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Abstract

Regressions of investment on Tobin's Q are misspecified in the presence of capital gestation lags because they don't distinguish between the value of existing capital and the value of capital at a future date. Current investment should be determined by the anticipated shadow value of capital at the gestation horizon. Under homogeneity conditions analogous to Hayashi [1982], this value is equal to the forecast of an adjusted version of Q. This misspecification helps to explain many pathologies in the literature: attenuated estimates of the coefficient on Q, low $\bar{R}^2$, and serially-correlated errors. Regressions using aggregate data suggest that (1) endogeneity problems associated with the standard regression of investment on Q can can be eliminated by reversing the regression, (2) forecastable changes in Q provide additional information about investment not captured in current Q, and (3) specifications that explicitly account for gestation lags yield capital adjustment costs of a more reasonable magnitude.

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I. Introduction

Perhaps the most appealing aspect of the convex adjustment cost Q-theory of investment is that it reduces a complicated dynamic optimization problem to a startlingly simple relationship between two observable variables. All of the factors that are relevant for determining current investment boil down a single measure: the shadow value of capital, or marginal q. When the conditions outlined in Hayashi [1982] hold, this shadow value is equal to Tobin's Q—a readily measurable variable. However, despite more than two decades of investment-Q regressions, empirical support for this theory continues to be mixed. The general consensus that has emerged from this work is that (1) the relationship between Q and investment is quantitatively small and sometimes statistically insignificant, and (2) that the lack of fit (as measured by $\bar{R}^2$) and apparent partial significance of other variables (such as current cash flow) in these regressions refutes the claim that Q is a sufficient statistic for investment.

Although there have been numerous attempts to explain these failures, little attention has been given to the effects of capital gestation lags.[1] Among other things, these lags represent the time required to prepare the designs, arrange external financing, take delivery and assemble, and to test the capital improvement.

Such lags are not well represented by typical adjustment cost models. These models share the characteristic that newly-purchased capital goods become productive with little or no delay. This may not be a reasonable assumption, since many varieties of capital (such as manufacturing plants and new aircraft) require an extensive planning and/or building process before they affect productive capacity. These requirements alter the timing of the costs and benefits associated with additional investment. In models without gestation lags, a unit of capital purchased today is a perfect substitute for a current unit of productive capital, and therefore has the same shadow value (current marginal q). However, when gestation lags are present, the stream of services associated with investment are delayed, so that current investment is associated with the forecasted shadow value of capital when it will be in place for production.

This paper develops a stochastic model of aggregate investment in which individual firms face distinct gestation lags for planning and building, and

[1] Recent literature has focused on the non-convexity in the adjustment cost function (such as regions of weak concavity and lump-sum transaction costs), and has focused primarily on firm-level analysis. See Abel and Eberly [1994] for a unified theoretical discussion. Caballero, Engel and Haltiwanger [1995] and Barnett and Sakellaris [1998] are representative of the empirical work in this area.

convex costs of capital adjustment. Under these conditions, the true sufficient statistic for current investment is the forecasted shadow value of productive capital at the gestation horizon. This forecast is not generally observable. However, when homogeneity assumptions analogous to Hayashi [1982] hold, I show that it is equal to the anticipated value of an adjusted Tobin's Q measure that can be formed using observable data.

The results of the model indicate that the regressions of investment on Q employed by Summers [1981] and countless others may be seriously misspecified because they use an inappropriate proxy for the anticipated shadow value of new capital. To the extent that the discrepancy between Q_t and the anticipated shadow value is pure noise, the problem can be corrected using the measurement error remedy employed by Erickson and Whited [2000], or by simply reversing the regression.[2] However, when viewed from the perspective of dynamic general equilibrium, the problem is more serious. As the aggregate economy adjusts towards its long run equilibrium, the shadow value of capital gravitates toward its steady state. Therefore, above-average values of Q should be associated with downward anticipated movements of Q, and vice versa. I show that these anticipated movements can be an additional source of endogeneity that (1) further attenuates the estimated coefficient on Q, and (2) causes serial correlation in the error term.

These claims are investigated using regressions on aggregate data. Preliminary tests indicate that regressions of investment on Q_t suffer from endogeneity. This endogeneity appears to be eliminated when the regression is run with Q as the dependent variable. This indicates that the empirical problems associated with investment - Q regressions may be largely attributable to mismeasurement of the shadow value of new capital. However, I argue that the discrepancy between the forward and reverse estimates is simply too large to be consistent with classical measurement error. Instead, I show that forecastable changes in the shadow value of capital are significant when added to the reverse regression specification, which suggests a role for gestation lags. Finally, I show that reverse OLS specifications that explicitly account for gestation lags yield a more reasonable magnitude of adjustment costs than those obtained in previous studies.

An outline for the paper is as follows. In Section II, I perform a preliminary regression analysis that highlights some of the problems associated with regressions of investment on Tobin's Q, and uses endogeneity tests to justify a reverse regression specification. This serves as a useful introduction to the discussion of gestation lags that is the focus of the remainder of the paper,

[2]The approach used by Abel and Blanchard [1986] is another potential remedy.

and as a benchmark for subsequent estimates. In Section III, a model of a firm's investment in the presence of gestation lags is developed that serves as a structural basis for the statistical analysis in Section IV. The final section offers concluding comments.

II. Should Investment be Regressed on Tobin's Q, or is it the Other Way Around?

This section discusses the relationship between the rate of investment and Tobin's Q in OLS regressions. In particular, I consider whether investment or Q should be considered the dependent variable in this relationship. This translates to a discussion of the alternative orthogonality restrictions that are implicit in a "forward" specification of the form:

$$\tilde{g}^K_{t+1} = a + bQ_t + u_{1t}, \tag{1}$$

and its "reverse" counterpart

$$Q_t = c + d\tilde{g}^K_{t+1} + u_{2t}, \tag{2}$$

where $\tilde{g}^K_{t+1} \equiv \frac{\tilde{K}_{t+1}}{\tilde{K}_t} - 1$ is the growth rate in the measured capital stock $\tilde{K}_t$.[3] In order to obtain consistent estimates, the first specification requires orthogonality between Q_t and u_{1t}, while the second requires orthogonality between $\tilde{g}^K_{t+1}$ and u_{2t}.

Both of these forms are motivated by the standard first order condition that links the investment rate to the current shadow value of capital in a model with convex capital adjustment costs. Specifically, let q_t denote the current shadow value of a unit of capital, where the price of new capital is fixed at one unit of the numeraire. Assume that markets are competitive, that production is linearly homogeneous in variable inputs and capital, and that time is continuous. Let capital adjustment costs take the quadratic and linearly homogeneous form

$$\Gamma(\dot{K}_t, K_t) = \frac{\gamma}{2}\left(\frac{\dot{K}_t}{K_t} - \mu\right)^2 K_t,$$

where the parameter γ governs the magnitude of adjustment costs. Then, the first order condition requires the shadow value of an additional unit of capital

[3]The practice of using capital growth as an investment measure differs slightly from previous studies. Using a direct measure of investment per unit of capital has a negligible effect on the investment results, since aggregate depreciation is roughly constant.

today to equal the marginal cost associated with purchasing and installing an additional unit of capital:

$$q_t = 1 + \gamma \left(\frac{\dot{K}_t}{K_t} - \mu \right). \tag{3}$$

Hayashi [1982] shows that the shadow value of capital (which is generally unobservable) is identical to Tobin's Q in this context. If we temporarily ignore the structural justification for the errors u_{1t} and u_{2t}, the first order condition can be easily manipulated to form either the forward or reverse specifications, where the slope coefficients b and d are equal to γ^{-1} and γ, respectively.

Previous studies have exclusively focused on the forward specification rather than the reverse specification. Among other things, this may be motivated by the fact that q_t (which is a function of market prices and technology after optimizing out all variable inputs) is effectively given from the perspective of a single competitive firm. Viewed from this narrow lens, dependency flows from Q_t to $\tilde{g}^K_{t+1}$, but not in the reverse. However, as many have pointed out, neither Q_t nor $\tilde{g}^K_{t+1}$ are exogenous from the viewpoint of dynamic general equilibrium. Therefore, it is not correct to think of causation flowing strictly from one variable to the other; they are mutually determined. On the other hand, the fact that the variables are mutually determined need not be a source of inconsistency, depending on the characteristics of the regression disturbance. For instance, if the disturbance is solely due to classical measurement error in Q_t, then using the reverse specification would be justifiable even though both variables are endogenous. This illustrates that there is no truly compelling reason to prefer either the forward or reverse specification. The choice should be dictated by the source of the regression disturbance.

At this point, I postpone discussion of the theoretical basis for the regression disturbance, and tackle the specification issue from a purely statistical standpoint. I estimate both specifications using a standard framework, which is applied to aggregate data. Then, I assess the practical viability of the two regression forms along two dimensions. First, I compare the magnitudes of the estimates to reasonable standards, such as the implied speed of capital adjustment. Second, I test for the presence of regression endogeneity, employing outside variables that should be exogenous according to theoretical considerations.

1. Data

This paper analyzes aggregate data on the time dimension, rather than longitudinal firm-level data. To a large extent, the puzzling relationship between investment and Q in the literature is ubiquitous to the choice of aggregate or firm-level data. To the extent that the choice is relevant, a number of factors seem to favor aggregate data. First, the assumptions of competitive markets and convex adjustment costs may better describe higher levels of integration. Arguably, the competitive markets assumption is better approximated by aggregate behavior, since idiosyncracies owing to market power, measurement, and other problems become less conspicuous. Second, the assumption of convex adjustment costs is probably more reasonable for the aggregate as well. Work by Doms and Dunne [1998], Abel and Eberly [1994] and others has established the importance of non-convex adjustment costs and heterogeneity for explaining lumpy investment behavior at the firm and plant levels. However, investment is much smoother in the aggregate, where it is disciplined by the effects of integration and consumption smoothing.[4] A final rationale is that the variance of measurement errors should be much smaller in the aggregate than at the firm level, since idiosyncratic factors become irrelevant.

