An Analysis of Government Spending in the Frequency Domain

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ABSTRACT

This paper utilizes frequency-domain techniques to identify and characterize economically important properties of government spending. Using post-war data for the United States, the paper first identifies peaks in the estimated spectra of the major components of fiscal spending. Second, the paper examines the relationship between these fiscal variables and various measures of aggregate economic activity. The analysis reveals that defense spending is best modeled as exogenous with respect to the aggregate economy, and nondefense spending (growth) appears to be white noise. Further, the unemployment rate has a very high coherency at the business cycle frequencies with unemployment insurance but far smaller coherency with other transfer payments. Finally, the paper finds a moderate degree of direct substitutability between certain types of government spending and private consumption and in the process illustrates how spectral techniques can be readily combined with a standard intertemporal optimizing model.

An Analysis of Government Spending in the Frequency Domain

I. Introduction

Recent legislative experience in the United States suggests a growing reliance on reductions on the spending side of the federal government budget to achieve the desired medium-term evolution of the budget surplus. Moreover, the long-term budget outlook under current law will be strongly influenced by explosive growth in social security and Medicare spending as the baby-boom generation retires. In addition to these dramatic current and prospective developments for government spending, recent papers by Blanchard and Perotti (1998) and by Eichenbaum and Fisher (1998) suggest renewed academic interest in the effects of government spending on the economy.

With such factors as motivation, this paper seeks to identify and characterize economically important properties of government spending both in isolation and in relation to other macroeconomic variables. To achieve this goal, the paper adopts a frequency-domain approach. In particular, the paper first will identify the important peaks of the estimated spectra of the major components of fiscal spending, using quarterly post-war data from the U.S. National Income and Product Accounts. This will allow determination of whether a substantial portion of the variance of a series can be attributed to components associated with business cycle and seasonal frequencies or with low frequencies, most likely related to demographic or other long-term, slowly evolving factors, and thus help create a set of stylized facts in the frequency domain. Second, the paper will examine the relationship between fiscal variables and various measures of aggregate economic activity using bivariate spectral techniques. This will shed light on the frequencies, if any, at which government spending is related to business cycle variables and relative prices. It also will allow us to examine the degree of direct substitutablility between certain types of government spending and private consumption and in the process illustrate how spectral techniques can be readily combined with a standard intertemporal optimizing model.

Of course, any structural interpretation of a relationship between two variables must be pursued cautiously. For example, a strong frequency domain relationship between unemployment insurance outlays and the unemployment rate can be given at least two interpretations. On the one hand, an increase in the latter will increase the former for a given structure of the unemployment insurance program; this effect is likely to show up prominently at the business cycle frequencies. However, expansions of the unemployment insurance program over time likely will boost the unemployment rate; such an effect arguably would show up at low frequencies if the underlying structural or trend level of the unemployment rate is altered, but also could show up at business cycle frequencies if the cyclical properties of the unemployment rate are affected as well.

The estimated population spectra of government spending variables will allow us to confirm certain well-known properties of these series in the time domain but not confirm others.

Although all second-moment properties would be identical in a very large sample, differences arise given small samples. Indeed, the existence of small samples helps to justify the study of government spending in the frequency, as well as, the time domain.[1] However, the spectral approach is not without its limitations. There typically are a lot of parameters to be estimated and it is difficult to identify low frequency components. The latter suggests difficulty, for example, in finding low-frequency components of government spending arising from slowly changing demographic factors such as retirement dynamics. In any case, based on the framework of this paper it is hoped that spectral techniques will be seen as much easier to use than probably most applied economists realize.

The remainder of the paper is organized as follows. Section IIA analyzes various components of government consumption and investment expenditures and their relationship to the unemployment rate. Section IIB examines government transfer payments and grants as well as their relationship to the unemployment rate and relative prices. Section IIC considers the issue of the degree of direct substitutability between private consumption and government spending. Section III offers some concluding thoughts. An appendix, written with the practitioner in mind, contains a summary of the main results from the spectral analysis of time series that are relevant for this paper.

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[1] Moreover, as shown in Engle (1976), spectral methods can be used to specify a time domain model.

II. Spectral Properties of Fiscal Variables

In this section, the properties of U.S. federal (and certain state and local) government seasonally adjusted fiscal variables taken from the National Income and Product Accounts (NIPAs) are discussed.[2] Most of these variables clearly are non-stationary in levels form and so they generally have been transformed into percent changes. Plots of the spectral densities and selected coherencies along with corresponding 95 percent confidence bands are included at the end of the paper.[3]

A. Government Consumption and Gross Investment Expenditures

We begin by examining consumption and investment expenditures on national defense in billions of current dollars, using quarterly NIPA data for the period from 1947:Q1 to 1997:Q1. The use of current dollars enables us to illustrate the "typical spectral shape" of macroeconomic variables identified by Granger (1966). Notice that the estimated spectrum has a prominent peak at very low frequencies, declines rapidly, and flattens out at high frequencies. Viewing this as evidence of nonstationarity,

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[2] There also are a few fiscal variables (not including transfer payments and grants) published on a not seasonally adjusted basis; in addition, these few variables are available only on a current dollar basis. In percent change form, their spectra (not displayed here) reveal dramatic spectral peaks at the 4-quarter and 2-quarter--or seasonal--frequencies. The spectrum of the percent change in nominal (nsa) federal consumption and investment spending also shows some spectral power at the business cycle frequencies.

[3] We utilize PROC SPECTRA from SAS to generate the basic spectral densities and squared coherencies. The respective 95 percent confidence bands, described in the appendix, were programmed by us.

we next examine the spectrum of defense consumption and investment expenditures as a percent of GDP, both in current dollars (Def/Y); the results are virtually identical if the variables are expressed in real terms. The spectrum of Def/Y also has roughly the typical spectral shape, suggesting that scaling by GDP may not be sufficient to achieve stationarity; an alternative interpretation is that Def/Y is a stationary series whose variance is primarily determined by very low frequency movements.[4] By contrast, the spectrum of the growth rate of defense consumption and investment expenditures does not have the typical spectral shape, suggesting that stationarity has been achieved. Moreover, the null hypothesis that the series is white noise is rejected by both the Fisher-Kappa (FK) and Bartlett-Kolmogorov-Smirnov (BKS) tests (as is the case for the other defense series discussed above).

Interestingly, the spectrum displays a prominent peak over the frequency range roughly between 0.15 and 0.30, corresponding to a period between 20 and 50 quarters. Because the spectral approach is nonparametric, with no explicit economic structure imposed, there is not a unique explanation of the spectral peak. One possibility is that it reflects recurring--if not exactly periodic--wartime (hot and cold) fluctuations in defense

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[4] Moreover, the spectra of federal government consumption and investment expenditures as a percent of GDP and total government consumption and investment expenditures as a percent of GDP (either in nominal or real terms) each have the typical spectral shape.

spending. Over this period there was the Korean War in the early 1950s, the Vietnam War in the late 1960s and early 1970s, the Carter-Reagan defense buildup of the 1980s, and the Gulf War in the early 1990s; these reasonably can be viewed as exogenous fluctuations and provide support for the common practice in macroeconomics of specifying defense expenditures as an exogenous variable or instrumental variable.

