Certainty Equivalence and the Non-Vertical Long Run Phillips-Curve*

Yvan Lengwiler[†]

June 18, 1998

Abstract

The certainty equivalence principle states that only the mean of a random variable is relevant to a rational decision maker facing uncertainty. This principle simplifies the application of the idea of rational expectations considerably. Yet, certainty equivalence does not in general apply outside of the special case of quadratic objective function subject to linear constraints. I use the standard augmented Phillips-Curve to demonstrate the significant effects that occur with the breakdown of certainty equivalence.

JEL classification codes. D82, E24, E31.

Keywords. rational expectations, expected utility maximization, certainty equivalence, Jensen's inequality, natural rate, long run Phillips-Curve.

1 Introduction

The fundamental axiom of rational expectations, according to Muth (1961), is that agents use all of their (imperfect) information when making decisions. The *certainty equivalence principle* simplifies the application of this idea by stating that only the mean of the random variables is relevant for the decision makers. Hence, in many macroeconomic models, agents are assumed to maximize utility using the conditional statistical mean of the random

*I thank Joseph Eisenhauer, Marc Giannoni, Daniel Heller, Athanasios Orphanides, Richard Porter, and Christof Stahel for interesting conversations and helpful comments. Parts of this paper were written while the author was on leave at the Board of Governors of the Federal Reserve System. The views expressed are not necessarily those of the Board of Governors of the Federal Reserve System or of the Swiss National Bank.

† Board of Governors of the Federal Reserve System and Swiss National Bank.

Please address correspondence to Yvan Lengwiler, Swiss National Bank, P.O. Box, CH–8022 Zürich, Switzerland, phone +41 1 631 3111, email yle@altavista.net

variables. I would like to call these kinds of models, where only the mean is of interest, "point expectations" models, because other aspects of the random variables (e.g., the variance, or the whole distribution) do not affect the behavior of the agents. Yet, the certainty equivalence principle is not always applicable.

If certainty equivalence does not apply, then it is not rational for the agents to ignore the additional information contained in the distribution (which presumably is known to them). Rather, a rational agent will maximize expected utility using all of his knowledge about the random variables. This additional information may make a difference: By definition, if certainty equivalence fails, then expected utility maximization and rational point expectations are not equivalent. In fact, I will show that a failure of certainty equivalence can have profound effects even in otherwise standard models.

To illustrate this claim I take the standard expectations augmented Phillips-Curve as proposed by Friedman and Phelps and recompute it using expected utility maximization, that is to say, the agents involved will maximize their expected utility, taking account not only of the mean, but of the whole distribution of the random price level. It will be seen that—at least for specific functional forms of the utility function, the production function, and the price distribution—this exercise leads in general to a positively or negatively sloped long run Phillips-Curve, contrary to the conventional model.

2 The textbook model

The expectations augmented Phillips-Curve rests on the idea that agents have to make decisions at a time when the relevant price level is unknown (see Friedman, 1968, and Phelps, 1968). A popular model that captures this idea assumes that nominal wages are sticky to some extent, and agents have to agree on a nominal wage without knowing for sure what the price level (and hence the real wage) will be. Such a model was used, for instance, in Gray (1976). Discussions of a model in this spirit can be found in several textbooks, such as Abel and Bernanke (1998, pp. 449–453), Dornbusch *et al.* (1997, p. 104), Mankiw (1997a, p. 694), and Stockman (1996, pp. 894–898). In particular, Mankiw (1997b, pp. 335–337) presents a model that is most similar to the one I will use here.

Consider a simple partial equilibrium model with one input (labor), denoted n, and one competitively produced output, y. The price p of the produced commodity is a serially uncorrelated random variable with some density function γ.[1] For some (unexplained) reason, the labor contract determines the nominal wage w, not the real wage w/p. It is assumed that employers are perfectly informed about the price level at the time the nominal wage is negotiated. Workers, by contrast, know only γ.

