Indeterminacy and Investment Adjustment Costs

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Abstract

It has been widely known that a neoclassical growth model with sufficient increasing returns in production may feature an indeterminate steady state. This note shows how investment adjustment costs increase the required degree of increasing returns for indeterminacy to arise. We also argue that sector-specific externalities are observationally equivalent to negative adjustment costs.

Key words: Indeterminacy, Investment Adjustment Costs, Two-Sector Models.
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1 Introduction

It has been shown that a neoclassical growth model with sufficient increasing returns in production may possess an indeterminate steady state, which can be exploited to generate endogenous business cycle fluctuations.[1] This note shows how investment adjustment costs interact with increasing returns in generating indeterminacy. The introduction of investment adjustment costs makes it difficult for indeterminacy to occur. That is, the required degree of increasing returns is higher in the presence of investment adjustment costs. Furthermore, models with large enough adjustment costs would never feature indeterminacy.

Some intuition for the existence of indeterminacy has been provided in the context of the labor market.[2] Increasing returns might justify households' optimism of market returns on labor, which would then induce both higher employment and a higher wage rate. In a model without investment adjustment costs, the necessary and sufficient condition for indeterminacy is that the labor demand schedule is upward sloping and is steeper than the labor supply curve.

The concept of adjustment costs has been widely used in the investment literature.[3] In a partial-equilibrium model without investment adjustment costs, the decision on the capital stock is static and so investment becomes infinitely volatile. General-equilibrium models have introduced various forms of investment adjustment costs to improve the properties of the model, such as the persistence.

This note shows that the introduction of investment adjustment costs, while leaving the labor market equilibrium condition intact, changes the necessary and sufficient condition for indeterminacy. That is, the intuition from the labor market does not hold in a model with investment adjustment costs. A more interesting result is that the presence of investment adjustment costs makes it less likely for indeterminacy to occur. The larger the adjustment costs, the larger the required degree of increasing returns.

The remainder of this note is organized as follows. Section 2 describes the model with investment adjustment costs. In section 3, the dynamics of the

[1] Benhabib and Farmer (1997) review the literature on indeterminacy and sunspots from the perspective of macroeconomics.

[2] See Benhabib and Farmer (1994) and the references in Benhabib and Farmer (1997).

[3] Kim (1997a) reviews the literature on investment adjustment costs from a macroeconomic perspective.

model are analyzed and a necessary and sufficient condition for indeterminacy is derived. We also relate the dynamics of sector-specific externalities to those of investment adjustment costs. Section 4 concludes.

2 The Economy

We develop a model that introduces investment adjustment costs into the continuous-time framework of Benhabib and Farmer (1994).

2.1 Firms

There is a continuum of identical firms, with the total number normalized to one. The aggregate production function is

$$Y = K^{\alpha} L^{\beta},\ \alpha > 0,\ \beta > 0, \tag{1}$$

where Y, K, and L represent total output, the aggregate stock of capital, and aggregate labor hours, respectively.[4] Unlike standard neoclassical growth models, we allow the two parameters α and β to sum to more than one. Benhabib and Farmer (1994) describe two environments that are consistent with (1). This note adopts the model with production externalities.[5]

Equation (1) represents the social technology and may be derived from the private technology of constant returns to scale:

$$Y_i = X K_i^{a} L_i^{b},\ a > 0,\ b > 0,\ a + b = 1, \tag{2}$$

where the subscript i denotes the individual firm. The term X represents production externalities that are taken as exogenous by each firm. We assume that the externalities are such that

$$X = \left(K^{a} L^{b}\right)^{\xi},\ \xi \geq 0. \tag{3}$$

When $\xi = 0$, the model reduces to the standard model with aggregate constant returns to scale. Following the macroeconomic convention of a symmetric equilibrium, we substitute (3) into (2) to obtain the social production

[4]For notational simplicity, the time dependence of the variables is suppressed.

