Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

Why and When do Spot Prices of Crude Oil Revert to Futures Price Levels?

Mark W. French

2005-30

Why and When do Spot Prices of Crude Oil Revert to Futures Price Levels?

Mark W. French
July 2005

Abstract

Recent studies of crude oil price formation emphasize the role of interest rates and convenience yield (the adjusted spot-futures spread), confirming that spot prices mean-revert and normally exceed discounted futures. However, these studies don't explain why such "backwardation" is normal. Also, models derived in these studies typically explain only about 1 percent of daily returns, suggesting other factors are important, too.

In this paper, I specify a structural oil-market model that links returns to convenience yield, inventory news, and revisions of expected production cost (growth of which is related to backwardation). Although its predictive power is only a marginal improvement, the model fits the data far better.

In addition, I find reversion of spot to futures prices only when backwardation is severe. Convenience yield behaves nonlinearly, but price *response* to convenience yield is also nonlinear. Equivalently, futures are informative about future spot prices only when spot prices *substantially* exceed futures.

Board of Governors of the Federal Reserve System
Division of Research and Statistics, Mail Stop 80
20th Street and Constitution Ave., NW
Washington, DC 20551

JEL CLASSIFICATION: G12, Q40
KEYWORDS: oil prices, oil futures, backwardation, convenience yield

For their helpful insights, I thank Ellen Dykes, Mike Feroli, Matt Pritsker, Bill Wascher, Jonathan Wright, and attendees at the 2003 conference of the International Association for Energy Economics in Mexico City. The views expressed are my own and do not necessarily reflect the views of the Board of Governors or the staff of the Federal Reserve System.

I. Introduction

Hotelling's seminal work in 1931 showed that in a simple equilibrium with no production cost or risk:

A. *Expected price growth for exhaustible commodities such as crude oil matches the opportunity cost of storage – that is, the nominal interest rate (r).*

Over the decades, however, prices of most storable commodities (including crude oil) have fallen in real terms, rather than rising as suggested by equilibrium condition A.[1] Furthermore, there have been periods of time when the discounted growth of oil prices was both non-zero and predictable – when prices reverted to an (evolving) trend rather than following a random walk with drift.[2] For example, during the Gulf War in 1990, futures prices suggested – correctly – that within about six months spot prices would fall drastically from their then-high levels. As I show below, the extent to which the spot price exceeds the futures price – "backwardation" – is an important predictor of mean-reversion in the spot price.

For years, economists have explained violations of Hotelling's rule as the effect on oil prices of "convenience yield," defined as the benefits from holding inventories beyond those associated with expected capital gains.[3] These additional benefits include production smoothing (Kaldor, 1939, and Working, 1948)[4], as well as the option value of holding inventories (Dixit and Pindyck [1994]).[5]

---

[1] Demonstrated by Adelman (1990) and Pindyck (1999), among others.

[2] More precisely, rather than following a submartingale (a random walk with drift and evolving variance).

[3] Williams and Wright (1991) point out that, even without the existence of convenience yield, inventories might be held profitably at times when futures prices appear insufficiently large (relative to spot prices) to compensate for average holding costs. Aggregation problems (due to cross-sectional differences in desired quality of crude, along with differences in transport and storage costs) could give the false impression that inventories are being held despite the expectation of net capital loss, when in fact they are being held with the correct expectation of positive capital gains. (Thus Williams and Wright prefer to use an equivalent term, "adjusted basis," in place of the value-laden term "convenience yield.") However, micro-level studies (e.g. Carter and Revoredo, 2001) discount the practical importance of aggregation problems in modeling commodity storage.

[4] There are costs to firms from rapidly shifting production and from rapid shifts (especially declines) in inventory holdings. Additional inventories offer the firm added flexibility to produce at the time that minimizes costs, by reducing the risk of stockouts.

[5] In effect, an option value exists because inventories allow the flexibility to choose the most profitable time to sell the commodity.

Allowing for convenience yield, the equilibrium condition for inventory demand is

B. *Expected capital loss (after borrowing costs) from holding an extra barrel of crude oil inventory exactly offsets the convenience gained, net of physical storage costs.*

As I show in the next section, the standard model derived from equilibrium condition B offers a partial explanation of mean-reversion in crude oil prices and a partial explanation for the tendency of oil prices to increase only slowly over the decades. But from an empirical standpoint, this model captures only about 1 percent of daily oil price variance and thus evidently omits the most important considerations driving oil price behavior. With some difficulty, it might be possible to improve the empirical fit of the model by separating revisions to expected convenience yield from the error term and including them in the empirical estimation.[6] But revisions to convenience yield are not the most intuitively plausible form of shocks to oil prices. Rather, geopolitical events frequently change the expected long-run cost of producing more oil, and new data on oil inventories, production, and consumption often cause sharp changes in spot and futures prices of crude oil. In practice, the standard model completely ignores the effects of this form of information on oil prices.

In addition to its lack of an explicit mechanism for processing new information about production cost and inventory surprises, the standard model makes systematic prediction errors that have important implications about the information contained in futures prices. In particular, the model predicts that spot oil prices will adjust to the (discounted) spread between spot and futures prices – whether that spread is positive or negative. In contrast, my empirical tests of this model suggest that spot prices respond to the spot-futures spread only when that spread is positive and very large.

---

[6] The revisions to expected convenience yield could be approximated by changes in the slope of the term structure of futures prices, after correcting for changes in futures risk premiums (which I will call $\rho_{t|t+i}$) at the corresponding maturities. The weighted sum of the revisions would have to equal 1, with the relative weights of near and far futures depending on interest rates and risk premiums. This particular extension of the standard model would require considerably more data (on futures prices at many maturities) than the extensions I introduce in my own model.

In response to these shortcomings, I present in section III a new and more complete approach to modeling oil price changes. My model, like the standard model, incorporates convenience yield. But my model also incorporates revisions to expectations about future production costs. In this regard, my model offers a more complete explanation of the spot price's tendency to exceed discounted futures than the standard model does.[7]

As shown in section IV, empirical tests of my model are promising. Revisions to expectations about long-run production costs appear to account for an additional 30 percent of the daily variance in oil returns. However, even with this improved specification, I still find an asymmetry in the response of oil returns to the spot-futures spread. In particular, I find that unless spot prices are sharply higher than futures prices (that is, unless unusually strong backwardation exists), oil returns appear not to respond to the measured convenience yield. This asymmetry can be explained in the context of my model by

- nonlinearity in the response of convenience yield to changes in inventory supply, or
- nonlinearity in the response of inventory supply to changes in oil prices, or even
- occasional large changes in risk premiums on far futures contracts.

Regardless of the cause of the asymmetry, for empirical purposes it is captured in a modified model that looks much like the old submartingale models (extensions of Hotelling's rule) except when spot prices are well above futures prices.