However, one might hypothesize that defense spending has a countercyclical component, involving, for example, increased recruitment during periods of cyclical downturns. This view receives a bit of support because there is a prominent peak in the (squared) coherency between the growth rate of real defense compensation and the change in the civilian unemployment rate at a frequency of about 0.7 or a period of 9 to 10 quarters; further the coherency at this frequency is statistically significant based on the Bloomfield test (indeed, values above roughly 0.3 are significantly different from zero). However, this evidence is far from overwhelming because the coherency itself is only about 0.4. Very similar results hold for the coherency between the growth rate of real defense investment spending and the change in the unemployment rate.

The estimated spectrum of the growth rate of real federal nondefense consumption and investment expenditures appears to have several prominent peaks including one at the business cycle frequencies (associated with a period of 5 years). However, based on the FK and the BKS tests, the hypothesis that this series is white noise cannot be rejected. The spectra for several of the components of total nondefense spending also are presented. Several have interesting looking spectra, especially the growth rate of nondefense services (compensation and other). However, in almost every case, the hypothesis of white noise cannot be rejected.

In addition to analyzing federal government spending, we have examined certain components of state and local government spending as well. For example, the hypothesis that the growth rate of real state and local consumption and investment spending is white noise is rejected using both the FK and BKS tests. Further, the spectrum of this series has power at low frequencies, reflecting long-run factors. It also has a modest peak at a period of about 20 quarters and the squared coherency between this series and the change in the unemployment rate is high (around 0.7) at business cycle frequencies corresponding to periods of about 12 to 14 quarters.

B. Government Transfer Payments and Grants

We now discuss the frequency domain properties of certain federal government transfer payments and grants, where the basic outlay data are expressed in nominal or current dollars. Unemployment insurance (regular plus extended), UI, is commonly viewed as the most important automatic stabilizer on the spending side of the budget. Its spectrum, based on quarterly data for the period 1959:Q1 to 1997:Q2, has the "typical spectral shape," showing no special spectral power at the business cycle frequencies. The spectrum of the unemployment rate has virtually the same shape. However, the (squared) coherency between these two series is very high at the business cycle frequencies, confirming the conventional view (and consistent with the methodological point in Sargent (1979), discussed in the appendix).

Moreover, the (squared) coherency between the growth in UI outlays and the change in the unemployment rate also is very high at the business cycle frequencies, and, indeed, the individual spectra of each series has a prominent peak at the business cycle frequencies.[5] Further, UI outlays as a share of GDP has a spectrum that does not display a peak at business cycle frequencies, although its coherence with the level of the unemployment rate is very high at these frequencies.

Finally, we can use the gain as a quantitative measure of the impact of an increase in the unemployment rate on UI outlays as a share of GDP. Over the range of business cycle frequencies, the gain is roughly constant with a value of about 0.18; this implies that a recession-induced increase in the unemployment rate of one percentage point will increase unemployment insurance outlays about $15 billion at an annual rate (as of 1997). This contrasts with recent estimates by the Office of Management and

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[5] We also have examined the spectrum of the growth in monthly, not seasonally adjusted unified budget outlays for unemployment insurance over the period January 1970 to January 1998. The spectrum displays no spectral power at the business cycle frequencies but clearly shows prominent peaks at the seasonal frequencies.

Budget (1998) that show total federal outlays increasing about $10 billion per point of unemployment (most of which results from higher unemployment insurance outlays). One possible explanation for the difference is that the gain at business cycle frequencies may be picking up the effect of expansions in the unemployment insurance program on the unemployment rate as well as the effect of unemployment on outlays.

Social security (OASDI) is the largest federal transfer program, comprising about 45 percent of total personal transfer payments in 1997. The FK test implies that the growth rate of social security payments over the period 1959:Q2 to 1997:Q1 is white noise, although the more powerful BKS test implies that the series is not white noise. Assuming that the series is not white noise, the spectrum has three interesting features.

First, there is a modest peak at business cycle frequencies around 0.6, corresponding to a period of 9 to 10 quarters. This suggests that social security may be a cyclically-sensitive program. However, conflicting evidence is provided by the (squared) coherency of the growth in OASDI spending and the change in the unemployment rate; the coherency is only about 0.2 at the business cycle frequencies. This contrasting evidence is consistent with the mixed results from the time domain which, when they have shown cyclical sensitivity of OASDI spending, have found a weak relationship.

Second, the spectrum of the growth rate of OASDI has prominent peaks at the seasonal frequencies (corresponding to periods of 4 and 2 quarters). At first glance, this might be surprising given that OASDI spending is a seasonally-adjusted series. However, the BEA has adopted the convention of "level adjusting" the series whenever a cost-of-living adjustment is made to social security payments; this adjustment has been made once every four quarters automatically since the mid-1970s and on an ad hoc basis in prior periods.[6]

Third, the spectrum displays power at very low frequencies which could reflect several factors such as general expansion of the program or demographic influences. The latter would not include the effects of the post-war baby-boom, baby-bust cycle because baby boomers have yet to retire. However, increases in life expectancy for retirees as well as increases in the number of retirees owing to population growth over the period may be captured in the low-frequency portion of the spectrum.[7]

Medicare has become the second largest federal transfer program since its inception in the mid-1960s; spending on Medicare amounted to about 25 percent of total transfer payments in 1997. Based on both the FK and the BKS tests, the growth rate

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[6] As might be expected, the seasonal peaks are eliminated in the spectrum of the 4-quarter growth rate of OASDI spending. Further, this spectrum has a modest peak at business cycle frequencies although at these frequencies the (squared) coherency between the 4-quarter growth rate of OASDI and the change in the unemployment rate is low.

[7] We also examine the properties of OASDI spending as a share of GDP. This variable has the "typical spectral shape," showing no peaks in the business cycle frequencies; moreover, the (squared) coherency with the level of the unemployment rate is low at these frequencies.

of Medicare over the period from 1967:Q4 to 1997:Q1 is not white noise. The spectrum displays most of its power at low frequencies, with no noticeable peak at the business cycle frequencies. Indeed, the squared coherency between the growth rate of Medicare and the change in the unemployment rate is very low at the business cycle frequencies, suggesting that this is not a cyclically-sensitive program, which generally accords with results from the time domain. Because the variance in Medicare growth apparently has been determined by slow moving factors, such as partially technology-driven changes in relative health care costs, it is natural to look for statistical evidence of such a relationship. Some supportive evidence is provided by the (squared) coherency between the growth rate of Medicare and relative medical services inflation (the growth rate of CPI, medical services minus the growth rate of the total CPI) which is high at low business-cycle frequencies, corresponding to periods between 20 and 30 quarters.

The spectrum of the growth rate of spending on the Food stamps program over the period 1963:Q2 to 1997:Q1 shows most of its power at very low frequencies, although two modest peaks appear at the business cycle frequencies. Moreover, the (squared) coherency between the growth rate of Food stamps and the change in the unemployment rate has a local maximum at business cycle frequencies; nevertheless, the maximum value is less than 0.3. Taken together the evidence suggests that the Food stamp program is cyclically sensitive, albeit with a sensitivity that is not quantitatively very large.

Medicaid is a federal grant-in-aid to state and local governments; indeed it is the largest grant, amounting to more than 40 percent of total grants in 1997. The spectrum of the growth rate of Medicaid spending has a small peak at the business cycle frequencies and large peaks at the seasonal frequencies, the latter again reflecting BEA conventions. Further, the coherency between the growth rate of Medicaid and the change in the unemployment rate has a local maximum at low business cycle frequencies; however, the value is low (around 0.3) suggesting that at best there is a quantitatively weak relationship.