[5]An alternative specification incorporates increasing returns in a monopolistically competitive economy. Kim (1997b) reviews three types of increasing returns from a critical perspective.

function (1) by defining

$$\frac{\alpha}{a} = \frac{\beta}{b} = 1 + \xi.$$

We also assume that externalities are not strong enough to generate endogenous growth, i.e. $\alpha < 1$.

Under the assumption that factor markets are perfectly competitive, profit maximization conditions are

$$\frac{ZK}{a} = \frac{WL}{b} = Y,$$

where Z is the rental rate of capital and W is the real wage.

2.2 Households without Adjustment Costs

The representative household maximizes

$$\int_0^{\infty} \left(\log C - \frac{L^{1+\chi}}{1+\chi} \right) e^{-\rho t} dt, \;\; \chi \geq 0, \;\; \rho > 0, \tag{4}$$

where C is consumption. Parameters χ and ρ represent the inverse of the intertemporal elasticity of substitution for labor supply and the discount rate, respectively. The household budget constraint is

$$C + I = ZK + WL, \tag{5}$$

where I is the gross investment. The household accumulates capital according to

$$\dot{K} = I - \delta K, \;\; 0 < \delta < 1, \;\; K(0) \text{ given}, \tag{6}$$

where δ is the capital depreciation rate. The factor prices are taken as given.

In this economy without adjustment costs, Benhabib and Farmer (1994) show that the necessary and sufficient condition for indeterminacy is

$$\beta - 1 > \chi. \tag{7}$$

This condition means that the labor demand schedule slopes up as a function of the real wage and is steeper than the labor supply curve. The labor market equilibrium condition is

$$bK^{\alpha} L^{\beta - 1} = W = CL^{\chi}. \tag{8}$$

This intuition from the labor market equilibrium has been used widely in the literature on indeterminacy. However, we will see that there are pitfalls in this interpretation. In a model with investment adjustment costs, the labor market equilibrium condition (8) remains unchanged but the condition for indeterminacy is different from (7).

2.3 Households with Adjustment Costs

The literature on investment adjustment costs has introduced the costs in various places. In the context of our model, the budget constraint (5) may incorporate such costs as follows,

$$C + \left[1 + h_a\left(\frac{I}{K}\right)\right] I = \left[1 - h_b\left(\frac{I}{K}\right)\right](ZK + WL),$$

and the capital accumulation equation (6) may be modified such that

$$\dot{K} = \left[1 - h_c\left(\frac{I}{K}\right)\right] I - \left[\delta + h_d\left(\frac{I}{K}\right)\right] K,$$

where the four cost functions follow certain regularity conditions, as stated in Abel and Blanchard (1983).

In this note, we adopt an alternative specification of constant elasticity as follows:

$$\dot{K} = \left[\delta\left(\frac{I}{\delta}\right)^{1-\phi} + (1-\delta) K^{1-\phi}\right]^{\frac{1}{1-\phi}} - K, \ \phi \geq 0. \tag{9}$$

A special case of $\phi = 0$ corresponds to the linear capital accumulation in (6). This constant-elasticity specification, combined with the budget constraint without adjustment costs (5), generates the dynamics equivalent to those from a model using h's.[6]

[6] Kim (1997a) shows that the equivalence holds when ϕ is a linear function of the four second derivatives of h's at the steady state. A key assumption is that $h(\delta) = h'(\delta) = 0$ for all four h's.

3 The Dynamics

3.1 Analysis of the Dynamics

The first-order conditions of the economy with adjustment costs are

$$CL^{\chi} = \frac{bY}{L}, \tag{10}$$

$$\frac{\dot{C}}{C} = \frac{aY}{K}K_I - \left[1 - (1-\delta)K_K\right] - \rho - \frac{\dot{K}_I}{K_I}, \tag{11}$$

where

$$K_K = K^{-\phi}\left[\delta\left(\frac{I}{\delta}\right)^{1-\phi} + (1-\delta)K^{1-\phi}\right]^{\frac{\phi}{1-\phi}},$$

$$K_I = \left(\frac{I}{\delta}\right)^{-\phi}\left[\delta\left(\frac{I}{\delta}\right)^{1-\phi} + (1-\delta)K^{1-\phi}\right]^{\frac{\phi}{1-\phi}}.$$

Equation (10) equates the slope of the household indifference curve to the real wage and (11) is the consumption Euler equation. The budget constraint (5) may be rewritten as

$$C + I = Y. \tag{12}$$

The equilibrium is characterized by five equations, (1) and (9)–(12), and a transversality condition.