The dataset is constructed using quarterly aggregates for non-farm non-financial U.S corporations over the period from 1959Q3 to 2002Q4. Series for Q, the measured growth rate in capital, and the rate of cash flow are formed using seasonally-adjusted aggregates from the Federal Reserve Board of Governors Flow of Funds Accounts, the Bureau of Economic Analysis (BEA), the Bureau of Labor Statistics (BLS), and Data Resources International (DRI). Data on hours growth, real hourly labor compensation, and output growth are directly from the BLS. The measured capital stock ($\tilde{K}_t$) is formed using quarterly fixed investment expenditures by iterating the standard capital accumulation identity

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + I_t,$$

which implicitly assumes a one period time to build.[5] Following Hall [2001], the aggregate market value of physical capital is the sum of the market values of equity and debt, less the value of all non-capital assets (including liquid assets), residential structures, and inventories. The value of debt is adjusted for changes in the interest rate using the algorithm outlined in Hall [2001]. The tax adjusted series for Q is corrected for the effect of investment tax credits and capital consumption allowances on the effective price of capital, and for the value of remaining depreciation allowances on existing capital.[6] Table 1 reports the first and second sample moments of the data for measured capital

[4]The work of Thomas [2002] favors this argument. Caballero, Engel and Haltiwanger [1995] provide a rationale for aggregate investment lumpiness owing to non-convexities at the firm level.

[5]To minimize the possibility of error associated with the choice of an initial capital value, my pre-sample begins in 1946Q4 at the BEA's measure of the capital stock.

[6]For specific details about these tax corrections, see the data appendix to Millar [2005].

growth and Q, with and without the tax adjustments.

2. Empirical Results for Forward and Reverse Regressions

I report separate estimates for regressions that employ the forward and reverse specifications, using both tax-adjusted and unadjusted series for Q. The forward regression results are shown in the top portion of Table 2. Durbin-Watson (DW) statistics for the both the adjusted and unadjusted data indicate a very high degree of positive serial correlation in the estimated errors. The coefficient of determination $\bar{R}^2$ is modest in both cases, ranging from 0.217 (unadjusted) to 0.244 (adjusted). This verifies that factors other than Q account for most of the variation in capital growth. The magnitude of the coefficients obtained using both the adjusted and unadjusted data are roughly in line with previous OLS estimates of previous studies. The b estimate of .0033 translates to about .013 at an annual frequency, which compares favorably with most previous OLS estimates using aggregate and firm-level data. The estimates for unadjusted and adjusted data imply elasticities of capital growth with respect to Q (at the sample mean) of 0.29 and 0.32, respectively. Although small in magnitude, the estimates are statistically significant at the one percent level or higher after making a heteroskedasticity and autocorrelation adjustment (HAC) to the standard errors. In order to assess the importance of small sample effects, the table reports bias-corrected estimates of the 90% confidence interval generated from a bootstrap simulation. Although the small sample distribution of the estimates is not as tight as the asymptotic approximation, the significance of the results is maintained with little evidence of bias.

The reverse regressions in the bottom portion of Table 2 portray the relationship between investment and Q quite differently. Like the forward specification, the fitted errors exhibit very high autocorrelation. Nonetheless, the magnitudes of the estimated coefficient d imply that the elasticity of capital growth to Q (at the sample mean) is about 1.33 for the unadjusted data, and 1.29 for the adjusted. This more than quadruples the elasticity estimate obtained using the forward specification. Using the more robust HAC standard errors, the estimates of d are significant at the five percent and one percent levels for the unadjusted and tax adjusted data, respectively. Bootstrap simulations indicate that the coefficient on capital growth may be slightly underestimated in a small sample. However, the simulations confirm that the estimates are significant at five percent after making a bias correction.

These results are more in line with previous estimates of γ in the literature than are the results from the forward specification. For instance, estimates in Gilchrist and Himmelberg [1995] imply an estimate for γ around twenty at an annual frequency, which is conspicuously smaller than most empirical estimates. In comparison, the point estimate of d for the unadjusted data implies that γ is around 16.5 for annual data. Another standard that can be used to assess the relative magnitudes of the estimates is the notion of doubling time introduced by Hall [2001]. According to this metric, the value of γ is roughly the number of periods required for capital growth to double in response to a doubling of q. Under this interpretation, the forward estimates suggest doubling times ranging from 75 years (using the unadjusted data), to 100 years (using the adjusted data). The corresponding durations implied by the reverse regression are dramatically lower, ranging from 17 years (unadjusted) to 25 years (adjusted). The latter set of estimates are much closer to independent estimates obtained by Shapiro [1986] and Hall [2004] using an alternative methodology that relies on non-financial data. These studies suggest that the doubling time is two years or lower.

The endogeneity tests reported in the table (labeled *NDG(p)*) also support the reverse specification. The rationale behind this test is that Q and investment should be sufficient for one another using the appropriate regression form. Therefore, no variables in the time t information set should help explain the fitted regression error. The variables I chose to satisfy this orthogonality condition are the current and lagged growth rates in aggregate labor hours, output, real wages, and federal defense expenditures, and the lagged rate of cash flow. The latter is a measure internal funds, which is a particularly familiar suspect for endogeneity. Many studies (beginning with Fazzari, Hubbard and Peterson [1988]) have demonstrated the partial significance of cash flow for explaining investment after controlling for Q_t. The results of the endogeneity test are reported as a p-value for a null of no endogeneity, for each specification. This null is rejected at five percent significance for the forward specification, but cannot be rejected for the reverse specification.[7] The inclusion of the current rate of cash flow in the set of exogenous variables did not substantively alter the test results.

From a purely statistical standpoint, these results provide support for using the reverse regression specification rather than the forward specification. However, the structural explanation for this result remains unclear. One possibility is that the first order condition holds exactly for the true values of Q and capital growth, but that Q is subject to classical measurement error as in Erickson and Whited [2000]. However, this interpretation is unpalatable in a number of respects. Not only must the measurement error exhibit very strong serial correlation to be consistent with the low Durbin-Watson statis-

[7] The test performed is a version of the Hausman test that is robust for the presence of autocorrelation and heteroskedasticity, as outlined in Wooldridge [2000].

tic, but it must be extremely large in relation to the true Q. Specifically, the ratio of signal-to-noise for Q must be around 1/3 in order to explain the discrepancy between the forward and reverse estimates.[8] The measurement error explanation is difficult to reconcile with other facts as well. By definition, the noise in Q must arise from errors in measuring the replacement value of capital, and/or errors in measuring the market value capital. If the error owes to capital mismeasurement, it is difficult to reconcile with the apparent lack of endogeneity in the reverse specification where capital growth is considered independent. If the error owes to market valuation, we must believe that the errors are systematic across firms, and serially correlated. These facts wrestle with the deeply-rooted notions of market efficiency and rational expectations.

An alternative explanation for these results is that there are gestation lags in the capital accumulation process. This explanation can justify measurement errors for Q that are serially correlated, systematic across firms, and large in magnitude. These ideas are developed more formally in the following sections.

III. The Gestation Lag Model

1. The Firm's Problem

The model developed in this section relates investment to Q for a single competitive firm that faces distinct gestation lags for building and planning and a convex adjustment technology for capital. The characterization of building is a special case of the setup in Kydland and Prescott [1982] with a planning stage similar to Christiano and Todd [1995]. Although the model is set in partial equilibrium, it is not possible to strictly limit the analysis to firm-level adjustment. When the assumptions of competitive markets and constant returns to scale hold, the variables that drive the dynamics of q are effectively determined at the market level of integration. Although this is always true under the Hayashi [1982] conditions, it is particularly important in the presence of gestation lags. This is because the firm's current investment decision are based on the anticipated shadow value of productive capital at the gestation

[8] The implied signal-to-noise ratio can be determined as follows. Let the variance of the true value of Q be σ^2, and the variance in its measurement error be σ_ε^2. Using well-known asymptotic formulas for classical measurement error, $\text{plim}(\hat{d}) = \gamma$,

$$\text{plim}(\hat{b}) = \gamma^{-1}\left(1 + \frac{\sigma_\varepsilon^2}{\sigma^2}\right)^{-1}, \qquad \text{and} \qquad \frac{\sigma^2}{\sigma_\varepsilon^2} = \frac{\text{plim}(\hat{b})\text{plim}(\hat{d})}{1 - \text{plim}(\hat{b})\text{plim}(\hat{d})}.$$

Plugging the estimates in Table 2 into this formula yields ratios of 0.28 and 0.33 for the unadjusted and adjusted data, respectively.

horizon. In order to consider the relationship between investment and current q, one must account for adjustments during the gestation phase that are driven by aggregate forces. Rather than develop the problem at both the firm and aggregate levels, my approach is to describe the optimization of a single competitive firm in detail, and to integrate the forces of aggregate adjustment in a stylized, reduced form manner.

Figure 1 depicts a time scale of the investment process under gestation lags. At t, the firm makes an irrevocable commitment to its quantity of productive capital at period $t+P+B$. After P planning periods, the firm purchases the new capital and a building phase of B periods commences.[9] After the total *gestation horizon* of $J=P+B$ periods is complete, the new capital is in place and available for production.

Let I_t represent the firm's investment expenditure and K_t its productive capital stock at time t. The capital accumulation condition takes the form

$$K_{t+i} = K_{t+i-1}\,(1-\delta) + I_{t+i-B}, \tag{4}$$

which incorporates the B period building lag depicted in Figure 1. Note that this accumulation scheme implies an important divergence between the true productive measure of capital K_t and the capital measure based on standard accounting $\tilde{K}_t$. The standard accounting, which maps investment to capital immediately after the expenditure, corresponds to the identity in equation (4) with $B=1$. Due to this construction, the measured stock tends to anticipate the true productive stock because it includes investment expenditures that are still within the building process. In the scheme depicted above, current investment joins the productive capital stock exactly $B-1$ periods after it joins the measured stock. Therefore, the productive measure maps to the accounting measure by the equation $\tilde{K}_{t+1} = K_{t+B}$. This scheme requires the firm to account for a total of J state variables in each period: its current productive capital stock K_t, and its investment commitments I_{t-B+j} for $j = 1,\ldots,J-1$. For simplicity, I represent this set of state variables by the vector $\mathcal{K}_t \equiv (K_t, I_{t-B+1}, \ldots, I_{t+P-1})'$.

