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Jonathan N. Millar

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Jonathan N. Millar*
Board of Governors of the Federal Reserve System

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Abstract

Investment models typically assume that capital becomes productive almost immediately after purchase and that there is no lead time needed to plan. In this case, marginal q is usually sufficient for investment. This paper develops a model of aggregate investment where competitive firms face no adjustment costs other than building and planning delays. In this context, both Tobin's Q and cash flow can be noisy indicators of investment because some shocks fail to outlast the combined gestation lag. The paper demonstrates some empirical facts that challenge prevailing theories of investment but are consistent with gestation requirements. Regressions using aggregate data suggest that it takes at least four quarters for investment to respond to technology shocks and as many as eight additional quarters before productive capacity is affected. Estimates from structural VARs show that only permanent shocks affect investment, but that cash flow and Q react to both permanent and transitory shocks.

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I. Introduction

Investment models typically assume that (1) capital expenditures occur immediately after the firm's investment decision, and (2) that purchased capital becomes productive with little or no delay. These features contrast with practical accounts of investment, where projects often require considerable periods of planning and building. The planning period encompasses the time needed for engineers to draw up the details, lawyers to obtain relevant permits, and management to arrange financing. Building involves the time needed for construction and for equipment to be ordered, delivered, and installed. Owing to these delays, there may be a considerable lag between the decision to increase capacity and the commencement of production in a new facility.

In the neoclassical world, the user cost adjusts to equate the (frictionless) demand for capital services with supply. In this environment, the current shadow value of a firm's capital yields no useful information for investment because its realized value is always equal to one. Although there is some evidence that this frictionless relationship holds in the very long run, economists have long recognized the shortcomings of this theory at higher frequencies.[1] Adjustment cost models have emerged as the dominant paradigm to fill this theoretical gap. In these models, deviations from the neoclassical capital equilibrium are the result of an optimizing process where firms weigh the costs and benefits of faster adjustment. When adjustment costs are convex, the process of capital adjustment is smooth and the current value of q completely encapsulates all of the firm's relevant investment considerations.[2] The empirical shortcomings of this framework have prompted more recent models that de-emphasize q as an investment indicator.[3] These models emphasize the lumpiness of investment at the plant and firm levels in the presence of non-convex adjustment costs.

However, the costs of capital adjustment are not always measured just in resource costs and lost production—they may also be reckoned in time. These lags cause complications for capital adjustment that are interesting and

[1]Caballero [1994] shows a long run relationship between the neoclassical user cost and the capital stock.

[2]Although q is not generally observable, Hayashi [1982] demonstrates that, under certain conditions, the current Tobin's Q is an exact measure of q.

[3]Some well-cited shortcoming of the convex adjustment cost model are that (1) investment is too lumpy at the plant and firm-level to be explained by convex adjustment costs (Doms and Dunne [1998]), (2) that cash flows seem to capture some relevant information for investment by financially-constrained firms that is not captured in Q (Fazzari, Hubbard, and Peterson [1988]), and (3) that Q is subject to measurement error (Erickson and Whited [2000]).

important in their own right. Because invested capital becomes productive with a delay, firms must base current investment decisions on forecasts of what variables like q and cash flow will be when the new capital comes on line. As a result, many of familiar contemporaneous linkages between investment, q, and the value of the capital service flow do not hold after the fact. Empirical testing is complicated by the fact that we observe realizations of variables like Q and cash flow rather than the anticipated values that are the basis of investment decisions. Further, time lags tend to spread out the response of investment and productive capacity to shocks, leading to richer dynamic effects.

These building and planning lags have some history in the real business cycle literature. The seminal work is Kydland and Prescott [1982], who add a time to build lag for capital to a calibrated RBC model. A more recent contribution by Christiano and Todd [1995] adds a planning phase to the Kydland and Prescott setup. These models suggest that capital gestation requirements can capture some empirical features of the business cycle more effectively than standard models with one building period or models with convex capital adjustment costs.[4] There are also some noteworthy attempts to consider gestation lags in the investment literature. Majd and Pindyck [1987] explore the implications of placing a ceiling on the amount of investment that can be undertaken each period in the process of assembling a single (irreversible) capital project. Investment outlays only continue when the anticipated discounted value of the completed project exceeds a minimum threshold. Altug [1993] takes a detailed look at capital pricing and investment decisions in the presence of Kydland and Prescott building requirements. She shows that additions to the capital stock depend on the forecast of marginal q after the building period, which may not be well proxied by the current Tobin's Q.

In the next section of this paper, I develop a model of aggregate investment in a competitive economy in which firms face distinct planning and building lags for new capital, but no other explicit adjustment costs. The economy is subject to temporary and permanent aggregate shocks that firms can distinguish at the moment they occur. These features yield important implications for investment, the rate of cash flow, and Tobin's Q. Investment only responds to shocks that are expected to outlast the gestation horizon, and then only after the planning phase is complete. In contrast, both the rate of cash flow and Q respond to all shocks throughout their duration, co-varying positively

[4] Christiano and Todd emphasize that a combined building and planning lag can account for the persistent effects of technological shocks, the tendency for business and structures investment to lag movements in output, and the leading relationship of productivity to hours worked.

with associated investment during the building period. As a result, both cash flow and Q tend to be noisy indicators of investment, where the correlation depends on the relative preponderance of temporary and permanent shocks in the data. The model also yields implications for the dynamic response of investment and productive capacity to shocks. The planning phase causes a delay in the response of investment spending, while building causes a lag between investment spending and the associated increase in production.

Section III performs some empirical analysis. First, data for "purified" Solow residuals are used to show that distinct planning and building lags exist, and to estimate their duration. Then, I estimate empirical impulse responses of aggregate investment, cash flow, and Tobin's Q to temporary and permanent shocks, and compare these responses to the predictions of the gestation lag model and other well-known alternatives from the investment literature. These impulse responses are estimated using a structural VAR that identifies temporary and permanent aggregate disturbances using the zero frequency restrictions of Shapiro and Watson [1988] and Blanchard and Quah [1989]. Among other things, the gestation lag model correctly predicts that aggregate investment is driven almost entirely by permanent shocks, while cash flow and Q respond to both shocks. In addition, aggregate investment exhibits a delayed response to permanent shocks that is consistent in character to the model's predictions, and inconsistent with models that have no gestation lag. Section IV concludes the paper with some discussion of the major results.

II. Model

Let *time to plan* denote the P periods that begin with the decision to add productive capital, and end when investment expenditures commence. *Time to build* denotes the B periods that begin with the first capital expenditure, and end when the new capital becomes productive. Following Kydland and Prescott [1982], assume that a proportion $\phi_j \in [0, 1]$ of the planned capital addition is acquired j periods before it becomes productive capital, so that $\sum_{j=0}^{B-1} \phi_{B-j} = 1$. These lags are depicted graphically in Figure 1. At time t, a firm commits to change its capital stock at period $t+P+B$. The P period planning phase then passes where there are no investment outlays associated with the plan. At $t+P$, the building phase begins, with the firm carrying out a non-negative proportion ϕ_{B-j} of the total expenditure associated with the plan at each period $t+P+j$, from $j = 0, \ldots, B-1$, with $\phi_B > 0$. The new capital becomes available for production at $t+P+B$, after a total *gestation lag* of $J=P+B$ periods.

Note that each investment plan is assumed to be *irrevocable* in the sense that the firm commits to a specific level of capital at the end of its gestation period. This assumption is necessary because the planning lag is meaningless when investment plans can be changed without cost. More specifically, the solution to any intertemporal optimization problem requires a plan for each control variable for every period in the problem horizon. However, the control variables can be changed costlessly when the problem is revisited in subsequent periods, so these plans are not binding. The irrevocability assumption makes this cost infinite for committed plans. Nonetheless, there is no restriction that investment plans be non-negative, so the irrevocability assumption is not the same as *irreversibility.* Firms can plan to dismantle their capital in subsequent periods, albeit with the same gestation requirement.

In the remainder of this section, I develop a model for the investment, cash flows, and value of an aggregate firm that faces the gestation lags described above. The firm operates in a competitive small open economy that is subject to temporary and permanent stochastic shocks to technology and the supply of labor. As such, all market prices are treated as given, and the interest rate exogenous. The competitive economy assumption is comparable to Hayashi [1982], which many cite as a justification for using Tobin's Q as a proxy for the shadow value of new capital. Yet unlike Hayashi, the unit of analysis is an aggregate firm. This is dictated by the fact that the optimal capital stock of an individual competitive firm is indeterminate when production exhibits constant returns to scale, so its optimal rate of investment is not well defined.[5] This indeterminacy is not an important issue for the aggregate firm, because equilibrium in the markets for other variable inputs pins down the aggregate capital stock.[6] The focus on a small open economy de-emphasizes a host of dynamic general equilibrium considerations that may not be relevant when the economy is open for trade in capital and goods. Moreover, this approach allows for a more transparent depiction of some issues related to gestation lags that have been largely neglected by previous work, such as the role of temporary capital scarcity in the investment-Q relationship.

The model development proceeds as follows. I begin by specifying the production technology for the aggregate firm and find optimal closed-form solutions for the capital growth rate, the rate of cash flow, and Tobin's Q in a decentralized equilibrium. Rather than explicitly solving the decentralized

[5] Although the scale of an individual firm in Hayashi's model is also indeterminate, its rate of investment is pinned down by a first order condition that links q to the marginal adjustment cost for capital.

[6] In his textbook, Romer [1996] adopts a similar approach for his discussion of investment with adjustment costs, albeit in reduced form.

problem to find these solutions, I employ a number of strategies to simplify the exposition. Since the decentralized solution will be efficient, I obtain the same optimality conditions for the production side by maximizing the value of an aggregate firm that treats prices as given, then imposing that the marginal product of each input equal its market rental rate. I do not bother to set out optimal household consumption conditions because these can be ignored in the small open economy according to the Fisher separation theorem. Finally, I incorporate household labor decisions in a stylized manner by introducing a reduced-form aggregate labor supply curve. The resulting model is used to describe in detail the interrelationships between cash flow, investment, and Q. I close the section by discussing measurement issues that arise from the existence of capital building requirements, and how they affect the interpretation of model results.

1. Cash Flow

For now, ignore the intertemporal aspects of the problem. Let current output be the numeraire. Assume that the aggregate firm enters the current period with a predetermined productive capital stock K and level of technology Z^T, and chooses the quantity of labor L that maximizes variable profits. Although the implications of the more general CES production function will also be considered, for expositional purposes it is useful (and considerably more tractable) to assume the Cobb-Douglas production function

$$F\left(K, Z^T L\right) = K^{1-\alpha}\left(Z^T L\right)^{\alpha}. \tag{1}$$

Units of labor can be hired at the given market wage rate w. After maximizing out the variable factor L, the aggregate firm's variable profit is

$$\bar{\Pi}\left(K|w, Z^T, \tau\right) = (1-\tau)\,\bar{h}\left(\frac{Z^T}{w}\right)^{\frac{\alpha}{1-\alpha}} K, \tag{2}$$

where τ is the corporate tax rate, and $\bar{h} \equiv (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}$. Let the *rate of cash flow* denote the average variable profit of capital:

$$\bar{\pi}\left(w, Z^T, \tau\right) = (1-\tau)\,\bar{h}\left(\frac{Z^T}{w}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3}$$

Since total cash flows are linear in K, this function is also the marginal product of capital. This equation can also be interpreted as a factor price possibility frontier that shows the negative relationship between the labor wage and the value of capital services with technology is held fixed. Since the Cobb-Douglas case embeds a unit elasticity of substitution between capital and labor the relationship between the factor prices is log-linear. In the more general CES case, the relationship between the factor prices becomes more convex as the elasticity of substitution between capital and labor diminishes. Therefore, the value of capital services will be more sensitive to changes in wages and technology as the degree of complementarity declines.

Ostensibly, equation (2) suggests that the marginal profit from capital is independent of the capital stock, so the aggregate demand for capital services seems to be undefined. However, capital demand can be pinned down by the aggregate labor market equilibrium. Aggregate labor demand can be obtained by applying Shephard's lemma to equation (2), yielding

$$L^d = -\frac{\partial \Pi \left(K|w, Z^T, \tau\right)}{\partial w} = \frac{\alpha}{1-\alpha} \frac{\bar{\pi}\left(w, Z^T, \tau\right)}{w} K. \tag{4}$$

This function is increasing in the quantities of technology and capital, and decreasing in the real wage and the tax rate. For simplicity, assume that the aggregate labor supply takes the form

$$L^s = w^\zeta Z^L, \tag{5}$$

where $\zeta \geq 0$ is the wage-elasticity of labor supply, and Z^L is a multiplicative labor supply shock. This formation can be interpreted as a log-linear approximation to the optimization condition that will govern aggregate labor supply in a dynamic general equilibrium model, where the process Z^L is a reduced form function of (among other things) population and the marginal utility of wealth. Under this interpretation, the parameter ζ is the Frisch wage elasticity of labor supply, and the process Z^L reflects a wide range of permanent and temporary influences that emanate from exogenous shocks and general equilibrium adjustment.

The market-clearing real wage can be determined by equating aggregate labor demand and aggregate labor supply. This wage can be substituted into (3) to yield the following equation for cash flow as a function of the aggregate capital stock:

$$\pi(K|Z, \tau) = h(1-\tau)^a \left(\frac{Z}{K}\right)^b \tag{6}$$

where

$$Z \equiv \left(Z^T\right)^{1+\zeta} Z^L, \qquad h \equiv (1-\alpha)\alpha^{b\zeta},$$

$$a \equiv \frac{(1-\alpha)(\zeta+1)}{\zeta(1-\alpha)+1} \in (0,1), \qquad \text{and} \qquad b \equiv \frac{\alpha}{\zeta(1-\alpha)+1} \in (0,\alpha).$$

This function represents the marginal contribution of capital services to variable profits in any given period, or the aggregate inverse demand curve for capital services. In a frictionless world, this is set equal to the neoclassical user cost to determine the current capital stock. The factor Z, which combines both shocks to technology and labor supply, neatly encapsulates the exogenous (non-tax) factors that shift the aggregate demand for capital services.

The parameter b represents the elasticity of cash flow with respect to the capital imbalance ratio K/Z, after accounting for endogenous movements in labor. In the Cobb-Douglas case, b is bounded in magnitude between zero and by labor's share α. For the more general CES production function, b is inversely related to the elasticity of substitution between labor and capital. Although a solution of the form in (6) is not generally available when the production function is CES, a log-linear approximation can be calculated for the special case where the labor supply elasticity ζ is zero. Then the elasticity of cash flow with respect to capital imbalance in the steady state is sh_L^*/σ, where σ is the constant substitution elasticity between capital and labor, and sh_L^* is labor's share of income in the steady state. Intuitively, this indicates that reduced substitutability between capital and variable inputs makes the value of capital more sensitive to its degree of aggregate scarcity.

2. Investment with Gestation Lags of Arbitrary Duration

Now consider the intertemporal aspects of the optimization problem relating to investment. This optimization determines a plan for the aggregate capital stock from the gestation horizon onward, subject to the constraints imposed by the predetermined quantities of capital for periods within the gestation horizon. Viewed from the perspective of the social planner, this path equates the ex ante value of capital services (the anticipated rate of cash flow) to its ex ante social cost. This is shorthand for the capital market equilibrium that would be determined, passively, by the interaction of atomistic decisions in the decentralized economy. From the perspective of the aggregate firm, the optimal plan maximizes its market value, taking as given the anticipated path of future prices and the rate of cash flow. The aggregate firm neglects the influence of its own capital stock on the rate of cash flow because its problem represents the accumulated decisions of individual firms that, in isolation, have a negligible influence on the value of capital. Consequently, the aggregate firm acts like a small firm that faces constant returns to scale in production, perceiving no well-defined solution for its optimal capital path. Instead, the optimal path of capital is pinned down by the capital market equilibrium.

