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Alternative Central Bank Credit Policies for Liquidity Provision in a Model of Payments

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I explore alternative central bank policies for liquidity provision in a model of payments. I use a mechanism design approach so that agents' incentives to default are explicit and contingent on the credit policy designed. In the first policy, the central bank invests in costly enforcement and charges an interest rate to recover costs. I show that the second best solution is not distortionary. In the second policy, the central bank requires collateral. If collateral does not bear an opportunity cost, then the solution is first best. Otherwise, the second best is distortionary because collateral serves as a binding credit constraint.

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1. INTRODUCTION

A primary role of a central bank is to facilitate a safe and efficient payments system. One source of inefficiency in payment systems is a potential shortage of liquidity. Central banks often respond by providing liquidity through the extension of credit. Because of this role, a central bank must manage its exposure to the risk that an agent does not repay. Some central banks, such as the European Central Bank, manage this risk by requiring borrowers to post collateral. Others, such as the Federal Reserve in the U.S., charge an explicit interest rate on credit and limit the amount any particular agent can borrow. In this paper, I explore these alternative credit policies in a theoretical model of payments and offer a rationale for why some central banks may choose one credit policy over another. I do this in a mechanism design framework, paying particular attention to the moral hazard issues associated with the repayment of debt that alternative credit policies aim to mitigate.

The payment systems most relevant to this paper are large-value payment systems which are mainly intraday, interbank payment systems. Many large-value payment systems are operated by central banks and are often real-time gross settlement (RTGS) systems. In an RTGS system, payments are made one at a time, with finality, during the day. Examples of RTGS systems include Fedwire operated by the Federal Reserve in the U.S. and TARGET, operated by the European Central Bank in the EMU.[3] Because payments are made one at a time, liquidity is needed to complete each transaction. If participants do not have enough liquidity to make a payment at a particular point in time, they can typically borrow funds

[3] TARGET is the collection of inter-connected domestic payment systems of the EMU that settle cross-border payments denominated in euros.

from the central bank by overdrawing on an account with the central bank, which they then pay back by the end of the day. The central bank faces a trade-off between supplying this intraday liquidity at little or no cost to enhance the efficiency of the system and accounting for moral hazard issues associated with the extension of credit. Of fundamental interest in this paper is how a central bank should design a credit policy for the provision of liquidity in an RTGS system to improve efficiency while dealing with moral hazard associated with debt repayment.

The main contribution of this paper is a framework with which to study the alternative credit policies of central banks. The key features of the framework are (i) default decisions of agents are endogenous, and (ii) mechanism design. The first is important to rigorously introduce a moral hazard problem that arises when debt is extended. The second is a useful approach to evaluate what good outcomes are achievable under alternative credit policies taking into account agents' incentives to default.

This framework is applied to a model of payments that is similar to that of Freeman (1996). Such a model captures some key features of large-value payment systems. These features are i) fiat money is necessary as a means of payment, ii) there is a need to acquire liquidity (in the form of fiat money) during the day to make such payments, and iii) money is also necessary to repay debts by the end of the day. These three features provide an endogenous role for an institution such as a central bank to provide liquidity to facilitate payments.

An important abstraction in Freeman's original model, however, is that there is costless enforcement that exogenously guarantees that debts are repaid. Such an abstraction has led to conclusions by Freeman (1996), Green (1997), Zhou (2000), Kahn and Roberds (2001) and Martin (2003) that a credit policy of free liquidity provision is optimal. These conclusions are immediate given that there is no explicit moral hazard problem in most of these models[4]. As a result, these models do not fully capture the trade-off between providing liquidity to facilitate payments and minimizing the exposure of credit risk associated with that provision. Moreover, Mills (2004) endogenizes the repayment decision of agents under costless enforcement in Freeman's model and shows that money is not necessary to repay debts if enforcement is too strong and so the need for liquidity in the model is questioned. As in Mills (2004), I shall depart from this abstraction so that the default decision of agents is not trivial.

In the context of the background environment, I look at two alternative credit policies that resemble some of the features of such policies in actual large-value payment systems. The first such policy is that of costly enforcement and pricing. The central bank invests in a costly enforcement technology that allows it to punish defaulters by confiscating some consumption goods. The second policy is that of requiring those who borrow from the central bank to post collateral. Under this policy, the central bank does not charge an explicit interest rate on debt. Collateral, however, may have an opportunity cost in that it cannot earn a return that it otherwise would have.

I use a mechanism design approach to see if the credit policies can achieve good allocations, which I define to be Pareto-optimal allocations. It is possible for both types of credit policies to implement these good allocations. In the case of the pricing policy, I find an example of where the optimal intraday interest rate is positive because of a requirement for the central bank to recover its costs of enforcement. This differs from the aforementioned literature and supports a

[4] Martin (2003) is an exception. See below.

suggestion made by Rochet and Tirole (1996) that the intraday interest rate be positive because monitoring and enforcement is costly. In the case of collateral, if it does not have an opportunity cost, such a policy can implement a good allocation that is first-best. If, on the other hand, there is a positive opportunity cost of collateral, requiring collateral adds binding incentive constraints that distort the allocation away from Pareto optimality. Collateral serves as an endogenous credit constraint.

This paper is most closely related to Martin (2003). Both papers are interested in evaluating how alternative credit policies address participants' moral hazard in a general equilibrium model where money is necessary. Martin (2003) models moral hazard by endogenizing some agents' choice of risk arising from a central bank's free provision of liquidity. Agents can choose a safe production technology or a risky one that exogenously leads to some default and central banks cannot enforce a choice of the safe asset. In his model, agents cannot strategically default. In this paper, agents do not have an opportunity to engage in risky behavior, but rather have the choice to strategically default. The central bank can enforce some repayment only after investing in a costly enforcement technology.