Assume that the level of output (gross of adjustment costs) is given by a concave and linearly homogeneous production function $F(K, L)$, where L is a variable input that can be purchased at the market wage $w(z)$. Although the firm considers the anticipated wage path to be given, its dynamics are consistent with equilibrium in the input market. In a dynamic general equilibrium model, these dynamics are closely related to the economy's divergence from

[9] More generally, the investment spending may be spread throughout the building period. See Millar (2004).

its aggregate steady state. These aggregate dynamics essentially govern the movement of

$$z \equiv \ln\left(\boldsymbol{K}_t/\boldsymbol{K}_t^*\right), \tag{5}$$

which measures the degree of departure of the aggregate capital stock $\boldsymbol{K}$ from its target in the absence of gestation lags and adjustment cost frictions, $\boldsymbol{K}^*$.[10] Dynamic stability of the economy ensures that the economy moves towards a steady state where these two values are equal, so z moves towards zero in the long run. Although individual firms account for these endogenous market dynamics in their capital acquisition decisions, they are treated as given. This is because the firm's actions, in isolation, have a negligible impact on the market.

The firm's capital is valued according to the flow of services that it generates throughout its usable life. The value of the service flow in each period (gross of adjustment costs) is the variable profit after accounting for optimal employment of variable inputs given the available capital stock:

$$\max_{L_t}\left[F(K_t, L_t) - w(z_t)L_t\right].$$

Since this problem is linearly homogeneous in (K_t, L_t), its value function takes the form $\pi(z_t)K_t$, where $\pi(z_t)$ represents the concentrated marginal product of capital after accounting for the optimal employment of the variable input. Since this is solely a function of market prices (and indirectly a function of z), it is considered given by the firm. In a steady state, $\pi(z_t)$ is equal to the frictionless rental rate for capital, $R^{B-1}(r+\delta)$. Otherwise, its magnitude is inversely related to z, rising when aggregate capital is scarce relative to a frictionless optimum, and declining when it relatively abundant.

The value of the firm is equal to the discounted sum of all future capital service flows net of the costs of capital adjustment. For simplicity, let adjustment costs in period t be represented by the quadratic function

$$\Gamma\left(I_{t-B+1}, K_t\right) = \frac{\gamma}{2}\left(\frac{I_{t-B+1}}{K_t} - \delta - \mu\right)^2 K_t, \tag{6}$$

which imposes that all the adjustment costs associated with a given investment occur in the period before it joins the productive capital stock. This

[10] In models that possess the balanced growth property, this frictionless target for the capital stock is equal to the stock of effective (or technology-augmented) labor. For example, see King, Plosser, and Rebelo [1988].

form directly associates the magnitude of the adjustment cost with the rate of growth in productive capital between t and $t+1$, since

$$\frac{I_{t-B+1}}{K_t} - \delta = \frac{K_{t+1}}{K_t} - 1$$

by equation (4). The parameter μ is the growth rate in productive capital at which marginal adjustment costs are zero. Letting R represent an appropriate discount factor, the *cum-dividend* value of the firm is given by the function

$$(7) \qquad V\left(\mathcal{K}_t | z_t\right) \equiv \max_{\{I_{t+P+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} R^{-i} E_t \left[\pi(z_{t+i}) K_{t+i} - \Gamma\left(I_{t-B+1}, K_t\right) - I_{t+i}\right],$$

where the maximization is subject to the state vector $\mathcal{K}_t$, and the capital accumulation condition (4). By the envelope theorem, each of the J state variables have an associated shadow value (or co-state variable). This differs from models without gestation lags, where the current capital stock and its shadow value are the only primal-dual pair. Denote the shadow values of $\mathcal{K}_t$ by the vector $\nabla_{\mathcal{K}} V$, and the specific element associated with productive capital by $q_{0,t}$.

After some tedious manipulation, one can show that the first order condition that governs investment at the planning horizon I_{t+P} is

$$(8) \qquad {}_t q_{0,t+J} = R^B + R\gamma\left(g_{t+J}^K - \mu\right),$$

where g_{t+J}^K is the growth rate in productive capital from $t+J-1$ to $t+J$ and ${}_t q_{0,t+J}$ is the expected shadow value of productive capital at $t+J$ given information at t. This condition equates the expected shadow value of the productive capital at the gestation horizon to the future value of the marginal costs associated with the investment plan. These costs include the future value of the outlay required to purchase the new capital at horizon P, and the future value of the marginal adjustment cost paid at horizon $J-1$. This marginal adjustment cost is a linear function of the growth rate in productive capital at the end of the gestation horizon, which maps exactly to the accounting growth rate $\tilde{g}_{t+P}^K$. This condition is a generalization of the more familiar first order condition in (3). In the special case of no planning or building ($P = B = 0$), it collapses to a discrete-time equivalent of the standard condition.

Note that today's investment expenditures are determined by the forecasted shadow value of capital at the building horizon, rather than the current shadow value of capital. This is because investment commitments do not convey the same future benefits as productive capital. Intuitively, $q_{0,t}$ represents the present value of all expected future benefits associated with a unit of productive capital from the current period onward. This can be shown formally by taking the partial derivative of the value function with respect to K_t to reveal that

$$q_{0,t} = \sum_{h=0}^{\infty} \left(\frac{R}{1-\delta} \right)^{-h} E_t \left[\pi_{t+h} - \frac{\partial \Gamma_{t+h}}{\partial K_{t+h}} \right], \tag{9}$$

where Γ_{t+h} is the adjustment cost at $t + h$.[11] The expected future benefit associated with capital ownership in each period is its concentrated marginal product π_{t+h}, net of the marginal effect of an additional unit of capital on the adjustment cost Γ_{t+h}. The discount factor R is divided by the factor $(1 - \delta)$ in order to account for physical depreciation in the quantity of capital over time. Since a unit of investment committed today does not become productive until the gestation horizon, its anticipated benefits are the same as a unit of productive capital in J periods, ${}_tq_{0,t+J}$.

These differences help to explain many of the empirical failures of regressions of investment on Q. For now, ignore any potential problems associated with using Q_t as a proxy for the shadow value of productive capital $q_{0,t}$, and assume that the two variables can be used interchangeably. Then, the first order condition (8) can be rearranged to yield the linear relationship

$$\tilde{g}^K_{t+1} = \left(\mu - \frac{R^{B-1}}{\gamma} \right) + \frac{1}{R\gamma} {}_{t-P}q_{0,t+B} \tag{10}$$

where I have substituted $\tilde{g}^K_{t+1}$ for the growth rate g^K_{t+B}. When the forward regression in (1) is interpreted under the lens of this structural relationship, the regression coefficients and errors are the reduced forms

$$a = \left(\mu - \frac{R^{B-1}}{\gamma} \right), \qquad b = \frac{1}{R\gamma}, \qquad \text{and} \qquad u_{1t} = b \left({}_{t-P}q_{0,t+B} - q_{0,t} \right).$$

If $q_{0,t}$ were orthogonal to the structural error u_{1t}, such a regression would yield a consistent estimate of b. However, this is unlikely. In a dynamic general equilibrium, q_0 tends to revert towards its steady state as part of the economy's broad adjustment to short-run macroeconomic disequilibrium (Romer [1996], Kimball [2003]). In this case, the steady state value of q is R^B,

[11] A finite solution for $q_{0,t}$ requires the transversality condition

$$\lim_{h \to \infty} \left(\frac{R}{1-\delta} \right)^{-h} E_t \left[\pi_{t+h} - \frac{\partial \Gamma_{t+h}}{\partial K_{t+h}} \right] \to 0.$$

which represents the replacement value of the initial investment outlay after B building periods. When q is above (below) this steady state, firms will expect this broad process of adjustment to pull it downward (upward). Hence, it is likely that $q_{0,t}$ covaries negatively with the structural error u_{1t}, which is closely related to the direction of its future movement. This endogeneity causes an attenuation of the coefficient b. Since anticipated changes in q are likely to be related across periods, it can also contribute to serial correlation in the fitted residuals.

As a simple illustration of this point, consider a case with only one building period and no planning requirement. Suppose that the dynamics of z_t are described by the $AR(1)$ process

$$z_{t+1} = \eta_{zz} z_t - \eta_{z\epsilon} \epsilon_{t+1}, \tag{11}$$

where $\eta_{zz} \in (0,1)$, ϵ_{t+1} is an $i.i.d$ disturbance to $\boldsymbol{K}^*_{t+1}$. To a first-order approximation, this is the process that would govern the economy's dynamic adjustment to a permanent technology shock in a standard RBC model with quadratic adjustment costs and $B = 1$, in the neighborhood of the balanced growth path.[12]. In Appendix B, I show that the anticipated dynamics of $q_{0,t}$ can then be represented by

$$_t q_{0,t+1} - R = \eta_{zz} \left(q_{0,t} - R \right), \tag{12}$$

to a first order approximation, in the vicinity of the balanced growth path. Therefore, the shadow value of capital reverts towards its steady state at a rate of decay equal to η_{zz} along the saddle path of adjustment for capital. Using this dynamic equation, the structural error $u_{1,t}$ for the one period time to build case is

$$u_{1t} = b(1 - \eta_{zz})(R - q_{0,t}).$$

This is negatively correlated to the regressor, which causes the estimate b to be biased downward. Indeed, for this special case, it can be shown that the true coefficient on $q_{0,t}$ is $\gamma\eta_{zz}$, so that the adjustment cost parameter is not strictly identified.

Gestation lags can pose other potential problems in the presence of temporary shocks. In particular, suppose that there are unanticipated disturbances to the factor z (emanating from the frictionless target $\boldsymbol{K}^*$) that have a duration shorter than the gestation horizon. Since such shocks affect the relative scarcity of capital in the short run, the market value of existing capital adjusts to reflect this scarcity. While the shock is active, this affects both the value

[12]For details, see Campbell [1994].

of the service flow $\pi(z_t)$, and the value of the stream of all future services $q_{0,t}$. However, it is unlikely that such shocks would prompt much investment, since their effect on cash flow and q dies off before new capital can be put in place. In Millar [2005], I provide evidence of temporary disturbances that do not affect investment, but increase Q and cash flow. These transitory shocks are likely to have two effects on the relationship between current investment and $q_{0,t}$. First, they serve as a source of noise in the relationship between investment and $q_{0,t}$ because they are orthogonal to investment. This effect acts like classical measurement error in the forward regression specification, attenuating the estimate of b and diminishing the $\bar{R}^2$. Second, they cause serial correlation in the fitted regression error to the extent that these fluctuations persist within the gestation horizon.

