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Growing Old Together: Firm Survival and Employee Turnover

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Abstract

Labor market outcomes such as turnover and earnings are correlated with employer characteristics, even after controlling for observable differences in worker characteristics. We argue that this systematic relationship constitutes strong evidence in favor of models where workers choose how much to invest in future productivity. Because employer characteristics are correlated with firm survival, returns to these investments vary across firm types. We describe a dynamic general equilibrium model where workers employed in firms more likely to survive choose to devote more time to productivity-enhancing activities, and therefore have a steeper earnings-tenure profile. Our model also predicts that quit rates should be lower in firms more likely to survive, and should tend to fall during slow times, while job destruction rates should rise. These predictions, we argue, are borne out by the existing empirical evidence.

Keywords: Firm survival; Firm size; Employee turnover; Firm specific human capital.

JEL classification: J24; J31; J63.

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1 Introduction

Labor market outcomes such as turnover and earnings are correlated with employer characteristics. These correlations could be explained by differences in worker productivity. If the evolution of productivity is exogenous, as in simple models of learning-by-doing (e.g., Jovanovic and Njarko, 1995), the correlation between firm characteristics, earnings, and worker turnover should disappear after controlling for the distribution of tenure. However, evidence in Abowd et al. (1999) and Anderson and Meyer (1994), among others, suggests that there remains considerable residual correlation after conditioning on worker tenures. If, instead, workers actively choose to invest in future productivity, then these decisions will lead to a systematic correlation between unobserved worker productivity and employer characteristics. A key factor that determines employer characteristics, and hence workers' willingness to invest in future productivity, is firm survival.

We formalize these arguments in a dynamic general equilibrium model in which employees split their time between accumulating firm-specific capital and delivering labor services, as in Jovanovic (1979).[1] Firms differ in only one respect: their likelihood of surviving from one period to the next. We show that workers employed by high survival firms invest more in firm-specific capital because it is more likely that they will benefit from these investments in the future. Therefore, earnings-seniority profiles are steeper in high survival firms. This is consistent with the evidence discussed in Abowd et al. (1999) who find, for instance, that seniority profiles are steeper in large firms. The model also predicts correctly that turnover rates should be lower in high survival firms, even after controlling for the distribution of tenure. Anderson and Meyer (1994), find that turnover rates fall with firm size, while Quintin and Stevens (2005) find that separation rates are higher in high exit rate industries, even after controlling for tenure differences across industries. If one thinks of firm-specific capital as the return to employer-specific training, as we do in the exposition, then our model predicts that training intensity should be positively correlated with firm characteristics that are associated with higher survival rates, such as age and size, which is consistent with the

[1] As we discuss later, our model only requires that this capital be firm-specific in part or, if most human capital is occupation specific, as argued by Kambourov and Manovskii (2002), that workers run the risk of not finding the same occupation when their employer dies.

evidence discussed by Frazis et al. (1995).

We qualitatively evaluate the dynamic properties of our model by computing the transition path between steady states following shocks to total factor productivity (TFP) and gross firm failure rates. Two outcomes of these experiments are particularly notable. First, as in the data, we find that quit rates are procyclical, because workers use slow times to retool (see DeJong and Ingram, 2001). Second, we find that job destruction rates are countercyclical provided gross failure rates for firms rise during recessions, even if the increase is very small as suggested by the existing evidence on corporate failure rates (see Platt and Platt, 1994, for a review). In summary, the cross-sectional and dynamic properties of our model provide compelling support for the broader use of models in which worker productivity evolves endogenously when analyzing issues related to firm survival and worker turnover.

2 The economy

We consider a discrete time, infinite horizon model, with three classes of agents: firms, workers, and the government. Firms differ only in their probability of survival. Type H firms survive to the next period with probability p_H, while type L firms survive with probability $p_L < p_H$. We will think of type as proxying for characteristics such as industry or geographic location that may affect an employer's perceived likelihood of survival. Our results extend immediately to environments with more firm types. Similarly, our assumption that survival rates are fixed and exogenous is strong but can be relaxed without altering our results.

A law of large numbers holds, therefore p_i is also the fraction of firms of type $i \in \{H, L\}$ that survive at the end of each period. In each period, a constant mass $\mu_i > 0$ of firms of type i are born. Firms of both types that have survived t periods can transform $n \geq 0$ units of labor into $(1+\eta)^t n^\alpha$ units of the unique consumption good, where $\alpha \in (0,1)$, and $\eta > 0$ is the exogenous rate of TFP growth. The following assumption bounds the average size of firms:

Assumption 1. $(1+\eta)^{\frac{1}{1-\alpha}} p_H < 1$

A constant measure of workers are born at the beginning of every period. Workers survive to the next period with probability β. We set the measure of newly born workers to $(1-\beta)$ so that the long-run population size is one. We assume that workers have linear preferences and, therefore, seek to maximize their expected lifetime labor income. All workers are assumed to own the same share of existing firms.[2]

At the beginning of each period, a worker is either employed by a firm or is unemployed. The maximum quantity of labor an employed worker can deliver to her employer depends upon her productivity level x, a random variable with values in $\{x_0, x_1, x_2\}$ where $x_0 < x_1 < x_2$.[3] A newly employed worker starts at productivity level $x = x_1$, regardless of her productivity history. In particular, we assume for simplicity that productivity levels are fully employer specific: workers at x_2 who lose or quit their job fall back to x_1 with certainty. The specificity of productivity can be relaxed in several ways. First, by complicating the notation a bit, productivity can be made partly general in nature (i.e., transferable across employers) without changing any results. Second, productivity can be made entirely general in nature provided it depreciates over time and there is a risk of unemployment. Lastly, productivity can be made occupation specific rather than firm specific, as long as workers who lose or quit their job run the risk of not finding employment in the same occupation.

If the worker and employer both survive to the next period, the evolution of the worker's productivity depends on the time $s \in [0, 1]$ she devotes to training. If her productivity level is not already x_2, it rises one notch in the subsequent period with probability $h(s) \in [0, 1)$; the function h is twice continuously differentiable and strictly concave with $h(0) = 0$ and $h'(0) > 0$. On the other hand, if her current productivity level is x_1 or x_2, she moves down one notch with likelihood δ.[4] We think of δ as the likelihood that the worker's productivity level depreciates, an event which can occur for idiosyncratic or firm-wide reasons, or for reasons common to a subset of firms that one could think of as an industry.[5]

Workers become unemployed if their employer dies, or if they choose to quit. In addition,

[2]The exact specification of ownership and the fact that we do not consider the possibility of workers trading ownership shares does not matter for any of our results, as we assume linear preferences.

[3]Allowing for an arbitrary number of productivity levels complicates notation and the derivation of results, but our basic results continue to hold. For instance, it remains the case that the time workers at productivity level x_1 devote to training rises with the likelihood of employer survival. Showing that workers train more at *all* productivity levels requires additional assumptions.

[4]Assuming that δ does not vary with s simplifies the exposition and the analysis, but does not alter any of our qualitative results.

[5]We assume that there is no aggregate uncertainty, but we do not require the evolution of worker productivity to be independent across workers or firms.

workers are born unemployed. Unemployment ends with probability $\phi < 1$ at the beginning of any given period, in which case workers may choose an employer of either type. In their first period of unemployment, previously employed workers receive unemployment benefits equal to a fraction $\rho \in (0, 1)$ of their labor income from the previous period. The government finances unemployment benefits through a fixed payroll tax rate τ and refunds excess fiscal revenues to workers in a lump-sum fashion. The following assumption makes verifying budget balance simpler:

Assumption 2. $\tau > \rho\beta\left[1 - p_L(1-\delta)\right](1-\phi)$

This assumption ensures that the tax rate on labor earnings exceeds the replacement rate times the probability that the benefits are claimed. A worker can claim benefits only if she survives, enters unemployment (the bracketed term), and does not instantaneously leave unemployment (the last term).

Our economy can generate several types of steady-state equilibria. For example, there are sets of exogenous parameters for which unemployed agents accept all job offers, and other sets for which unemployed workers only accept job offers after their benefits have expired. We make three more assumptions that simplify the exposition by enabling us to concentrate on one type of equilibrium. Relaxing these assumptions complicates arguments but does not change our basic results. Assumption 3 ensures that, in steady state, unemployed workers are always better off accepting job offers than turning them down.[6]

Assumption 3. $\rho x_2 < x_1$

In other words, the benefits received by workers at the highest productivity level are bounded above by the income that could be earned by accepting a new job offer.