The system can be reduced to a bivariate first-order system as follows. First, L is eliminated from the system by combining (1) and (10). Denoting lower-case letters for the logarithmic of the variables, we have

$$y - k = \lambda + \kappa k + \gamma c,$$

where

$$\lambda = \frac{\beta \log b}{\chi - (\beta - 1)},$$

$$\kappa = \frac{\alpha(1+\chi)}{\chi - (\beta - 1)} - 1,$$

$$\gamma = \frac{-\beta}{\chi - (\beta - 1)}.$$

Using this relation and (12), we approximate (9) and (11) as an autonomous system of k and c:

$$\begin{aligned}\dot{k} &= e^{\lambda+\kappa k+\gamma c} - e^{-k+c} - \delta, \\ \dot{c} &= ae^{\lambda+\kappa k+\gamma c}e^{-\phi_\delta \dot{k}} - \left[1-(1-\delta)\, e^{\phi \dot{k}}\right] - \rho + \phi_\delta \dddot{k},\end{aligned}$$

where

$$\phi_\delta = \left(\frac{1-\delta}{\delta}\right)\phi.$$

Replacing $\dddot{k}$ with the derivative of the first equation and linearizing the system, we have

$$\begin{aligned}\dot{k} &= \tilde{\kappa}\left(k-\bar{k}\right) + \tilde{\gamma}\,(c-\bar{c})\,, \\ -\phi_\delta\tilde{\kappa}\dot{k} + (1-\phi_\delta\tilde{\gamma})\,\dot{c} &= [(\rho+\delta)\,\kappa - \phi_\delta\rho\tilde{\kappa}]\left(k-\bar{k}\right) + [(\rho+\delta)\,\gamma - \phi_\delta\rho\tilde{\gamma}]\,(c-\bar{c})\,,\end{aligned}$$

where

$$\begin{aligned}\tilde{\kappa} &= \frac{\rho+\delta}{a}\kappa + \frac{\rho+b\delta}{a} = \frac{\rho\chi+\delta\,(\beta-1)+(\rho+\delta)}{\chi-(\beta-1)}, \\ \tilde{\gamma} &= \frac{\rho+\delta}{a}\gamma - \frac{\rho+b\delta}{a} = \frac{(\rho+b\delta)\,\chi+a\delta\,(\beta-1)+(\rho+\delta)}{a\,[\chi-(\beta-1)]}.\end{aligned}$$

3.2 Necessary and Sufficient Condition

Premultiplying the inverse of the coefficient matrix of the dotted variables with the coefficient matrix of the level variables, we have the Jacobian:

$$\frac{1}{1-\phi_\delta\tilde{\gamma}}\begin{bmatrix} \tilde{\kappa}\,(1-\phi_\delta\tilde{\gamma}) & \tilde{\gamma}\,(1-\phi_\delta\tilde{\gamma}) \\ (\rho+\delta)\,\kappa - \phi_\delta\tilde{\kappa}\,(\rho-\tilde{\kappa}) & (\rho+\delta)\,\gamma - \phi_\delta\tilde{\gamma}\,(\rho-\tilde{\kappa}) \end{bmatrix}.$$

