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Abstract

This paper uses a dynamic general-equilibrium model with a nominal tax system to consider the effects of temporary partial expensing allowances on investment and other macroeconomic aggregates.

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1 Introduction

In recent years, the use of forward-looking general-equilibrium models to analyze the conduct of monetary policy has become a commonplace of the macroeconomics literature. By contrast, considerably less progress has been made in employing these models to examine questions related to *fiscal* policy.[1]

In this paper, we incorporate a nominal tax system into an otherwise standard sticky-price monetary business cycle (MBC) model, and use the resulting framework to examine the effect of a temporary partial expensing allowance on investment expenditures, real activity, and government revenues.[2] From a technical standpoint, temporary expensing allowances provide an excellent candidate for this kind of approach: There is significant scope for the general-equilibrium effects of these policies to differ from what a partial-equilibrium analysis would predict; moreover, the fact that these tax changes are temporary requires us to explicitly consider how agents' behavior today is affected by their expectations of future events.[3] In addition, expensing allowances figured prominently in the fiscal stimulus packages that were enacted in the wake of the most recent recession; hence, an analysis of the effects of these policies has important topical relevance.

Besides analyzing the general-equilibrium effects of investment incentives, a broader goal of this paper is to contribute to our understanding of how the canonical MBC model responds to fiscal policy changes. Previous research has provided us

[1]There is an irony here inasmuch as one of the earliest calls for a structural approach to policy modelling—Lucas's 1976 paper "Econometric Policy Evaluation: A Critique"—invoked a fiscal policy example (the establishment of an investment tax credit) to make its point.

[2]Partial expensing allowances permit firms to deduct a fraction of the cost of newly purchased capital goods from their taxable income. An expensing allowance is therefore similar to an investment tax credit (ITC) in that it allows a firm to raise its posttax income through purchases of capital goods; importantly, however, a firm is not allowed to claim any future depreciation allowances for its expensed capital (under an ITC, such a restriction is partly or wholly absent).

[3]Previous analyses of investment tax policies have not typically employed a framework that permits the simultaneous treatment of these issues (recent work by House and Shapiro, 2005, is an important exception). For example, Elmendorf and Reifschneider (2003) use a rational-expectations macromodel (the Federal Reserve Board's FRB/US model) to examine a *permanent* change in an investment tax credit, but are unable to treat the effect of a temporary credit. Similarly, Cohen, Hansen, and Hassett (2002) provide a careful study of the partial-equilibrium impact of an expensing allowance on user costs, while Abel (1982) examines the partial-equilibrium effects of temporary and permanent changes in tax incentives for investment.

with a relatively broad understanding of the model's strengths and shortcomings as a tool for monetary policy evaluation. However, the model's successes (or failures) in illuminating monetary policy issues need not translate to a corresponding degree of success in the fiscal policy context. In particular, this focus on monetary policy (as well as these models' inherent complexity) has often led researchers to place less emphasis on capturing features of the economy—such as the capital formation process—that are likely to matter much more when fiscal policy concerns are paramount. We therefore provide a relatively detailed description of how the model responds to the particular fiscal policy changes we consider, and identify those components of the model's structure that most profoundly influence our results.

In the remainder of the paper, we derive our theoretical model under various assumptions as to the type of costs faced by firms in adjusting their capital stock and labor inputs, the nature of the economy's aggregate supply relation, and the way in which household saving is determined. Our motivation for considering a number of alternative investment specifications stems from the fact that the fiscal policies we consider involve current *and prospective* changes to the tax system; given the forward-looking nature of the problem, then, the nature of the investment adjustment costs faced by firms will have an important effect on their capital expenditures. Similarly, we demonstrate that the presence of a nominal tax system implies that real *and* nominal interest rates will have an important influence on the model's real responses; hence, it is important to consider the degree to which the model's predicted effect of changes in tax-based investment incentives depends on its implied dynamics for real interest rates and expected inflation.

We also use the model to explore a practical question concerning the relative effects of two types of tax-based investment incentives; specifically, we examine what happens to output, government revenue, and capital formation when expensing allowances are increased with the corresponding response of these variables following a reduction in capital taxes. This type of "bang-for-the-buck" calculation is similar to that discussed by Abel (1978) for permanent tax changes in a partial-equilibrium setting; however, our own treatment represents the first time this topic has been addressed within the context of a fully specified microfounded dynamic general-equilibrium framework.

2 A Sticky-Price Model with Nominal Taxation

Our model economy is characterized by three sets of agents: households, firms, and the government. Households consume output, supply (homogeneous) labor, and purchase goods that are then transformed into capital and rented to firms. There are two classes of firms: a continuum of monopolistically competitive intermediate-goods producers, each of whom hires labor and capital to produce a differentiated good, and a single final-good producer who aggregates the intermediate goods to produce output for final demand. Finally, the government consists of a fiscal authority, who levies taxes that are rebated to households as lump-sum transfers, and a monetary authority who sets interest rates according to a Taylor rule.

With the exception of our treatment of taxation and investment, our theoretical setup is quite similar to the sticky-price monetary business cycle models used by Woodford (2003) and others to analyze monetary policy. We therefore devote most of this section to a detailed examination of those features of the model that are affected by the introduction of a nominal tax system, and relegate a more complete description of the model to the Appendix.

2.1 Households

The preferences of household i (where $i \in [0,1]$) are represented by the utility function

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[\frac{1}{1-\sigma} \left(C_t^i \right)^{1-\sigma} - \frac{1}{1+s} \left(H_t^i \right)^{1+s} \right] \right\}, \tag{1}$$

where C_t^i is defined as household i's consumption, H_t^i is its labor supply, and δ and s denote the household's discount factor and labor supply elasticity, respectively.

The household's budget constraint—which reflects its role in accumulating physical capital—is given by

$$A_{t+1}^i / R_t^f = A_t^i + R_t^k K_t^i - F_t^k \left(R_t^k K_t^i - X_t P_t I_t^i - \sum_{v=1}^{\infty} \kappa (1-\kappa)^{v-1} (1 - X_{t-v}) P_{t-v} I_{t-v}^i \right)$$
$$+ \left(1 - F_t^h \right) \left(W_t H_t^i + \mathit{Profits}_t^i \right) + T_t^i - P_t C_t^i - P_t I_t^i, \tag{2}$$

where

$$R_t^f = R_t - F_t^h \left(R_t - 1 \right). \tag{3}$$

The variable A_t^i denotes the nominal value of household i's bond holdings at the beginning of period t; W_t is the nominal wage paid on labor; R_t^k is the rental rate paid to household i for the use of its capital stock K_t^i (where K_t^i depreciates geometrically at the rate κ); $Profits^i$ represents the profits disbursed (as dividends) to households from the monopolistically competitive intermediate-goods producers; T_t^i are lump-sum transfers from the fiscal authority; P_t is the price of final output; I_t^i denotes the household's current-period purchases of investment goods; and R_t is the gross *pretax* nominal interest rate between periods t and $t+1$.

The fiscal system that we assume taxes all forms of nominal personal income (that is, income from financial assets, dividends, and labor) at the rate F_t^h, and taxes capital income at the rate F_t^k.[4] Hence, households receive an after-tax return R_t^f on their financial assets that is given by equation (3).[5] In addition, two types of deductions are permitted against capital income: depreciation charges and expensing allowances. The presence of depreciation allowances reflects the fiscal authority's recognition that part of the payment capital owners receive from renting out their capital stock merely reflects compensation for the depreciation of the stock from its use in production. An expensing allowance, meanwhile, represents a (partial) rebate of the purchase price of a new capital good. Unlike a pure subsidy or credit, however, future depreciation of the portion of the new investment good that is expensed may not later be deducted from taxable income. Thus, an expensing allowance can be loosely thought of as a completely "front-loaded" depreciation allowance.

We make the standard simplifying assumption that households directly own all capital in the economy and rent it out to firms; hence, tax provisions on investment are directly reflected in the budget constraint (2), as follows. First, an expensing allowance X_t is applied to household i's time-t nominal expenditure on new capital

[4] We are making an arbitrary (but ultimately unimportant) distinction here between the "profits" that appear in equation (2)—which represent a pure surplus over the payments to the factors of production that is distributed as a dividend to firm owners—and payments to households in their capacity as owners of the capital stock, which serve as the base of the corporate income tax. While it is somewhat artificial to assume that the former payments are not considered profits by the tax code, this assumption has no substantive effect on our analysis because monopoly profits have the same effect on household budget constraints as a lump-sum payment (and are zero in equilibrium).

[5] Note that the form of this expression reflects the fact that only interest—not principal—is subject to taxation.

goods, $P_t I_t^i$. Second, the dollar value of depreciation at time-t from all previous purchases of capital is given as $\sum_{v=1}^{\infty} \kappa(1-\kappa)^{v-1} P_{t-v} I_{t-v}^i$. However, because previously expensed capital may not receive a depreciation allowance, each term $P_{t-v} I_{t-v}^i$ in the sum in equation (2) must be multiplied by $(1 - X_{t-v})$. In addition, under the U.S. tax code depreciation is computed using *historical* cost; as a result, the investment price in the depreciation term is written with a $t - v$ subscript.[6]

In practice, depreciation allowances are based on a legislated schedule of depreciation rates, not the true (economic) depreciation rate κ. In our model, using legislated depreciation rates to compute depreciation allowances would merely involve replacing $\sum_{v=1}^{\infty} \kappa(1-\kappa)^{v-1} P_{t-v} I_{t-v}^i$ in equation (2) with $\sum_{v=1}^{V} \kappa_v^{irs} P_{t-v} I_{t-v}^i$, where V denotes the tax-life of the capital stock—which averages around 5-1/2 years (22 quarters) for equipment investment—and κ_v^{irs} denotes the rate of depreciation for tax purposes (specified by the tax code) in the vth period of the capital stock's life. However, this extension significantly increases the number of state variables in the model, and complicates our interpretation of the resulting first-order conditions for investment. In addition, it turns out that few of the model's qualitative results are affected by our equating tax depreciation with economic depreciation.[7] We therefore assume that $\kappa_v^{irs} = \kappa(1-\kappa)^{v-1}$ throughout.

In the absence of adjustment costs on capital or investment spending, the capital accumulation process is given by

$$K_{t+1}^i = (1-\kappa) K_t^i + I_t^i. \tag{4}$$

For our baseline model, we assume that is it costly to adjust firms' capital stocks, with adjustment costs taking a quadratic form. This yields the following capital

[6]The difference between a partial expensing allowance and a pure investment subsidy can be easily described in the context of equation (2). Under partial expensing, when the household deducts its allowed proportion of current investment spending from current capital income future depreciation allowances are scaled back accordingly (hence the term $1 - X_t$ multiplying the depreciation allowance terms). By contrast, under an investment subsidy the allowance today would leave future depreciation allowances unaffected, so that allowable deductions to taxable income would be given by $X_t P_t I_t^i - \sum_{v=1}^{\infty} \kappa(1-\kappa)^{v-1} P_{t-v} I_{t-v}^i$.

[7]Intuitively, reasonable changes to the assumed pattern of capital depreciation have a very small effect on the cost of capital relative to the effect that obtains from the presence or absence of an expensing allowance. Hence, it is this latter factor that is the dominant influence on the contour of the model's impulse response function for investment.

evolution equation:

$$K_{t+1}^i = (1-\kappa)K_t^i + I_t^i \exp\left[-\frac{\chi^k}{2}\left(\frac{K_{t+1}^i}{K_t^i}-1\right)^2\right], \tag{5}$$

where the parameter χ^k controls the curvature of the adjustment-cost function.

In the baseline model, then, the household takes as given its initial bond stock A_0^i, the expected path of the gross nominal interest rate R_t, the price level P_t, the wage rate W_t, the rental rate R_t^k, profits income, and the legislated personal income tax rates and expensing allowances (F_t^h, F_t^k, and X_t), and chooses $\{C_t^i, H_t^i, I_t^i, K_{t+1}^i\}_{t=0}^{\infty}$ so as to maximize equation (1) subject to the budget constraint (2) and the capital evolution equation (5).

2.2 Intermediate- and Final-Goods Producers

The monopolistically competitive firm j chooses labor H_t^j and capital K_t^j to minimize its cost of producing output Y_t^j, taking as given the wage rate W_t, the rental rate R_t^k, and the production function. Specifically, firm j solves:

$$\min_{\{H_t^j, K_t^j\}_{t=0}^{\infty}} W_t H_t^j + R_t^k K_t^j \text{ such that } \left(H_t^j\right)^{1-\alpha}\left(K_t^j\right)^{\alpha} - FC \geq Y_t^j, \tag{6}$$

where α is the elasticity of output with respect to capital and and FC is a fixed cost (set equal to $FC = \frac{Y_*}{\theta-1}$) that is assumed in order to preclude positive steady-state profits. The cost-minimization problem implies labor- and capital-demand schedules for each firm as well as an expression for the firm's marginal cost MC_t^j. We bring sticky prices into the model by assuming that intermediate-goods producers are Calvo price-setters: In any period, a fraction $(1-\eta)$ of firms can reset their price, while the remaining fraction η are constrained to charge their existing price (which is indexed to the steady-state inflation rate).

We also assume a representative final-good producing firm who takes as given the prices $\{P_t^j\}_{j=0}^1$ that are set by each intermediate-good producer, and chooses intermediate inputs $\{Y_t^j\}_{j=0}^1$ to minimize its cost of producing aggregate output Y_t subject to a Dixit-Stiglitz production function:

$$\min_{\{Y_t^j\}_{t=0}^{\infty}} \int_0^1 P_t^j Y_t^j \, dj \text{ s.t. } Y_t \leq \left(\int_0^1 Y_t^{j\,\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}. \tag{7}$$

This cost-minimization problem yields demand functions for each intermediate good that are given by $Y_t^j = Y_t(P_t^j/P_t)^{-\theta}$, where P_t, the price of final output, is defined as $P_t = (\int_0^1 (P_t^j)^{1-\theta} dz)^{\frac{1}{1-\theta}}$.

2.3 The Monetary Authority

The central bank sets the nominal interest rate according to a Taylor-style feedback rule. Specifically, the target nominal interest rate $\bar{R}_t$ is assumed to respond to deviations of output and the (gross) inflation rate from their respective target levels $\bar{\Pi}$ and $\bar{Y}$:

$$\bar{R}_t = (\Pi_t/\bar{\Pi})^{\beta} (Y_t/\bar{Y})^{\gamma} R_*, \tag{8}$$

where R_* denotes the economy's steady-state (equilibrium) interest rate. For simplicity, we will assume that the central bank targets the economy's steady-state level of output, implying that $\bar{Y} = Y_*$. Policymakers smoothly adjust the actual interest rate to its target level:

$$R_t = (R_{t-1})^{\rho} (\bar{R}_t)^{1-\rho} \exp[\xi_t^r], \tag{9}$$

where ξ_t^r represents a policy shock.