II. Today's "industry standard" model of oil pricing

The standard model of oil pricing can be derived from the optimal control problem for a price-taking company that

- produces crude oil at a cost that varies with extraction rates

---

[7] Even when a short-term oversupply of inventories causes spot price to fall below futures prices for delivery a few months ahead (as in mid-2005), there is still a tendency for discounted futures prices beyond the short maturities to decline as months to maturity increases.

The expectations revisions in my model are not predetermined. Thus, even though my model has more descriptive power than the standard model, it does not necessarily have more predictive power. Nonetheless, the extra descriptive power probably makes it an improved framework for testing the apparent asymmetry of oil returns in response to the spot-futures spread.

● sells some or all of its production to customers (who will be annoyed when the firm has insufficient oil on hand to supply their needs)

● freely adjusts its sales and inventory levels, unless sales rise to the level of production plus starting inventories (that is, unless stockout occurs)

● pays a physical holding cost for each barrel of its (aboveground) inventories.

Considine and Larson (2001) show that the competitive solution to this problem is the standard inventory equilibrium condition B above.

Convenience yield: a partial explanation for puzzles in oil price behavior

Convenience yield is a slippery concept, and a bit of care must be taken in using it to explain oil price movements. It seems obvious that on average, oil inventories provide convenience (beyond capital gains) to at least some inventory holders, but this average convenience yield has no bearing on the rate of oil price growth or on the tendency of oil prices to mean-revert. Rather, what matters is the convenience of inventories at the margin, defined as follows:

● "Marginal Convenience Yield" is the convenience gained from holding an extra barrel of inventories;

● "Net Marginal Convenience Yield" (cy_t) is the marginal convenience yield net of physical holding costs;

● "Percentage Net Marginal Convenience Yield" ($\%cy_t$) is the net marginal convenience yield (cy_t) divided by the spot price of the commodity (P_t).

Because the marginal convenience from holding an added barrel of oil typically outweighs the physical cost of holding that extra barrel, those who buy inventories are in effect buying a "dividend stream" of future convenience yield. This stream of additional benefits causes the price of oil, like the price of dividend-bearing stocks, to increase more slowly than the overall required return. As a result, the expected long-run growth of oil prices is less than as predicted by Hotelling's rule.

To illustrate, suppose that a sharp drop in the supply of oil inventories suddenly increases the risk of stockout. An extra barrel of oil inventories will then offer a typical oil company a much greater convenience benefit than it did previously. (In other words, net marginal convenience yield (cy_t) has risen sharply from its usual level ($cy^{equilib}$)). Oil companies will therefore demand more inventories, driving up the spot price. If long-run production costs are unchanged,[8] this rise in spot price implies that the expected future increase in spot price is smaller. In effect, expected oil returns $[P_{t|t+1} - P_t * (1+r_t+\rho_{t|t+1})]$ have fallen in response to a rise in convenience yield. But over time, oil inventories will tend to move back to normal levels, and correspondingly, marginal convenience yield also moves back to normal. (Such mirror-image movements in convenience yield and oil inventories can be seen in the historical data, as shown in figure 1.) As convenience yield returns to normal, oil prices will also readjust, reverting to a trend growth path. Thus, incorporating convenience yield into the model (via inventory equilibrium condition B) offers a theoretical explanation for the two best-known empirical problems with Hotelling's rule: mean-reversion and slow trend price growth over the decades.

To make the model operational, net marginal convenience yield (cy_t) must be defined in terms of observed data. This is done using equilibrium condition B from section I, plus the following identity -- the equilibrium condition for futures markets:

C. *A futures price ($F_{t/t+i}$) equals the expected future spot price ($P_{t/t+i}$) plus a CAPM-style risk premium ($\rho_{t|t+i}$) for holding futures.*[9]

Identities B and C combine to give the operational means for constructing an empirical measure of convenience yield:

---

[8] And if the risk premium ($\rho_{t|t+1}$) on holding futures contracts is also unchanged.

[9] The sign of this risk premium will depend on considerations like the following. Are inventory-holders (such as oil companies) able to fully diversify their portfolios? If not, then own-price risk is a valid consideration. However, if inventory-holders' portfolios are diversified and some variant of the CAPM model holds, then the sign of the risk premium will depend on whether supply or demand shocks predominate. During oil supply shocks, negative covariance with the market portfolio means that, on average, holding a small additional claim on oil (that is, a futures contract) reduces the variance of one's overall portfolio (assuming that investors can fully diversify their portfolios). As a charge for that risk reduction, sellers of oil futures would offer them at a delivery price above the expected future spot price. Therefore, futures prices are likely biased upward a bit during oil supply shocks.

D. *Net marginal convenience yield per barrel (cy_t) is defined as*
spot price (P_t) marked up for interest yield ($r_t * P_t$) minus futures price ($F_{t|t+1}$).

Given the data for convenience yield as derived in identity D, solutions to the equilibrium condition B can be estimated empirically.[10] Several authors (Pindyck, 1993, Litzenberger and Rabinowicz, 1995, Schwarz, 1997, and Considine and Larson, 2001) have estimated variants[11] of the following solution to equilibrium condition B:

$$(1)\ P_{t+1} - P_t * (1 + r_t + \rho_{t|t+1} - \%cy^{equilib}) = -(cy_t - \%cy^{equilib} * P_t) + \mu_{t+1} ,$$

where

$P_{t|t+1}$ ≡ the expectation at time t of spot price at time t+1,
cy_t ≡ net marginal convenience yield from holding oil from time t to time t+1,
$\%cy^{equilib}$ ≡ the long-run equilibrium percentage net marginal convenience yield,
r_t ≡ the risk-free interest rate,
$\rho_{t|t+1}$ ≡ a risk premium specific to crude oil, [12]
(Together r_t and $\rho_{t|t+1}$ constitute the required rate of return RR_t.)
μ_{t+1} ≡ discounted revisions to expectations of future convenience yields.
(In practice, however, μ_{t+1} becomes a catchall error term that includes measurement noise, aggregation errors, nonlinearities related to interest rate variation, and so forth.)

---

[10] Equilibrium condition B amounts to an expectational difference equation (or differential equation, in some studies). Pindyck (1993) shows how to solve this difference equation to get a version of equation 1.

[11] Some authors used continuous-time error-correction models (Ornstein-Uhlenbeck processes), whereas others used discrete-time models. Some simply assumed the form of the price equation, whereas others derived it from optimal control exercises. Some separately identified the trend percentage convenience yield ($\%cy^{equilib}$), whereas others did not. Pindyck (1993) estimated equation 1 at a monthly frequency, for heating oil rather than crude. He found significant error correction to the discounted spread between near futures and year-ahead futures, over the period October 1980 through February 1990.