2. The Relationship between the Shadow Value of Productive Capital and Tobin's Q in the Presence of Gestation Lags

Now consider the relationship between Tobin's Q and the current shadow value of productive capital in the presence of gestation lags. Hayashi [1982] established the equivalence of marginal q and Tobin's Q in continuous time with no gestation lags, in the special case where output and adjustment costs are linearly homogeneous in capital and markets are competitive. As it turns out, this result cannot be applied to models with gestation lags without some modification. This stems from the fact that units of committed investment and units of productive capital are not perfect substitutes. Each has a distinct shadow value that is reflected in the current market value of the firm. As a result, the standard measure of Tobin's Q, which is formed as the current market value of all capital divided by the replacement value of all capital, is an adulterated measure of $q_{0,t}$.

More formally, let $q_{j,t}$ represent the shadow value of an investment committment that is j periods from becoming productive, for $j = 1, \ldots, J-1$. In Appendix A, I show that the value of the firm can be decomposed into parts associated with productive capital and each investment commitment that remains in its gestation process:

$$V_t = \begin{cases} q_{0,t}K_t + \sum_{j=1}^{J-1} q_{j,t} I_{t-B+j} & J > 1, \\ q_{0,t}K_t & J = 1. \end{cases}$$

This reveals that the value of the firm differs from the value of productive capital stock by the value associated with the firm's investment commitments.[13]

[13]Note that the current investment choice I_{t+P} is not reflected in firm value because the first order condition sets its marginal contribution firm value at zero.

Hence, $q_{0,t}$ generally differs from measures of average Q (for example, V_t/K_t or $V_t/\tilde{K}_t$), except in the special case where $J=1$.

Fortunately, there is an amended version of Q that will give a pure reflection of the value of productive capital under certain conditions. In the appendix, I show that the amended measure $\bar{Q}_t$, defined as

$$\bar{Q}_t \equiv \begin{cases} \frac{V_t}{K_t} & 0 \leq B \leq 1 \\ \frac{V_t}{K_t} - \frac{\sum_{j=1}^{B-1} R^{B-j} I_{t-B+j}}{K_t} & B > 1, \end{cases}$$

will, under certain assumptions, have the same anticipated value as the true shadow value of productive capital at horizons of J or longer:

$${}_t q_{0,t+J+i} = {}_t \bar{Q}_{t+J+i},$$

for all $i \geq 0$. Note that this equivalence only holds in expectation for horizons of J or longer.[14] Although this is a much weaker equivalence than the result of Hayashi [1982], it can still be exploited to explain investment at the planning horizon. This is because the forecast error in $\bar{Q}_{t+J}$ is orthogonal to the investment plan, which depends only on information available at time t.

The amended measure $\bar{Q}$ is formed by deducting the current value of all purchased capital that is within the building process (after compensating for foregone interest), then dividing by the stock of productive capital. The intuition for this result is that the current shadow value of a firm's productive capital should be measured using only the portion of the firm's market value that is associated with productive capital. The value of the firm includes the replacement value of all capital that has been acquired, whether productive or unproductive. Therefore, the anticipated replacement value of all capital commitments that are within their building phase must be deducted from firm value in order to obtain an appropriate measure of the value of productive capital. Note that realizations of the components of $\bar{Q}$ are readily observable after the fact. Therefore, an econometrician armed with knowledge of the building horizon could form $\bar{Q}$ using readily available data, and the appropriate accounting scheme for productive capital. In the following section, I exploit this idea to form regressions of Q and investment that account for gestation lags of varying duration.

[14] In the appendix, I show that the discrepancy between the two forecasts at horizons shorter than J is a weighted average of anticipated forecast errors in $q_{0,t}$.

IV. Empirical Analysis

In this section, I tackle two issues using regression analysis. First, I provide evidence for gestation lags, by showing forecasted changes in Q provide explanatory power in a reverse regression of Q on measured capital growth. This provides evidence for the gestation lag story that distinguishes it from pure measurement error. In the second part, I perform regressions of Q on capital growth that explicitly account for the effects of gestation lags.

1. Evidence for Gestation Lags

In order to test for the existence of gestation lags it is necessary to determine whether expected future values of Q contain information for investment that is not contained in current Q. The null hypothesis is that the standard linear Q model without gestation lags holds, and that the orthogonality condition implicit in equation (2) is appropriate. The alternative is a specification that is consistent with gestation lags. In this case, a simple rearrangement of equation (10) suggests the backwards linear relationship

$$q_{0,t} = R\left(R^{B-1} - \gamma\mu\right) + R\gamma\tilde{g}^K_{t+1} - \left(q_{0,t+B} - q_{0,t}\right) + \left(q_{0,t+B} - {}_{t-P}q_{0,t+B}\right). \tag{13}$$

This specification suggests that one might test for the existence of gestation lags using regressions of the form

$$Q_t = \check{a} + \check{b}\tilde{g}^K_{t+1} + \check{c}\left(\Delta^B Q_{t+B}\right) + \check{u}_t, \tag{14}$$

for alternative values of $B > 1$, where Δ^d is the d^{th} difference operator $1 - L^d$. A rejection of a null hypothesis that $\check{c}$ is zero would offer evidence against the validity of the forward specification. An estimate of $\check{c}$ statistically indistinguishable from a value of negative one would be consistent with (13), offering direct evidence for gestation lags as the source of this failure.

Note that the structural specification in equation (13) is not valid for OLS regression. Although the forecast error is orthogonal to capital growth under the assumptions of the gestation lag model, it is never orthogonal to $q_{0,t+B}$. This motivates an instrumental variables approach, where valid instruments can include $\tilde{g}^K_{t+1}$ and any variable available at time $t-P$, with the exception of Q_t.[15] Alternative estimates are obtained using separate instrument sets for the cases $P=0$ and $P=2$. The instruments selected from each information

[15]Note that Q_t could be included for the case where $P=0$ under the specification in (13). This is because the form in (13) allows no structural justification for the error under the null. To be fair to the null, an *i.i.d* structural error is allowed for by never including Q_t in the instrument set.

set are $\tilde{g}^K_{t+1}$, the nearest lag of Q_t, and the nearest eight lags of growth in real hourly labor compensation. Note that this set overidentifies the coefficient $\check{c}$. Overidentification tests are performed to test both the reasonableness of the chosen restrictions and the validity of the time to plan restriction implicit in the choice of P.[16]

Estimates of equation (14) are reported in Table 3 for no planning period and Table 4 for a planning period of two quarters. Each table reports results for building horizons ranging from one to eight quarters. In each case, separate estimates are obtained using IV and a two-step efficient GMM estimator that employs a HAC weighting matrix. For brevity, only estimates using the tax adjusted Q are reported. Overall, the IV results are roughly similar to those obtained from the more efficient GMM. All specifications feature a high degree of positive serial correlation in the fitted errors. Evidently, there is some information content in these residuals. Although both estimators are consistent, the GMM estimates of the parameter $\check{b}$ are lower (and more precisely estimated) than IV estimate in every case. IV estimates of $\check{b}$ range from around 87 to as low as 58.7, while GMM estimates range from 82.38 to as low as 40.64. These estimates are often insignificant using the robust HAC errors, except for low values of B, and are rarely significant at any level according to bootstrap simulations. However, this lack of significance should not be interpreted negatively. Among other things, it may be a symptom of multicollinearity between the instrumented $\Delta^B Q_{t+B}$ and $\tilde{g}^K_{t+1}$. If this is the case, the lack of precision in the estimate may reflect the ability of the anticipated forward change in Q to explain variation in capital growth. This is entirely consistent with the gestation lag theory, and largely inconsistent with the alternative.

Although a high degree of autocorrelation in the regression error is often viewed in a negative light, it is a feature that one would expect in the presence of a multi-period gestation lag. This is because the fitted errors have a structural interpretation as the cumulative forecast error in $q_{0,t}$ relative to information at $t - P$, which is serially correlated for $J > 1$ provided that the underlying process z is persistent. Serially correlated residuals are inconsistent with most convex formulations of the Q model.[17]

Broadly, the estimates of $\check{c}$ seem to favor the existence of gestation lags.

[16]For the IV regressions, the test of the overidentifying restrictions is a version of the score tests outlined in Wooldridge [1995] that is HAC robust. The test for the GMM estimates is a J-test that employs a HAC form of the weighting matrix.

[17]It is not impossible, however. Structural explanations include serially correlated measurement error, or an autocorrelated "target" shock to the adjustment costs function. Autocorrelation could also be induced by a combination of estimation bias (perhaps owing to endogeneity) and persistence in $\tilde{g}^K_{t+1}$.

The estimates obtained for both IV and GMM are negative in every case, and tend to decline in absolute magnitude as the horizon B is increased. Most of the values are reasonably close to negative one, particularly those obtained using GMM. For the no planning horizon case, the estimates are statistically significant at ten percent or higher using HAC robust standard errors out to $B = 5$ for the IV estimate, and significant at one percent for all values of B using robust GMM. There is steady improvement in the precision of the estimate as the building horizon increases, undoubtedly due to a concurrent improvement in instrument power. Although they are not nearly as precise, the bootstrap confidence intervals largely confirm that these results are applicable to a small sample. Indeed, the simulations reveal that there is a (positive) small sample bias in the estimate of $\check{c}$ at each building horizon that may work against rejection of the null. Results obtained using a two period planning horizon are generally less emphatic. The IV estimates are much less precisely estimated (and smaller in magnitude) beyond $B=3$, although the instruments appear to have more explanatory power in terms of the first-stage F statistic. Despite this drop in precision, the GMM estimates are still significant at levels of five percent or higher out to $B=7$. The overidentifying restrictions cannot be rejected in any of the regressions.