2.4 The Fiscal Authority

To keep the number of fiscal distortions in the model to a minimum, we assume a role for government that is as simple as possible; namely, one in which the fiscal authority merely raises revenues via taxation and then rebates these revenues as lump-sum transfers T_t^i to households. Hence, the government faces the following budget constraint:

$$\begin{aligned} \int_0^1 T_t^i di \; = \; Revenue_t = \int_0^1 F_t^h W_t H_t^i di + \int_0^1 F_t^k R_t^k K_t^i di + \int_0^1 F^h Profits_t^i di \\ + \int_0^1 F_t^h (R_{t-1} - 1) \left(A_t^i / R_{t-1}\right) di - \int_0^1 F_t^k X_t P_t I_t^i di - \int_0^1 F_t^k Liab_t^{i,\kappa} di. \end{aligned} \tag{10}$$

The government's depreciation allowance liability to household i in period t, $Liab_t^{i,\kappa}$, is given by:

$$Liab_t^{i,\kappa} = \sum_{v=1}^{\infty} \kappa (1-\kappa)^{v-1} (1 - X_{t-v}) P_{t-v} I_{t-v}^i = \kappa (1 - X_{t-1}) P_{t-1} I_{t-1}^i + (1-\kappa) Liab_{t-1}^{i,\kappa}$$

under our assumption that depreciation allowances equal true economic depreciation.[8] Note that if the net stock of bonds in the economy is zero (as it will be when all bonds are domestic and privately issued), then the first term in the second line of equation (11) drops out.

An additional variable that we define here (since it will prove useful when we attempt to score different tax policies) is the present discounted value of revenues. This is given as:

$$PDV_t^{rev} = E_t \left[\sum_{v=0}^{\infty} \frac{\delta^v MU_{t+v}/P_{t+v}}{MU_t/P_t} Rev_{t+v} \right] = Rev_t + E_t \left[\frac{\delta MU_{t+1}/P_{t+v}}{MU_t/P_t} PDV_{t+1}^{rev} \right] \quad (11)$$

where the dependence on the marginal utility of consumption, MU_t, reflects the use of a stochastic discount factor to value future income.

Finally, we note in passing that changes in tax policy in our framework can be equated with shocks to suitably specified exogenous processes for the fiscal variables. For example, the introduction of a *permanent* partial expensing allowance is captured by a one-time shock to X_t, where the expensing allowance is assumed to follow an $AR(1)$ process with a unit autoregressive root:

$$X_t = X_{t-1} + \epsilon_t^x. \quad (12)$$

Similarly, a temporary (n-period) partial expensing allowance can be treated as an innovation to X_t under the assumption that the allowance follows an $MA(n-1)$ process:

$$X_t = \epsilon_t^x + \epsilon_{t-1}^x + \cdots + \epsilon_{t-n+1}^x. \quad (13)$$

Naturally, shocks to other fiscal variables (such as F_t^k) can be treated in a parallel fashion.

2.5 The Model's First-Order Conditions

We only consider the first-order conditions that are directly affected by the presence of nominal taxation; other first-order conditions are described in the Appendix.

The household's utility-maximization problem yields an intertemporal Euler equation along with a supply schedule for labor:

$$\frac{1}{C_t^\sigma P_t} = \delta E_t \left[\frac{R_t^f}{C_{t+1}^\sigma P_{t+1}} \right] \quad (14)$$

[8] With legislated depreciation rates, this liability equals $\sum_{v=1}^{V} \kappa_v^{irs} \left(1 - X_{t-v}\right) P_{t-v} I_{t-v}^i$.

and

$$\frac{W_t(1-F_t^h)}{P_t} = H_t^s C_t^\sigma. \tag{15}$$

The solution to the household's maximization problem also yields a capital supply condition; however, when adjustment costs are present, this expression is relatively complicated. We therefore relegate it to the Appendix (as equation 45), and instead give the capital supply equation that obtains when there are no adjustment costs for capital or investment, namely:

$$E_t\left[\frac{R_{t+1}^k(1-F_{t+1}^k)}{P_{t+1}}\right] = E_t\left[\frac{R_t^f}{\Pi_{t+1}}\left(1-F_t^k X_t - PDV_t^\kappa\,(1-X_t)\right)\right] \\ - E_t\left[(1-\kappa)\left(1-F_{t+1}^k X_{t+1} - PDV_{t+1}^\kappa\,(1-X_{t+1})\right)\right], \tag{16}$$

where the variable R_t^f is defined by equation (3). The variable PDV_t^κ in equation (16) is the present discounted value of future depreciation allowances that households can deduct from their tax liability; when depreciation allowances for tax purposes are equal to true economic depreciation, this is given by

$$PDV_t^\kappa = E_t\left\{\sum_{v=1}^{\infty}\frac{\delta^v MU_{t+v}/P_{t+v}}{MU_t/P_t}\kappa\,(1-\kappa)^{v-1}\,F_{t+v}^k\right\}, \tag{17}$$

where we again use a stochastic discount factor to value future income streams.[9]

In addition, factor demand schedules (in which labor and capital demand is expressed as a function of output and factor-price ratios) are obtained from the intermediate-goods producers' problem, while the final-goods producer's problem yields demand functions for intermediate goods and an expression for the aggregate price level. These relations (along with the economy's market-clearing condition) are described in detail in the Appendix.

2.6 The Log-Linearized Model Equations

We obtain a linear model by log-linearizing the model equations about a deterministic steady state. Again, we mainly focus on describing and interpreting those equations that are directly affected by the presence of a nominal tax system; other log-linearized model equations are presented in the Appendix.

[9]When allowances are based on legislated depreciation rates, the $\kappa(1-\kappa)^{v-1}$ term in equation (17) is replaced by κ_v^{irs}.

The household's Euler equation (14) becomes

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(r_t^f - E_t \pi_{t+1}\right), \tag{18}$$

with π defined as the log-difference of the price level (here and elsewhere, we use lower-case letters to denote log deviations of variables from their steady-state values). As is clearly evident from this equation, consumption growth is a function of the real *posttax* interest rate. The log-linearized posttax *nominal* interest rate is given by

$$r_t^f = \frac{\frac{\bar{\Pi}}{\delta} - F_*^h}{\frac{\bar{\Pi}}{\delta}} r_t - \frac{\frac{\bar{\Pi}}{\delta} - 1}{\frac{\bar{\Pi}}{\delta}} \cdot \frac{F_*^h}{1 - F_*^h} E_t f_{t+1}^h, \tag{19}$$

where an asterisk in lieu of a time subscript denotes a variable's steady-state value. Finally, the household's labor supply condition log-linearizes to

$$w_t = \frac{F_*^h}{1 - F_*^h} f_t^h + \sigma \cdot c_t + s \cdot h_t. \tag{20}$$

When capital adjustment costs are present, the capital supply condition yields the following log-linear expression for the user cost:

$$\begin{aligned} E_t r_{t+1}^k = & \left[\frac{F_*^k}{1 - F_*^k}\right] f_{t+1}^k + \left[\frac{1}{1 - \delta(1-\kappa)}\right]\left(r_t^f - E_t \pi_{t+1}\right) \\ & - \left[\frac{1}{1 - \delta(1-\kappa)} \cdot \frac{PDV_*^\kappa}{1 - PDV_*^\kappa}\right]\left(pdv_t^\kappa - \delta(1-\kappa) E_t pdv_{t+1}^\kappa\right) \\ & - \left[\frac{1}{1 - \delta(1-\kappa)} \cdot \frac{F_*^k - PDV_*^\kappa}{1 - PDV_*^\kappa}\right]\left(X_t - \delta(1-\kappa) E_t X_{t+1}\right) \\ & - \left[\frac{\chi^k \cdot \kappa}{1 - \delta(1-\kappa)} \cdot \frac{1}{1 - PDV_*^\kappa}\right]\left(\delta E_t k_{t+2} - (1+\delta) k_{t+1} + k_t\right), \end{aligned} \tag{21}$$

with

$$pdv_t^\kappa = \left(\delta/\bar{\Pi}\right)(1-\kappa) E_t pdv_{t+1}^\kappa + \left(1 - \left(\delta/\bar{\Pi}\right)(1-\kappa)\right) E_t f_{t+1}^k - r_t^f. \tag{22}$$

(Note that equation 21 is the log-linearized version of equation 45 from the Appendix.)

As can be seen from these equations, there are two important ways in which the presence of a nominal tax system affects aggregate demand determination. First, consumption growth and the user cost are both functions of the real *posttax* interest rate, which will not move one-for-one with changes in the nominal interest rate when income taxes are nonzero. Second, because depreciation allowances are valued at historic cost, they will be worth less in current-dollar terms when inflation is positive—put differently, the nominal nature of depreciation allowances implies that *nominal* interest rates determine their discounted present value. Hence, an increase in nominal interest rates raises the user cost of capital in two ways: first by raising the posttax real interest rate, and second by lowering the expected present value of depreciation allowances.[10]

The other components of the log-linearized model are quite standard. Capital and labor demand are given by

$$k_t = \left(\frac{\theta - 1}{\theta}\right) y_t + (1 - \alpha)\, w_t - (1 - \alpha)\, r_t^k \tag{23}$$

and

$$h_t = \left(\frac{\theta - 1}{\theta}\right) y_t - \alpha\, w_t + \alpha\, r_t^k, \tag{24}$$

respectively, while the log-linearized aggregate supply relation is a new-Keynesian Phillips curve of the form

$$\pi_t = \delta\, E_t \pi_{t+1} + \frac{(1 - \eta)\,(1 - \eta\delta)}{\eta} mc_t. \tag{25}$$

Finally, the log-linearized monetary policy rule is

$$r_t = \rho r_{t-1} + (1 - \rho)\,(\beta E_t \pi_{t+1} + \gamma y_t) + \xi_t^r, \tag{26}$$

which combines equations (8) and (9).[11]

2.7 Calibration

The structural parameter values that we use in order to calibrate the baseline model are summarized in the table below. The values for α, σ^{-1}, and θ are set so as to match Kimball's (1995) preferred calibration; δ is taken from Clarida, Galí, and Gertler (2000, p. 170); and κ is computed from the depreciation rates and nominal

[10]These intersections of the tax system with aggregate demand determination change the conditions required for the existence of a determinate and stable rational-expectations equilibrium; see Edge and Rudd (2002) for a discussion.

[11]The Appendix details the model's remaining log-linearized equations.

stocks in Katz and Herman (1997). None of these is particularly controversial.[12] For χ^k, we choose a value that gives our capital adjustment cost function the same curvature properties as Kimball's specification; more concretely, the adjustment costs under this calibration are such that, following a permanent shock (and in *partial* equilibrium), the capital stock adjusts 30 percent of the way to its desired level after one year.[13] Finally, our assumed value for η implies that firms' prices are fixed for one year on average, which is again standard; conditional on this value for η, our assumed (inverse) labor supply elasticity s is then chosen so as to yield an elasticity of inflation with respect to output that is similar to what Clarida, *et al.* employ in their work.[14]

Calibrated Values of Common Structural Parameters

Parameter	Description	Value
α	Elasticity of output with respect to capital	0.30
σ^{-1}	Intertemporal elasticity of substitution	0.20
θ	Elasticity of substitution of intermediates	11
κ	Depreciation rate	0.034
δ	Households' discount factor	0.99
χ^k	Curvature parameter in adjustment cost function	500
$(1-\eta)$	Probability firm can reset price	0.25
s	Inverse labor supply elasticity	2.75
F_*^h	Steady-state tax rate on noncapital income	0.30
F_*^k	Steady-state tax rate on capital income	0.48
$\bar{\Pi}$	Inflation target	1.00

[12] Note that our assumed value of θ implies an equilibrium markup of 10 percent. In addition, the depreciation rate κ and discount factor δ are expressed at a quarterly—not annual—rate; for example, our assumed value for depreciation equals 13 percent per year.

[13] Kimball's calibration is particularly relevant for our purposes since it is informed by the results of Cummins, Hassett, and Hubbard's (1994) study, which uses variation in business tax rates (including ITC provisions and depreciation allowances) to identify and estimate structural investment equations.

[14] With this value of s, a 2.75 percent increase in wages is required to raise hours supplied by one percent (all else equal). While this implies a labor supply curve that is steeper than what is commonly employed by RBC modellers, it is quite consistent with the range of values found in the micro-labor literature (see, for example, Abowd and Card, 1989, table 10); it also yields a much more realistic implication for the representative consumer's marginal expenditure share of leisure (*c.f.* the discussion in Kimball, 1995, pp. 1267-69).

For the policy-related parameters, the assumed values for F_*^h and F_*^k are intended to capture the average marginal tax rates on noncapital and capital income that are implied by the current U.S. tax code; a detailed description of how these values were chosen (together with a discussion of how sensitive our results are to different assumptions about F_*^h and F_*^k) is provided in the Appendix. The $\bar{\Pi}$ value we specify implies an inflation target of zero—which is the assumed steady-state value of inflation in the model—while the parameter values we set in our Taylor rule are $\beta = 1.80$, $\gamma = 0.0675$, and $\rho = 0.79$, which are the post-1979 values estimated by Orphanides (2001) using real-time data.

3 Effects of Partial Expensing Allowances

In this section, we use the baseline model to examine the effects of permanent and temporary changes in the expensing allowance on capital investment, with a particular focus on the way in which the general-equilibrium character of the model influences its response to fiscal shocks. We then discuss how the model's basic predictions change when the firm faces alternative adjustment-cost specifications for its capital stock or investment spending.

To provide a useful benchmark, we first present results from a *partial*-equilibrium model that uses the same neoclassical investment specification that underpins the general-equilibrium model. Hence, any difference in results that obtains under the general-equilibrium framework arises because of the effects that changes in investment demand have on output, real interest rates, and consumption demand. In addition, when we compare the results from our general-equilibrium setup to those that obtain in a partial-equilibrium analysis, we use a version of the baseline model in which prices are assumed to be fully flexible (since aggregate price rigidities are irrelevant when output is exogenous). Later, this will permit us to separately identify the role played by sticky prices in our framework.