Martin (2003) compares alternative central bank credit policies and concludes that a collateral policy with a zero intraday interest rate is preferred to debt limits in mitigating the credit risk. The collateral in his model is debt issued by private agents who exogenously commit to repayment (i.e. there is no choice to strategically default) and does not bear an opportunity cost. In this paper, the issuers of collateral do have the opportunity to strategically default and collateral does bear an opportunity cost. Moreover, Martin (2003) does not consider costly monitoring of agents receiving central bank liquidity and how that might compare to a collateral policy.

Finally, in this paper the default decision of agents is endogenous, but the liquidity shortage is exogenous. This is complementary to an area in the literature by Bech and Garratt (2003), Angelini (1998) and Kobayakawa (1997). These papers endogenize the liquidity shortage by focusing on the incentives agents have to coordinate the timing of payments given alternative credit policies, but do not endogenize the need for such credit policies.

The paper is organized as follows. Section 2 presents the environment while Section 3 provides a benchmark of optimal allocations. Sections 4 and 5 contain the main results as pertains to the credit policy with pricing and collateral, respectively. Section 6 extends the analysis to include exogenous default and central bank losses. Section 7 concludes.

2. THE ENVIRONMENT

The model is a variation of both Freeman (1996) and Mills (2004). It is a pure exchange endowment model of two-period-lived overlapping generations with two goods at each date, *good 1* and *good 2*. The economy starts at date $t = 1$. There is a $[0, 1]$ continuum of each of two types of agents, called *creditors* and *debtors*, born at every date.[5] These two types are distinguished by their endowments and preferences.

Each creditor is endowed with y units of good 1 when young and nothing when old. Each debtor is endowed with x units of good 2 when young and nothing when old.

Let $c^t_{zt'}$ denote consumption of good $z \in \{1, 2\}$ at date t' by a creditor of

[5] The name given to creditors is a bit misleading because these agents never lend in equilibrium.

generation t. The utility of a creditor is $u(c^t_{1t}, c^t_{2,t+1})$, where $u : \Re^2_+ \rightarrow \Re$. Notice that a creditor wishes to consume good 1 when young and good 2 when old. The function u is strictly increasing and concave in each argument, is C^1, and $u'(0) = \infty$ and $u'(\infty) = 0$.

Let $d^t_{zt'}$ denote consumption of good $z \in \{1, 2\}$ at date t' by a debtor of generation t. The utility of a debtor born at date t is $v(d^t_{1t}, d^t_{2t})$ where $v : \Re^2_+ \rightarrow \Re$. Hence, a debtor wishes to consume both good 1 and good 2 when young. A debtor does not wish to consume either good when old. The function v is strictly increasing and concave in each argument, is C^1, and $v'(0) = \infty$ and $v'(\infty) = 0$.

At date $t = 1$, there is a [0,1] continuum of initial old creditors. These creditors are each endowed with M divisible units of fiat money.

It is assumed that agents cannot commit to trades and that there is no public memory of trading histories. It is also assumed that agents do not consume any goods until the end of the period.

There is also an institution called a central bank that has three technologies unique to it.[6] The first technology is the ability to print fiat money. The second is a record-keeping technology that enables the central bank to keep track of individual balances of both money and goods that a private agent may have with it. The third technology is an enforcement technology that can be acquired at a real resource cost $\gamma > 0$ per period.[7] The enforcement technology allows the central bank to punish defaulters by confiscating goods. The resource cost γ can be thought of as the cost of monitoring and the use of channels to confiscate goods to satisfy repayment.

[6] What I call a central bank may also be interpreted as a private clearinghouse that is separate from the other agents. As noted in Green (1997), it remains an open question as to whether the liquidity-providing institution in the model should be a public or private one and is beyond the scope of this paper.

[7] In Freeman (1996), $\gamma = 0$.

There are four stages within a period. At the first stage, young debtors meet the central bank. As we shall see, young debtors may seek liquidity from the central bank at this time. At the second stage, young debtors and young creditors meet. This is the only opportunity for young debtors to acquire good 1. At the third stage, young debtors and old creditors meet. This is the only opportunity for old creditors to acquire good 2. Finally, at the fourth stage, young debtors are reunited with the central bank. At this time, young debtors have an opportunity to repay the central bank for any liquidity provided by it at the first stage.

Debtors are endowed with an investment technology that allows them to invest some of their endowment ($I \leq x$) at the end of the first stage, that yields with certainty, RI units of good 2 at the beginning of the third stage, where $R \geq 1$. The sequence of events for each date is summarized in Figure 1.

The setup captures some key elements of large-value RTGS payment systems. In such systems, banks use funds in central bank accounts to make payments. In the model, fiat money (in the form of currency) is necessary as a means of payment if trade is to take place because of the timing of trading opportunities within a period and the fact that there is no commitment and no public memory.[8] In actual RTGS systems, banks may face a liquidity problem during the day because of the mismatch between payments received and payments made. In central bank-operated systems, banks may borrow funds from the central bank by overdrawing on their accounts to make payments. The liquidity problem is approximated in the model via the timing of events within a period; because young debtors are not endowed with fiat money they must first acquire some via a credit relationship with the central bank. In actual central-bank operated RTGS systems, overdrafts are

[8] See Kocherlakota (1998) for a general discussion and Mills (2004) for one in the context of this type of model.

repaid via deposits into central bank accounts. This is approximated in the model by requiring young debtors to repay the central bank at the final stage within a period so that it may retire the same amount of money that it injected into the economy at the beginning of the period. Moreover, as in Townsend (1989) money serves as a communication device that signals to the central bank the past behavior of debtors. Thus, money is essential for the repayment of debt in the model.

The model does abstract from explicitly modelling banks as intermediaries that make payments on behalf of customers. What is important for the analysis, however, is that the model contain a liquidity problem in the payment system that can be alleviated by the central bank. By not modeling banks explicitly, I am assuming that inefficiencies arising from an interbank payment system have real effects on the economy.

3. BENCHMARK: OPTIMAL ALLOCATIONS

Before describing the alternative credit policies, I first define some optimal allocations. A first-best allocation is one that maximizes ex-ante expected steady-state utility of debtors and creditors subject to a limited set of feasibility constraints. This limited set abstracts from incentive constraints which will be important for implementation.