Let $s_{k,t}$ represent, at time t, the planned addition to the productive capital stock in k periods. Then, the productive capital stock evolves according to the accumulation condition

$$K_{t+i} = K_{t+i-1}(1-\delta) + s_{1,t+i-1}, \tag{7}$$

where δ is the depreciation rate. This differs from the standard accumulation identity because the addition to the productive stock is dictated by the plan from J periods earlier rather than current investment. Committed plans evolve such that this period's planned addition at horizon k equals the next period's plan for horizon $k-1$, so that

$$s_{k-1,t+i+1} = s_{k,t+i}, \tag{8}$$

for $k=2,\ldots,J$. The total investment flow in each period is the sum of spending on all committed plans that are in the building process:

$$I_{t+i} = \sum_{j=1}^{B} \phi_j s_{j,t+i}. \tag{9}$$

Consequently, the investment flow is not generally associated with any specific plan; rather, it a moving average of planned additions over the next B periods.

Given this structure, there are many state variables that must be considered in the optimization problem. At time t, the firm inherits its current productive capital stock, along with planned additions for the next $J-1$ periods, yielding a total of J state variables. Note that these plans are relevant to the problem, although they are not yet part of the productive capital stock, because they will affect the optimal capital addition at the gestation horizon.

Now consider the problem from the perspective of the aggregate firm. For simplicity, the appropriate discount factor is constant at $R = 1 + r$, where r is the interest rate. New units of capital can be purchased for a fixed price of $\bar{p}$, which is net of the value of any government tax incentives.[7] Since anticipated rates of cash flow are considered given, the appropriate notion of variable profit is $\bar{\pi}K$, where $\bar{\pi}$ represents the function (3). The market valuation of firm is the discounted total of all future expected cash flows, net of investment outlays under the optimal plan:

$$V(K_t, \{s_{j,t}\}_{j=1}^{J-1} | \bar{p}, \{_t\bar{\pi}_{t+i}\}_{i=0}^{\infty}) \equiv \max_{\{s_{J,t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} R^{-i} \left[{}_t\bar{\pi}_{t+i} K_{t+i} - \bar{p} I_{t+i} \right], \tag{10}$$

[7]This includes both an investment tax credit ι and the present value of capital consumption allowances z. These incentives effectively reduce the price of new capital by a factor $(1-\iota-z)$, where $\iota + z$ is the tax wedge.

subject to the constraints (7) through (9). The firm solves this problem by planning additions to its capital stock from period $t{+}J$ onward. However, only the plan for $t{+}J$ binds future decisions.[8] Note that $\bar{\pi}_{t+i}$ is a function of the given (but not exogenous) market real wage w_{t+i}, so the valuation problem reflects expected conditions in the labor market (and, by implication, the anticipated path of Z) contingent on current information.

It is useful to restate this problem as a series of unrelated intratemporal problems. Tedious manipulation that (among other things) utilizes equations (7) through (9) to eliminate the flow variables I_{t+i} and $s_{J,t+i}$ for $i>0$ yields:

$$\text{(11)}\quad V(K_t, \{s_{j,t}\}_{j=1}^{J-1}\,|\bar{p}, \{{}_t\bar{\pi}_{t+i}\}_{i=0}^{\infty}) \equiv \bar{p}q_0^* K_t + \bar{p}\sum_{i=1}^{B-1} q_i^* s_{i,t} + \sum_{i=0}^{J-1} R^{-i}\left[\frac{{}_t\pi_{t+i}}{\bar{p}} - u^*\right]\bar{p}K_{t+i} + R^{-J}\max_{\{K_{t+J+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} R^{-i}\left[\frac{{}_t\pi_{t+J+i}}{\bar{p}} - u^*\right]\bar{p}K_{t+J+i},$$

where u^* is defined as the steady state user cost of capital, and q_i^* is the steady state shadow value of capital that is $i=0,\ldots,B-1$ periods from joining the productive capital stock, reckoned in terms of new capital. These values are considered given because they are functions of the interest rate and parameters. This depiction of the problem can be interpreted as follows. The first two terms collectively represent the value of all funds committed to productive capital and ongoing construction. The shadow values, which are calculated as

$$\text{(12)}\qquad q_i^* \equiv \sum_{j=i+1}^{B} R^{j-i}\phi_j \qquad \text{for } i = 0, \ldots, B-1,$$

represent the future value of all the outlays that were necessary to acquire the capital at its current stage of completion. For instance, to obtain a unit of completed productive capital today ($i = 0$), the firm must purchase ϕ_j units of new capital at time $t-j$, which is worth $\phi_j R^j$ in today's terms after compensating for foregone interest. These payments are summed for $j=1$ to B to obtain the total shadow value q_0^*. It can easily be seen that q_0^* exceeds one. Intuitively, this compensates for the interest foregone on capital outlays during the unproductive building period. Also, note that the capital outlays

[8] Equation (10) can be amended to incorporate the personal taxes and depreciation allowances the firm holds for its existing capital. Let t^g and t^d represent the tax rates on capital gains and dividends, respectively, and let $\tilde{Z}_t$ represent the present value of the remaining capital consumption allowances on the firm's existing capital. Then, the value of the firm becomes $\tilde{V} = \frac{R(1-t^d)}{R-t^g}(V + \tilde{Z}_t)$, where $R = 1 + \frac{r}{1-t^g}$ in (10).

associated with a given plan do not affect the value of the firm until the outlay has taken place.

The third set of terms in (11) captures the value of the quasi-rents that the firm expects to earn on its productive capital during the gestation period. These anticipated rents occur because the firm cannot adjust its productive capital to reflect new information until the end of the gestation horizon. The rent in each period is the difference between the cash flow (reckoned in terms of capital) and the steady state user cost of capital u^*, multiplied by the acquisition value of the capital. The steady state user cost is given by

$$u^* \equiv q_0^* - (1-\delta)R^{-1}q_0^*, \tag{13}$$

which represents the total opportunity cost of obtaining a unit of capital services for the current period only. This is the steady state value of a unit of productive capital q_0^* today less proceeds that could be obtained from selling the undepreciated portion of the installed capital next period.

The final set of terms in (11) represents the value of the quasi-rents that the firm expects to earn from the current gestation horizon onward. At this point, it is useful to temporarily consider the problem from the social planner's perspective. From this viewpoint, it is optimal for these anticipated rents to be zero so that the marginal social cost of capital is equal to its marginal social benefit. This requires the steady state user cost of capital to equal the anticipated cash flow from the gestation horizon onward, so that

$$\frac{{}_t\bar{\pi}_{t+J+i}}{\bar{p}} = u^* \qquad \text{for all } i \geq 0. \tag{14}$$

This implies that cash flow is always expected to return to its long run benchmark of u^* at the end of the gestation horizon. The firm's optimal plans must be consistent with this anticipated market equilibrium, so this condition effectively pins down the path of cash flows (and, in turn, productive capital) from the gestation horizon onward. Consequently, the final set of terms in (11) are always zero, so they drop out of the problem.

Condition (14) can be used to determine the equilibrium aggregate capital stock for period $t+J$ and non-binding plans for the aggregate stock in subsequent periods. Using equations (6) and (14), one can determine that:

$$K_{t+J} = \left[\frac{h(1-\tau)^a}{\bar{p}u^*}\right]^{\frac{1}{b}} E_t[Z_{t+J}^b]^{\frac{1}{b}}. \tag{15}$$

Note that the quantity of capital is based upon a forecast of Z, rather than its realization. Therefore, the capital stock can only respond to unanticipated movements in the factor Z with a lag. Moreover, transitory movements in Z that are not expected to outlast the gestation horizon will never affect the capital stock. Despite this, the capital stock does move roughly in proportion with the demand for capital services in the long run.[9]

These statements can be established formally by assuming that the capital demand factor Z_t follows an exogenous process. For simplicity, I approximate a finite-order *ARIMA* using the *IMA* form

$$\ln Z_{t+1} = \ln Z_t + \mu + \Gamma(L)\psi_{t+1} + \Theta(L)\epsilon_{t+1}, \tag{16}$$

where ϵ_{t+s} and ψ_{t+s}, are normally distributed *iid* shocks with zero mean and unit variance. The parameter μ is (approximately) the expected rate of growth in the level of frictionless capital demand. $\Theta(L)$ and $\Gamma(L)$ are the following polynomials in the lag operator L:

$$\Theta(L) \equiv \sum_{i=0}^{n_T} \theta_i L^i, \quad \text{and} \quad \Gamma(L) \equiv \sum_{i=0}^{n_G} \gamma_i L^i, \tag{17}$$

where n_T is a positive integer, and n_G is a non-negative integer. It is assumed that there is a unit root in the *MA* polynomial Θ, which ensures that only the ψ_t shocks have a permanent effect upon the sequence $\{Z_{t+s}\}_{s=0}^{\infty}$.[10]

Now consider the rate of growth in the capital stock, given the exogenous process for Z described above. Although it need not be generally true, assume for expositional purposes that $\Gamma(L) = \gamma_0 = \gamma$, so that the permanent portion of Z is a random walk. Let $g_t^K = \Delta \ln K_t$ denote the growth rate in the capital stock at t, where Δ is the first-difference operator $1-L$. By equation (15), this growth rate is

$$g_{t+J}^K = \frac{1}{b}\left(\ln E_t[Z_{t+J}^b] - \ln E_{t-1}[Z_{t+J-1}^b]\right). \tag{18}$$

Substituting the conditional expectations of Z_{t+j}^b for $j = J$ and $j = J-1$ into this equation yields

$$g_{t+J}^K = \mu + \gamma\psi_t + \epsilon_t \sum_{i=0}^{\min(J,n_T)} \theta_i + \sum_{i=1}^{\min(n_T-J,0)} \theta_{J+i}\epsilon_{t-i}.$$

The growth rate in productive capital at $t+J$ is equal to the unconditional growth rate μ, plus adjustments for the anticipated effect of shocks dated t

[9] Note that by Jensen's inequality, $E_t[Z_{t+J}^b]^{\frac{1}{b}} < E_t[Z_{t+J}]$, so $E[K_t] \not\sim E[Z_t]$.

[10] Further, assume that the cumulative sum of the *MA* coefficients in $\Theta(L)$ and $\Gamma(L)$ are never negative up to any lag. This ensures that the cumulative effect of each shock is always in one direction.

and earlier on the rate of growth in the capital demand factor Z. The dynamic effects of these shocks are summarized by the impulse responses

$$\frac{\partial g^K_{t+j}}{\partial \epsilon_t} = \begin{cases} 0 & j < J \\ \sum_{i=0}^{\min(J,n)} \theta_i & j = J \\ \theta_j & j > J \end{cases} \qquad \text{and} \qquad \frac{\partial g^K_{t+j}}{\partial \psi_t} = \begin{cases} 0 & j < J \\ \gamma & j = J \\ 0 & j > J \end{cases}.$$

Shocks never affect productive capital growth until the end of the gestation horizon J, because they were not observable when the capital plans were committed. At the gestation horizon ($j = J$), capital growth generally has a large catch-up response to the anticipated cumulative effect of the shock on the demand for capital services. For a permanent shock, the response of productive capital growth is confined to horizon J. No further adjustment is required at subsequent horizons, because the extra demand for capital is fully reflected in the capital stock. In comparison, a temporary shock may not affect the growth rate of capital at all if it is sufficiently short-lived (so that $n_T < J$). More generally, a temporary shock will prompt productive capital growth at horizon J if it outlasts the gestation horizon. However, this will eventually be accompanied by negative capital growth in subsequent periods, since the temporary shock has no effect on the frictionless demand for capital services in the long run.[11] As the length of the gestation horizon increases, it becomes increasingly unlikely that temporary shocks will outlast the gestation horizon and prompt investment. Provided that the gestation horizon is sufficiently long, capital growth will only be associated with permanent shocks.

3. The Relationship of Investment to Cash Flow and Tobin's Q

In this section I consider the relationship between the growth rate in productive capital and two variables that are commonly used as indicators for investment, the rate of cash flow and Tobin's Q.

By equation (14), the rate of cash flow is always expected to return to the steady state user cost at the end of the current gestation horizon. Despite this, the realized demand for capital services at this long run user cost will not generally be equal to the fixed flow of capital services available to the firm in any given period. This is because the quantity of productive capital was determined J periods earlier, using incomplete information. As a result, the

[11] This fact can be demonstrated as follows:

$$\lim_{j\to\infty} \frac{\partial \ln K_{t+j}}{\partial \epsilon_t} = \lim_{j\to\infty} \sum_{i=1}^{j} \frac{\partial g^K_{t+i}}{\partial \epsilon_t} = \lim_{j\to\infty} \sum_{i=0}^{\min(j,n_T)} \theta_i = \Theta(1) = 0.$$

shadow value of capital services adjusts to equal the true economic value of capital after the fact. This can be demonstrated by using equations (6) and (14) to yield

$$\frac{\pi_t}{\bar{p}} = \frac{Z_t^b}{E_{t-J}\left[Z_t^b\right]} u^*. \tag{19}$$

The realized cash flow does not generally equal the long run user cost because of errors in forecasting the capital demand factor Z. If this expectational error is positive, the demand for capital services at the long run user cost exceeds the capital stock, so the rate of cash flow rises to reflect the relative scarcity of capital. If the expectational error is negative, there is a surplus of capital relative to the demand for capital services at the long run user cost, so the rate of cash flow declines.

Indeed, once the capital stock has been established for any given period, the shadow user cost of capital must adjust to equilibrate the demand for capital services with the fixed supply.[12] To accomplish this, the anticipated shadow value of capital adjusts to satisfy the Euler condition

$$\frac{{}_t\pi_{t+j}}{\bar{p}} = {}_tq_{0,t+j} - (1-\delta)R^{-1}{}_tq_{0,t+j+1}, \tag{20}$$

for all $0 \leq j < J$, where $q_{0,t+j}$ is the shadow value of productive capital at $t+j$.[13] Intuitively, this ex ante shadow user cost represents the internal cost to the firm of foregoing one unit of capital services in period $t+j$: the anticipated shadow value of productive capital at $t+j$ less the shadow value of a unit of productive capital in the following period after depreciation. Figure 2 shows a graphical depiction of this process. At period t, the capital stock for $t+J$ is determined by the intersection of the demand and supply curves for capital services. Demand is equal to the cash flow at each K, conditional on current expectations for Z. Supply is perfectly elastic at the long run user cost. This initial decision fixes the supply of capital services at $t+J$. An unanticipated increase in capital demand at $t+J$ increases the demand for capital services at each user cost. Hence, the anticipated shadow user cost must rise to u' to equilibrate anticipated demand with supply. To satisfy (20), the anticipated shadow user cost at $t+J$ must rise relative to its value in the following period.

Intuitively, cash flow responds immediately to all forecast errors in Z, regardless of the duration of the disturbance. The shock will continue to affect

[12]The concept of an ex post shadow price of capital (or temporary equilibrium with capital fixity) has been explored by Berndt and Fuss [1986] and Hulten [1986].

[13]This can be calculated using the envelope theorem, by putting (10) in an iterative (Bellman) form, then calculating the partial derivative $q_{0,t} \equiv V_K/\bar{p}$. The result for periods $t+j>0$ follows by the law of iterated expectations.

the rate of cash flow until the capital stock has had a chance to fully adjust to the additional capital demand. This can be established formally by using the exogenous process for Z in (16) and equation (19) to calculate the following impulse responses for temporary and permanent shocks:

$$\frac{\partial \ln \pi_{t+j}}{\partial \epsilon_t} = \begin{cases} b\sum_{i=0}^{\min(j,n)} \theta_i & 0 \le j < J \\ 0 & \text{otherwise} \end{cases}, \quad \text{and} \quad \frac{\partial \ln \pi_{t+j}}{\partial \psi_t} = \begin{cases} b\gamma & 0 \le j < J \\ 0 & \text{otherwise} \end{cases}.$$

Generally, the effect of any shock on cash flow depends on the magnitude of the shock and the elasticity factor b. At impact, a shock raises cash flow by the product of b and the impact MA coefficient. To the extent that it persists, a shock can affect future cash flows up to the gestation horizon. For a horizon of j periods after the shock, the effect depends on the cumulative sum of the MA coefficients up to lag j. Intuitively, this sum represents the cumulative effect of the shock on the forecast error for Z^b. Neither temporary nor permanent shocks affect cash flow at the gestation horizon or beyond, once capital has the ability to adjust. The ex post rents caused by shocks are always unanticipated and transitory, as one would expect in a competitive market.