3.1 Effect of a Permanent Partial Expensing Allowance

We first consider the effects of a permanent 30 percent expensing allowance.[15] Figure 1 shows the predicted responses of the capital stock, gross investment, and

[15] We choose 30 percent for our example because it corresponds to the size of the (temporary) expensing allowance that was instituted under the 2002 Job Creation and Worker Assistance Act.

the real rental rate from the partial-equilibrium model, while Figure 2 gives the corresponding responses from the flexible-price version of the baseline model. (As consumption is an endogenous variable in the general-equilibrium model, we also plot its response in Figure 2.)[16] In both models, the presence of the expensing allowance makes new capital a more attractive investment. Households therefore immediately begin to purchase capital goods, which raises the economy's capital stock; the aggregate rental rate then falls as this new capital is added to the economy.

A closer comparison of the two sets of results reveals some important differences, however. In the partial-equilibrium model, the only constraint agents face in adding to the capital stock is the presence of adjustment costs. By contrast, in a general-equilibrium framework, additional capital spending can only occur if more output is produced and/or a greater share of output is devoted to investment. In the model, this process is mediated by higher real interest rates (not shown), which induce households both to give up some of their consumption and to supply more labor (thus raising output).[17]

It is also important to note that essentially all of the sluggishness of the response of the capital stock in the general-equilibrium model reflects the endogenous reaction of the other variables in the model. This can be most clearly seen by comparing the path of the capital-output ratio in the baseline general-equilibrium model to its path in a version of the model in which adjustment costs are completely absent, which we do in Figure 3. As is evident from this plot, capital adjustment costs have a relatively small incremental effect on the path of the capital-output ratio that obtains in the general-equilibrium model. This point can also be illustrated by noting that the capital-output ratio eventually rises about five percent above its baseline level as a result of the expensing allowance. In the partial-equilibrium model, therefore, the capital stock has moved roughly three-fourths of the way to its long-run value after twenty quarters. In the general-equilibrium setup, however,

[16] All variables are expressed as percentage deviations from their steady-state values, with the exception of the rental rate, which is given as a percentage-point deviation at a quarterly rate.

[17] Note that the rise in real rates actually pushes the economywide real rental rate above its baseline level for several periods after the expensing allowance comes into effect. (Intuitively, the rise in aggregate demand that results from the increased demand for investment goods makes installed capital more valuable.) Even so, there is still an incentive to invest, since the expensing allowance implies that *new* capital remains attractive even with the rise in real rates.

the capital-output ratio has moved about a third of the way to its long-run level after the same period of time has elapsed.

3.2 Effect of a Temporary Partial Expensing Allowance

We now turn to an examination of the effects of partial expensing allowances that are put into place for a limited period of time. This adds an important forward-looking aspect to the model, since firms' current behavior will anticipate the expected future change in tax policy. As a result, the model's dynamic responses will be richer, and will further highlight how the general-equilibrium nature of the analysis influences the results. The specific experiment we consider is the introduction of a 30 percent expensing allowance that lasts for three years; all agents are assumed to fully understand and believe the temporary nature of the allowance.

Panel A of Figure 4 plots the predicted responses of capital, investment, and the rental rate from the partial-equilibrium model (with capital adjustment costs) following the introduction of the temporary expensing allowance. As before, the new allowances make new capital investment (temporarily) more attractive, thereby leading to a gradual increase in the capital stock and an immediate jump in investment expenditure; over time, as more capital is added to the economy, the aggregate rental rate declines. Interestingly, however, in this case the temporary nature of the allowances induces firms to "pull forward" their investment spending (as can be seen from the figure, the path of the capital stock following a *permanent* increase in the expensing allowance—plotted here as a dotted line—lies below the response from the temporary-allowance case for the first four years). Later, when the expensing allowance expires, the capital stock lies above its steady-state level. Disinvestment is costly, however (there are adjustment costs), and so takes place over an extended period. The result is a persistent investment "pothole," as the level of investment falls *below* its steady-state level.

The responses of these variables (and consumption) in the flexible-price general-equilibrium model are plotted in panel B of Figure 4. As is apparent from a comparison with the partial-equilibrium case, the responses of capital and investment are smaller in the general-equilibrium model; in addition, there is no longer an investment pothole inasmuch as investment remains above its steady-state level even after the expensing allowance comes off (though we still obtain a sharp drop in the level of investment—and thus a reduction in its growth rate—in the period that the allowance expires). Once again, the source of this more muted response of investment is the endogenous response of real interest rates and consumption to changes in investment demand.[18] In general equilibrium, higher aggregate demand pushes up real interest rates (this is needed in order to call forth more saving), which acts to attenuate the increase in investment and the capital stock. Then, when the expensing allowance comes off, the resulting decline in aggregate demand is partly buffered by a reduction in real rates. Both of these factors imply that the resulting overcapacity (and desire to disinvest) is not as severe.

It is worth noting that very little investment is pulled forward under a temporary allowance in the general-equilibrium case (this can be seen from the leftmost plot in panel B of the figure, which also plots the response of capital following a permanent expensing allowance). Put differently, the usual conclusion that a temporary investment tax incentive will have a greater (short-term) effect on investment than a permanent tax change—an insight that is readily drawn from the partial-equilibrium framework—need not be correct once general-equilibrium considerations are taken into account.[19]

3.3 Expensing Allowances When Prices are Sticky

Up to this point, we have examined a version of the general-equilibrium model in which prices were assumed to be fully flexible (this was done in order to permit a direct comparison with the partial-equilibrium setup). We now assume that prices are sticky by incorporating the log-linearized aggregate supply relation (equation 25) into the model with costly capital adjustment.

Panel A of Figure 5 plots the responses of capital, consumption and investment, and the real rental rate from this model following the introduction of a three-year, 30 percent expensing allowance. Comparison with panel B of Figure 4 reveals that

[18] Note that, under our calibration, the contribution of consumption to output growth (which here is analogous to nonfarm business output) is a little less than four times as great as that for investment.

[19] This is not, of course, a completely general result: As Auerbach (1989) demonstrates, the differential effects on investment of temporary and permanent tax changes (or any change to the cost of capital) depends on the nature of the adjustment-cost function. However, our result obtains for all of the adjustment-cost specifications that we consider under reasonable calibrations. Moreover, there is invariably a pronounced difference between the partial- and general-equilibrium predictions of the model, which is the point that we are seeking to establish.

adding sticky prices changes the model's predictions in several ways. First, the response of investment is greater than in the flexible-price case (though it is still smaller than the investment response from the partial-equilibrium model). Second, investment now temporarily falls below its steady-state level after the expensing allowance expires (so in this sense, we once again obtain an investment pothole).

The intuition for both of these findings is relatively straightforward. Under sticky prices, firms commit to meeting all demand for their output at their fixed, posted price. Since output is partly demand-determined, there is less need for consumption to be crowded out through an increase in real interest rates, since a positive aggregate demand shock is partly met by increased supply. In addition, this makes firms more concerned with their capacity (now and in the future), since an increase in demand will cause a sharp rise in their real marginal costs—and, hence, a decline in their real profits—unless they increase their capital stock. Likewise, under sticky prices there is an incentive to disinvest more rapidly in the face of a slump in demand, since firms are not able to make up for a demand shortfall by cutting prices. Finally, the movements in the rental rate for capital reflect the interaction of these swings in capital demand with currently available capital supply.[20]

3.4 Alternative Adjustment Cost Specifications

In the preceding analysis, the presence of costly capital adjustment added an important forward-looking element to firms' and households' decisionmaking. However, recent work on investment dynamics has moved away from using this type of adjustment-cost mechanism to model the frictions facing firms when they seek to adjust their inputs. In this section, therefore, we consider two other specifications for factor-adjustment costs: one in which it is costly to adjust capital *and* the capital-output ratio (which can be thought of as a convex approximation to a putty-clay technology), and one in which it is costly to change the level of investment (which captures elements of time-to-build).[21]

[20]Sticky prices also affect the model's response to a *permanent* change in expensing allowances (not shown). As noted earlier, the fact that depreciation allowances are calculated using historical costs implies that the *nominal* interest rate has an independent influence on the cost of capital (by determining the present value of future depreciation allowances). A permanent expensing allowance yields a permanently higher level of the capital stock, which in turn implies permanently lower marginal costs and persistently lower inflation. As a result, nominal interest rates and the cost of capital both decline, which generates a larger eventual response of the capital stock.

[21]See section A.3 of the Appendix for the log-linearized versions of these equations.

Costly factor-ratio adjustment: Under costly factor-ratio adjustment, the production technology in the intermediate-goods sector becomes

$$Y_t^j = \left(H_t^j\right)^{1-\alpha} \left(K_t^j\right)^{\alpha} \exp\left[-\frac{\chi^f}{2}\left(\frac{K_t^j/H_t^j}{K_{t-1}^j/H_{t-1}^j} - 1\right)^2\right] - FC \tag{27}$$

(again, we also assume that it is costly to adjust the capital stock as well as the factor mix).[22] Panel B of Figure 5 plots the results from this version of the model. The additional source of inflexibility implies that there is now less benefit from adjusting the capital stock independently; as a result, the investment response in this version of the model is more muted than what we obtained in the model with capital-adjustment costs only (in particular, investment now reaches its peak a little earlier, and also remains above its steady-state level after the expensing allowance comes off). However, we also now see a much larger swing in the rental rate for capital than before. Previously, firms facing changes in demand for their output were able to change their production by altering the amount of labor they hired. Here, though, this avenue is partly closed off, as it is now also costly to adjust labor inputs; the result is a more pronounced swing in demand for installed capital, which shows up as a relatively larger change in the rental rate.[23]

Investment adjustment costs: Both the capital and factor-ratio adjustment-cost specifications imply a sharp initial jump in investment spending when expensing allowances are first introduced. In practice, however, firms' ability to rapidly change their capital expenditure plans is likely to be hampered by the presence of time-to-plan and time-to-build considerations. One straightforward way to capture this is to assume that *investment*—rather than the capital stock—is costly to adjust.[24] We do this by assuming the following form for the capital evolution equation:

$$K_{t+1}^i = (1-\kappa)K_t^i + I_t^i \exp\left[-\frac{\chi^a}{2}\left(\frac{I_t^i}{I_{t-1}^i} - 1\right)^2\right], \tag{28}$$

[22]We set χ^f in equation (27) equal to 25, which roughly halves the swing in the capital-labor ratio that occurs around the expiration date of the temporary expensing allowance.

[23]Note, however, that the responses of pre- and posttax nominal interest rates (not shown) are considerably smoother than the response of the rental rate, which reflects swings in the marginal product of installed capital and the markup of prices over marginal costs.

[24]Investment adjustment costs have been used by a number of authors; see Christiano, Eichenbaum, and Evans (2001) for a recent example.

where the adjustment cost parameter χ^a is set equal to 1.75 in order to match the estimated value found in Edge, Laubach, and Williams (2003). Panel C of Figure 5 plots results from this version of the model. The investment response in this case is smoother and shows a more pronounced hump (with the peak of the hump occurring about a year before the expensing allowance expires); in addition, the decline in investment that occurs immediately after the expiration of the expensing allowance is not as sharp and is spread over a longer period of time. Similarly, the swings in the rental rate are more muted as well.

4 Additional Extensions to the Baseline Model

In addition to the specific form of adjustment costs we assume, several other features of our model can conceivably affect the predicted response of investment to a change in tax policy. For example, the responses of saving and hours worked to changes in the real interest rate will obviously influence the response of investment spending to a tax shock; similarly, the independent role of the *nominal* interest rate on capital demand (which arises as a result of the nominal character of depreciation allowances) yields an additional way in which our characterization of the model economy's aggregate supply relation affect the model's predicted responses. Finally, a less obvious aspect of the model's specification that turns out to have an interesting effect on our results is our implicit assumption that labor and capital can be used to produce either consumption or investment goods (which in turn reflects the single-good nature of our baseline theoretical framework). In this section, therefore, we consider how our results are affected by employing alternative specifications for household consumption and aggregate supply, and also extend the model to incorporate sector-specific factor inputs.

4.1 Modelling Habit Persistence in Consumption

Within the macromodelling literature, it has become increasingly common to advocate a specification for household preferences that yields habit persistence in consumption.[25] We add "external" habit persistence to our model by assuming the

[25] See Fuhrer (2000) for a representative example.

following alternative representation of preferences:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[\frac{1}{1-\sigma} \left(C_t^i - bC_{t-1} \right)^{1-\sigma} - \frac{1}{1+s} \left(H_t^i \right)^{1+s} \right] \right\}, \tag{29}$$

in the sticky-price model with investment adjustment costs.[26] (We use the model with investment adjustment costs as our jumping-off point because we view its real responses—specifically, the hump-shaped response of output and investment that obtains following a shock—to be the most realistic.) The new first-order conditions for the household's consumption and labor supply decisions are given in the Appendix; in log-linearized form, the consumption Euler equation is given by

$$\left(\frac{1}{1-b} \right) (c_t - bc_{t-1}) = \left(\frac{1}{1-b} \right) (E_t c_{t+1} - bc_t) - \frac{1}{\sigma} \left(r_t^f - E_t \pi_{t+1} \right) \tag{30}$$

and the household's labor supply curve is

$$w_t = \frac{F_*^h}{1-F_*^h} f_t^h + \frac{\sigma}{1-b} (c_t - bc_{t-1}) + s \cdot h_t. \tag{31}$$

Panel A of Figure 6 plots the responses to a three-year, 30 percent temporary expensing allowance from this alternative version of the model against the baseline model's responses. As is evident from the figure, incorporating habit persistence has essentially no effect on the predicted path for investment. The source of this result can be readily seen if we re-write the log-linearized labor supply curve in the following equivalent fashion:

$$w_t = \frac{F_*^h}{1-F_*^h} f_t^h + \frac{\sigma}{1-b} ((1-b)c_t + b\Delta c_t) + s \cdot h_t \tag{32}$$

(note that we are making use here of the identity $c_t \equiv (1-b)c_t + bc_t$). In the baseline model *without* habit persistence, the response of consumption is already relatively smooth (see panel C of Figure 5). As a result, $\Delta c_t \approx 0$, which implies that equation (32) approximately reduces to the labor supply curve from the baseline model. Hence, there is little scope for habit persistence to influence the labor supply decision—and, thus, the response of the real economy—in this context.

[26] This particular specification of habit formation is taken from Christiano, Eichenbaum, and Evans (2001). We set b in equation (29) equal to 0.8, which implies a relatively large degree of habit persistence.