Next we assume that in steady state the rate of exogenous technological growth is such that the growth in labor demand by surviving firms always exceeds the average rate of productivity growth of existing employees, so that all firms have to hire new workers in every period. Let $\bar{s} = \arg\max_{s\in[0,1]} h(s)$ and $\bar{h} = h(\bar{s})$. A condition sufficient to guarantee that all firms need to hire new workers in all periods is:

[6]In most states, the law prohibits workers who receive unemployment benefits from turning down "acceptable" offers. The degree to which that requirement is enforced, however, is unclear.

Assumption 4. $\beta\left[\bar{h}\frac{x_2}{(1-\bar{s})x_1}+1-\delta-\bar{h}\right]<(1+\eta)^{\frac{1}{1-\alpha}}$

The left-hand side of the inequality is a bound on the average productivity growth of returning workers, while the right-hand term is the rate of growth of labor demand by firms who survive from one period to the next. This assumption simplifies our existence proof because checking labor clearing then amounts to checking that labor demand and supply coincide for each firm type. In addition, by ruling out the possibility that firms enter a given period with excess labor, we avoid making an arbitrary assumption about which workers firms would lay off.

Finally, we set x_0 low enough so that workers who reach productivity level $x = x_0$ choose to quit in equilibrium (see proposition 3 below).

Assumption 5. $x_0 < \left[\frac{\phi}{1-\beta(1-\phi)} - \beta p_H\right] x_1$

We now turn to defining and characterizing steady state equilibria in our economy.

3 Steady-state equilibria

We will study equilibria in which firms behave competitively and pay workers the value of their marginal product (net of taxes) each period. Given linear preferences, this payment scheme is always weakly optimal, but many alterative compensation schemes are also optimal. For instance, workers could be paid the value of their expected average lifetime marginal product in every period. Our main results are independent of the specific payment scheme adopted by workers and firms. The only exception is proposition 6 which compares the shape of the seniority profile across firm types. That result is based on the premise that current earnings and current productivity are positively correlated.[7] Similarly, different assumptions on the relative bargaining power of firms and workers would not change our results.

Denote by w_i the wage rate offered by a firm of type $i \in \{H, L\}$. Workers at productivity level x who devote time s to training earn $(1-s)xw_i$ in type i firms. A type i firm of age

[7] Note that such a correlation must exist when human capital is in part general. While we only consider firm-specific capital, our results hold unchanged in a version of the model where human capital is partially general.

$t \geq 0$ chooses effective labor input n to maximize profits:

$$\max_{n \geq 0} \ (1+\eta)^t n^\alpha - n w_i (1+\tau).$$

Total labor demand for each firm type is the sum of optimal labor demands across firms of that type. As for labor supply, employed workers of a given productivity level split their time between training and delivering labor so as to maximize their expected lifetime labor income. In the appendix, we formally state the corresponding optimization problem. Steady-state equilibria, in this context, are constant wage rates for each firm type such that labor markets clear and the government's budget is balanced in every period. We establish in the appendix that a unique steady state equilibrium pair of wage rates exists in this economy for all sets of exogenous parameters that satisfy assumptions 1 to 5. Furthermore, because high survival firms offer better training opportunities and a lower unemployment risk, general equilibrium considerations imply that the wage rate (the price of each unit of labor provided by employees) is higher in firms whose survival is less likely. Our first proposition records these findings.

Proposition 1. *A unique steady-state equilibrium exists. Furthermore,* $w_L > w_H$.

Proof. See appendix. □

Our next result is that type H firms tend to be larger than type L firms in terms of employment, provided the survival probability of type H firms is high enough. This proposition gives the sense in which size and survival rates are positively correlated in our model, as they are in the data.

Proposition 2. *Given other parameters, there exists* $\underline{p} < (1+\eta)^{\frac{-1}{1-\alpha}}$ *such that if* $p_H > \underline{p}$ *then type* H *firms employ more workers on average than type* L *firms in steady state.*

Proof. See appendix. □

As p_H rises to $(1+\eta)^{\frac{-1}{1-\alpha}}$, the average labor demand of type H firms grows without bound. Even though the average amount of labor delivered by workers may be larger in type H firms, the labor demand differential between firm types becomes large enough to guarantee that type H firms employ more workers on average. In this sense, our model predicts that firms more likely to survive tend to be larger in employment terms. Since wage rates are lower in those firms (proposition 1), this appears to imply that large firms pay less than small firms in our model, which is counterfactual. But the model makes no such prediction. Earnings are the product of three terms: time devoted to work, productivity, and wage rates. While wage rates are lower and employees devote more time to training in firms more likely to survive, they are also more productive on average than workers employed in low survival firms. In fact, simple algebra shows that when there is no unemployment ($\phi = 1$), average earnings must be the same in steady state across firm types. For the parameters we use in the numerical section, average earnings rise monotonically with size for each firm type. Furthermore, the economy-wide correlation between firm size and average worker earnings is positive.

We now wish to compare quit rates and job destruction rates across firm types. We begin by recording the fact that given assumption 5, workers remain with their employer as long as their productivity is at least x_1 and as long as their employer survives.

Proposition 3. *In steady state, workers quit if and only if their productivity is x_0.*

Proof. See appendix □

Therefore, to compare quit rates across firm types we need only compare average productivity levels and hence training policies. Workers whose productivity is x_2 do not devote any time to training in either firm type. Denote by s^i the fraction of time workers at productivity level x_1 devote to training when their employer is of type $i \in \{H, L\}$. The following result says that workers employed in high survival firms devote more time to training than other workers.

Proposition 4. *In steady state, $s^L \leq s^H$, and $s^L < s^H$ if $s^L \in (0,1)$ or $s^H \in (0,1)$.*

Proof. See appendix. □

The intuition behind this result is simple. Since the opportunity cost of training (x_1 times the wage rate) is lower in type H firms, it is sufficient to show that the return to training is higher in type H firms. It may seem obvious that returns to training must be higher in type H firms, as initial earnings are higher in type L firms than in type H firms. However, the fact that workers in type H firms face a lower unemployment risk could, in principle, be enough to compensate workers for the initial wage differential. We argue in the appendix that the unemployment risk differential is not enough: Training returns must be higher in type H firms for workers to be indifferent between firms. This result can only be established in a general equilibrium model; a partial equilibrium model would require ad hoc assumptions about how compensation profiles vary across firm types.

We can now characterize the seniority profiles of productivity and earnings in both firm types. Given proposition 3, the evolution of the productivity level of a worker employed in a firm of type $i \in \{H, L\}$, *conditional on the continuation of the employment relationship*, is governed by a Markov chain with two states, x_1 and x_2, and with transition matrix:

$$\begin{bmatrix} 1 - h_i & h_i \\ \delta & 1 - \delta \end{bmatrix}$$

where $h_i \equiv \frac{h(s^i)}{1-\delta}$ is the probability that a worker employed in a firm of type i moves up to productivity level x_2. For $i \in \{H, L\}$ and $t \geq 0$, denote by $E_i^t(x)$ the average productivity level of workers with t periods of tenure in type i firms. The following result characterizes the evolution with tenure of this average.

Proposition 5. *In steady state,*

- $\lim_{t \mapsto +\infty} E_i^t(x) = \frac{\delta}{h_i+\delta} x_1 + \frac{h_i}{h_i+\delta} x_2$ *for* $i \in \{H, L\}$,
- *If* $h_i < 1 - \delta$, $E_i^{t+1}(x) > E_i^t(x)$ *for* $i \in \{H, L\}$ *and all* $t \geq 0$,
- $E_H^t(x) \geq E_L^t(x)$ *for all* $t \geq 0$, *with a strict inequality when* $s^L \in (0, 1)$ *or* $s^H \in (0, 1)$.

Proof. The first two items are standard results for two-state Markov chains. The last item is a direct consequence of proposition 4. □

Therefore, with tenure, average productivity rises to an invariant value. This convergence is monotonic when productivity levels are persistent in the sense that a worker's expected productivity level in the next period, conditional on the worker keeping the same employer, rises with the current level of productivity. When that condition is not met, productivity oscillates ever closer to its invariant value.