2. Estimates Using a Modified Statistical Approach

A final empirical objective is to develop a specification for the investment - Q relationship that does not suffer from the problems related to gestation lags. Rearranging the first order condition (8), and using the results of Corollary 2 (Appendix A) to replace $q_{0,t+J}$ yields the linear specification

$$\bar{Q}_t = \bar{a} + \bar{b} g_t^K + \bar{u}_t,$$

where the coefficients $\bar{a}$ and $\bar{b}$ have the structural form

$$\bar{a} = R\gamma + R^B - \mu, \qquad \bar{b} = R\gamma, \qquad \bar{u}_t = \bar{Q}_t - {}_{t-J}\bar{Q}_t,$$

and where g_t^K is the growth rate in productive capital entering period t. Since the growth rate in productive capital corresponds to investment B periods earlier, this is a projection of current investment onto a future realization of Q. The primary merit of this formulation is that it is amenable to OLS estimation under the assumptions of the model, since the cumulative forecast error in $\bar{Q}$ over the gestation horizon is orthogonal to current productive capital growth. Note that the dependent variable is the adjusted measure $\bar{Q}$, which is calculated after deducting the replacement value of investment commitments that are within the building process.

Admittedly, the adjustment cost parameter γ is not strictly identified in equation 2. However, provided that the discount factor takes a reasonable value (which should be close to one at the quarterly frequency of the data), it should be approximately equal to γ.[18] Hence, one metric for model evaluation should be the size of the $\bar{b}$ estimate, which should suggest a reasonable magnitude of adjustment costs.

OLS results obtained using both tax-adjusted and unadjusted data for $\bar{Q}$ are reported in Table 5, for building horizons of up to eight quarters. Table 6 reports corresponding measures of fit and endogeneity tests. Each specification characteristically has a modest $\bar{R}^2$, and fitted errors that exhibit a high degree of autocorrelation. Given the structural interpretation of the model, these properties suggest that the cumulative forecast error in $\bar{Q}$ accounts for most of its variation. Since serial correlation in the cumulative forecast error would be expected, this provides some support for the model. Endogeneity tests conducted using the same exogenous variables as the estimates in Table 2 do not suggest any problems with the assumption of orthogonality between the residual and capital growth.

It is striking that the point estimates of $\bar{b}$ rapidly decrease in magnitude with the building horizon, and are consistently lower than the estimates using current q. This property need not favor a gestation lag view over standard convex adjustment costs, since the declining coefficients are observationally equivalent to the anticipated movements in q along a negatively-sloped adjustment path. Nonetheless, this steady pattern of decline provides some broad support for the notion of convex adjustment on the aggregate. The estimates using unadjusted data decline from about 62 at $B = 1$ to 39 at $B = 8$. The estimates roughly suggest an elasticity of capital growth to the q forecast (at the sample mean) between 1.40 and 2.04. The corresponding estimates using adjusted data are higher, ranging between 93 (1.39 elasticity) to 61 (2.08 elasticity). Although the precision of the estimates for $\bar{b}$ are similar for all values of B, the estimates seem to decline in significance as the building horizon is raised. Estimates using unadjusted data and adjusted data are statistically significant at 10 percent or lower out to $B = 6$ and $B = 8$, respectively. Despite an apparent negative small sample bias in the estimates, the corrected bootstrap simulations broadly confirm the inferences from the large sample approximations after making an appropriate adjustment for autocorrelation.

[18] The initial assumption of a constant ex ante discount factor R is maintained throughout.

V. Discussion

This paper presents a model of investment in the presence of gestation lags and convex capital adjustment costs with gestation lags. The model addresses many of the previous empirical criticisms of Q theory. According to the model, the forecast of Q at the true gestation horizon is the true sufficient statistic for current investment, while current Q is a noisy indicator. Among its merits are (1) the ability to explain why Q might be noisy at high frequencies, yet still have a strong low frequency relationship to investment, and (2) why regressions of investment on Q yield results that seem inconsistent with models that do not incorporate gestation.

Although the assumption of a common gestation lag for all types of capital may be strong, the model does provide a useful framework for thinking about the empirical problems posed by gestation lags. Millar [2005] discusses how gestation lags can remain important in the presence of capital goods without short building lags, provided that these goods are imperfect substitutes for other capital goods with long horizons.

So how reasonable are the statistical results of this paper compared to the standard approach? One possible metric is Hall's notion of the doubling time for capital growth in response to a doubling of q. The OLS results in Table 2 corresponding to the standard forward specification suggest a doubling time of about 75 years for the unadjusted data, and about 100 years using the adjusted data. The corresponding figures in Table 5 suggest much more modest doubling times, ranging from between $15\frac{1}{2}$ and $9\frac{3}{4}$ years for the unadjusted data, and between $23\frac{1}{4}$ and $15\frac{1}{4}$ years for the adjusted data. This is a substantial improvement, even after accounting the building horizon. But what is a reasonable benchmark for the quadratic adjustment cost parameter? The estimates of Shapiro [1986] and Hall [2004], which rely on GMM estimates of dynamic Euler conditions, suggest "doubling times" of less than two years. Although these estimates are a good bit lower than the estimates listed in Table 5, they are not completely out of line with the confidence bands associated with these estimates. Therefore, the chasm between the adjustment cost estimates obtained by directly estimating Euler equations and the estimates obtained using Q regression may not be as wide as previously reckoned.

Nonetheless, the results in this paper fall well short of a complete reconciliation. There remains a fairly large gap between what many would consider a reasonable magnitude of adjustment costs and the point estimates obtained in this paper. The statistical framework is also somewhat dissatisfying because it provides little guidance for discerning the appropriate duration of the gestation lag. On this last point, there is little hope for determining an appropriate gestation lag using a framework that relies solely on investment and Q data. This is because the key feature of the results in Table 5—a positive relationship between investment and leads of Q that diminishes with the time horizon—is observationally equivalent to what one would expect in a convex adjustment cost model without lags. Nonetheless, estimates of the building lag obtained using other methodologies can provide guidance for determining which estimate of the adjustment cost is most appropriate. Millar [2005] argues that the key insight for estimating the building lag is that it corresponds in duration to the time between the first outlay associated with a given capital addition and the time when the completed capital addition begins to affect production. Using aggregate data, this delay is estimated to be as long as eight quarters. Assuming that this estimate of the building lag is accurate, the lowest estimates of the adjustment cost parameter obtained in this paper may not be unreasonable.

A. Proofs

Theorem 1 *Assume that the firm's value function takes the form in (7), where the value function for gross cash flow resulting from the intra-temporal problem is linearly homogeneous in K_t. Further, let $\Gamma(I_{t-B+1}, K_t)$ be convex in its first argument and linearly homogeneous in both of its arguments. Then the value function is linearly homogeneous in the vector $\mathcal{K}_t$.*

Proof: Consider the following Bellman representation of the problem in (7):

$$V\left(\mathcal{K}_t|z_t\right) \equiv \pi(z_t)K_t - \Gamma\left(I_{t-B+1}, K_t\right) - I_t + R^{-1}E_t\left[V\left(\mathcal{K}_{t+1}|z_{t+1}\right)\right],$$

where I allow Let the value function at $t+1$ be linearly homogeneous in $\mathcal{K}_{t+1}$. Given this, it is sufficient to show that $V\left(\beta\mathcal{K}_t \mid z_t\right) = \beta V\left(\mathcal{K}_t \mid z_t\right)$ for any positive β. Multiply the vector $\mathcal{K}_{t+1}$ by β. By linear homogeneity, this multiplies V_{t+1} by β, and also multiplies the vector $\mathcal{K}_t$ by the proportion β.[19] *By assumption, each of the first three terms in V_t are linearly homogeneous in $\mathcal{K}_t$, so each increases by the multiple β. Therefore, all four of the terms that compose V_t increase by the proportion β, which establishes the result.*

Corollary 1 *Let the assumptions of Theorem 1 hold for a non-zero gestation horizon J. Then*

$$\begin{cases} q_{0,t}K_t + \sum_{j=1}^{J-1} q_{j,t}I_{t-B+j} & J > 1, \\ q_{0,t}K_t & J = 1. \end{cases}$$

Proof: This follows from Theorem 1 by applying Euler's theorem for homogeneous functions to yield that $V = \nabla_{\mathcal{K}}V \cdot \mathcal{K}$, using the definition of $\mathcal{K}$, and defining $q_{j,t} = \frac{dV_t}{dI_{t-B+j}}$ for $j = 1, \ldots, J-1$.

Corollary 2 *Let the assumptions of Theorem 1 hold for a non-zero gestation horizon J. Define:*

$$\bar{Q}_t \equiv \begin{cases} Q_t & 0 \leq B \leq 1 \\ Q_t - \frac{\sum_{j=1}^{B-1} R^{B-j} I_{t-B+j}}{K_t} & B > 1, \end{cases}$$

where $Q_t \equiv \frac{V_t}{K_t}$. Then, the following holds conditionally for any $i \geq 0$:

$$_tq_{0,t+J+i} = {}_t\bar{Q}_{t+J+i}.$$

[19] The latter claim follows directly for the investment commitments I_{t-B+j}, $j = 2, \ldots, J-1$. Proportional changes in K_t and I_{t+B+1} must also result because $K_{t+1} = (1-\delta)K_t + I_{t-B+1}$.