Denote the steady state levels of consumption of both good 1 and good 2 by d_z for a debtor and c_z for a creditor for $z \in \{1, 2\}$. The problem is then to maximize

$$u(c_1, c_2) + v(d_1, d_2) \tag{1}$$

with respect to I, d_1, d_2, c_1, c_2 and subject to the following feasibility constraints:

$$x \geq I \tag{2}$$

$$y \geq d_1 + c_1 \tag{3}$$

$$RI + (x - I) \geq d_2 + c_2. \tag{4}$$

Define u_z as the partial derivative of creditor utility with respect to good z and v_z as the partial derivative of debtor utility with respect to good z for $z \in \{1, 2\}$. Optimal allocations require that (2)-(4) are satisfied at equality. The first order conditions, then, which satisfy the Kuhn-Tucker conditions for necessity and sufficiency simplify to:

$$\frac{u_1}{u_2} = \frac{v_1}{v_2} \tag{5}$$

Condition (5) states that optimal allocations are those that are Pareto optimal. Thus, in what follows, I shall look for implementable allocations (ones that take into account the incentives of agents) that satisfy (5).

4. LIQUIDITY PROVISION WITH COSTLY ENFORCEMENT AND PRICING

In this section, I provide an example of a payment mechanism where a central bank provides liquidity with a credit policy of paying a real cost $\gamma > 0$ for the enforcement technology and charging an intraday interest rate (or price) for liquidity. I characterize a set of implementable allocations via the mechanism as those that satisfy a set of incentive constraints. Allocations are implementable if they are subgame perfect equilibrium allocations. Finally, I show that the second-best optimal allocation that is implementable via the pricing mechanism is Pareto-optimal.

Recall that investment in the enforcement technology enables the central bank to confiscate goods from a defaulting debtor. The central bank can effectively choose some combination of goods 1 and 2 to confiscate so that, in equilibrium, debtors will choose not to default.[9] This costly enforcement is meant to model the opportunities a central bank may have when it monitors the behavior of payment system participants. The exogenous parameter, γ, is a proxy for the real cost of monitoring agents and the costs associated with the potential liquidation of assets in the event of a default.

The central bank charges an intraday interest rate, $r \geq 0$ proportional to the amount borrowed. The interest payment is payable in units of good 2. This is convenient so as to provide an easy comparison with the collateral policy of the next section. Some central banks, such as the Federal Reserve, have a mandate to fully recover costs of the operation of their payment services. Such an assumption in the context of the model is that the central bank charge an intraday interest rate such that the nominal value of the interest payment be at least as large as the nominal value of the cost of enforcement. I shall describe the payment game that agents play under the assumption that the central bank must fully recover its costs.

The game is as follows for any date t. At the first stage of a period, generation-t debtors choose whether or not to seek liquidity from the central bank. Those that seek such credit acquire M units. Let $\delta_1^t \in [0,1]$ be the fraction of debtors who seek credit from the central bank. The generation-t debtor then invests the entire amount of good 2 ($I = x$). At the second stage, the mechanism suggests that generation-t creditors who want to participate in exchange each offer $\overline{d_1}$ of good 1 and that generation-t debtors who want to participate offer M units of fiat

[9] In Section 6, I shall consider the possibility that some debtors may exogenously default.

money. The creditors and debtors simultaneously choose whether to participate in exchange or not. Let $\kappa_2^t \in [0,1]$ be the fraction of generation-t creditors who agree to offer $\overline{d_1}$ of good 1 and $\delta_2^t \in [0, \delta_1^t]$ be the fraction of generation-t debtors who agree to exchange M units of money for some consumption of good 1. Each debtor who agrees to trade M units of money for consumption receives $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1. Each creditor that agrees then receives $\frac{\delta_2^t}{\kappa_2^t}M$ units of money and has $c_1^t = y - \overline{d_1}$ units of good 1 left for consumption. Those that disagree leave with autarky.

At the third stage of date t, the generation-t debtors' investments pay off and each now has Rx units of good 2. The mechanism then suggests that generation-t debtors who want to participate in exchange each offer $\overline{c_2}$ of good 2 and that generation-$t-1$ creditors who have $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and who want to participate in exchange offer up $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. Generation-$t-1$ creditors with no money are not able to participate in exchange. The debtors and those creditors able to participate in exchange simultaneously choose to participate or not. Let $\delta_3^t \in [0, \delta_2^t]$ be the fraction of generation-t debtors who agree to offer $\overline{c_2}$ of good 2 and $\kappa_3^{t-1} \in [0, \kappa_2^{t-1}]$ be the fraction of generation-$t-1$ creditors who agree to exchange money for some consumption of good 2. Each creditor who agrees to trade money for consumption receives $\frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}$ units of good 2. Each debtor that agrees then receives $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and has $Rx - \overline{c_2}$ units of good 2 left over. Those that disagree leave with autarky.

At the final stage, if a generation-t debtor who has borrowed money at the first stage now has $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money, then the debtor may choose to repay the central bank $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money plus an interest payment, i_2, in units of good 2. The nominal value of the interest payment is determined by the nominal value of the amount borrowed. Specifically, let p_1 and p_2 represent the prices of goods 1 and 2 in terms of money, respectively. Then $p_2 i_2 = rM = rp_1 \frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ or $i_2 = r\frac{p_1}{p_2}\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$. I can express i_2 in terms of good 2, $i_2 = r\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2}$ by noting that the nominal value of good 1 acquired by generation-$t-1$ creditors is $p_2\frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2} = \frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ so that $p_2\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2} = M = p_1\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ where the latter equality represents the nominal value of good 1 acquired by the generation-t debtors. The central bank then removes the $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money from circulation and the debtor has $d_2^t = Rx - \overline{c_2} - r\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2}$ units of good 2 for consumption. If the young debtor does not have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and does not offer $i_2 = r\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2}$ units of good 2 to the central bank, then that debtor is punished by surrendering the amount of good 1 he acquired at stage 2.