[12] Over the past twenty years, the estimated covariance of spot oil returns (month-end to month-end) with gross returns on the S&P 500 (including dividend yield) was significant only during the first Gulf War and in the late summer of 2004. Similarly, Schwarz (1997) found no significant risk premium in near futures for crude.

After estimating their preferred variants of equation 1, both Pindyck (1993) and Schwartz (1997) observed that, empirically, the net marginal percentage convenience yield of crude oil ($\%cy^{equilib}$) has a positive equilibrium value. But why is that equilibrium value greater than zero? Equilibrium conditions B and C for storage and futures markets *per se* do not tell us whether the gross added convenience from an additional barrel of oil inventory will normally exceed the cost of physical storage for that last barrel. To answer this question, one must go further than these earlier studies.[13] Equilibrium conditions B and C must be solved jointly with behavioral equations for production, consumption, and convenience yield – while accounting for market expectations – as I do in section III below.

An empirical test of the "standard" model of oil prices

Although equation 1 does not explicitly model the determinants of convenience yield, nothing in the analysis so far indicates that the specification is actually incorrect. As it turns out, however, estimates of equation 1 are not robust and thus are potentially misleading. In particular, the estimated basic version of equation 1 is

$$(2) \qquad P_t - P_{t-1} * (1 + r_t) = \underset{(2.9)}{.038} \; - \underset{(3.2)}{.010} * \; cy_{t-1} \; + \; \varepsilon_t$$

where $cy_{t-1} \;=\; [(1+r_t)^* P_{t-1} - F_{t-1\,|\,t+one\,year}]$ and r_t is the one-year treasury bill rate.[14]

Business-day frequency, sample period 16mar1989 to 24Mar2005
t-statistics in parentheses
R-bar squared = .003
Standard error = 58.9 cents per barrel
Durbin-Watson = 2.0

---

[13] This is not to say that equation 1 is useless as it stands. Equation 1 and the empirical definition of convenience yield (identity D) imply that

$$P_{t+1} \;=\; F_{t|t+1} \; + \rho_{t|t+1} P_t \; + \; \mu_{t+1}$$

Given an appropriate measure of the risk to holding oil futures, one can test the semi-strong form efficiency of oil pricing by verifying that additional variables known at time t do not help predict P_{t+1} in the framework of this equation.

[14] I followed Pindyck (1993) in choosing year-ahead maturity for the futures price in this specification: The consistent availability of this series since January 1989 allows a longer sample period than more-distant maturities would.

However, when equation 2 is reestimated with the inclusion of dummy terms specific to a regime in which the size of the convenience yield is normal or smaller than normal (that is, less than \$4 per barrel or negative), the results are strikingly different:[15]

$$(3)\quad P_t - P_{t-1} * (1 + r_t) = \underset{(4.9)}{.26} \;-\; \underset{(5.5)}{.044} * cy_{t-1}$$

$$-\; \underset{(4.5)}{.24}\, Dumlow_{t-1} \;+\; \underset{(4.7)}{.049} * Dumlow_{t-1} * cy_{t-1} + \varepsilon_t$$

($Dumlow_{t-1}$ = 1.0 if net convenience yield (cy_{t-1}) < \$4 per barrel, and = 0.0 otherwise)

Business-day frequency, sample period 16mar1989 to 24Mar2005
R-bar squared = .007
Standard error = 58.7 cents per barrel
Durbin-Watson = 2.0

Equation 2 has a constant negative coefficient on the lagged convenience yield.[16] However, in equation 3 the coefficient appears to differ across regimes, with the coefficient much smaller (near zero) when the size of the convenience yield is normal or small. This result seems to suggest that crude oil futures prices only have predictive

---

[15] I preselected \$4 per barrel as the level of convenience yield to split the sample–before I did any estimation. Inspecting figure 1, I chose this split point so that nearly all local peaks in convenience yield lay above the split, while keeping below the split all troughs or "normal" stretches (that is, time periods lacking a large peak or trough). After the fact, I found that no other choice of split point noticeably improved the fit of equation 3. (Note that in equations 2 and 3, median(cy_t)=\$2.46/barrel, mean(cy_t)=\$2.89, max(cy_t)=\$16.46, and min(cy_t)=-\$4.70)

Some care must be taken with the split-regime specification, as pointed out by Williams and Wright (1991, p. 180). Even if the true error-correction coefficient is always the same nonzero figure, one cannot easily reject a null hypothesis that "the error-correction coefficient is significant when convenience yield is large and zero when convenience yield is small," because in the latter case the information in the convenience yield is small relative to the variance of oil returns. (However, under the alternative, with one common error-correction coefficient, the estimated confidence interval for the error-correction coefficient in the low-convenience regime will usually encompass both zero and the true error-correction coefficient.)

Testing of a different null hypothesis--that the error-correction coefficient is the same across regimes–may also have low power but will be very informative if the test rejects the null. One can test this null hypothesis by adding terms involving the product of the independent variables with a dummy equal to one in an alternative regime where convenience yield is normal or low. If the t-statistics of these additional dummied terms are significant, the null (of one common error-correction coefficient) can be rejected.

[16] Shown as identically equal to -1.0 in equation 1, but in practice the coefficient is a constant tied to the frequency of the dependent variable relative to the time to maturity of the futures quote embedded in the convenience yield. I chose to estimate the coefficient freely.

information (beyond that contained in spot crude prices) when convenience yield is substantially larger than normal.[17] However, the poor fit of both equations 2 and 3 suggests that one should be cautious in interpreting these results. Thus, I later revisit these two equations in the context of a model with a better fit.

III. A New Model of Crude Oil Price Formation

Rather than treat convenience yield as predetermined or exogenous, as do most of the studies cited in section II, I choose to close the model--by specify behavioral equations for convenience yield and the components of inventory supply. The additional equations illustrate how convenience yield helps to bring production, consumption and demand for inventories into balance.

Behavioral equations for convenience yield, production and consumption

As discussed in sections I and II, the benefits that make up convenience yield in holding inventories are:

i. facilitating the smooth flow of the commodity from producer to processor to the ultimate consumer.
ii. enabling producers to smooth output over time--that is, they reduce the expense of a large surprising shift in demand--especially a sudden increase in demand that producers may not be able to meet through an immediate increase in production (stockout avoidance is an important part of this convenience)
iii. allowing producers to choose the optimum time to sell their commodity, thereby maximizing revenues from the sale (the option value of holding inventories).