C. Private Consumption and Government Spending

We now utilize spectral techniques to address the issue of the degree of direct substitutability between aggregate private consumption expenditures, C_t, and government spending, G_t. There has been limited empirical analysis of this issue in the time domain. Aschauer (1985), for example, estimates that an additional dollar of government consumption expenditures directly crowds out about 25 cents worth of private consumption. Kormendi and Meguire (1995) reach substantially the same conclusion, although Graham (1995) finds a value only half as large.

To help in interpreting subsequent results of our empirical cross spectral analysis, we provide a slightly modified version of the Aschauer framework. Agents maximize the expected present discounted value of utility of "effective" consumption, C^*_t, where $C^*_t = C_t + \psi G_t$. That is, for a given level of effective consumption, an additional unit of government spending will induce the agent to reduce private consumption by ψ units.

Subject to quadratic utility, rational expectations, and equality of the rate of time preference and the real interest rate, Aschauer establishes that:

$$\Delta C_t = \psi G_{t-1} - \psi E_{t-1}G_t + u_t \tag{1}$$

where $E_{t-1}G_t$ denotes the expected value of period t government spending based on information available at time t-1; u_t is a zero mean, white noise random variable. Further, for concreteness, we assume that G_t is an AR(2) process:

$$G_t = g_0 + g_1G_{t-1} + g_2G_{t-2} + \xi_t \tag{2}$$

where we impose $g_1 + g_2 = 1$.[8] Also, ξ_t is white noise with mean zero and variance, σ_ξ^2. It follows that $E_{t-1}G_t = g_0 + g_1G_{t-1} + g_2G_{t-2}$ and thus (9) can be rewritten as:

$$\Delta C_t = \alpha + \theta\Delta G_{t-1} + u_t \tag{3}$$

where $\alpha = -\psi g_0$ and $\theta = \psi(1 - g_1)$. From (2), it also follows that:

$$\Delta G_t = g_0 + g\Delta G_{t-1} + \xi_t \quad \text{or} \quad \Delta G_{t-1} = g_0 + g\Delta G_{t-2} + \xi_{t-1} \tag{4}$$

where $g = g_1 - 1$. That is, $\Delta G_{t-1} = X_t$ is an AR(1) process. Let $Y_t = \Delta C_t$. Combining expressions, we get:

$$Y_t = \alpha + \theta X_t + u_t \tag{5}$$

and we assume that Y_t and X_t are stationary.

We now analyze this model in the frequency domain,

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[8] We note that the assumed nonstationarity ($g_1+g_2 = 1$) of G_t is consistent with results using time domain techniques. For example, based on the augmented Dickey-Fuller test, one cannot reject the null hypothesis that combined real government consumption expenditure has a unit root over the post-war period.

ultimately deriving the squared coherency to aid in interpreting our empirical results. Chatfield (1996) shows that for an AR(1) process such as that for X_t, $s_x(\omega) = \sigma_\xi^2/(2\pi)(1 - 2g\cos\omega + g^2)$, where $s_z(\omega)$ denotes the spectrum of Z. Also, since u_t is white noise, its spectrum $s_u(\omega) = \sigma_u^2/2\pi$ as discussed in the appendix. In our example, the cross covariance function, $\gamma_{xy}(k) = COV(X_t, Y_{t+k})$ $= (\theta g^k \sigma_\xi^2/(1-g^2)$ and $\gamma_{xy}(k) = \gamma_{xy}(-k)$ for $k = 0, 1, 2, \ldots.$ Using (A6), the cross spectrum, $S_{xy}(\omega)$, is given by:

$S_{xy}(\omega) = (1/2\pi)(\theta/(1-g^2))\sigma_\xi^2[1 + 2g\cos\omega + 2g^2\cos2\omega + \ldots].$

In this case, the cross spectrum is real (i.e., the quadrature spectrum is zero) because of the property of our model that $\gamma_{xy}(k) = \gamma_{xy}(-k)$.[9] This implies that the co-spectrum equals the cross spectrum and that the gain, from (A8), equals $S_{xy}(\omega)/s_x(\omega)$. From (A7), the (squared) coherency for X and Y is:

$C(\omega) = (S_{xy}(\omega))^2/s_x(\omega)s_y(\omega).$

From the appendix we know that the spectrum of Y is given by:

$s_y(\omega) = (\text{gain})^2\, s_x(\omega) + s_u(\omega) = (S_{xy}(\omega))^2/s_x(\omega) + s_u(\omega).$

Thus, the coherency is given by:

$C(\omega) = 1/(1 + Z)$ where $Z = s_x(\omega)s_u(\omega)/(S_{xy}(\omega))^2$.

Recalling that $\theta = \psi(1 - g_1)$, it follows mainly from the expression for $S_{xy}(\omega)$ that:

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[9] The symmetry of the cross-covariance function depends on the assumption that ΔG_t is a stationary AR(1) process as in equation (4). If, instead, ΔG_t were a stationary AR(2) process, the symmetry property would not hold in general and, hence, the cross spectrum would not be real, making clean closed-form results difficult to derive.

$$\lim_{\psi \to 0} C(\omega) = 0 \quad \forall\omega$$

This result accords with intuition. If there is very little substitutability between private consumption and government spending ($\psi \approx 0$), we would expect that a measure of correlation between Y_t and X_t such as the coherency should be very low, as indeed it is at all frequencies. Technically, the coherency is zero because, for $\psi = 0$, $Y_t = \Delta C_t$ depends on the random process u_t, but not on ξ_t as well (because the ΔG_{t-1} term has dropped out of equation (3)). However, perhaps surprisingly, it is not the case, if the degree of substitutability is quite high ($\psi \approx 1$), that the coherency converges to unity or even to a high value. This is because Y_t and X_t are driven by different random processes: Y_t now depends directly on u_t and indirectly on ξ_t through its dependence on X_t, while X_t depends only on ξ_t.

By substitution of the expressions for $s_x(\omega)$, $s_u(\omega)$, and $(S_{xy}(\omega))^2$ into the expression for the squared coherency and by use of the properties of geometric series, it follows that if $\psi > 0$ and $g < 0$, then $C_{xy}(\omega=0) < C_{xy}(\omega=\pi/2) < C_{xy}(\omega=\pi)$; for $g > 0$, the inequality signs are reversed. This monotonicity property of the coherency across the three frequencies is useful for empirical purposes.

To make this result more concrete, we now compute sample values of the squared coherency at the three frequencies and for different values of the key parameters. Also it is assumed for simplicity that $\sigma_u^2 = \sigma_\xi^2$.