Its trace and determinant are given by

$$\begin{aligned}\texttt{Tr} &= \frac{a\,(\rho+\delta)\,(1+\chi)\,\xi}{a\,[\chi-(\beta-1)]+\phi_\delta\,[(\rho+b\delta)\,\chi+a\delta\,(\beta-1)+(\rho+\delta)]} + \rho, \\ \texttt{Det} &= -\frac{(1+\chi)\,(1-\alpha)\,(\rho+\delta)\,(\rho+b\delta)}{a\,[\chi-(\beta-1)]+\phi_\delta\,[(\rho+b\delta)\,\chi+a\delta\,(\beta-1)+(\rho+\delta)]}.\end{aligned}$$

Now we are ready to calculate the necessary and sufficient condition. Indeterminacy arises if and only if the parameter values satisfy the following two conditions:

$$\beta - 1 > \chi + (1+\chi)\left(\frac{\rho+\delta}{a\delta}\right)\frac{(1-\delta)\,\phi}{1-(1-\delta)\,\phi},$$

and

$$\phi < (1-\delta)^{-1}.$$

Since the determinant passes through minus infinity to plus infinity, the condition involving the trace is not binding.

The first condition may be rewritten as

$$\xi > \left(\frac{1+\chi}{b}\right)\left[1+\left(\frac{\rho+\delta}{a\delta}\right)\frac{(1-\delta)\,\phi}{1-(1-\delta)\,\phi}\right] - 1 \equiv \xi^*,$$

which says that the degree of externalities required for indeterminacy to occur (ξ^*) is an increasing function of the degree of adjustment costs (ϕ). Intuitively speaking, less flexibility due to adjustment costs should be offset by more flexibility in increasing returns for optimism to be self-fulfilling. Furthermore, the required degree increases very fast since it is a rational function in the degree of adjustment costs. The second condition says that the degree of adjustment costs should be less than $(1-\delta)^{-1}$. In terms of q-regressions, the elasticity of (I/K) with respect to q should be larger than unity. As the degree of adjustment costs approaches $(1-\delta)^{-1}$, the required degree of externalities diverges to infinity.

Wen (in press) also shows the positive relationship between the required degree of externalities (ξ^*) and the degree of adjustment costs (ϕ). However, the relation is not analytically derived and so the paper does not comment on the upper bound of ϕ for indeterminacy to arise. Furthermore, his conclusion that the oscillation mechanism is preserved even under a unique equilibrium depends on the specification that adjustment costs are a function of the first difference in investment. In our model where the change of the capital stock determines the costs, the oscillation mechanism disappears together with indeterminacy.

To provide an idea of the sensitivity of the required degree of externalities with respect to the degree of adjustment costs, consider the following parameterization, which is favorable for indeterminacy to arise.[7]

[7] This parameterization follows Benhabib and Farmer (1996) except for the indivisible-labor specification ($\chi = 0$).

Parameter	a	b	χ	ρ	δ
Calibrated value	0.3	0.7	0	0.05	0.1

As reviewed in Hamermesh and Pfann (1996), the estimates of investment adjustment costs are few and vary widely. So we analyze the space of ϕ and ξ for indeterminacy to occur instead of calibrating the two parameters.

With the calibrated parameter values in the previous table, the mapping from the degree of adjustment costs to the required degree of externalities is as follows:[8]

ϕ	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.5	1
ξ^*	-0.42	-0.16	0.12	0.43	0.77	1.14	1.54	6.27	64

It is easy to see that the required degree of externalities increases very fast as the degree of adjustment costs increases. As the degree of adjustment costs approaches unity, which is a Cobb-Douglas specification used in Hercowitz and Sampson (1991), it is almost impossible for indeterminacy to occur under any reasonable parameter values. According to our calibration, when the degree of investment adjustment costs is as low as 0.25, it is impossible for indeterminacy to arise. The lower bound of empirical estimates for ϕ is about 0.2. Even for a lower degree of 0.15, the required degree of externalities is 1.54 which is too large to rationalize from empirical grounds.

3.3 Sector-Specific Externalities and Adjustment Costs

Benhabib and Farmer (1996) show that sector-specific externalities make it easy for indeterminacy to occur under a reasonable parameterization. In this subsection, we show that sector-specific externalities are equivalent to negative adjustment costs. This equivalence suggests that empirical relevance of indeterminacy needs more analysis from both theoretical and empirical grounds.