A mere counterexample to the result that the long run Phillips-Curve is necessarily vertical should suffice to make my point, so I do not aim for the greatest possible generality.

[1] The Phillips-Curve usually relates unemployment to the rate of inflation, not to the price level. In the following the inflation rate will not appear, but the model can easily be adapted to reflect a relationship between unemployment and the inflation rate by a simple reinterpretation of the variables: Let P be the "end of last period" price level, and let π be the inflation rate, assumed to be a serially uncorrelated random variable. Then $p := P \cdot (1 + \pi)$ is next period's price level. Building expectations on π is equivalent to building expectations on p.

For that reason I introduce concrete functional forms for all the relationships involved, and I choose these functional forms such as to make solving the model as straightforward as possible. In this spirit, let us assume that p is uniformly distributed,

$$\gamma(p) := \begin{cases} 1/2s & \text{if } \mu - s \leqslant p \leqslant \mu + s, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

μ is the expected value of p, and s is some measure of its volatility.

Firms operate under the production function

$$f(n) := \frac{1}{\beta} \cdot n^{\beta}. \tag{2}$$

Labor demand is derived from competitive profit maximization, which yields the usual first-order condition that the marginal product, $f'(n) = n^{\beta-1}$, must equate the real wage, w/p. Thus, labor demand is given by

$$n^d := \left(\frac{w}{p}\right)^{-1/(1-\beta)}. \tag{3}$$

Workers have a quasi-linear utility function

$$u(y, n) := y^{\alpha} - n. \tag{4}$$

Let $E(p)$ denote the workers' expectation of the price level. Labor supply is derived from constrained utility maximization, given this expectation, so

$$\max_{y,n} \; u(y, n) \qquad \text{s.t.} \quad E(p) \cdot y \leqslant w \cdot n. \tag{5}$$

The presumption is that the decision about how much to work preceeds the consumption decision. Thus, if the expectation turns out to be wrong, $E(p) \neq p$, then the worker will have to adapt consumption. For instance, if the price level turns out to be higher than expected, then planned consumption will not be feasible and the workers' consumption plans will be frustrated. If the price level turns out to be lower than expected, then the real wage will be higher than expected and the worker will be able to consume more. Of course, in a more complete model, using the savings and loans market, the worker would smooth such windfall gains and losses. In the static setup of the present model, though, the worker is constrained to use up his budget every period. The maximization problem thus becomes simpler. In (5) we can substitute y in the utility function for the budget constraint, and we can use the quasi-linear utility function (4), yielding

$$\max_{n} \left(n \frac{w}{E(p)}\right)^{\alpha} - n. \tag{6}$$

There is no difference between (5) and (6) in the point expectations model, but this change will be relevant in the expected utility model. The solution to (6) is the labor supply function,

$$n^s := \alpha^{1/(1-\alpha)} \cdot \left(\frac{w}{E(p)}\right)^{\alpha/(1-\alpha)}. \tag{7}$$

The nominal wage is determined by labor market equilibrium, $n^d = n^s$, which requires

$$\hat{w} := \alpha^{-(1-\beta)/(1-\alpha\beta)} \cdot p^{(1-\alpha)/(1-\alpha\beta)} \cdot E(p)^{\alpha(1-\beta)/(1-\alpha\beta)}, \tag{8}$$

and equilibrium employment is

$$\hat{n} := \alpha^{1/(1-\alpha\beta)} \cdot \left(\frac{p}{E(p)}\right)^{\alpha/(1-\alpha\beta)}. \tag{9}$$

We can rearrange (9) so that it becomes clearer that this equation is a Phillips-Curve,

$$p = E(p) \cdot \alpha^{-1/\alpha} \cdot \hat{n}^{(1-\alpha\beta)/\alpha}. \tag{10}$$

This expression is the *short run Phillips-Curve.* As expected, the Phillips-Curve is increasing in $\hat{n}$ (hence decreasing in leisure or unemployment) and homogenous of degree one with respect to $E(p)$, indicating that a change of price expectations leads to a one-to-one shift of the Phillips-Curve.

