Optimal Discretion

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Abstract

This paper investigates the desirability of adopting a rule in favor of discretionary monetary policy in a model exhibiting Kydland and Prescott's dynamic inconsistency problem. We deviate from earlier work by adopting assumptions regarding policymaker preferences and inflation dynamics that are compatible with empirically motivated models used for macroeconomic policy evaluation. In particular, we dispense with the notion of a fundamental incompatibility between the policymaker's price stability and full employment objectives and allow for stickiness in the determination of inflation. In this setting, we show that if discretion provides a policy flexibility benefit, adoption of a rule remains optimal but only under certain circumstances. If the central bank's preference to contain inflation is fully credible, then a rule is optimal only when inflation exceeds an endogenously determined threshold. This setup gives rise to a discretionary policy zone for inflation with the central bank taking more drastic action towards stabilizing inflation when inflation veers outside the zone. We also examine optimal policy when the central bank's inflation fighting determination is not fully credible. Then, adopting a rule becomes optimal even when inflation is lower. This result provides a reconciliation of the theory regarding the optimality of adopting a rule with the empirical observation that policymakers appear more willing to abandon discretion when facing either low credibility or high inflation but are less inclined to do so otherwise.

Keywords: Rules, discretion, credibility, dynamic inconsistency, disinflation, inflation targeting.

JEL Classification System: E58, D82, C72.

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1 Introduction

Despite significant theoretical advances since Kydland and Prescott's celebrated demonstration of the dynamic inconsistency problem faced by discretionary policymakers, resolution of the debate regarding the desirability of adopting a fixed rule for the conduct of monetary policy as opposed to allowing policymakers the flexibility of discretionary decisions appears no closer now than twenty years ago.[1] Many reasons can be cited for this predicament. Participants on opposite sides of the debate analyze the question using different models, hold different beliefs regarding the goals of policy, disagree about the relevance of constraining the behavior of public institutions and so on. And, of course, these differences are reflected in analyses which build on different and often mutually exclusive assumptions.

This state of affairs is rather unfortunate. The polarization of the debate fails to provide helpful guidance to policymakers as to how either the structure of central banks or the conduct of monetary policy ought to change to facilitate the realization of better policy outcomes. Presently, monetary policy in every major industrialized country is formulated and implemented with a healthy dose of discretion. Nonetheless, there have been historical episodes, particularly ones associated with stabilization from high or at least moderate inflation, when policymakers explicitly chose to be bound by more or less explicit rules, such as would be recommended in Kydland and Prescott's world. From the existing theoretical work, however, it is difficult to evaluate whether this occasional and rather selective adoption of rules coincides or is even remotely compatible with good policy. By and large, the polarized arguments suggest that rule adoption is either always or never preferable to discretion and either always or never feasible.

The purpose of this paper is to narrow the existing chasm in the current debate and attempt to understand the circumstances under which policymakers should abandon discre-

[1] Starting with Barro and Gordon (1983) a large literature developed the implications of Kydland and Prescott's (1977) work for monetary policy. Blackburn and Christensen (1989) and Fischer (1990) provide extensive surveys. McCallum (1997) and Clarida, Gertler and Gali (1999) discuss more recent developments.

tionary policy in favor of a rule. We build a simple model that is compatible with the central dynamic inconsistency argument in Kydland and Prescott in favor of designing institutions that eliminate discretion. But our model deviates from work designed to present the most favorable case for the adoption of rules in two ways. First, we abandon the assumption that policymakers somehow try to achieve the incompatible targets of maintaining employment above the economy's potential while seeking price stability. While this assumption has served well its purpose of illustrating the temptation described by Kydland and Prescott, its relevance has been the subject of much justified dispute. Consequently, models based on the premise of such inconsistent preferences may not be particularly useful for either understanding or guiding actual central bank behavior.[2] Rather, we posit that policymaker preferences are consistent with the bliss state of price stability and full employment and concentrate on the problem of stabilizing the economy in the presence of stochastic disturbances that move the economy away from its equilibrium.[3] An additional benefit is that with this modification our analysis becomes consistent with the framework employed in recent discussions of inflation targeting. This is of special interest given the increasing popularity of inflation targeting for actual policy design. (Bernanke and Mishkin, 1997, Bernanke, Laubach, Mishkin and Posen, 1998 and Svensson, 1997.) Second, we abandon the assumption that prices are perfectly flexible and can be costlessly and instantaneously adjusted. Rather we allow inflation to exhibit some stickiness. Again, our motivation is to move away from the comfort of a toy model of price determination that is extremely convenient in theory but questionable in practice, and towards a description of inflation closer to compatibility with empirically estimated macroeconometric models relevant for policy analysis.[4]

With these assumptions, we investigate under what conditions policymakers may seek

[2]Blinder (1997), McCallum (1995) and Taylor (1996) have recently elaborated this critique.

[3]This modification, of course, does not change the fundamental insight in Kydland and Prescott, that by influencing the formation of expectations, adoption of a rule may alter the tradeoff faced by a policymaker. Clarida, Gertler and Gali (1999) and Woodford (1999) elaborate on this point.

[4]See Taylor (1999) for a recent collection of models employed for policy evaluation in practice and alternative formulations of price stickiness.

to adopt a rule instead of maintaining a discretionary regime that might offer policy flexibility benefits. Clearly, if the economy were to remain at the deterministic bliss of price stability and full employment, no policy action would ever be needed and the choice between discretionary policy or policy bound by a rule would be of no relevance. However, adopting a rule continues to have the usual benefit of centering inflation expectations around the path consistent with following the policy required by the rule. This benefit is useful to policymakers when inflation deviates from the bliss of price stability. Therefore, in the context of a stochastic economy in which aggregate supply or demand shocks cause fluctuations in inflation, the option of adopting a rule as a means to stabilize inflation remains important despite the absence of any fundamental incompatibility between the central bank's long run price stability and full employment objectives. Adopting a rule would always be optimal, of course, unless discretionary policy provides something else, namely the flexibility of allowing the central bank to engage in fine-tuning by adjusting policy to developments that could not be foreseen when the rule was adopted. If the central bank expects to possess sufficient information to make such attempts at fine-tuning worthwhile, discretion may remain optimal. Our model is consistent with Milton Friedman's argument that if such reliable information is not available to the central bank, discretion ought to be abandoned in favor of a rule.[5]

With our simple model, we show that even if discretionary policy offers some flexibility for fine-tuning the economy by responding to unexpected developments whose realizations could not be foreseen when policy is set following a rule, the adoption of a rule will be optimal when inflation exceeds an endogenously determined threshold. This result defines a discretionary policy zone for inflation which provides valuable guidance in understanding why policymakers appear much more inclined to adopt a rule in an environment of high inflation than when inflation is low and stable. By investigating the comparative statics properties of the implicit threshold that determines the discretion zone, we also shed light on the role

[5]See his 1969 collected essays. Meltzer (1987) also elaborates on this issue.

of the policymaker's preferences, inflation stickiness and the quality of information required for fine-tuning for the desirability of adopting a rule.

We then turn our attention to the role of credibility. In our setting, no central bank "likes" inflation. However, the speed of disinflation, and by implication, the stability of inflation in a stochastic environment depend crucially on the determination with which the central bank is willing to pursue its inflation stabilization goal at the expense of output stability. Greater anti-inflation resolve coincides with lower variability of inflation at the expense of a higher variability of output. In this setting, credibility plays an important role for inflation stabilization by influencing public inflation expectations whenever inflation deviates from its target. When the resolve of the central bank is in question, unfavorable expectations of a slower disinflation than would be expected if the resolve of the central bank were beyond dispute raise the cost of disinflation under a discretionary policy regime. Under these circumstances, a central bank with high anti-inflation resolve will abandon discretion in favor of a rule even for levels of inflation for which it would not consider doing so had its credibility not been in question.

Our analysis, thus, confirms the desirability of adopting a policy rule in order to stabilize inflation when a central bank faces credibility problem. In summary, our analysis suggests that when inflation is near the long run goal of price stability and the government's credibility is high, it is not optimal for the policymaker to be constrained by a rule. But policymakers should be willing to abandon discretion in favor of a rule if inflation substantially deviates from the price stability objective or the credibility of the central bank to stabilize inflation is in question.

