where the incorrect decision can be a false alarm (d_1 selected when H_0 is true) or a miss (d_0 selected when H_1 is true).

The decision maker is assumed to select the limits -z_0 and z_0, such that H_0 is accepted whenever $-z_0 \leq z \leq z_0$. The probability that z is outside this range, even though it is merely noise is defined as

$$\alpha(z_0) = \int_{-\infty}^{-z_0} p(z|H_0)dz + \int_{z_0}^{\infty} p(z|H_0)dz,$$

and the probability that z is inside the range even though there is a signal is defined as

$$\beta(z_0) = \int_{-z_0}^{z_0} p(z|H_1)dz.$$

Therefore, the probability of a false alarm is

$$p(d_1, H_0) \;=\; \Pi_0\,\alpha,$$

and the probability of a miss is

$$p(d_0,\, H_1) \;=\; \Pi_1\,\beta.$$

IV.2.a. A Bayesian Criterion for Evaluating the Likelihood Ratio

Let R denote the linear Bayes risk function

$$R \;=\; c_0\,\alpha\,\Pi_0 \,+ c_1\,\beta\,\Pi_1,$$

where c_0 and c_1 are positive and finite constants expressing the marginal cost of a false alarm and a miss, respectively. The first-order condition with respect to z_0 can be solved such that

$$L(z_0) \;=\; \frac{\beta(z_0)}{\alpha(z_0)} = \frac{c_0\Pi_0}{c_1\Pi_1} = K$$

defines a likelihood ratio. If $L(z_t) < K$, the decision is that z_t is noise, and that as a consequence one does not react to it in the manner exemplified by (25) or even (22), and vice versa, if $L(z_t) \geq K$. Observe that the likelihood of interpreting an observation as pure noise is greater if the prior probability of noise, Π_0, or the subjective cost, c_0, of a false alarm is relatively large, since in both these cases K is larger.

If noisy information is treated as described above, the optimal feedback rule for money is a CMG rule during periods when $L(z_t) < K$, assuming a_t, b_t, and Ey_{t-1} to be fixed. That is, suppose the policy authority follows the feedback rule (25). In periods when $L(z_t) < K$, the authority sets money growth as a function of the prior expected value of y_{t-1} only, i.e.,

$$\hat{m}_t = -\frac{a_t}{b_t} E y_{t-1}, \tag{26}$$

which becomes a CMG when a_t, b_t, and Ey_{t-1} are constant. The linear quadratic Gaussian model with noisy information generally implies the continuous feedback rule (25). However, policy may switch intermittently between continuous reaction to data and no reaction at all when there is uncertainty about whether an observed change in the data measuring nominal income growth reflects a change in fundamental trend growth or is purely transient.

IV.2.b. <u>A Minmax Criterion for Evaluating the Likelihood Ratio</u>

Alternatively, the threshold may itself be selected via the more conservative minmax criterion by minimization of the maximum ''Bayes loss,'' R, if Knightian uncertainty attaches to the prior probability of noise,

Π_0. Noting that Π_0 = 1 - Π_1, R may be rewritten as

$$R \;=\; [c_0\,\alpha(z_0)\;-c_1\,\beta(z_0)]\,\Pi_0\;+c_1\,\beta(z_0).$$

Suppose nature chooses Π_0 maliciously. Then the decision maker should choose z_0 to make the Bayes loss, R, independent of Π_0, namely such that

$$c_0\,\alpha(z_0\,)\;\;=\;\;c_1\,\beta(z_0).$$

Given this, the ''minmax'' likelihood criterion becomes

$$L^*(z_0) = \frac{\beta(z_0)}{\alpha(z_0)} = \frac{c_0}{c_1} = K^*,$$

where, comparing with the previously computed K,

$$K^* \geq K \; if \; \Pi_0 \leq .5,$$

if Π_0 were, in fact, known. The minmax criterion is thus equivalent to assigning a 50 percent probability to either H_1 or H_0. Observe, too, that since $L^*(z_0)$ is an increasing function of z_0, the minmax policy implies wider CMG bands than the minimized Bayes risk function only if $\Pi_0 < .5$. Thus, if under the minimum risk criterion, the prior probability of pure noise is believed to be equal to or smaller than one half, the implied CMG range of inaction $[-z_0, z_0]$ is equal to or smaller than the range obtained under a minmax criterion. Conversely, the minmax criterion leads to a smaller range of inaction, if Π_0 is believed to exceed one half, given the Bayes policy. This example is further evidence that robust policies derived in the face of Knightian uncertainty are not necessarily more restrained than policies derived under less intimidating circumstances.[16]