Benefits i and ii suggest that convenience yield will increase with a decline in inventories relative to production or consumption--that is, "days supply." Benefits ii and iii suggest that convenience yield will increase with a rise in the volatility of inventory supply (that

---

[17] More precisely, futures usually contain no more information than spot crude oil prices adjusted for trend price growth--which is linked in turn to interest rates, risk premiums, and the equilibrium percentage convenience yield.

is, the volatility of production minus consumption in this simple model), which corresponds to a rise in the volatility of spot price itself. I will call this conditional volatility of spot price "s_t".[18] Benefit iii suggests that convenience yield will increase with the level of spot price, other things being equal (as noted by Pindyck, 2001a).[19]

To capture these influences, convenience yield can be modeled with the following local linear approximation:[20]

$$(4)\quad cy_t \;=\; a_0 P_t \;+ a1 * s_t \;- a2 * (inventory_t - target\ inventory_t)$$

To make equation 4 operational, one must define $inventory_t$ and $target\ inventory_t$:[21]

$$(5)\quad inventory_t \;=\; inventory_{t-1} \;+\; production_t - consumption_t$$

$$(6)\quad (target\ inventory)_t \;=\; a3 * equilibrium\ production = a3 * b0$$

For ease in solving the model, production and consumption behavior are defined in simple linear equations:[22]

$$(7)\ Production_t \;=\; b0 + b1 * (P_{t-1\,|\,t} \;-\; LRMC_{t-1\,|\,t}) + \;u_{1t}$$

$$(8)\ Consumption_t = \;\; c0 - \; c1 * (P_{t-1\,|\,t} \;- P_{substitutes\;\; t-1\,|\,t}\,) + u_{2t}$$

---

[18] Financial theory says that, in general, the conditional standard deviation of price, s_t , need not be closely related to $\rho_{t|t+1}$, the risk to portfolios from holding oil inventories.

[19] Considine and Larson (2001) formally combined these considerations in a maximization problem for a single firm. However, they ignored the supply-side implications and derived only an equation like eqn 1.

[20] Globally, however, the change in convenience yield for a given change in inventories depends on the level of inventories, as shown for example by Working (1948). Thus, the simplifying assumption that a2 is a constant will be relaxed later.

This specification of the structural equation for convenience yield is almost identical to the one chosen by Pindyck (2001b). However, Pindyck's system did not include additional structural equations for inventory supply (incorporating expectations)–a crucial difference, as it turns out.

[21] For purposes of exposition, I ignore net imports in defining inventories, as do all of the above-cited studies.

[22] In the production equation, LRMC stands for long-run marginal cost (that is, full cost including exploration and development costs); $P_{substitutes}$ is the price of substitutes in consumption (converted to $/bbl). In equations 7 and 8, the assumption of linear response to price may be acceptable for small price changes. However, results in section IV below suggest that globally, the response of production and consumption to price changes may not be approximately linear.

Reduced form and solution of the structural model

My simple "structural" model of oil markets includes equations 4 through 8 as well as equilibrium conditions B and C. Combining these structural equations and transforming the coefficients yields an expectational difference equation describing oil price formation:[23]

(9) $P_t = \alpha_0 P_{t\,|\,t+1} + \alpha_1 P_{t-1} + \alpha_2 P_{t-1\,|\,t} + x_t$.[24]

In terms of the original structural coefficients, the solution to equation 9 is:

(10) $P_t - P_{t-1} * (1 + RR - \%cy^{equilib}) = -K * ((1+RR - \%cy^{equilib})* P_{t-1} - P^{alt}_{t-1\,|\,t})$

$- InventorySurprise_t + 1.0 * ExpectationRevisions_t + u_t$ [25]

where:

$InventorySurprise_t \equiv a_2 * (u_{1t} - u_{2t})$

$ExpectationRevisions_t \equiv \sum_{i=0}^{\infty} w_{t+i} * (P^{alt}_{t\,|\,t+i} - P^{alt}_{t-1\,|\,t+i}),\ (\sum_{i=0}^{\infty} w_{t+i} = 1)$ [26]

$P^{alt} \equiv$ the elasticity-weighted average of [long-run marginal production cost] and [price of substitutes in consumption].

---

[23] To get an analytic solution to the reduced-form equation, I make the usual assumption that the change in required return over short periods is negligible compared with changes in spot price and convenience yield. (This assumption, made by Pindyck, 1993, for example, will be relaxed later.) Thus, $RR_t \equiv RR$.
I transformed the structural coefficients so that equation 9 would match the corresponding equation in Blanchard and Fischer (1989, pp. 264-66). They derive its solution.

[24] The coefficients of equation 9 are defined in terms of the coefficients of equations 4 to 8 as follows:
$\alpha_0 \equiv 1 / (1 + RR - a_0)$ $\alpha_1 \equiv 1$ $\alpha_2 \equiv -(1 + a_2 * (b_1 + c_1)) / (1 + RR - a_0)$
$x_t \equiv a_2 * (b_1 + c_1) / (1 + RR - a_0) * P^{alt}_{t-1\,|\,t} + Expectation\ Revisions_t + InventorySurprise_t + u_t$,
where $u_t \equiv a_1 * \Delta s_t$

[25] For ease of exposition, equation 10 introduces the weighted average price $P^{alt}_{t-1\,|\,t}$:
$P^{alt}_{t-1\,|\,t} \equiv [b_1 / (b_1 + c_1)] * LRMC_{t-1\,|\,t} + [c_1 / (b_1 + c_1)] * P_{substitutes\ t-1\,|\,t}$ Also,

$K \equiv a2*(b1 + c1) / (1 + a2*(b1 + c1) + \lambda_1)$, where λ_1 is the stable root of equation 9.
$\lambda_1 = (1+[RR - \%cy^{equilib} + a2*(b1 + c1)])/2 - 1/2 *((1+RR - \%cy^{equilib} + a2*(b1 + c1))^2 - 4*[1+RR - \%cy^{equilib}])^{1/2}$

[26] The weight w_{t+i} on revisions to expectations of P^{alt} is defined in terms of λ_1, the stable root of equation 9 presented in the previous footnote: $w_{t+i} \equiv (1 - \lambda_1) * \lambda_1^{i}$

Boundary condition for equation 10: as $k \rightarrow \infty$, $\lim (P_{t|t+k} - P^{alt}_{t|t+k}) = 0$

Roughly speaking, the boundary condition says that the spot crude price, long-run marginal cost and the price of substitutes are all expected to converge eventually. (Also, equilibrium condition C implies that their expected value in the distant future is the corresponding far futures price net of any futures risk premium.)

Equation 10 indicates that:

- trend growth of spot crude oil prices equals the required rate of return minus equilibrium net marginal percentage convenience yield
- deviations of spot price growth from trend are due to one of the following causes:
    - i. deviations of lagged price from LRMC or from prices of substitutes in consumption (or from both);
    - ii. revisions to expectations about future levels of LRMC or the price of substitutes in consumption
    - iii. surprises in the amount of current inventory accumulation

Equation 10 and its associated boundary condition imply that the equilibrium percentage convenience yield is identically equal to the difference between the required rate of return and the growth rate of long-run marginal production cost. They also imply that the equilibrium percentage convenience yield is identically equal to the difference between the required rate of return and the growth rate of the price of substitutes in oil consumption.