The following table summarizes the results. The first column shows that the squared coherency is zero when there is no substitutability between private and government consumption (ψ = 0). The second column shows low coherencies even if the degree of substitution is at the high end of the range of time domain estimates (ψ = .25). Comparing the first two columns suggests that it will be quite difficult to differentiate empirically between low and modest rates of substitutability using these

Table: Squared Coherencies

Frequencies	Parameter values			
	$g = -.5$ $\psi = 0$	$g = -.5$ $\psi = .25$	$g = -.5$ $\psi = .5$	$g = -.5$ $\psi = 1$
	(1)	(2)	(3)	(4)
$\omega = 0$	0	.007	.027	.11
$\omega = \pi/2$	0	.012	.048	.17
$\omega = \pi$	0	.059	.20	.50

spectral techniques. By contrast, the final two columns, which show much larger coherencies at higher frequencies, suggest that it is possible to distinguish between very low and very high degrees of substitutability. This example is meant to help interpret cross spectral estimates in terms of an economic model, but more generally to illustrate how spectral techniques can be combined with a standard intertemporal optimizing model of a representative consumer. Of course, other economic models presumably would generate different spectral patterns.[10]

With this in mind, we now turn to our empirical results. The private consumption variable of our model is proxied by per capita NIPA real personal consumption expenditures on nondurable goods and services; this measure omits spending on durables which has an investment component. Government spending is proxied for by per capita NIPA real federal plus state and local government consumption expenditures.[11] Each variable is in first differenced form and the resulting government variable is lagged one period to conform with the model; examination of the spectra suggests that each first-differenced variable is stationary. With the exception of a few isolated frequencies, the squared coherency is low; indeed, it is quite low at the three key frequencies examined in the theoretical discussion above ($\omega = 0$, $\pi/2$, π), allowing us to strongly discount the possibility of a high degree of substitutability. Moreover, as mentioned above, it is difficult to differentiate between low and moderate degrees

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[10] The spectral approach also might be useful in analyzing the relationship of government spending to variables other than private consumption. For example, future research might examine the potentially important relationship between government spending and private investment identified in Baxter and King (1993). They argue that a higher path of government spending (financed by current or future lump sum taxes) has a positive wealth effect on labor supply which in turn raises the marginal product of capital schedule and, hence, investment demand.

[11] Because NIPA government consumption spending includes an estimate of the consumption services of fixed government capital (CFC) whereas NIPA private consumption does not include the service flow from the stock of private durables, we also examine the relationship between private consumption and government consumption, excluding CFC. The results are essentially the same.

of substitutability. On the one hand, the coherencies at the three key frequencies are not significantly different from zero using the Bloomfield test, suggesting that there is no direct substitutability between aggregate private and public consumption expenditures. On the other hand, because the estimated coherencies are .001, .025, and .069 at frequencies 0, $\pi/2$, and π, respectively, it is tempting to conclude that ψ is approximately .25 based on the second column of the above table, implying that there is a moderate degree of direct substitutability between aggregate private and public consumption.

Intuitively, we might expect greater substitution to occur between more narrowly specified groups of goods and services. To examine this possibility, we consider the relationship between two government transfer-in-kind goods--Medicare and Food stamps--and their private consumption counterparts. We compute coherencies using both nominal and real magnitudes; we use the PCE implicit price deflator for medical care to deflate Medicare spending and the deflator for food to deflate Food stamp spending.

The (squared) coherency between the change in real Medicare spending per population member aged 65 and over and the change in real personal consumption expenditures on medical care (net of Medicare outlays) per total population member exceeds 0.5 for low frequencies corresponding to periods in excess of ten years and for business cycle frequencies corresponding to periods around 3 years. With the variables expressed in nominal terms, the coherency is around 0.5 only at very low frequencies. Although the results are somewhat mixed, on balance, the evidence in light of the results from our optimizing model above, although not strictly applicable, suggests at least a moderate degree of direct substitutability between Medicare and private health care expenditures.

Moreover, the (squared) coherency between the change in per capita real Food stamps and the change in per capita real personal consumption expenditures on food generally is large enough to suggest at least a moderate degree of substitution, although again at most frequencies the coherencies are not significantly different from zero. Similarly mixed results hold when Food stamps and private food consumption are expressed in nominal terms.

IV. Conclusions

This paper characterizes several economically important properties of government spending and, in the process, shows that frequency domain techniques can be a useful means of confirming or refuting standard time domain results. Further, the paper shows that spectral techniques can be combined with an optimizing model of the representative consumer; more specifically, this combination is used to shed light on the proposition that government spending directly substitutes for private consumption expenditures. The main empirical findings are as follows.

Defense spending is best modeled as exogenous with respect to the aggregate economy, and nondefense spending on goods and services appears to be white noise. Also, the influence of demographics, such as the baby-boom, baby-bust cycle on government transfer payments occurs over too long a period to be captured by the frequency domain approach applied here to the post-war period. Further, the unemployment rate has very high coherency at the business cycle frequencies with unemployment insurance but far smaller coherency with other transfer payments. Finally, there appears to be a moderate degree of direct substitutability between changes in government consumption and private consumption (excluding durables) expenditures, although the evidence is decidedly mixed. Stronger evidence of at least a moderate degree of substitutability exists for the case of transfer-in-kind goods, such as Medicare and Food stamps, and private consumption expenditures of the associated good.

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Appendix: Review of the Frequency Domain Approach

A. Introduction

While economists are quite familiar with describing a stationary stochastic process in terms of its autocovariance or autocorrelation function, they are less familiar with analysis of the process in terms of its frequency properties. This section presents the basic results and intuition of frequency domain analysis utilized throughout the paper. The section is written with the practitioner in mind; it draws on more rigorous presentations in Bloomfield (1976), Brillinger (1975), Chatfield (1996), Granger and Newbold (1986), Hamilton (1994), Jenkins and Watts (1968), Priestly (1982),and Sargent (1979). The rest of this appendix can be skipped by those familiar with spectral techniques.

The basic idea of frequency domain analysis is that a covariance stationary process, Y_t , can be described as the sum of uncorrelated sine and cosine waves of differing frequency and amplitude. The goal is to identify frequencies that contain a lot of information about the variance of Y. For example, the variance of an economic variable may be dominated by variations at the seasonal and business cycle frequencies. Using frequency domain techniques, Granger (1966) argues that the typical macro-economic variable in level form, such as real GDP or the industrial production index, is dominated by low frequency or trend-like variation; as discussed below, this could reflect a stable underlying autoregressive process or, alternatively, a nonstationary process (for which our basic theory below does not apply unless the variable is appropriately transformed).

When considering the relationship between two variables, frequency domain analysis provides a measure of the linear correlation of the variables frequency by frequency as well as a measure that is interpretable as the regression coefficient of one variable on the other at a given frequency. Moreover, one can find relationships between corresponding bands or groups of frequencies of two variables, such as low-frequency groupings or high-frequency groupings, by the method of band regression, as developed in economics by Engle (1974, 1979), although this method is not discussed further here.

B. The Univariate Case

Frequency domain analysis begins by defining the (power) spectral density function, or spectrum, of Y_t at frequency ω, $s_y(\omega)$; the spectrum is the Fourier transform of the autocovariance function, γ_j, scaled by 2π:

$$s_y(\omega) = (1/2\pi) \sum_{j=-\infty}^{\infty} \gamma_j e^{-i\omega j} = (1/2\pi)\left[\sum_{j=-\infty}^{\infty} \gamma_j \cos(\omega j)\right] \qquad \text{(A1)}$$

where $\gamma_j = E(Y_t - \mu)(Y_{t-j} - \mu)$ and $E(Y_t) = \mu$.

The second expression in (1) uses De Moivre's theorem, standard properties of trigonometric functions, and $\gamma_j = \gamma_{-j}$ for stationary time series. This expression shows that the spectrum at frequency ω is real and is an infinite weighted sum of cosine terms. Further, as suggested by both expressions in (1), the theoretical spectrum of Y_t at every frequency ω utilizes an infinite number of leads and lags of the data (since γ_j is evaluated for values of j from $-\infty$ to $+\infty$).