2 The model

Recognizing the dual objective of stable prices and full employment that is faced by many central banks we follow earlier work and specify that the central bank has preferences over the output gap and over the inflation rate. Following Kydland and Prescott, and Barro and Gordon, much of the earlier work on rules versus discretion is based on the assumption that the central bank's preferences are somehow twisted, in that the central bank prefers a higher output level than the economy's potential. These preferences are usually justified by suggesting that there are externalities and other imperfections in the markets that make potential output lower than the socially optimal level of output. Thus, it is argued, a benevolent central bank will try to boost output above potential. This attempt is, of course, futile on average, resulting only in an inflationary bias. In contrast to this assumption, we posit that the central bank's preference has a bliss point at potential output and at zero inflation. The reason for this is that central bankers are aware of the fact that they cannot reliably boost output above potential. Hence, they are content with containing inflation and keeping output close to potential.[6] Yet, the relative weight a central bank gives to each of these two objectives is open to question. We capture this idea by positing the following loss function for the central bank,

$$loss := \omega\pi^2 + (1-\omega)y^2. \tag{1}$$

π is the rate of inflation and y is the output gap. The parameter ω provides a measure of the central bank's relative valuation of the losses associated with inflation and output outside the bliss state of price stability and full employment. A higher value of ω indicates greater willingness to accept output losses to achieve quick disinflation. In this sense, ω provides a

[6]Commenting on the assumed inflationary temptation in the Barro-Gordon model, Blinder observes that during his tenure as vice-chairman at the Federal Reserve "... I never once witnessed nor experienced this temptation. Nor do I believe my colleagues did. I firmly believe that this theoretical problem is a nonproblem" (1997, p. 13). On the other side of the Atlantic, Bank of England Deputy Governor King points out that "[b]y relating monetary policy to macroeconomic goals, there is no 'inflation bias' and hence no obstacle to the achievement of price stability" (1996, p. 61).

measure of the *anti-inflation resolve* of the central bank. A central bank with a high ω might be called an "inflation hawk," a central bank with a low ω could accurately be described as an "inflation dove." Implicit in our specification is a target of zero inflation, but any other choice could also be accommodated in the model.

There is a Phillips curve linking output to inflation,

$$\pi := \lambda\pi^e + (1-\lambda)\pi_0 + \alpha y + \varepsilon. \tag{2}$$

ε is some random supply shock with zero mean. π_0 is "initial inflation," or "last period's inflation," that is to say, it is the rate of inflation the economy inherits. $\lambda \in [0,1]$ is a measure of inflation stickiness: The smaller λ is, the more inflation is dominated by the auto-regressive term, and the less quickly it can change. A large λ, on the other hand, means that realized inflation is not much influenced by its past realizations, but more dominated by expectations. The classic assumption is to set $\lambda = 1$. Then, switching the equation around offers the interpretation of a Lucas supply curve instead of a Phillips curve. At the other extreme, when $\lambda = 0$ the equation becomes an accelerationist Phillips curve. Empirically, the evidence suggests rather that an intermediate degree of stickiness appears consistent with the data. There are at least two justifications for $\lambda < 1$. One, consistent with the evidence presented by Fuhrer and Moore (1995), points to the possibility that the structure of wage setting may be responsible for this stickiness. The other rests on the hypothesis that, in the presence of costly information accumulation, a process of adaptive learning may reflect more realistically the formation of expectations than rational expectations. In that case, expectations may be captured better as a weighted average of the mathematical expectations based on our assumed structure of the economy, π^e, and in part on the focal level provided by current inflation, π_0. Roberts (1997) provides evidence pointing towards this explanation. Of course under either explanation but particularly the second one, λ might not be a fixed parameter but might actually increase with inflation, as both wage setting adjusts at higher frequency and the benefits of information gathering rise relative to costs especially as inflation becomes very rapid. For simplicity, however, we assume that maintaining a fixed value for λ that is independent of inflation is a reasonable approximation for the low and moderate levels of inflation that we believe are relevant for the task at hand.

The central bank's control variable, m, has some influence on aggregate demand y, but demand is also subject to random shocks, v,

$$y := m + v. \tag{3}$$

The randomness of v expresses the inability of the central bank to perfectly control output. This formulation is obviously inspired by the quantity equation, but is only exact if m reflects real money balances. We will nevertheless refer to m as "money supply" and to v as "velocity shock." The addition of a "money demand" equation linking money to interest rates would allow description of policy in terms of an interest rate rule with no fundamental change in our analysis so that is also omitted. Part of the velocity shock can be observed by the central bank. Let

$$v := x + \nu. \tag{4}$$

where x and ν are independent random variables with zero mean and some positive variance. The central bank can privately observe x during the period but does not observe ν. The public cannot observe either component of the velocity shock. The motivation for this assumption is to introduce a potential benefit for the policy flexibility associated with discretion. This formulation draws from Rogoff's (1985) analysis. By restricting attention to central bank policy, however, our model does not incorporate the policy flexibility associated with a government's power to overrule the central bank's decision in response to adverse developments, as developed by Lohmann (1992).

By *discretionary policy* we mean setting money supply so as to minimize the central bank's loss function, using all the information available to the central bank including, in particular, its observation of x, which becomes available during the period. We call this policy discretionary because the public cannot verify if the central bank has indeed followed the best possible policy, because the policy is conditioned on information that is unavailable to the public. An equally valid characterization of such a policy would be to say that the central bank is *fine tuning* the economy using information the public has no access to.

By a *policy rule* we mean setting money supply so as to minimize the central bank's loss function, but using only publicly available information. So a central bank that has committed to a rule will not be able to condition money supply on the privately observed part of the velocity shock x. We believe that a policy which is not verifiable by the public cannot be called a rule, because the public would never be able to distinguish rule-abiding behavior from discretionary deviations. Thus, we say that a policy is a rule if it is verifiable by the public. In our model this is synonymous to the requirement that the money supply cannot be conditioned on x.

Unlike the famous $k\%$ rule we do, however, allow the central bank to specify a rule that is state dependent, that is we allow a policy commitment to depend on inflation if such a dependence is optimal. In that sense, our definition of rule follows McCallum (1997) and Taylor (1993) more closely than Friedman (1969).

Our definition of a rule is also distinct from the terminology used in the Barro and Gordon tradition. This literature has been concerned with the difference between period-by-period re-optimization ('discretion') versus intertemporal multi-period optimization ('rule' in the Barro-Gordon terminology). We do not consider multi-period issues at all in our model, so the Barro-Gordon terminology does not apply.

The policy "game" unfolds as follows:

1. The economy starts with some inherited inflation $\pi_0 > 0$, which is publicly observed.

2. The central bank decides on whether to adopt a rule or to pursue a discretionary policy. If it decides to use a rule, it also announces the money supply, m^r (for $r$ule).

3. The public forms expectations π^e.

4. The central bank observes part of the velocity shock, x.

5. If the central bank has decided to use a rule, it provides the announced money supply m^r. If it has decided to do a discretionary policy, it decides at this instant on the money supply m^d (for $d$iscretion), taking x into account.

6. y and π are observed.

The central bank moves twice, in stages 2 and 5. Its strategy space is the Cartesian product of all the choices the central bank has along the game tree. A generic strategy consists of two components. The first component is either $\emptyset$, if the central bank does not commit to a rule in stage 2 (i.e. opts for discretion), or it is a real number, m^r, indicating the money supply the central bank commits to. The second component is a function m^d, mapping the additional information the central bank has at that stage, (π^e, x), to the discretionary money supply.

This concludes the specification of the game. We look for a subgame perfect equilibrium and therefore solve the game backwards.