The degree of co-movement between the rate of cash flow and investment depends on the nature of the shock. For permanent shocks, the co-movement is positive. Cash flow responds to the shock immediately, and continues to be affected to the end of the gestation horizon. Although growth in the productive capital stock is postponed to the gestation horizon and beyond, investment spending commences at the planning horizon P. Therefore, both cash flow and investment respond in the same direction during the building period. Temporary shocks with a duration shorter than the gestation horizon do not prompt investment, so there is no positive co-movement. Temporary shocks that outlast the gestation horizon may cause investment to co-move positively or negatively with cash flow. In order for a temporary shock to affect capital growth, it must also affect cash flow up to the end of the gestation horizon, in the same direction as the shock. If this is the case, the investment response at the building horizon is in the same direction as cash flow. However, since the temporary shock cannot affect the level of the capital stock in the long run, the positive initial response of investment must eventually be reversed with negative investment. Some of this negative investment may occur while cash flow remains elevated within the building phase. Nonetheless, it is reasonable to expect the correlation between investment and cash flow to be positive, on balance, with the strength of the correlation depending on the relative preponderance of temporary and permanent shocks in the economy.

Tobin's Q is usually calculated as the current market value of a firm divided by the replacement value of its current capital stock. For now, assume that the replacement value of capital is measured as the replacement value of the productive capital stock. Then (11) can be used to determine that

$$Q_t = q_0^* + \sum_{i=1}^{B-1} \frac{s_{i,t}}{K_t} q_i^* + \sum_{i=0}^{J-1} R^{-i} \left[\frac{{}_t\pi_{t+i}}{\bar{p}} - u^* \right] \frac{K_{t+i}}{K_t}. \tag{21}$$

The first two terms in (21) represent the value of the funds committed to productive capital and ongoing building efforts, per unit of productive capital. The unconditional value of these two terms generally exceeds one, for two reasons. As demonstrated earlier, the unconditional shadow values incorporate compensation for foregone interest during the gestation period. As well, the planned capital additions $s_{i,t}$ are generally positive owing to economic growth. Therefore, when there are gestation lags, this measure of Q should exceed one in the long run. The final term shows that Q reflects the anticipated value of economic rents looking forward over the entire gestation horizon.

Since Q reflects both the value of productive capital and of committed plans, it is not equivalent to the shadow value of productive capital $q_{0,t}$. In Appendix A, I demonstrate that

$$Q_t = q_{0,t} + \sum_{i=1}^{J-1} q_{i,t} \frac{s_{i,t}}{K_t}, \tag{22}$$

where $q_{i,t}$ is the current shadow value of $s_{i,t}$. Further, I show that the shadow value of productive capital is its steady state value, plus the discounted value of all anticipated rents during the gestation period:

$$q_{0,t} = q_0^* + \sum_{j=0}^{J-1} \left(\frac{R}{1-\delta} \right)^{-j} \left[\frac{{}_t\pi_{t+j}}{\bar{p}} - u^* \right], \tag{23}$$

where the discount factor includes $(1 - \delta)$ in order to compensate for the opportunity cost of depreciation. This confirms that both q and Q reflect the same economic rents that affect cash flow. As filtrations of the same shock process they provide similar economic information.

Moreover, neither Q_t nor $q_{0,t}$ consistently provide reliable information about current investment. Recall that the capital stock is determined by equating the anticipated demand for capital services to the long run user cost of capital u^*. This corresponds to setting ${}_tq_{0,t+J}$ equal to the fixed long run shadow value q_0^*. Consequently, the deviation between the realization of $q_{0,t+J}$ and q_0^* is a forecast error that must be orthogonal to productive capital growth at $t+J$. However, current values of Q_t and $q_{0,t}$ co-move with investment to the extent that shocks to Z create unanticipated rents during the building phase of the gestation period. In addition, there will be a response in Q owing to the direct effect of investment expenditures on the value of the firm during the building process. Intuitively, a permanent shock to Z affects Q_t (and $q_{0,t}$) on impact, by causing anticipated rents during the entire gestation horizon. Both variables respond in the direction of the shock throughout the gestation period because rents persist over time. At the planning horizon, investment expenditures begin to respond to the shock. Therefore, both Q_t and $q_{0,t}$ covary positively with investment during the building process. However, as with cash flow, this co-movement breaks down for temporary shocks that are not sufficiently long-lived to prompt investment. When this is the case, movements in Q_t (and $q_{0,t}$) are unrelated to investment.

These claims can be confirmed formally using impulse responses for the log-linearized value of Q_t. Using the log-linearization in Appendix B, the impulse responses can be calculated using the responses for cash flow and capital growth, yielding

$$\frac{\partial \ln Q_{t+j}}{\partial \psi_t} \approx \sum_{i=1}^{B-1} \chi_i \frac{\partial g^K_{t+j+i}}{\partial \psi_t} + \sum_{i=0}^{J-1-j} \omega_i \frac{\partial \ln \pi_{t+j+i}}{\partial \psi_t}$$

$$= \begin{cases} b\gamma \sum_{i=0}^{J-1-j} \omega_i & 0 \leq j \leq P \\ \gamma \chi_{J-j} + b\gamma \sum_{i=0}^{J-1-j} \omega_i & P < j < J-1 \\ 0 & j \geq J \end{cases}, \qquad \text{and}$$

$$\frac{\partial \ln Q_{t+j}}{\partial \epsilon_t} \approx \sum_{i=1}^{B-1} \chi_i \frac{\partial g^K_{t+j+i}}{\partial \epsilon_t} + \sum_{i=0}^{J-1-j} \omega_i \frac{\partial \ln \pi_{t+j+i}}{\partial \epsilon_t}$$

$$= \begin{cases} \sum_{i=0}^{J-1-j} \omega_i b \sum_{i=0}^{\min(j,n)} \theta_i & 0 \leq j \leq P \\ \chi_{J-j} \sum_{i=0}^{\min(J,n)} \theta_i + \sum_{k=1}^{\max(j-P-1,0)} \chi_{J-j+k} \theta_{J+k} + \sum_{i=0}^{J-1-j} \omega_i b \sum_{i=0}^{\min(j,n)} \theta_i & P < j < J-1 \\ \sum_{k=1}^{B-1} \chi_k \theta_{j+k} & j \geq J \end{cases}.$$

Here, χ_i is the (semi-)elasticity of Q_t with respect to capital growth at horizon i. In the appendix, I demonstrate that this elasticity is decreasing in i. The parameter ω_i is the elasticity of Q with respect to cash flow at horizon i, which also decreases in i for reasonable calibrations. These responses reflect three clear phases. The first phase coincides with the planning horizon, with Q increasing to reflect the present value of anticipated rents throughout the remainder of the gestation period. The value of these anticipated rents eventually decline as a new long equilibrium becomes imminent, because the horizon over which they occur becomes smaller. However, this effect need not be strongest on impact. If the shock has sufficient duration to prompt investment, there is a second phase in which Q rises to reflect the value of non-productive capital as it is accumulated throughout the building phase. This effect becomes stronger as the new long run equilibrium approaches. Finally, a third phase may arise for temporary shocks that outlast the gestation horizon. In this phase, the capital stock continues to adjust downward as the shock dies out over time. In this phase, there are no rents, but Q declines to reflect the value of ongoing disinvestment.

4. A Reconciliation Between Measured Capital and Productive Capital

The results above require productive capital to be measured using an accounting scheme that correctly accounts for building lags. In practice, measures of the capital stock are usually formed under the assumption of one building period.[14] Therefore, the estimate of the capital stock at any point in time includes both completed and incomplete capital. Consequently, standard statistical measures of Q_t, capital growth, and cash flow do not coincide with the true productive measures described above.

One strategy for dealing with this problem is to use investment expenditures to construct measures of the capital stock that account for alternative building lags. However, this is unsatisfactory because it imposes a lag structure on the data. The strategy adopted in this paper is to find a mapping from accounting measure to the unobserved measure of productive capital. This mapping can then be incorporated into the interpretation of statistical results, allowing the data to tell its story.

Let $\tilde{K}_t$ denote the accounting measure of the capital stock at t, formed using a standard one period time to build capital accumulation identity. In Appendix C, I show that the productive measure of capital, K, maps to this accounting measure by the lag polynomial

$$(24) \qquad \tilde{K}_{t+1+i} = \phi(L) K_{t+B+i}, \qquad \text{where} \qquad \phi(L) = \sum_{j=0}^{B} \phi_{B-j} L^j.$$

This is simply a generalization of the standard accounting relationship, which correctly measures the productive capital stock in the special case where $J=$

[14] With the exception of electric light and power structures, the BEA does not make an explicit attempt to adjust for building lags for most types of capital. The practice is justified by the fact that the aggregate value of uncompleted plants has been a small and stable proportion of the value of completed plants through time (see BEA [1999]).

$B = 1$ and $\phi_1 = 1$.[15] In this general case, the accounting measure $\tilde{K}_{t+1}$ is a weighted average of the planned stocks of productive capital from $t{+}1$ to $t{+}B$, with the weight at horizon j equal to the spending proportion ϕ_{B-j}. Provided that there is a building period, the accounting measure incorporates changes in the productive capital stock before they occur.

It is also useful to determine a mapping from the accounting measure of capital growth to the true productive measure. Define $\tilde{g}^K_{t+i}$ as the rate of growth in accounting capital, $\Delta \ln \tilde{K}_{t+i}$. The appendix shows that this maps to the true productive measure by the lag polynomial

$$\tilde{g}^K_{t+1+i} \approx \tilde{\phi}(L) g^K_{t+B+i}, \qquad \text{where} \qquad \tilde{\phi}(L) = \sum_{j=0}^{B} \tilde{\phi}_{B-j} L^j, \tag{25}$$

and $\tilde{\phi}(1) = 1$. Again, this generalizes the standard condition, which correctly measures productive capital in the special case where $J = B = 1$. The growth rate in the statistical measure is approximately a weighted average of the growth rates in the productive capital stock over the next B periods. The weights $\tilde{\phi}_j$ are closely related to the true spending weights ϕ_j.[16]

The responses described earlier in this section can be translated to cases where capital is measured using the standard accounting. Accounting capital growth never reflects shocks until the planning horizon is complete. Thereafter, a planned addition to productive capital works its way through the building phase, affecting the observed measure of capital growth by the amount that it changes spending in each period. This can be seen using the equations

$$\frac{\partial \tilde{g}^K_{t+j}}{\partial \psi_t} \approx \begin{cases} 0 & j \leq P \\ \tilde{\phi}_{J-j+1} \frac{\partial g^K_{t+J}}{\partial \psi_t} & J > j > P \\ 0 & j > J \end{cases}, \qquad \text{and}$$

$$\frac{\partial \tilde{g}^K_{t+j}}{\partial \epsilon_t} \approx \begin{cases} 0 & j \leq P \\ \sum_{i=0}^{\min(j-P, B-1)} \tilde{\phi}_{B-i} \frac{\partial g^K_{t+B+j-i-1}}{\partial \epsilon_t} & j > P \end{cases}.$$

These responses show that the measured capital growth associated with any plan is spread throughout the building period. For example, consider a permanent shock ψ_t that increases g^K_{t+J} by one percentage point. Due to the

[15] Note that this also embeds a special case where there is no time to build, so $\phi_0 = 1$. For this case, the end-of-the-period statistical measure, after current investment, is the actual quantity of productive capital *during* the period.

[16] The weights $\tilde{\phi}_j$ give slightly more importance to spending at longer horizons j than ϕ_j, and less importance to shorter horizons.

planning lag, the shock doesn't affect observed capital growth up to $t+P+1$. During the building phase, the shock affects observed capital growth by $\tilde{\phi}_j$ percentage points in each period, where j is the number of periods to the end of the gestation horizon. Once the building period is complete, there are no further effects on observed capital growth.

Since $\tilde{K}_t$ is used to calculate measures of cash flow and Q, discrepancies between productive capital and the accounting measure also affect how these variables respond to shocks. The accounting measures of cash flow and Q are related to their true productive measures by

$$\text{(26)} \qquad \ln \tilde{\pi}_{t+i} = \ln \pi_{t+i} - \ln \frac{\tilde{K}_{t+i}}{K_{t+i}} \qquad \text{and} \qquad \ln \tilde{Q}_{t+i} = \ln Q_{t+i} - \ln \frac{\tilde{K}_{t+i}}{K_{t+i}}.$$

Applying a linear approximation yields that

$$\text{(27)} \qquad d \ln \frac{\tilde{K}_{t+i}}{K_{t+i}} \approx \tilde{\Phi}(L) dg^K_{t+B+i}, \qquad \text{where} \qquad \tilde{\Phi}(L) = \sum_{j=0}^{B-1} \tilde{\Phi}_{B-j} L^j,$$

and $\tilde{\Phi}_{B-j} = \tilde{\phi}_B + \ldots + \tilde{\phi}_j$. Therefore, the change in the "error" associated with mismeasurement of the capital stock is related to a distributed lag of the growth rates in productive capital over the building horizon.

This measurement discrepancy affects the interpretation of the impulse responses for cash flow and Q as follows. Up to the end of the planning horizon P, the error has no effect. Intuitively, this is because the accounting measure of the capital stock has not yet reacted to the shock. If the shock has sufficient duration to prompt investment, the accounting measure of the capital stock rises over the course of the building period. Therefore, it has a progressively negative influence on the impulse response. If b is below one, this effect eventually becomes strong enough to outweigh the positive influence of rents on cash flow and Q, so the response becomes negative at sufficiently long horizons.

III. Empirical Evidence

1. Data

I constructed my dataset using quarterly aggregates for non-farm non-financial U.S corporations from 1959Q2 to 2002Q4. Series for Tobin's Q, cash flows, and the growth rate in capital were constructed using seasonally-adjusted data from the Flow of Funds Accounts of the Federal Reserve Board, the Bureau of Economic Analysis (BEA), the Bureau of Labor Statistics (BLS), and Data Resources International (DRI). The accounting measure of the capital stock was generated iteratively using quarterly fixed investment expenditures and a one period time-to-build capital accumulation identity.[17] Following Hall [2001], I calculate the measure of the aggregate market value of physical capital as the value of equity and debt, less the value of all non-capital assets (including liquid assets), residential structures, and inventories. Both Tobin's Q and cash flows are adjusted to account for corporate income taxes and the influence of investment tax credits and depreciation allowances on the effective price of capital. A detailed description of the data construction is given in Appendix F.

Time plots of the data are shown in Figures 3 to 5. Table 1 contains sample moments. Figure 3 demonstrates the considerable volatility in capital growth, which exhibits many prolonged movements around a mean of about 1.1 percent per quarter. Cash flows and Tobin's Q are plotted in Figures 4 and 5, respectively. Since the tax correction for the price of capital goods decreases the replacement value of the accounting measure of capital, it causes a noticeable increase in both series. The measure of Q is very volatile, and does not seem to revert to a discernable long run level. Rather, the series is characterized by its many high-frequency movements around prolonged, low-frequency trends. Note from Table 1 that the sample average of the tax-corrected measure is well above one, which is consistent with the gestation model for capital. Cash flow seems to cycle around a stable long run mean, with movements resembling the business cycle. Although there are periods where cash flow and Q exhibit coherence with investment, neither is a consistent indicator. Despite this, the three variables have positive mutual correlation, which is apparent by inspection of the plots.