Notice that equations (2)-(3) are satisfied at equality by the mechanism but that (4) is not because $i_2 = r\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2} \geq \gamma > 0$ represents a real resource cost borne by the private agents. I first characterize the set of allocations that are implementable via the payment mechanism with pricing.

PROPOSITION 1. *A steady-state allocation is implementable if it satisfies the following participation constraints:*

$$v[y - c_1, Rx - (1+r)c_2] \geq v[0, Rx] \tag{6}$$

for debtors and

$$u[c_1, c_2] \geq u[y, 0] \tag{7}$$

for creditors.

The proof of Proposition 1 is in the appendix. Proposition 1 gives two simple conditions that an allocation must meet for it to be implementable. They essentially ensure that both debtors and creditors wish to participate in exchange. The next proposition characterizes the second-best optimal allocation via the payment mechanism with pricing and shows that it is Pareto-optimal. The optimal allocation is always second-best because the enforcement technology combined with the cost-recovery constraint reduces the amount of good 2 available to the agents.

PROPOSITION 2. *The optimal allocation implementable via the payment mechanism with pricing satisfies (5).*

Proof. The optimization problem can be written as maximizing (1) with respect to I, d_1, d_2, c_1, c_2, r and subject to (2)-(3), (6)-(7), and

$$RI + (x - I) \geq d_2 + (1 + r)c_2 \tag{8}$$

$$rc_2 \geq \gamma \tag{9}$$

where (8) replaces (4) and (9) is the cost recovery constraint of the central bank. Given that (2)-(3) and (8)-(9) hold at equality, and substituting these relationships into the optimization problem, the first order conditions, which satisfy the Kuhn-Tucker conditions for necessity and sufficiency simplify to

$$\frac{(1+\lambda_d)v_1}{(1+\lambda_d)v_2} = \frac{(1+\lambda_c)u_1}{(1+\lambda_c)u_2} \tag{10}$$

$$\lambda_c\{u[c_1, c_2] - u[y, 0]\} = 0 \tag{11}$$

$$\lambda_d\{v[y - c_1, Rx - \gamma - c_2] - v[0, Rx]\} = 0 \tag{12}$$

$$\lambda_c, \lambda_d \geq 0 \tag{13}$$

where λ_c and λ_d are the multipliers for the creditor and debtor participation constraints, respectively. Inspection of (10) reveals that (5) is satisfied regardless of whether the participation constraints (6) and (7) bind or not. ∎

While the intraday interest rate may influence whether or not debtors participate in trade (for trade to take place at all under this credit policy, it is important that $r = \frac{\gamma}{c_2}$ is not too high that constraint 6 is violated), it does not create a wedge between the ratios of marginal rates of substitution and so is Pareto-optimal. This is because debtors do not have to pay the interest rate until stage 4 so that the cost in terms of good 2 can be shared among debtors and creditors.

One interpretation of $r = \frac{\gamma}{c_2} > 0$ is that it is the optimal risk-free intraday interest rate. This is because (i) a positive interest rate is necessary for central bank liquidity provision (because of the cost recovery constraint) and (ii) investment in the enforcement technology eliminates the risk that a debtor defaults. This is a departure from the case where $r = 0$ (free intraday liquidity), which has been found to be optimal in papers such as Freeman (1996), Green (1997), Zhou (2000) Kahn and Roberds (2001) and Martin (2003). In each of those cases, it is implicitly assumed that $\gamma = 0$ so that there was no social cost attached to providing intraday liquidity. The positive optimal risk-free interest rate found here supports a recommendation suggested by Rochet-Tirole (1996) that costly monitoring of agents is necessary and liquidity providers should be compensated.

5. LIQUIDITY PROVISION WITH COLLATERAL

In this section, I provide an example of a payment mechanism where a central bank provides liquidity with a credit policy of requiring collateral. As in the previous section, I characterize a set of implementable allocations via the mechanism as those that satisfy a set of constraints. Finally, I show that the second-best optimal allocation implementable via the collateral policy is not Parteo optimal if collateral bears an opportunity cost. The first-best optimal allocation is achieved, however, if there is no opportunity cost to posting collateral.

The young debtors, when they seek liquidity from the central bank, pledge some of their endowment of good 2 as collateral which they will then buy back from the central bank at the end of the period (during the fourth stage). Recall that young debtors can invest their endowment of good 2 and receive a certain return of $R \geq 1$. Because the amount of good 2 they pledge is transferred to the central bank, there is an opportunity cost in that the collateral is no longer available to invest whenever $R > 1$.

In terms of actual large-value payment systems, one can think of the opportunity cost of collateral in the following way.[10] Suppose that participants of the system can post only a limited set of assets as collateral. These assets are generally viewed as safe from the point of view of the liquidity-provider. Typically, such safe assets have lower (expected) returns. To the extent that participants seeking intraday liquidity hold more of these safe assets than they otherwise would without the need to post them as collateral, one could argue that there is an opportunity cost to pledging collateral.

The game is as follows for any date t. At the first stage of a period, generation-t debtors choose whether or not to seek liquidity from the central bank. Those that seek such credit acquire M units and deposit $\sigma \leq x$ units of good 2 at the central bank as collateral. Generation-t debtors then invest their remaining supply of good 2 ($I = x - \sigma$). Let $\delta_1^t \in [0, 1]$ be the fraction of debtors who seek credit from the central bank. At the second stage, the mechanism suggests that generation-t creditors who want to participate in exchange each offer $\overline{d_1}$ of good 1 and that

[10] Zhou (2000) also makes this argument.

generation-t debtors who want to participate offer M units of fiat money. The creditors and debtors simultaneously choose whether to participate in exchange or not. Let $\kappa_2^t \in [0,1]$ be the fraction of generation-t creditors who agree to offer $\overline{d_1}$ of good 1 and $\delta_2^t \in [0, \delta_1^t]$ be the fraction of generation-t debtors who agree to exchange M units of money for some consumption of good 1. Each debtor who agrees to trade M units of money for consumption receives $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1. Each creditor that agrees then receives $\frac{\delta_2^t}{\kappa_2^t}M$ units of money and has $c_1^t = y - \overline{d_1}$ units of good 1 left for consumption. Those that disagree leave with autarky.