Alternatively, Hamilton (1994) shows that the autocovariance at lag k is the inverse Fourier transform of the spectral density function:

$$\gamma_k = \int_{-\pi}^{\pi} s_y(\omega) e^{i\omega k} d\omega = \int_{-\pi}^{\pi} s_y(\omega)\cos(\omega k) d\omega \qquad \text{(A2)}$$

Together with (1), this implies that the spectrum and the infinite sequence of autocovariances contain exactly the same information. Also, from (2) it follows that the variance of Y, γ_0, is given by:

$$\gamma_0 = \int_{-\pi}^{\pi} s_y(\omega) d\omega \qquad \text{(A3)}$$

so that the area under the spectrum is the variance; so, loosely, the variance of a series is the sum of the spectra over all frequencies. Because the spectrum is symmetric about the zero frequency, it is common to plot $s(\omega)$ against ω only for $0 \le \omega \le \pi$. Also, the area between two frequencies, ω_1 and ω_2, is the contribution to the variance of the series from frequencies between ω_1 and ω_2.

The spectrum can take many shapes. If the spectrum is flat or constant at all frequencies, then each contributes a like amount to the variance and the series is called white noise (just as the color white is formed by an equal contribution from all colors of the light spectrum); from (1), the spectrum of a white noise variable is $\gamma_0/2\pi$. Chatfield (1996) shows that the spectrum for an AR(1) or an MA(1) process with positive coefficients declines monotonically as frequency increases from 0 to π. If a series is integrated of order 1, and hence is not stationary, its spectrum is proportional to ω^{-2} at low frequencies (see Engle and Granger (1987)); hence, the spectrum is not finite at the zero frequency and declines rapidly for small values of ω.

A spectrum also can have prominent peaks at frequencies associated with business and seasonal cycles. Sargent (1979) specifies business cycles generally as having periods of length 2 to 4 years (NBER short cycles) or 8 years (NBER major cycles). Baxter and King (1997) consider business cycles with periods between 6 and 32 quarters. Granger and Newbold (1986) specify a seasonal as that component of a time series that causes the spectrum to have peaks at or around seasonal frequencies, i.e., frequencies $\omega_k = 2\pi k/p$ where $k = 1,2,3,\ldots$ and p is the period of the principal seasonal, taking the value 4 for quarterly data, 12 for monthly data, and so forth. Thus, for quarterly data the seasonal peaks in the spectrum would show up at (or near) the principal and secondary seasonal frequencies $\pi/2$ and π (with corresponding periods, $2\pi/\omega$, of length 4 and 2 quarters), respectively; for monthly data, peaks would show up at (or near) frequencies $\pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, $5\pi/6$, and π (with corresponding periods of length 12, 6, 4, 3, 2.4, and 2 months, respectively).

Further, Chatfield (1996) shows that transforming or filtering a series will alter the spectrum of the basic series. For example, a simple moving average of the series, commonly used in the time domain to reduce high-frequency variation, is a low-band-pass filter that magnifies the spectrum of the underlying series at low frequencies and shrinks the spectrum at high frequencies. Conversely, first-differencing the series is a high-band-pass filter that filters out most of the low frequency variation and magnifies high frequency variation (indeed, for a stationary series, first-differencing completely eliminates the contribution to the variance at the zero frequency, although this generally will not be the case for a non-stationary series). These two filters are not "ideal" in that they do not sharply cut off or eliminate variation from a specific frequency band.

The empirical estimation of the population spectrum from a finite sample is known as spectral analysis. Nonparametric or kernel techniques are used in the text to estimate the spectrum of several fiscal variables. First, however, it should be noted that for a sample of size T, the longest period or wavelength that can be detected in the data is of length T; thus, for example, one cannot discover a fifty-year baby-boom, baby-bust demographic cycle in fiscal data that do not span at least a fifty-year period. Indeed, because the period or wavelength of a sinusoidal cycle is $2\pi/\omega$, the lowest frequency that can be detected in the data is $2\pi/T$ (i.e., a frequency that completes one cycle in the whole length of the time series). Also, as seen in (A2) above, the highest frequency considered is $\omega = \pi$, which implies that the shortest period that can be detected is 2 quarters (assuming quarterly data).

We denote $\hat{s}_y(\omega)$ as the sample periodogram (even though s_y is a function of frequency rather than period) or sample spectrum for a sample of size T. Its expression is given by:

$$\hat{s}_y(\omega) = (1/2\pi) \sum_{j=-T+1}^{T-1} \hat{\gamma}_j e^{-i\omega j} = (1/2\pi)[\hat{\gamma}_0 + 2 \sum_{j=1}^{T-1} \hat{\gamma}_j \cos(\omega j)] \qquad \text{(A4)}$$

where $\hat{\gamma}_j$ is the sample autocovariance.

To provide a bit more insight into periodogram calculations, we evaluate $\hat{s}_y(\omega)$ at three different frequencies, using the final expression above for the sample periodogram (and the property that $\gamma_j = \gamma_{-j}$). At the lowest or zero frequency, $\hat{s}_y(\omega=0)$ is the sum of the sample variance and all sample autocovariances (divided by 2π). At the highest frequency, $\omega=\pi$, $\hat{s}_y(\omega=\pi)$ is the sum of the sample variance and all the sample autocovariances alternating in sign (divided by 2π). At an intermediate frequency, such as the business cycle frequency of $\pi/4$ (corresponding to a period of 8 quarters with quarterly data) $2\pi\hat{s}_y(\omega=\pi/4) = \hat{\gamma}_0+2(.7\hat{\gamma}_1-.7\hat{\gamma}_3-.7\hat{\gamma}_5+.7\hat{\gamma}_7+...) + 2(-\hat{\gamma}_4+\hat{\gamma}_8-\hat{\gamma}_{12}+\hat{\gamma}_{16}+...)$. Thus, at each frequency, the sample periodogram utilizes all sample autocovariances--and, hence, utilizes all the data--although the weights on the sample autocovariances depend on the frequency.

As Brillinger (1975) shows, the sample periodogram is not a consistent estimator of the population spectrum (essentially because the number of parameters to be estimated equals the number of observations). This problem can be attenuated by the use of kernel estimates. This approach assumes that $s_y(\omega)$ is close to $s_y(\lambda)$ when the frequencies ω and λ are close to each other. The kernel estimate of the spectrum at frequency ω_j is a weighted average of the sample periodograms at frequencies near ω_j, where the weights sum to one:

$$\hat{s}_y(\omega_j) = \sum_{b=-h}^{h} \kappa(\omega_{j+b}, \omega_j)\hat{s}_y(\omega_{j+b}) \qquad \text{where}$$

$$\sum_{b=-h}^{h} \kappa(\omega_{j+b}, \omega_j) = 1. \tag{A5}$$

Here, b is a bandwidth parameter indicating the number of frequencies used in the estimation of the spectrum at ω_j. Generally h = 4 in the applied work in the text, where the kernel function $\kappa(\omega_{j+b}, \omega_j)$ is given by $[h+1-|b|]/(h+1)^2$. As shown in Chatfield (1996), using kernel estimates, the $100(1-\alpha)$ percent confidence band for $s_y(\omega)$ is:

$$\frac{2(2h+1)\hat{s}_y(\omega_j)}{X^2_{\alpha/2}(4h+2)} \le s_y(\omega) \le \frac{2(2h+1)\hat{s}_y(\omega_j)}{X^2_{1-\alpha/2}(4h+2)}$$

where $X^2(d)$ is a chi-squared variable with d degrees of freedom.