Visually, it appears that the Q series may be non-stationary. Table 2 explores this possibility using the Augmented Dickey Fuller test and the Variance Ratio test. The Dickey Fuller test rejects a unit root in $\tilde{g}_t^K$ and $\tilde{\pi}_t$, but fails to reject for $\tilde{Q}_t$. This is problematic for most investment theories, since Q should revert to a well-defined long run level. Inspection of Figure 5 suggests that this failure may reflect very low-frequency movements in the level of Q, which could be explained by a number of factors, including, for example, changes in the effective tax rate on capital gains and dividends, or changes in components of firm value that are outside of the model, such as intangible capital

[17] A pre-sample for the capital stock was generated for the period 1947Q1 to 1959Q1 in order to minimize the possibility of errors associated with an appropriate starting value. The initial value for the end of 1946 was set equal to the BEA's estimate of the real capital stock.

(Hall [2001]).[18] Variance ratios for Q diminish considerably at longer horizons, which provides evidence against a unit root.

2. The Existence and Duration of Gestation Lags

In this section, I use tests involving Solow residuals and labor hours growth to consider two distinct issues. The first issue is whether there is a delayed response of investment to aggregate shocks, which I interpret as a planning lag. The second issue is whether there is a delayed response of productive capital to investment, which would be associated with a building lag.

John Fernald kindly provided quarterly Solow residuals for the period 1965Q2 to 2001Q4 that are corrected for measurement errors owing to changes in labor quality and variable factor utilization using the methodology in Basu, Fernald, and Shapiro [2000].[19] The Solow residuals are divided by a labor share of $\alpha = 2/3$ to convert to units of labor-augmenting technological progress. Quarterly data for aggregate labor hours of non-financial corporations are from the BLS. Figure 7 shows a time plot of the purified Solow residuals and the growth rate in aggregate labor hours.

i. Evidence from Previous Work

There have been a few attempts to measure the duration of the gestation period using case studies at the plant and firm level, and other non-parametric methods. Estimates by Koeva [2000], Mayer [1960] and Krainer [1968] suggest that the capital gestation lag ranges between one and two years. Mayer [1960] and Jorgenson and Stephenson [1967] obtain estimates of the planning duration ranging between six months and a year.[20]

ii. Planning

Most prominent models do not feature a delayed response of capital growth to shocks. To illustrate this, consider the effect of a positive permanent shock.

[18] The valuation data are not adjusted to reflect changes in the tax rate on dividends and the capital gains rate, so there may be some trends owing to this mismeasurement. Summers [1981] and McGrattan and Prescott [2002] demonstrate that changes in these tax rates can have large effects on firm value.

[19] This study makes an additional adjustments for capital adjustment costs and for the reallocation of resources across sectors, which I remove for the purpose of my calculations.

[20] This evidence is supported by structural VAR estimates by Erceg and Levin [2003] using aggregate data, who find a seven quarter lag in the response of business investment to monetary shocks.

In the frictionless neoclassical model, investment should respond to the shock immediately, with the maximum rate of response at impact. In a model with convex adjustment costs, the investment response is also largest on impact, but is more drawn out over time. Models with fixed adjustment costs, such as Caballero and Engel [1999], and irreversibility, such as Abel and Eberly [1993], tie the likelihood of investment to the degree of departure from the frictionless demand for capital services. Provided that the shock is not too large, generally some firms will invest, and some will not. This implies that aggregate capital growth depends on the distribution of the capital imbalances for all firms in the economy. Since some firms are prompted to invest in response to an aggregate shock, neither of these issues complicate the initial timing of the aggregate response, only the magnitude. To get the maximum benefit, most firms that do adjust should do so immediately.[21]

To investigate whether there is a planning lag in response to technology shocks, I estimated the following equation using OLS:

$$\tilde{g}^K_{t+1} = c_0 + \sum_{i=0}^{n_{sr}} d_i \tilde{sr}_{t-i} + e_{1t}, \tag{28}$$

where $\tilde{sr}_{t-i}$ is the purified Solow residual at lag i. To conserve degrees of freedom, I chose a maximum lag length of 14 quarters, which seems a reasonable bound for the total gestation horizon given the previous research discussed above. This specification nests all possible planning and building combinations as special cases. Given the generalized form of the growth rate in the accounting measure of capital in (25),

$$\frac{d\tilde{g}^K_{t+1}}{d\tilde{sr}_{t-P-i}} = \sum_{j=0}^{B} \tilde{\phi}_{B-j} \frac{dg^K_{t+B-j}}{d\tilde{sr}_{t-P-i}} = d_{P+i}, \qquad \text{for } i = 0, \ldots, B. \tag{29}$$

Most investment models make the implicit assumption that $P = 0$ and that either $\tilde{\phi}_0 = 1$ or $\tilde{\phi}_1 = 1$. Since these models suggest an immediate response of productive capital growth at the building horizon, d_0 should be positive. If there is a planning lag, d_P should be the first positive coefficient, and P should be an estimate of the planning horizon.

The magnitude of the estimated coefficients can be given a structural interpretation in the gestation lag model for a special case where technology and labor supply disturbances are uncorrelated, and the technology process is a

[21] Although it is possible that some firms might reach their investment trigger faster in the following periods (due to depreciation), it seems doubtful that this effect would compose most of the response.

random walk. For this case, the results of Appendix D show that

$$d_{P+i} = \tilde{\phi}_{B-i} \left(1 + \zeta\right).$$

Empirical estimates of the wage-elasticity of aggregate labor supply ζ range between 0 and 1. For the special case where $\zeta = 0$, the coefficient d_{P+i} should be a direct estimate of the spending share $\tilde{\phi}_{B-i}$.

Results for this regression are shown in Table 3. Coefficient estimates are insignificant up to the fourth lag, which is significant at ten percent. This suggests a planning period for investment of one year, which is at the high end of previous estimates by Mayer [1960] and Jorgenson and Stephenson [1967]. Thereafter, the coefficients for lags five through ten are each significant at levels of five percent or lower. This suggests a planning lag of four or five quarters. The magnitude of the coefficients at lags four through seven indicate that about 13 percent of the investment expenditures associated with a given plan occur during this time window. If the tenth lag is interpreted as the end of the building horizon, the estimates suggest a total gestation period of ten quarters, with a building phase from period four to period ten. However, this interpretation is subject some important caveats. In principle, the building period should be measured by the delay between the initial change in investment spending and the time it begins to affect productive capital. Since it is possible for building to continue with little or no expenditures, this may not accurately reflect the length of the building horizon. A second concern with this interpretation is that the significance of the lagged coefficients beyond the initial planning stage may reflect a planning period combined with convex adjustment costs for capital and/or prolonged general equilibrium adjustment.

In the absence of labor supply endogeneity, the coefficients d_i, $i = P, \ldots, J$ should sum to one over the building period. The tests reported in the bottom portion of Table 3 show that the cumulative sum of the coefficients up to the tenth lag is about one fourth, falling well short of the required benchmark in terms of magnitude and significance. Among other things, this failure may reflect inconsistency in the regression estimates owing to measurement error or endogeneity. Another plausible explanation is the presence of external adjustment costs in general equilibrium. In dynamic general equilibrium models that exhibit the balanced growth property, it is well known that permanent technology shocks prompt an equivalent cumulative response in capital growth. However, due to the smoothing of consumption and labor, the response will tend to be drawn out over time even in the absence of internal adjustment costs and/or capital gestation lags. Reasonably calibrated RBC models suggest that it takes the economy between three to six quarters to complete one-fourth of the total capital growth mandated by a permanent technology shock. In the benchmark case of Campbell [1994], which features Cobb-Douglas production, fixed labor, and unit intertemporal substitution elasticity, the economy takes about seven quarters to complete one-fourth of the mandated capital growth.[22]

Indeed, this smoothing effect becomes more pronounced as the duration of the gestation period increases. Figure 9 shows the response of measured capital growth to a permanent technology shock in a calibrated RBC model, for building lags ranging from one to nine quarters. In each case, it is assumed that expenditures are distributed evenly throughout the building period. Details of the model setup and calibration are outlined in Appendix E. According to these simulations, the time required to complete one fourth of the total adjustment increases exponentially with the building horizon, rising from seven quarters with TTB=1, thirteen quarters with TTB=5, to 104 quarters with TTB=9. Given these results, the magnitude of the estimated coefficients are not unreasonable, nor is the notion that they could be a reasonable outcome for an economy without explicit internal adjustment costs for capital.

The regression results provide evidence for a substantial planning lag. Not only is there a delayed response of investment to shocks, but the cumulative response up to the third lag is not significantly different from zero. As well, the character of the response is inconsistent with other models. The response is actually hump-shaped, peaking at the seventh lag. Most models without planning would tend to have the largest response on impact, or, most favorably, a flat response out to some horizon. The estimated response is inconsistent with these possibilities. To wit, the estimated cumulative response from the fourth lag to the third lag is statistically larger than the estimated cumulative response from impact to the third lag. Although these facts are challenging to other models, they can easily be reconciled with planning and building lags.

iii. Building

Building involves the transformation of capital goods to productive capital. The time required for this transformation is not easily estimated, because productive capital is not directly measurable. The strategy employed in this section is based on the principle that changes in productive capital contribute directly to output growth. Therefore, some portion of observed output growth must be attributable to growth in the productive capital stock.

Applying the standard techniques of growth accounting to the simplified Cobb-Douglas production function (1), one can obtain the following implicit

[22] Adding a labor supply decision tends to extend the period of adjustment, but not dramatically.

measure of the growth rate in productive capital and technology:

$$\tilde{m}_t \equiv \tilde{g}_t^Y - \alpha \tilde{g}_t^H = (1-\alpha) g_t^K + sr_t, \tag{30}$$

where $\tilde{g}_t^Y$ and $\tilde{g}_t^H$ are the measured growth rates in output and labor, and sr_t is true technological growth. This suggests a structural equation of the form:

$$\tilde{m}_t = \mu_{sr} + (1-\alpha) g_t^K + \psi_{sr,t} \tag{31}$$

where $\psi_{sr,t}$ is a mean-zero random disturbance. In principle, this equation is a valid regression specification provided that the true technology shock is orthogonal to the growth rate in productive capital, which is satisfied if there is at least a one-period time to build for capital. This suggests that one might uncover the length of the building period by regressing values of $\tilde{m}_t$ on lags of $\tilde{g}_{t-j}^K$, where the significant lagged coefficient at the longest lag is an estimate of the building horizon.[23]

Unfortunately, the above specification has undesirable properties that make the results difficult to interpret. Generally, there is no one-to-one mapping between productive capital growth and measured capital growth. By inspection of (25), such a mapping only exists for a special case where $\phi_B = 1$, so that $g_{t+B}^K = \tilde{g}_{t+1}^K$.[24] For this special case, such a regression would correctly estimate the building horizon. This special case holds for any investment model with a standard one period building horizon ($B=1$). For other cases, the characteristics of the mapping depend on the unobserved roots $\{x_j\}_{j=1}^{B-1}$ of the lag polynomial $\tilde{\Phi}(x)$. Generally, there can be stable solutions for g_t^K forward and backward (or both) in the observed measure $\tilde{g}_t^K$, where the roots can be negative, positive, or complex. This leads to counterintuitive results that complicate the interpretation of the estimates.

This can be illustrated using some simple examples. Consider a case where $B=2$, with $\tilde{\phi}_2=2/3$ and $\tilde{\phi}_1=1/3$. In this case, the polynomial $\tilde{\Phi}(L)$ is simply $(1+.5L)$, which has a stable root of -2. For this very simple case, the mapping is

$$g_t^K = \frac{3}{2} \sum_{j=0}^{\infty} \left(-\frac{1}{2}\right)^j \tilde{g}_{t-1-j}^K.$$

Here, coefficients on the lags of measured capital growth are non-zero from the first lag onward and have signs that oscillate from negative to positive

[23]The fact that we are looking for the longest lag can easily be seen in Figure 1. Expenditures join the capital stock sooner as the firm nears the end of the building period.

[24]Note that I assume that $\phi_B > 0$ in order for the building horizon to be distinguishable from planning. This rules out other one-to-one mappings.

at successive lags. Although the coefficients attenuate in magnitude at larger lags, it is highly plausible that estimates would yield significant coefficients for $j \geq B$. Therefore, the highest lag with a significant coefficient cannot be interpreted as an estimate of the building horizon. As a further example, consider a case where $B = 2$ but $\tilde{\phi}_2 = 1/3$ and $\tilde{\phi}_1 = 2/3$. In this case, the root of the lag polynomial $\tilde{\Phi}(L)$ is unstable at -0.5, and the mapping is

$$g_t^K = -\frac{3}{2}\sum_{j=0}^{\infty}\left(-\frac{1}{2}\right)^j \tilde{g}_{t+j}^K.$$

The suggested regression would have a significant impact coefficient, but no significant coefficients at any other lag. The results would incorrectly point to a one period building horizon.

A less problematic structural specification can be obtained by combining equations (25) and (31) to obtain the following specification:

$$\tilde{g}_{t+1}^K = b_0 + \sum_{j=1}^{B} b_j \tilde{m}_{t+j} + e_t, \qquad \text{where} \tag{32}$$

$$b_0 \equiv -\frac{\mu_{sr}}{1-\alpha}, \quad e_t \equiv -\sum_{j=1}^{B}\frac{\tilde{\phi}_j}{1-\alpha}\tilde{\psi}_{sr,t+j}, \quad \text{and} \quad b_j \equiv \frac{\tilde{\phi}_j}{1-\alpha}$$

for $j = 1, \ldots, B$. Here, observed capital growth depends on forward values of the implicit measure of capital growth and the technology disturbance. By construction, the implicit measure $\tilde{m}_t$ is negatively correlated to the error term because it contains the technology shock ψ_{sr}. However, potential endogeneity problems can be avoided by instrumenting for the forward values of $\tilde{m}_{t+j}$.

Finding an appropriate set of instruments is a thorny issue. First, the regression requires a lot of instruments. To avoid inconsistency, the number of included leads of $\tilde{m}_t$ should be no smaller than the building lag. To satisfy the order condition, at least one instrument must be included for each lead. Second, although the set of valid instruments contains the entire time t information set, most choices are likely to have limited strength because the variation in each regressor is partially attributable to an unforecastable technology shock. Nonetheless, some success was achieved using current and lagged values of the measured growth rates in capital and labor hours. These choices were motivated by theoretical considerations. In order to identify all the spending shares $\tilde{\phi}_j$, information about the growth rates in the productive capital stock from $t+1$ to $t+B$ must be included. Provided that technology shocks are exogenous, serially uncorrelated, and unforecastable, anything in the time t information set is uncorrelated to e_t. According to equation (25), measured capital growth at t reflects the growth rate in productive capital from t to $t+B-1$, while lags up to $t+B-1$ contain additional identifying information. However, these measures provide no information on g^K_{t+B}, leaving $\tilde{\phi}_{T+B}$ unidentified. Under the model, planned additions to the productive capital stock reflect information on labor growth from J periods earlier. Provided that $P>0$, labor growth from periods t to $t-J+1$ should contain the needed information, plus overidentifying information about the growth rates from $t+1$ to $t+B-1$.

Unfortunately, the estimates using this specification are likely to suffer from a significant small sample bias. This is because the reduced-form disturbances e_t are autocorrelated at lags up to $B-1$, which violates the Gauss-Markov assumptions. Therefore, tests that rely on asymptotic distributions will give misleading results. To correct for this problem, I generate bias-corrected confidence intervals for the estimated parameters, using a bootstrap technique.[25]

The results of the regression are shown in Table 4. The set of explanatory variables contains twelve forward values of the $\tilde{m}_{t+j}$, which are instrumented using measured rates of growth in capital and labor hours for lags ranging from zero to thirteen quarters. After correcting for small-sample bias using a bootstrap, the estimated coefficients are statistically significant at leads $+2$ and from $+4$ through $+8$ at significance levels of ten percent or higher.[26] The estimates at the remaining leads are not different from zero at significance levels of at least ten percent. This could indicate a lack of power against the null, which is a reasonable assertion when the result is considered in conjunction with the other estimates. Nonetheless, the presence of zero coefficients at these leads is not inconsistent with the theory. The partial R^2 (Shea [1997]) for each of the regressors is about 0.10, which raises the possibility of the size distortions owing to weak instruments that are discussed by Bound et al. [1989], Stock, Wright, and Yogo [2002], and others. These distortions may cause the true significance level of the tests to be understated. Notwithstanding these possible distortions, the fact that the partial R^2 does not decline with the forward lead offers partial support for the gestation lag story, as does the apparent effectiveness of deep lags as instruments. Considered collectively, the estimates are suggestive of an eight-quarter building horizon, which is in line with the estimates that Koeva [2000] obtained using a non-parametric methodology. This estimate, combined with the planning estimate of one year obtained in the previous section, suggests a total gestation lag for new capital

[25] In order to preserve the autocorrelation structure of the estimates in the bootstrap simulation, I re-sample blocks of twelve adjacent observations.