At the third stage of date t, the generation-t debtors' investments pay off and each now has $R(x-\sigma)$ units of good 2 available at this stage. The mechanism then suggests that generation-t debtors who want to participate in exchange each offer $\overline{c_2} \leq R(x-\sigma)$ of good 2 and that generation-$t-1$ creditors who have $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and who want to participate in exchange offer up $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. Generation-$t-1$ creditors with no money are not able to participate in exchange. The debtors and those creditors able to participate in exchange simultaneously choose to participate or not. Let $\delta_3^t \in [0, \delta_2^t]$ be the fraction of generation-t debtors who agree to offer $\overline{c_2}$ of good 2 and $\kappa_3^{t-1} \in [0, \kappa_2^{t-1}]$ be the fraction of generation-$t-1$ creditors who agree to exchange money for some consumption of good 2. Each creditor who agrees to trade money for consumption receives $\frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}$ units of good 2. Each debtor that agrees then receives $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money and has $R(x-\sigma) - \overline{c_2}$ units of good 2 left. Those that disagree leave with autarky.

At the final stage, if a generation-t debtor who has borrowed M units at the first stage, now has $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money, then the debtor may choose to repay the central bank $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money in exchange for the return of the σ units of good 2 that served as collateral. The central bank then removes the $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$

units of money from circulation and the debtor has $d_2^t = R(x-\sigma) - \overline{c_2} + \sigma$ units of good 2 for consumption. If the young debtor does not have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money, then the central bank does not return the collateral.

Notice that (3) and (4) are satisfied at equality by the mechanism but that (2) is not when there is an opportunity cost of collateral ($R > 1$). Rather $I = x - \sigma$. The opportunity cost of collateral is then $(R-1)\sigma$ which is the difference between Rx and $R(x-\sigma)+\sigma$. There is also an additional feasibility constraint that requires $c_2 \leq R(x-\sigma)$. This constraint reflects the fact that the amount of good 2 that generation-$t-1$ creditors can consume must be less than the total amount available at the third stage.

I now characterize the set of allocations that are implementable via the payment mechanism with collateral.

Proposition 3. *A steady-state allocation is implementable if it satisfies the following incentive constraints:*

$$v[y-c_1, R(x-\sigma)-c_2+\sigma)] \geq v[0, Rx] \tag{14}$$

$$v[y-c_1, R(x-\sigma)-c_2+\sigma)] \geq v[y-c_1, R(x-\sigma)] \tag{15}$$

for debtors and

$$u[c_1, c_2] \geq u[y, 0] \tag{16}$$

for creditors.

The proof of Proposition 3 is in the appendix. Compared with Proposition 1, Proposition 3 has an additional incentive constraint beyond participation. This constraint, (15), essentially requires that the amount of collateral that a debtor buys back from the central bank must be at least as much as the amount of good 2 a creditor is expected to receive ($\sigma \geq \overline{c_2}$). Otherwise, a debtor, after acquiring some of good 1, would prefer not to exchange with old creditors to acquire money and so default on his debt to the central bank.

The following proposition states that the payment mechanism under a credit policy with collateral cannot achieve Pareto-optimal allocations when there is an opportunity cost of collateral.

PROPOSITION 4. *When $R > 1$, the optimal allocation implementable via the payment mechanism with collateral does not satisfy (5).*

Proof. The optimization problem can be written as maximizing (1) with respect to $I, d_1, d_2, c_1, c_2, \sigma$ and subject to (3)-(4), (14)-(16), and

$$x - \sigma \geq I \tag{17}$$

$$RI + (x - \sigma - I) \geq c_2. \tag{18}$$

where (17) replaces (2) from the benchmark problem and (18) is an additional feasibility constraint for stage 3. Given that (3),(4), and (17) will hold at equality, and substituting these relationships into the optimization problem, the first order conditions, which satisfy the Kuhn-Tucker conditions for necessity and sufficiency simplify to

$$
\begin{aligned}
\frac{(1+\lambda_d)v_1}{(1+\lambda_2+\lambda_d)v_2+\lambda_1} &= \frac{(1+\lambda_c)u_1}{(1+\lambda_c)u_2} && (19)\\
(1+\lambda_d)v_2(R-1)+\lambda_1 R &= \lambda_2 v_2 && (20)\\
\lambda_1\{R(x-\sigma)-c_2\} &= 0 && (21)\\
\lambda_2\{v[d_1,R(x-\sigma)-c_2+\sigma)]-v[d_1,R(x-\sigma)]\} &= 0 && (22)\\
\lambda_c\{u[c_1,c_2]-u[y,0]\} &= 0 && (23)\\
\lambda_d\{v[y-c_1,R(x-\sigma)-c_2+\sigma]-v[0,Rx]\} &= 0 && (24)\\
\lambda_1,\lambda_2,\lambda_c,\lambda_d &\geq 0 && (25)
\end{aligned}
$$

where λ_1 is the multiplier for (18), λ_2 is the multiplier for (15), and λ_c and λ_d are the multipliers for the creditor and debtor participation constraints, (14) and (16), respectively. Condition (20) is the first-order condition with respect to σ.

For (19) to equal (5), it must be the case that $\lambda_1=\lambda_2=0$ which is the case if (15) and (17) do not bind. If $\lambda_1=\lambda_2=0$, then (20) reduces to $(1+\lambda_d)v_2(R-1)=0$ implying that $v_2=0$, which violates the assumptions about debtor preferences.[11] Therefore, a solution to this optimization problem cannot have both λ_1 and λ_2 be equal to 0. ∎

The intuition for Proposition 4 is as follows. Because there is an opportunity cost to pledging collateral, a solution to the optimization problem should minimize the amount of collateral required. For such an allocation to be incentive feasible for debtors, $\sigma \geq \overline{c_2}$. Thus, an optimal allocation should have $\sigma=\overline{c_2}$ so that (15) binds. But if (15) binds, then it turns out that debtors are credit constrained.

