Measurement Error and Time Aggregation:

A closer look at estimates of output-labor elasticities

Marcello Estevão*

Board of Governors of the Federal Reserve System

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Abstract

This paper analyzes the effect of time aggregation on estimates of the elasticities of output with respect to employment and to average hours of work. The main goal is to get accurate estimates of production function parameters. Low frequency data generate better estimates of output-employment elasticity while high frequency data generate better estimates of output-average hours elasticity. This result comes from the fact that time aggregation increases (decreases) the bias in the estimate of the elasticity with respect to average hours (employment). Estimations of these elasticities at different data frequencies and numerical simulations illustrate this point. In addition, this estimation methodology shows that the elasticity of output with respect to employment is bigger than the elasticity of output with respect to average hours, as theory predicts, contradicting an established result in the literature.

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1 Introduction

In general, the most acceptable specification of total labor effort in production function estimat es is some combination of employment and average hours, where both variables are included separately in the estimation process. Empirical tests of this specification have almost uniformly found that the elasticity of output with respect to average hours of work is at least as large as the elasticity of output with respect to employment. Given this finding, an increase in average hours and a compensatory decrease in employment, having total hours of work constant, generates more output.

This result is far from being uncontroversial This paper explains such results and provides new estimates for both elasticities. A re-evaluat ion of previous estimates is important for a number of reasons. Besides their import ante in the literature of production function est imat ion, the sign and the size of the difference between both elasticities may help to explain the cyclical behavior of labor market variables, such as labor productivity y and labor marginal cost. Another motivation for such a study comes from unemployment policy issues. The effect of labor-sharing policies, which are based on an exogenous cut in average hours of work in an attempt to increase the employment level, also depends on this difference.

The next section presents a simple model that highlights the puzzling implications of high output-average hours elasticity estimates. Briefly, if these estimates are right, firms and workers are not taking advantage of the increasing returns of average hours on labor productivity. In addition, if the hourly wage function is not highly sensitive to variations in average hours, firms should hire an infinite amount of average hours and almost no workers. Therefore, firms and workers have economic incentives to change the number of average hours worked until the increasing returns on total hours productivity of rearrangements in average hours and employment are exhausted. In summary, there are strong reasons to suspect that these estimates are not correct.

In this case, what would explain the high estimates for the output-average hours elasticity found by previous works? Table 1 lists these results. With the exception of Leslie and White (1980), they all find that the output-employment elasticity is smaller than the output-average hours elasticity. I argue here 3, that the instrumental variables used in these papers are not good and that estimates of the difference between the two elasticities are inconsistent. In fact, it is hard to find instrumental variables for aggregate production function estimates. Because labor effort and capacity utilization are unobservable and firms hoard labor along the business cycle, as shown by Fay and Medoff (1985), instrumental variables related to exogenous demand shocks are not acceptable.[1] Furthermore, instruments that may not be correlated to omitted supply or technological shocks, such as lagged endogenous variables, are correlated to the error term since it exhibits serial correlation.

Even without good instrumental variables the estimates can be improved by carefully selecting the data frequency. Because employment and average hours variability and their correlation with unobserved variables differ across data sampling frequencies, the large sample bias of estimates of both elasticities also changes with data periodicity. In this paper, I document the relevance of this effect and provide some guideline to decide what the optimal data frequency for estimation of output-labor elasticities is.

The paper is organized as follows: The next section establishes in more detail the puzzle

[1] So military expenditures, as used in Hall (1988), or demand shift variables based on input-output tables, as proposed by Shea (1993), are not appropriate here.

Table 1: Previous Results

Papers	Output-employment elasticity	Output-hours elasticity	Data
Feldstein (1967)	0.75-0.90	1.10-2.55	Annual data Cross-section UK
Craine (1973)	0.68-0.80	1.89-1.98	Annual data Time series US
Hart and McGregor (1987)	0.31	0.81	Semi-annual data Pooling of c.s. and t.s., Germany
Leslie and White (1980)	0.64	0.64	Same as Feldstein
Shapiro (1986)	1.00	1.06	Quarterly data Time Series US

in the previous results. Section 3 presents evidence on the dynamic behavior of employment and average hours to exogenous output shocks and provides estimates for both elasticities at different data frequencies. I show that estimates of the elasticity of output with respect to employment decrease with time aggregation while the elasticity of output with respect to average hours of work increases. Section 4 considers the time aggregation effect explicitly and reports Monte- Carlo simulations showing that there is a bias in the aggregation process that explains the results obtained here and in the literature. The estimation bias of the output-average hours elasticity increases with time aggregation while the bias in estimates of the elasticity of output with respect to employment falls. Once monthly data is used to estimate the elasticity of output with respect to average hours and annual data is used to estimate the output-employment elasticity we find that the former is smaller than the latter, as expected. The final section concludes this work.

2 Labor effort and hours of work

2.1 A model

This section studies the relationship between labor effort and the output-employment and output-average hours elasticities in steady state. First, for the sake of simplicity, assume the production function has the following form:

$$Y_t = A_t\, S(N_t, H_t)^{\alpha} K_t^{\beta} \tag{1}$$

where, α and $\beta \leq 1$ and $S(\cdot) = N_t H_t J(H_t)$ is the total effort function, or services of labor function. $J(H_t)$, the effort function, also measures how far the average hours elasticity is from the employment elasticity. [2]

A general functional form for $J(H_t)$ is used: [3]

$$J = J(H_t),\ \text{such that,} \begin{cases} \frac{\partial J(H_t)}{\partial H_t} > 0 & \text{if } H_t < \overline{H} \\ \frac{\partial J(H_t)}{\partial H_t} < 0 & \text{if } H_t > \overline{H} \end{cases} \tag{2}$$

This function traces out the relationship between average hours of work and labor effort. There are two effects at work in determining the shape of this function. For low values of H, the fixed amount of time spent on warming up, beginning and finishing production, meals, instructions, and so on, generates a positive relationship between average hours and the $J(\cdot)$ function (Feldstein (1967)). Therefore, when firms decrease employment and increase the number of hours hired from each worker, keeping constant the level of total hours hired, the total labor

[2] The output-average hours elasticity can be written aa $\alpha(1 + \epsilon_H^J)$ and the output-employment elasticity, α. So, the difference bet ween both elasticities depends on the elasticity of effort with respect to average hours (ϵ_H^J).

[3] See also Estevão (1993).