Workers are assumed to have rational expectations (in the sense of not making biased forecasts), so they set $E(p) = \mu$, which is statistically the best guess they can make. Expectational errors, $\mu - p$, are not systematic. Define the *long run Phillips-Curve* to be the locus of all (p, n)-pairs that are such that the employees make no forecast error, i.e. $p = E(p)$. Because, as we observed before, the short run Phillips-Curve is homogenous of degree one with respect to the expected price, all (p, n)-pairs on the long run Phillips-Curve share a common level of employment, n^*, which is called the *natural rate of employment.* Employment can deviate from n^* only due to expectational errors. Since expectational errors are not systematic, n will not systematically deviate from n^*. Furthermore, because the workers over- and underestimate inflation with equal probability, n^* is the median level of employment. The natural rate n^* can easily be computed by equating p and $E(p)$ in equation (9),

$$n^* := \alpha^{1/(1-\alpha\beta)}. \tag{11}$$

n^* is uniquely defined, so the *long run Phillips-Curve* is indeed a vertical line.

3 Expected utility maximization

Let us turn to slightly more sophisticated workers now. They know that they are imperfectly informed, so rationality requires them to take this fact into account. The right thing for them to do is to maximize *expected utility*, given their knowledge of the distribution of p, thereby using the information they have more fully.

Let us assume that the decision problem of the employees satisfies the von Neumann-Morgenstern axioms, so that the utility function u as introduced in (4) is additively separable with respect to probabilities. Agents now maximize expected utility, and the new objective function presents itself as[2]

$$\int_{\mu-s}^{\mu+s} u\left(n\frac{w}{p}, n\right)\gamma(p)\,dp. \tag{12}$$

Using the specifications, (1), (2), and (4), and making the same steps as before, we find the market clearing wage and employment,

$$\tilde{w} := p^{(1-\alpha)/(1-\alpha\beta)} \cdot \left[\frac{\alpha}{1-\alpha} \cdot \frac{(\mu+s)^{1-\alpha}-(\mu-s)^{1-\alpha}}{2s}\right]^{-(1-\beta)/(1-\alpha\beta)}, \tag{13}$$

$$\tilde{n} := p^{\alpha/(1-\alpha\beta)} \cdot \left[\frac{\alpha}{1-\alpha} \cdot \frac{(\mu+s)^{1-\alpha}-(\mu-s)^{1-\alpha}}{2s}\right]^{1/(1-\alpha\beta)}, \tag{14}$$

or again rearranging (14), we get the short run Phillips-Curve

$$p = \tilde{n}^{(1-\alpha\beta)/\alpha} \cdot \left[\frac{\alpha}{1-\alpha} \cdot \frac{(\mu+s)^{1-\alpha}-(\mu-s)^{1-\alpha}}{2s}\right]^{-1/\alpha}. \tag{15}$$

Clearly, the short run Phillips-Curve increases in n (decreases in unemployment or leisure), as in the case of point expectations. However, a shift in the expected price level, μ, does *not* lead to a one-to-one shift of the Phillips-Curve.

We defined the long run Phillips-Curve to be the locus of all (p, n)-pairs that are such that the employees make no forecast error. The employees' forecast of p is μ. Equating μ and p in (14) yields the natural rate of unemployment as a function of μ and s,

$$n^{**} := \mu^{\alpha/(1-\alpha\beta)} \cdot \left[\frac{\alpha}{1-\alpha} \cdot \frac{(\mu+s)^{1-\alpha}-(\mu-s)^{1-\alpha}}{2s}\right]^{1/(1-\alpha\beta)}. \tag{16}$$

This equation is the long run Phillips-Curve if workers choose labor supply such as to maximize expected utility. It is *not* a vertical line: the natural rate (i.e. the employment level that materializes if the agents make no forecast error) depends on μ and s. Again, employment n will not systematically deviate from the natural rate n^{**}, and n^{**} is the median employment, but in this model, the natural rate is a function of the average inflation and of its variability.