Proof: *First, consider the case where* $i = 0$. *Move the result of Corollary 1 forward* J *periods, and taking the expectation conditional on time* t *information obtains that*

$$ {}_t q_{0,t+J} = \frac{{}_t V_{t+J} - \sum_{j=1}^{J-1} E_t\left[q_{j,t+J} I_{t+P+j}\right]}{K_{t+J}}, $$

where K_{t+J} *is outside of the expectation because it is known at* t. *Since the right hand side of this equation is the time* t *expectation of* $\bar{Q}_{t+J}$, *it is sufficient to show that*

$$ E_t\left[q_{j,t+J} I_{t+P+j}\right] = \begin{cases} R^{B-j} E_t\left[I_{t+P+j}\right] & j = 1, \ldots, B-1 \\ 0 & j \geq B. \end{cases} $$

Applying the law of iterated expectations to $E_t\left[q_{j,t+J} I_{t+P+j}\right]$ *yields that*

$$ E_t\left[q_{j,t+J} I_{t+P+j}\right] = E_t\left[\left({}_{t+j} q_{j,t+J}\right) I_{t+P+j}\right], $$

for $j = 1, \ldots, J-1$, *since* I_{t+P+j} *is observable at time* $t+j$. *Using the envelope theorem and recursive substitution, the shadow values associated with the firm's investment commitments at* $t + J$ *can be derived as*

$$ q_{j,t+J} = \begin{cases} R^{-j}\left[{}_{t+J} q_{0,t+J+j} - R^B - R\Gamma_{1,t+J+j-1}\right] & j \geq B \\ R^{-j}{}_{t+J} q_{0,t+J+j} - R^{-(j-1)}\Gamma_{1,t+J+j-1} & j = 1, \ldots, B-1. \end{cases} $$

Taking the conditional expectation of $q_{j,t+J}$ *at* $t + j$ *for each case, applying the law of iterated expectations, and using the equilibrium condition* (8) *yields that*

$$ {}_{t+j} q_{j,t+J} = \begin{cases} R^{-j}\left[{}_{t+j} q_{0,t+j+J} - R^B - R\Gamma_{1,t+j+J-1}\right] = 0 & j \geq B \\ R^{-j}\left[{}_{t+j} q_{0,t+j+J} - R\Gamma_{1,t+j+J-1}\right] = R^{B-j} & j = 1, \ldots, B-1, \end{cases} $$

where the conditional expectation operator is not applied to $\Gamma_{1,t+j+J-1}$ *because it is known at* $t+j$. *Substituting the above into* $E_t\left[q_{j,t+J} I_{t+P+j}\right]$ *for* $j = 1, \ldots, J-1$, *and simplifying, proves the desired result for* $i = 0$.

To prove the result for $i > 0$, *take the time* t *expectation of each side of the equality* ${}_{t+i} q_{0,t+J+i} = {}_{t+i}\bar{Q}_{t+J+i}$, *and use the law of iterated expectations.*

COROLLARY 3 *Let the assumptions of Theorem 1 hold for a non-zero gestation horizon* J. *Define*

$$ e_t^{q_0} \equiv q_{0,t} - {}_{t-J} q_{0,t} $$

as the cumulative forecast error in $q_{0,t}$ *relative to information at* $t + J$. *Then,*

$$ {}_{t+i}\bar{Q}_{t+J} - {}_t\bar{Q}_{t+J} = \begin{cases} {}_{t+i} e_{t+J}^{q_0} & i = 1 \\ {}_{t+i} e_{t+J}^{q_0} + \sum_{j=1}^{i-1} R^{-j}{}_{t+i} e_{t+J+j}^{q_0} \frac{I_{t+P+j}}{K_{t+J}} & 1 < i \leq J. \end{cases} $$

Proof: Using the results of Corollary 1, the results in equation (2), and the first order condition (8), the value of the firm at $t+J$ reduces to

$$V_{t+J} = q_{0,t+J}K_{t+J} + \sum_{j=1}^{J-1} R^{-j} \left({}_{t+J}q_{0,t+J+j} - {}_{t+j}\, q_{0,t+J+j}\right) I_{t+P+j} + \sum_{j=1}^{B-1} R^{-j} I_{t+P+j}.$$

Simplifying using the definition of $\bar{Q}_{t+J}$ yields

$$\bar{Q}_{t+J}K_{t+J} = q_{0,t+J}K_{t+J} + \sum_{j=1}^{J-1} R^{-j} \left[{}_{t+J}q_{0,t+J+j} - {}_{t+j}\, q_{0,t+J+j}\right] I_{t+P+j}.$$

Using the facts that $I_{t+P+j} \in \Omega_{t+i}$ for $i \leq j$ and the law of iterated expectations, it can be established that

$$E_{t+i}\left[\left({}_{t+J}q_{0,t+J+j} - {}_{t+j}\, q_{0,t+J+j}\right) I_{t+P+j}\right] = \begin{cases} E_{t+i}\left[{}_{t+J}q_{0,t+J+j} - {}_{t+j}\, q_{0,t+J+j}\right] I_{t+P+j} & i < j \\ E_{t+i}\left[I_{t+P+j}E_{t+j}\left({}_{t+J}q_{0,t+J+j} - {}_{t+j}\, q_{0,t+J+j}\right)\right] = 0 & i \geq j. \end{cases}$$

Taking the conditional expectation of this expression at $t+i$, noting that $K_{t+J} \in \Omega_t$, and using the facts above can obtain that

$${}_{t+i}\bar{Q}_{t+J}K_{t+J} = {}_{t+i}\, q_{0,t+J}K_{t+J} + \sum_{j=1}^{i-1} R^{-j} E_{t+i}\left[{}_{t+J}q_{0,t+J+j} - {}_{t+j}\, q_{0,t+J+j}\right] I_{t+P+j},$$

Deducting the result ${}_t\bar{Q}_{t+J} = {}_tq_{0,t+J}$ from both sides, and dividing by K_{t+J} shows that

$${}_{t+i}\bar{Q}_{t+J} - {}_t\bar{Q}_{t+J} = {}_{t+i}\, q_{0,t+J} - {}_tq_{0,t+J} + \sum_{j=1}^{i-1} R^{-j} \left[{}_{t+i}q_{0,t+J+j} - {}_{t+j}\, q_{0,t+J+j}\right] I_{t+P+j}.$$

The desired result can then be determined by noting that ${}_{t+i}e^{q_0}_{t+J+j} = {}_{t+i}q_{0,t+J+j} - {}_{t+j}q_{0,t+J+j}$ for all $i \geq j \geq 0$ by applying the law of iterated expectations to the definition of the cumulative forecast error $e^{q_0}_t$.

B. System Dynamics for the One Period Time to Build Case

As a first step, equation (9) can be rearranged into the iterative form

$$q_{0,t} = \pi\left(z_t\right) - \gamma\left(g^K_{t+1} - \mu\right)\left(g^K_{t+1} + \delta\right) + \frac{\gamma}{2}\left(g^K_{t+1} - \mu\right)^2 + (1-\delta)R^{-1}{}_tq_{0,t+1}.$$

Using the first order condition (8), this reduces to the nonlinear difference equation:

$$\begin{aligned} q_{0,t} &= \tilde{f}^q\left(z_t, {}_tq_{0,t+1}\right) \\ &= \pi\left(z_t\right) + \left(\frac{R}{2\gamma} - \mu - \delta\right) + \frac{1}{R}\left(\mu + \delta - \frac{R}{\gamma}\right){}_tq_{0,t+1} \\ &+ (1-\delta)_t q_{0,t+1} + \frac{1}{2\gamma R^2}\left({}_tq_{0,t+1}\right)^2. \end{aligned}$$

For simplicity, assume that $\pi(z_t) = R^{B-1}(r+\delta)e^{-az_t}$, which can be regarded as a first-order log-linear approximation of the function around its steady state. It can be shown that the two variable system composed of (11) and $q_{0,t}$ (above) has steady states

$$(z^{\mathrm{SS1}}, q^{\mathrm{SS1}}) = (0, R) \qquad \text{and} \qquad (z^{\mathrm{SS2}}, q^{\mathrm{SS2}}) = (0, R[R + 2\gamma(r-\mu)]).$$

The first of these steady states is saddle path stable provided that $r > \mu$, with q serving as the jump variable. The second steady state is a source, with both q and z acting as historical variables. Since q is naturally a forward-looking variable, the first steady state is the relevant one.[20] According to the implicit function theorem, a function

$$_tq_{0,t+1} = f^q(z_t, q_{0,t})$$

exists in some neighborhood of this steady state, since $\tilde{f}_2^q(0, R) = \frac{1+\mu}{1+r} \neq 0$.

Linearizing the system in the vicinity of the first steady state using the implicit function theorem, one obtains

$$\begin{bmatrix} _tz_{t+1} \\ _tq_{0,t+1} - R \end{bmatrix} \approx \begin{bmatrix} \eta_{zz} & 0 \\ f_1^q & f_2^q \end{bmatrix} \begin{bmatrix} z_t \\ q_{0,t} - R \end{bmatrix},$$

where $f_1^q = -\frac{R(r+\delta)}{1+\mu} > 0$ and $f_2^q = \frac{1+r}{1+\mu} > 1$ are the partial derivatives of f^q with respect to its first and second arguments. This system has a stable eigenvalue $\lambda_1 = \eta_{zz}$ and an unstable eigenvalue $\lambda_2 = f_2^q$. Decoupling this system for the saddle path corresponding to this stable eigenvalue justifies equation (12).

[20]It can be shown that this is the sole steady state for the aggregated economy.

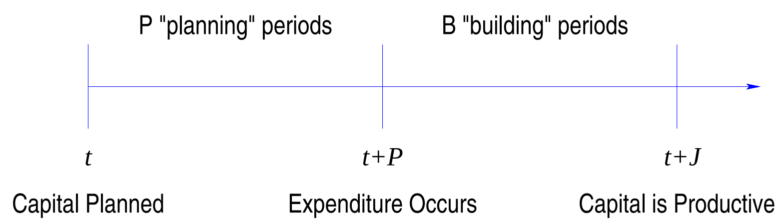

Figure 1: Time scale depiction of investment with gestation lags.