3 Perfect credibility

In this section we assume that the inflation fighting resolve of the central bank is known to the public, that is to say, the central bank has no credibility problem: A "hawk" (high ω type) is known by the public to be a hawk; a "dove" (low ω type) is also known for his low resolve. The following proposition establishes that under these circumstances the central bank will adopt a rule if and only if initial inflation exceeds some threshold. This result is due to the tradeoff that commitment to a rule entails. On the one hand, a rule constrains the central bank to ignore valuable information (namely x). On the other hand, committing to a rule stabilizes inflation expectation, and therefore makes it easier to disinflate. This advantage becomes relatively more important compared to the aforementioned disadvantage the greater initial inflation is. Thus, if initial inflation is sufficiently high, the advantage of the rule dominates, and the central bank optimally adopts a rule.[7]

Proposition 1 *There exists some threshold value $\pi^* > 0$ such that the central bank will commit to a rule if and only if initial inflation exceeds this threshold, $\pi_0 > \pi^*$.*

The proof in appendix A provides an explicit solution for π^*. The proof exploits the following fact: Disinflation is costly because inflation is assumed not to be perfectly flexible ($\lambda < 1$). The expected loss associated with this fact is smaller with the rule than under discretion because the announcement of the rule stabilizes expected inflation, thus allowing the central bank to disinflate more rapidly. One might say that the announcement is "half the way" to disinflation. The rule, however, has the drawback that it produces a fixed positive expected loss due to the fact that under the rule the central bank cannot adjust to observations of x.

Another way to interpret proposition 1 is as defining an inflation zone around the central bank's inflation target. Within the zone, that is when inflation does not deviate more than π^* from the central bank's target, the central bank follows discretionary policy and attempts to fine-tune the economy while progressing towards its inflation target. Outside this zone, however, it adopts a rule abandoning the potential benefit of fine-tuning in favor of a less costly disinflation. Consistent with this strategy, the expected speed of disinflation is greater outside the zone than inside it. Figure 1 demonstrates this, and shows that this

[7] Obviously, our argument is symmetric. If the economy is in deflation and the deflation is sufficiently severe, a rule will be adopted. For clarity, we state our results in terms of positive inflation rates only.

basically simple model is sufficient to give rise to nonlinear, discontinuous policy functions. Interestingly, the resulting nonlinearity shares the heightened attention to inflation when inflation is outside the "zone," in discussions of inflation zone targeting (as in Bernanke and Mishkin, 1997, and Orphanides and Wieland, 1999).

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figure 1 about here

Given the central role of the inflation threshold, π^*, we next describe the influence of key parameters for its determination. Obviously, the adoption of a rule is more costly, and therefore less likely, the more informative the central bank's signal of the velocity shock is. Thus, the larger the variance of the observed part of the shock, σ_x, the higher the threshold, π^*.

The comparative statics with respect to inflation stickiness, λ, is not monotonic. In fact π^* diverges to infinity when λ tends to either zero or one. That means that a policy rule is never adopted if λ is close to either of the extreme values of zero and one. A small λ (close to zero) means that inflation is strongly dominated by past inflation. Expectations are unimportant for realized inflation, and thus do not have a large impact on the expected loss. For this reason, if $\lambda \to 0$, the ability of the rule to stabilize inflationary expectations eventually becomes worthless to the central bank, and it will therefore never be adopted, that is, $\pi^* \to \infty$. One the other hand, as $\lambda \to 1$, the central bank can immediately and *costlessly* disinflate. In that situation, the anti-inflation resolve of the central bank is irrelevant (as long as ω is strictly greater than zero). Any central bank will aim for the bliss point, which is zero expected inflation and zero expected output. The public knows that and accordingly expects zero inflation even if the central bank has not committed to a rule. Adoption of a rule can obviously not improve upon this outcome (there are no inflationary expectations to be stabilized). Consequently, there is no point in adopting a rule if $\lambda = 1$. The top left panel of figure 2 illustrates this relationship for an example economy. With the parameters shown, even a small deviation from rational expectations (λ slightly smaller than one) changes π^* dramatically.

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figure 2 about here

The comparative statics with respect to the anti-inflation resolve, ω, of the central bank is also not monotonic. Again, π^* diverges to infinity when ω tends to either zero or one. It is easily seen why $\pi^* \to \infty$ when $\omega \to 0$: The less the central bank cares about inflation, the smaller is the value of the expectational effect of adopting a rule to the central bank. Thus, the less likely is the central bank to prefer a rule over the flexibility of discretion which allows the central bank to stabilize output better. It is somewhat more difficult to see why $\omega \to 1$ implies $\pi^* \to \infty$ as well. This result means that a central bank whose sole interest is to control inflation will also never adopt a rule. The reason is that output affects inflation through the Phillips curve (2). Thus, an extreme hawk (ω close to one) does not really care about the output loss that is necessary to overcome the stickiness of inflation. Such a central bank will simply set m such that $(1-\lambda)\pi_0 + \alpha y = 0$, and thus force inflation to zero (except for the unobservable shocks, ε and ν, the central bank can do nothing about). In order to do that, the central bank wants to be able to control output well, which precludes committing to a rule. In other words, such a central bank does not adopt a rule because it wants to be able to choose the deepness of the recession precisely, $y = -\pi_0(1-\lambda)/\alpha$. The top right panel of figure 2 illustrates these comparative statics. As can be seen there, π^* is decreasing in ω except when ω becomes very high.

In summary, rules are not for central banks with extreme preferences, be they doves or hawks. They are of interest for central banks with moderate preferences only. And they become interesting only if expectations are neither just forward nor just backward looking.

Finally, we note that the comparative statics of π^* with respect to the slope of the Phillips curve, α, are also not monotonic. If the short-run Phillips curve is almost vertical,

$\alpha \to \infty$, disinflation is almost costless. Accordingly, tying ones hand by a rule has almost no advantages. Consequently, it is very unlikely that a rule is adopted, that is, $\pi^* \to \infty$. On the other hand, if the Phillips curve is flat, $\alpha = 0$, then expected inflation is independent of output and of monetary policy. As a consequence, adoption of a rule cannot stabilize inflation expectations, and provides therefore no advantage over discretion. As a result, adoption of a rule cannot be optimal if the Phillips curve is flat, or formally, $\pi^* \to \infty$ as $\alpha \to 0$. The bottom left panel of figure 2 depicts the relationship between α and π^*.

4 Imperfect credibility

Imperfect credibility of the central bank means that the public is not completely sure about the anti-inflation resolve, ω, of the central bank. We investigate the simplest case of this incomplete information problem, following the examples presented in Backus and Driffill (1985) and Barro (1986). We assume that there are only two possible types of central banks, namely $\overline{\omega}$ and $\underline{\omega}$, with $0 < \underline{\omega} < \overline{\omega}$. We call the central bank with preference parameter $\overline{\omega}$ the *hawk* and the one with parameter $\underline{\omega}$ the *dove*. The public believes that $\omega = \overline{\omega}$ with probability p, and $\omega = \underline{\omega}$ with probability $1 - p$. We assume $0 < p < 1$, so that information is incomplete. Using proposition 1, we can determine the initial inflation thresholds for which each type commits to a rule under complete information. Let $\underline{\pi}^*$ and $\overline{\pi}^*$ denote these thresholds for the types $\underline{\omega}$ and $\overline{\omega}$, respectively. As we noted earlier, unless a central bank's anti-inflation resolve, ω, is extremely high, this threshold is decreasing with it. As we consider this to be the most empirically relevant case, we concentrate our attention to this case for the analysis of the imperfect credibility problem. A sufficient condition is that $\overline{\omega} \leq (1 - \lambda)/(1 - \lambda + \alpha^2)$. Then, $\underline{\omega} < \overline{\omega}$, implies that $\underline{\pi}^* > \overline{\pi}^*$. The following proposition characterizes the resulting equilibrium. (A proof is provided in appendix B.)

Proposition 2 *Suppose $\overline{\omega} \leq (1-\lambda)/(1-\lambda+\alpha^2)$. Then, there exists a threshold $\overline{\pi}^{**}$ with $0 < \overline{\pi}^{**} < \overline{\pi}^{*} < \underline{\pi}^{*}$ such that the equilibrium is characterized as follows:*

$$
\begin{array}{llcl}
i) & \pi_0 < \overline{\pi}^{**} & \Longrightarrow & \textit{both central bank types use discretion,} \\
ii) & \overline{\pi}^{**} < \pi_0 < \underline{\pi}^{*} & \Longrightarrow & \textit{the hawk commits to a rule, the dove uses discretion,} \\
iii) & \underline{\pi}^{*} < \pi_0 & \Longrightarrow & \textit{both central bank types adopt a rule.}
\end{array}
$$

Importantly, when discretion is chosen by both types, that is when $\pi_0 < \overline{\pi}^{**}$, the equilibrium is a pooling one. By contrast, at higher inflation rates separating equilibria obtain. It is most convenient to describe the outcome by comparison to proposition 1. Consider the behavior of the inflation hawk, $\omega = \overline{\omega}$. According to proposition 1 a rule is adopted only when inflation is high, when $\pi_0 > \overline{\pi}^{*}$. Proposition 2 suggests that a rule will also be adopted at lower inflation levels if the central bank suffers from imperfect credibility. The credibility problem makes the adoption of a rule more attractive to the hawk ($\overline{\pi}^{**} < \overline{\pi}^{*}$).