[11] $v_2=0$ if and only if $d_2=\infty$ which is not feasible.

That is to say they cannot borrow "enough" from the central bank to acquire the desired amount of good 1 from young creditors. Thus, when collateral bears an opportunity cost, it serves as an endogenous credit constraint. This result is consistent with other papers on the use of collateral, such as Lacker (2001).

Given that the constraint (15) binds, the central bank credit is fully collateralized. To see this, note that $p_1 d_1 = M$, i.e., the nominal value of debtor consumption of good 1 equals the money supply. This is also true of the nominal value of creditor consumption of good 2, $p_2 c_2 = M$. Because $\sigma = c_2$, we have $p_1 d_1 = p_2 \sigma$, or the nominal value of the amount of good 1 financed by credit from the central bank equals the nominal value of the collateral.

Finally, it is worth exploring the case when there is no opportunity cost of collateral. This may be the case in actual large-value payment systems when the central bank accepts a wide range of assets as collateral, mitigating the need to have an asset portfolio with a heavier than optimal weight on safe assets.

PROPOSITION 5. *When $R = 1$, the optimal allocation implementable via the payment mechanism with collateral satisfies (5) if the allocation has $\overline{c_2} < \frac{x}{2}$.*

Proof. When $R = 1$, the first order conditions from Proposition 4 simplify to

$$\frac{(1+\lambda_d)v_1}{(1+\lambda_2+\lambda_d)v_2+\lambda_1} = \frac{(1+\lambda_c)u_1}{(1+\lambda_c)u_2} \tag{26}$$

$$\lambda_1 = \lambda_2 v_2 \tag{27}$$

$$\lambda_1\{x-\sigma-c_2\} = 0 \tag{28}$$

$$\lambda_2\{v[d_1, x-c_2] - v[d_1, x-\sigma]\} = 0 \tag{29}$$

$$\lambda_c\{u[c_1, c_2] - u[y, 0]\} = 0 \tag{30}$$

$$\lambda_d\{v[y-c_1, x-c_2] - v[0, x]\} = 0 \tag{31}$$

$$\lambda_1, \lambda_2, \lambda_c, \lambda_d \geq 0 \tag{32}$$

As before, I need $\lambda_1 = \lambda_2 = 0$ which is the case if (15) and (17) do not bind. Such a condition does not violate (27) and is met when $\overline{c_2} < \sigma < x - \overline{c_2}$ or $\overline{c_2} < \frac{x}{2}$. ∎

This gives sufficient conditions for which the debtor incentive constraint (15) does not bind. In this case, because there is no opportunity cost of collateral, the optimum does not require σ to be small. Thus it is possible to choose from a range of σ that does not lead to any credit constraints.

Notice that when there is no opportunity cost of collateral, the Pareto-optimal allocations are first-best. This is because the use of collateral in this case does not add any additional social cost. Only a subset of such allocations, however, are achievable because of the need to satisfy the incentive constraint of debtors.

6. EXOGENOUS DEFAULT

Up to this point in the analysis, the only type of default that is possible is strategic default. As a result, the central bank is assured of no equilibrium credit losses under either policy because both effectively address the moral hazard issues associated with the repayment of debt. The banks that use central bank liquidity, however, are typically complex financial institutions[12]. Although credit policies may be designed so that a payment system does not provide an incentive for default, there may be other factors exogenous to the system that could lead a bank to fail to repay its debt, potentially leading to central bank losses. For example, a bank could become insolvent prior to repaying the central bank. In addition, the monitoring and enforcement technology that the central bank employs in the pricing policy may be less effective than has been assumed here. It may not be able to completely identify institutions that are more likely to become insolvent, and in the event of a liquidation, the central bank may face uncertainty about its claims.

In this section, I extend the model to capture these concerns by introducing exogenous default. Specifically, assume that between stages 3 and 4 within a period, a debtor receives a shock with probability ε that he loses all of the money and goods that he has in his possession. With probability $1-\varepsilon$, the debtor enters stage 4 as previously assumed, with everything he had at the end of stage 3. Neither the agents nor the central bank know which debtors will receive the shock until after it is realized.

If the debtor receives the shock, he is unable to repay the central bank. Under the pricing policy, the central bank is unable to recover anything from an exogenously defaulting debtor. Under the collateral policy, the central bank keeps the collateral.

I choose to model exogenous default this way because I wish to focus exclusively on the prospect of central bank losses. As noted earlier, the extension of central

[12] See Bliss (2003) for a discussion of the complexities of bankruptcy procedures for financial institutions.

bank liquidity arises in large-value payment systems that are real-time gross settlement systems with payment finality. Payment finality is a guarantee by the central bank that once a payment is made from one party to another, it cannot be undone. In the context of this paper, this means that a debtor defaults only on the central bank at stage 4 and not on old creditors at stage 3. Thus, I model exogenous default so that the incentive constraints that must be satisfied for implementation under either policy remains unchanged. To see this note that at stage 4, with probability $1-\varepsilon$, a debtor faces the same decision as to whether to repay the central bank as before, while with probability ε he receives nothing and so defaults. Thus, in each preceding stage, a debtor's incentive constraint takes the following form

$$(1-\varepsilon)v[\text{accepting proposed trade}]+\varepsilon v[0,0] \geq (1-\varepsilon)v[\text{rejecting proposed trade}]+\varepsilon v[0,0]$$

which reduces to the debtor incentive constraints in Propositions 1 and 3 and their proofs.