4.2 More Inertial Price-Setting

The new-Keynesian Phillips curve that we employ in the baseline model has been criticized on the grounds that it implies a too-rapid response of inflation to real shocks. We therefore gauge the influence of this assumption on our results by considering a "hybrid" Phillips curve (due to Christiano, Eichenbaum, and Evans, 2001) in which inflation is partially indexed to its own lag.[27] In log-linearized form, this aggregate supply relation is given as:

$$\pi_t = \frac{1}{1+\delta}\pi_{t-1} + \frac{\delta}{1+\delta}E_t\pi_{t+1} + \frac{(1-\eta)(1-\eta\delta)}{(1+\delta)\eta}mc_t. \tag{33}$$

Panel B of the figure gives the results from this version of the model. In broad terms, the predicted responses for investment are quite similar across the two specifications: Although the hybrid inflation equation does in fact yield a smaller *initial* response of inflation (not shown), after a few quarters the path of the inflation rate is similar to what is obtained in the baseline model (in addition, because we assume that the monetary authority tries to smooth its policy rate, the path of nominal interest rates is also quite similar across the two models). However, because the hybrid Phillips curve imparts more inertia to price setting, the path of inflation (and nominal interest rates) remains higher over a longer period in the alternative model. As a result, the response of investment is attenuated slightly relative to the baseline case.[28]

4.3 Multisector Production with Limited Factor Mobility

Up to this point, the models that we have examined have all implicitly assumed a one-sector production structure in which labor and (existing) capital can be instantaneously and costlessly allocated to the production of either consumption or capital goods. As a result, a large portion of any increase in investment demand in our baseline model is accommodated by an increase in output, as households supply more hours to the economy's single production sector.

[27]We would not want to leave the impression that we are advocating this model of price-setting behavior, since there is compelling evidence that it too has difficulty in capturing observed inflation dynamics (see Rudd and Whelan, 2003, for a discussion). Rather, our motivation for employing this specification stems from its representing the most commonly cited alternative to a purely forward-looking inflation equation.

[28]Recall that nominal interest rates have an independent influence on investment through their effect on the present discounted value of depreciation allowances.

A more realistic production structure would involve separate sectors for the production of consumption and investment goods and would take into account the fact that capital and labor inputs tend to be sector specific (particularly over short horizons). In such an economy, it will be more difficult to rapidly increase production in a given sector; in particular, we would expect the rise in investment demand that results from the introduction of a temporary expensing allowance to be only partially met. As a result, we will tend to see a slower response of aggregate investment to a change in tax policy.

We model sector-specific labor inputs by assuming that households incur a (convex) adjustment cost whenever they change the number of hours that they supply to the consumption or investment sector; this implies the following alternative representation for household preferences:

$$U_0 = E_0 \sum_{t=0}^{\infty} \delta^t \left[\frac{1}{1-\sigma} \left(C_t^i \right)^{1-\sigma} - \frac{1}{1+s} (H_t^{c,i} + H_t^{k,i})^{1+s} \exp \left[\frac{\chi^l}{2} \left(\frac{H_t^{c,i}/H_t^{k,i}}{H_{t-1}^{c}/H_{t-1}^{k}} - 1 \right)^2 \right] \right], \tag{34}$$

where the k and c superscripts on hours index the investment- and consumption-goods sectors, respectively. We set the adjustment-cost parameter χ^l equal to a relatively low value (namely unity).[29]

We allow for sector-specific capital by assuming distinct accumulation processes for the capital stocks employed in the consumption- and investment-goods sectors. Specifically, household i's holdings of sector-n capital evolve according to

$$K_{t+1}^{i,n} = (1-\kappa) K_t^{i,n} + I_t^{i,n} \exp \left[-\frac{\chi^a}{2} \left(\frac{I_t^{i,n}}{I_{t-1}^{i,n}} - 1 \right)^2 \right] \quad (n = k, c), \tag{35}$$

which, except for the n superscript, has an identical form to the investment adjustment-cost specification from the one-sector model (equation 28).[30]

Panel C of Figure 6 plots the model's response to a three-year, 30 percent expensing allowance. As is evident from this panel, the presence of sector-specific factor

[29] This calibration, along with the other parameter values we assume, implies that wages in the consumption-goods sector must move 1-3/4 percent above wages in the capital-goods sector in order to have one percent of the economy's aggregate labor supply shift into the production of consumption goods (and *vice-versa*).

[30] The household faces a slightly different budget constraint in this version of the model; we describe it in detail in the Appendix. In addition, note that firms' production functions will now reflect the sector-specific nature of factor inputs.

supplies further mutes investment's response to a temporary expensing allowance relative to the baseline model.

5 Effects of Changes in the Capital Income Tax Rate

An alternative policy that is often suggested as a means of stimulating investment spending involves reducing the tax rate on capital income. We next examine the effect of this policy on investment and output, and compare it with the effect of an expensing allowance that has the same impact on government revenues.

Model responses: Figure 7 plots the usual set of model responses (under several different adjustment-cost specifications) following a three-year, 20 percentage point reduction in the capital tax rate F^k. Qualitatively, a temporary cut in the capital tax rate yields a path for investment spending that is much more front-loaded than the path that obtains under an expensing allowance. The reason is that the benefits from a reduction in capital taxes are received for as long as the policy is in place; as a result, purchasing and holding a unit of capital for the full three-year period yields the greatest gains. By contrast, an expensing allowance represents a one-time boon (in the quarter that the capital is purchased) that is worth roughly as much at the start of the three-year period as toward the end.

Revenue Impact of Alternative Tax Policies: One of the most useful features of our model is its ability to assess the revenue consequences of alternative tax policies—in particular, we can compare the investment responses induced by a capital tax cut and an expensing allowance, where each policy is constrained to have an identical impact on government revenue.

In Figure 8, we compare the effect of a temporary capital tax cut with that of a temporary expensing allowance, where each policy is set so as to yield the same change in the present value of government revenues.[31] (We focus on the variant of the baseline model of Section 3 that incorporates investment adjustment costs; results for the other specifications are similar.) The present value is computed over a ten-year (or 40-quarter) period—this corresponds to the width of the "budget window" that is typically used to score the revenue effects of fiscal policy changes—

[31] Specifically, we compare a 30 percent temporary (three-year) expensing allowance with a 19.5 percentage point three-year reduction in the capital income tax.

using the following expression,

$$pdv^{rev}(10)_t = \left(\frac{1-\delta}{1-\delta^{40}}\right) E_t \left[rev_t - \sum_{v=1}^{39} \delta^v \left(rev_{t+v} - \sum_{j=0}^{v-1} (r^f_{t+j} - \pi_{t+1+j}) \right) \right], \quad (36)$$

which corresponds to a log-linearized, finite-period version of equation (11). As can be seen from the figure, the expensing allowance yields a uniformly higher response of investment (and output). Intuitively, since a capital tax applies to the income from *all* capital while an expensing allowance applies to expenditures on new capital only, the former represents a relatively expensive way to call forth additional investment spending.[32]

6 Conclusions and Directions for Future Work

This paper has attempted to analyze tax-based investment incentives in the context of a fully specified general-equilibrium model. Our analysis revealed two rather surprising results, and confirmed another that had been well-established by previous researchers working in a partial-equilibrium framework.

- First, our findings highlight the need to take general-equilibrium considerations seriously when thinking about the impact of tax incentives on investment. For example, the standard (and intuitive) view that temporary changes in expensing allowances induce a larger response of investment spending than do permanent changes turns out to depend on whether saving is endogenous.

- Second, while the development of models that incorporate habit persistence and alternative specifications for price-setting behavior has led to important improvements in our ability to credibly assess *monetary* policy, we find that these extensions carry considerably less importance in the context of fiscal policy evaluation. By contrast, whether we model the multisector nature of production—which receives almost no attention in the monetary policy literature—turns out to have a relatively important effect on the model's response to a tax change. In intuitive terms, the fiscal shocks that we are

[32] Note also that the capital stock is much higher at the end of the three-year period under the expensing allowance. Thus, if the revenue consequences of each policy were considered over a longer (or infinite) period, expensing allowances would appear even more attractive.

considering here are more like real disturbances, so the model's responses—and the determinants of its dynamics—are often more akin to those of a *real* (as opposed to monetary) business cycle model.

- Finally, our analysis allowed us to confirm the result—previously only considered in a partial-equilibrium setup—that for policies with the same effect on the present value of government revenues, a change in the rate of capital taxation represents a relatively less efficacious way of stimulating investment than does a change in the expensing rate for newly purchased capital.

A natural next step is to refine the framework developed here into one that can be used for *quantitative* simulations. Attaining this goal would require advancing our analysis along at least three fronts. First, any serious quantitative assessment of expensing allowances must recognize the fact that these tax provisions pertain to equipment investment only. Constructing a fully specified model in which different types of capital are used in production would require making difficult decisions about the degree of substitutability across capital types. However, for the purposes of short-term analysis, it might be sufficient to consider a model in which the stock of structures is assumed to be fixed; this would permit the model to generate more realistic predicted responses of output to tax-induced changes in (equipment) capital without requiring us to explicitly model the investment decision for structures.

Extending the baseline model to an open-economy setting would also represent an important refinement. With no external sector, the endogenous response of the real interest rate is larger following a tax-induced change in investment, since only domestically produced output can be used to meet the additional demand for physical capital. For the U.S. economy, this might be a reasonable first approximation (though see Auerbach, 1989, for a contrary view), as only about a third of the equipment purchased for investment in the U.S. is produced abroad. Nevertheless, an explicit treatment of external considerations in this context—while difficult given the current state of open-economy dynamic general-equilibrium modelling—would yield a framework with even greater practical relevance.

More fundamentally, any model that purports to inform real-world decisionmaking should be able to demonstrate a reasonable degree of empirical validity. For the application considered here, formal empirical justification is likely to be complicated by the fact that the effect of tax changes on investment—let alone on interest rates, consumption, and inflation—is probably very difficult to parse out; moreover, relatively few historical examples of these sorts of tax changes exist. This suggests that considering tax changes alone will not allow us to identify all of the model's parameters (though it might be possible to estimate these parameters by examining the model's predicted response to other shocks).

Finally, an additional useful extension would involve constructing an apparatus that would permit the assessment of uncertain future policies. In practice, the likelihood and/or length of proposed tax policies are typically not known with certainty, and this should act to attenuate the economy's response to announced changes in tax policy. Whether the effects of such uncertainty could be quantified in a linear framework, however, is far from clear.

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A Detailed Model Derivations

This section of the Appendix gathers together the first-order conditions from the baseline model of section 2 (and its later versions) that are not discussed in the text, and explicitly describes the model's equilibrium and steady-state solution.

A.1 Omitted First-Order Conditions

The **intermediate-goods producers'** cost-minimization problem (6) yields factor demand schedules for each firm; these have the form:

$$H_t^j = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(Y_t^j + FC\right) \left(\frac{R_t^k/P_t}{W_t/P_t}\right)^{\alpha} \text{ and} \tag{37}$$

$$K_t^j = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(Y_t^j + FC\right) \left(\frac{W_t/P_t}{R_t^k/P_t}\right)^{1-\alpha}. \tag{38}$$

In addition, this problem implies a marginal cost function (which is identical for all firms) that is given by:

$$\frac{MC_t^j}{P_t} = \left(\frac{W_t/P_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t^k/P_t}{\alpha}\right)^{\alpha}. \tag{39}$$

An intermediate-goods producing firm that is able to reset its price in period t takes as given its nominal marginal cost MC_t^j, the aggregate price level P_t, and aggregate output Y_t and solves:

$$\max_{\{P_t^j\}} \sum_{k=0}^{\infty} \eta^k E_t \left[\frac{\delta^k MU_{t+k}/P_{t+k}}{MU_t/P_t} \left(\left(P_t^j - MC_{t+k}^j\right) Y_{t+k}^j - P_t FC\right)\right] \tag{40}$$

such that

$$Y_{t+k}^j = Y_{t+k} \left(\frac{P_t^j}{P_{t+k}}\right)^{-\theta}, \tag{41}$$

where MU_t denotes the marginal utility of consumption. This implicitly defines an optimal price P_t^j for firms who do change their prices in period t, which is expressed as:

$$P_t^j = \frac{\sum_{k=0}^{\infty} \eta^k E_t \left[((\delta^k MU_{t+k}/P_{t+k})/(MU_t/P_t)) MC_{t+k}^j \theta Y_{t+k}\right]}{\sum_{k=0}^{\infty} \eta^k E_t \left[((\delta^k MU_{t+k}/P_{t+k})/(MU_t/P_t))\left(\theta - 1\right) Y_{t+k}\right]}. \tag{42}$$

The **final-good producing firm's cost-minimization problem** (equation 7) yields a demand function for each of the intermediate goods:

$$Y_t^j = Y_t \left(P_t^j/P_t\right)^{-\theta}. \tag{43}$$

The demand functions for the intermediate goods imply that the competitive price P_t for the final (actual) good is defined implicitly as:

$$P_t = \left(\int_0^1 (P_t^j)^{1-\theta} dz \right)^{\frac{1}{1-\theta}} . \tag{44}$$

The economy's **goods-market clearing** condition implies that $C_t + I_t = Y_t$, where I_t denotes actual spending on capital goods.