In the applied work in the text, use is made of two frequency domain tests for white noise. The Fisher-Kappa (FK) test is designed to detect one sinusoidal component buried in white noise; the test statistic is the ratio of the largest periodogram ordinate to the average of all the ordinates. The Bartlett Kolmogorov-Smirnov (BKS) test is designed to detect departures from white noise over the whole range of frequencies and is more powerful than the Fisher-Kappa test. At each frequency ω_k the sum of periodogram ordinates from ω_1 to ω_k is divided by the sum of all periodogram ordinates; the ratio is compared to the cumulative distribution function of a uniform (0,1) random variable.

C. The Bivariate Case

The cross spectrum, $s_{xy}(\omega)$, is utilized when examining the relationship between two stationary variables. It is defined as the Fourier transform of the cross-covariance function, $\gamma_{xy}(k)$:

$$s_{xy}(\omega) = (1/2\pi) \sum_{k=-\infty}^{\infty} \gamma_{xy}(k) e^{-i\omega k} \qquad \text{(A6)}$$

where $\gamma_{xy}(k) = COV(X_t, Y_{t+k}) = COV(Y_t, X_{t-k})$.

In general, the cross spectrum is complex; its real part is called the co-spectrum, $c(\omega)$, and its imaginary part is called the quadrature spectrum, $q(\omega)$. These functions are difficult to interpret and so it is usual to focus on other functions derived from them. Denoting $s_x(\omega)$ and $s_y(\omega)$ as the spectral density functions of X and Y, respectively, the (squared) coherency is:

$$C(\omega) = [c^2(\omega) + q^2(\omega)]/s_x(\omega)s_y(\omega) \quad . \tag{A7}$$

This quantity measures the square of the linear correlation between the two variables at frequency ω and is analogous to the square of the usual correlation coefficient. It is shown in Brillinger (1975) that $0 \leq C(\omega) \leq 1$. Thus, if the coherency is near one at frequency ω, it means that the ω-frequency components of the two series are highly related, but a value near zero means that the corresponding frequency components are not closely related.[12] Bloomfield (1976) provides a test of the null hypothesis that the coherency is zero. If the level of significance of the test is α, than an estimated coherency less than $1 - (1-\alpha)^{g/(1-g)}$ should be regarded as not significantly different from zero, where

$$g = \sum_{b=-h}^{h} [(h+1-|b|)/(h+1)^2]^2$$

Jenkins and Watts (1973) presents the 95 percent confidence interval for the (squared) coherency. It is given by:

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[12] Granger and Weiss (1983) point out that if Y and X are integrated of order one and are cointegrated, then the coherency between ΔX_t and ΔY_t is one at the zero frequency.

$$\frac{[\hat{\xi}_{xy}(\omega)-\tanh\ (1.96/\nu^{.5})]^2}{[1-\hat{\xi}_{xy}(\omega)(\tanh(1.96/\nu^{.5}))]^2} \le C_{xy}(\omega) \le \frac{[\hat{\xi}_{xy}(\omega)+\tanh\ (1.96/\nu^{.5})]^2}{[1+\hat{\xi}_{xy}(\omega)(\tanh(1.96/\nu^{.5}))]^2}$$

Here, $\hat{\xi}_{xy}(\omega)$ denotes the square root of the estimated (squared) coherency; tanh denotes the hyperbolic tangent; and ν denotes the degrees of freedom equal to 2(2h+1). Note the close analogy between the confidence interval for the coherency and for the standard correlation coefficient.

As a practical matter, we see examples in the text in which the spectrum of a fiscal variable does not have a prominent peak at the range of frequencies associated with the business cycle but does have a high coherence with business cycle variables (such as the unemployment rate) at business cycle frequencies. Sargent (1979) cautions that lack of a prominent peak in the spectrum should not be taken to mean necessarily that the series does not experience any fluctuations associated with the business cycle; in such cases, one should carefully examine coherencies.

Another useful function is the gain spectrum, defined as:

$$G_{xy}(\omega) = [c^2(\omega) + q^2(\omega)]^{1/2}/s_x(\omega) \qquad \text{(A8)}$$

which is interpretable as the regression coefficient of the process Y_t on the process X_t at frequency ω.

Jenkins and Watts (1968) show that $s_y(\omega) = G^2_{xy}(\omega)s_x(\omega)$ in a time-invariant linear system with no noise (i.e., if Y_t is a linear distributed lag (and lead) function of the input, X_t).

The addition of uncorrelated noise, N_t, to the system implies that $s_y(\omega) = G^2_{xy}(\omega) s_x(\omega) + s_N(\omega)$. The first term on the right hand side is called the output signal and the ratio of the first term to the second term is called the signal-to-noise ratio, S/N. A little algebra establishes that $C(\omega) = [1/(1+(S/N)^{-1})]$. Thus a large signal-to-noise ratio implies a large coherency.

Figure 1.
Nominal Defense Consumption and Investment
1946:2 to 1997:1

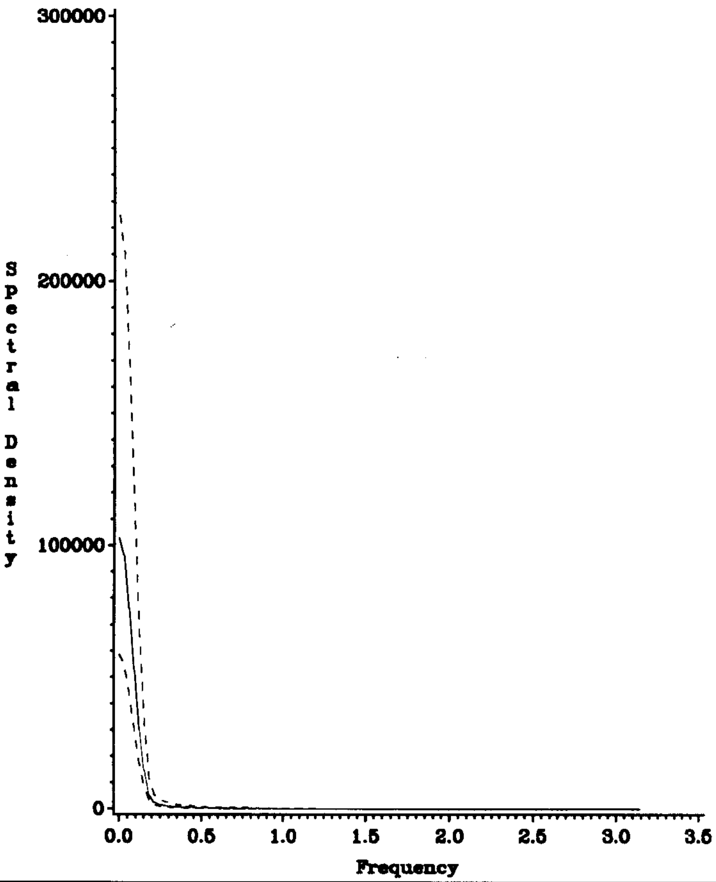

Figure 2.
Nominal Defense Consumption and Investment
(Percent of Nominal GDP)
1947:2 to 1997:1