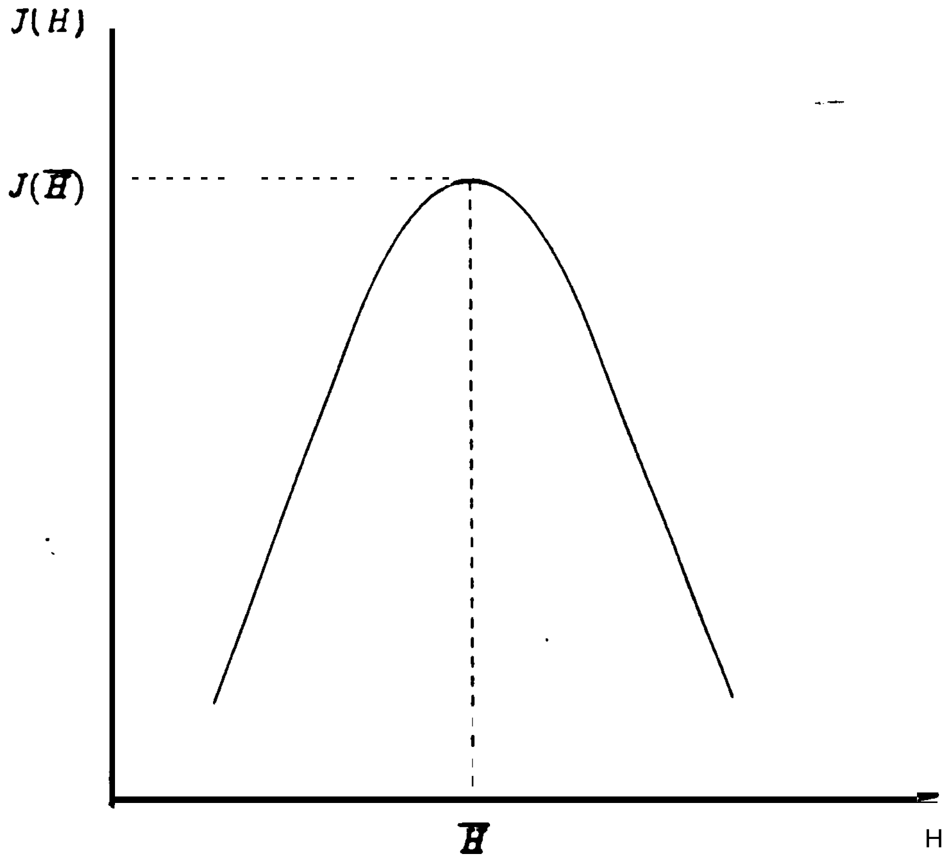

Figure 1: $J(H_t)$ function

services used in the production process increases. Eventually, fatigue caused by long hours of work overcomes the initial increasing returns of average hours of work on total effort and generates a negative relationship between H and $J(H)$. At $\overline{H}$ both effects cancel out. $J(\cdot)$ has the form given in (2) and shown in figure 1.

In addition, it is assumed that firms face:

1. A downward sloping demand function:

$$P_t = \left(\frac{Z_t}{Y_t}\right)^{1-\frac{1}{\mu}} \tag{3}$$

where, μ = markup; Z_t = demand shift parameter.

2. Two types of labor cost: $W(H_t)$ = hourly wage function (~ >0, $\frac{\partial^2 W}{\partial H^2} > 0$) and $C =$

cost per worker.[4]

So, firms will maximize profit given equations (l), (3), the labor effort function and the labor cost variables. Thechoice variables areemployment andaverage hours of work. The level of capital is given, by assumption.

Max. $\Pi = P(Y_t)\,Y_t - W(H_t)\,N_t H_t - C N_t$

s.t. (1) and (3)

From the first-order conditions:

$$\frac{W(H_t^*)\,H_t^*}{C} = \frac{1+\epsilon_H^J}{\epsilon_H^W - \epsilon_H^J} \tag{4}$$

where, $\epsilon_H^J = \frac{\partial J(H_t^*)}{\partial H_t^*}\frac{H_t^*}{J(H_t^*)}$ and $\epsilon_H^W = \frac{\partial W(H_t^*)}{\partial H_t^*}\frac{H_t^*}{W(H_t^*)}$, and

$$N_t^* = N(H_t^*, Z_t, A_t, K_t, \mu) \tag{5}$$

Notice that the optimal number of hours per person does not depend on demand parameters. Given the ratio between variable and fixed labor costs and the elasticities of the wage and the effort functions, firms decide the optimal number of hours hired from each employee. The level of employment is determined by the demand parameter, the mark-up, the technology parameter, the stock of capital, and the optimal hours of work.[5] In a dynamic version of the model, with costs to adjust labor, demand shocks would cause changes in average hours of work to compensate for employment stickiness.

[4]This parameter captures all expenses related to the labor force that are independent of the number of hours worked. I will assume that this variable is constant and given exogenously by some institutional arrangement. See Hart (1984) for an exhaustive discussion of the variables represented by C.

[5]This fact is already well-known in the literature. Ehrenberg (1971) has proved this result for effort functions that are separable in N and H.

Equation (4) also shows that firms choose average hours of work in the region where labor effort elasticity is positive if the wage function is locally sufficiently sensitive to variations in hours.[6] In this case, the output-average hours elasticity ($\alpha(1 + \epsilon_H^J)$) will be greater than the output-employment elasticity (α). Otherwise, they will hire average hours until the increasing returns on the hourly productivity of a worker is exhausted, the effort elasticity is negative and the elasticity of output with respect to average hours will be smaller than the output-employment elasticity.

2.2 Are firms hiring too few hours?

The results reported in table 1 can be explained in the context of the present model if one of the two following conditions hold:

- the wage function is very steep at the hired level of average hours of work;
- firms would like to be hiring a larger number of work hours, but are prevented from doing so by some institutional constraint.

The first explanation suggests that firms do not hire more average hours of work because the disutility of an increase in labor effort causes an extra hour of work to be too expensive. Firms would increase the average number of hours hired if workers were willing to work more

[6] The second-order conditions are, after some manipulations:

$$\alpha < \mu \tag{6}$$

the well-known condition establishing that the scale elasticity has to be smaller than the mark-up for the existence of an interior maximum. And,

$$\left(1+\epsilon_H^J\right)\left(1+\epsilon_H^W\right)\left(\epsilon_H^W-\epsilon_H^J\right) > \left(1+\epsilon_H^W\right)\frac{\partial \epsilon_H^J}{\partial H}H - \left(1+\epsilon_H^J\right)\frac{\partial \epsilon_H^W}{\partial H}H \tag{7}$$

Since, in general, $\frac{\partial \epsilon_H^J}{\partial H} \leq 0$ and $\frac{\partial \epsilon_H^W}{\partial H} \geq 0$, $\epsilon_H^W > \epsilon_H^J$ is also a sufficient condition.

at the same hourly wage rate. This is the *labor supply constraint case.*

The problem with this argument is that existing evidence shows that individuals are constrained to work fewer hours than the desirable level at the observed wage rate.[7] Because workers are hours-constrained in their supply of labor, the actual hourly wage function elasticity at this point should be close to zero. In addition, Altonji and Paxson (1986) show that workers tend to move from one job to another in order to vary the number of hours worked. This evidence gives stronger support to the hypothesis that average hours of work at each job is determined by firms. Therefore, firms should stop hiring only if there is a negative effect on hourly effort.