The first-order condition for **capital supply** for the model with **capital adjustment costs** is given by:

$$\begin{aligned} & E_t \left[\frac{R^k_{t+1}\left(1-F^k_{t+1}\right)}{P_{t+1}} \right] \qquad (45) \\ = \; & E_t \left[\frac{R^f_t}{\Pi_{t+1}} \left(1-\exp\left[-\frac{\chi^k}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 \right] \left(F^k_t X_t + PDV^\kappa_t (1-X_t) \right) \right) \right. \\ & \left. \times \exp\left[\frac{\chi^k}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 \right] \left(1 + \chi^k \left(\frac{K_{t+1}}{K_t} - (1-\kappa) \right) \left(\frac{K_{t+1}}{K_t} - 1 \right) \right) \right] \\ & + E_t \left[\frac{R^f_t}{\Pi_{t+1}} \left(F^k_t X_t + PDV^\kappa_t (1-X_t) \right) \chi^k \left(\frac{K_{t+1}}{K_t} - (1-\kappa) \right) \left(\frac{K_{t+1}}{K_t} - 1 \right) \right] \\ & - E_t \left[\left(1-\exp\left[-\frac{\chi^k}{2} \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 \right] \left(F^k_{t+1} X_{t+1} + PDV^\kappa_{t+1} (1-X_{t+1}) \right) \right) \right. \\ & \left. \times \exp\left[\frac{\chi^k}{2} \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 \right] \left((1-\kappa) + \chi^k \left(\frac{K_{t+2}}{K_{t+1}} - (1-\kappa) \right) \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} \right) \right] \\ & - E_t \left[\left(F^k_{t+1} X_{t+1} + PDV^\kappa_{t+1} (1-X_{t+1}) \right) \chi^k \left(\frac{K_{t+2}}{K_{t+1}} - (1-\kappa) \right) \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} \right] \end{aligned}$$

while the corresponding expression for capital supply under **investment adjustment costs** is given by:

$$\begin{aligned} E_t \left[\frac{R^k_{t+1}(1-F^k_{t+1})}{P_{t+1}} \right] = \; & E_t \left[\frac{R^f_t}{\Pi_{t+1}} \left(\frac{Q_t}{P_t} - F^k_t X_t - PDV^\kappa_t (1-X_t) \right) \right] \\ & - E_t \left[(1-\kappa) \left(\frac{Q_{t+1}}{P_{t+1}} - F^k_{t+1} X_{t+1} - PDV^\kappa_{t+1} (1-X_{t+1}) \right) \right] \end{aligned}$$

where

$$\begin{aligned} & E_t \left[\frac{Q_{t+1}}{P_{t+1}} I_{t+1} \exp\left[-\frac{\chi^i}{2} \left(\frac{I_{t+1}}{I_t} - 1 \right)^2 \right] \chi^i \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} - 1 \right) \right] \\ & \quad = E_t \left[\frac{R^f_t}{\Pi_{t+1}} \left(I_t - \frac{Q_t}{P_t} I_t \exp\left[-\frac{\chi^i}{2} \left(\frac{I_{t+1}}{I_t} - 1 \right)^2 \right] \left(1 - \chi^i \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} - 1 \right) \right) \right) \right] . \end{aligned}$$

Finally, when it is costly to adjust factor ratios (*i.e.*, the production function takes the form of equation 27), the first-order conditions from the intermediate-good producing firm's cost-minimization problem become:

$$
\begin{aligned}
&\frac{(W_t/P_t)H_t^j}{(MC_t/P_t)(Y_t^j+FC)} - (1-\alpha) \\
&\quad = \chi^f \left(\frac{K_t^j/H_t^j}{K_{t-1}^j/H_{t-1}^j} - 1\right)\left(\frac{K_t^j/H_t^j}{K_{t-1}^j/H_{t-1}^j}\right) \\
&\qquad -\chi^f E_t\left[\frac{\Pi_{t+1}}{R_t^f}\cdot\frac{(MC_{t+1}/P_{t+1})(Y_{t+1}^j+FC)}{(MC_t/P_t)(Y_t^j+FC)}\left(\frac{K_{t+1}^j/H_{t+1}^j}{K_t^j/H_t^j}-1\right)\left(\frac{K_{t+1}^j/H_{t+1}^j}{K_t^j/H_t^j}\right)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{(R_t^k/P_t)K_t^j}{(MC_t/P_t)(Y_t^j+FC)} - \alpha \\
&\quad = -\chi^f \left(\frac{K_t^j/H_t^j}{K_{t-1}^j/H_{t-1}^j} - 1\right)\left(\frac{K_t^j/H_t^j}{K_{t-1}^j/H_{t-1}^j}\right) \\
&\qquad +\chi^f E_t\left[\frac{\Pi_{t+1}}{R_t^f}\cdot\frac{(MC_{t+1}/P_{t+1})(Y_{t+1}+FC)}{(MC_t/P_t)(Y_t+FC)}\left(\frac{K_{t+1}^j/H_{t+1}^j}{K_t^j/H_t^j}-1\right)\left(\frac{K_{t+1}^j/H_{t+1}^j}{K_t^j/H_t^j}\right)\right].
\end{aligned}
$$

A.2 Steady-State Equilibrium

In deriving the model's steady-state equilibrium, we first note that the steady-state value of the inflation rate, Π_*, is assumed to equal the central bank's inflation target, $\bar{\Pi}$. The steady-state values of all other variables in the model are functions of the model's parameters as well as of the steady-state inflation rate and the steady-state value of the tax variables (F_*^h, F_*^k, and X_*).

From equations (3) and (14), the steady-state pretax and posttax nominal interest rates are given by:

$$R_* = \left(\frac{\bar{\Pi}}{\delta} - F_*^h\right)\frac{1}{1-F_*^h} \quad \text{and} \tag{46}$$

$$R_*^f = \frac{\bar{\Pi}}{\delta}. \tag{47}$$

The steady-state value of real marginal cost is given by the inverse of the markup, while equations (16) and (39) imply that the steady-state values of the factor prices are given by:

$$\frac{MC_*^j}{P_*} = \frac{MC_*}{P_*} = \frac{\theta - 1}{\theta}, \tag{48}$$

$$\frac{R_*^k}{P_*} = \left(\frac{1 - PDV_*^{\kappa}}{1 - F_*^k}\right)\left(\frac{1}{\delta} - (1-\kappa)\right), \text{ and} \tag{49}$$

$$\frac{W_*}{P_*} = (1-\alpha)\left(\frac{MC_*}{P_*}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{R_*^k/P_*}\right)^{\frac{\alpha}{1-\alpha}}. \tag{50}$$

The variable PDV_*^{κ} is equal to either

$$PDV_*^{\kappa} = F_*^k \cdot \frac{\kappa}{\frac{\bar{\Pi}}{\delta} - (1-\kappa)} \quad \text{or} \quad PDV_t^{\kappa^{irs}} = F_*^k \sum_{v=1}^{V}\left(\frac{\delta}{\bar{\Pi}}\right)^v \kappa_v^{irs},$$

depending on whether we use economic depreciation or the legislated tax schedule for depreciation allowances. Implicit in the definition of R_*^k/P_* is the assumption that there are no expensing allowance provisions in the steady-state (which characterizes the U.S. tax code since 1986); as a result, $X_* = 0$ and $X_t - X_* = X_t$.

The steady-state ratios $\frac{H_*^j}{Y_*} = \frac{H_*}{Y_*}$, $\frac{K_*}{Y_*}$, $\frac{I_*}{Y_*}$, and $\frac{C_*}{Y_*}$ can be derived from equations (4), (37), (38), and the market-clearing condition. This yields:

$$\frac{H_*^j}{Y_*} = \frac{H_*}{Y_*} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{R_*^k/P_*}{W_*/P_*}\right)^{\alpha}, \tag{51}$$

$$\frac{K_*}{Y_*} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{W_*/P_*}{R_*^k/P_*}\right)^{1-\alpha}, \tag{52}$$

$$\frac{I_*}{Y_*} = \kappa \cdot \frac{K_*}{Y_*} = \kappa\left(\frac{\theta}{\theta-1}\right)\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{W_*/P_*}{R_*^k/P_*}\right)^{1-\alpha}, \tag{53}$$

$$\frac{C_*}{Y_*} = 1 - \frac{I_*}{Y_*} = 1 - \kappa\left(\frac{\theta}{\theta-1}\right)\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{W_*/P_*}{R_*^k/P_*}\right)^{1-\alpha}. \tag{54}$$

Equations (51) and (54), together with the steady-state version of equation (15), yield the steady-state solution for real output:

$$\begin{aligned} Y_* &= \frac{(W_*/P_*)^{\frac{1}{\sigma+s}}(1-F_*)^{\frac{1}{\sigma+s}}}{(H_*/Y_*)^{\frac{s}{\sigma+s}}(C_*/Y_*)^{\frac{\sigma}{\sigma+s}}} \qquad (55) \\ &= \frac{(W_*/P_*)^{\frac{1}{\sigma+s}}(1-F_*)^{\frac{1}{\sigma+s}}}{\left(\frac{\theta}{\theta-1}\right)^{\frac{s}{\sigma+s}}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{s\alpha}{\sigma+s}}\left(\frac{R_*^k/P_*}{W_*/P_*}\right)^{\frac{s\alpha}{\sigma+s}}\left(1-\kappa\left(\frac{\theta}{\theta-1}\right)\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{W_*/P_*}{R_*^k/P_*}\right)^{1-\alpha}\right)^{\frac{\sigma}{\sigma+s}}}. \end{aligned}$$

Together with equations (51) through (54), equation (56) yields solutions for the steady-state values of H_*, K_*, I_*, and C_*.

Finally, in the steady state real revenue is:

$$\frac{Rev_*}{P_*} = F_*^h Y_* + (F_*^k - F_*^h)\frac{R_*^k}{P_*}K_* - F_*^k \frac{Liab_*^\kappa}{P_*}, \tag{56}$$

where real depreciation allowance liabilities are

$$\frac{Liab_*^\kappa}{P_*} = \frac{\kappa}{\bar{\Pi} - (1-\kappa)} I_*, \tag{57}$$

when we assume that firms deduct true economic depreciation and

$$\frac{Liab_*^\kappa}{P_*} = \sum_{v=1}^{V} \kappa_v^{irs} \left(\frac{1}{\bar{\Pi}}\right)^v I_*, \tag{58}$$

when deductions follow the legislated schedule of allowances. The steady-state present discounted value of real revenues is given by:

$$\frac{PDV_*^{rev}}{P_*} = \left(\frac{1}{1-\delta}\right)\frac{Rev_*}{P_*}.$$

A.3 Additional Log-Linearized Model Equations

The log-linear expression for economy-wide **marginal cost** is

$$mc_t = (1-\alpha)\, w_t + \alpha\, r_t^k. \tag{59}$$

The log-linear **government revenue** expression is given by

$$rev_t = \frac{F_*^h Y_*}{Rev_*/P_*}\left(f_t^h + y_t\right) + \frac{(F_*^k - F_*^h)(R_*^k/P_*)K_*}{Rev_*/P_*}\left(r_t^k + k_t\right) + \frac{F_*^k (R_*^k/P_*)K_*}{Rev_*/P_*} f_t^k$$
$$- \frac{F_*^h (R_*^k/P_*)K_*}{Rev_*/P_*} f_t^h - \frac{F_*^k I_*}{Rev_*/P_*} X_t - \frac{F_*^k (Liab_*^\kappa/P_*)}{Rev_*/P_*}\left(f_t^k + liab_t^\kappa\right),$$

where

$$liab_t^\kappa = \left(\frac{\bar{\Pi} - (1-\kappa)}{\bar{\Pi}}\right) I_* \left(i_{t-1} - \pi_t - X_{t-1}\right) + \left(\frac{\bar{\Pi} - (1-\kappa)}{\bar{\Pi}}\right)\left(liab_{t-1}^\kappa - \pi_t\right),$$

for the case where economic depreciation is used to compute firms' depreciation allowances, and

$$liab_t^{\kappa^{irs}} = \frac{1}{\sum_{v=1}^{V}\left(\frac{1}{\bar{\Pi}}\right)^v \kappa_v^{irs}} \sum_{v=1}^{V}\left(\frac{1}{\bar{\Pi}}\right)^v \kappa_v^{irs}(i_{t-v} - X_{t-v})$$
$$+ \frac{1}{\sum_{v=1}^{V}\left(\frac{1}{\bar{\Pi}}\right)^v \kappa_v^{irs}} \sum_{w=1}^{V}\left(\sum_{v=w}^{V}\left(\frac{1}{\bar{\Pi}}\right)^v \kappa_v^{irs}\right)\pi_{t+w-1}^k \tag{60}$$

for the case where legislated depreciation rates are used. We can then log-linearize equation (11), which yields:

$$pdv_t^{rev} = (1-\delta)rev_t - \delta(r_t - E_t\pi_{t+1}) + \delta E_t pdv_{t+1}^{rev}. \tag{61}$$

In addition, note that if depreciation rates for tax purposes are given by a legislated schedule (and, so, not equivalent to economic depreciation), then the expression for the log-linearized present value of depreciation allowances (equation 22) becomes

$$pdv_t^{\kappa^{irs}} = \frac{1}{\sum_{v=1}^{V}\left(\frac{\delta}{\bar{\Pi}}\right)^v \kappa_v^{irs}} \sum_{v=1}^{V}\left(\frac{\delta}{\bar{\Pi}}\right)^v \kappa_v^{irs} E_t f_{t+v}^k \tag{62}$$

$$+ \frac{1}{\sum_{v=1}^{V}\left(\frac{\delta}{\bar{\Pi}}\right)^v \kappa_v^{irs}} \sum_{w=1}^{V}\left(\sum_{v=w}^{V}\left(\frac{\delta}{\bar{\Pi}}\right)^v \kappa_v^{irs}\right) E_t r_{t+w-1}^k. \tag{63}$$

For the model without capital or investment adjustment costs, the capital supply condition yields the following log-linear expression for the **user cost**:

$$\begin{aligned} E_t r_{t+1}^k = & \left[\frac{F_*^k}{1-F_*^k}\right] f_{t+1}^k + \left[\frac{1}{1-\delta(1-\kappa)}\right]\left(r_t^f - E_t\pi_{t+1}\right) \\ & - \left[\frac{1}{1-\delta(1-\kappa)} \cdot \frac{PDV_*^\kappa}{1-PDV_*^\kappa}\right](pdv_t^\kappa - \delta(1-\kappa)E_t pdv_{t+1}^\kappa) \\ & - \left[\frac{1}{1-\delta(1-\kappa)} \cdot \frac{F_*^k - PDV_*^\kappa}{1-PDV_*^\kappa}\right](X_t - \delta(1-\kappa)E_t X_{t+1}). \end{aligned} \tag{64}$$