The last item of proposition 5 says that at equal tenure, the average productivity of workers is higher in high survival firms. A natural question to ask is whether the average earnings of workers are higher in high survival firms beyond some threshold level of tenure. These workers are at a higher productivity level on average than workers employed in low survival firms, but they also spend more time in training when at productivity level x_1, and have a lower wage rate. By proposition 5, workers employed in type H firms eventually earn more, on average, than workers employed in type L firms if and only if

$$\frac{\delta}{h_L+\delta}(1-s^L)x_1w_L + \frac{h_L}{h_L+\delta}x_2w_L < \frac{\delta}{h_H+\delta}(1-s^H)x_1w_H + \frac{h_H}{h_H+\delta}x_2w_H \tag{3.1}$$

The following result says that whenever p_L is small enough and ϕ, the hazard rate out of unemployment, is high enough, condition (3.1) holds in steady state.

Proposition 6. *Given other parameters, there exist $(\underline{\phi}, \bar{p}) \in (0,1)^2$ such that workers eventually earn more, on average, in type H firms than in type L firms in steady state if $p_L < \bar{p}$ and $\phi > \underline{\phi}$.*

Proof. See appendix □

The argument we provide in the appendix consists of showing that inequality (3.1) must hold in steady state when $\phi = 1$ and $p_L = 0$. The proposition is then obtained with a continuity argument.

We now invoke the last item of proposition 5 to demonstrate that steady state quit and job destruction rates are higher in type L firms than in type H firms. Quit rates are the fraction of workers who decide to quit at the beginning of a given period. The job destruction rate for a given firm type is the sum of quits and involuntary separations, i.e. quits plus jobs lost due to firm death, divided by total employment.[8] The following results also make note of an obvious corollary to proposition 5: Quit rates are inversely related to tenure in this economy as in Jovanovic (1979) and as in the data.

[8]Specifically, let ω_i be the fraction of workers whose productivity level is x_1 in firms of type $i \in \{H, L\}$. The turnover (quit) rate in industry i is $\omega_i\delta$ while the job destruction rate is $1 - \beta p_i(1 - \omega_i\delta)$.

Proposition 7. *In steady state,*

- *Average quit rates are higher in type L firms than in type H firms;*
- *Average quit rates are also higher in type L firms at each tenure level;*
- *Quit rates decrease with tenure in both firm types;*
- *The job destruction rate is higher in type L firms than in type H firms.*

Proof. Proposition 5 implies that, at all tenure levels, workers employed in type H firms are less likely to fall to productivity level x_0. This result implies the first three items of the proposition. As for the fourth, note that $p_L < p_H$ implies that involuntary separations are more frequent for workers employed in type L firms than workers employed in type H firms. □

Our model, therefore, predicts that employee turnover and firm survival should be correlated, even at equal tenure. The fundamental force behind this result is that workers can influence the evolution of their productivity. Assuming instead that productivity evolves exogenously (i.e., $h(s) = h > 0$ for all $s \in [0, 1]$)) would make our model very similar to the learning-by-doing model described by Jovanovic and Njarko (1995). In that case, it is easily shown that turnover rates continue to be lower in high survival firms, as workers in those firms tend to have higher productivity levels. Also, earnings-seniority profiles continue to rise to an invariant value. What, then, distinguishes this exogenous worker productivity model from our endogenous accumulation model? First, in the exogenous model, earnings-seniority profiles do not differ across firms: The growth rate of productivity is the same in all firms at all tenure levels. While the early panel evidence of Barron et al. (1987) is ambiguous on this question, Abowd et al. (1999, table 11) find with much more detailed panel data on French workers that returns to seniority unambiguously rise with firm size.[9] Second, unlike our model, the exogenous model implies that turnover rates are identical across firm types once we control for tenure. That implication is inconsistent with the results of Anderson and Meyer (1994, see table 6), Topel and Ward (1992), and Quintin and Stevens (2005). Anderson and Meyer, for instance, find that turnover rates fall with firm size, whether or not one

[9] We wish to thank David Margolis for very helpful comments on this issue.

controls for tenure effects, as predicted by our model. Third, the exogenous model makes no prediction about the optimal quantity of training workers receive whereas the endogenous model predicts that workers employed in firms more likely to survive should receive more training on average, as they do according to the 1995 survey of employer-provided training described by Frazis et al. (1998). Fourth, unlike the exogenous model, the endogenous model allows for empirically relevant dynamics, such as procyclical quit rates and countercyclical job destruction rates, an issue we explore in the next section.

4 Dynamic implications

In this section we consider equilibria where wage rates vary over time. Equilibria, as before, are sequences of wage rates such that in each period, firms and workers behave optimally, labor markets clear, and budget balance is obtained. Our main objective in this section is to check that our model is qualitatively consistent with the fact that in the U.S., quit rates are procyclical while job destruction and training rates are countercyclical. In our dynamic experiments, we will assume that our economy is initially in steady state and consider the effects of unexpected shocks to an exogenous parameter. First, however, we review parameter selection and some selected steady state statistics.

4.1 Parameter choices

We will think of a period as a quarter and select parameters to match the appropriate features of U.S. data. Details may be found in appendix B. Broadly speaking, although we will not emphasize the precise quantitative implications of our model, the parameters shown in table 1 are such that the steady state generated by our model matches empirical estimates of the duration of unemployment, the time devoted to training, returns to training, the average earnings loss following an involuntary separation, and other U.S. statistics.

Table 1: Parameter choices

Survival rates		Unemployment		Production technology		Training technology	
β	0.98	ϕ	0.50	α	0.64	x_1	1.00
μ_1	0.0003	ρ	0.60	η	0.004	x_2	1.10
μ_2	0.0003	τ	0.21			δ	0.02
p_L	0.94					$h(s)$	$1.30s - 1.95s^2$
p_H	0.98						

Table 2: Steady State Summary Statistics by Firm Type

	Firm type:	
	Low survival	High survival
Average firm size	26.96	54.39
Fraction of workers at x_2	41.85	67.94
2-year growth rate of earnings	7.17	15.08
Quit rate	1.16	0.64
Job destruction rate	8.95	4.58

Notes: All values are expressed as percentages, except for average firm size which is in number of workers. Rates are quarterly, except where noted. High and low survival correspond to type H and type L firms.

4.2 Steady state statistics

Summary statistics for the steady state obtained using these parameters are in table 2. As summarized by proposition 2, high survival firms are larger because they are more productive on average in total factor terms. Given our parameterization, type H firms have roughly twice as many employees as type L firms. Table 2 also illustrates proposition 5: The two-year earnings growth profile is twice as steep in high survival firms as in low survival firms. The quarterly quit rate in low survival firms is nearly double the rate in high survival firms. Higher quit rates together with a lower firm survival rate imply that the job destruction rate in type L firms is also much higher. Proposition 7 also says that turnover rates must be higher in low survival firms after controlling for tenure, a result that is illustrated in figure 1.

4.3 Cyclicality of turnover and job destruction

In order to assess the cyclical behavior of turnover and job destruction, we shock exogenous parameters and compute the transition path back to steady state.[10] For each shock, we plot the percent deviation from steady state for select variables of interest—wages, output per worker, training intensity, quit rates, job destruction rates, and the unemployment rate.

We first shock total factor productivity (TFP)—which in our model is $(1+\eta)^t$ for firms of age t—by multiplying it by

$$1 + \theta_1 \theta_2^i$$

where $\theta_1 \in (-1, 1)$ is the magnitude of the shock, $\theta_2 \in [0, 1)$ is the shock's persistence, and $i \geq 0$ denotes the number of periods since the shock. The persistence parameter was set to 0.92; this value corresponds to a half-life of 3-1/2 years, a standard estimate of the average half-life of business cycle shocks in the U.S. The results of a 1 percent shock to TFP are illustrated in figure 2. On impact, firms find themselves with too much labor, and so wage rates must fall in order to clear the labor market. Workers, anticipating higher future wages, devote more time to training and less time to production. This behavior is consistent with workers "retooling" during recessions (DeJong and Ingram, 2001).[11] Lower TFP and more time spent on training imply that output per worker must fall. Consequently, aggregate output also drops. With more training, fewer workers drop to x_0, and so fewer workers choose to quit, as in the data. During slow times, workers train more and quit less often. Unlike the evidence from U.S. data, however, these procyclical quit rates lead to procyclical job destruction.