[26] I report bias-corrected intervals at a 90% significance level. Intervals were also calculated for significance levels of 95% and 99%, for which I only report significance.

of about three years.

According to the structural specification in (32), the coefficients b_j should sum to $(1-\alpha)^{-1}$ over the building horizon. Since capital's share of output is roughly 1/3 in aggregate data, the estimates should sum to about three. A model with a standard one-period time to build capital accumulation identity should satisfy the restriction that $b_1 = 3$. The fact that this null is easily rejected provides evidence against this alternative. However, the null that the cumulative sum of the estimated coefficients is three cannot be rejected using the bootstrapped confidence intervals, for significance levels of ten percent or higher. If the building horizon is interpreted as eight periods, the 90% upper significance level is about 3.03, while at eleven periods, the upper limit rises to about 3.78.[27] This reinforces the building horizon estimate of eight quarters.

The reasonableness of an eight quarter building horizon can also be assessed using an alternative test. According to the generalized process for measured capital growth in (25), the unconditional autocorrelation of measured capital growth at a given lag cannot be zero unless that lag exceeds the building horizon.[28] Beyond the building horizon, this correlation should eventually go to zero, although it may extend beyond the building horizon if the growth rate in productive capital is serially correlated. Therefore, an upper bound on the building horizon is the lag at which the unconditional autocorrelation of measured capital growth is statistically zero. Consider the correlogram for $\tilde{g}_t^K$ in Figure 8. These correlations suggest that the measured growth rate in capital is unconditionally autocorrelated up to ninth lag, at ten percent significance. This suggests an upper bound for the building period of nine quarters, slightly higher than the estimate obtained above.

3. Temporary and Permanent Innovations

This section estimates impulse responses to temporary and permanent aggregate shocks that are identified using the zero-frequency restrictions employed in other contexts by Shapiro and Watson [1988] and Blanchard and Quah [1989]. According to this scheme, only permanent shocks can affect the capital stock in the long run. This is a very weak restriction that should be satisfied in any model that converges to a neoclassical capital market equilibrium in the long run. This includes standard investment models with convex

[27] Note that there may be a potential bias owing to endogeneity between the approximation error in (25) and the instrumented regressors. Simulations by the author using a calibrated system (which can be obtained upon request) suggest that this causes a very small negative bias in large samples.

[28] This holds because $cov\left(\tilde{g}_t^K, \tilde{g}_{t-j}^K\right) > 0$ for $j = 1\ldots, B$, provided that $\phi_j > 0$.

and non-convex adjustment costs, transaction costs, and irreversibility. Since the identifying restriction is reasonable for most models, the properties of the estimated impulse responses can be compared to their respective predictions.

It is important to assess the models using reasonable standards, due to the nature of small-sample VAR estimation. It is typical for identified VARs to estimate a smooth impulse response, even if the data-generating process has more well-defined characteristics. Moreover, most of the well-known results from alternative models are demonstrated in a partial equilibrium setting. Price adjustment in general equilibrium should tend to smooth results compared to these predictions.[29] Therefore, it is important to judge the models by their consistency with the general character of the estimated responses.

Some reasonable implications of the gestation lag model are as follows. Due to the planning lag, measured capital growth should respond sluggishly to shocks. If there is a significant gestation horizon, measured capital growth should react much more strongly to permanent innovations than to temporary innovations of the same magnitude. Given the sizable gestation lags estimated above, it would be reasonable to expect little or no response of capital growth to temporary shocks. Consequently, almost all of the variance of investment should be attributable to permanent shocks. In comparison, Q and the rate of cash flow should respond immediately to both disturbances, with the response limited to the duration of the gestation horizon. The complications that arise due to the mismeasurement of productive capital should also be considered. For shocks that prompt investment, cash flow should decline monotonically over the course of the building period, because the measured capital stock (in its denominator) anticipates the actual productive stock. This effect should not occur for Q, since it is roughly offset by the effect of the firm's ongoing accumulation of capital during the building period on market value. Significant proportions of the variation in Q and cash flow should be attributable to both temporary and permanent shocks.

Alternative models broadly imply that aggregate investment should respond immediately to permanent shocks. In a model with convex adjustment costs, the response attenuates over time. Other models, such as fixed adjustment costs and irreversibility, imply a more concentrated response. Broadly, these models are well protected by a null that the response to the permanent shock is not upward-sloping over some horizon. The response to temporary shocks for alternative models are more difficult to assess. A non-positive response is evi-

[29] For example, Thomas [2002] demonstrates that the lumpiness of micro-level investment suggested by a partial equilibrium model with transaction costs will be smoothed considerably in aggregate general equilibrium.

dence against a convex adjustment cost model–if the shock affects q, it should prompt investment. In models with fixed adjustment costs or irreversibility, it is sensible to think that firms are reluctant to adjust to temporary shocks. However, there is little reason to believe that such firms would reduce investment. With this in mind, a null that the response is non-negative is more than adequate to protect these models.

I estimate two separate bivariate structural VARs. Specification (1) combines measured capital growth (in annual percentage terms) and the log of measured cash flow ($Y_t^1 = [\tilde{g}_{t+1}^K, \ln \tilde{\pi}_t]'$), while specification (2) combines the capital growth measure and the log of Tobin's Q ($Y_t^2 = [\tilde{g}_{t+1}^K, \ln \tilde{Q}_t]'$). For each system, I estimate a VAR of the form

$$Y_t^j = B_0^j + B_1^j Y_{t-1}^j + \ldots + B_p^j Y_{t-p}^j + e_t^j, \qquad \text{where} \qquad E\left[e_t^j e_t^{j'}\right] = \Sigma^j,$$

and p is the number of lags.[30] The estimated VARs for $j = 1, 2$ are then converted to structural moving average form

$$Y_t^j = \mu_Y^j + \sum_{i=0}^{\infty} \Xi_i^j \left[\psi_{t-i}^j, \epsilon_{t-i}^j\right]', \qquad \text{where} \qquad \mu_Y^j = E\left[Y_t\right], \tag{33}$$

where ψ_{t-i}^j and ϵ_{t-i}^j are the permanent and temporary shocks at time $t-i$, and the Ξ_i^j are (2x2) matrices of structural coefficients. The identification of each system rests upon the assumption that

$$\lim_{s\to\infty} \frac{\partial \ln K_{t+s}}{\partial \epsilon_t} = \lim_{s\to\infty} \sum_{j=0}^{s} \frac{\partial g_{t+1+j}^K}{\partial \epsilon_t} = 0,$$

so that the changes in the measured capital growth prompted by the temporary shock sum to zero.

Generally, the identification methodology requires both variables in Y_t to be stationary, and the results are sensitive to departures from this condition. This sensitivity is a common empirical problem associated with zero-frequency constraints. For instance, Blanchard and Quah [1989] make adjustments for non-stationarity in the unemployment rate, from which they remove a fitted linear time trend. Recall that the evidence for stationarity of Q is ambiguous: Dickey-Fuller tests fail to reject a unit root, while the variance ratio test suggests stationarity. To avoid problems associated with the potential non-stationarity of Q, I eliminated a very low frequency trend using an HP filter.[31]

[30] The lag length is chosen according to the AIC.

[31] The filter was estimated with $\lambda = 99999$. The results seem fairly robust to other choices.

The rationale behind applying this filter is that the scope of the theory is limited to movements in Q up to the gestation lag. Arguably, the lower frequencies reflect mismeasurement of tax effects, intangibles, and other things that are outside of the theory. Figure 6 shows the fitted trendline against actual Tobin's Q, in logs. The detrended series of (logged) Q is the actual series minus the trendline.

For robustness, I report 90 percent confidence intervals estimated using two alternative methods. The first intervals, which are denoted with "$\cdot \bullet \cdot$," are calculated using the asymptotic (normal) distribution of the impulse response (see Lütkepohl [1993]). The second set of intervals, denoted with "$--$", employ the bootstrap-after-bootstrap method of Kilian [1998], which is more robust in small samples.[32]

Figures 10 and 11 show the impulse response estimates for specifications $j=1,2$. In many respects, the character of these responses is consistent with the predictions of the gestation lag model. In both specifications, capital growth has a hump-shaped response to the permanent shocks that, although significant on impact, peaks at a lag of three to four quarters. The timing of this peak is roughly consistent with the planning lag estimated in the previous section. Both responses remain positive and significant at lags of up to 15 quarters. Investment exhibits no significant response to the temporary shock in either specification. This is consistent with the gestation lag model, but may also be consistent with irreversibility or transaction cost models. Cash flow and Q respond to both temporary and permanent innovations in a similar manner, peaking near the time of impact, then attenuating to zero over time. The response of cash flow to the permanent shock declines faster than the corresponding response for Q, which is roughly consistent with the model's prediction. Despite this, there is no evidence that the cash flow response eventually declines below zero over the course of the building horizon.

It is notable that the magnitude of the responses of cash flow and Q to the temporary and permanent shocks seem implausibly large relative to the measured capital response given the predictions of the gestation lag model. According to the derivations in Section II, the response of measured cash flow to a permanent shock should be no larger than $b\gamma$ within the gestation period. In annual percentage terms, the response of measured capital growth at horizon $P \leq j < J$ should be $400\gamma\tilde{\phi}_{J-j}$. Since the expenditure shares sum to one, this suggests that the ratio of the responses should average $400/bJ$ over the course

[32] In all cases, I perform 5000 replications of the first stage of the bootstrap of the procedure (which corrects for bias in the estimated VAR coefficients), and 5000 replications of the second stage of the procedure (which uses the corrected coefficient estimates).

of the building horizon.[33] When the production function is Cobb-Douglas, b can be no larger than labor's income share. Assuming that this share is about 2/3, the ratio of the capital growth and cash flow responses should exceed $400\tilde{\phi}_{B-j}/b \approx 600\tilde{\phi}_{B-j}$ at any given horizon, and should be larger than $600/J$, on average. Cursory examination of Figure 10 suggests that the ratio falls well below this magnitude for any reasonable gestation horizon. For instance, the ratio of the two responses averages around 25 for a gestation horizon of ten quarters, well below the minimal benchmark of 60.

One possible explanation for this discrepancy is that the degree of factor substitutability imposed by Cobb-Douglas is too strong, which dampens the magnitude of the ex post rents predicted by the theory. A greater degree of complementarity would magnify the sensitivity of cash flow (and Q) to imbalances between the frictionless demand for capital services and the fixed ex post supply of productive capital. Recall that when the production function is CES and labor supply is inelastic, the magnitude of b is inversely proportional to the elasticity of substitution between capital and labor. For this case, a substitution elasticity of 2/5 would be roughly sufficient to justify the relative magnitudes of the capital and cash flow responses in the preceding example for $J = 10$. A low elasticity of substitution would also explain why measured cash flow and Q fail to decline below zero over the course of the building horizon, because this only occurs when $b < 1$.

The upper left-hand panels of Figures 12 and 13 test, for each specification, whether the upward-sloping character of the investment response to the permanent shocks is statistically significant. For each specification, these figures show the first-difference of the response, and the 10 percent lower tail for the distribution of this difference. A lower tail above zero corresponds to a rejection of the null that the first-difference is non-positive at 10 percent significance. In both specifications, this null can be rejected for the first few lags of the response. Figures 14 and 15 are an alternative test for a delayed response. These figures plot the impulse response net of the impact effect, along with the 10 percent lower tail of the distribution of this statistic. These tests confirm that the delayed response of investment to permanent shocks is statistically significant, which provides evidence against models that have no planning delay. The lower left-hand panels in each figure indicate that there are no significant delays in the responses of cash flow and Q, which is also consistent with the model.

The forecast error variance decompositions for each variable largely conform to the predictions of the gestation lag model. Figures 16 and 17 report forecast

[33]Note that γ is not identified in either response.

error variance decompositions for specifications $j = 1, 2$. In each panel, the area below the line is the proportion of the forecast error variance at a given horizon that is attributable to the permanent shock. The left panels of the two figures indicate that almost all of the forecast error variance of capital growth at each horizon can be attributed to permanent shocks. As the forecast horizon becomes large, this proportion approaches the proportion of the unconditional variance that can be attributed to the permanent shock. The plots suggest that permanent shocks account for almost all of the forecast error variance of capital growth at all horizons, and that this proportion increases with the horizon length. The right panels in each figure show forecast error variance decompositions for cash flow and Q. These indicate that temporary shocks account for a more than one-half of the forecast error variance of each variable. This suggests that temporary shocks are an important part of the variation in Q and cash flow, but not important for the variation of investment.[34]

A final concern is whether the responses reported in this section are robust to other identification schemes for temporary and permanent shocks. In order to address this concern, I re-estimated the impulse responses in Figures 10 and 11 using a two-stage procedure that identifies the temporary and permanent disturbances using independent data. In the first stage, I estimated temporary and permanent innovations using a separate bivariate system containing quarterly data on output growth and the (ex post) real rate on 90-day treasury bills.[35] As in the previous systems, identification was achieved by only allowing the permanent shock to affect the long run level of output. The estimated structural innovations from this system were then used to obtain impulse responses for capital growth, cash flow, and Q, using the technique suggested in Chang and Sakata [2003]. Specifically, the response of each variable at lag n is obtained by regressing the variable on the nth lag of the structural innovations estimated from the first system.

Figure 18 shows the impulse response of output growth and the interest rate to the temporary and permanent disturbances. The two left-hand panels depict responses to the permanent shock, which causes the real interest rate and output growth to rise on impact and then fall off over time. These effects are consistent with the typical characteristics of a favorable technological innovation. The two right-hand panels depict responses to the temporary disturbance. These responses resemble the effects of a contractionary monetary disturbance, raising the real interest rate and reducing output growth.

[34] This feature is not imposed by the identification scheme. In principle, the temporary shock can compose an arbitrarily large portion of the forecast error variance of capital growth.

[35] Inflation was proxied using the rate of GDP price inflation for non-financial corporations.

Figure 19 graphs the impulse responses of capital growth, cash flow, and Q to the temporary and permanent innovations. Confidence intervals for these responses were generated using a bootstrap-after-bootstrap technique that corrects for small sample biases in both the identifying VAR and the OLS impulse response estimates.[36] The top two panels show that capital growth has a hump-shaped response to the permanent disturbance that peaks between the five- and ten-quarter horizons, and a response to the temporary disturbance that is close to zero at all horizons. Neither response is statistically significant at standard significance levels.[37] Among other things, this lack of significance could reflect a loss of power from utilizing the two-stage procedure rather than the more direct one-stage methodology. The responses of Q to the favorable permanent disturbance and the adverse temporary disturbance are (roughly) mirror images, with each response peaking near impact, then following a rough pattern of attenuation. The cash flow responses in the middle two panels are slightly more nebulous. Cash flow declines in response to the adverse temporary shock, albeit with a hump shape that peaks at five quarters. The response of cash flow to the favorable permanent disturbance is positive on impact, and attenuates to zero after about seven quarters. Although these results are not as clean as those obtained using the more direct methodology utilized earlier in this section, they seem to give some cautious support for those estimates.