Thus, normal backwardation exists not simply because there is an equilibrium percentage convenience yield; rather, normal backwardation and equilibrium percentage convenience yield both exist because long-run marginal costs and prices of substitutes increase at a rate slower than the required rate of return (perhaps because of technical improvement).[27]

---

[27] Litzenberger and Rabinowitz (1995) show from the production side that slow growth of long-run marginal cost can induce normal backwardation. They do not provide, as I do, a more general formulation that also includes slow growth of prices of substitutes in oil consumption, nor do they examine the effect of news on expectations and prices.

To make equation 10 empirically useful, I replace P^{alt}, *ExpectationRevisions*, and *InventorySurprise* with observed approximations:

- Because market predictions of weekly DOE inventory change are typically not very accurate, I proxy *InventorySurprise* using each Wednesday's actual DOE figure for the previous week's inventory change.[28]
- P^{alt} is approximated by incorporating two assumptions:
  - i. The equilibrium price growth rate, $(1+RR - \%cy^{equilib})$, is roughly equal to the growth rate of futures prices near the far futures date t+k, $([F_{t-1|t+k} / F_{t-1|t+k-12}]^{k/12})$;
  - ii. The expected growth rate of P^{alt} from today onward is its long-run equilibrium growth rate $(1+RR - \%cy^{equilib})$.[29]

With these assumptions added, the error-correction term of equation 10 [that is, $(1+RR - \%cy^{equilib})* P_{t-1} - P^{alt}_{t-1\,|\,t}$] becomes essentially the difference between spot price and the discounted far futures price (where the monthly discount factor is measured by $[F_{t-1|t+k} / F_{t-1|t+k-12}]^{1/12}$, a number that is usually slightly less than 1).[30]
Also, the *ExpectationRevisions* term of equation 10 becomes simply the discounted revision to far futures price, still with a coefficient of exactly 1. The model variables are now observable, and parameters K_0, K_1 and K_2 can be estimated in equation 11:

---

[28] This variable equals 0 every day except Wednesday, when it equals the value reported by the Department of Energy for private U.S. crude oil inventory change in the week ending the previous Friday. The assumption that actual inventory change approximately equals the shock to inventories is consistent with the inaccuracy of autoregressive projections of inventory change. In my regression test over a two-decade sample, a weekly autoregressive equation for inventory change showed a tendency for changes to be reversed in the following week–probably due in part to sampling errors in the DOE inventory data. Also, the equation showed slow mean-reversion in the series. However, these considerations explained only about 2 percent of the variance of inventory change.

[29] In assumption i, lower case "k" is defined as months to maturity of the far futures contract. Assumption ii is fairly strong; but since P^{alt} is unobserved in the short run, this assumption is the best possible for near-term growth of P^{alt}.

[30] Plus the discounted risk premium on far futures price, proxied by K_0. Over the past fourteen years, the empirical measure of equilibrium oil price growth $[F_{t-1|t+k} / F_{t-1|t+k-12}]^{k/12}$ averaged about 4-1/2 percentage points lower than the 3-month treasury-bill rate. However, the equilibrium growth rate was more volatile than the t-bill rate. In particular, the series had a fair number of spikes lasting only 1 to 3 days. These spikes were not correlated with movements in treasury bill rates, and I view them as noise, not as a signal about the equilibrium growth rate. Thus one could smooth the series for equilibrium price growth. However, one would have to use a one-sided filter in such smoothing. A two-sided filter, like the Hodrick-Prescott filter, could in principle invalidate the results by putting information about future equilibrium price growth into the lagged convenience yield and into the lagged estimate of the equilibrium growth rate.

(11) $P_t - P_{t-1} *[F_{t-1|t+k} / F_{t-1|t+k-12}]^{1/12} \quad = \quad K_0$

$-K_1 *([F_{t-1|t+k} / F_{t-1|t+k-12}]^{1/12} * P_{t-1} - F_{t-1\,|\,t+k} / [F_{t-1|t+k} / F_{t-1|t+k-12}]^{k/12})$

$-K_2$ * Wednesday series for *InventoryChange*$_t$ in the week ending previous Friday

$+ \quad R_t \quad + \quad \varepsilon_t$

where

K_0 is the discounted risk premium on far futures $[K_1 * \rho_{t-1\,|\,t+k}/(1+RR - \%cy^{equilib})^{t+k-1}]$

R_t is the discounted change in far futures price $(F_{t\,|\,t+k} - F_{t-1\,|\,t+k}) / [F_{t-1|t+k} / F_{t-1|t+k-12}]^{k/12}$

and at monthly frequency,
$(1+RR - \%cy^{equilib}) \approx [F_{t-1|t+k} / F_{t-1|t+k-12}]^{1/12}$
(k defined as the number of months between near futures and far futures months).[31]

IV. Empirical Results

I first estimate equation 11 at a daily frequency over the sample period from December 6, 1990, through February 4, 2005.[32] To test the specification, I initially treat the discounted risk premium on far futures $[\rho_{t-1\,|\,t+k} / (1+RR - \%cy^{equilib})^{t+k-1}]$ as a constant intercept term K_0, but later find that this intercept takes on a different value when backwardation is large.

To simplify the presentation, I let $r^{eq}_t \equiv (1+RR - \%cy^{equilib})_t$
and define net convenience yield (cy_{t-1}) as $cy_{t-1} \equiv (r^{eq}_{t-1} * P_{t-1} - F_{t-1\,|\,t+k} / [(r^{eq}_{t-1})^{t+k-1}])$,
where both of the terms in the definition of cy_{t-1} have been converted from monthly to daily frequency.[33] The estimation results are (t-statistics in parentheses):

---

[31] To estimate equilibrium growth at business-day frequency, raise the right-hand side of this equation to the power 12/261.

[32] I proxy for spot prices using the nearest futures quote. The number of months to maturity of the far futures contract increases over the course of the sample, but with little effect on the results, when I split the sample. I start the sample in December 1990 because the far futures month moved from 2 years ahead to 3 years ahead at that time.

[33] In equations 12 and 13, median(cy_t)=\$1.93/barrel, mean(cy_t)=\$3.12, max(cy_t)=\$14.74, and min(cy_t)=-\$6.28

(12) $P_t - P_{t-1} * r^{eq}_t = \underset{(1.8)}{.015} \; - \underset{(2.6)}{.0048} * cy_{t-1}$

$\underset{(4.8)}{-.019}$ * Wednesday series for weekly DOE *Inventory Change*$_t$

$+ \quad R_t \quad + \quad \varepsilon_t$ [34]

where R_t is the discounted change in far futures price, still with coefficient equal to 1.