4 Comparing the two models

In this section, we compare the results under the assumptions of utility maximization with rational point expectations and expected utility maximization. Under point expectations, the short run Phillips-Curve is given by equation (7), and the long run Phillips-Curve is a vertical line through n^*. Let us analyze the properties of the expected utility maximization case now.

[2]Again we assume that the agents decide first how much to work. Consumption is then determined by the real income this labor supply generates, which is a random variable, given that p is random. A somewhat less intuitive alternative would be to assume that the agents first decide how much to consume. Labor supply would then be computed as the amount necessary to finance this consumption, and would therefore be a random variable (given that p is random). These alternatives are not equivalent.

First, note that s is a measure of the volatility of p. When s approaches zero, the distribution of p approaches a unique mass point on μ. In the limit, the problem under expected utility maximization coincides with the problem under rational point expectations, so we can expect a result like the following.

Observation. *As s approaches 0, the short run and the long run Phillips-Curves under expected utility maximization converge point-wise to the respective Phillips-Curves under rational point expectations.*

Malinvaud (1969) proves a general result that implies this observation. A direct proof for the specific model of this paper is straightforward, so I give it here.

Proof. Using de l'Hospital's rule we find that $\tilde{n}$ (employment in the expected utility maximization model) converges to $\hat{n}$ (employment in the point expectations model) as s approaches 0,

$$\lim_{s \to 0} \tilde{n} \;=\; \alpha^{1/(1-\alpha\beta)} \cdot \left(\frac{p}{\mu}\right)^{\alpha/(1-\alpha\beta)} \;=\; \hat{n}.$$

The convergence of n^{**} to n^{*} as $s \to 0$ is proved in the same way. QED

Even though the expected price is μ, the expected real wage $E(w/p)$ is larger than w/μ due to Jensen's inequality, so it is in fact the non-linearity of $1/p$ that causes certainty equivalence to fail. Because of that, agents that maximize expected utility supply more labor than agents that operate under point expectations. Thus, the natural rate of emploment in the expected utility maximization model is always bigger than in the point expectations model, $n^{**} > n^{*}$. Whether the effect of Jensens' inequality increases or decreases with average inflation depends on the ratio s/μ (which is by definition between zero and one). If μ is large compared to s (and hence s/μ is close to zero), the distribution of p is close to a mass point on μ. In this case, expected utility maximization is almost the same as utility maximization under point expectations. The curve resulting from expected utility maximization therefore approaches the curve of the point expectations model. Conversely, as μ approaches s (and thus s/μ approaches unity), the variability of the real wage becomes larger, and Jensen's inequality becomes more relevant. The real wage varies between at least $w/(\mu + s)$ and at most $w/(\mu - s)$. Hence, if μ is only slightly bigger than s, there is some chance that the employees will earn the low real wage $w/(\mu + s)$. But this bad prospect is more than offset by the prospect of earning the much higher real wage $w/(\mu - s)$. This is the supremum of the support of the distribution of the real wage. It diverges to infinity as μ approaches s. It turns out that this positive prospect dominates: the smaller s/μ is, the more labor is supplied in equilibrium, and hence the higher is employment.