IV. Discussion

This paper demonstrates that gestation lags are both theoretically important and empirically relevant. A simple model of aggregate investment with distinct gestation lags for planning and building can allow movements in cash flow and Q that are noisy indicators of investment. This relationship owes to the fact that both Q and cash flow adjust to reflect the short run scarcity of productive capital. All unanticipated disturbances in the economy cause actual holdings of productive capital to diverge from what firms would hold

[36]Each sample was generated by estimating a VAR(8) model for a vector containing output growth, the real interest rate, capital growth, cash flow, and Q. Then, estimates of the permanent and temporary innovations were obtained using a bivariate VAR containing output growth and the real interest rate. Finally, the responses of capital growth, cash flow, and Q to each innovation were obtained by regressing each variable, separately, on the vector of estimated innovations for each lag. Each of these three stages were corrected for small-sample bias using bias estimates from preliminary bootstrap experiments. The bias corrections for each stage were estimated in stages, using bias-corrected replications of the data from the previous stage.

[37]The response to the permanent shock at a horizon of eight quarters is significant at 25 percent. It is significant at less than ten percent when cash flow is excluded from the bootstrap simulation.

in a frictionless equilibrium. This causes a short-term divergence between the economic value of new capital goods and the value of capital that is already in place for production. Since it takes time to add new capital, such divergences do not always signal investment. Though all shocks create similar price signals, only disturbances that are expected to outlast the gestation period prompt new investment. The empirical estimates in this paper suggest a planning horizon of one year, which is followed by a building horizon of two years. Hence, there is considerable scope for temporary shocks to create noise in the relationship between investment and Q.

These findings provide some insight into the empirical shortcomings of Q regressions, including the claim that investment is excessively sensitive to cash flows. With investment lags, the typical regression of current investment on the current Q is misspecified. Both cash flows and Q are correlated to investment because they contain rents that reflect the relative scarcity of capital.[38] Consequently, the coefficients on Q that are estimated by researchers may reflect the imperfect co-movement of rents with investment, rather than the magnitude of quadratic adjustment costs. It is easy to see why cash flows might perform well as an additional variable in such regressions, even without financing constraints, because they contain similar information. Millar [2005] demonstrates how gestation lags are problematic for the standard regression of investment against Tobin's Q, and proposes alternative specifications that account for such lags in the presence of convex adjustment costs for capital.

The results of this paper are also relevant for a number of other areas of study. For instance, building lags have important implications for growth accounting because they entail the mismeasurement of productive capital and technological progress, and for business cycle theorists because they complicate the economy's dynamic response to shocks. For the investment literature, I have demonstrated an alternative model that is characterized by deviations from the frictionless equilibrium at higher frequencies, but obeys the neoclassical equilibrium in the long run. This specification has some promising properties. It explains why investment is slow to respond to shocks, despite the economic incentive for firms to adjust rapidly. It also allows for a range of characteristics that are consistent with empirical facts about aggregate investment. These include lumpiness, serial correlation, and a lagged relationship to output at business cycle frequencies.

[38] This finding is similar in flavor to Abel and Eberly [2002], who show that the cash flows and Q of monopolistic firms reflect rents that indicate growth opportunities.

A. Shadow Value Derivations

It is easily verified that the value function (11) is linearly homogeneous in the state variables K_t and $\{s_{i,t}\}_{i=1}^{J-1}$. Therefore, by Euler's theorem, it must be true that

$$\frac{V_t}{\bar{p}} = q_{0,t}K_t + \sum_{i=1}^{J-1} q_{i,t}s_{i,t},$$

where

$$q_{0,t} \equiv \frac{V_{K_t}}{\bar{p}} \qquad \text{and} \qquad q_{i,t} \equiv \frac{V_{s_{i,t}}}{\bar{p}}.$$

Applying the envelope theorem to (10), these derivatives can be calculated as

$$q_{0,t} = \sum_{j=0}^{\infty} \left(\frac{R}{1-\delta}\right)^{-j} \frac{{}_t\bar{\pi}_{t+j}}{\bar{p}},$$

$$q_{i,t} = \begin{cases} R^{-i}\left({}_tq_{0,t+i} - q_0^*\right) + q_i^* & 1 \le i < B-1 \\ R^{-i}\left({}_tq_{0,t+i} - q_0^*\right) & B \le i < J. \end{cases}.$$

In taking these derivatives, it is useful to note that capital accumulation identity can be iterated to obtain that

$$K_{t+j} = (1-\delta)^j K_t + \sum_{i=1}^{j} (1-\delta)^{i-j} s_{1,t+i}$$

Some additional results are also useful. Taking the conditional expectation of $q_{0,t}$ at $t-J$, and using the equilibrium condition (14), it can be shown that

$${}_{t-J}q_{0,t} = E[q_t] = q_0^*,$$

which verifies that q_0^* is the unconditional shadow value of productive capital. Further, it can be shown that

$$q_{0,t} = q_0^* + \sum_{j=0}^{J-1} \left(\frac{R}{1-\delta}\right)^{-j} \left[\frac{{}_t\bar{\pi}_{t+j}}{\bar{p}} - u^*\right],$$

where the summation is limited to the horizon $J-1$ because rents are always anticipated to be zero beyond the gestation horizon J.

B. Log-Linearization of Q_t

The log-linearized value of Q_t around the unconditional mean can be calculated from (21) as:

$$d\ln Q_t \approx \sum_{i=1}^{B-1} \chi_i dg_{t+i}^K + \sum_{i=0}^{J-1} \omega_i d\ln {}_t\pi_{t+i}$$

where

$$\begin{aligned}
\omega_i &\equiv R^{-i} G_i \frac{u^*}{E[Q]}, \quad G_i \equiv E\left[\frac{K_{t+i}}{K_t}\right] && \text{for all } i \geq 0, \\
\chi_i &\equiv \chi_{i+1} + \left[q_i^* - (1-\delta) q_{i+1}^*\right] \frac{G_i}{E[Q]}, && \text{for } i = 1, \ldots, B-2, \\
\chi_{B-1} &\equiv G_{B-1} \frac{q_{B-1}^*}{E[Q]}, && \text{and} \\
E[Q] &= q_0^* + \sum_{i=1}^{B-1} \left[G_i - (1-\delta) G_{i-1}\right] q_i^* > q_0^* > 1.
\end{aligned}$$

The parameter χ_i is the elasticity of Q with respect to capital growth at $t+i$, and ω_i is the elasticity of Q with respect to anticipated cash flows at horizon i. Note that there is are no cash flow terms beyond the gestation lag, because cash flows are always expected to reset to u^*.

Note that $G_i > G_{i-1}$ for $i > 1$ since the process for Z in (16) implies positive expected growth. From this, and the fact that $q_i^* > q_{i+1}^*$, it can be easily shown that $\chi_i > \chi_{i+1}$. Therefore, Q has a higher elasticity with respect to productive capital growth at shorter horizons.

The cash flow elasticity ω_i may be increasing or decreasing i, depending on the size of the growth rate G_i, and the magnitude of R. However, outside considerations suggest that $R^i > G^i$. For instance, according to the Ramsey Model, steady states where the real interest rate is less than the rate of growth in the capital stock are inefficient. Therefore, a reasonable calibration would have ω_i decrease in the forecast horizon i.

C. Mapping from Productive Capital to Accounting Capital

Let $\tilde{K}_{t+i}$ denote the accounting measure of capital at $t+i$, constructed using a one period capital accumulation identity. The one period capital accumulation identity can be iterated backwards to yield that

$$\tilde{K}_{t+i+1} = d(L) I_{t+i}, \qquad \text{where} \qquad d(L) = \sum_{j=0}^{\infty} (1-\delta)^j L^j.$$

Using the arbitrary time to build accounting in equations (7) to (9), investment can be stated as

$$I_{t+i} = g(L) K_{t+J+i}, \qquad \text{where} \qquad g(L) = \sum_{j=0}^{B} g_{J-j} L^j,$$

and

$$g_j \equiv \begin{cases} \phi_B & j = 0 \\ \phi_{B-j} - (1-\delta)\phi_{B-j+1} & 0 < j < B \\ -(1-\delta)\phi_1 & j = B \end{cases}.$$

By substitution, these two equations suggest that

$$\tilde{K}_{t+i+1} = m(L) K_{t+J+i}, \qquad \text{where} \qquad m(L) \equiv d(L) g(L)$$

is the product of the two lag polynomials. Performing this multiplication yields that

$$m(L) = L^P \phi(L), \qquad \text{where} \qquad \phi(L) = \sum_{j=0}^{B-1} \phi_{B-j} L^j.$$

Therefore, the accounting measure of capital maps to the productive measure by

$$\tilde{K}_{t+i+1} = \phi(L) K_{t+B+i}.$$

The accounting measure of capital growth is

$$\Delta \ln \tilde{K}_{t+1} = \phi(L) \Delta \ln K_{t+B}.$$

Log-linearizing this around the unconditional expectation yields that

$$\Delta \ln \tilde{K}_{t+1} \approx \tilde{\phi}(L) g^K_{t+B},$$

where

$$\tilde{\phi}(L) = \sum_{j=0}^{B-1} \tilde{\phi}_{B-j} L^j, \qquad \tilde{\phi}_j = \frac{\phi_j G_j}{\sum_{k=1}^{B} \phi_k G_k} \geq 0, \qquad \text{and} \qquad \sum_{j=1}^{B} \tilde{\phi}_j = 1.$$

D. Interpretation of Regression Coefficients in Planning Lag Regression

Given the solution for the growth rate of the productive capital in (18),

$$\frac{dg^K_{t+B}}{d\tilde{sr}_{t-P}} = \frac{1}{b}\frac{d\ln E_{t-P}[Z^b_{t+B}]}{d\tilde{sr}_{t-P}}$$

In the special case where the technology and labor supply factors are independent, the form of Z in (6) can be used to show that

$$\ln E_{t-P}[Z^b_{t+B}] = \ln E_{t-P}\left[\left(Z^T_{t+B}\right)^{b(1+\zeta)}\right] + \ln E_{t-P}\left[\left(Z^L_{t+B}\right)^{b}\right],$$

The standard Solow residual is related to labor-augmenting technological growth by $\tilde{sr}_{t-P} = \Delta \ln Z^T_{t-P}$. Assume that Z^T is a random walk, so the Solow residuals represent permanent shocks. Taking the conditional expectation at $t-P$, then evaluating the partial derivative with respect to $\tilde{sr}_{t-P}$, one arrives at

$$\frac{1}{b}\frac{d\ln E_{t-P}[Z^b_{t+B}]}{d\tilde{sr}_{t-P}} = \frac{1}{b}\frac{d\ln E_{t-P}\left[\left(Z^T_{t+B}\right)^{b(1+\zeta)}\right]}{d\tilde{sr}_{t-P}} = 1+\zeta.$$

Substituting this into the first equation, then substituting and rearranging (29) shows the desired result.

E. Specification of a Simple RBC Model with Gestation Lags

Assume that the social planner chooses a consumption and investment plan to maximize the expected value of the lifetime utility

$$U = \sum_{j=t}^{\infty} \beta^{j-t} E_t\left[\ln C_{t+j}\right],$$

subject to the feasibility constraint that $F\left(K_{t+j}, Z^T_{t+j}L_{t+j}\right) = C_{t+j} + I_{t+j}$ for all $j \geq 0$, where F takes the specification in (1). The evolution of capital is described by equations (7) to (9) with $P=0$, and L_{t+j} is normalized to one for all periods. The log of technology follows a random walk with a constant drift equal to the rate of growth $\ln G$, so that $\ln Z^T_{t+1} = \ln G + \ln Z^T_t + \psi_t$.

The optimization problem reduces to the intertemporal first order condition

$$\sum_{j=0}^{B} \beta_j g_j E_t\left[\frac{1}{C_{t+j}}\right] = \beta^B E_t\left[(1-\alpha)\left(\frac{Z^T_{t+B}}{K_{t+B}}\right)^{\alpha}\frac{1}{C_{t+B}}\right],$$

and the constraint that

$$C_{t+s} = K_{t+s}^{1-\alpha}\left(Z_{t+s}^{T}\right)^{\alpha} - \sum_{j=0}^{B} g_{B-j} K_{t+j}$$

where g_j is as defined in Appendix C. This nonlinear system links the current and future values of the endogenous variables C_t and K_{t+B} to the set of state variables $(K_t, \ldots, K_{t+B-1}, Z_t)$. An approximate solution to this system was obtained by log-linearizing the first order conditions around the steady state, then finding linear feedback rules for C_t and K_{t+B} by the method of undetermined coefficients, as described in Campbell [1994]. That is, I solved for the coefficients $[\eta_{..}]$ in the following equations:

$$\ln C_t = \sum_{j=0}^{B-1} \eta_{cj} \ln K_{t+j} + \eta_{cz} z_t, \qquad \ln K_{t+B} = \sum_{j=0}^{B-1} \eta_{kj} \ln K_{t+j} + \eta_{kz} z_t,$$

such that the first order conditions were satisfied, using a numerical solver algorithm. Generally there will be many linear solutions of this form that satisfy the optimization conditions for a given $B > 0$. The solution set was limited to those that implied a stable AR process for capital, *i.e.*, those for which the largest modulus of the roots of the polynomial

$$1 - \sum\nolimits_{j=0}^{B-1} \eta_{kj} L^{B-j}$$

was outside the unit circle.

The parameters of the model were set to resemble a typical RBC calibration. The steady state growth factor G was set to 1.005, which amounts to about 2 percent in annual terms. The discount factor β was set to $G/1.015$, which ensures a risk-free interest rate of 1.5% per quarter. The rate of depreciation δ was set to 2.5%. For simplicity, it is assumed that expenditures are spread evenly throughout the building period, so that $\phi_{B-j} = B^{-1}$ for $j = 0, \ldots, B-1$.

F. Data Appendix

1. Investment and the Capital Stock

A time series was constructed for flows of real quarterly aggregate investment from 1946Q4 to 2002Q4 using data from the Federal Reserve Board's Flow of Funds (FF) and capital stock estimates from the Bureau of Economic Analysis (BEA). Each quarter's investment was determined by dividing the

FF figure for non-financial, non-farm corporations (NFNFC) by the implicit price deflator for quarterly nonresidential fixed investment from BEA Table 7.6.

To obtain the series for the real capital stock, I iterated the capital accumulation identity (7) for a one period time to build. The initial estimate of the real capital stock was determined by dividing the nominal value of nonresidential fixed capital for non-financial corporations at the end of 1946 by the price deflator for the last quarter. In order to minimize the error owing to the estimate of the initial capital stock, I do not use the capital stock estimates prior to 1959Q3. Depreciation rates were calculated as a weighted average of the BEA depreciation rates for structures, equipment, and IT capital, with weights set equal to the share of each category in the nominal value of the aggregate stock of nonresidential capital for non-financial corporations. The depreciation rate for each quarter is set to the corresponding annual rate.

2. Tax-Adjusted Price of Capital

The tax-adjusted price of capital for each quarter is calculated using the equation

$$\bar{p}_t \equiv p_t(1 - \iota_t - z_t)$$

where p_t is the pretax price, ι_t is the investment tax credit, and z_t is the present value of deprecation allowances per dollar of capital. The pretax series is the implicit price deflator for quarterly nonresidential fixed investment from BEA Table 7.6. The present value of capital consumption allowances (z_t) was determined using data from DRI on the value of capital consumption allowances for different types of capital, with the share of each type in total nominal nonresidential investment expenditure as weights. This figure was then multiplied by the corporate income tax rate. Data on the average ITC for equipment and structures were obtained from DRI for 1959 to 2002. A weighted average was then calculated using the shares of nominal (nonresidential) fixed investment from BEA Table 5.4.

3. Tobin's Q

Aggregate Tobin's Q is calculated as the market value of all non-residential fixed capital divided by the replacement value of non-residential fixed capital, where the replacement value is calculated using the previous period's tax-adjusted price of aggregate capital:

$$Q_t \equiv \frac{V_t}{\bar{p}_{t-1} K_t}.$$

The market value of non-residential fixed capital (V_t) is determined by deducting the value of all assets except equipment and nonresidential structures from the total market value of the firm. To illustrate, consider the balance sheet in Table 5, which shows the major liabilities and assets of firms. Assets include the firm's financial assets (FA_t), the present value of the depreciation shields for its existing capital (PVCCA_t), inventories (INV_t), residential capital (RESK_t), and non-residential capital ($Q_t \bar{p}_{t-1} K_t$). The collective value of these assets is equal to the market value of all the claims on these assets: debt (DEBT_t) and equity (EQU_t). Therefore, to determine an appropriate market value of non-residential capital, one must calculate

$$V_t = \text{EQU}_t + \text{DEBT}_t - \text{FINAS}_t - \text{INV}_t - \text{RESK}_t - \text{PVCCA}_t.$$

These components were determined from Flow of Funds data for NFNFC, as follows:

$\mathbf{EQU}_t$ is the FF figure for the aggregate market value of equity.