The second explanation is the *institutional constraint case.* The Standard Fair Act (1935) mandates that firms pay a 50% overtime premium for employees working more than 40 hours per week. Even if employees were willing to work additional hours at the same wage rate, firms would be prevented from hiring more hours because of the high marginal cost of an extra hour. A simplistic interpretation of this law is that the extreme sensitivity of the wage function to variations in hours of work in this region prevents firms from hiring over 40 hours per week from each worker in steady state.

The basic problem with this explanation is that it assumes the law is effective and neglects the economic incentives firms have to reorganize the production process to avoid the extra overtime costs.[8] For inst ante, firms could hire 10 hours of work per day for four days and

[7] See Kahn and Lang (1988) and Dickens and Lundberg (1993), for instance.

'For the sake of simplicity, the model presented above does not make a distinction between daily and weekly hours. In practice, firms can rearrange daily average hours of work and the number of days worked. An example of such rearrangement can be found in The New York Times, 5/16/93: "Lacking enough demand, some manufacturers are finding ways to avoid both hiring and overtime. The Quaker Oats Company of Chicago, which now employs 11, 000 Americans, down slightly in recent years, has shifted many of them to 10-hour daily shifts, four days a week - giving up overtime . . . ."

with no overtime premium, instead of hiring 8.5 hours per day during five days and paying the overtime premium for the extra 2.5 hours.[9]

Finally, firms and workers have the option of negotiating a lower straight-time wage to compensate the overtime premium. As argued in Hall (1980), the long term relationship between firms and their workers allows them to negotiate compensation schemes that map the disutility of effort (a continuous function of average hours) better than the discontinuous scheme proposed by the law. In this case, the shadow price of an overtime hour will be substantially smaller than the one stated in the law. Trejo (1991) documents this fact.

The issues discussed above lead us to expect that firms hire hours in the downward sloping region of the effort function.[10] But ,the evidence presented in table 1 contradicts this a *priori* expectation. The next sections address this puzzle.

3 Empirical strategy and results

3.1 The dynamics of employment and average hours adjustment to exogenous output shocks

The adjustment path of employment and hours to their steady state level depends on how expectations are formed, the nature of the economic shock (if it is permanent or temporary), and the type of labor adjustment costs. In general, average hours respond immediately to output

[9]The idea is that firms use less total hours of work to produce a given amount of output since they will be exhausting all the "increasing returns" coming from increases in average daily hours/worker. In this example it is assumed that firms can produce the same level of output using 40 hours/worker per week and 10 hours/worker daily (what makes $\epsilon_H^J \leq 0$) or using 42.5 hours/worker per week and 8.5 hours/worker daily (what makes $\epsilon_H^J > 0$).

[10]One last reason for this prior is just casual observation. In general, people perceive a decrease in their hourly work productivity at the end of a regular working day.

shocks while employment adjustment is more sluggish due to the existence of adjustment costs. [11] In the long run, average hours are scale independent and should return to their steady-state level once the effect of the output shock is over.

In order to see if this dynamic response is present in the data used here, I ran a reduced-form VAR system of output, employment, and average hours, represented in equations (8) and (9). Employment and output are defined in first differences while the level of average hours is used in the system. This specification captures the fact that employment and output are integrated of order 1 but average hours is integrated of order zero. Similar results are obtained if the first difference of average hours is used. This system imposes a smooth adjustment process on the data and, while *ad hoc*, it captures the raw correlations between output and employment and output and average hours at both high frequency and after the adjustment process is completed .[12]

$$An = c_1 + \Theta_1(L)h + \Psi_1(L)\Delta n + \Lambda_1(L)\Delta y + \nu_1 \quad (8)$$

$$h = C_2 + \Theta_2(L)h + \Psi_2(L)\Delta n + \Lambda_2(L)\Delta y + \nu_2 \quad (9)$$

Figure 2 shows that the initial response of employment to a one percent permanent shock in output is small but it increases over the course of the adjustment process. On the other hand, average hours are initially quite sensitive to variations in output, but as the shock works through the system, they return to their initial level as illustrated in figure 3.13

[11] Here I am considering output as an exogenous variable to simplify the argument. For instance, firms are demand constrained.

[12] In equations (8) and (9), $x = log(X)$ and Ax = first difference of x. The estimations use monthly data for the American manufacturing sector and 6 lags for each of the three variables. The result is robust to changes in the lag order. The appendix describes the database used in this paper.

[13] Notice that these responses do not allow for a feedback in output. The approach used here considers output