[10] Specifically, the algorithm for computing a transition path is as follows. The initial and final steady states are computed using standard methods; in the examples considered here the initial and final steady states are the same. We assume that the transition is complete after T periods, for large T, and we guess an initial path for w_L. We then repeat the next two steps $50,000$ times at which point the transition path has been solved (i.e., the labor market clears in each period and the policy functions solve the appropriate maximization problem):

(1) Starting at period $t = T - 1$ the value and policy functions are computed iteratively back to period $t = 1$; w_H is chosen so that newly hired workers are indifferent between the two industries.

(2) Labor clearing is not guaranteed at this point, as w_L was fixed for each t. Therefore, for each period $t = 1, \ldots, T-1$, we adjust wages by a very small amount in the direction needed to clear the labor market.

[11] In the model there are no fixed costs associated with labor, therefore firms do not fire workers. Rather, workers supply less labor by training more.

The reason for this counterfactual result is that a TFP shock, as we have modeled it, has no effect on firm survival rates and therefore all deviations from steady state reflect only the behavior of worker quits. In a downturn, we also expect firm survival to decline. Indeed, existing empirical evidence on business failures documents a small, but significant, increase in failures during downturns (Platt and Platt, 1994). Therefore, we redo the above analysis for a joint shock to both TFP and firm survival rates. The TFP shock is the same as before, but we now divide p_L and p_H by a common factor. Although data on business failure rates suggest that shocks to survival rates are less persistent than shocks to TFP, we chose to use the same degree of persistence. This assumption biases our experiment *against* finding procyclical quit rates, as a more persistent shock to survival probabilities increases the likelihood of countercyclical quits. As before, we consider a 1 percent shock to TFP; the survival rate shock is 0.01 percent (i.e., 1 out of every 10,000 businesses per quarter). The impact of this joint shock is summarized in figure 3. Wages, output per worker, and training are similar to the TFP-only shock, and we still find procyclical quit rates. Importantly, because of the increase in firm failures, job destruction rates are now countercyclical. In other words, while procyclical quit rates tend to lower job destruction, this effect is more than offset by a rise in involuntary separations due to the lower survival probabilities of firms, even though the survival shock is very small. The net effect is that unemployment rises.

4.4 Sensitivity analysis

The fact that our model's dynamic behavior is consistent with turnover facts in the U.S. remains true for other reasonable sets of exogenous parameters. Although all the evidence suggests that important resources are devoted to training, the existing data on training intensity and returns are imprecise (see Barron et al., 1997). We experimented with a wide variety of parameters choices (within the confines of assumptions 1 to 5), and found our findings to be robust to those changes. Another potential concern is that the choice for δ suggested by the microeconomic evidence reviewed by Mincer (1991) leads to an economy-wide quarterly quit rate of 1 percent, which is much below the 4.5 percent average calculated by Hamermesh and Pfann (1996) for the 1960-1981 time period in the U.S. Raising δ from

0.02 to 0.05 yields a quit rate closer to its empirical counterpart, and does not affect our results.

Finally, the shape of the production technology in the type of model we laid out is a source of debate. Like Hopenhayn and Rogerson (1993), we set $\alpha = 0.64$ to match the labor income share in the U.S., but Atkeson et al (1996), among others, argue that the implied returns to scale are too low. Raising α to near the upper bound implied by assumption 1 did not change our findings: Quits continue to fall during recessions, while job destruction rises.

5 Conclusion

Our paper characterizes the impact of firm survival on the evolution of worker productivity in a dynamic general equilibrium model. Quite intuitively, workers employed in firms highly likely to survive choose to invest more in future productivity than their counterparts in low survival firms. These investment patterns have several implications for the features of turnover and earnings across firm types in steady state and the evolution of turnover rates following business cycle shocks that are consistent with the relevant empirical evidence.

The correlation between firm characteristics and labor market outcomes thus constitutes strong support for models in which employee productivity evolves endogenously. Importantly, the intuition we develop in this paper does not depend on one's view of how worker decisions affect future productivity. If, for instance, productivity depends on unobservable effort (as in Lazear, 1981) workers may invest in "relationship collateral" by accepting lower initial earnings in return for actuarially fair payments later in the life of the contract. The intuition we develop in this paper does, however, depend on workers' ability to condition their decisions on the expected survival of their employer. In models with contractual frictions, employer survival affects the expected duration of the contract and hence the willingness of workers to invest in the relationship with their employer. Thus, our framework can accommodate a wide range of approaches to modeling workers' investments in productivity.

Our model could be generalized to allow for time-varying or endogenous firm survival rates. Assuming for example that survival rates follow a first-order Markov process would complicate the analysis by adding a new state variable to the worker's problem, but should not alter the basic correlation between expected employer survival and productivity investments. One could also allow for some feedback between worker decisions and firm survival. Firms with a more productive workforce could be more likely to survive. Our model's main predictions should stand up to this generalization. In a rational expectations equilibrium, workers must form expectations on their employer's likelihood of survival, and those expectations must prove correct. Furthermore, in such an equilibrium, atomistic workers treat the likelihood of survival as exogenous, so that the nature of their optimization problem should change little. Establishing the existence of a rational expectation equilibrium in this context may be challenging, but employer survival rates should have a similar effect on worker decisions as in our simpler framework.

A Proofs

A.1 Government budget balance

Assume that we have found a pair of constant wage rates (w_L, w_H) such that the labor market clears in every period. We will show that government revenues exceed aggregate unemployment benefits, so that we have in fact found a steady state. (Recall that excess tax revenues are rebated in a lump-sum fashion to workers.)

Workers at productivity level x_1 in type L firms earn $(1-s^L)x_1 w_L$ in a given period, where s^L is the optimal training policy given wage rates. The tax revenue associated with these workers is $\tau(1-s^L)x_1 w_L$. The fraction of workers that die at the end of the period is $(1-\beta)$. The fraction of surviving workers that quit or lose their job in each period and fail to find a new job immediately is $[1-p_L(1-\delta)](1-\phi)$. The corresponding unemployment benefit for these workers is $\beta[1-p_L(1-\delta)](1-\phi)\rho(1-s^L)x_1 w_L$. But since $\tau(1-s^L)x_1 w_L > \beta[1-p_L(1-\delta)](1-\phi)\rho(1-s^L)x_1 w_L$ under assumption 2, this type of worker increases tax revenues more than government expenses. Similar arguments show that this is the case for all possible types of workers, which establishes that tax revenues exceed total unemployment benefits, as claimed.

A.2 Statement of the worker's problem

We will now characterize the decisions of workers given wage rates. Fix (w_L, w_H). For $i \in \{H, L\}$ and $j \in \{0, 1, 2\}$, denote by V_j^i the expected lifetime labor income of a worker employed by a firm of type i and whose productivity level is x_j, so that, for instance, V_0^L denotes the expected lifetime income for a worker of productivity level x_0 who works for a firm of type L. In any steady state we must have $V_1^L = V_1^H$, as otherwise one firm type would not be able to hire workers, which is incompatible with the fact that both firm types have positive labor demand at all wage rates.

A.2.1 Value function of unemployed workers

Consider a worker who just became unemployed and whose current benefits are $b \geq 0$ (that is, their earnings in the previous period were $\frac{b}{\rho}$.) Denote by $V^U(b)$ their expected lifetime income. Then, for all $b \geq 0$,

$$V^U(b) = \phi \max\{V^U(b), V_1^L\} + (1-\phi)[b + \beta V^U(0)]$$

where

$$V^U(0) = \frac{\phi}{1-(1-\phi)\beta} V_1^L.$$

Indeed, they find a job with probability ϕ and accept it when $V_1^L \geq V^U(b)$. With probability $1-\phi$ they do not get a job offer, get the benefits in the current period, remain unemployed, and benefits expire after one period. We will now argue that assumption 3 implies that $V^U(b) < V_1^L$. Note first that assumption 3 implies that $b < x_1 \max\{w_L, w_H\}$. Then,

$$V^U(b) = b + \beta V^U(0) < x_1 \max\{w_L, w_H\} + \beta V^U(0) \leq V_1^L.$$

The last inequality follows from the fact that a feasible policy for newly employed workers consists of devoting no time to training in the first period and quitting after one period, so that $V_1^L \geq x_1 w_L + \beta V^U(\rho x_1 w_L)$. Having established that $V^U(b) < V_1^L$ for all workers in equilibrium, we will write for simplicity and without any loss of generality that for all $b \geq 0$:

$$V^U(b) = \phi V_1^L + (1-\phi)[b + \beta V^U(0)] = (1-\phi)b + \frac{\phi}{1-(1-\phi)\beta} V_1^L \tag{A.1}$$

A.2.2 Proof of proposition 3

To simplify the statement of the worker's problem, we begin by proving proposition 3.