The comparative statics of $\overline{\pi}^{**}$ are qualitatively the same as those of $\overline{\pi}^{*}$, the threshold for $\overline{\omega}$ with complete information (see section 4). With respect to the credibility parameter p, we find

$$\frac{d\overline{\pi}^{**}}{dp} = \frac{d\overline{\pi}^{**}}{d\tilde{\omega}} \cdot \frac{d\tilde{\omega}}{dp} > 0$$

(see item (*ii*) of step 4 of part 3 of the proof of proposition 2). Not too surprisingly, the adoption of a rule becomes less likely (the threshold $\overline{\pi}^{**}$ is greater) the better the credibility of the high resolve central bank is (the larger p is).

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figure 3 about here

Although in the model monetary policy is formulated in terms of the money supply, m, it is easier to gain an intuitive understanding of the policy outcomes by translating policy decisions to their corresponding implied expected inflation rate, $E(\pi)$. Figure 3 depicts expected inflation as a function of initial inflation π_0. Consider first the hawk. At low initial inflation rates he disinflates to some extent $[E(\pi) < \pi_0]$, in accordance with the optimal discretionary speed of disinflation, and given the fact that his credibility is not perfect. The pace of disinflation increases discontinuously once the hawk switches to the rule $[\pi_0 > \overline{\pi}^{**}]$. Two things happen here: First, the equilibrium changes from a pooling to a separating one, so that the credibility of the hawk becomes perfect. Second, the commitment to the rule stabilizes inflation expectations. This allows the hawk to disinflate faster.

At low inflation rates, the dove also disinflates, though to a lesser extent. The policy stance changes discontinuously once initial inflation exceeds $\overline{\pi}^{**}$ and the equilibrium switches from pooling to separating. At this point a hawk would switch to a rule and the very fact that the central bank in charge does *not* commit to a rule reveals to the public a low anti-inflation resolve. Recognizing that a dove runs the central bank, the public discontinuously increases its expectation about the inflation rate and the central bank adjusts to this fact by providing more liquidity in order to avoid a recession. Thus, at the threshold level $\overline{\pi}^{**}$, the dove discontinuously deflates at a slower rate. Once initial inflation exceeds $\underline{\pi}^{*}$ the dove switches to a rule, which allows it to disinflate faster, producing a second discontinuity in its policy reaction function.

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figure 4 about here

Proposition 2 is restricted to the case where the hawk is not too extreme, that is when $\overline{\omega} \leq (1-\lambda)/(1-\lambda+\alpha^2)$. The shaded area in Figure 4 shows parameter combinations that are ruled out by this restriction on $\overline{\omega}$. If the restriction does not hold, then other types of situations might arise.[8] However, these cases are less likely to be relevant empirically because they require an apparently implausible degree of anti-inflation resolve, one that indicates almost complete disregard for the impact of monetary policy on output stability.[9]

[8] The characterization of the equilibrium given in proposition 2 might still be valid for some set of parameters that violate this restriction. The restriction is a sufficient, but not a necessary condition for this characterization.

[9] For instance, consider $\sigma_x = 1$, $\alpha = 0.5$, $\lambda = 0.5$, $\underline{\omega} = 0.6$, $\overline{\omega} = 0.9$, and $p = 0.1$. This parameter combination violates the restriction because $\overline{\omega} > (1-\lambda)/(1-\lambda+\alpha^2) = 0.67$. This outcome is basically

5 Conclusion

Our analysis confirms that policymaker aversion to surrendering policy to a predetermined rule may be justified in an environment of low and stable inflation when discretion can yield the benefit of allowing the central bank to react to unexpected developments and dampen the impact of shocks to the economy. But even with the best of intentions, aggregate demand and supply disturbances may still result in an occasional buildup of inflation. When that occurs, the option of switching to a rule becomes a valuable policy tool for focusing expectations around a rapid disinflation path and reducing the costs of disinflation. Under those circumstances, adopting a rule becomes optimal. Adopting a rule becomes even more important for containing the costs for disinflation if a central bank's commitment towards a rapid return to price stability is in question. Then, the potential benefits for fine-tuning associated with discretion can be easily overshadowed by the restoration of credibility offered by adopting a rule and bringing the economy to an environment of price stability.

The Volcker disinflation following the Great Inflation of the 1970s in the United States, offers a rather useful example of how the intuition developed in our model may relate to monetary policy in practice. The short period from October 1979 to October 1982 is perhaps the only period since the gold standard when monetary policy in the United States could possibly be described as following a rule. Bernanke and Mihov (1998) provide evidence of a change in regime in that period. That the regime was adopted with Volcker's assuming the Chair of the Board of Governors is easily identified with a desire to boost the credibility of the

a contest among two "inflation maniacs" (very high ω's) in which the hawk has very low credibility (small p). This specification yields the following thresholds: $\underline{\pi}^* = 4.3$, $\overline{\pi}^* = 7.0$, $\underline{\pi}^{**} = 6.6$, and $\overline{\pi}^{**} = 2.2$. If $6.6 < \pi_0 < 7.0$, then we have an unusual separating equilibrium in which the dove commits to a rule, but the hawk uses discretion. In this extreme situation, the public can infer that the hawk is in charge if the central bank does *not* commit to a rule. The reason is that by not adopting a rule the hawk signals to the public that he does not care much about the cost of disinflation and therefore has little appreciation for the expectations stabilizing effect of a rule. With the same parameter values, if $4.3 < \pi_0 < 6.6$, then the dove wants to commit to a rule if and only if the hawk does ($\underline{\pi}^* < \pi_0 < \underline{\pi}^{**}$). The hawk, however, wants to commit to a rule if and only if the dove does not ($\overline{\pi}^{**} < \pi_0 < \overline{\pi}^*$). So this payoff structure produces a "matching pennies game" (Gibbons, 1992, p. 29ff) and therefore no pure strategy equilibrium exists (although there is one in mixed strategies).

anti-inflation resolve of the Federal Reserve System. But continuing adherence to a policy rule was not considered as essential once inflation was contained and the Federal Reserve's credibility restored. Policy returned to a discretionary regime during the remaining years of the Volcker era. Our analysis suggests that this sequence of decisions may have been entirely appropriate.

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Appendix A: Proof of Proposition 1

Proof. Since we are interested in subgame perfect equilibria only, we solve the game backwards.

Step 1, "expectations if the central bank is committed to a rule." We first consider that part of the game tree in which the central bank has chosen to commit to a rule in stage 2. In that case, there is nothing left for the central bank to decide in stage 5. It simply implements the announced money supply m^r. In stage 3, the public forms unbiased expectations, $\pi^e = E(\pi|\pi_0, m^r)$, which are given by (2), (3), and (4),

$$\pi^e = \lambda\pi^e + (1-\lambda)\pi_0 + \alpha m^r = E(\pi|\pi_0, m^r).$$

This has a unique solution for π^e, namely

$$\pi^e = \pi_0 + \frac{\alpha}{1-\lambda} m^r. \tag{5}$$

Step 2, "choosing the optimal rule in stage 2." The central bank will choose m^r such as to minimize expected loss. Using the above equation for expected inflation, and using (2), (3), (4), we can compute inflation and output as a function of m^r and of the shocks x, ν, and ε. Putting this into the loss function gives us the minimization problem of the central bank (subject to its commitment to a rule),

$$m^r := \arg\min_m E\left((1-\omega)\cdot(m+x+\nu)^2 + \omega\cdot(\pi_0 + m\alpha/(1-\lambda) + \alpha(x+\nu)+\varepsilon)^2\right).$$

The first order condition of this problem yields

$$m^r = -\frac{\alpha(1-\lambda)\omega}{(1-\lambda)^2(1-\omega)+\alpha^2\omega}\pi_0. \tag{6}$$

Step 3, "collapsing steps 1 and 2." We substitute (6) into (5) and find

$$\pi^e = \frac{(1-\lambda)^2}{(1-\lambda)^2 + \alpha^2\omega/(1-\omega)}\pi_0. \tag{7}$$