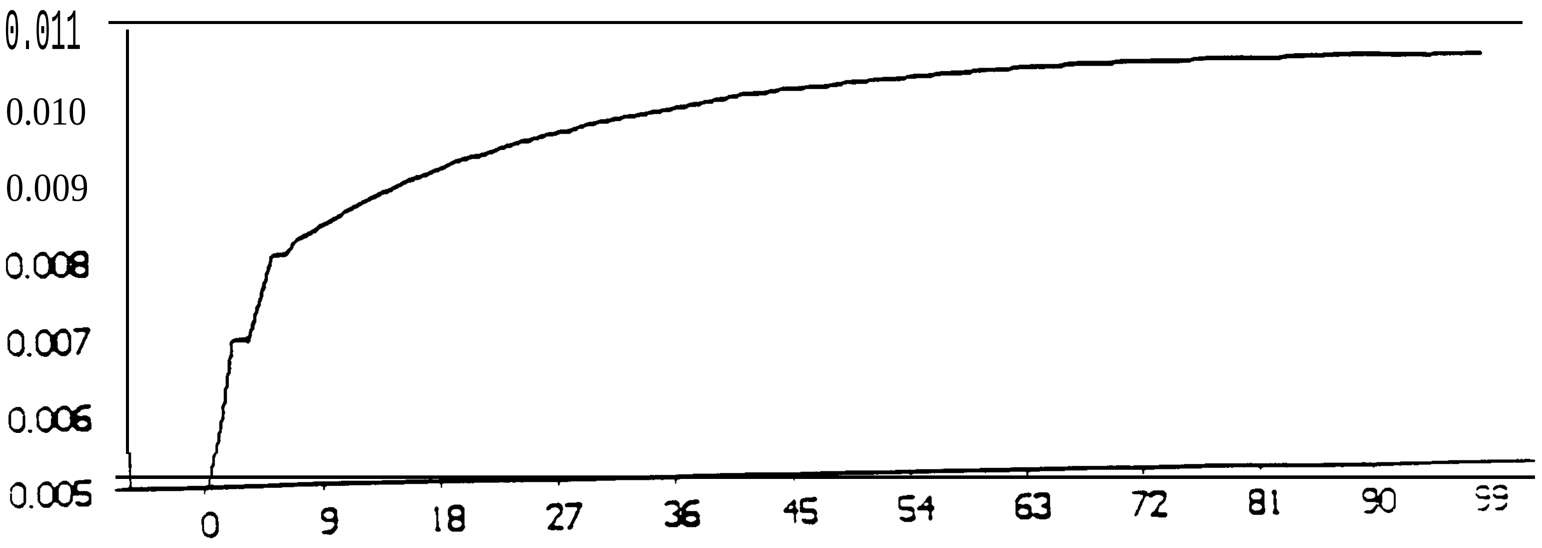

Figure 2: Employment responses to a one percent permanent shock in output

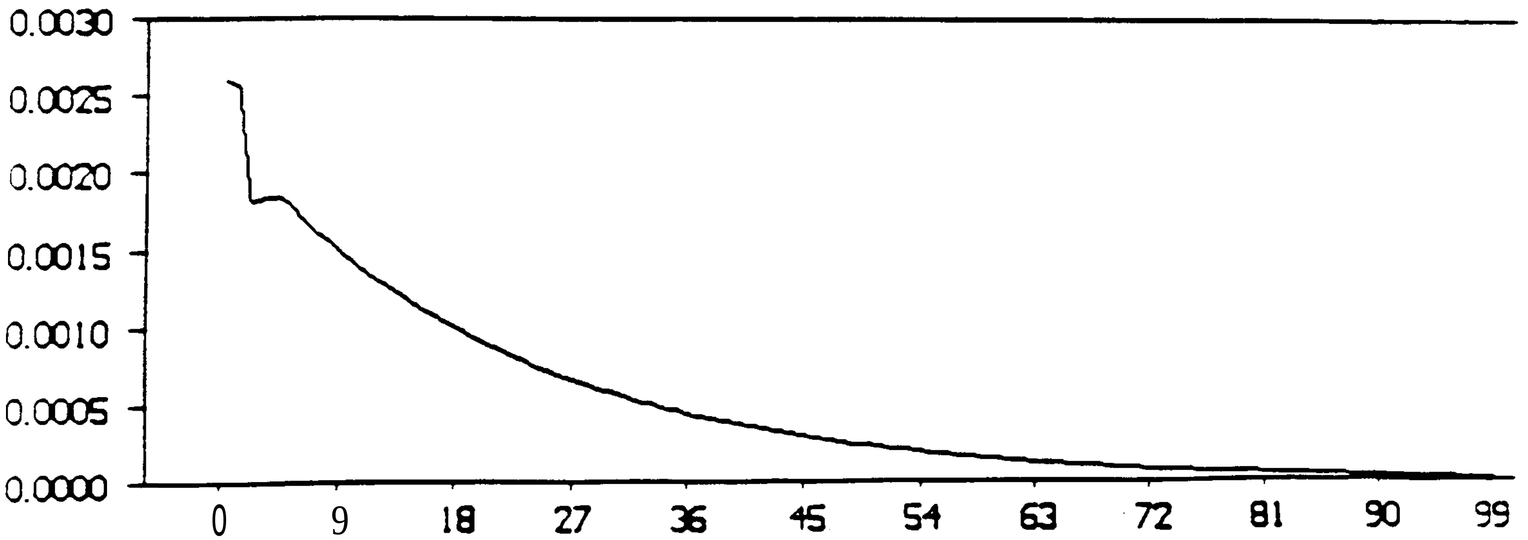

Figure3: Average hours responses to aone percent permanent shock in output

'This type of behavior implies that, first, the variance of average hours of work is larger at high frequency sampling while employment variance is larger at low frequencies. In addition, average hours of work should track more closely the behavior of variables that present the same pattern of adjustment, for instance, worker effort and capacity utilization. Employment, on the other hand, should present weaker correlation to these variables. When annual data is used in production function estimations, most of the short-run dynamics of average hours, worker effort and capacity utilization is eliminated, but employment will still be correlated to intermediate inputs and the level of capital stock.

The dynamic effects mentioned above cause variations in the bias in OLS estimates depending on the data frequency chosen. This bias can be written as,

$$bias(\beta_{OLS}) = \frac{f(cov(n,\epsilon), cov(h,\epsilon), \cdot)}{g(var(n), var(h), \cdot)} \tag{lo}$$

where ϵ is the regression error. Equation (10) represents the "signal-to-noise" ratio in the estimation of both elasticities. At the same time that monthly data present a higher covariance between average hours of work and excluded variables (work effort and capacity utilization), and consequently, a larger numerator in (10), it also preserve the high variance in hours or a larger denominator in (10) that diminishes significantly with time aggregation. Changes in the covariance between employment and the residual term and in its variance at different degrees of time aggregation add more uncertainty on the behavior of this ratio. Given that the instrumental variables used in this literature are not perfect, the problem cannot be solved with IV estimation. In summary, there is no reason to get equal parameter estimates when using

as an exogenous variable. The short and long-run responses of employment and average hours to output shocks are not affected if a feedback in output is allowed.

different data frequencies, if the instrumental variables are not perfect. The next subsection provides estimations that support this view. Section 4 will show numerically that the pattern of the signal-to-noise ratio at different data frequencies will give guidance on the best data periodicit y for estimation purposes.

3.2 Econometric methodology and results

In order to test the effect of time aggregation on estimation bias, I will estimate output-employment and output-average hours elasticities at different data frequencies. [14]

Let us write the production function as:

$$Y_t = F(A_t, S(N_t, \tilde{H}_t), O_t) \tag{11}$$

where N_t is the employment level, $\tilde{H}_t$ is the average hours of work, O_t represents other inputs used in the production process; $A_t = e^{ct+r_t}$, wit h c as a constant, and r_t a mix of technology and other exogenous shocks. For estimation purposes, it is assumed that $J(\cdot)$ can be locally approximated by a constant elasticity function:

$$J\left(\tilde{H}_t\right) = \tilde{H}_t^{\theta} \tag{12}$$

Letting $x = \log(X)$, $F_X = \frac{\partial F}{\partial X}$, and taking total derivatives of (11), yields:

[14]This is also a test on th quality of the instruments used at different frequencies. As argued in the past section, if the instruments were perfect the estimates at each frequency would be identical. Because I use the same instrumental variables used in the papers listed in table 1, I will be able to say something about the quality of previous results.

$$dy_t = \frac{F_S S_t}{Y_t} ds_t + \frac{F_O O_t}{Y_t} do_t + \frac{F_A A_t}{Y_t} da_t$$

Assuming that $\frac{F_S S_t}{Y_t} = \alpha, \frac{F_O O_t}{Y_t} = \frac{F_A A_t}{Y_t} = 1$, gives:

$$dy_t = c\,dt + \alpha\,dn_t + \alpha(1+\theta)\,d\tilde{h}_t + do_t + dr_t$$

Then, assuming that O_t is non-observable, the discrete time version of this equation is:

$$\Delta y_t = c + \alpha\,\Delta n_t + \lambda\,\Delta \tilde{h}_t + e_t \qquad (13)$$

where, et = $\Delta o_t + \Delta r_t$; and, $\lambda = \alpha(1+\theta)$.