In the model as developed in sections 2 and 3 it was assumed that s was some exogenously given constant. Thus, as μ increases, s/μ decreases and the effect of Jensen's inequality vanishes. For that reason, the long run Phillips-Curve will be *upward sloping*. As μ becomes large, the curve becomes asymptotically vertical at the natural employment level of the point expectations model. Conversely, as μ approaches s, the curve becomes asymptotically horizontal. An example of such a relationship can be found in figure 1, right panel.

figure 1 about here

To assume that the variability of inflation is some fixed number, however, conflicts with the established empirical fact that countries with high average inflation also experience highly volatile inflation.[3] So, suppose s (a measure of volatility) was some affine function of μ,[4]

$$s := \Theta_0 + \Theta_1 \cdot \mu. \tag{17}$$

The ratio s/μ then equals $\Theta_0/\mu + \Theta_1$. This expression is either increasing or decreasing in μ depending on the sign of Θ_0. If Θ_0 is negative, then the effect of Jensen's inequality increases with μ, and a downward sloping, or *keynesian*, long run Phillips-Curve emerges (figure 1, left panel). If Θ_0 is zero, then the long run Phillips-Curve becomes vertical, or *classical* (figure 1, mid panel). However, the natural rate of employment n^{**} will be bigger than the one identified in the point expectations model n^*. Finally, if Θ_0 is positive, then we have a downward sloping, or one could say, "*super-classical*," long run Phillips-Curve. So the theory proposed here links the sign of the intercept Θ_0 to the sign of the slope of the long run Phillips-Curve. Whether, in fact, Θ_0 is positive or negative or zero is largely an empirical question.

5 Conclusion

This paper argues that the idea of rational expectations formation is not appropriately modelled by the assumption that the expectations simply reflect the conditional mean of a random variable. If the imperfectly informed agents know something about the distribution, they should also use this information when maximizing their expected utility.

Using the standard expectations augmented Phillips-Curve it is demonstrated that this more complete treatment of the maximization problem has significant implications for the shape of the long run Phillips-Curve. Specifically, the long run Phillips-Curve is shown to be generally not vertical, but—for the specification I have chosen—positively or negatively sloped (or, exceptionally, vertical). This, I think, is by itself an interesting finding. Incidentally, Milton Friedman (1977) in his Nobel Lecture presents some tentative empirical evidence in favor of a positively sloped Phillips-Curve. More recently, however, King and Watson (1992) find no such evidence.

The other, more general point of the paper is that modelling the idea of rational expectations as point expectations is not appropriate, because with this hypothesis agents forego relevant information. Rather we should model agents as expected utility maximizers that take account of all the information they have, including the information on the distribution of random variables. Doing this instead of taking certainty equivalence for granted might produce significant results also in other areas of macroeconomics.

[3] See Okun (1971) and Ball and Cecchetti (1990) for empirical studies of the phenomenon.

[4] This is only a first order approximation, inspired by the scatter plots in Okun and in Ball and Cecchetti. The true relationship is probably more complicated.

References

Abel, Andrew B. and Ben S. Bernanke. *Macroeconomics*, 3rd ed. Addison-Wesley, 1998.

Ball, Laurence and Stephen G. Cecchetti. "Inflation and Uncertainty at Short and Long Horizons." *Brooking Papers on Economic Activity* 1 (1990), 215–245.

Dornbusch, Rudiger, Stanley Fischer, and Richard Startz. *Macroeconomics*, 7th ed. McGraw-Hill, 1997.

Friedman, Milton. "The Role of Monetary Policy." *American Economic Review* 58 (March 1968), 1–17.

Friedman, Milton. "Inflation and Unemployment." *Journal of Political Economy* 85 (1977), 451–472.

Gray, Jo Anna. "Wage Indexation: A Macroeconomic Approach." *Journal of Monetary Economics* 2 (1976), 221–235.

King, Robert G. and Mark W. Watson. "Testing Long Run Neutrality." *NBER Working Paper* #4156 (1992).

Malinvaud, Edmond. "First Order Certainty Equivalence." *Econometrica* 37 (October 1969), 706–718.

Mankiw, N. Gregory. *Principles of Economics*, 1st ed. The Dryden Press, 1997a.

Mankiw, N. Gregory. *Macroeconomics*, 3rd ed. Worth Publishers, 1997b.