Step 4, "optimal discretionary policy in stage 5." Next we analyze the other half of the game tree in which the central bank has not committed to a rule. In this case, it can chose the optimal discretionary policy in stage 5. The central bank's problem is to set money supply such as to minimize its expected loss,

$$m^d(\pi^e, x) = \arg\min_m E((1-\omega)y^2 + \omega\pi^2 | \pi_0, \pi^e, x), \qquad \text{s.t.} \quad (2),\ (3),\ (4).$$

The first order condition of this problem yields

$$m^d(\pi^e, x) := -\frac{\alpha\omega}{1-\omega(1-\alpha^2)}(\lambda\pi^e + (1-\lambda)\pi_0) - x. \tag{8}$$

Step 5, "expectations under discretion." Given (8), inflation is determined by the Phillips curve (2) and the quantity equation (3),

$$\pi = (\lambda\pi^e + (1-\lambda)\pi_0) + \alpha(m^d(\pi^e, x) + x + \nu) + \varepsilon.$$

Taking expectations yields,

$$\pi^e = \left(1 - \frac{\alpha^2\omega}{1-\omega(1-\alpha^2)}\right)(\lambda\pi^e + (1-\lambda)\pi_0). \tag{9}$$

Again, there is a unique π^e that solves this,

$$\pi^e = \frac{1-\lambda}{1-\lambda+\alpha^2\omega/(1-\omega)}\pi_0. \tag{10}$$

Comparing this equation with (7) reveals that expected inflation is reduced by the simple fact of committing to the rule.

Step 6, "collapsing steps 4 and 5." We substitute (10) into (8) and find

$$\tilde{m}^d(x) = m^d\left(\frac{1-\lambda}{1-\lambda+\alpha^2\omega/(1-\omega)}\pi_0, x\right) = -\frac{\alpha(1-\lambda)\omega}{(1-\lambda)(1-\omega)+\alpha^2\omega}\pi_0 - x. \tag{11}$$

Step 7, "commit or not commit?" We finally reach the root of the tree and ask if the central bank will optimally commit to a rule in stage 2 or not. From (2), (3), (4), (6), and (7), we can compute output and inflation if the central bank commits to a rule, and we can substitute this into the loss function,

$$(1-\omega)\cdot\left(-\frac{\alpha(1-\lambda)\omega}{(1-\lambda)^2(1-\omega)+\alpha^2\omega}\pi_0 + x + \nu\right)^2 + \omega\cdot\left(\frac{(1-\lambda)^2(1-\omega)}{(1-\lambda)^2(1-\omega)+\alpha^2\omega}\pi_0 + \alpha(x+\nu) + \varepsilon\right)^2.$$

Similarly, using (10) and (11), we can compute the loss associated with discretion,

$$(1-\omega)\cdot\left(-\frac{\alpha(1-\lambda)\omega}{(1-\lambda)(1-\omega)+\alpha^2\omega}\pi_0+\nu\right)^2+\omega\cdot\left(\frac{(1-\lambda)(1-\omega)}{(1-\lambda)(1-\omega)+\alpha^2\omega}\pi_0+\alpha\nu+\varepsilon\right)^2.$$

Taking expectations yields, respectively,

$$\begin{aligned} E(loss|rule) &= \frac{(1-\lambda)^2\omega(1-\omega)}{(1-\lambda)^2(1-\omega)+\alpha^2\omega}\pi_0^2+\omega\sigma_\varepsilon^2+(1-\omega(1-\alpha^2))\sigma_\nu^2 \\ &\qquad +(1-\omega(1-\alpha^2))\sigma_x^2, \qquad (12) \\ E(loss|discretion) &= \frac{(1-\lambda)^2\omega(1-\omega)(1-\omega(1-\alpha^2))}{((1-\lambda)(1-\omega)+\alpha^2\omega)^2}\pi_0^2+\omega\sigma_\varepsilon^2+(1-\omega(1-\alpha^2))\sigma_\nu^2. \qquad (13) \end{aligned}$$

The former is smaller than the latter if and only if

$$\pi_0>\pi^*:=\sigma_x\cdot\sqrt{(1-\omega(1-\alpha^2))((1-\lambda)^2(1-\omega)+\alpha^2\omega)}\cdot\frac{(1-\lambda)(1-\omega)+\alpha^2\omega}{\alpha\lambda(1-\lambda)\omega(1-\omega)}. \qquad (14)$$

Thus, it pays for the central bank to commit to a rule if and only if $\pi_0 > \pi^*$. QED

The components of (12) and (13) can be interpreted as follows: The first component is the expected loss which is due to the fact that inflation is not completely flexible. For that reason the central bank cannot immediately and costlessly disinflate. This component is smaller with the rule than under discretion because the announcement of the rule allows the central bank to disinflate more rapidly because the announcement of the rule has an effect on expectations. One might say that the announcement is "half the way" to disinflation. This effect can most clearly be seen by comparing (7) with (10). The differences between the first components in (12) and (13) captures the effect of this difference on the expected loss. The second and third components are the expected losses due to the shocks that the central bank cannot observe, i.e. the supply shock ε and the unobserved part of the velocity shock ν. The fourth component in (12) is the loss due to that part of the velocity shock that the central bank can observe in stage 4 of the game, x. Under commitment to a rule, the central bank has to decide on the money supply before knowing x, so this shock also causes an expected loss. Under discretion, the central bank accommodates this shock by adjusting the money supply accordingly. As a result, x causes no loss and the corresponding term is absent from (13).

Appendix B: Proof of Proposition 2

Proof. We distinguish two types of equilibria. We call an equilibrium *pooling* if both types of central bank play the same strategy in stage 2, because then the public cannot tell for sure what type of central bank is in charge when they form expectations. If the two central bank types play different strategies in stage 2, we say that the equilibrium is *separating.* For instance, if one type commits to a rule but the other type does not, the public is able to determine which type of central bank is in charge simply by observing whether the central bank has adopted a rule or not.

We call the hawk the *opponent* of the dove, and vice versa. Depending on the opponent's choice in stage 2 of the game, the central bank faces different possible replies with different implications. Part 1 of the proof deals with the central bank's decision problem if the opponent does not adopt a rule. Part 2 investigates the central bank's options if the opponent does commit to a rule. Part 3 computes the equilibrium.

In the following, $L_\omega(R, D)$ refers to the minimal expected loss for the central bank of type ω if it does commit to a rule (the first argument, R), but its opponent does not commit to a rule (the second argument, D). $L_\omega(R, R)$, $L_\omega(D, R)$, and $L_\omega(D, D)$ are defined likewise.

PART 1: If the opponent does not adopt a rule…

$\rightarrow$ *possible reply: "do also not adopt a rule."* Since both central bank types do the same thing in stage 2 (they opt for discretion), the public cannot tell the type of the central bank. If this is an equilibrium, it is a pooling one.

In stage 5, the central bank will choose the discretionary policy that minimizes its expected loss, given expected inflation. This is precisely the same problem as with complete information (step 4 of the proof of proposition 1), so the result is also the same, namely equation (8). With two types, this gives rise to two separate functions—one for each type—defining the optimal discretionary policy,

$$m^d_\omega(\pi^e, x) := -\frac{\alpha\omega}{1-\omega(1-\alpha^2)}(\lambda\pi^e + (1-\lambda)\pi_0) - x, \qquad \omega = \underline{\omega}, \overline{\omega}. \tag{15}$$

Of course, these two money supplies are compatible with to different inflation rates, which can be found in the same way as in step 5 of the proof of proposition 1, giving rise to two equations like (9), except that ω is replaced by $\overline{\omega}$ and $\underline{\omega}$, respectively. The public's expected inflation is a weighted average of the expectations consistent with each type of central bank,

$$\pi^e = pE(\pi|\omega = \overline{\omega}) + (1-p)E(\pi|\omega = \underline{\omega}).$$

Solving this for π^e yields (after some transformations),

$$\pi^e \;=\; \frac{1-\lambda}{1-\lambda+\alpha^2\tilde{\omega}/(1-\tilde{\omega})} \cdot \pi_0, \tag{16}$$

$$\text{with} \quad \tilde{\omega} := \frac{(1-\overline{\omega}(1-\alpha^2))\underline{\omega} + p(\overline{\omega}-\underline{\omega})}{(1-\overline{\omega}(1-\alpha^2)) + p(\overline{\omega}-\underline{\omega})(1-\alpha^2)}. \tag{17}$$

π^e here is just the same as with perfect credibility, except that the anti-inflation resolve known with certainty, ω in equation (10), is replaced by $\tilde{\omega}$. Inspection of the definition of $\tilde{\omega}$ reveals that, as we increase p from 0 to 1, $\tilde{\omega}$ monotonically increases from $\tilde{\omega} = \underline{\omega}$ to $\tilde{\omega} = \overline{\omega}$. This makes sense intuitively: The public expects the same inflation rate from a discretionary central bank that is imperfectly credible as it would expect from a perfectly credible central bank with type $\tilde{\omega}$, where $\tilde{\omega}$ is some convex combination of the dove's resolve $\underline{\omega}$ and of the hawk's resolve $\overline{\omega}$.