Proof. Consider an employee in a type H firm (for concreteness) whose productivity reaches x_0. If she stays with her employer, her expected lifetime income is bounded above by $x_0 w_H + \beta p_H V_1^H$. On the other hand, if she quits, her expected lifetime income is bounded below by:

$$\phi V_1^H + \beta(1-\phi)\phi V_1^H + [\beta(1-\phi)]^2 \phi V_1^H + \ldots = \frac{\phi}{1-\beta(1-\phi)} V_1^H$$

The worker will quit provided the upper bound from staying is less than the lower bound from quitting, which is the case (for instance) if:

$$\begin{aligned} x_0 w_H + \beta p_H V_1^H &< \frac{\phi}{1-\beta(1-\phi)} V_1^H \\ \iff \qquad x_0 &< \left[\frac{\phi}{1-\beta(1-\phi)} - \beta p_H\right] \cdot \frac{V_1^H}{w_H}. \end{aligned}$$

But this inequality follows from assumption 5, and the fact that $V_1^H > x_1 w_H$. The second item of the proposition was established in subsection A.2.1. □

A.2.3 Value function of employed workers

Having established proposition 3, we need only consider employed workers whose productivity level is x_1 or x_2. Fix $i \in \{H, L\}$. Expected incomes for workers employed in type i firms must satisfy the following conditions in steady state:

$$\begin{aligned} V_1^i &= \max_{s\in[0,1]} (1-s)x_1 w_i + \beta p_i[(1-h(s)-\delta)V_1^i + h(s)V_2^i] \\ &+ \beta[(1-p_i) + p_i\delta]V^U(\rho(1-s)x_1 w_i) \qquad \text{(A.2)} \\ V_2^i &= x_2 w_i + \beta p_i[\delta V_1^i + (1-\delta)V_2^i] + \beta(1-p_i)V^U(\rho x_2 w_i) \qquad \text{(A.3)} \end{aligned}$$

To see this, consider first equation (A.2). A worker whose current productivity level is x_1 and chooses to devote time s to training receives $(1-s)x_1 w_i$ in labor income in the current period. As for future periods, assume first that the firm survives, which occurs with likelihood p_i. The worker moves up to productivity level x_2 with probability $h(s)$, in which case she remains with the firm and expects income V_2^i. With probability δ her human capital falls to level x_0, she quits, and expects future income $V^U(\rho(1-s)x_1 w_i)$. With probability $1-\delta-h(s)$, she remains at productivity level V_1^i. If the firm fails, which occurs with probability $(1-p_i)$, the agent becomes unemployed, and expects future income $V^U(\rho(1-s)x_1 w_i)$. Condition (A.3) is justified in the same fashion, and uses the fact that workers do not devote any time to human capital accumulation when they reach level x_2.

Solving equation A.3 for V_2^i yields $V_2^i = \frac{1}{1-\beta p_i(1-\delta)}\left\{x_2 w_i + \beta[p\delta V_1^i + (1-p)V^U(\tau x_2 w_i)]\right\}$. Plugging this into A.2 now gives:

$$
\begin{aligned}
V_1^i &= \max_{s\in[0,1]} (1-s)x_1 w_i + \beta p[(1-h(s)-\delta)V_1^i \\
&+ h(s)\frac{1}{1-\beta p_i(1-\delta)}\left\{x_2 w_i + \beta[p\delta V_1^i + (1-p)V^U(\tau x_2 w_i)]\right\}] \\
&+ \beta[(1-p)+p\delta]V^U(\tau(1-s)x_1 w_i) \qquad \text{(A.4)}
\end{aligned}
$$

Given expression (A.1), the right-hand side of equation (A.4) defines a mapping on $I\!R_+$ in V_1^i. That mapping has a unique fixed point, as we now argue.

Lemma 1. *The right-hand side of equation (A.4) defines a contraction mapping on* $I\!R_+$.

Proof. Fix $i \in \{H, L\}$. Consider the mapping $T : I\!R_+ \mapsto I\!R_+$ which to every value V_1 associates the right-hand side of (A.4). By construction, solutions of (A.4) and fixed points of T coincide. We will now argue that T is a contraction. T is clearly monotonic. Now note that expression (A.1) implies $\frac{\partial V^U(b)}{\partial V_1^L} = \frac{\phi}{1-(1-\phi)\beta} < 1$. Furthermore, $\frac{\delta}{1-\beta p_i(1-\delta)} < 1$ and $\frac{1-p_i}{1-\beta p_i(1-\delta)} < 1$. These observations and some algebra imply that for all $V_1 > 0$ and $c > 0$, $T(V_1 + c) < T(V_1) + \beta c$. Therefore, T is a contraction with modulus β. □

Given $i \in \{H, L\}$, the optimal training policy s^i is uniquely defined, because h is continuous and strictly concave. Furthermore, s^i is continuous in all parameters by the Theorem of the Maximum. In particular, it is easy to see that, for $i \in \{H, L\}$, V_1^i is linear in wages and the solution s^i to (A.4) is independent of wage rates, a fact upon which we will rely to establish that a unique pair of steady state wage rates exists.[12] In turn this implies that s^L is independent of p_H and that s^H is independent of p_L. More precisely, if (s^L, s^H) is the steady state pair of human capital policies given (p_L, p_H) and $(s^{L'}, s^{H'})$ is the steady state pair of human capital policies given (p'_L, p'_H), $p'_L = p_L$ implies $s^{L'} = s^L$. We will use this last fact in the proof of proposition 2.

A.3 Proof of proposition 1

Given assumption 4, it can never be the case that the demand for labor on the part of a given firm falls short of the intended supply of labor by existing employees. Therefore, labor

[12] Formally, since V^U is linear in V_1^i, T preserves linearity. Since the set of linear functions of w_i is a closed subset of the set of bounded real valued functions equipped with the supnorm topology, standard dynamic programming arguments imply that the fixed point of T is linear in wage rates. Dividing both sides of (A.4) by w^i now shows that optimal training is independent of the wage rate in steady state. This is also evident from first order condition (A.5) since $\frac{V_2^i - V_1^i}{w_i}$ is a constant.

market clearing requires only that overall labor demand equal overall labor supply for each firm type. We now show that this obtains for a unique pair of wage rates.

Proof. Fix w_L. Since V_2^L rises without bound with w_H, there is a unique wage rate $w_H = g(w_L)$ such that $V_1^L = V_1^H$. Furthermore, g is continuous and rises with w_L. Let $D^i(w_i)$ be the aggregate labor demand by firms of type i when the wage rate is w_i. By assumption 1, $D^i < +\infty$ for $i \in \{H, L\}$. Furthermore, both demand functions are continuous and strictly decreasing on $\mathbb{R}_+$. We will construct an equilibrium where workers always work for the same type of firms. Since expected incomes must be equal in equilibrium across firm types, such a policy is always (weakly) optimal.

For $i \in \{H, L\}$, denote by S^i the *average* supply of labor by agents who work for firms of type i during their lifetime given optimal human capital accumulation policies s^i. As we argued above, s^i does not depend on wage rates. Let $\sigma_L(w_L) = \frac{D^L(w_L)}{S^L}$ and $\sigma_H(w_L) = \frac{D^H(g(w_L))}{S^H}$. If $\sigma_L(w_L) + \sigma_H(w_L) = 1$, we can construct an equilibrium by assigning fraction $\sigma_L(w_L)$ of workers to type L firms and fraction $\sigma_H(w_L)$ to type H firms.[13]

To see that such a value for w_L exists observe first that σ_L and σ_H are continuous since labor demand functions are continuous. Moreover, $\sigma_L(w_L) + \sigma_H(w_L) > 1$ when w_L is small enough since $D^L(w_L)$ diverges to $+\infty$ as w_L gets small. On the other hand, for w_L large enough, $\sigma_L(w_L) + \sigma_H(w_L) < 1$ since $D^L(w_L)$ and $D^H(g(w_L))$ converge to zero when w_L gets large. By the intermediate value theorem, a solution exists. Because labor demand functions are strictly decreasing while s^i and therefore S^i are independent of wage rates, the solution is unique.