Business-day frequency, sample period 6Dec1990 to 3Feb2005
R-squared = .314
Standard error = 46.1 cents per barrel
Durbin-Watson = 1.8 [35]

Equation 12 is innovative in two important ways:

- The discount rate (r^{eq}_t) is derived from the evolving expected growth of LRMC, as embedded in far futures prices.[36] It thus implicitly nets out an evolving long-run equilibrium percentage convenience yield. Other studies do not incorporate changes in equilibrium percentage convenience yield. At best, they lump the equilibrium convenience yield into the equation's constant term, or worse, they set the equilibrium percentage convenience yield to zero. These simplifications lead to overestimates of the size of the (expected) convenience yield–a bias that can be very large if convenience yield is measured over the coming year or more, rather than simply over the coming month or two.
- The equation also includes discounted changes in far futures price (R_t) as a proxy for revisions to expected long-run marginal production cost. This term, whose coefficient

---

[34] The coefficient of the error-correction term (-.0040) is small but significant at the 95% level.

[35] The Durbin-Watson statistic suggests that a lagged dependent variable would contribute little, on average, to the equation's fit. Serial correlation in daily returns, when it occurs, is captured at least in part by serial correlation in the convenience yield.

[36] For this approach to be valid, daily NYMEX closing prices for the December far futures contract (and for the contract maturing 12 months earlier) should not often be notional. Preferably, the NYMEX settlement committee should have some trades, or at least bid and ask prices, on which to base their closing price quote for these far future contracts. Since February 1990, open interest for the far futures contract (and for that far maturity less 12 months) has always been positive, and more important, daily trading volumes for these far futures prices have rarely been zero. Trading volumes have occasionally been very thin (just a few thousand barrels), but usually volumes for the December far futures quotes are comfortably large–on a par with volumes for some of the shorter-maturity contracts.

must equal 1, is not useful for forecasting purposes. However, it increases the R^2 from a level near zero to a level over .3; the drastically improved fit provides a better framework for hypothesis testing than equation 2 does.[37]

In other respects, the equation should not surprise most energy economists these days. As one would expect, spot crude prices error-correct (albeit slowly) in response to the size of the net convenience yield. Results were very similar when the start of the sample period was moved up to 19mar1991 (to exclude any Gulf War effects) or to 24jan1997 (when the far futures month was first extended an extra three years).

In light of the regime shifts in equation 3, I also tested the robustness of equation 12. Specifically, I split the sample, using a dummy variable to estimate an additional set of coefficients for the periods in which net marginal convenience yield is less than \$4 per barrel. As in equation 3, the coefficients of equation 13 are strikingly different in this regime:

$$(13)\quad P_t - P_{t-1} * r^{eq}_t = \underset{(4.0)}{.168} \; - \underset{(4.5)}{.024} * cy_{t-1} \; \underset{(4.5)}{-.032} * \textit{Wednesday inventory dummy}_t$$

$$- \underset{(3.8)}{.161}\, \textit{Dumlow}_{t-1} \quad \underset{(3.0)}{+.022} * \textit{Dumlow}_{t-1} * cy_{t-1}$$

$$\underset{(2.2)}{+.019} * \textit{Dumlow}_{t-1} * \textit{Wednesday inventory dummy}_t \quad + R_t \quad + \quad \varepsilon_t$$

where R_t is the discounted change in far futures price, with coefficient equal to 1.

Business-day frequency, sample period 6Dec1990 to 3Feb2005
R-squared = .317
Standard error = 46.0 cents per barrel
Durbin-Watson = 1.8

---

[37] In practice, there is no problem of simultaneity of R_t with the inventory change variable. When R_t is omitted, the coefficient of the inventory variable changes only from -.019 to -.020.

When net convenience yield exceeds $4 per barrel, spot crude prices error-correct more strongly in response to the size of the net convenience yield--and the intercept ("risk premium") becomes significant. However, when net convenience yield is below $4 per barrel, the offsetting coefficients imply that, on net, the estimated response to the convenience yield is small or zero, and the intercept is very small. As shown in the appendix, this dichotomy is equally evident when the start of the sample period is moved up to 19mar1991 or to 24jan1997. The split is also evident with different lags on the convenience yield (an appendix equation shows similar results replacing the one-day lag with a one-month lag). Choice of futures month in constructing the convenience yield also appears to make no difference to the results: The dichotomy is just as clear with year-ahead futures (equation 3) as with far futures (equation 13).

Implications of estimation results

In equation 13, the estimated coefficient values for $Dumlow_{t-1}$ and $Dumlow_{t-1} * cy_{t-1}$ appear significantly different from zero. Working backward to the specification in equation 11, this result implies in turn that

● changes in the discounted risk premium on far futures contracts (embedded in coefficient K_0) may be non-negligible: that risk premium could be an autoregressive process, perhaps varying with the lagged level or change in the spot-futures spread.

● the response of spot price to the lagged convenience yield (coefficient K_1) varies with the size of that spread between spot price and discounted far futures price. This in turn implies that in specification 10, the coefficient K on the spread between spot price and the price of alternatives (P^{alt}) may not be constant. That coefficient depends in turn on parameters from the structural equations for convenience yield, production and consumption.[38]

In summary, my model structure suggests three possible reasons why the estimated response of spot price to convenience yield might vary with the size of the convenience yield:

● Changes in the risk premium on far futures contracts may be non-negligible;

---

[38] That is, parameters a_2 from equation 4, b_1 from equation 7, and c_1 from equation 8.

- The response of convenience yield to inventory changes may be larger when inventory levels are smaller; and
- The response of inventory supply to changes in lagged prices may not be approximately linear, if those price changes are large.

Ex post predictive accuracy of spot and year-ahead futures prices

The empirical results from equations 3 and 13 suggest that oil futures prices may be no more accurate than spot prices in predicting future spot prices–except when current spot prices are well above current futures prices. As shown in the table below, this conclusion is borne out by the data (for most of the estimation period).

Errors in forecasting year-ahead spot price: March 1989 – September 2003

(standard errors, in dollars per barrel)

	Predictor	
	Spot price (P_t)	Futures Price ($F_{t\|t+12}$)
Full sample	6.08	5.85
$P_t - F_{t\|t+12}$ > \$4/barrel	7.35	5.73
$P_t - F_{t\|t+12} \leq$ \$4/barrel	5.81	5.88

In the past two or three years, however, spot prices have outpredicted futures prices even when the spread between spot and year-ahead futures prices has been high. My guess is that this more recent phenomenon is likely due to an unusual string of upward revisions to expected long-run marginal cost, rather than a signal that year-ahead futures prices will be irrelevant under any circumstances from now on. Indeed, as shown in figure 2, far futures prices have risen at an annual rate of 50 percent since September 2003, orders of magnitude faster than the estimated equilibrium increase in oil prices over the same period (0.3 percent at an annual rate).[39]

V. Conclusions

In this paper, I developed a model of crude oil price formation that is more complete than the common alternative model. It demonstrates more clearly the causes of oil price changes and provides a better framework for hypothesis testing because it explains far more of the movement in oil prices.