The existence of labor hoarding (see Fair(1969) and Fay and Medoff (1985)) implies that actual average hours of work will be different from the paid (or observed) average hours. Labor hoarding smoothes variations in the actual hours of work. This effect is modeled here as a measurement error in the actual hours worked, where the error is negatively correlated to the actual variable. Formally, let h_t be the (log of) hired average hours, and $\sigma_{xy} = Cov(\Delta x, \Delta y)$ then:

$$h_t = \tilde{h}_t + \xi_t$$

where $\sigma_h^2 < \sigma_{\tilde{h}}^2$; $\sigma_{\tilde{h}\xi}$, $\sigma_{h\xi}$ and $\sigma_{n\xi}$ are negative.

Finally, the equation estimated is:

$$\Delta y_t = c + \alpha\,\Delta n_t + \lambda\,\Delta h_t + \nu_t \tag{14}$$

where, ν_t = et – $\lambda\,\Delta\xi_t$.

The OLS estimators for both elasticities will be inconsistent for several reasons. First, output and inputs will be adjusted simultaneously when input prices change or when there is a technological shock. The direction of this simultaneity bias is not known *a* priori and I will assume that the instrumental variables chosen for the estimations solve this problem.

Second, the existence of labor hoarding creates a positive correlation between the error term and the regressors. The absence of other relevant inputs or capacity utilization also causes a positive correlation bet ween the regressors and the errors. Equations (15) and (16) are the probability limits of the OLS estimator. It is also assumed that E [An] = $E[\Delta h]$ = O for the sake of simplicity.

$$plim\,(\alpha_{ols} - \alpha) = \frac{\sigma_h^2\sigma_{n\nu} - \sigma_{nh}\sigma_{h\nu}}{\sigma_n^2\sigma_h^2 - \sigma_{nh}^2} \tag{15}$$

$$plim\,(\lambda_{ols} - \lambda) = \frac{\sigma_n^2\sigma_{h\nu} - \sigma_{nh}\sigma_{n\nu}}{\sigma_n^2\sigma_h^2 - \sigma_{nh}^2} \tag{16}$$

where the denominators of these expressions are positive by the Schwarz inequality.

Without more information on the magnitudes of each term in both equations, the direction of the bias caused by the existence of labor hoarding and omitted inputs is not clear. 1 will analyze the direction of the bias at different data frequencies in section 4. For now it is enough to know that the estimation of equation (14) requires the use of instruments for the regressors.

Table 2 shows estimations of equation (14) using time series of US aggregate manufacturing data.[15] The output-average hours elasticity is smaller than the output-employment elasticity if the equation is estimated using monthly data. Furthermore, the size of this elasticity increases the more temporally- aggregated is the data. The estimations using annual data match previous comparable results.[16] Time aggregation seems also relevant for estimations of α, although they follow an opposite pattern, where $\hat{\alpha}$ decreases with time aggregation.[17] [18]

The estimations using instrument al variables generate the same result. The first set of estimates uses lags of employment, average hours and real wages as instruments.[19] While the R^2 of the first stage regressions ranges from .2 to .55, the residuals of the estimated equation are serially correlated indicating that these variables are, possibly, not good instruments. One possible source of serial correlation could be omitted inputs that face adjustment costs. Previous papers also use lags of explanatory variables as instruments.[20]

Rank variables for employment and average hours and lags of the real wage variable are

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[15] The data appendix discusses problems with the database. It focuses on measurement error and its impact on the estimates.

[16] See Crane (1973) in table 1, for instance. He also uses data for the American manufacturing sector.

[17] This pattern remains the same if data aggregated into 4 and 6-months frequencies are used. These results were excluded because these data periodicities are not commonly used.

[18] Notice that the R^2 reported in the first three rows of table 2 increases with time aggregation. As it was first proved by Zellner and Montmarquette (1971), this is purely a mathematical result generated by the aggregation process. For instance, it can be shown, using the aggregation matrix described in the next section, that the R^2 of a univariate version of the first difference model presented above, when using quarterly data (R_q^2), can be written as:

$$R_q^2 = \frac{R_m^2}{R_m^2 + \Upsilon(1 - R_m^2)}$$

where, $R_m^2 = R^2$ obtained by monthly estimations of the model and

$$\Upsilon = \left(1 + \frac{32}{19\rho_1} + \frac{20}{19\rho_2} + \frac{8}{19\rho_3} + \frac{2}{19\rho_4}\right)^{-1}$$

The ρ_i's are the autocorrelation coefficients for the independent variable. Since these autocorrelations are positive in most economic time series, the result is immediate.

[19] I use three lags for the monthly and quarterly estimations and just one lag of these variables for the annual estimations. The pattern of the estimates remains the same independent of the number of lags chosen for the instruments.

[20] See, Feldstein (1967), Leslie and White (1980), and Abott, Griliches and Hausman (1988).

Table 2: Regressions for Aggregate Manufacturing Data

Frequency	Constant	α	λ	Method	R^2
Monthly	.0031	1.0384	.5573	OLS	.7338
	(11.59)	(30.96)	(12.71)		
Quarterly	.0093	.9506	1.1286	OLS	.9011
	(15.38)	(22.22)	(10.37)		
Annual	.0375	.8365	1.4670	OLS	.9713
	(20.99)	(16.11)	(8.28)		
Monthly	.0031	1.0560	.3874	IV	First set of instruments
	(11.43)	(16.21)	(3.52)		
Quarterly	.0092	.8133	1.4534	IV	
	(14.56)	(14.16)	(7.36)		
Annual	.0376	.9701	1.2189	IV	
	(19.49)	(10.45)	(4.24)		
Monthly	.0031	.9717	.6405	IV	Second set of instruments
	(11.43)	(24.99)	(12.40)		
Quarterly	.0093	.9316	1.1525	IV	
	(15.14)	(19.62)	(9.85)		
Annual	.0375	.8517	1.3970	IV	
	(20.96)	(15.00)	(7.23)		

Notes:

1. t-statistics in parenthesis.
2. Sample sizes: monthly estimations, 1947:02 to 1992:11; quarterly estimations, 1947:2 to 1992:3; annual estimates: 1948 to 1991.
3. The first set of instruments includes a constant, lags of employment, average hours and real wage. The second set of instruments is composed of a constant, rank variables for employment and average hours, and lag of order 1 for the real wage.

also used as instruments. [21] The rank variables are certainly correlated to the regressors but nothing definite can be said about their correlation with the error term. The results show the same pattern as the OLS estimations.