$\mathbf{DEBT}_t$ was determined by adding the aggregate book value of non-bond debt liabilities to the aggregate market value of outstanding corporate bonds. The market value of outstanding corporate bonds was estimated using an algorithm outlined by Hall [2001], which corrects the book value of debt for changes in market interest rates. This algorithm is available on Hall's website.

$\mathbf{FINAS}_t$ is the aggregate book value of financial assets.

$\mathbf{INV}_t$ is the FF figure for the aggregate replacement value of inventories.

$\mathbf{RESK}_t$ is the FF figure for the aggregate replacement value of all residential capital.

$\mathbf{PVCCA}_t$ was calculated under the assumption that capital consumption allowances can be well approximated using a sum-of-years-digits (SYD) method. The stream of allowable depreciation allowances for each quarter's expenditure were calculated using the SYD formula, assuming an average asset life of 15 years. To determine the present value of the remaining consumption allowances for any given quarter, I discounted the depreciation allowances remaining using the BAA corporate rate net of the income tax rate for corporations.

4. Rate of Cash Flow

Cash flows per unit of capital were determined by dividing after tax cash flows by the replacement value of productive capital:

$$\pi_t \equiv \frac{\bar{\Pi}_t}{\bar{p}_{t-1} K_t}.$$

After tax cash flows ($\bar{\Pi}_t$) for each quarter were calculated using FF data for NFNFC, and BEA quarterly aggregates. To determine the cash flow, I added the book value of after-tax profits from FF, the value of capital consumption allowances from FF, and net interest payments. Net interest payments were calculated using BEA data, by deducting an estimate of net interest for corporate farms from the net interest figure for non-financial corporations. This nominal figure was then adjusted for inflation during the quarter using the rate of increase in the quarterly GDP deflator for non-financial corporations.

Table 1: Sample moments of the tax-corrected data

	$\tilde{g}^K_{t+1}$	$\ln\tilde{Q}_t$	$\ln(\tilde{\pi}_t/\bar{p})$
mean	.0111	1.4505	.0628
stdev	.0029	.5969	.0062
$corr(\tilde{g}^K_{t+1},\cdot)$	$\cdot$	.5028	.5432
$corr(\ln\tilde{Q}_t,\cdot)$	$\cdot$	$\cdot$	.3659

Sample Period: 1959Q3 to 2002Q4 (174 observations). $\tilde{g}^K_{t+1}$ *represents the growth rate in measured capital, while* $\tilde{Q}$ *and* $\tilde{\pi}$ *denote the measured values of Tobin's* Q*, and the rate of cash flow, respectively.*

Table 2: Unit root tests

Variance Ratio (horizon)	$\tilde{g}^K_{t+1}$	$\ln\tilde{Q}_t$	$\ln(\tilde{\pi}_t/\bar{p})$
VR(5)	2.0822	0.9832	1.3895
VR(10)	2.0108	0.7471	1.3904
VR(25)	1.2637	0.5910	0.8012
VR(50)	0.5612	0.5182	0.2500
VR(100)	0.1870	0.1230	0.1596
ADF T-Statistic	-2.9487‡	-1.8442	-3.8938$^{\#}$

Significance Levels: ‡*5%;* $^{\#}$*1%. Sample Period: 1959Q3 to 2002Q4 (174 observations).* VR(n) *denotes the value of the variance ratio at a horizon of* n *periods. The* ADF *T-Statistic is for an augmented Dickey-Fuller test where the null is the existence of a unit root.*

Table 3: Planning Horizon Estimates by OLS

Measured Capital Growth on Lags of Purified Solow Residual					
lag	$\hat{d}_i$	se^r	*lag*	$\hat{d}_i$	se^r
0	-.0052	.0203	8	.0351	.0168‡
1	.0026	.0246	9	.0282	.0129‡
2	.0092	.0222	10	.0275	.0133‡
3	.0195	.0171	11	.0232	.0146
4	.0269	.0143†	12	.0171	.0158
5	.0328	.0148‡	13	.0201	.0144
6	.0349	.0169‡	14	.0146	.0134
7	.0377	.0172‡	*const*	.0101	.0008$^{\#}$

Selected Tests Using Estimated Coefficients			
sum lags	*coef*	se^r	CI_{90}^{bc}
0 to 10	.2492	.1551	-.0231,.6214
4 to 10	.2463	.0993	.0480,.4408
4 to 7	.1324	.0575	.0105,.2603
0 to 3	.0261	.0795	-.1001,.1896
(4 to 7) - (0 to 3)	.1063	.0532	-.0132,.1873

Significance Levels: †*10%,* ‡*5%,* $^{\#}$*1%;* $\bar{R}^2$ *= .0068;* dw *= .0680. Sample Period: 1965Q3 to 2002Q2 (150 observations). Standard errors are robust for heteroskedasticity and autocorrelation (Newey-West maximum lag = 15).*

Table 4: Building Horizon Estimates by Instrumental Variables

Measured Capital Growth on Instrumented Leads of Implicit Productive Capital Growth									
lead	$\hat{h}_i$	se^r	CI_{90}^{bc}	R_p^2	*lead*	$\hat{h}_i$	se^r	CI_{90}^{bc}	R_p^2
1	.0706	.1282	-.0037,.4323	.1186	7	.2458	.0747$^{\#}$	.2482,.6667$^{\#}$	.1137
2	.1372	.1466	.0670,.5651‡	.1060	8	.2201	.0698$^{\#}$	.2014,.6304$^{\#}$	.1183
3	.0706	.1239	-.0443,.3716	.0992	9	.1049	.1396	-.0228,.2956	.0851
4	.1056	.1076	.0094,.3917†	.1171	10	.1092	.1131	-.0860,.2833	.0952
5	.2314	.1482	.2182,.6620$^{\#}$	.0843	11	.0759	.1112	-.1375,.2391	.1162
6	.2539	.1101‡	.2390,.6845$^{\#}$	.1208	12	-.0088	.1089	-.3566,.0771	.1040

Selected Tests Using Estimated Coefficients			
linear combination	*coef*	se^r	CI_{90}^{bc}
$h_1 + \ldots + h_8$	1.3352	.5886	1.2942, 3.0308
$h_1 + \ldots + h_9$	1.4401	.5989	1.3643, 3.3450
$h_1 + \ldots + h_{10}$	1.5493	.5762	1.4664, 3.6412
$h_1 + \ldots + h_{11}$	1.6251	.5598	1.5424, 3.7840

Significance Levels: †10%, ‡5%, $^{\#}$1%. Sample Period: 1965Q3 to 2002Q2 (150 observations). Instruments (28): $g_t^H, \ldots, g_{t-13}^H$, $\tilde{g}_t^K, \ldots, \tilde{g}_{t-13}^K$. IV standard errors are robust for heteroskedasticity and autocorrelation (Newey-West maximum lag = 15). R_p^2 is the "partial R^2" outlined in Shea [1997], which measures the squared correlation between the portion of the regressor that is orthogonal to the other regressors, and the portion of the fitted regressor that is orthogonal to the other fitted regressors. The overidentifying restrictions cannot be rejected at 1% significance, after correcting for small-sample bias. Bias-corrected intervals were constructed using 50,000 bootstrap replications, with re-sampling in blocks of 12 adjacent observations. An included constant is not reported.

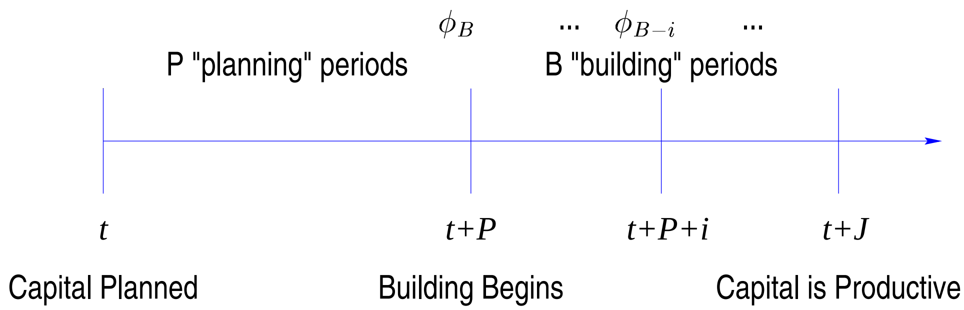


Figure 1: Time scale depiction of the investment process with gestation lags.

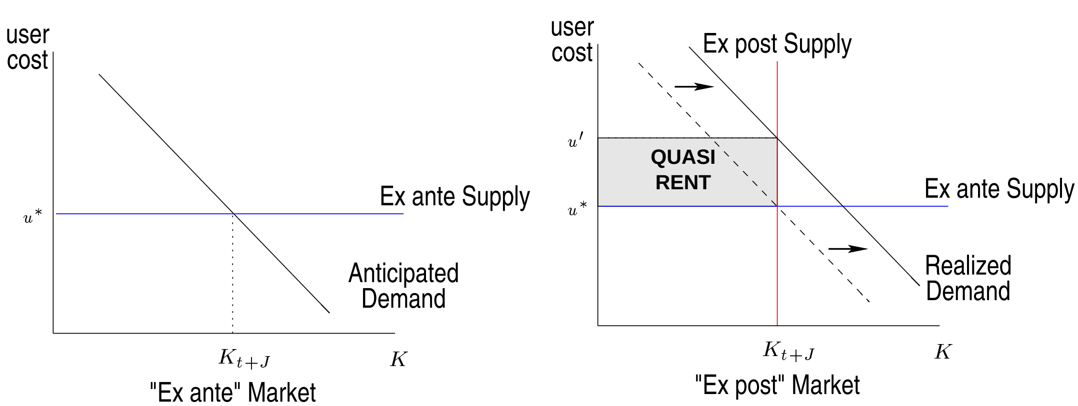


Figure 2: Diagram of equilibrium in the ex ante and ex post capital services market.

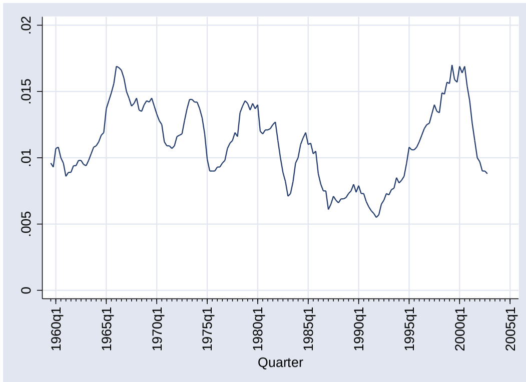


Figure 3: Plot of measured capital growth.

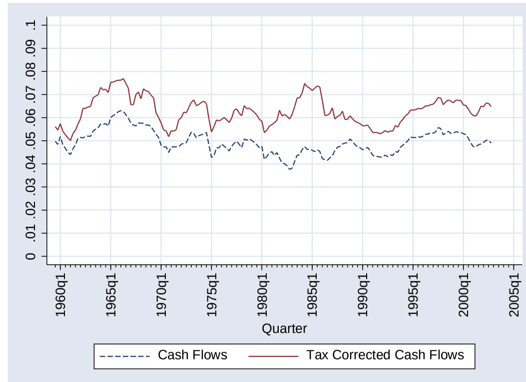


Figure 4: Plot of the rate of cash flow.

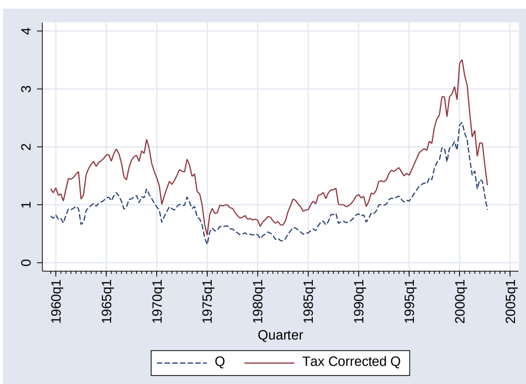


Figure 5: Plot of Tobin's Q.

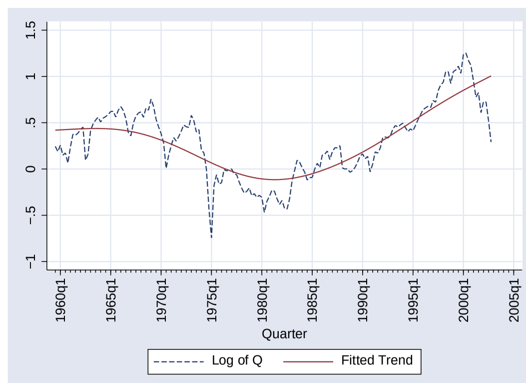


Figure 6: Log of Tobin's Q and fitted trend

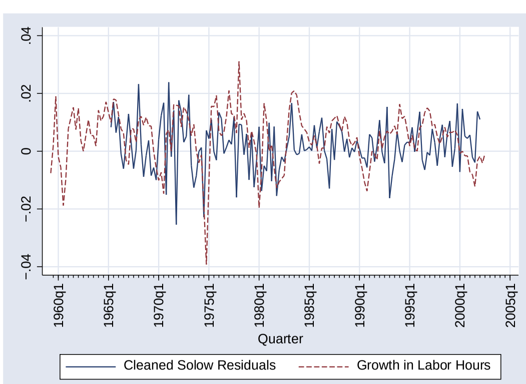


Figure 7: Hours growth and purified Solow residuals.

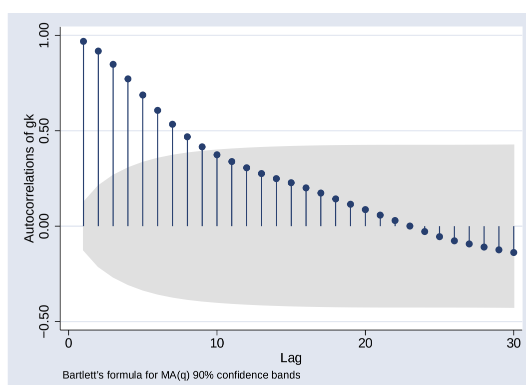


Figure 8: Correlogram of measured capital growth.