Statistical tests suggest that the "industry standard" model, even in the more complete form presented in Section III, does not adequately explain short-run oil price behavior when futures markets are in weak backwardation or contango (when futures prices exceed spot). This conclusion is not just a repetition of the known result that, when the net marginal convenience yield is small, no information exists in futures prices beyond that contained in current spot prices. Rather, it seems that the net marginal convenience yield has little bearing on price movements unless that yield is *well above* its long-run average.

To understand whether nonlinearities or evolving risk premiums (or both) caused the standard model to fail for the period from 1990 to 2005, one needs to model the risk premium on oil futures in more detail than does equation 13. For example, does covariance with the market portfolio matter more, or are oil companies more concerned with own price risk, because of an inability to fully diversify? In the latter case, the magnitude of the risk premium is related to the size of the convenience yield. A multivariate garch study is called for, and I plan to follow up.

The role of noncommercial traders in oil futures markets also needs more study. When net marginal convenience yield is very low, it becomes much less dangerous for these traders to take a position in the futures market. If many uninformed traders take a

---

[39] In particular, far futures prices rose at an extremely rapid rate in October 2003, August 2004, October 2004 and March 2005. However, even while these price levels soared, each day's far futures quote was almost identical to the same day's quote for 12 months before the far futures month.

position when convenience yield is low, futures prices may deviate from the normal futures market equilibrium condition.

BIBLIOGRAPHY

Adelman, Morris (1990). "Mineral Depletion, with Special Reference to Petroleum," Review of Economics and Statistics, vol. 72 no. 1 (February), pp. 1-10.

Baker, Malcolm, E. Scott Mayfield and John Parsons (1998). "Alternative Models of Uncertain Commodity Prices for Use with Modern Asset Pricing Methods," Energy Journal, vol. 19 no. 1, pp. 115-148.

Blanchard, Olivier, and Stanley Fischer (1989). Lectures on Macroeconomics. Cambridge, Mass: MIT Press.

Carter, Colin, and Cesar Revoredo (2001). "The Working Curve and Commodity Storage under Backwardation," Working Paper, Department of Agricultural and Resource Economics, University of California at Davis.

Considine, Timothy, and Donald Larson (2001). "Uncertainty and the convenience yield in crude oil price backwardations," Energy Economics, vol. 23 (September), pp. 533-548.

Cortazar, Gonzalo, and Eduardo Schwartz (2003). "Implementing a stochastic model for oil futures prices," Energy Economics, vol. 25 No. 3 (May), pp. 215-238.

Danthine, Jean-Pierre (1977). "Martingale, Market Efficiency and Commodity Prices," European Economic Review, vol. 10, no. 1 (October), pp. 1-17.

Deaton, Angus, and Guy Laroque (1992). "On the Behaviour of Commodity Prices," Review of Economic Studies, vol. 59, pp. 1-23.

Dixit, Avinash, and Robert Pindyck (1994). Investment under Uncertainty. Princeton NJ: Princeton University Press.

Gibson, Rajna, and Eduardo Schwartz (1990). "Stochastic Convenience Yield and the Pricing of Contingent Claims," Journal of Finance, vol. 45 no. 3, pp. 959-976.

Hotelling, Harold (1931). "The Economics of Exhaustible Resources," Journal of Political Economy, vol. 39 no. 2 (April), pp. 137-175.

Kaldor, Nicholas (1939). "Speculation and Economic Stability," Review of Economic Studies, vol. 7, pp. 1-27.

Litzenberger, Robert, and Nir Rabinowitz (1995). "Backwardation in Oil Futures Markets:Theory and Empirical Evidence," Journal of Finance, vol. 50 no. 5 (December), pp. 1517-1545.

Pindyck, Robert (1993). “The Present Value Model of Commodity Pricing,” The Economic Journal, vol. 103 (May), pp. 511-530.

Pindyck, Robert (1999). “The Long-Run Evolution of Energy Prices,” Energy Journal, vol. 20 no. 2, pp. 1-27.

Pindyck, Robert (2001a). “The Dynamics of Commodity Spot and Futures Markets: A Primer,” Energy Journal, vol. 22 no. 3, pp. 1-29.

Pindyck, Robert (2001b). “Volatility and Commodity Price Dynamics,” Massachusetts Institute of Technology, working paper, August 19.

Routledge, Bryan, Duane Seppi and Chester Spatt (2000). “Equilibrium Forward Curves for Commodities,” Journal of Finance, vol. 55 no. 3 (June), pp. 1297-1338.

Samuelson, Robert (1967). “Proof that Properly Anticipated Prices Fluctuate Randomly,” Industrial Management Review vol. 6 (Spring), pp. 41-49.

Schwartz, Eduardo (1997). “The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging,” Journal of Finance, vol. 52 no. 3 (July), pp. 923-973.

Williams, Jeffrey, and Brian Wright (1991). Storage and Commodity Markets. Cambridge, England: Cambridge University Press.

Working, Holbrook (1948). “The theory of the price of storage,” American Economic Review, vol. 39, pp. 1254-1262.

APPENDIX: TESTING EQUATION 13 FOR ROBUSTNESS

The Gulf War

Including the Gulf War in the sample did not distort the results in equation 13. A sample that excluded the Gulf War (3/19/91–2/3/05) gave results similar to equation 13[40]:

(13a) $P_t - P_{t-1} * r^{eq}_t \;=\; \underset{(3.8)}{.148}$

$- \underset{(4.1)}{.020} * cy_{t-1} \quad - \underset{(5.2)}{.034} *$ Wednesday $inventory\ dummy_t$

$- \underset{(3.5)}{.142}\ Dumlow_{t-1} \quad + \underset{(2.9)}{.020} * Dumlow_{t-1} * cy_{t-1}$

$+ \underset{(2.6)}{.021} * Dumlow_{t-1} *$ Wednesday $inventory\ dummy_t \quad + R_t \quad + \quad \varepsilon_t$

where R_t is the discounted change in far futures price.

Business-day frequency, sample period 19Mar1991 to 3Feb2005
R-squared = .328
Standard error = 42.5 cents per barrel
Durbin-Watson = 1.8

---

[40] As in the main text, t-statistics are in parentheses.