When **investment adjustment costs** are present, the corresponding expression for the user cost is given by:

$$\begin{aligned} E_t r_{t+1}^k = & \left[\frac{F_*^k}{1-F_*^k}\right] f_{t+1}^k + \left[\frac{1}{1-\delta(1-\kappa)}\right]\left(r_t^f - E_t\pi_{t+1}\right) \\ & - \left[\frac{1}{1-\delta(1-\kappa)} \cdot \frac{PDV_*^\kappa}{1-PDV_*^\kappa}\right](pdv_t^\kappa - \delta(1-\kappa)E_t pdv_{t+1}^\kappa) \\ & - \left[\frac{1}{1-\delta(1-\kappa)} \cdot \frac{F_*^k - PDV_*^\kappa}{1-PDV_*^\kappa}\right](X_t - \delta(1-\kappa)E_t X_{t+1}) \\ & + \left[\frac{1}{1-\delta(1-\kappa)} \cdot \frac{1}{1-PDV_*^\kappa}\right](q_t - \delta(1-\kappa)E_t q_{t+1}), \text{ and} \end{aligned} \tag{65}$$

$$q_t = \chi^a(1+\delta)i_t - \chi^a \cdot i_{t-1} - \chi^a \cdot E_t\delta i_{t+1}. \tag{66}$$

while the log-linearized first-order conditions under **costly factor-ratio adjustment** are:

$$
\begin{aligned}
\left(h_t - \left(\frac{\theta-1}{\theta}\right) y_t\right) &= \frac{\chi^f}{1+(1+\delta)\chi^f}\left(h_{t-1} - \left(\frac{\theta-1}{\theta}\right) y_{t-1}\right) \\
&+ \frac{\delta\chi^f}{1+(1+\delta)\chi^f} E_t\left(h_{t+1} - \left(\frac{\theta-1}{\theta}\right) y_{t+1}\right) \\
&- \frac{\alpha}{1+(1+\delta)\chi^f}(w_t - r_t^k), \text{ and} \\
\left(k_t - \left(\frac{\theta-1}{\theta}\right) y_t\right) &= \frac{\chi^f}{1+(1+\delta)\chi^f}\left(k_{t-1} - \left(\frac{\theta-1}{\theta}\right) y_{t-1}\right) \\
&+ \frac{\delta\chi^f}{1+(1+\delta)\chi^f} E_t\left(k_{t+1} - \left(\frac{\theta-1}{\theta}\right) y_{t+1}\right) \\
&+ \frac{1-\alpha}{1+(1+\delta)\chi^f}(w_t - r_t^k).
\end{aligned}
$$

Finally, the economy's **goods-market clearing condition** log-linearizes to

$$
y_t = \frac{C_*}{Y_*} c_t + \frac{I_*}{Y_*} i_t. \tag{67}
$$

A.4 Additional Equations for the Habit-Persistence Model

With habit-persistence the consumption Euler equation and labor supply curve become:

$$
\frac{1}{(C_t - bC_{t-1})^\sigma P_t} = \delta E_t\left[\frac{R_t^t}{(C_{t+1} - bC_t)^\sigma P_{t+1}}\right]
$$

and

$$
\frac{W_t(1-F_t^h)}{P_t} = H_t^s (C_t - bC_{t-1})^\sigma.
$$

A.5 Additional Equation for the Hybrid New-Keynesian Phillips Curve

Under the assumption that firms who cannot reset their prices index to lagged inflation equation (42) becomes:

$$
\left(MU_t\left(\frac{P_t^j}{P_t}\right)^{2-\theta}(\theta-1)\,Y_t + \sum_{k=0}^{\infty}\gamma^{k+1}\beta^{k+1}E_0\left[MU_{t+k+1}\left(\frac{P_t^j}{P_{t+k+1}}\prod_{l=0}^{k}\Pi_{t+l}\right)^{2-\theta}(\theta-1)\,Y_{t+k+1}\right]\right)^{-1}
$$
$$
\times\left(MU_t\frac{MC_t^j}{P_t}\left(\frac{P_t^j}{P_t}\right)^{1-\theta}\theta Y_t\right.
$$

$$+\sum_{k=0}^{\infty}\gamma^{k+1}\beta^{k+1}E_0\left[MU_{t+k+1}\frac{MC^j_{t+k+1}}{P_{t+k+1}}\left(\frac{P^j_t}{P_{t+k+1}}\prod_{l=0}^{k}\Pi_{t+l}\right)^{1-\theta}\theta Y_{t+k+1}\right]\Bigg) = 1. \tag{68}$$

When we log-linearize this, we obtain the hybrid new-Keynesian Phillips curve (equation 33).

A.6 Additional Equations for the Sector-Specific Factors Model

With two sectors, the labor and capital demand schedule, marginal cost function, and pricing equation all generalize in a straightforward manner (there is now one for each sector). The labor supply schedules are more complicated, however. For $H^{c,i}_t$ the first-order condition is given by:

$$\exp\left[\frac{\chi^l}{2}\left(\frac{H^{c,i}_t/H^{k,i}_t}{H^c_{t-1}/H^k_{t-1}}-1\right)^2\right] \tag{69}$$
$$\times\left(\left(H^{c,i}_t+H^{k,i}_t\right)^s+\frac{1}{1+s}\left(H^{c,i}_t+H^{k,i}_t\right)^{1+s}\chi^l\left(\frac{H^{c,i}_t/H^{k,i}_t}{H^c_{t-1}/H^k_{t-1}}-1\right)\frac{1/H^{k,i}_t}{H^c_{t-1}/H^k_{t-1}}\right),$$

while the first-order condition for $H^{k,i}_t$ is:

$$\exp\left[\frac{\chi^l}{2}\left(\frac{H^{c,i}_t/H^{k,i}_t}{H^c_{t-1}/H^k_{t-1}}-1\right)^2\right] \tag{70}$$
$$\times\left(\left(H^{c,i}_t+H^{k,i}_t\right)^s+\frac{1}{1+s}\left(H^{c,i}_t+H^{k,i}_t\right)^{1+s}\chi^l\left(\frac{H^{c,i}_t/H^{k,i}_t}{H^c_{t-1}/H^k_{t-1}}-1\right)\frac{H^{c,i}_t}{H^{k,i}_t}\cdot\frac{-1/H^{k,i}_t}{H^c_{t-1}/H^k_{t-1}}\right).$$

These log-linearize as follows:

$$w^c_t = \frac{F^h_*}{1-F^h_*}f^h_t+\sigma\cdot c_t+s\left(\frac{H^c_*}{H^c_*+H^k_*}\cdot h^c_t+\frac{H^k_*}{H^c_*+H^k_*}\cdot h^k_t\right) \tag{71}$$
$$+\frac{1}{1+s}\left(\frac{H^c_*+H^k_*}{H^c_*}\right)\chi^l\left(\left(h^c_t-h^k_t\right)-\left(h^c_{t-1}-h^k_{t-1}\right)\right)$$
$$w^k_t = \frac{F^h_*}{1-F^h_*}f^h_t+\sigma\cdot c_t+s\left(\frac{H^c_*}{H^c_*+H^k_*}\cdot h^c_t+\frac{H^k_*}{H^c_*+H^k_*}\cdot h^k_t\right)$$
$$+\frac{1}{1+s}\left(\frac{H^c_*+H^k_*}{H^k_*}\right)\chi^l\left(\left(h^k_t-h^c_t\right)-\left(h^k_{t-1}-h^c_{t-1}\right)\right).$$

B Calibrating the Steady-State Tax Rates

This portion of the Appendix describes how the effective tax rates on income are calibrated, and discusses how our main results are affected by different assumed values for the capital tax rate.

B.1 Calibration of F_*^h

We follow Edge and Rudd (2002) in using tabulations from the *Statistics of Income* (Table 3.4) to compute average marginal Federal tax rates on earned income. For 2001 (the most recent year for which these data are available), we obtain an average marginal rate that is a little more than 25 percent. We then adjust this figure to reflect income taxation by state and local governments; specifically, data from the National Income and Product Accounts (NIPAs) indicate that state and local personal income taxes represented about 2-1/2 percent of overall personal income in 2001. As this is an average (not marginal) rate, we double it to capture the progressive nature of most state and local tax systems. The sum of these two rates yields the 30 percent average marginal tax rate that we assume.

B.2 Calibration of F_*^k

We require an estimate of the average marginal tax rate on capital income. Excluding depreciation, net capital income can be divided into three categories: dividends, retained earnings, and interest payments. If the corporate income tax rate is given by F_*^c, and if dividends (and capital gains) are taxed at the rate F_*^d, then the effective tax rate on capital income F_*^k is implicitly defined by

$$1 - F_*^k = (1 - \omega)(1 - F_*^d)(1 - F_*^c) + \omega(1 - F_*^h), \tag{72}$$

where ω denotes the share of net interest payments in overall capital income. Under current law, the Federal corporate income tax rate is 35 percent, while the Federal tax rate on dividends and capital gains is 15 percent. (We add an additional 5 percentage points to these rates to reflect taxation at the state and local level.) Using NIPA data, we estimate that 17.5 percent of the capital income share is paid out as net interest. All together, these figures imply a capital tax rate of 48 percent, which is the value we assume for F_*^k in our baseline model.

The preceding assumes that the double taxation of dividends (at the corporate and personal level) matters in determining the cost of capital. Under the so-called "new view" of dividend taxation, however, the taxation of dividend income at the personal level is immaterial as far as the cost of capital is concerned.[33] In this case, the first tax term in parentheses on the right-hand side of equation (72) equals one, implying that the effective tax rate on capital income is 38 percent.

Finally, the simplest possible case arises when firms are financed exclusively through debt (in which case taxable corporate income is zero). This implies that all capital income is taxed at the personal tax rate, or that $F_*^k = F_*^h = 30$ percent.

To assess how sensitive our results are to alternative assumptions about F_*^k, the table below gives the long-run change in the real rental rate of capital (expressed as a percent deviation from its steady-state level) following a permanent 30 percent expensing allowance for various assumed values of F_*^k.[34] Based on the figures in the table (and given the log-linear structure of the model), assuming a value of F_*^k consistent with dividend taxation's having no effect on the cost of capital would reduce the model's responses by about a third, while assuming that firms are purely debt-financed would scale them down by about a half.

Long-Run Percent Change in Real Rental Rate

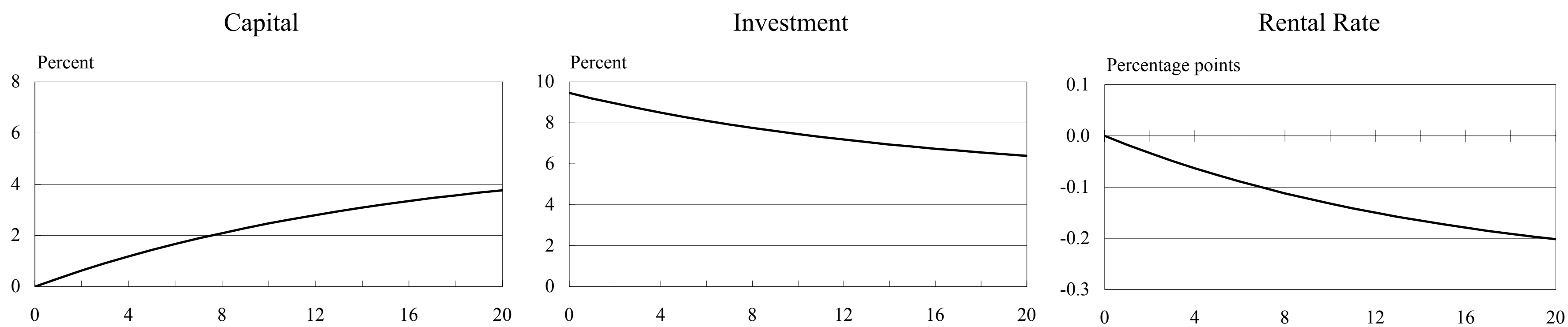
Tax rate F_*^k	Description	Change
48 percent	Baseline assumption	-5.21
38 percent	"New view" of dividend taxation	-3.68
30 percent	Fully debt-financed firms	-2.67

[33]See Auerbach (1979) and Bradford (1981) for discussions of this issue.

[34]The figures in the table give the *direct* effect on the rental rate that obtains from a change in the expensing allowance under the specified tax rate; they do not incorporate any general-equilibrium effects.

FIGURE 1.: Permanent Expensing in a Partial-Equilibrium Model with Capital Adjustment Costs

A.: 20 Quarters



B.: 200 Quarters

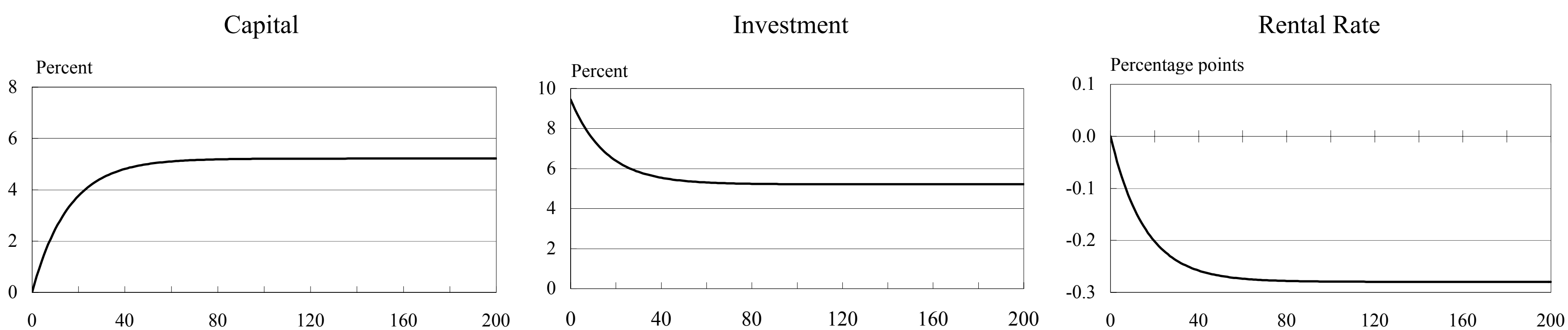
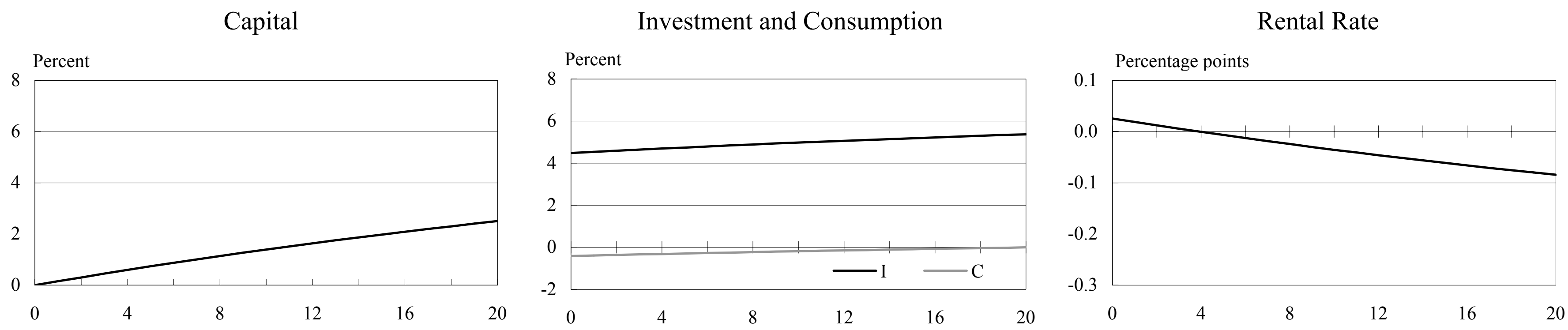