Table 1: Sample Moments of Measured Capital Growth and Tobin's Q

	$\tilde{g}^K_{t+1}$	Q_t	Q^{tc}_t
mean	0.0114	1.01	1.47
stdev	0.0030	0.42	0.60
$corr(\tilde{g}^K_{t+1}, \cdot)$	–	0.44	0.46

Sample Period: 1959Q3 to 2002Q4 (174 observations). Q^{tc} *is Tobin's* Q *calculated using the net price of new capital goods after deducting investment tax credits and the present value of tax shields associated with future depreciation allowances.*

Table 2: Capital Growth and Q: Comparison of Forward and Reverse Regressions

	Unadjusted Data		*Tax-Adjusted Data*	
$\tilde{g}^K_{t+1}$ *onto* Q_t	a	b	a	b
coef	0.0081	0.0033	0.0078	0.0025
se	$0.0005^{\#}$	$0.0005^{\#}$	$0.0005^{\#}$	$0.0003^{\#}$
se^r	$0.0012^{\#}$	$0.0009^{\#}$	$0.0005^{\#}$	$0.0007^{\#}$
bias	-0.0001	0.0001	-0.0001	0.0001
CI^{bc}_{90}	$0.0052{,}0.0101^{\#}$	$0.0008{,}0.0037^{\ddagger}$	$0.0052{,}0.0101^{\#}$	$0.0008{,}0.0037^{\ddagger}$
DW	0.074		0.080	
$\bar{R}^2$	0.217		0.241	
NDG(p)	0.046		0.040	
Q_t *onto* $\tilde{g}^K_{t+1}$	c	d	c	d
coef	0.25	66.21	0.33	99.50
se	$0.11^{\ddagger}$	$9.50^{\#}$	$0.16^{\ddagger}$	$13.34^{\#}$
se^r	0.28	$27.56^{\ddagger}$	0.37	$37.70^{\#}$
bias	-0.00	-2.47	-0.00	-3.39
CI^{bc}_{90}	-0.31,0.68	$20.72{,}119.31^{\ddagger}$	-0.44,0.93	$39.26{,}172.31^{\ddagger}$
DW	0.075		0.089	
$\bar{R}^2$	0.217		0.241	
NDG(p)	0.771		0.795	

Significance Levels: †*10%,* ‡*5%,y* $^{\#}$*1%. Sample Period: 1959Q3 to 2002Q3 (172 observations). "bias" is an estimate, from the bootstrap, of the small-sample bias associated with estimating the coefficient. HAC standard errors (denoted* se^r*) are estimated with a maximum lag length of ten.* NDG(p) *is the p-value for a null of no endogeneity, with robustness for serially correlated and heteroskedastic errors.* CI^{bc}_{90} *denotes bootstrapped 90 percent confidence intervals with correction for small-sample bias. These were generated using 10,000 bootstrap replications, where the data were re-sampled in blocks of twelve observations to account for autocorrelation.*

Table 3: Relevance of Forecastable Change in Q for Capital Growth: No Planning

$$Q_t = \check{a} + \check{b}\tilde{g}^K_{t+1} + \check{c}\left(\Delta^B Q_{t+B}\right) + \check{u}_t$$

B		$\check{a}$ IV	$\check{a}$ GMM	$\check{b}$ IV	$\check{b}$ GMM	$\check{c}$ IV	$\check{c}$ GMM		
+1 qtr									
	coef	0.47	0.73	87.68	66.08	-1.80	-1.63	F_1	2.675
	se	0.19‡	–	16.06$^{\#}$	–	0.79‡	–	DW_I	0.273
	se^r	0.40	0.27$^{\#}$	41.52‡	26.94‡	0.76‡	0.51$^{\#}$	DW_G	0.217
								$Ov_I(p)$	0.202
	CI^{bc}_{90}	-0.12,1.48	0.37,2.03‡	-10.89,148.66	-66.78,107.84	-9.87,-0.46†	-9.81,-0.63$^{\#}$	$Ov_G(p)$	0.506
+2 qtr									
	coef	0.57	0.77	79.00	62.85	-1.46	-1.43	F_1	2.960
	se	0.21$^{\#}$	–	17.88$^{\#}$	–	0.58‡	–	DW_I	0.183
	se^r	0.43	0.33‡	43.75†	33.43†	0.67‡	0.44$^{\#}$	DW_G	0.167
								$Ov_I(p)$	0.187
	CI^{bc}_{90}	-0.02,1.78	0.44,2.27‡	-39.97,139.37	-79.49,103.91	-5.99,-0.41†	-5.78,-0.70‡	$Ov_G(p)$	0.566
+3 qtr									
	coef	0.74	0.86	64.36	52.80	-1.73	-1.59	F_1	3.085
	se	0.25$^{\#}$	–	21.00$^{\#}$	–	0.54$^{\#}$	–	DW_I	0.188
	se^r	0.46	0.34‡	47.86	33.81	0.70‡	0.48$^{\#}$	DW_G	0.163
								$Ov_I(p)$	0.181
	CI^{bc}_{90}	0.23,2.16‡	0.56,2.52$^{\#}$	-66.23,120.53	-111.27,88.77	-5.82,-1.25‡	-4.75,-1.27$^{\#}$	$Ov_G(p)$	0.464
+4 qtr									
	coef	0.74	0.89	64.30	50.41	-1.37	-1.09	F_1	4.147
	se	0.23$^{\#}$	–	19.63$^{\#}$	–	0.38$^{\#}$	–	DW_I	0.122
	se^r	0.23	0.36‡	47.86	34.75	0.55‡	0.35$^{\#}$	DW_G	0.082
								$Ov_I(p)$	0.572
	CI^{bc}_{90}	0.22,2.00‡	0.57,2.38$^{\#}$	-53.33,122.30	-100.55,88.43	-3.94,-1.01‡	-3.19,-0.65‡	$Ov_G(p)$	0.562
+5 qtr									
	coef	0.73	0.95	65.00	45.61	-1.08	-1.09	F_1	5.255
	se	0.22$^{\#}$	–	18.70$^{\#}$	–	0.29$^{\#}$	–	DW_I	0.084
	se^r	0.47	0.35$^{\#}$	47.72	33.32	0.53‡	0.32$^{\#}$	DW_G	0.080
								$Ov_I(p)$	0.110
	CI^{bc}_{90}	0.20,1.97‡	0.66,2.45$^{\#}$	-46.57,125.15	-99.96,80.18	-2.68,-0.67‡	-2.82,-0.95$^{\#}$	$Ov_G(p)$	0.650
+6 qtr									
	coef	0.80	1.02	58.70	40.64	-1.07	-1.11	F_1	5.751
	se	0.23$^{\#}$	–	19.23$^{\#}$	–	0.27$^{\#}$	–	DW_I	0.074
	se^r	0.51	0.36$^{\#}$	50.86	33.13	0.55†	0.28$^{\#}$	DW_G	0.076
								$Ov_I(p)$	0.546
	CI^{bc}_{90}	0.24,2.07‡	0.78,2.54$^{\#}$	-54.99,121.10	-111.93,70.59	-2.52,-0.41‡	-2.54,-1.08$^{\#}$	$Ov_G(p)$	0.612
+7 qtr									
	coef	0.71	0.96	66.32	45.18	-0.76	-0.94	F_1	6.798
	se	0.21$^{\#}$	–	17.51$^{\#}$	–	0.21$^{\#}$	–	DW_I	0.048
	se^r	0.50	0.35$^{\#}$	50.03	32.64	0.51	0.27$^{\#}$	DW_G	0.058
								$Ov_I(p)$	0.570
	CI^{bc}_{90}	0.13,1.92†	0.67,2.48$^{\#}$	-37.24,130.35	-96.84,80.93	-2.24,0.04	-2.32,-0.70‡	$Ov_G(p)$	0.625
+8 qtr									
	coef	0.66	0.89	69.48	50.66	-0.58	-0.75	F_1	7.723
	se	0.20$^{\#}$	–	16.71$^{\#}$	–	0.19$^{\#}$	–	DW_I	0.046
	se^r	0.49	0.33$^{\#}$	48.78	30.39	0.50	0.29$^{\#}$	DW_G	0.048
								$Ov_I(p)$	0.562
	CI^{bc}_{90}	0.06,1.72†	0.51,2.18$^{\#}$	-19.77,136.70	-68.38,95.47	-2.08,0.13	-1.91,-0.25‡	$Ov_G(p)$	0.619

Significance Levels: †10%, ‡5%, $^{\#}$1%. Sample Period: 1959Q3 to 2002Q4 (174 observations). HAC standard errors (denoted se^r*) are estimated with a maximum lag length of ten.* Ov$_I$(p) *and* Ov$_G$(p) *are p-values for the overidentification test in the IV and GMM specifications. F_1 is the F-statistic for the first-stage regression.* CI$^{bc}_{90}$ *denotes bootstrapped 90 percent confidence intervals with correction for small-sample bias. These were generated using 10,000 bootstrap replications, where the data were re-sampled in blocks of twelve observations to account for autocorrelation.*

Table 4: Relevance of Forecastable Change in Q for Capital Growth: 2Q Planning

$$Q_t = \breve{a} + \breve{b}\tilde{g}^K_{t+1} + \breve{c}\left(\Delta^B Q_{t+B}\right) + \breve{u}_t$$