Months to maturity of the far futures contract

For the first half of the sample period of equation 13, the far futures contract matured in only 3 years, which is not truly "far". However, a sample beginning when the NYMEX extended the far futures quote to 6 years ahead (1/24/97) yielded the same basic results.

$$\begin{array}{llll}
\text{(13b)}\quad P_t - P_{t-1} * r^{eq}_t = & .161 & -\ .021 * cy_{t-1} & -.042 * \text{Wednesday } \mathit{inventory\ dummy}_t \\
 & (3.0) & (3.2) & (4.7)
\end{array}$$

$$\begin{array}{ll}
-\ .150\ \mathit{Dumlow}_{t-1} & +.021 * \mathit{Dumlow}_{t-1} * \text{cy}_{t-1} \\
(2.7) & (2.2)
\end{array}$$

$$\begin{array}{l}
+.023 * \mathit{Dumlow}_{t-1} * \text{Wednesday } \mathit{inventory\ dummy}_t \quad + R_t \quad + \quad \varepsilon_t \\
(1.7)
\end{array}$$

where R_t is the discounted change in far futures price.

Business-day frequency, sample period 24Jan1997 to 3Feb2005
R-squared = .324
Standard error = 51.4 cents per barrel
Durbin-Watson = 1.8

Length of lag on convenience yield

The derivation in the text does not specify periodicity, so the question arises whether large changes in lag time on convenience yield affect the asymmetric result of equation 13. I find that the asymmetric result still holds when using a one-month lag on the convenience yield rather than a one-day lag--although t-statistics deteriorated. With the longer lag, the magnitude of the coefficient for price response to net convenience yield fell significantly (at the 95 percent confidence level) for the subsample where the net convenience yield was less than \$4 per barrel.[41]

(13c.) $$P_t - P_{t-1} * r^{eq}_t = \underset{(2.7)}{.106} \; \underset{(3.0)}{- .0149} * cy_{t-22} \; \underset{(5.1)}{-.034} * \text{Wednesday } \mathit{inventory\ dummy}_t$$

$$\underset{(2.5)}{- .100}\; \mathit{Dumlow}_{t-22} \; \underset{(1.8)}{+.0124} * \mathit{Dumlow}_{t-22} * cy_{t-22}$$

$$\underset{(2.6)}{+.021} * \mathit{Dumlow}_{t-22} * \text{Wednesday } \mathit{inventory\ dummy}_t + R_t + \varepsilon_t$$

where R_t is the discounted change in far futures price.

Business-day frequency, sample period 19Mar1991 to 3Feb2005
R-squared = .327
Standard error = 42.5 cents per barrel
Durbin-Watson = 1.8

---

[41] With the longer lag, the sample period is shortened to 19mar1991 – 4feb2005, as in equation 13a.

FIGURE 1

Net Convenience Yield vs DOE crude inventories

(Units: dollars per barrel and days supply)
(Horizontal line at $4/bbl divides ’large’ from ’normal’ convenience yield)

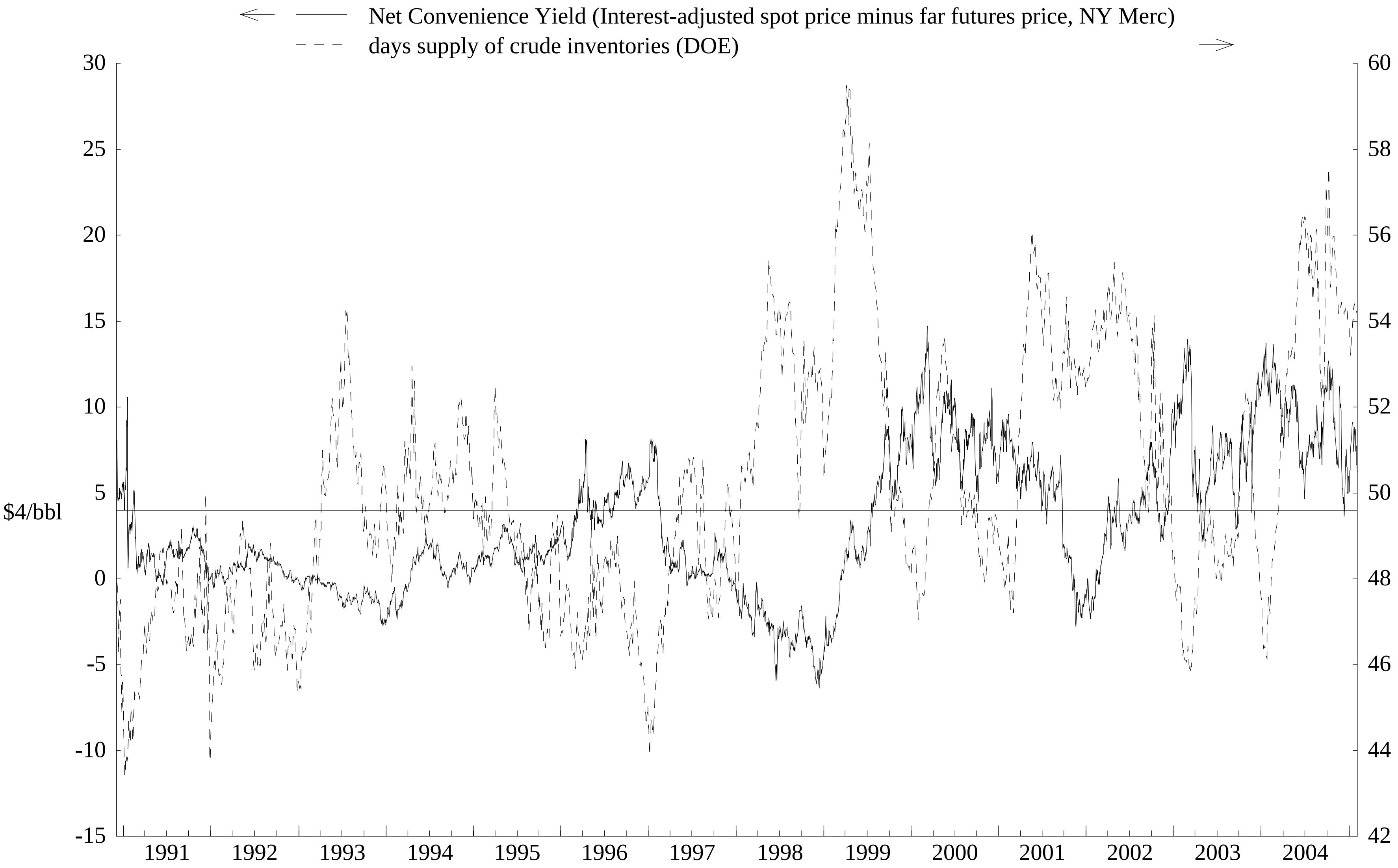

FIGURE 2

Actual vs. equilibrium growth path of far futures price for crude oil

(NY Merc, monthly, dollars per barrel)