To see that $w_L > w_H$ in steady state, assume by way of contradiction that a steady state exists with $w_H \geq w_L$. Workers in type H firms can choose to set $s^H = s^L$. In this case, their average labor income when employed is at least as high as that of workers in type L firms, but the expected time they spend in unemployment is lower. Therefore, $V_1^L < V_1^H$, a contradiction. This completes the proof. □

As we point out in the proof above, workers whose unemployment spell ends are indifferent between the two employer types, and so employer choice policies are indeterminate. But the average labor supply to each firm type is independent of the specification of employer choice policies, as this average only depends on the tenure distribution of employees; in turn, the tenure distribution only depends on training policy functions and the survival probabilities of firms and workers. Therefore, the exact specification of the employer choice policy cannot affect steady state equilibrium wage rates, and so wage rates are unique.

A.4 Proof of proposition 2

We now show that for p_H high enough, type H firms are larger than type L firms in employment terms in all steady states.

[13]Because the size of the population of workers is one, σ_i is also the number of workers assigned to type i firms for $i \in \{H, L\}$.

Proof. For $i \in \{H, L\}$, the average labor demand by type i firms is given by:

$$\frac{\sum_{t=0}^{+\infty} \mu_i [p_i(1+\eta)^{\frac{1}{1-\alpha}}]^t \left(\frac{\alpha}{w_i}\right)^{\frac{1}{1-\alpha}}}{\mu_i/(1-p_i)} = \frac{1-p_i}{1-p_i(1+\eta)^{\frac{1}{1-\alpha}}} \left(\frac{\alpha}{w_i}\right)^{\frac{1}{1-\alpha}}.$$

Note that $\mu_i/(1-p_i)$ is the long-run number of firms of type i. We showed earlier that in any steady state, $w_L > w_H$. Recall also that the labor supply of a worker in a type H firm is bounded above by x_2, so the average labor supply by workers in type H firms, S^H, is also bounded by x_2. Finally, note that the average labor supply in type L firms, S^L, does not depend on p_H. Then, taking the ratio of the average employment size of type H firms to the average employment size of type L firms,

$$\left(\frac{S^L}{S^H}\right)\left(\frac{1-p_H}{1-p_L}\right)\frac{1-p_L(1+\eta)^{\frac{1}{1-\alpha}}}{1-p_H(1+\eta)^{\frac{1}{1-\alpha}}}\left(\frac{w_L}{w_H}\right)^{\frac{1}{1-\alpha}} > \left(\frac{S^L}{x_2}\right)\left(\frac{1-p_H}{1-p_L}\right)\frac{1-p_L(1+\eta)^{\frac{1}{1-\alpha}}}{1-p_H(1+\eta)^{\frac{1}{1-\alpha}}}.$$

Now fix p_L. As $p_H \mapsto (1+\eta)^{\frac{-1}{1-\alpha}}$, S^L is unaffected, and the ratio diverges to $+\infty$ which establishes proposition 2. □

A.5 Proof of proposition 4

Next we establish that in steady state, workers employed in type H firms devote more time to training than workers employed in type L firms.

Proof. For the purpose of this proof, we find it more convenient to work with equations (A.2-A.3) than their reduced version (A.4). Consider a steady state pair of wage rates and assume that both s^L and s^H are interior; other cases are trivial. Given (A.2), the first order condition for s^i for $i \in \{H, L\}$ is:

$$x_1 = \beta p_i h'(s^i)\frac{V_2^i - V_1^i}{w_i} - \beta[(1-p_i) + p_i\delta](1-\phi)\rho x_1 \tag{A.5}$$

Now, $[(1-p_i) + p_i\delta]$ falls with p_i, hence i. Therefore, a sufficient condition for $s_1 < s_2$ is that $\frac{p_i(V_2^i - V_1^i)}{w_i}$ rises with i. We will establish that this condition holds in steady state. For all $p \in [p_L, p_H]$, denote by $V_1(w,p)$ and $V_2(w,p)$ the solutions to (A.2-A.3) when $p_i = p$ and $w_i = w$. Also, for all $p \in [p_L, p_H]$, denote by $w(p)$ the unique wage rate such that $V_1(w(p), p) = V_1^L = V_1(w_L, p_L)$. In particular, note that $w(p_H) = w_H$, by construction. We will argue that for all $p \in [p_L, p_H]$,

$$\frac{\partial\left(\frac{p(V_2(w(p),p) - V_1(w(p),p))}{w(p)}\right)}{\partial p} \geq 0 \tag{A.6}$$

The left-hand side of condition (A.6) can be written as the sum of three terms:

$$a(p) = \frac{(V_2(w(p),p) - V_1(w(p),p))}{w(p)},$$

$$b(p) = p\frac{\partial}{\partial w}\left(\frac{(V_2(w(p),p) - V_1(w(p),p))}{w(p)}\right)\frac{\partial w(p)}{\partial p},$$

$$\text{and } c(p) = \frac{p}{w(p)}\left(\frac{\partial V_2}{\partial p}(w(p),p) - \frac{\partial V_1}{\partial p}(w(p),p)\right).$$

But $b(p) = 0$ for all p since, given p, V_2 and V_1 are linear in w. So we only have to show that $a(p) + c(p) \geq 0$ for all p. Using the envelope theorem and dropping the arguments of function h to curb notation,

$$\begin{aligned}
\frac{\partial V_2}{\partial p} &= \beta[\delta V_1 + (1-\delta)V_2] + \beta p[\delta\frac{\partial V_1}{\partial p} + (1-\delta)\frac{\partial V_2}{\partial p}] \\
&- \beta V^U(\rho x_2 w) + \beta(1-p)\frac{\phi}{1-(1-\phi)\beta}\frac{\partial V_1}{\partial p} \\
\frac{\partial V_1}{\partial p} &= \beta[(1-\delta-h)V_1 + hV_2] + \beta p[(1-\delta-h)\frac{\partial V_1}{\partial p} + h\frac{\partial V_2}{\partial p}] \\
&- \beta(1-\delta)V^U(\rho(1-s)x_1 w) + \beta[p\delta + (1-p)]\frac{\phi}{1-(1-\phi)\beta}\frac{\partial V_1}{\partial p}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial V_2}{\partial p} - \frac{\partial V_1}{\partial p} &= \beta(1-\delta-h)(V_2 - V_1) + \beta\delta V_1 \\
&+ \beta p(1-\delta-h)\left(\frac{\partial V_2}{\partial p} - \frac{\partial V_1}{\partial p}\right) + \beta p\delta\frac{\partial V_1}{\partial p} \\
&- \beta[V^U(\rho x_2 w) - V^U(\rho(1-s)x_1 w)] - \beta p\delta\frac{\phi}{1-(1-\phi)\beta}\frac{\partial V_1}{\partial p}
\end{aligned}$$

Now note that[14]

$$V^U(\rho x_2 w) - V^U(\rho(1-s)x_1 w) = (1-\phi)\rho[x_2 - (1-s)x_1]w \leq V_2 - V_1$$

Furthermore, $\frac{\partial V_1}{\partial p} > 0$ (as can be seen by partially differentiating equation A.2) and $\beta p\delta > \beta p\delta\frac{\phi}{1-(1-\phi)\beta}$. Therefore,

$$p\left(\frac{\partial V_2}{\partial p} - \frac{\partial V_1}{\partial p}\right) \geq -\frac{\beta p(\delta+h)}{1-\beta p(1-\delta-h)}(V_2 - V_1) \geq -(V_2 - V_1)$$

since $\frac{\beta p(\delta+h)}{1-\beta p(1-\delta-h)} < 1$ and, in turn,

$$c(p) > -\frac{(V_2(w(p),p) - V_1(w(p),p))}{w(p)} = -a(p).$$

[14] In approximate terms, this inequality says that the fact that the unemployment risk is lower in high survival firms is not sufficient to compensate workers for the initial wage differential. Returns to training must also be higher.

This completes the proof. □

A.6 Proof of proposition 6

We now demonstrate that for p_L small enough and ϕ high enough, workers employed in type H firms earn more than small firm workers past a certain tenure threshold.