I test the restrictions that both elasticities are the same. The F-test rejects the null hypothesis of equality between them when monthly and annual data are used. [22] It is therefore relevant to consider both variables separately in the production function, at least at those frequencies.

Table 3 presents annual regressions including the capital stock. The results remain more or less the same. Estimates of the elasticity of output with respect to employment are slightly smaller than the ones presented in table 2 but the output-average hours elasticity is basically the same. Because there are no data on monthly capital stock and the quarterly data are not reliable, I did not run regressions including the capital stock at these frequencies. The results presumably would not change much if these regressions were performed since the capital stock is not very sensitive to high frequency shocks.

Table 4 reports estimates using a pooling of time series and panel data for 2 digit-manufacturing sectors. The coefficients for employment and average hours are forced to be the same in each sub-sector and sectoral dummies are introduced to capture sectoral differences in productivity y. [23] These dummies correspond to the time slope dummies in Leslie and White (1980), since the equation estimated here is specified in first differences, while they estimate a similar equation in levels. Rank variables of the regressors and lags of the real wage were used as instruments. The results are basically the same if lagged endogenous variables are used as instruments. Although

[21] The same variables were used as instruments in Feldstein (1967), Leslie and White (1980) and Hart and McGregor (1988). Rank variables go from 1 to T, the number of observat ions, with step size equal to one, and order the respective data series from its smallest to its highest value.

[22] The statistics at the three data frequencies in the OLS estimations are: $F_m(1, 547) = 61.96$, $F_q(1, 179) = 1.55$ and $F_a(1, 41) = 8.15$. The constraint is equally rejected for the IV estimations.

[23] The coefficients for the dummy variables are omitted to save space.

Table 3: AnnualRegressions for Aggregate Manufacturing Data with Capital Stock

Constant	α	λ	Output-Capital elasticity	Method	R^2
.0315 (12.57)	.790 (16.95)	1.458 (9.43)	.215 (3.46)	OLS	.9804
.0312 (10.61)	.813 (12.63)	1.396 (7.96)	.209 (2.47)	IV	First set of instruments
.0319 (11.79)	.791 (15.75)	1.434 (8.63)	.2046 (2.92)	IV	Second set of instruments

Notes:

1. t-statistics in parenthesis.
2. Sample size: 1948to 1991.
3. The first set of instruments includes a constant, lags of order 1 of employment, average hours and real wage, and lags of order 1 to 8 of the capital stock. The second set of instruments is composed of a constant, rank variables for employment, average hours and capital, and lag of order 1 for the real wage.

an F-test rejects the restriction that the coefficients are the same for each sector, the estimated coefficients give some information on the average output-employment and output-average hours of work elasticities.

This set of regressions tries to capture some of the effect of sectoral aggregation on elasticity estimates. Using a British database, Leslie and White (1980) test the hypothesis that more productive sectors also hire more hours from their workers. In this case, the coefficient of average hours of work will be biased upwards. They found that, once sectoral aggregation is taken explicitly into account, the elasticity of output with respect to average hours of work declines to the level of the output-employment elasticit y (see table 1). I could not replicate the same effect for the manufacturing sector in the US, although the estimates for the output-employment elasticity are smaller at each frequency than the ones reported in table 2. Hart

Table 4: Restricted Regressions for Panel of 2-Digit Sectors

Frequency	α	λ
Monthly	.9159	.6574
	(29.58)	(15.14)
Quarterly	.8041	1.1741
	(24.66)	(14.10)
Annual	.7441	1.6262
	(35.00)	(27.13)

Notes:

1. t-statistics in parenthesis.
2. Sample size: monthly estimations, 1947:2 to 1992:03; quarterly estimations, 1947:02 to 1992:01; annual estimations, 1948 to 1991.
3. The set of instruments is composed of a constant, rank variables for employment and average hours, and lag of order 1 for the real wage.

and McGregor (1988) could not find a sect oral aggregation effect on the output-average hours elasticity for the German manufacturing sector either. But, they also found that the elasticity of output with respect to employment falls. The estimated elasticities in table 4 follow the same pattern obtained when using aggregate manufacturing data.

As discussed in the beginning of this section, differences in the elasticity estimates at each data frequency are consistent with the fact that changes in employment and average hours have different variances and covariances with the residual at different frequencies. This result is also evidence that the instrumental variables used here and in previous papers are correlated to the error term in at least two of the three data frequencies. In this case, the IV estimations just replicate the OLS estimations. Therefore, the usual claim in this literature that IV estimates are similar to OLS results should be viewed with caution.[24] In the next section, I will show

[24] See Leslie and White (1980), Feldstein (1967), and Hart and McGregor (1988).

that the pattern obtained for estimates of both elasticities at different data frequency can be explained by the effect of time aggregation on the signal-to-noise ratio.

4 The temporal aggregation effect

The basic problem in analyzing the signal-to-noise ratio at each frequency is the unobservability of the correlation between the regressors and the error term. To overcome this problem, I model explicitly the time aggregation process. The final bias equation will depend on the variances, autocovariances, and covariances between lags and leads of the regressors and the residuals when evaluated using monthly data. The advantage of this methodology is that, given the autocorrelation and cross-correlation functions for these variables at monthly frequency, it is possible to assess the bias at lower frequencies and evaluate the effect of time aggregation without having to assume explicitly a different value for the correlation between the regressors and the residuals.

First, let us write equation (14) in matrix format:

$$\Delta y = X\beta + \nu \tag{17}$$

$X = [\ 1\ \Delta n\ \ \text{Ah}];\ \nu = e -\ \text{Q/3};\ Q = [\ 0\ 0\ \ \Delta\xi\ ]$ and $\beta = [\ c\ \ \alpha\ \lambda\ ]'$.

Define $M_{G\times T}$, as the aggregation matrix:

$$M = \begin{bmatrix} 1 & 2 & \dots & m-1 & m & m-1 & \dots & 1 & 0 & \dots & 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & \dots & m-1 & m & \dots & 1 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & & & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & & & \cdot & \cdot \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 1 & \dots & m-1 & m & m-1 & \dots & 1 \end{bmatrix}$$

This matrix takes the T observations of a first difference data at periodicity 1 and transforms them into G observations of first difference data at periodicity m. So, the equation to be estimated using data with periodicity m is:

$$M\Delta y = MX\beta + M\nu \tag{18}$$

For instance, the M matrix for the quarterly aggregation case is:

$$M = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 \end{bmatrix}$$

Equation (18) represents estimations using data observed at each m periods. When m = 1, $M = I_{T\times T}$. The OLS estimator for β using aggregated data is:

$$\beta_{ols} = (X'PX)^{-1}X'Py = \beta - [X'PX]^{-1}[(X'PQ)\beta + X'Pe]$$

where, $P = M'M$.