[16]When deriving dynamic policies, thresholds such as those defined above are determined sequentially by some updating rule. Thus, in the i-th

Finally, if the monetary rule chosen by the authority is the linear-quadratic feedback policy (25), then, in periods when $L(z_t) < K$, the optimal value of m_t under the minimum Bayes risk criterion is

$$m_t^c = -\frac{a_t}{b_t}\left(1 - \frac{\sigma_y^2}{\sigma_y^2 + \sigma_\varepsilon^2}\right)Ey_{t-1}.$$

Observe, that in the limit, as σ_ϵ^2 becomes very large, m_t^c is ruled in large part by the prior expectation, Ey_{t-1}, while under the minmax rule, (22), m_t depends on the maximum and minimum possible error. Thus, while Bayesian policy merely reduces the influence of data when σ_ε^2 becomes large, the minmax rule is always keyed to the midpoint between the boundaries of the support of the unknown process driving trend growth.

V. CONCLUDING COMMENT

It might be objected that the outcome of static mental experiments biases the conclusion. Certainly, static solutions are not necessarily optimal in a dynamic sense. The static approach neglects future uncertainty. But as pointed out by Craine (1979), in dynamic situations, future uncertainty (empheasized by Friedman) tends to raise the current level of the average policy response, while current uncertainty (emphasized by Brunner (1980)), lowers the response. In this sense, the static examples in this paper are not necessarily biased in favor of the activist paradigm. Moreover, in the examples described here, static and dynamic solutions are identical.

A reading of the monetarist literature suggests that a principal motivation for wanting a fixed money growth rule is a sense that

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period, the prior probability, Π_0, of noise is updated sequentially, based on information gained in the preceding period. Interestingly, such an updating procedure does not exist if the thresholds are determined by a minmax strategy, since, as just shown, signal extraction based on a minmax strategy is independent of Π_0. The minmax criterion precludes learning about probability distributions, and vice versa.

discretionary policy causes pro-cyclical uncertainty. This paper takes up this particular reasoning and, to load the result as much in favor of the monetarist stance as possible, considers the case of Knightian model and data uncertainty. Even so, and perhaps surprisingly, nonactivist behavior as a response to extreme uncertainty seems to be the exception rather than the norm. When comparing standard Bayesian, linear-quadratic feedback rules with corresponding minmax rules, one finds the former tend to be neither more nor less attenuated than the latter, and that all depends on a comparison of the means and variances of the parameter distribtions in Bayesian cases and the underlying limits on the supports of the parameter space assumed in the case of Knightian uncertainty.

The value of the preceding exercises is perhaps best appreciated if one realizes the extreme skepticism with which typical policy makers regard almost any description of the economy, formal or not. Another way of stating this is to assert that policy makers view almost all parameters of a given model economy as extremely uncertain. Fixed rate monetary growth rules may have been adopted as the most robust-seeming policy in the hope of insulating the consequences of policy from errors arising from uncertainty. The question that has been posed here is whether the desire for robustness, indeed, requires the extreme of a fixed rate of growth in the instrument. The apparent conclusion is that this monetarist prescription really applies only in limited and possibly rare circumstances.

James Tobin (1980), expressing doubts about the value of fixing the rate of growth of money, points out that policy makers do not need full structural information to formulate intelligent activist macroeconomic policies. Indeed, Kalchbrenner and Tinsley (1975, 1976, 1977), have argued that through proper use of information, optimal stabilization strategies can be efficiently adapted to stochastic environments confronting the authority. Even the most extreme version of the contrary belief that government can do little to inform itself, let alone improve social welfare, does not necessarily imply inaction or even diminished action. The difference in views thus seems to depend less on philosophical or technical disagreements than on a divergence of opinions regarding the competence of central banks.

If central bankers are generically unable to use discretion or follow rules, a regime denying them discretion and rules would be optimal, since potential harm is thus minimized. But a broad-brush characterization of incompetence is clearly unproved and, indeed, unprovable as an empirical proposition; and if it is false, an unnecessary adherence to perpetual inaction would lead to welfare losses.

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