Proof. Fix p_H and assume that $\phi = 1$ and $p_L = 0$. Let w_L^* and w_H^* be the corresponding steady state wage rates. Since $\phi = 1$ and $p_L = 0$, $V_1^L = \frac{x_1 w_L^*}{1-\beta}$ while some algebra shows that:

$$\begin{aligned}(1-\beta)V_1^H &= \left(1 - \frac{\beta p_H h(s^H)}{1-\beta p_H(1-\delta-h(s^H))}\right)(1-s^H)x_1 w_H^* + \left(\frac{\beta p_H h(s^H)}{1-\beta p_H(1-\delta-h(s^H))}\right)x_2 w_H^* \\ &< \frac{\delta}{\delta+h(s^H)}(1-s^H)x_1 w_H^* + \frac{h(s^H)}{\delta+h(s^H)}x_2 w_H^* \\ &< \frac{\delta}{h_H+\delta}(1-s^H)x_1 w_H^* + \frac{h_H}{h_H+\delta}x_2 w_H^*.\end{aligned}$$

The first equality follows from manipulations of (A.2-A.3) when $\phi = 1$. The first inequality uses the fact that $\beta p_H < 1$ while the second inequality uses the fact that $h_H \equiv \frac{h(s^H)}{1-\delta} > h(s^H)$. Then, if equation (3.1) does not hold, we have $V_1^L > V_1^H$, which cannot hold in equilibrium. Given the continuity of policy functions in survival probabilities and in ϕ, steady state wages vary continuously with p_L and ϕ. Since (3.1) holds when $p_L = 0$ and $\phi = 1$, it must then continue to hold for p_L small enough and ϕ high enough, as claimed. □

B Parameter selection

We set the hazard rate ϕ out of unemployment to 0.5 so that unemployment spells last $\frac{1-.5}{.5} = 1$ quarter on average. This is the average time between separation and re-employment estimated by Anderson and Meyer (1994) using data from eight state unemployment systems between 1978 and 1984. We set the replacement rate ρ to 60 percent, the average U.S. replacement rate (OECD, 1997). In our simulations, we assume that benefits are available for two quarters, as they are in most U.S. states.[15] We set τ to 21 percent, the overall payroll tax rate for the 1989 to 1994 period in the U.S. (Nickell and Layard, 1999).

We use data on plant deaths by 4-digit NAICS industries to set p_L and p_H. The data were created by the Census Bureau using the 1998 and 1999 County Business Patterns data. We only consider industries with at least 1,000 establishments. The minimum and maximum annual plant death rates in the resulting sample were 4.6 percent and 20.1 percent. We set p_L and p_H to the corresponding quarterly survival rates of approximately 94 percent and 98 percent. To set β, the fraction of workers who remain in the labor force, we assume that transitions from employment to out of the labor force are only permanent for individuals over the age of 55. Fallick and Fleischman (2002), using Current Population Survey (CPS)

[15] With two quarters rather than one, assumption 2 and the first-order conditions defining the optimal training policies differ slightly from the previous sections, but the adjustment is trivial and does not alter any of our results.

data from 1994, 1996–2001, find that individuals over the age of 55 account for 13.5 percent of employment in the U.S., and that 4.3 percent of those individuals, on average, leave the labor force each month. We therefore set our survival rate β to $.865 + .135 \times (1 - 0.043)^3 \simeq$ 98 percent.

The degree α of strict concavity of the production function is set to 0.64. This value is roughly equal to the average share of labor income in U.S. Gross National Product between 1960 and 1990 implied by standard real business cycle calculations (see Cooley, 1995). Setting η, the quarterly rate of growth of output per unit of labor, is more difficult. Standard measures of labor productivity would overestimate that number since they do not take into account the fact that average labor quality rises over time in surviving firms due to training. Total factor productivity (TFP) is inadequate for the same reason. On the other hand, TFP may underestimate η since it controls for growth in the capital stock, which is not in our model. For lack of better data, we use Baily et al's (1992) estimates of TFP growth in manufacturing plants between 1972 and 1987 as a rough guide. Conveniently for our purposes, they produce separate TFP growth estimates for plants who remain in the sample (i.e. do not fail) between census years. For these plants, they calculate (see table 2, p. 210) a compound TFP growth of 27 percent between 1972 and 1987 which translates into an average quarterly growth rate of 0.4 percent, our selection for η.

We set the quarterly depreciation rate for on-the-job training, δ, to 2 percent. This value is the midpoint of the 4 to 12 percent range of annual rate estimates reported by Mincer (1991). For our specification of the function h, we assume that it is quadratic. Specifically, $h(s) = as - bs^2$ for all $s \in [0, 1]$ where $a, b > 0$. We choose the ratio a/b to match the elasticity of labor productivity to the on-the-job training of newly hired employees reported by Barron et al. (1987). Barron et al. use data from a 1982 survey financed by the National Institute of Education and the National Center for Research in Vocational Training. The survey collected data for 659 firms on the on-the-job training received by newly hired workers in the first three months of employment, and the productivity and wages of those same workers after two years of employment. In each firm, a manager or firm owner provided training data on two recently employed workers (see Bishop, 1987, for a detailed description of the data). The fact that the survey focuses on new hires is convenient for our purposes since new hires are unambiguously at productivity level x_1 in our model. Barron et al. estimate that at the mean time devoted to training in their sample (151 hours in the first 3 months, roughly 30 percent of an average hire's hours worked), a 10 percent increase in training raises productivity (output per worker) by 3 percent after two years. Separately, Bishop (1991) calculates that roughly half of this productivity gain occurs during the first quarter of employment.[16] We therefore choose a/b so that at $s = .3$, $\frac{sh'(s)}{h(s)} = .5 \times .3$, or, after some algebra, $\frac{a}{b} \simeq \frac{2}{3}$.

This leaves us with two parameters to set: a and x_2. We choose these parameters jointly to match two statistics: 1) the average share of time devoted to on-the-job training by U.S. employees and 2) the average loss of earnings by *high earners* (see definition below) in the U.S. economy following an involuntary separation. The first statistic is notoriously

[16]At least two caveats are in order however. First, productivity estimates are derived from answers to qualitative questions inquiring about the performance of workers compared to their peers, and are subject to the standard criticism. Second, the survey combines firm-specific and general training.

Table 3: Average loss of earnings among top earners following an involuntary separation

Gender	Age	College education	1984	1986	1988	1990	1992	1994	1996	1998	2000
Male	>40	No	29.82	28.39	29.81	22.27	21.44	25.58	20.35	17.45	30.99
			(111)	(141)	(133)	(138)	(114)	(61)	(62)	(32)	(36)
Male	>40	Yes	21.79	19.04	21.65	11.31	21.02	17.08	12.96	6.47	7.21
			(75)	(107)	(105)	(87)	(113)	(81)	(72)	(41)	(43)
Male	≤40	No	30.27	24.88	26.28	27.96	27.14	31.87	20.35	24.52	24.29
			(58)	(85)	(80)	(81)	(72)	(38)	(37)	(29)	(34)
Male	≤40	Yes	19.46	16.49	30.48	17.25	33.43	28.05	21.91	13.80	20.79
			(24)	(45)	(55)	(48)	(82)	(62)	(58)	(42)	(55)
Female	>40	No	8.86	12.67	9.32	19.85	13.73	14.70	15.66	0.90	11.29
			(37)	(55)	(54)	(66)	(52)	(22)	(29)	(15)	(18)
Female	>40	Yes	25.03	20.81	23.91	20.50	13.96	15.07	10.30	11.32	27.86
			(29)	(41)	(55)	(45)	(62)	(44)	(47)	(31)	(32)
Female	≤40	No	28.07	25.03	24.75	20.48	19.86	41.88	28.50	24.18	17.93
			(30)	(37)	(42)	(54)	(45)	(22)	(25)	(20)	(23)
Female	≤40	Yes	13.76	19.09	17.04	10.44	30.53	10.21	18.87	19.04	13.79
			(12)	(21)	(22)	(22)	(44)	(25)	(35)	(30)	(33)
All top earners			24.49	22.65	25.04	19.72	22.70	21.44	18.21	14.04	19.00
			(376)	(532)	(546)	(541)	(584)	(355)	(365)	(240)	(274)
Memo: Entire sample			0.86	5.94	5.18	4.59	8.24	8.01	5.44	2.98	6.32
			(1567)	(2102)	(2162)	(2122)	(2326)	(1411)	(1258)	(942)	(930)

Notes: The sample consists of workers who lost a job in the previous 5 years (3 years after 1994). All of the table entries, with the exception of those in the memo line, are for "top earners," defined as workers whose earnings in the lost job were above the 75th percentile in their gender-age-education category. The numbers in parentheses denote the number of observations used to compute each statistic.

difficult to obtain (see e.g. Barron et al., 1997). Based on a 1995 survey of employer-provided training of 1,074 employees from establishments with 50 or more employees, Frazis et al. (1998) calculate that, on average, employees receive 44.5 hours of formal and informal training during a 6 month period, roughly 4.5 percent of hours worked. This is the fraction we will match. For the second statistic, we use data from the displaced worker supplement to the January Current Population Survey which is available every other year between 1988 and 2000. We only consider workers between the ages of 16 and 65 who report having lost a job in the past 5 years (3 years in supplements after 1994), who were employed full-time in their previous jobs and are employed full-time in their current jobs, and who had at least one year of tenure in their previous job. We then classify workers into gender-age-education cells according to whether their age exceeds 40, and whether they have some college education.[17] We focus on those workers whose CPI-deflated hourly earnings in their previous jobs exceeds the 75th percentile in their respective cells, our practical definition of high earners.[18] In our model, those observations would correspond to workers whose productivity level at the time they lost their job was x_2. We then compute the CPS-weighted average earnings loss in each

[17]The limited size of our sample forces us to use rather coarse categories. Nevertheless, using finer categories does not appear to significantly alter our main results.