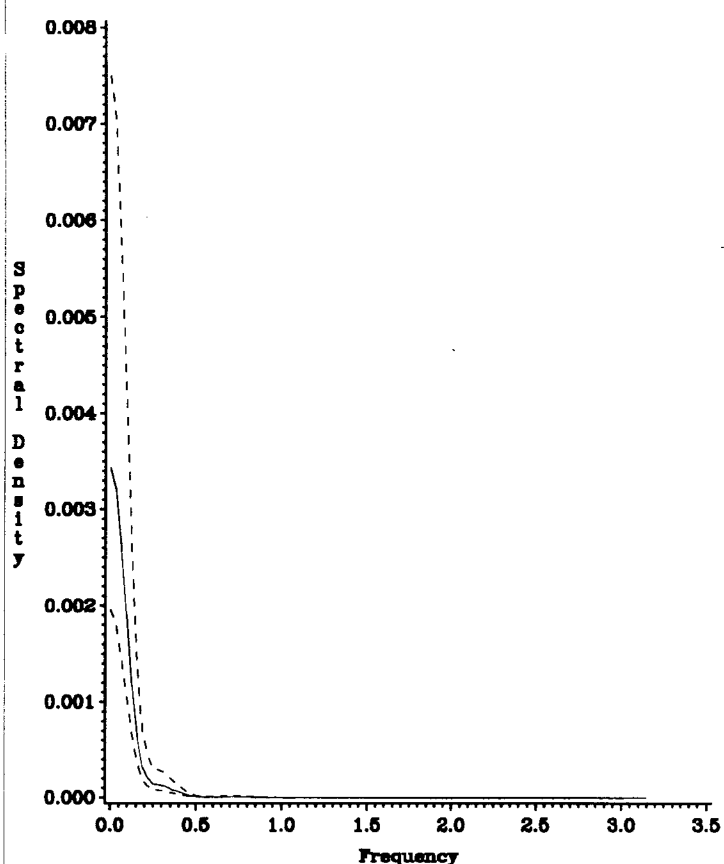

Figure 3.
Real Defense Consumption and Investment
(Percent of Real GDP)
1947:2 to 1997:1

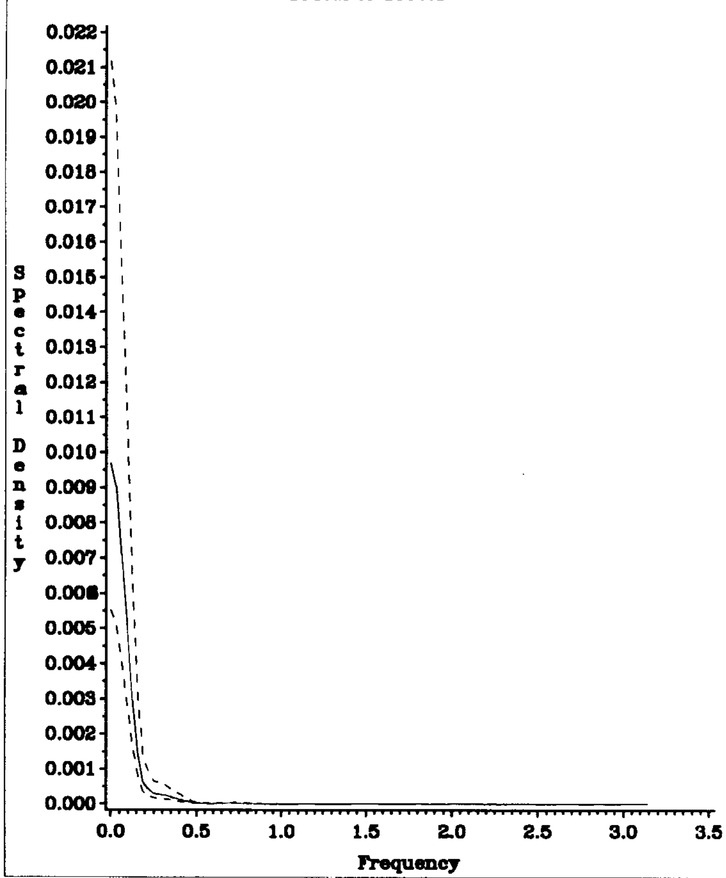

Figure 4.
Growth Rate of Real Defense Consumption and Investment
1947:2 to 1997:1

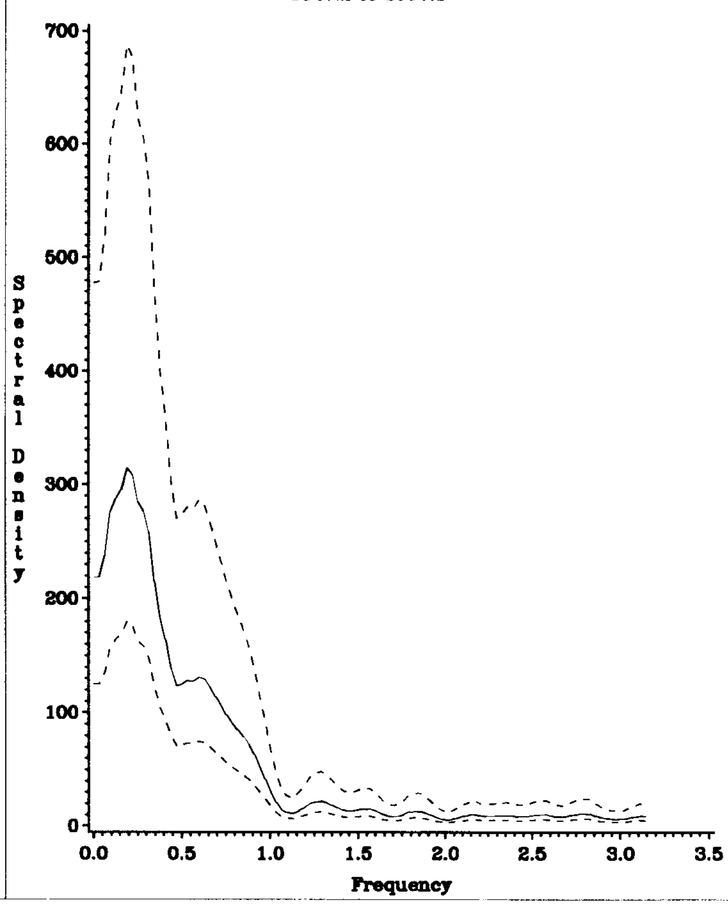

Figure 5.
Growth Rate of Real Defense Compensation
and the Change in the Unemployment Rate
1948:2 to 1997:1

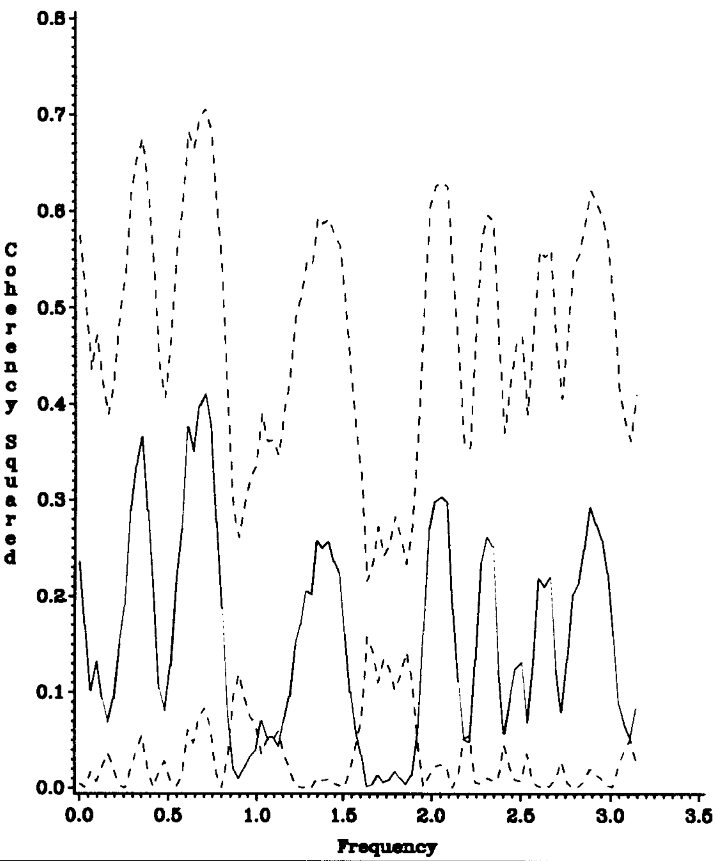

Figure 6.
Total Real Government Consumption and Investment
(Percent of Real GDP)
1947:2 to 1997:1