Under the collateral policy, the central bank is protected fully from losses because, as shown in Section 5, borrowing is fully collateralized.[13] Under the pricing policy, however, the central bank must recover its costs. The presence of exogenous default changes the cost recovery constraint of the central bank, (9), to

$$(1-\varepsilon)rc_2 \geq \gamma \tag{33}$$

[13]This is a bit contrived because it is assumed that there is no real cost to the central bank of suffering a default under the collateral policy. This may not be true in practice, for example, because a default could lead to an unanticipated increase in the money supply. The selling of collateral can help undo this unanticipated increase and so reduce or eliminate the cost of default.

which, given that the constraint will bind, translates to an intraday interest rate

$$r = \frac{\gamma}{(1-\varepsilon)c_2}. \tag{34}$$

Exogenous default, therefore, adds a risk premium to the risk-free intraday interest rate for a given level of creditor consumption of good 2. Alternatively, should the central bank wish to continue to charge the risk-free rate, implementable allocations would have a higher level of creditor consumption of good 2 and, by (5) would lead to a lower level of debtor consumption of good 1. In other words, exogenous default leads to "less" borrowing by debtors in terms of the amount of good 1 they can buy.

Moreover, exogenous default tightens the debtor incentive constraint, (6). If the probability of default is high enough then the pricing policy could lead to a distortion that is more serious than that generated by the collateral policy when collateral has an opportunity cost; access to central bank liquidity would be shut down and there would be no trade. While this result may seem extreme, the point to take from the analysis is that less effective monitoring and enforcement powers of the central bank may lead to a rationing of access to central bank liquidity under the pricing policy which may be distortionary and perhaps even more distortionary than what is generated by the collateral policy when collateral bears an opportunity cost.[14]

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[14] I thank an annoymous referee for pointing this out.

7. CONCLUSION

The above analysis sheds some light on why different central banks may have different credit policies for RTGS systems. Collateral is preferred if there is no opportunity cost of collateral, such as may be the case when a wide range of assets are accepted as collateral. This is because it can achieve first-best allocations. If collateral does have an opportunity cost, comparison of the relative cost (in terms of good 2 in the model) is important. For example, the European Central Bank does not have monitoring authority over participating banks. Thus, it may be difficult to coordinate monitoring and enforcement authorities. In the context of the model, this is a high enough γ so that collateral may be the preferred option. On the other hand, the Federal Reserve already has supervisory authority over depository institutions it serves over Fedwire, so that economies of scope are likely to yield a low γ so that pricing may be the preferred option. In the case where the cost of both policies would be the same ($\gamma = (R-1)\sigma$), the pricing policy would clearly be preferred due to the result that collateral adds a binding endogenous borrowing constraint that does not permit a Pareto-optimal allocation.

Another credit policy tool that has not been modeled here is that of setting limits or caps to the amount a debtor can borrow. The Federal Reserve, for example, sets net debit caps that limit the amount that Fedwire participants can borrow to limit the Fed's exposure to credit risk. In the context of the model, such binding constraints could have a similar effect on the pricing policy as what takes place under exogenous default when the central bank held the intraday interest rate fixed at the risk-free rate. That is, it would reduce a debtor's consumption of good 1, but Pareto-optimal allocations may still be achievable. This would simply reduce the set of Pareto-optimal outcomes that would be achievable at the expense of debtors and for the gain of creditors.

The results of the paper suggest that the existence of an opportunity cost of collateral is key to that type of credit policy leading to inefficient allocations. Thus, it is important to empirically understand whether or not there is an effective opportunity cost of intraday collateral.

Collateral in this model is riskless to the central bank. An extension might involve the introduction of a range of collateral that varies according to riskiness. This complicates matters in that the central bank may have to decide what types of risky assets are acceptable as collateral. The conjecture here is that as a central bank accepts a wider range of assets, the opportunity cost to the participant of posting collateral is less, but collateral provides less protection to the central bank in the event of defaults unless the value of the collateral is discounted appropriately.

Finally, this paper restricts itself only to two credit policies designed to replicate actual central bank policies. A more generalized study may reveal that a third policy may be more appropriate especially when some of the aforementioned complications are present in the model.

APPENDIX A: PROOFS

Proof of Proposition 1. The proof solves for subgame perfect equilibria of the game via backwards induction. The equilibria are those where every agent agrees at every stage.

Begin with stage 4 within a period at date t. Generation-t debtors who have agreed up to this stage have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. They will choose to return the money and pay i_2 units of good 2 if

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, Rx - c_2 - r\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2}] \geq v[0, Rx - c_2]. \tag{35}$$

Note that the right-hand side of (35) represents utility after the central bank confiscates the debtor's amount of good 1 he previously acquired.

Now turn to stage 3. A creditor from generation $t-1$ enters this stage with either $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ or 0 units of money which is private information. Suppose that all other agents agree in the third stage. If the creditor does not have any money then she cannot trade. If she has $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money then it is trivial that she will want to agree to trade as well because

$$u[y - \overline{d_1}, \frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}] \geq u[y - \overline{d_1}, 0]. \tag{36}$$

Thus, $\kappa_3^{t-1} = \kappa_2^{t-1}$.

A generation-t debtor enters the third stage with either $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ or 0 units of good 1, which is private information. Suppose that all other agents that can participate in trade will agree in the third stage. If the debtor has $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1, he will also agree if

$$\max\{v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, Rx - \overline{c_2} - r\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2}], v[0, Rx - c_2]\} \geq v[0, Rx]. \tag{37}$$

The left hand side of the expression represents a debtor's stage 4 decision. The right hand side takes into account the fact that if a debtor disagrees, he will not receive money and will then be punished by losing $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ at stage 4. Because

$v[0, Rx - c_2]\} < v[0, Rx]$ (37) reduces to

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, Rx - \overline{c_2} - r\frac{\delta_3^t}{\kappa_3^{t-1}}\frac{\kappa_2^{t-1}}{\delta_2^{t-1}}\overline{c_2}] \geq v[0, Rx] \tag{38}$$

and (35) is satisfied if (38) is satisfied. If the debtor has none of good 1, it is trivial that he chooses not to agree to trade.

Now consider an arbitrary generation-t debtor at stage 2. If the debtor disagrees at this stage, he enters the third stage with 0 units of good 1 and will disagree in the third stage as well. Thus, he will receive only autarkic utility, $v[0, Rx]$. If the debtor agrees when everyone else does, then his second-stage participation constraint is identical to his third-stage participation constraint, (38). This implies that all of the generation-t debtors who agree at stage 2 will also agree at stage 3, that is, $\delta_3^t = \delta_2^t$.