[18]Each observation is weighted using the weights provided by the Census Bureau.

cell, and the CPS-weighted average earnings loss across cells. The results are shown in table 3. The average loss across cells turns out to be near 20 percent for most years. Given a set of parameters, the average earnings loss in our model is endogenous, as it depends on training decisions. We searched over wide grids for a and x_2 and found that setting $a = 1.3$ and $x_2 = 1.1$ produces the desired steady state statistics. Strictly speaking, these values together with the other parameters we chose do not satisfy assumption 4, but they bound average worker productivity growth sufficiently to imply that all firms must hire new workers in all periods in steady state equilibrium.

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Figure 1: Quit Rates by Length of Tenure

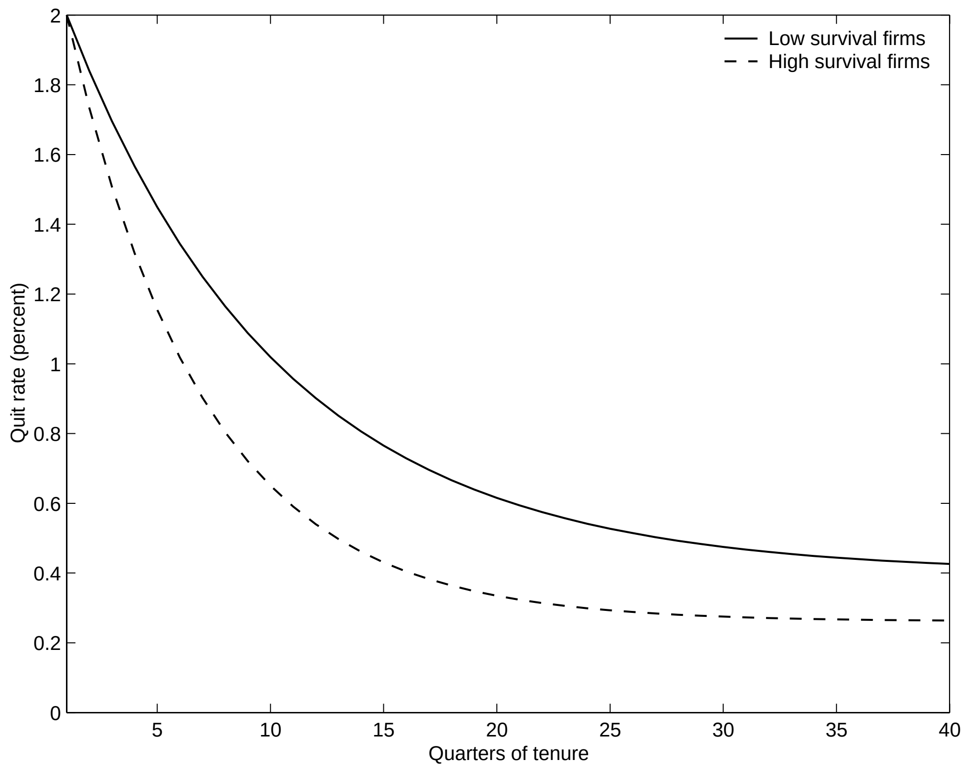

Figure 2: Percent Deviations from Steady State Following a Temporary Shock to TFP

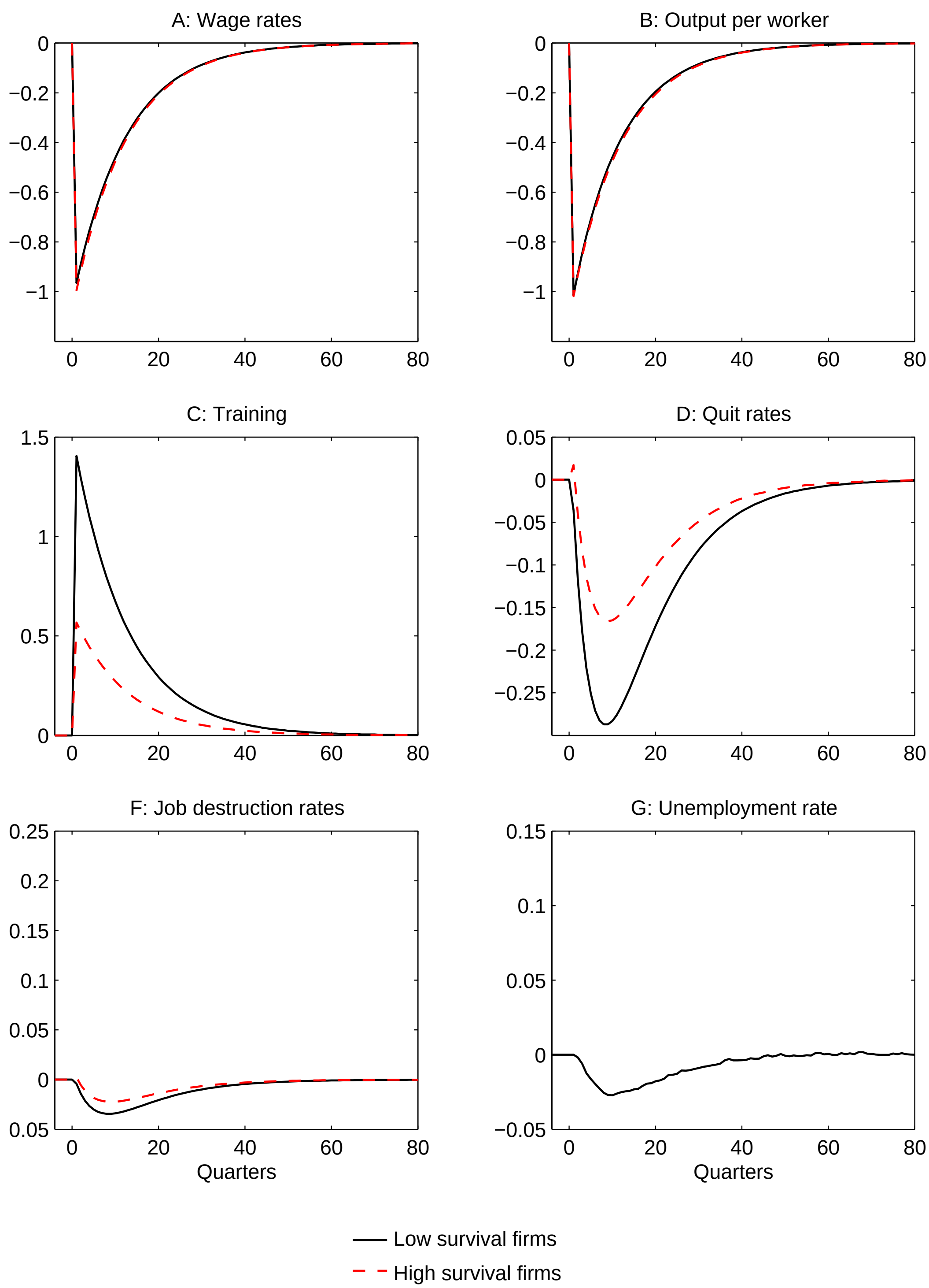

Figure 3: Percent Deviations from Steady State Following a Temporary Shock to Both TFP and Firm Survival Rates