Combining (15) and (16) yields the optimal discretionary policy of both types as a function of the observed velocity shock only,

$$\tilde{m}^d_\omega(x) := -\frac{\alpha(1-\lambda)\omega}{(1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega}} \cdot \frac{1-\tilde{\omega}(1-\alpha^2)}{1-\omega(1-\alpha^2)} \cdot \pi_0 - x, \qquad \text{for} \quad \omega = \underline{\omega}, \overline{\omega}. \tag{18}$$

This is the optimal discretionary policy if the public has incomplete information about the central bank's type (i.e. in a pooling situation).

In order to compute the expected loss to the central bank from this policy, we combine (2), (3), and (18) to compute y and π, which we can then put into the loss function (1),

$$(1-\omega)\cdot\left(-\frac{\alpha(1-\lambda)\omega}{(1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega}} \cdot \frac{1-\tilde{\omega}(1-\alpha^2)}{1-\omega(1-\alpha^2)} \cdot \pi_0 + \nu\right)^2 + \\ \omega\cdot\left(\frac{(1-\lambda)(1-\omega)}{(1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega}} \cdot \frac{1-\tilde{\omega}(1-\alpha^2)}{1-\omega(1-\alpha^2)} \cdot \pi_0 + \alpha\nu + \varepsilon\right)^2.$$

Taking expectations yields

$$L_\omega(D,D) := \frac{(1-\lambda)^2\omega(1-\omega)(1-\tilde{\omega}(1-\alpha^2))^2}{((1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega})^2(1-\omega(1-\alpha^2))} \cdot \pi_0^2 + \omega\sigma_\varepsilon^2 + (1-\omega(1-\alpha^2))\sigma_\nu^2. \tag{19}$$

→ *possible reply: "do adopt a rule (force separation)."* In that case, money supply in stage 5 of the game is determined by the rule the central bank has committed to, and the type of the central bank becomes irrelevant to the public. Expected inflation is simply a function of the announced money supply m^r—just as in the complete information case. Hence, the optimal rule is given by equation (6), and expected inflation is given by (7). The expected loss of this policy to the central bank is defined in (12), which we repeat here for completeness,

$$L_\omega(R, D) := \frac{(1-\lambda)^2\omega(1-\omega)}{(1-\lambda)^2(1-\omega)+\alpha^2\omega}\pi_0^2 + \omega\sigma_\varepsilon^2 + (1-\omega(1-\alpha^2))\sigma_\nu^2 + (1-\omega(1-\alpha^2))\sigma_x^2. \quad (12)$$

PART 2: If the opponent does commit to a rule...

→ *possible reply: "adopt a rule as well."* The only way to avoid separation is to "keep up appearances" by adopting *the same rule* as the opponent. Yet, when the central bank commits to a rule, the resulting policy becomes independent of the central bank's type; it is simply a function of the rule itself that has been adopted. As a consequence, the type of the central bank becomes irrelevant to the public. Accordingly, when a central bank adopts a rule, it might just as well adopt the rule that minimizes its own expected loss, no matter if this rule causes separation or not. This minimal expected loss is again independent of the strategy of the opponent. Hence,

$$L_\omega(R, R) := L_\omega(R, D). \quad (20)$$

→ *possible reply: "use discretion."* This causes separation; the public can tell the type of the central bank that is in charge because one type commits to a rule and the other does not. Hence, the problem is the same as with complete information, and the solution is also the same, see equations (10) and (11). The expected loss is defined in (13), which we repeat here,

$$L_\omega(D, R) := \frac{(1-\lambda)^2\omega(1-\omega)(1-\omega(1-\alpha^2))}{((1-\lambda)(1-\omega)+\alpha^2\omega)^2}\pi_0^2 + \omega\sigma_\varepsilon^2 + (1-\omega(1-\alpha^2))\sigma_\nu^2. \quad (13)$$

PART 3: Equilibrium.

Step 1: reduce strategy space. The strategy space is infinite, since it consists of the collection of pairs (m^r, m^d) whose first component is either $\emptyset$ or a real number, and the second component is a function from the real plane to the real line. However, when the opponent chooses not to commit to some rule, the only relevant strategies for the central bank to consider are forcing separation by choosing the optimal rule, or not forcing separation by choosing the optimal discretionary policy. Likewise, when the opponent does commit to a rule, the only alternatives the central bank has to choose from are the optimal rule and the optimal discretionary policy. In this way we can reduce this infinite game to a finite game. Figure 5 shows the normal form of this reduced game.

---

figure 5 about here

Step 2: compute threshold if opponent uses discretion. The gain to central bank-type ω from committing to a rule is the expected loss from discretion minus the expected loss from commitment to a rule. If the opponent uses discretion, this amounts to

$$L_\omega(D,D) - L_\omega(R,D) = \Bigg(\frac{(1-\tilde{\omega}(1-\alpha^2))^2}{((1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega})^2(1-\omega(1-\alpha^2))} - \frac{1}{(1-\lambda)^2(1-\omega)+\alpha^2\omega} \Bigg)(1-\lambda)^2\omega(1-\omega)\pi_0^2 - (1+\omega(1-\alpha^2))\sigma_x^2.$$

A few transformations reveal that $L_\omega(D,D) - L_\omega(R,D) > 0$ if and only if

$$\delta_\omega \cdot \pi_0^2 \; > \; \sigma_x^2 \cdot \frac{(1-\tilde{\omega}(1-\alpha^2))^2((1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega})^2((1-\lambda)(1-\omega)+\alpha^2\omega)}{(1-\lambda)^2\omega(1-\omega)}$$

$$\text{with} \quad \delta_\omega := (1-\tilde{\omega}(1-\alpha^2))^2((1-\lambda)^2(1-\omega)+\alpha^2\omega) - (1-\omega(1-\alpha^2))((1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega})^2.$$

If $\delta_\omega \leq 0$, this inequality cannot hold, in which case the central bank always prefers discretion. Furthermore, if $\pi_0 = 0$, then also this inequality is necessarily violated. Hence, no central bank (no matter what its anti-inflation resolve) will commit to a rule if initial inflation is zero. If $\delta_\omega > 0$ this equation has a unique positive root at

$$\pi^{**} := \sigma_x \cdot \frac{(1-\omega(1-\alpha^2))((1-\lambda)(1-\tilde{\omega})+\alpha^2\tilde{\omega})}{1-\lambda} \cdot \sqrt{\frac{(1-\lambda)^2(1-\omega)+\alpha^2\omega}{\omega(1-\omega)\delta_\omega}}. \tag{21}$$

If $\omega = \underline{\omega}$, this equation defines $\underline{\pi}^{**}$, the dove's threshold; if $\omega = \overline{\omega}$, this equation defines $\overline{\pi}^{**}$. If the dove opts for discretion, the hawk will commit to a rule if and only if $\pi_0 > \overline{\pi}^{**}$, and analogously for the dove.

Step 3: if $\overline{\omega} \leq \Omega := (1-\lambda)/(1-\lambda+\alpha^2)$, then $\overline{\pi}^* < \underline{\pi}^*$. Differentiating π^* as defined in (14) with respect to ω reveals that π^* is decreasing as a function of ω for $\omega < \Omega$, and achieves a minimum at $\omega = \Omega$. Thus, if $\underline{\omega} < \overline{\omega} \leq \Omega$, then $\underline{\pi}^* > \overline{\pi}^*$.