Because, $corr(X,$ Q) and tom-(X, e) # O, β_{ols} will be an inconsistent estimator of β. The probability limits for α_{ols} and λ_{ols} when time-aggregated data are used are:

$$plim(\alpha_{ols} - \alpha) = \frac{\sigma_h^2 \sigma_{n\nu} - \sigma_{nh}\sigma_{h\nu} + \phi_1}{\sigma_n^2 \sigma_h^2 - \sigma_{nh}^2 + \phi_2} \tag{19}$$

$$plim(\lambda_{ols} - \lambda) = \frac{\sigma_n^2 \sigma_{h\nu} - \sigma_{nh}\sigma_{n\nu} + \phi_3}{\sigma_n^2 \sigma_h^2 - \sigma_{nh}^2 + \phi_4} \tag{20}$$

The σ_{ij} terms are the variances and covariances of the high frequency observations. The bias term is the same as before except for the ϕ_is which involve terms composed of cross-correlations and autocorrelations of lags and leads of regressors and errors, multiplied by coefficients determined by the degree of temporal aggregation. These coefficients depend on matrix M. The ϕ_is will determine the direction of the aggregation bias. For instance, if An, Ah, $\Delta\xi$ and e are i.i.d., the size of the bias is the same for all the data frequencies used. In general, the bias in estimates of β will vary at each frequency.

One way to check if time aggregation matters is to compute the probability limits for both elasticities when estimated at different frequencies. These limits can be evaluated using the observable variances and covariances between the regressors and their lags, and the assumptions on their correlation with the error term. There are two problems with this approach. First, it is computationally burdensome. The ϕ terms in equations (19) and (20) are more complex and involve higher-order lag correlations the larger is the degree of time aggregation. Second, the result is valid when the number of observations tends to infinity but imperfect in small samples.

In order to handle both problems, I perform Monte-Carlo simulations. Their basic structure can be described as follows:

- first, 550 random data points are generated for An, Δh and ν assuming that these variables have a joint multivariate normal distribution with a variance-covariance matrix, Σ;

- second, Δy is generated under the assumption that α = .75 and λ = .40. Different initial values for β were used but the pattern of the estimates at different data frequencies is the same. At this point, equation (18) is estimated "at monthly frequency";

- third, using matrix M, I obtain the quarterly and the annual data for Δy, An and Δh and perform estimations of β at these frequencies;

- finally, this process is repeated 1000 times.

In order to build Σ some identifying hypotheses have to be made. The variances and covariances of An and Ah and their lags and leads are observed. The variance of v were such that it matches the variance of the residuals of the estimated equation for the manufacturing sector. The basic identifying restriction is that estimates of both elasticities at monthly frequency are positively biased. Using the observed values of σ_n^2, σ_h^2 and σ_{nh} to establish restrictions on $\sigma_{n\nu}$ and $\sigma_{h\nu}$ such that (15) and (16) are positive leads to :

$$\frac{1.23}{6.85} < \frac{\sigma_{h\nu}}{\sigma_{n\nu}} < \frac{4}{1.23}$$

So, monthly estimates of both elasticities will not be upward biased if one of the covariances is substantially bigger than the other one. Given that at monthly frequency both variables are expected to be significantly correlated to labor hoarding and capacity utilization this alternative does not seem plausible. Finally, I impose different identifying assumptions on the covariances between regressors and the residual term at monthly frequency. This assumptions are less arbitrary than they may seem at first sight. Because Σ is constrained to be positive definite and the variance of v is given by the data, many of the stochastic processes assumed for v were discarded. In particular, the autocorrelation coefficients of ν are required to go to zero fast as the autocorrelation order increases.

The pattern of the estimates reported in the last section are replicated for any paramet er randomly chosen in the region defined by the identifying restrictions. Table 5 reports a represent ative result m the region were $\sigma_{h\nu} > \sigma_{n\nu}$. The only criteria used to choose this specific simulation is that $\sigma_{..}$ and $\sigma_{h\nu}$ are of the same order of magnitude as σ_{nh}. Obviously, if both covariances are too small there is no significant bias in the estimates. In this case, the estimates at different frequencies should not differ from each other what contradicts the OLS results presented before. [25]

Simulations in the region where $\sigma_{h\nu} < \sigma_{n\nu}$ can be represented by the simulation reported in table 6.[26] The results show the same pattern as the ones presented in table 5. In fact simulations in this region tend to fit the data better, since the higher covariance between employment and the error term increases (decreases) the bias in monthly estimates of output-employment (average hours) elasticity. [27]

[25] The variance-covariance matrix used in table 5 was built using the observed variance-covariance matrix for n and h, assuming that $\sigma_{n\nu} = 5 * 10^{-6}$, $\sigma_{h\nu} = 1 * 10^{-5}$, and $\sigma_\nu^2 = 4 * 10^{-5}$ = residual variance observed in the data. Additionally, the lag correlations between the residual and the regressors were assumed to decline fast to zero. Different assumptions on the shape of these cross-correlations generated minor changes in the results.

[26] If $\sigma_{h\nu}$ is very low such that th output-average hours elasticity is underestimated at monthly frequency, the simulations show that the use of time-aggregated data will cause the bias to be even more negative. The simulations will also change if the correlation between employment and the residual are too low. But, as said before these caaes do not seem plausible.

[27] The variance-covariance matrix used in table 6 is basically the same as the one used in table 5. The difference is that I assume that $\sigma_{n\nu} = 1 * 10^{-5}$ and $\sigma_{h\nu} = 5 * 10^{-6}$. The results are also robust to different assumptions on the ν process.

Table 5: Monte-Carlo Simulations - $\sigma_{n\nu} < \sigma_{h\nu}$

	β_i	β_1	β_3	β_{12}
constant	.003	.003	.028	.445
		(.000)	(.003)	(.043)
α	.75	.806	.723	.721
		(.034)	(.045)	(.064)
λ	.40	.764	.939	1.126
		(.050)	(.162)	(.349)
R^2	—	.604	.714	.825
Number of ohs.	—	550	182	44

Note:Std. deviations of the coefficients in parenthesis.

Table 6: Monte-Carlo Simulations - $\sigma_{n\nu} > \sigma_{h\nu}$

	β_i	β_1	β_3	β_{12}
constant	.003	.003	.028	.448
		(.000)	(.002)	(.036)
α	.75	.856	.767	.731
		(.035)	(.037)	(.048)
λ	.40	.475	.732	1.212
		(.040)	(.127)	(.312)
R^2	—	.654	.766	.860
Number of ohs.	—	550	182	44

Note:Std. deviations of the coefficients in parenthesis.