Now, consider an arbitrary generation-$t-1$ creditor at the second stage of date $t-1$. If the generation $t-1$ creditor disagrees at the second stage when young, she enters the third stage when old with no money and, therefore, receives autarkic utility, $u[y, 0]$. She also knows that $\delta_3^t = \delta_2^t$. If she agrees at the second stage of date $t-1$, she will enter the third stage of date t with money and agree so that she receives $u[y - \overline{d_1}, \frac{\delta_2^t}{\kappa_3^{t-1}}\overline{c_2}]$. She will agree if

$$u[y - \overline{d_1}, \frac{\delta_2^t}{\kappa_3^{t-1}}\overline{c_2}] \geq u[y, 0] \tag{39}$$

where δ_2^t is substituted for δ_3^t.

Finally, consider generation-t debtors at stage 1 of date t. Here, if all other debtors agree to borrowing from the central bank, then $\delta_1^t = \delta_2^t = \delta_3^t = 1$ and an arbitrary debtor also agrees if

$$v[\kappa_2^t \overline{d_1}, Rx - \overline{c_2} - r\frac{\kappa_2^{t-1}}{\kappa_3^{t-1}}\overline{c_2}] \geq v[0, Rx] \tag{40}$$

which turns out to be the debtor participation constraint for stage 2 and stage 3. As a result, $\kappa_2^{t-1} = \kappa_2^t = \kappa_3^{t-1} = 1$ because (39) is now satisfied because (7) is satisfied by hypothesis. Thus, generation-t debtor constraints at stages 2 and 3 reduce to (6), the stage 1 generation-t constraint is then trivially satisfied so that $\delta_4^t = 1$, and all nontrivial creditor constraints reduce to (7), both of which are satisfied. ∎

Proof of Proposition 3. The proof solves for subgame perfect equilibria of the game via backwards induction. The equilibria are those where every agent agrees at every stage.

Begin with stage 4 within a period at date t. Generation-t debtors who have agreed up to this stage have $\frac{\kappa_3^{t-1}}{\delta_3^t}\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money. They will choose to return the money in exchange for collateral if

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma) - c_2 + \sigma] \geq v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma) - c_2] \tag{41}$$

which trivially holds.

Now turn to stage 3. A creditor from generation $t-1$ enters this stage with either $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ or 0 units of money which is private information. Suppose that all other agents agree in the third stage. If the creditor does not have any money then she cannot trade. If she has $\frac{\delta_2^{t-1}}{\kappa_2^{t-1}}M$ units of money then it is trivial that she will want to agree to trade as well because

$$u[y - \overline{d_1}, \frac{\delta_3^t}{\kappa_3^{t-1}}\overline{c_2}] \geq u[y - \overline{d_1}, 0]. \tag{42}$$

Thus, $\kappa_3^{t-1} = \kappa_2^{t-1}$.

A generation-t debtor enters the third stage with either $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ or 0 units of good 1, which is private information. Suppose that all other agents that can participate in trade will agree in the third stage. If the debtor has $\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}$ units of good 1, he will also agree if

$$v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma) - \overline{c_2} + \sigma] \geq v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma)]. \tag{43}$$

The right hand side of the expression takes into account the fact that if a debtor disagrees, he will not receive money and will then not be able to reclaim his collateral at stage 4. If the debtor has none of good 1, it is trivial that he chooses not to agree to trade.

Now consider an arbitrary generation-t debtor at stage 2 who has borrowed from the central bank. If the debtor disagrees at this stage, he enters the third stage with 0 units of good 1 and will disagree in the third stage as well. Thus, he will receive only autarkic utility, $v[0, R(x-\sigma)]$. If the debtor agrees when everyone else does, then his second-stage participation constraint is

$$\max\{v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma) - \overline{c_2} + \sigma], v[\frac{\kappa_2^t}{\delta_2^t}\overline{d_1}, R(x-\sigma)]\} \geq v[0, R(x-\sigma)] \tag{44}$$

which is trivially satisfied. Thus, $\delta_2^t = \delta_1^t$. Those that have not borrowed from the central bank will not be able to agree to trade.

Now, consider an arbitrary generation-$t-1$ creditor at the second stage of date $t-1$. If the generation $t-1$ creditor disagrees at the second stage when young, she enters the third stage when old with no money and, therefore, receives autarkic utility, $u[y,0]$. If she agrees at the second stage of date $t-1$, she will enter the third stage of date t with money and agree so that she receives $u[y-\overline{d_1}, \frac{\delta_2^t}{\kappa_3^{t-1}}\overline{c_2}]$. She will agree if

$$u[y-\overline{d_1}, \frac{\delta_3^t}{\kappa_2^{t-1}}\overline{c_2}] \geq u[y,0] \tag{45}$$

where κ_2^{t-1} is substituted for κ_3^{t-1}.

Finally, consider generation-t debtors at stage 1 of date t. Here, if all other debtors agree to borrowing from the central bank, then $\delta_1^t = \delta_2^t = 1$ and an arbitrary debtor also agrees if

$$\max\{v[\kappa_2^t\overline{d_1}, R(x-\sigma)-\overline{c_2}+\sigma], v[\kappa_2^t\overline{d_1}, R(x-\sigma)]\} \geq v[0, Rx]. \tag{46}$$

Now it remains to be shown that $\kappa_2^{t-1} = \kappa_2^t = \delta_3^t = 1$ is supported in an equilibrium. If $\kappa_2^{t-1} = \kappa_2^t = 1$, then (46) trivially holds given that (14) holds by hypothesis. Thus, $\delta_1^t = \delta_2^t = 1$ and (43) reduces to (15) which is satisfied and $\delta_3^t = 1$. If $\delta_1^t = \delta_2^t = \delta_3^t = 1$, then it is obvious that $\kappa_2^{t-1} = \kappa_2^t = 1$ because (45) is satisfied given that (16) is. ∎

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FIG. 1 Sequence of Events in a Period