FIGURE 2.: Permanent Expensing in a General-Equilibrium Flexible-Price Model with Capital Adjustment Costs

A.: 20 Quarters



B.: 200 Quarters

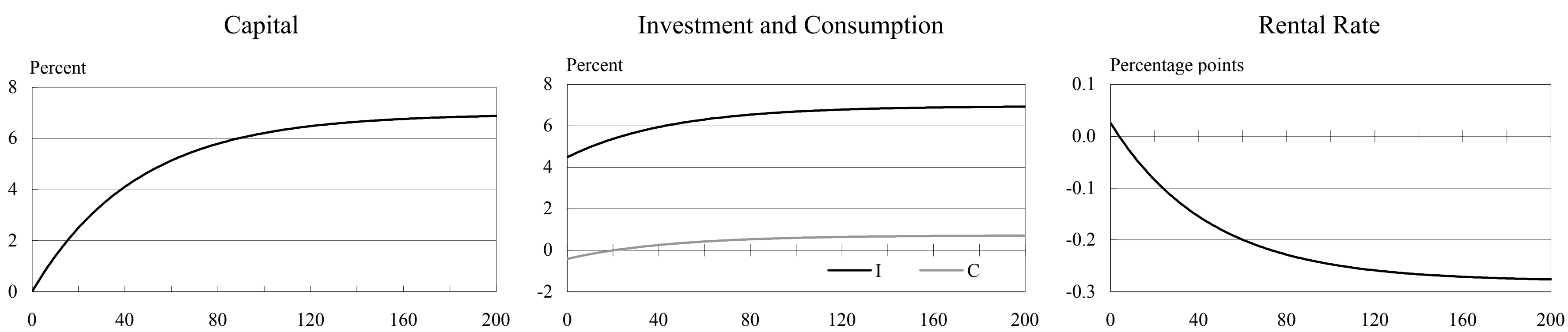


FIGURE 3.: Effect of Permanent Expensing on the Capital-Output Ratio
in a General-Equilibrium Flexible-Price Model

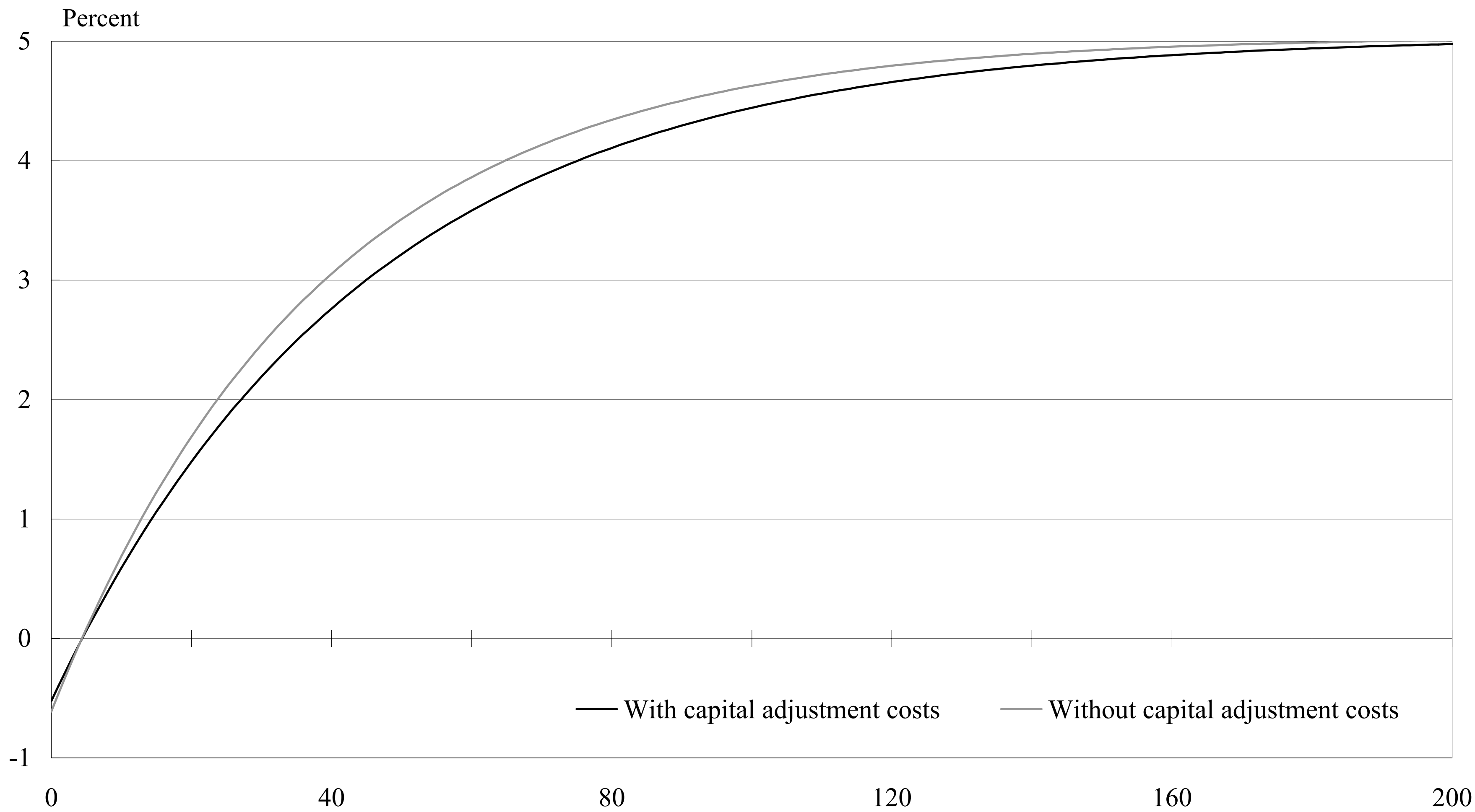
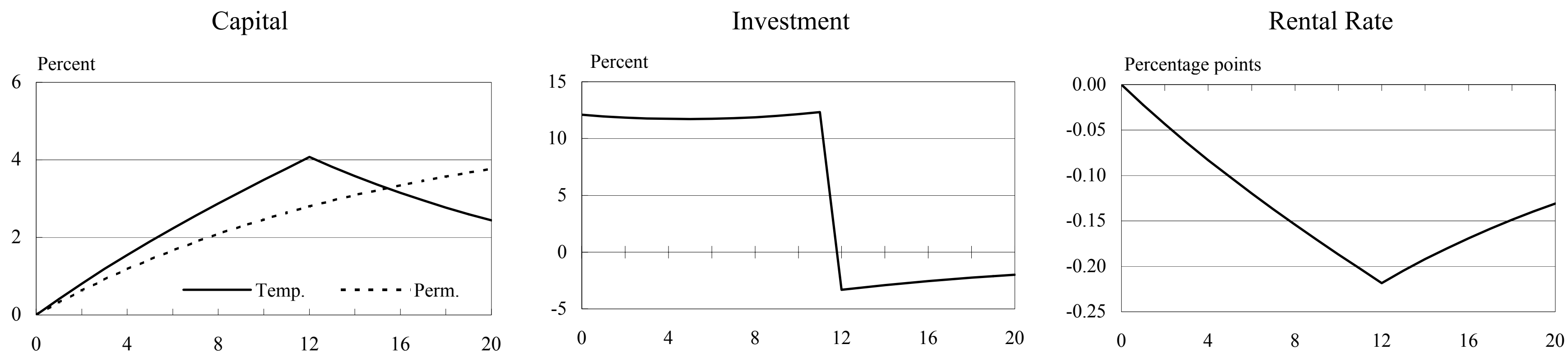


FIGURE 4.: Temporary Expensing with Capital Adjustment Costs

A.: Partial-Equilibrium Model



B.: General-Equilibrium Flexible-Price Model

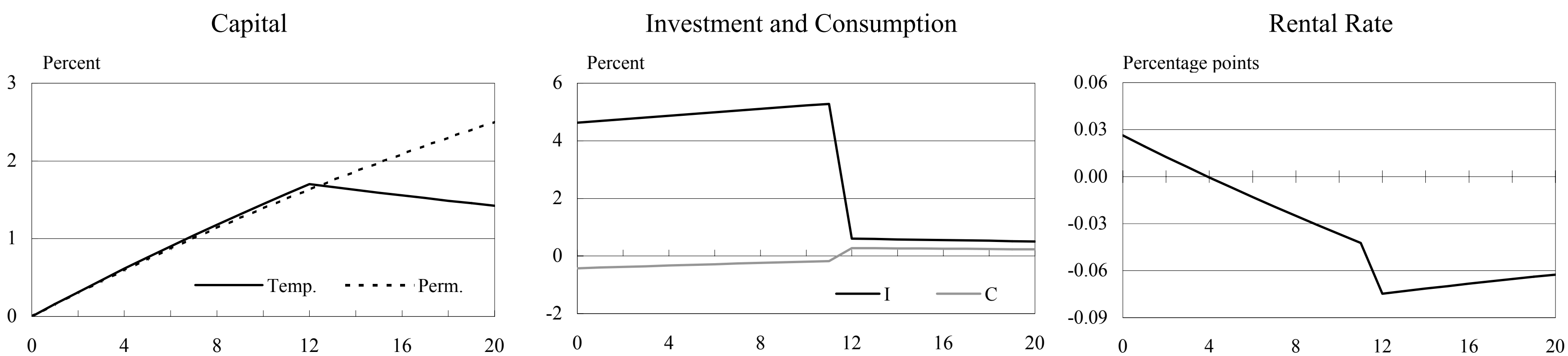
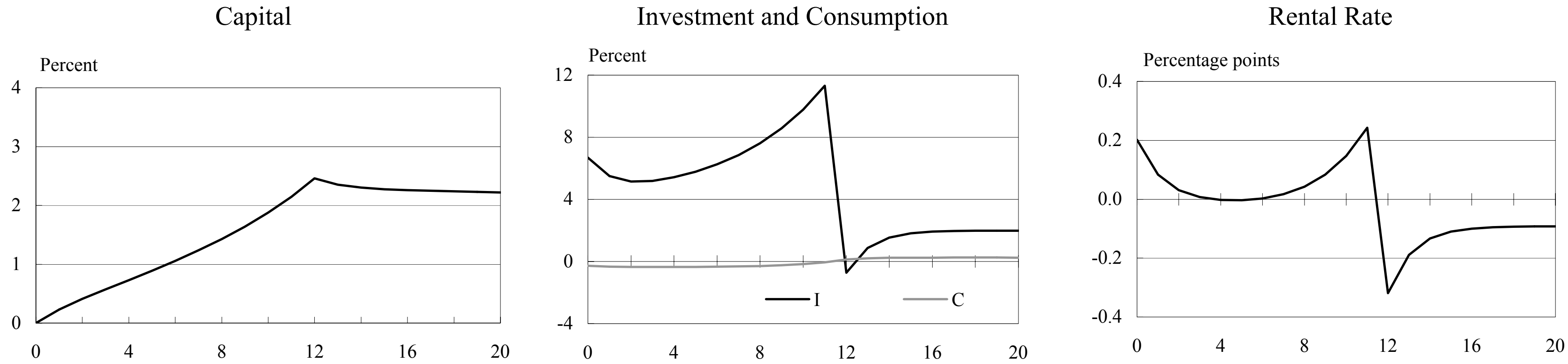
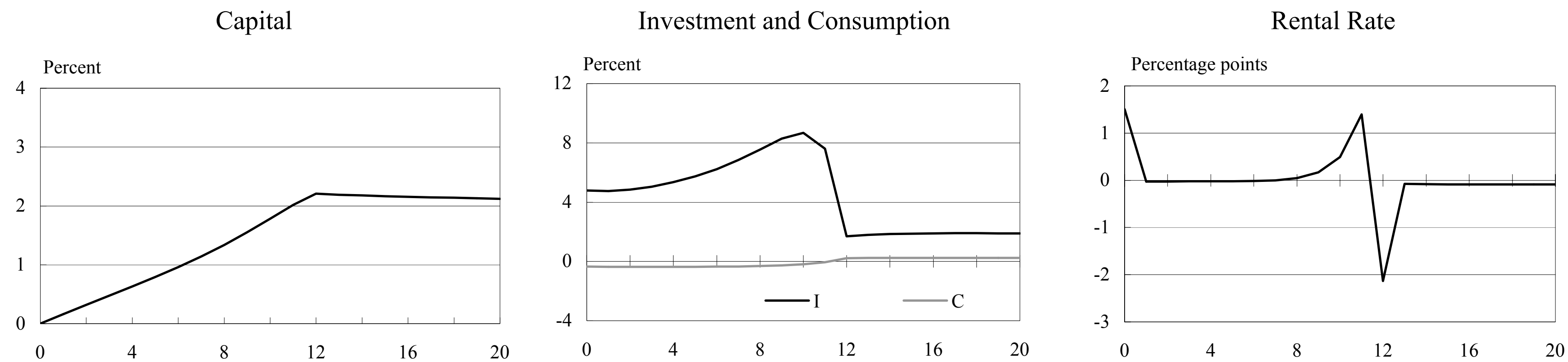


FIGURE 5: Temporary Partial Expensing, Alternative Adjustment-Cost Specifications