Step 4: to show, $\underline{\pi}^* \leq \underline{\pi}^{**}$ and $\overline{\pi}^{**} \leq \overline{\pi}^*$. (i) Note that, as $p \to 0$, we have $\tilde{\omega} \to \underline{\omega}$ and $\delta_{\underline{\omega}} \to \alpha^2\lambda^2\underline{\omega}(1-\underline{\omega})(1-\underline{\omega}(1-\alpha^2))$. Substituting this into (21) yields $\lim_{p\to 0} \underline{\pi}^{**} = \underline{\pi}^*$. Similarly, $\lim_{p\to 1} \overline{\pi}^{**} = \overline{\pi}^*$.

(ii) Next, note that $d\pi^{**}/dp = d\pi^{**}/d\tilde{\omega} \cdot d\tilde{\omega}/dp$. Furthermore, using (17) we can compute that $d\tilde{\omega}/dp > 0$. It remains to determine the sign of $d\pi^{**}/d\tilde{\omega}$.

It is hard to see the sign of $d\pi^{**}/d\tilde{\omega}$, but because $\pi^{**} > 0$, multiplying π^{**} with itself is a positive monotonic transformation, so that $\text{sign}(d\pi^{**}/d\tilde{\omega}) = \text{sign}(d(\pi^{**})^2/d\tilde{\omega})$. This is easier to evaluate. We first rewrite $(\pi^{**})^2$ using (21) as follows,

$$(\pi^{**})^2 = a \cdot \frac{((1-\lambda)(1-\tilde{\omega}) + \alpha^2\tilde{\omega})^2}{\delta_w},$$

where a is some positive number unrelated to $\tilde{\omega}$. We can compute

$$\frac{d(\pi^{**})^2}{d\tilde{\omega}} = a \cdot \frac{2((1-\lambda)^2(1-\omega) + a^2\omega)}{\alpha^2\lambda \cdot b^2} \cdot (1-\tilde{\omega}(1-\alpha^2)) \cdot ((1-\lambda)(1-\tilde{\omega}) + \alpha^2\tilde{\omega}),$$

where b is some arbitrary real number (which does depend on $\tilde{\omega}$). This expression is unambiguously strictly positive, thus $d\pi^{**}/d\tilde{\omega} > 0$, and therefore $d\pi^{**}/dp > 0$.

(iii) From (i) and (ii) it follows that $\underline{\pi}^* \leq \underline{\pi}^{**}$ and $\overline{\pi}^{**} \leq \overline{\pi}^*$.

Step 5: types of equilibria. In the previous two steps we have established that $0 < \overline{\pi}^{**} < \overline{\pi}^* < \underline{\pi}^* < \underline{\pi}^{**}$. The relation of π_0 to $\underline{\pi}^{**}$ and $\overline{\pi}^{**}$, respectively, tells us the best reply of both central bank types if the opponent uses discretion (i.e. should the central bank force separation by adopting a rule or not). Similarly, the relation of π_0 to $\underline{\pi}^*$ and $\overline{\pi}^*$, respectively, tells us each central bank type's best reply if the opponent does force separation (i.e. adopts a rule) so that information is complete. This gives rise to five different types of equilibria, as shown in figure 6.

figure 6 about here

If $\pi_0 < \overline{\pi}^{**}$, then discretion is a dominant strategy for both central bank types. If $\overline{\pi}^{**} < \pi_0 < \overline{\pi}^{*}$, then discretion remains a dominant strategy for the dove, but the hawk adopts a rule because $\overline{\pi}^{**} < \pi_0$. If $\overline{\pi}^{*} < \pi_0$, then commitment to a rule is a dominant strategy for the hawk. The dove replies with discretion if $\pi_0 < \underline{\pi}^{*}$; it adopts a rule otherwise. If $\underline{\pi}^{**} < \pi_0$, then adoption of a rule is a dominant strategy for both central bank types. QED

Equations (16) and (17) are the key to this proof. Inspection of (17) reveals that, in a pooling equilibrium, the public treats the imperfectly credible central bank as having a type $\tilde{\omega}$ that is some convex combination of the dove's and the hawk's anti-inflation resolve. $\tilde{\omega}$ monotonically increases with p, and accordingly, by (16), expected inflation decreases the more the public believes the hawk is in charge. Comparing (16) with (10) also reveals that the hawk is penalized by this credibility problem. The dove, on the other hand, has an advantage from the incompleteness of information. This explains why the dove never forces separation. The hawk, however, may have an incentive to buy the advantageous expectations of the policy rule, given by (5), and pay the price of the reduced flexibility.

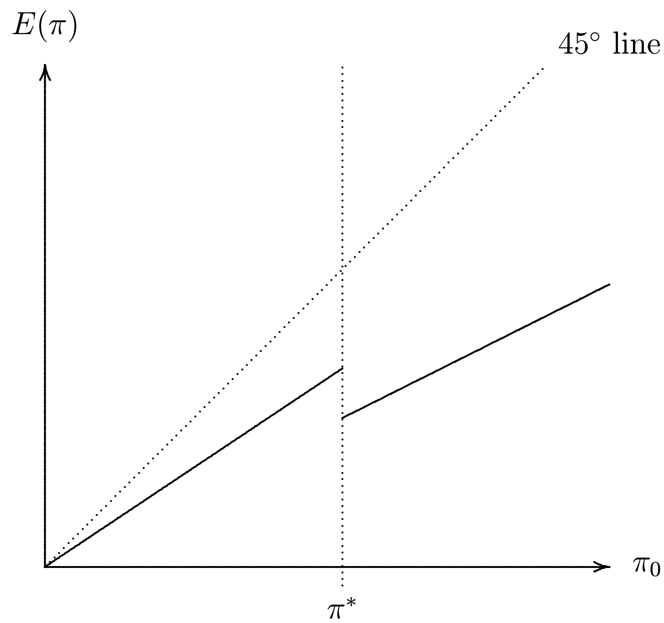

Figure 1. Policy function in terms of expected disinflation, with $\alpha = 0.5$, $\lambda = 0.5$, $\omega = 0.5$, and $\sigma_x = \sigma_\nu = \sigma_\varepsilon = 1$.