For expositional simplicity, we assume that the capital stock is accumulated in a linear way as in (6). Following Benhabib and Farmer (1996), the two sectoral production functions and the aggregation identities are as

[8] Negative values of ϕ are included for the next subsection of sector-specific externalities. The upper bound of ξ is 7/3 to exclude endogenous growth, but bigger values are displayed for expositional purpose.

follows:

$$\begin{aligned} C_i &= X_C \left(K_{C,i}\right)^a \left(L_{C,i}\right)^b, \\ I_i &= X_I \left(K_{I,i}\right)^a \left(L_{I,i}\right)^b, \\ K_i &= K_{C,i} + K_{I,i}, \\ L_i &= L_{C,i} + L_{I,i}. \end{aligned}$$

Subscripts C and I denote the consumption and the investment sector. Production externalities are such that

$$\begin{aligned} X_C &= \left(\frac{\bar{C}}{\bar{Y}}\right)^{-\theta} \left[\left(K_C\right)^a \left(L_C\right)^b\right]^\theta \left[K^a L^b\right]^\sigma, \\ X_I &= \left(\frac{\bar{I}}{\bar{Y}}\right)^{-\theta} \left[\left(K_I\right)^a \left(L_I\right)^b\right]^\theta \left[K^a L^b\right]^\sigma. \end{aligned}$$

Two parameters θ and σ represent sector-specific and aggregate externalities, respectively, and the upper bars denote the steady state in the previous one-sector model.[9]

Assuming symmetry, the transformation between consumption and investment has a constant elasticity and its slope is unity at the steady state. Algebraically,

$$\left(\frac{\bar{C}}{\bar{Y}}\right)^{\frac{\theta}{1+\theta}} C^{\frac{1}{1+\theta}} + \left(\frac{\bar{I}}{\bar{Y}}\right)^{\frac{\theta}{1+\theta}} I^{\frac{1}{1+\theta}} = \left(K^a L^b\right)^{\frac{1+\theta+\sigma}{1+\theta}}.$$

It is shown in Kim (1997a) that this specification of constant elasticity of transformation between consumption and investment is observationally equivalent to the specification of constant elasticity of substitution between investment and the capital stock, as in (9).

To put this concretely, an observational-equivalence result holds between two models. The first is the one-sector model with investment adjustment costs and production externalities and the second is the two-sector model with both sectoral and aggregate externalities. Equivalence holds under the following condition:

$$\begin{aligned} \xi &= \theta + \sigma, \\ \phi &= -\theta. \end{aligned}$$

[9] This specification is different from that of Benhabib and Farmer (1996) due to the coefficient terms involving the steady states, which enable the exact equivalence between sector-specific externalities and investment adjustment costs.

Now it is clear why sector-specific externalities are more powerful than aggregate externalities in generating indeterminacy. Sector-specific externalities not only increase the degree of overall externalities (ξ), but also decrease the degree of investment adjustment costs (ϕ). In contrast, aggregate externalities merely increase the degree of overall externalities. Benhabib and Farmer (1996) calibrate a model with sector-specific externalities with $\theta = 0.15$. In this case, the previous table shows that indeterminacy would would occur even with significant decreasing returns. We also can see that the critical value of θ for indeterminacy with overall constant returns is between 0.05 and 0.1.

The result in this subsection adds another dimension to the discussion on empirical relevance of indeterminacy. When we discuss the plausibility of indeterminacy, as in the two-sector model of Benhabib and Farmer (1996), we should consider other avenues which may render it difficult for indeterminacy to arise. An example is the presence of investment adjustment costs provided in this note.

4 Conclusion

We have shown how the presence of positive investment adjustment costs makes it less likely for indeterminacy to occur. The larger the adjustment costs, the larger the required degree of increasing returns. It would be interesting to explore the empirical plausibility of indeterminacy in an economy with investment adjustment costs, preferably combined with sector-specific externalities.

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