Since nothing was assumed explicitly about thecorrelation between theregressors and the residual term at lower frequencies, this exercise provides strong evidence in favor of a time aggregation bias. The larger is the degree of temporal aggregation, the more distorted are estimations of the output-average hours elasticity. On the other hand, the larger is the degree of temporal aggregation the less distorted are estimations of output-employment elasticity. The simulations show that, alt bough the bias in estimates of output-employment elasticit y can be very large at monthly frequency, they are very stable at annual frequency. The simulated value for this elasticity using annual aggregation does not depend much on parameter choice.

The slight negative bias in the simulations of the output-employment elasticity at quarterly and annual frequency should be viewed as an exclusive consequence of temporal aggregation. Notice that the Monte-Carlo simulations do not take into account variables that are positively correlated with employment at this frequency but are not included in the regression and could be driving upward the estimates in section 3.2. As table 3 shows, for instance, when the capital stock is included in the estimated regression at annual frequency the output-employment elasticity decreases slightly. [28]

5 Conclusions

The simulations described in the past section show that, assuming a positive bias in monthly estimates of output-employment and output-average hours elasticities, a time aggregation effect can explain the results obtained in section 3.2. Temporal aggregation increases the positive bias

[28] In fact the same argument can be made for the elasticity of output with respect to average hours. Feldstein (1967), for instance, shows that the estimate of this elasticity falls when measures of capacity utilization are included in his annual regressions, but it is still much bigger than output-employment elasticity.

in estimates of output-average hours elasticity. On the other hand, it decreases the positive bias in estimates of output-employment elasticity. In this case, the monthly estimate of the elasticity of output with respect to average hours is closer to the actual parameter. The output-employment elasticity is more accurately estimated when annual data is used. Therefore, using the elasticities estimates at these frequencies reported in section 3.2, this paper shows that the output-average hours elasticity is smaller than the output-employment elasticity. This finding contradicts the evidence of previous works. Additionally, these previous results can be explained by the time aggregation effect since they were built using annual, semi-annual or quarterly data and the same instrumental variables used here.

The estimation problems studied in this paper have a general flavor. Whenever the regressors in a equation present a dynamic pattern of adjustment, which means that their covariance, as well as, their variances change at different data frequencies, the parameter estimates will also change as long as there is a correlation between the regressors and the residual term. The problem is even bigger if there is a chance that this correlation also varies at different data frequencies. If the instrumental variables used in the estimation are not perfect or close to perfect ion, the problem will not go away with IV estimations. Specifications using different data frequencies can reveal information on the quality of the instrumental variables used and on the direction of the bias. If the results are insensitive to time aggregation the researcher is using good instruments. If time aggregation seems to be an issue, simulations using the observed variance- covariance matrix of the regressors and identifying assumptions on the origin of the correlation between regressors and the residual term, may point to the direction of the time aggregation bias.

Appendix

Data Problems and Database Description

The database used in this paper is composed of

- a sectoral production index calculated by the Board of Governors of the Federal Reserve System (FED). This seasonally adjusted index is available since January, 1947;
- data on production workers, average weekly hours of production workers, gross average hourly earnings of production workers and consumer price index calculated by the Bureau of Labor Statistics (BLS). These data are available since January of 1947. Only data on employment and average hours is seasonally adjusted.
- the capital stock series is the "Equipment and Structures" annual series for the manufacturing sector calculated by the Department of Commerce, BEA.

The data on seasonally-adjusted real hourly earnings were obtained by regressing real gross average hourly earnings on seasonal dummies and then using the forecast errors, weighted by a smoothing parameter to get a smoothed series.

There are some basic problems with the data. First, there is measurement error in the output data. The FED index of industrial production tries to capture changes in physical volume in each sector. Since there is no measure of physical volume for some industrial goods, however, the information for these categories was inferred from input data: production-hours worked (BLS), employment (BLS) and kilowatt-hours consumed *(FED).* Data based On total hours of work or employment correspond to 25.2% of total industrial production, while 30.0% are based on kilowatt-hours consumed and 42.970 on pure physical output data. The remaining uses a combination of employment, total hours of work, and kilowatt-hours consumed. Some sectors have a higher proportion of imputed information than others. The results reported here do not seem to be biased in a particular way for this reason. Additionally, sectoral regressions present the same pattern as the ones reported in the paper, independently of the proportion of imputed information in the output series.[29]

The same regressions were performed for selected 2-digit sectors and the whole manufacturing sector using an alternative data for physical output. This data was built using the Bureau of Commerce data on sales and inventories variation. The results follow the same pattern as the ones reported here, but present much stronger serial correlation problems and the estimated parameters are more sensitive to the choice of instruments.

Second, there are important methodological differences among the series used. Components of each sectoral output index are adjusted for monthly differences in the number of working days. Reported product data are converted to a daily average basis by adjusting for the number of working days in the reporting period. The employment and average hours data include all full-time and part-time workers who received pay for any part of the pay period that includes the 12th day of the month. This causes a mismatch between these data, basically a weekly data, and the output data, that refer to the whole month.[30] So, output data is a smoother version of the appropriate data (if it is considered the sampling methodology for employment and hours data as "appropriate)'). Furthermore, holidays are seasonal. Since, employment and average hours series are weekly samples of monthly data it is possible to have "bad seasonality" in the sense that variation in output per month may not be reflected in the labor data. Given these problems it is crucial to deseasonalize the data.

[29] The sectoral results are available upon request.

[30] A similar discussion can be found in Sims (1974).

It is worth noticing that the use of deseasonalized data in production function estimates is not indisputable. Fair (1969), for inst ante, argues that the production function is a represent a-tion of the technical relationship between inputs and output and not between "deseasonalized input" and "deseasonalized output)'. But, the data structure does not give a better research alternative and all estimations are carried on with the deseasonalized data provided by the FED and the BLS.

This difference in the sampling methodology causes an additional problem. Bresnaham and Ramey (1994) show that firms frequently use production shutdowns as a way to achieve production goals. If there is a production stoppage in the week that contains the twelfth day of the month, but production is subsequently resumed, the output data will bear a weak relationship with the labor input data. The problem will be more serious the smaller is the degree of data unit aggregation. In Bresnaham and Ramey (1994) this was a crucial point since they work with plant level data. I work with 2-digit sectors data and in this case the intermittent production problem is much less relevant since aggregation should wash out macroeconomic idiosyncrasies.

Furthermore, in order to sign the direction of the bias caused by the sampling methodology it is necessary to know, for instance, if firms tends to (de)accelerate production at the beginning or at the end of the month to meet monthly production. Or, if there is any regular pattern for the production shutdowns for inventories adjustment. This can be an interesting research question. Right now, there is no reason to assume that the sampling problem causes bias in any particular direction.

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