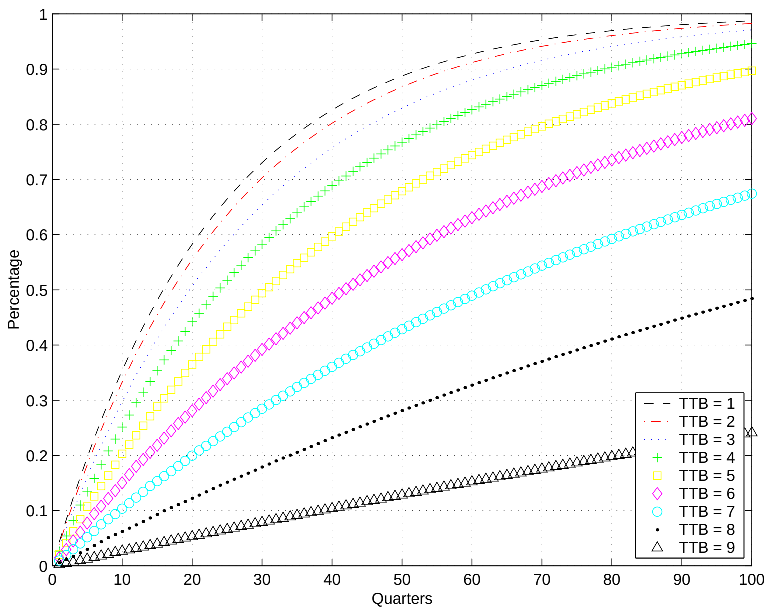
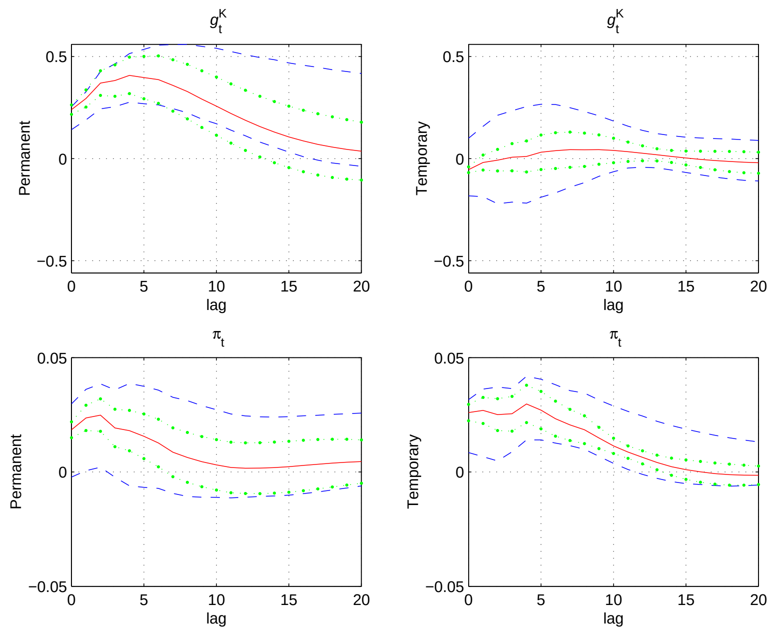
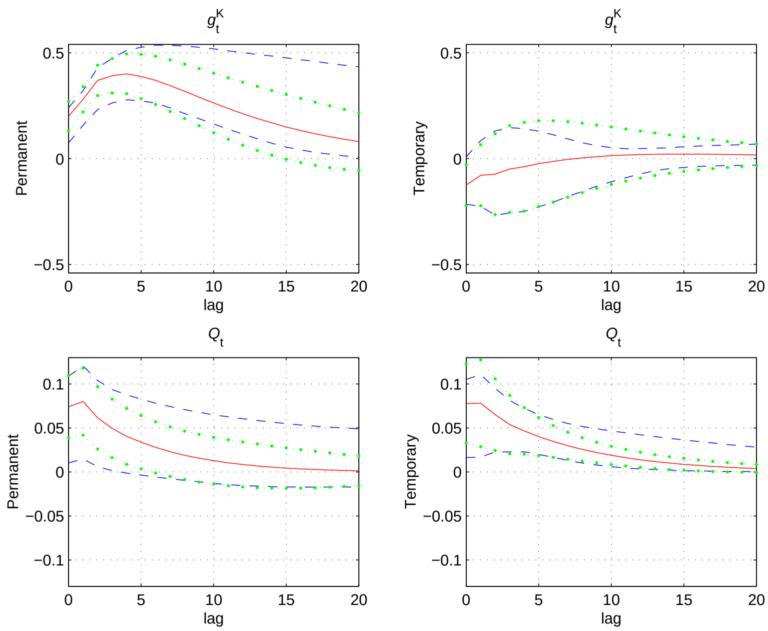


Figure 9: Dynamic response of the measured capital stock to a permanent one percentage point technology shock in calibrated RBC models featuring time to build of increasing duration.



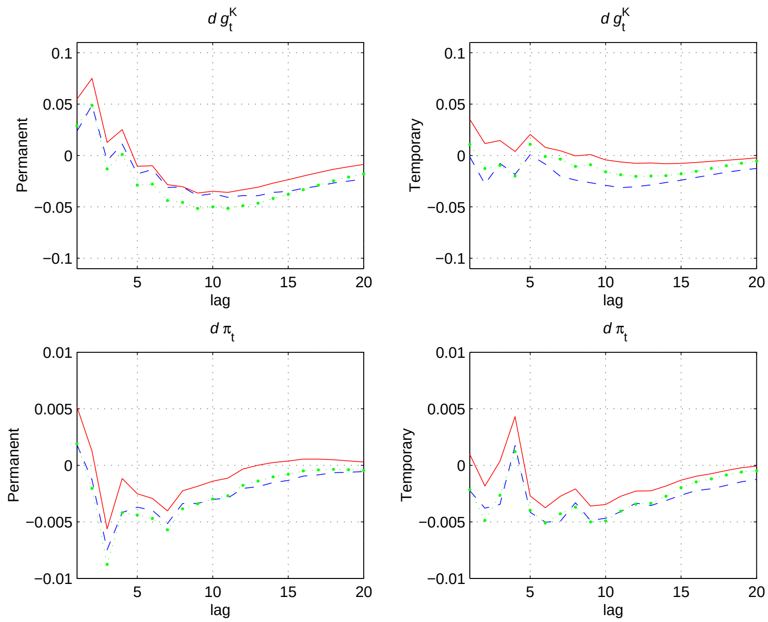
Asymptotic normal confidence intervals are denoted with "$\cdot \bullet \cdot$".
Bootstrap-within-bootstrap confidence intervals are denoted with "$--$".

Figure 10: Impulse responses of the logged rate of cash flow and measured capital growth to temporary and permanent structural innovations.



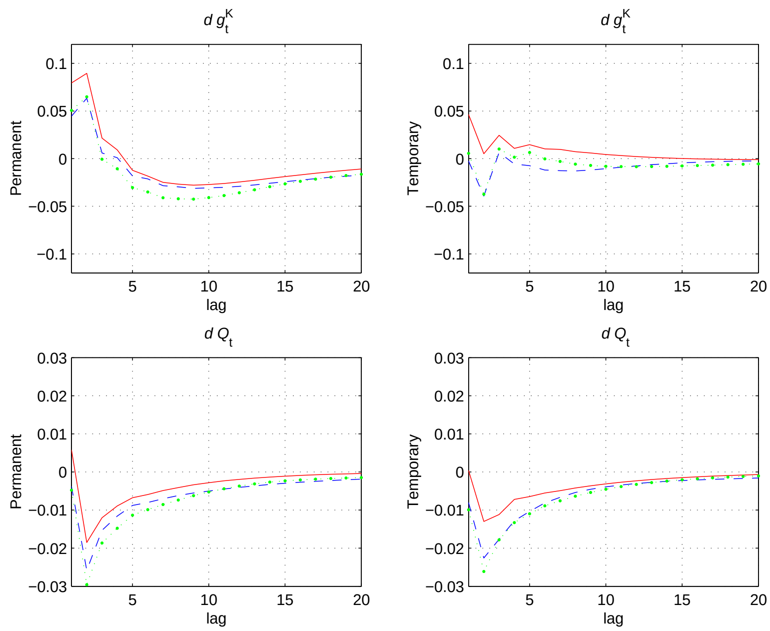
Asymptotic normal confidence intervals are denoted with "· • ·".
Bootstrap-within-bootstrap confidence intervals are denoted with "−−".

Figure 11: Impulse response of logged Tobin's Q and measured capital growth to temporary and permanent structural innovations.



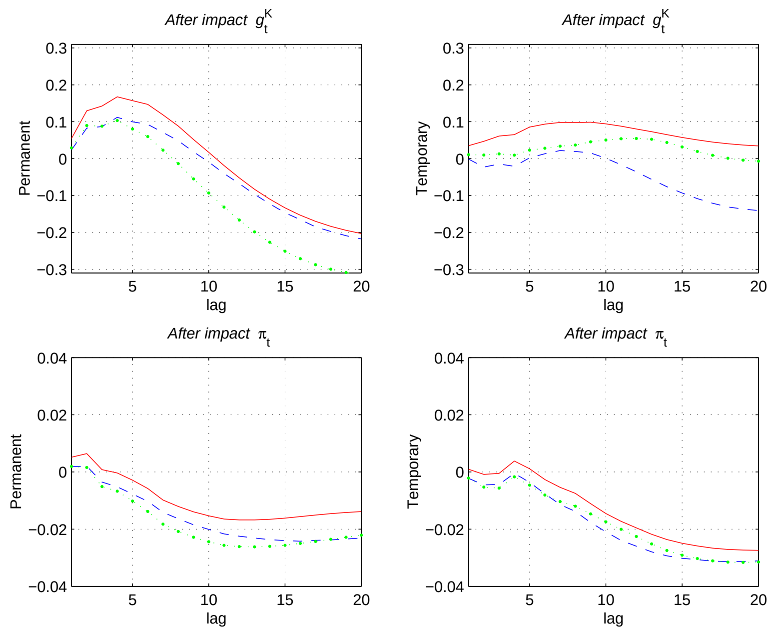
Asymptotic normal confidence intervals are denoted with "$\cdot \bullet \cdot$".
Bootstrap-within-bootstrap confidence intervals are denoted with "−−".

Figure 12: First-difference of the impulse responses of the logged rate of cash flow and measured capital to temporary and permanent structural innovations.



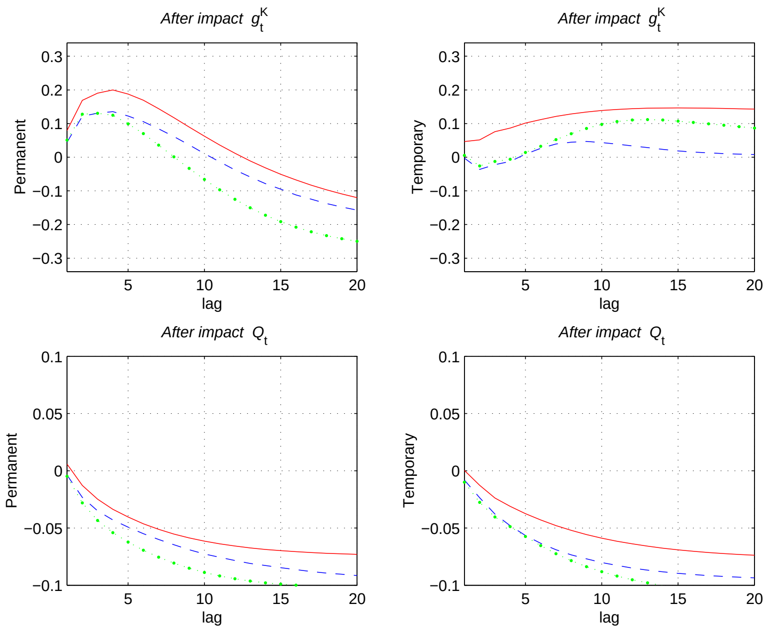
Asymptotic normal confidence intervals are denoted with "$\cdot \bullet \cdot$".
Bootstrap-within-bootstrap confidence intervals are denoted with "$--$".

Figure 13: First-difference of the impulse responses of logged Tobin's Q and measured capital growth to temporary and permanent structural innovations.



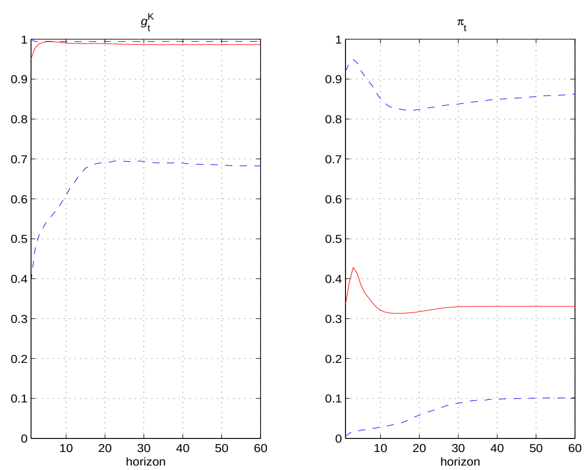
Asymptotic normal confidence intervals are denoted with "· • ·".
Bootstrap-within-bootstrap confidence intervals are denoted with "−−".

Figure 14: Net-of-impact impulse response of the logged rate of cash flow and measured capital growth to temporary and permanent structural innovations.



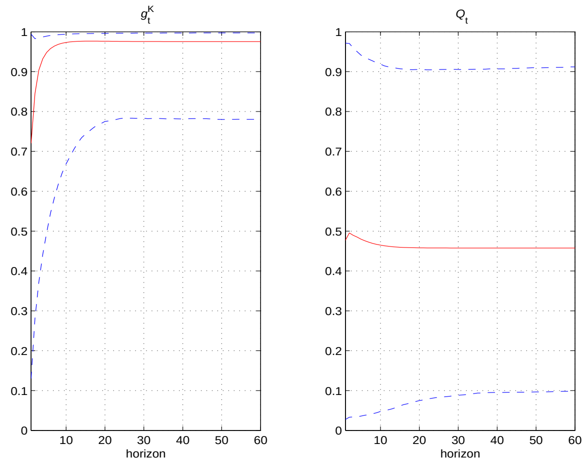
Asymptotic normal confidence intervals are denoted with "· • ·".
Bootstrap-within-bootstrap confidence intervals are denoted with "−−".

Figure 15: Net-of-impact impulse response of logged Tobin's Q and measured capital growth to temporary and permanent structural innovations.



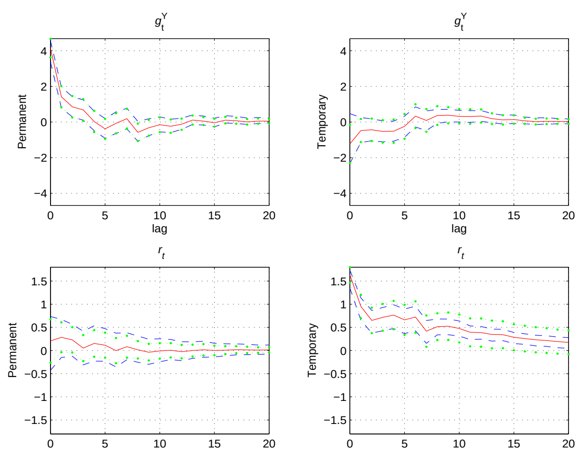
The line represents the proportion of the forecast error at each horizon that is accounted for by the permanent shock.

Figure 16: Forecast error variance decomposition for temporary and permanent shocks in the system with the logged rate of cash flow and measured capital growth.



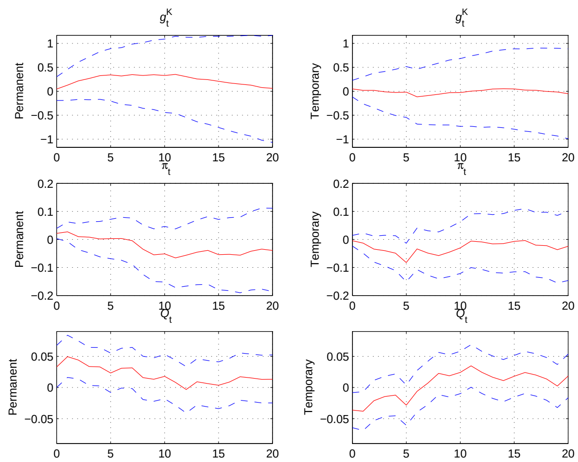
The line represents the proportion of the forecast error at each horizon that is accounted for by the permanent shock.

Figure 17: Forecast error variance decomposition for temporary and permanent shocks in the system with logged Tobin's Q and measured capital growth.



Asymptotic normal confidence intervals are denoted with "$\cdot \bullet \cdot$".
Bootstrap-within-bootstrap confidence intervals are denoted with "$--$".

Figure 18: Impulse response of output growth and the real interest rate to temporary and permanent structural innovations.



Asymptotic normal confidence intervals are denoted with "$\cdot \bullet \cdot$".
Bootstrap-within-bootstrap confidence intervals are denoted with "$--$".

Figure 19: Impulse response of measured capital growth, the logged rate of cash flow and logged Tobin's Q to separately-identified temporary and permanent structural innovations.

Table 5: Stylized Balance Sheet

Assets	*Liabilities + Equity*
FINAS_t	DEBT_t
PVCCA_t	
INV_t	
RESK_t	
$Q_t \bar{p}_{t-1} K_t$	EQU_t
Total Market Value	*Total Market Value*

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