B		$\breve{a}$ IV	$\breve{a}$ GMM	$\breve{b}$ IV	$\breve{b}$ GMM	$\breve{c}$ IV	$\breve{c}$ GMM		
+1 qtr									
	coef	0.47	0.55	87.24	82.38	-1.92	-1.146	F_1	2.371
	se	0.20‡	–	16.46$^{\#}$	–	0.84‡	–	DW_I	0.309
	se^r	0.40	0.26‡	42.32‡	26.89$^{\#}$	1.11†	0.63†	DW_G	0.112
								Ov_I(p)	0.453
	CI^{bc}_{90}	-0.10,1.37	0.08,1.53†	-2.62,147.68	-26.34,134.36	-8.74,-0.52†	-6.51,0.15	Ov_G(p)	0.462
+2 qtr									
	coef	0.55	0.67	80.86	72.27	-1.56	-0.84	F_1	3.148
	se	0.21$^{\#}$	–	16.46$^{\#}$	–	0.57‡	–	DW_I	0.156
	se^r	0.45	0.30‡	42.32‡	28.90‡	0.66‡	0.45†	DW_G	0.068
								Ov_I(p)	0.589
	CI^{bc}_{90}	-0.03,1.65	0.21,1.83‡	-25.93,142.01	-52.66,125.89	-6.67,-0.55‡	-5.67,0.06	Ov_G(p)	0.631
+3 qtr									
	coef	0.56	0.79	79.38	61.09	-1.58	-1.07	F_1	4.117
	se	0.21$^{\#}$	–	17.40$^{\#}$	–	0.53$^{\#}$	–	DW_I	0.087
	se^r	0.50	0.35‡	47.10	33.98†	0.70‡	0.43‡	DW_G	0.084
								Ov_I(p)	0.181
	CI^{bc}_{90}	-0.00,1.78	0.45,2.40‡	-34.93,139.98	-94.07,102.47	-4.14,-0.40‡	-4.37,-0.62‡	Ov_G(p)	0.623
+4 qtr									
	coef	0.62	0.79	74.83	63.19	-0.99	-0.91	F_1	4.896
	se	0.21$^{\#}$	–	17.61$^{\#}$	–	0.32$^{\#}$	–	DW_I	0.072
	se^r	0.52	0.34‡	52.47	31.19‡	0.73	0.32$^{\#}$	DW_G	0.061
								Ov_I(p)	0.476
	CI^{bc}_{90}	0.05,1.90†	0.39,2.20‡	-43.00,137.20	-72.66,114.92	-3.57,-0.29†	-3.32,-0.40‡	Ov_G(p)	0.652
+5 qtr									
	coef	0.60	0.81	76.07	60.02	-1.77	-0.86	F_1	5.797
	se	0.21$^{\#}$	–	17.15$^{\#}$	–	0.26$^{\#}$	–	DW_I	0.055
	se^r	0.53	0.38‡	52.86	35.96†	0.67	0.40‡	DW_G	0.058
								Ov_I(p)	0.713
	CI^{bc}_{90}	-0.03,1.81	0.40,2.25‡	-33.61,143.33	-72.56,110.62	-2.87,0.05	-2.86,-0.35‡	Ov_G(p)	0.624
+6 qtr									
	coef	0.58	0.93	76.80	45.77	-0.64	-0.86	F_1	6.735
	se	0.20$^{\#}$	–	16.76$^{\#}$	–	0.22$^{\#}$	–	DW_I	0.047
	se^r	0.53	0.39‡	52.31	37.64	0.60	0.36‡	DW_G	0.054
								Ov_I(p)	0.154
	CI^{bc}_{90}	-0.06,1.74	0.59,2.54$^{\#}$	-24.80,147.16	-105.53,87.29	-2.64,0.13	-2.56,-0.47‡	Ov_G(p)	0.633
+7 qtr									
	coef	0.57	0.89	77.58	49.46	-0.52	-0.79	F_1	7.519
	se	0.20$^{\#}$	–	16.38$^{\#}$	–	0.19$^{\#}$	–	DW_I	0.041
	se^r	0.52	0.40‡	51.12	38.39	0.55	0.37‡	DW_G	0.047
								Ov_I(p)	0.391
	CI^{bc}_{90}	-0.08,1.61	0.56,2.55$^{\#}$	-10.41,150.34	-101.10,89.99	-2.39,0.15	-2.41,-0.32‡	Ov_G(p)	0.648
+8 qtr									
	coef	0.52	0.74	80.73	63.05	-0.36	-0.48	F_1	8.637
	se	0.19$^{\#}$	–	15.87$^{\#}$	–	0.17‡	–	DW_I	0.050
	se^r	0.49	0.33‡	48.61	30.73‡	0.48	0.30	DW_G	0.044
								Ov_I(p)	0.530
	CI^{bc}_{90}	-0.23,1.35	0.28,1.79‡	12.73,159.29†	-39.14,113.90	-2.21,0.22	-1.73,0.16	Ov_G(p)	0.635

Significance Levels: †*10%,* ‡*5%,* $^{\#}$*1%. Sample Period: 1959Q3 to 2002Q4 (174 observations). HAC standard errors (denoted* se^r*) are estimated with a maximum lag length of ten.* Ov$_I$(p) *and* Ov$_G$(p) *are p-values for the overidentification test in the IV and GMM specifications.* F_1 *is the F-statistic for the first-stage regression.* CI$^{bc}_{90}$ *denotes bootstrapped 90 percent confidence intervals with correction for small-sample bias. These were generated using 10,000 bootstrap replications, where the data were re-sampled in blocks of twelve observations to account for autocorrelation.*

Table 5: OLS Regression of Forward Q on Capital Growth

$$\bar{Q}_{t+J} = \bar{a} + \bar{b} g_t^K + \bar{u}_t$$

J		*Unadjusted Data* $\bar{a}$	*Unadjusted Data* $\bar{b}$	*Tax-Adjusted Data* $\bar{a}$	*Tax-Adjusted Data* $\bar{b}$
+1 qtr					
	coef	0.30	61.97	0.41	92.71
	se	$0.11^{\#}$	$9.67^{\#}$	$0.16^{\ddagger}$	$13.62^{\#}$
	se^r	0.28	$28.13^{\ddagger}$	0.37	$38.61^{\ddagger}$
	bias	0.01	-2.81	0.02	-4.37
	CI_{90}^{bc}	-0.28,0.74	$15.23,116.66^{\ddagger}$	-0.35,1.01	$27.88,165.51^{\ddagger}$
+2 qtr					
	coef	0.32	59.39	0.45	88.92
	se	$0.12^{\#}$	$9.94^{\#}$	$0.17^{\#}$	$14.05^{\#}$
	se^r	0.28	$28.60^{\ddagger}$	0.38	$39.30^{\ddagger}$
	bias	0.03	-4.52	0.05	-6.84
	CI_{90}^{bc}	-0.29,0.76	$12.67,116.05^{\ddagger}$	-0.34,1.05	$23.88,164.65^{\ddagger}$
+3 qtr					
	coef	0.33	57.53	0.48	86.20
	se	$0.12^{\#}$	$10.17^{\#}$	$0.17^{\#}$	$14.41^{\#}$
	se^r	0.29	$29.37^{\ddagger}$	0.39	$40.45^{\ddagger}$
	bias	0.04	-5.84	0.07	-8.73
	CI_{90}^{bc}	-0.31,0.77	$9.66,116.23^{\dagger}$	-0.35,1.08	$20.09,165.51^{\ddagger}$
+4 qtr					
	coef	0.34	55.47	0.51	86.30
	se	$0.12^{\#}$	$10.38^{\#}$	$0.17^{\#}$	$14.71^{\#}$
	se^r	0.29	$29.40^{\dagger}$	0.39	$40.52^{\ddagger}$
	bias	0.06	-6.91	0.09	-10.23
	CI_{90}^{bc}	-0.32,0.77	$7.62,115.23^{\dagger}$	-0.33,1.11	$16.99,164.60^{\ddagger}$
+5 qtr					
	coef	0.36	51.89	0.56	78.54
	se	$0.12^{\#}$	$10.60^{\#}$	$0.18^{\#}$	$15.01^{\#}$
	se^r	0.36	$28.94^{\ddagger}$	0.38	$39.88^{\dagger}$
	bias	0.06	-7.23	0.10	-10.61
	CI_{90}^{bc}	-0.28,0.80	$4.08,112.12^{\dagger}$	-0.27,1.15	$11.48,159.52^{\dagger}$
+6 qtr					
	coef	0.39	48.06	0.62	73.30
	se	$0.13^{\#}$	$10.79^{\#}$	$0.18^{\#}$	$15.26^{\#}$
	se^r	0.28	$28.16^{\dagger}$	$0.37^{\dagger}$	$38.81^{\dagger}$
	bias	0.06	-6.65	0.09	-9.64
	CI_{90}^{bc}	-0.23,0.83	$0.56,106.62^{\dagger}$	-0.20,1.21	$6.24,152.71^{\dagger}$
+7 qtr					
	coef	0.43	43.63	0.68	67.20
	se	$0.13^{\#}$	$10.99^{\#}$	$0.18^{\#}$	$15.51^{\#}$
	se^r	0.27	27.18	$0.35^{\dagger}$	$37.48^{\dagger}$
	bias	0.05	-5.90	0.08	-8.46
	CI_{90}^{bc}	-0.16,0.87	-3.54,101.23	$-0.10,1.28^{\dagger}$	-0.09,144.52
+8 qtr					
	coef	0.46	39.38	0.74	61.48
	se	$0.13^{\#}$	$11.22^{\#}$	$0.18^{\#}$	$15.81^{\#}$
	se^r	$0.26^{\dagger}$	26.22	$0.34^{\ddagger}$	$36.13^{\dagger}$
	bias	0.04	-5.40	0.07	-7.70
	CI_{90}^{bc}	-0.11,0.89	-7.32,95.69	-0.01,1.33	-4.50,137.57

Significance Levels: †*10%,* ‡*5%,* $^{\#}$*1%. Sample Period: 1959Q3 to 2002Q4 (174 observations). HAC standard errors (denoted* se^r*) are estimated with a maximum lag length of ten.* CI_{90}^{bc} *denotes bootstrapped 90 percent confidence intervals with correction for small-sample bias. These were generated using 10,000 bootstrap replications, where the data were re-sampled in blocks of twelve observations to account for autocorrelation.*

Table 6: Fit and Endogeneity Tests: OLS Regression of Forward Q on Capital Growth

	Unadjusted Data				*Tax-Adjusted Data*			
Quarter	+1	+2	+3	+4	+1	+2	+3	+4
DW	0.083	0.091	0.083	0.074	0.098	0.106	0.098	0.089
$\bar{R}^2$	0.189	0.169	0.154	0.140	0.209	0.186	0.170	0.156
NDG(p)	0.884	0.966	0.969	0.940	0.736	0.632	0.551	0.508
Quarter	+5	+6	+7	+8	+5	+6	+7	+8
DW	0.080	0.074	0.075	0.073	0.094	0.088	0.087	0.085
$\bar{R}^2$	0.120	0.101	0.082	0.064	0.136	0.117	0.097	0.079
NDG(p)	0.866	0.808	0.718	0.658	0.431	0.374	0.317	0.279

Sample Period: 1959Q3 to 2002Q4 (174 observations). NDG(p) *is the p-value for the null of no endogeneity, with robustness for serially correlated and heteroskedastic errors. For the endogeneity test, added exogenous variables were current and lagged changes in (1) growth in government defense expenditures, (2) growth in labor hours, (3) output growth, (4) real hourly labor compensation, and (5) lagged after-tax cash flow per unit of capital.*

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