A.: Capital Adjustment Costs



B.: Factor-Ratio Adjustment Costs



C.: Investment Adjustment Costs

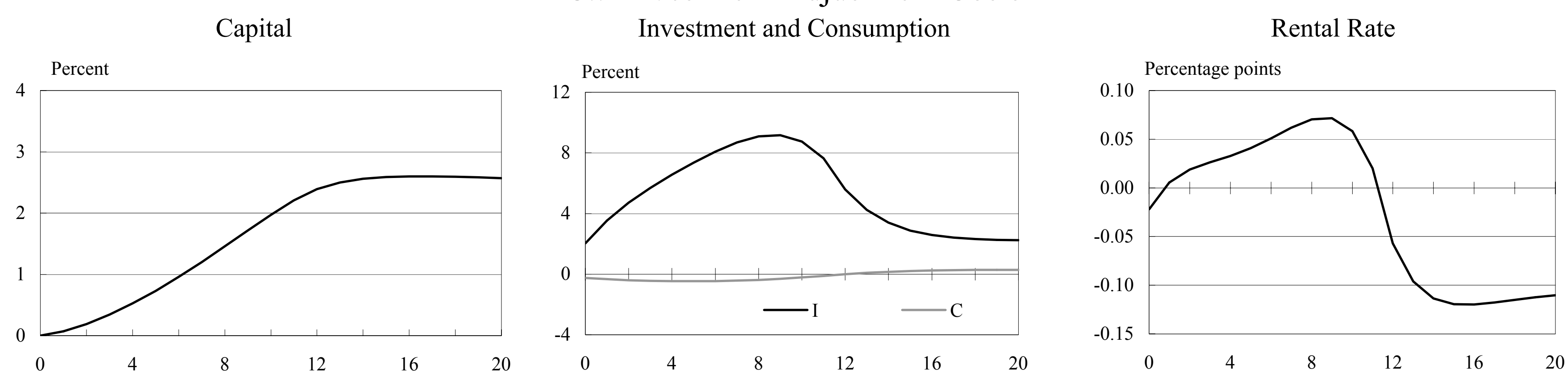
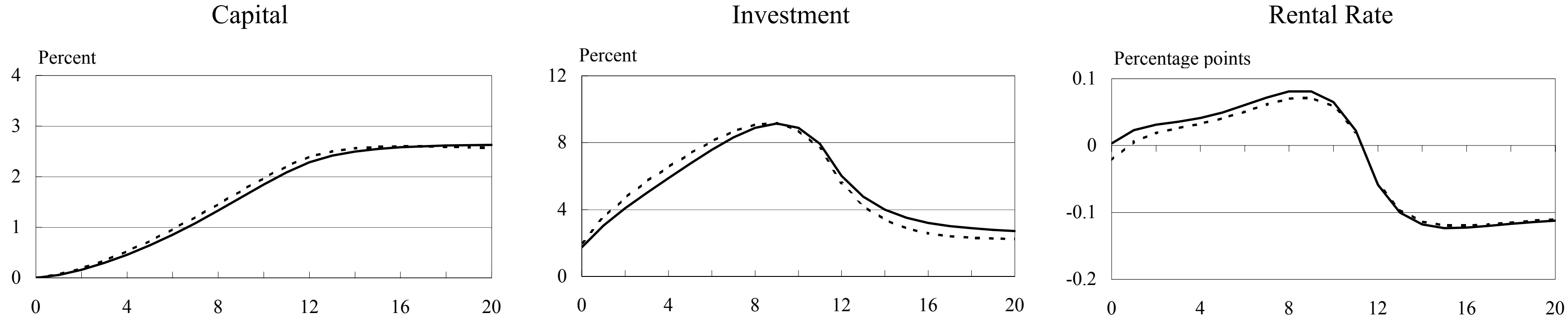


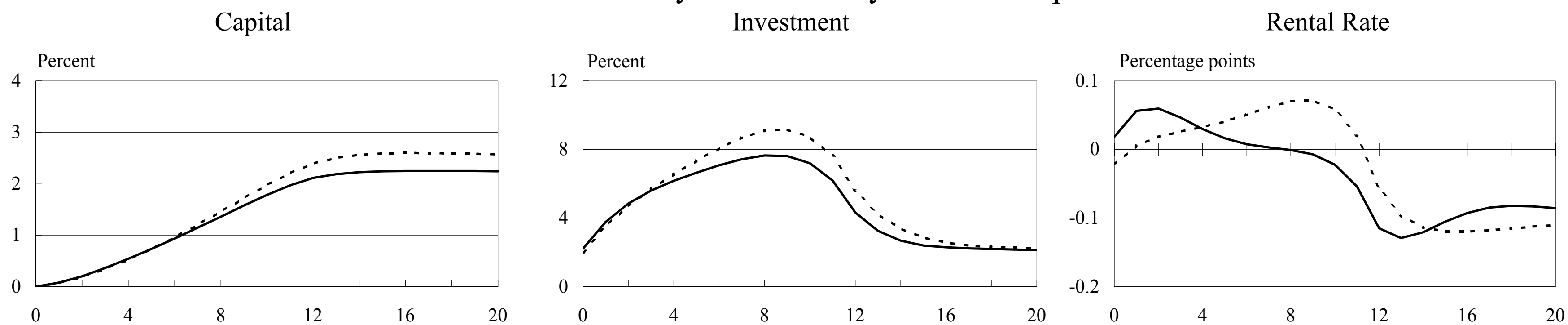
FIGURE 6.: Temporary Expensing Allowance, Alternative Models

A.: Model with Habit-Persistence in Consumption



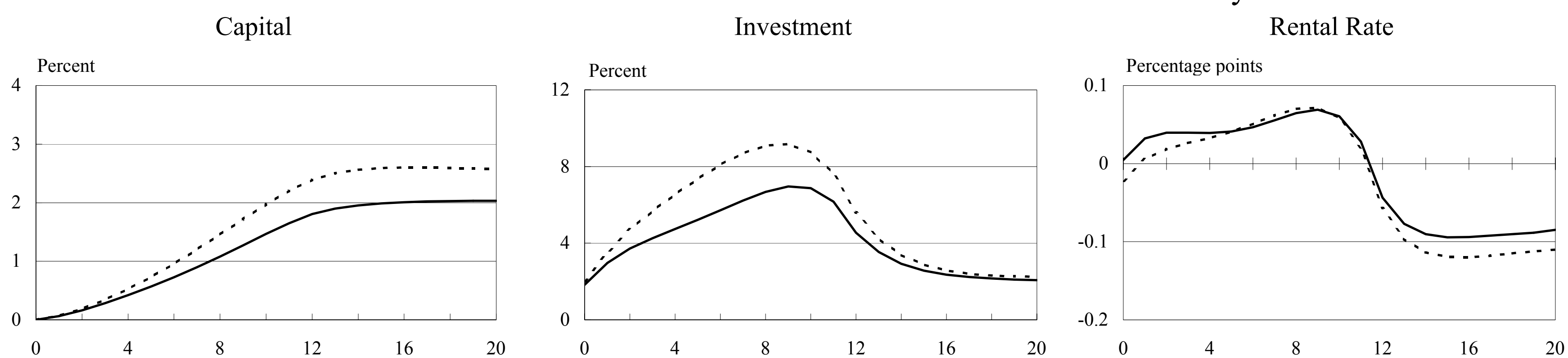
Solid line: Model with habit-persistence; Dotted line: Benchmark investment adjustment cost model.

B.: Model with a Hybrid New-Keynesian Phillips Curve



Solid line: Model with hybrid new-Keynesian Phillips curve; Dotted line: Benchmark investment adjustment cost model.

C.: Two-sector Model with Limited Cross-sectoral Factor Mobility



Solid line: Two-sector model; Dotted line: Benchmark investment adjustment cost model.

FIGURE 7: Capital Tax Rate Cut, Alternative Adjustment-Cost Specifications

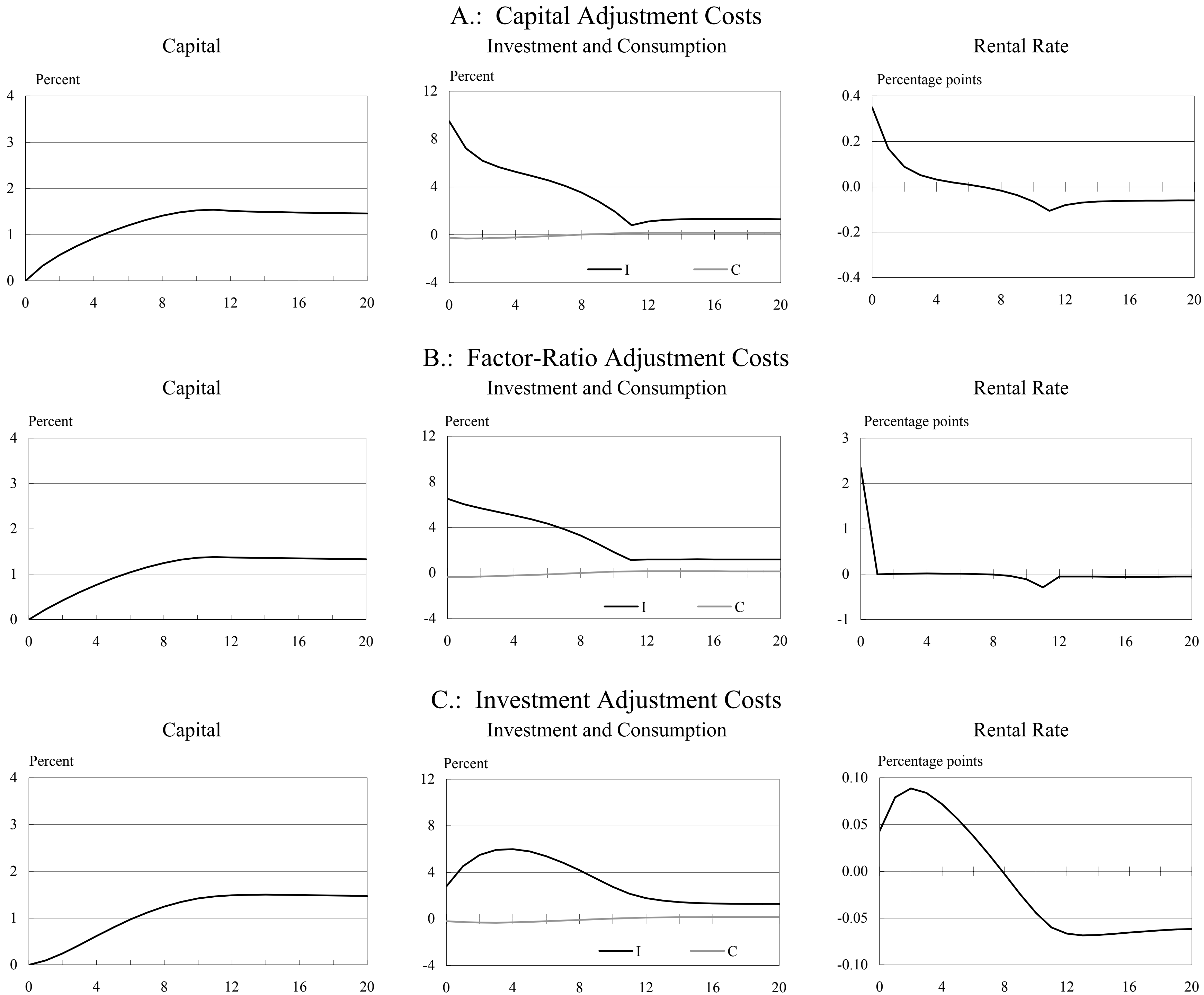
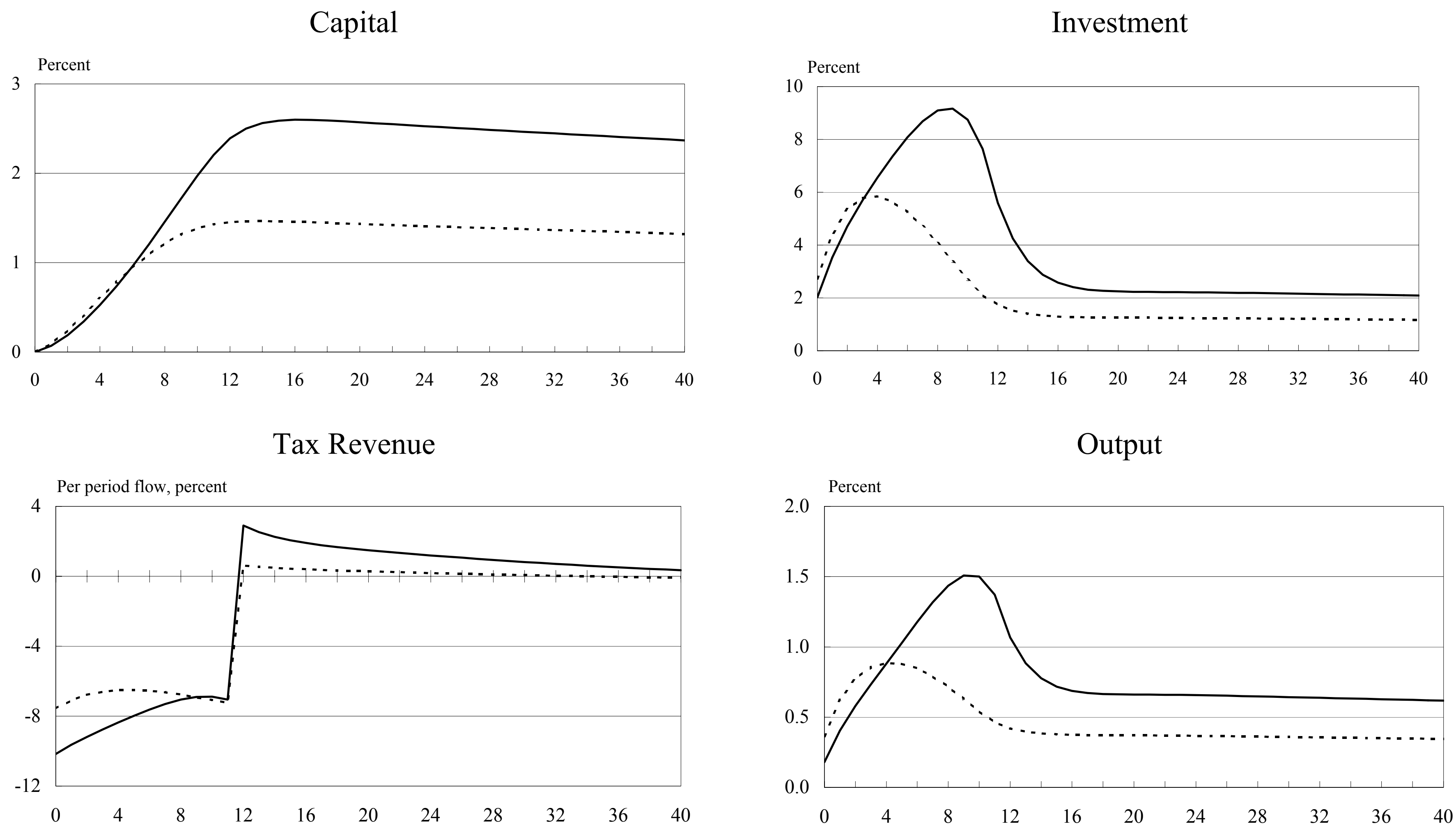


FIGURE 8: Comparison of Two Temporary Equal-Revenue Investment Incentive Policies



Solid line: 30 percent partial expensing allowance
Dotted line: 19.5 percentage point cut in the capital tax rate