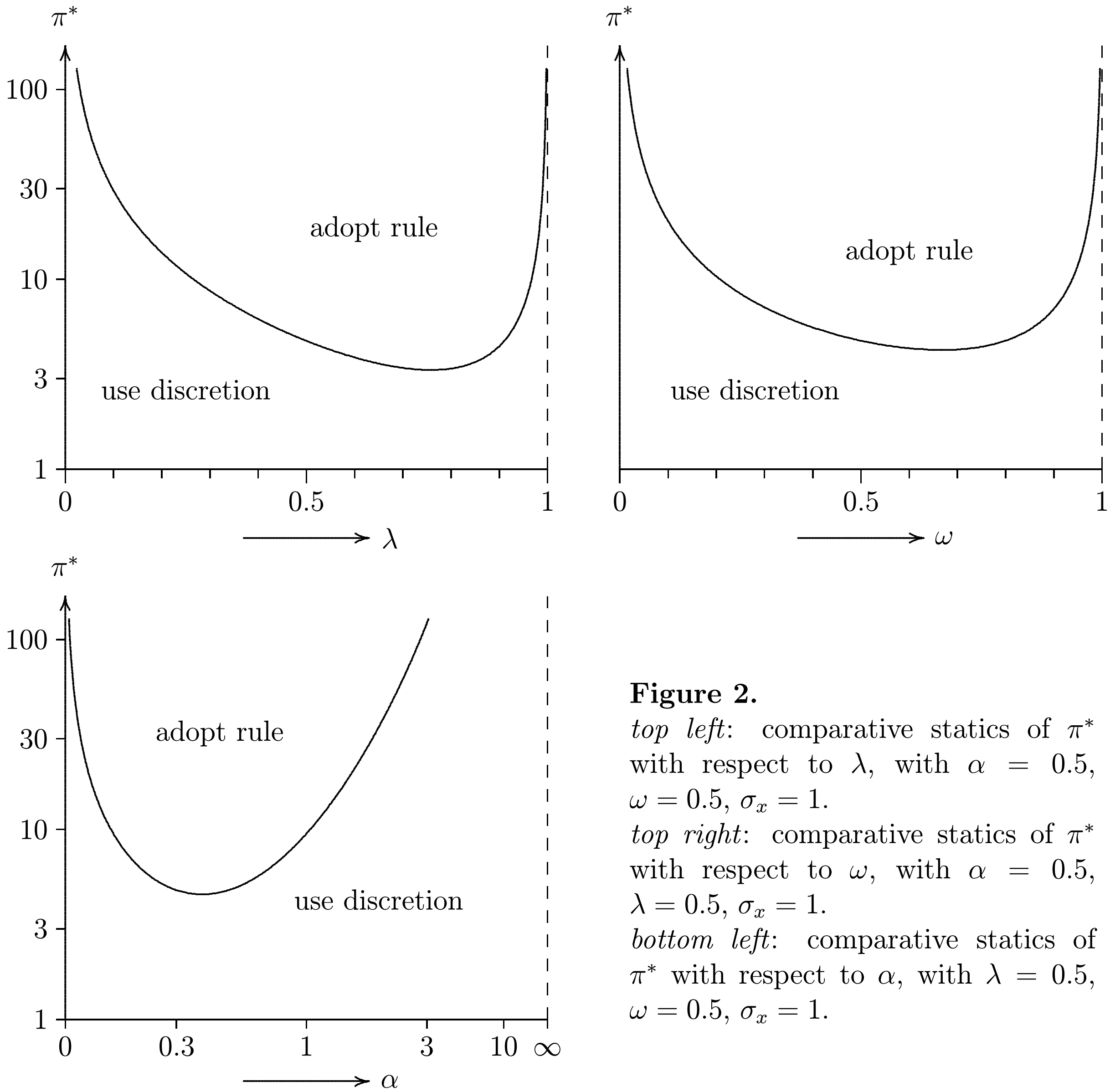

Figure 2.
top left: comparative statics of π^* with respect to λ, with $\alpha = 0.5$, $\omega = 0.5$, $\sigma_x = 1$.
top right: comparative statics of π^* with respect to ω, with $\alpha = 0.5$, $\lambda = 0.5$, $\sigma_x = 1$.
bottom left: comparative statics of π^* with respect to α, with $\lambda = 0.5$, $\omega = 0.5$, $\sigma_x = 1$.

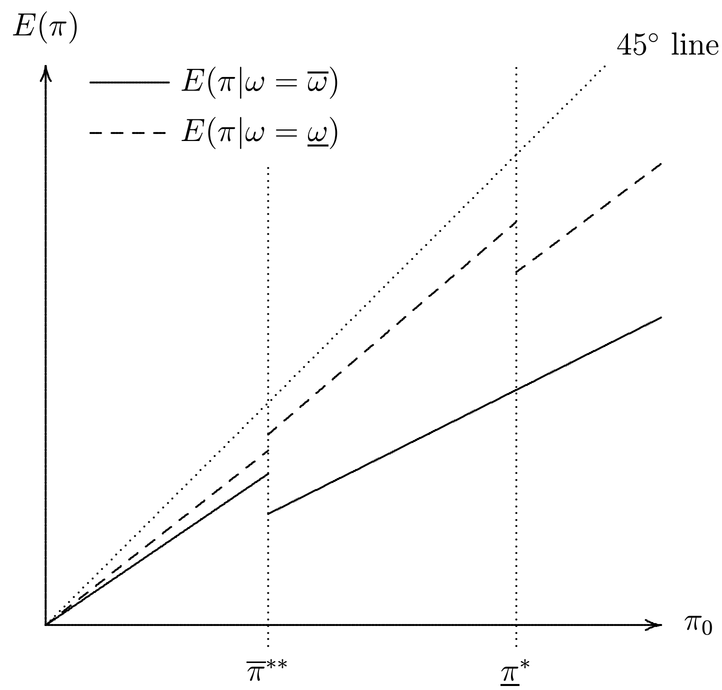

Figure 3. Policy functions in terms of expected disinflation of both central bank types, with $\alpha = 0.5$, $\lambda = 0.5$, $\underline{\omega} = 0.25$, $\overline{\omega} = 0.5$, $p = 0.8$, and $\sigma_x = \sigma_\nu = \sigma_\varepsilon = 1$.

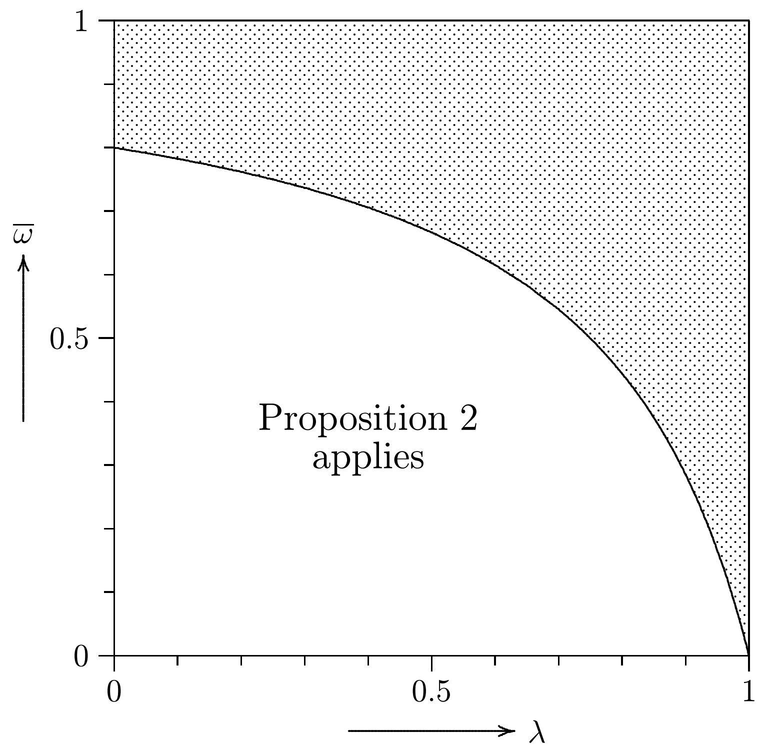

Figure 4. Range of applicability of proposition 2 with $\alpha = 0.5$.

		$\overline{\omega}$	
		R	D
$\underline{\omega}$	R	$L_{\underline{\omega}}(R, R)$, $L_{\overline{\varpi}}(R, R)$	$L_{\underline{\omega}}(R, D)$, $L_{\overline{\varpi}}(D, R)$
	D	$L_{\underline{\omega}}(D, R)$, $L_{\overline{\varpi}}(R, D)$	$L_{\underline{\omega}}(D, D)$, $L_{\overline{\varpi}}(D, D)$

Figure 5. Normal form of the reduced game with imperfect credibility.

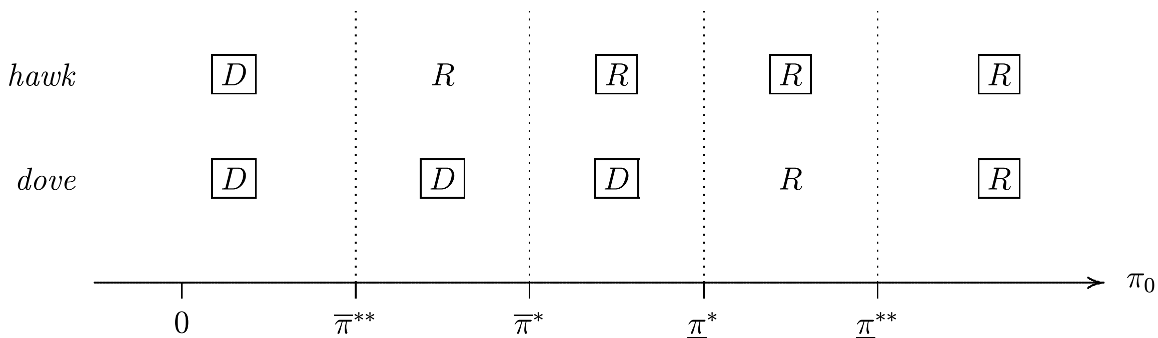

Figure 6. Five types of equilibria and their regions. $\boxed{D}$ means that discretion is a dominant strategy; $\boxed{R}$ means that commitment to a rule is a dominant strategy; R means that adopting a rule is the best reply in equilibrium, but is not a dominant strategy.