Muth, John F. "Rational Expectations and the Theory of Price Movements." *Econometrica* 29 (June 1961), 315–335.

Okun, Arthur M. "The Mirage of Steady Inflation." *Brooking Papers on Economic Activity* 2 (1971), 485–498.

Phelps, Edmund S. "Phillips Curves, Expectations of Inflation, and Optimal Unemployment over Time." *Economica* 34 (August 1967), 254–281.

Stockman, Alan C. *Introduction to Economics*, 1st ed. The Dryden Press, 1996.

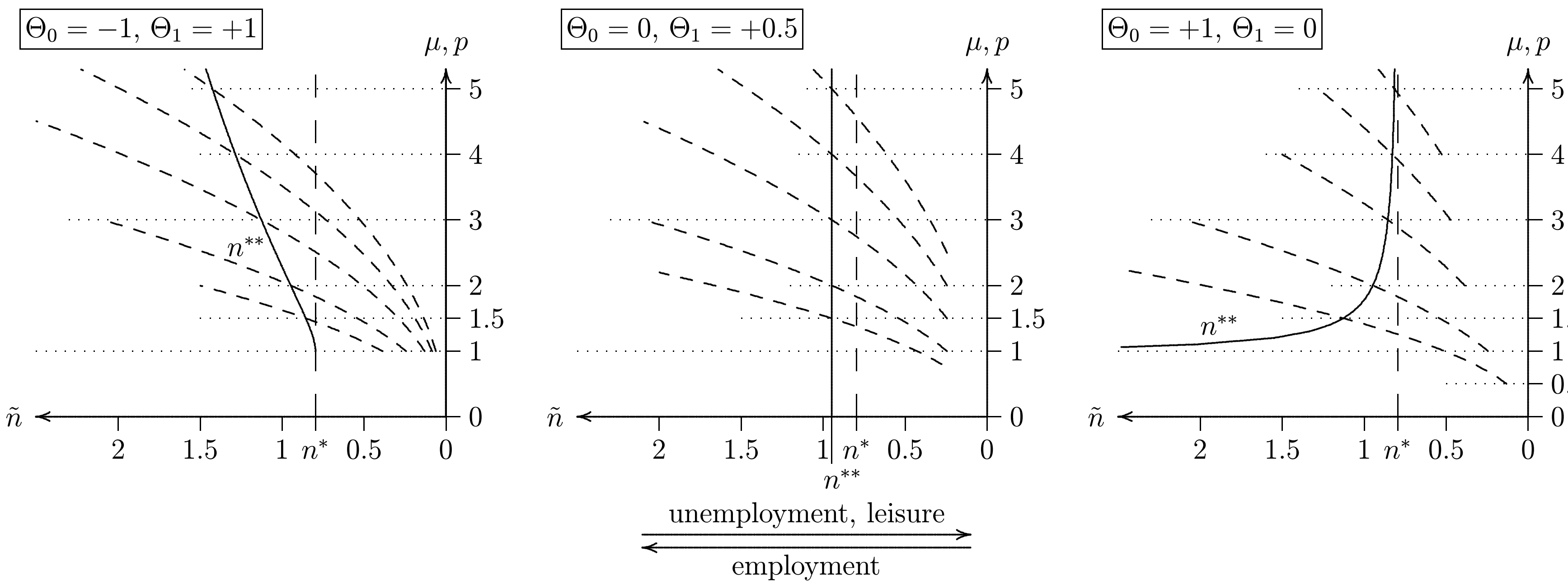

Figure 1. *Long run Phillips-Curves for $\alpha = 0.9$, $\beta = 0.6$, and variable Θ_0 and Θ_1. The abscissa (measuring employment) points to the left so that the Phillips-Curves are drawn with respect to unemployment, as it is usually done. The dashed lines are short run Phillips-Curves for μ equal to 1.5, 2, 3, 4, and 5, respectively.*