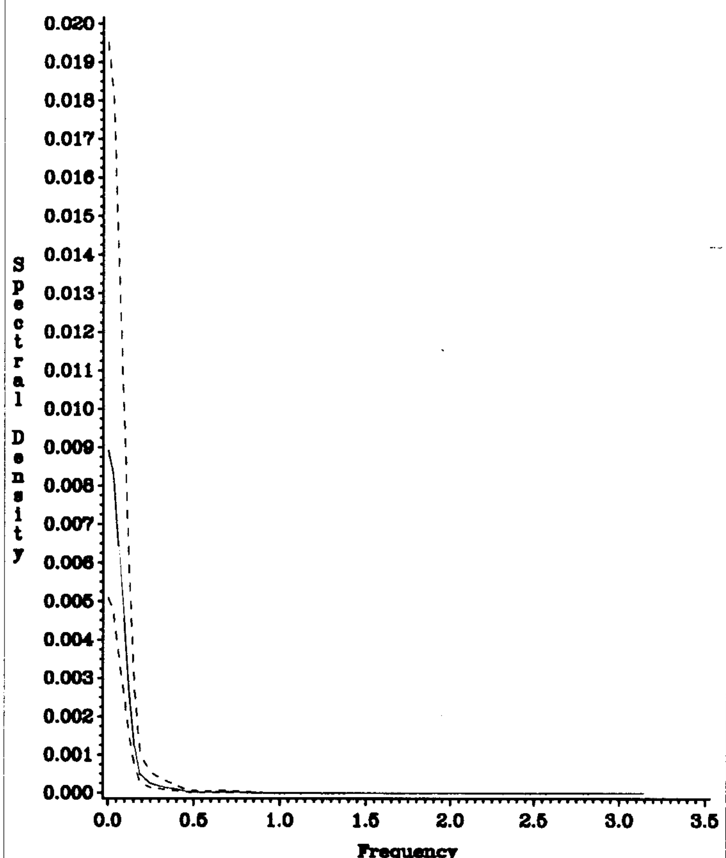

Figure 7.
Real Federal Government Consumption and Investment
(Percent of Real GDP)
1947:2 to 1997:1

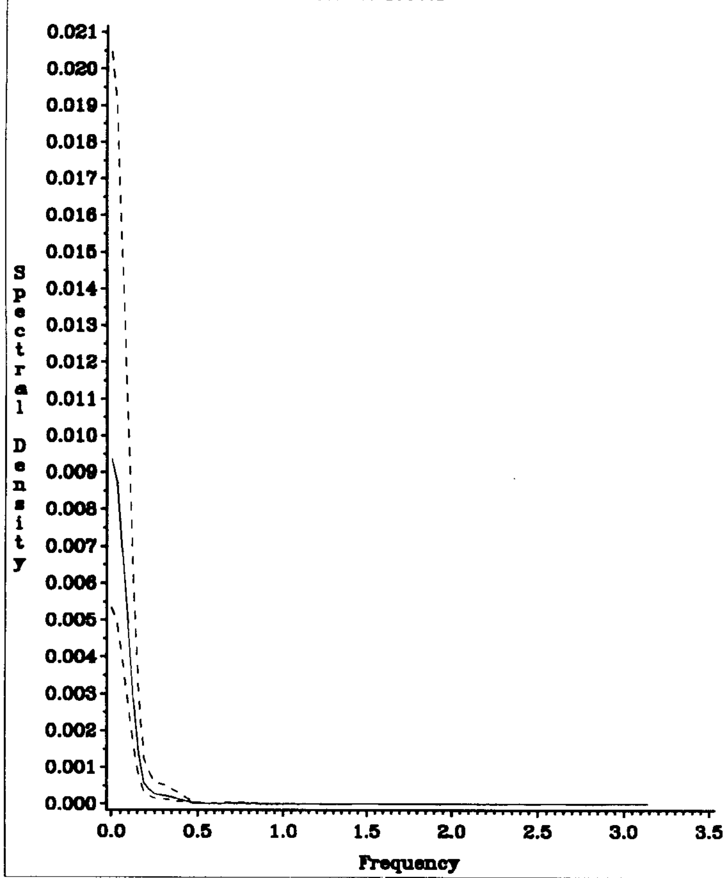

Figure 8.
Total Nominal Government Consumption and Investment
(Percent of Nominal GDP)
1947:2 to 1997:1

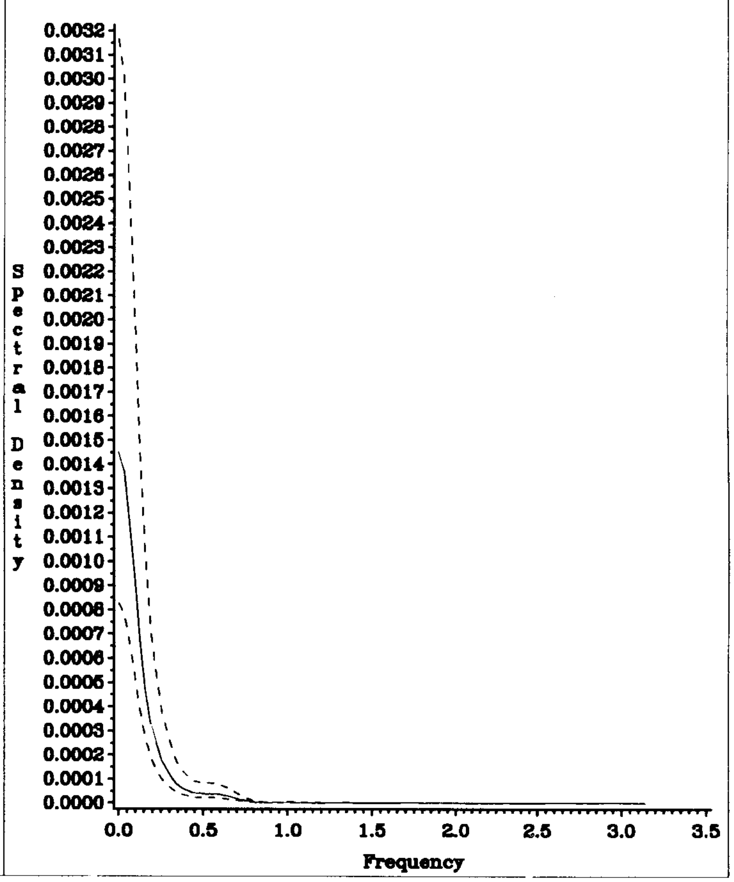

Figure 9.
Nominal Federal Government Consumption and Investment
(Percent of Nominal GDP)
1947:2 to 1997:1

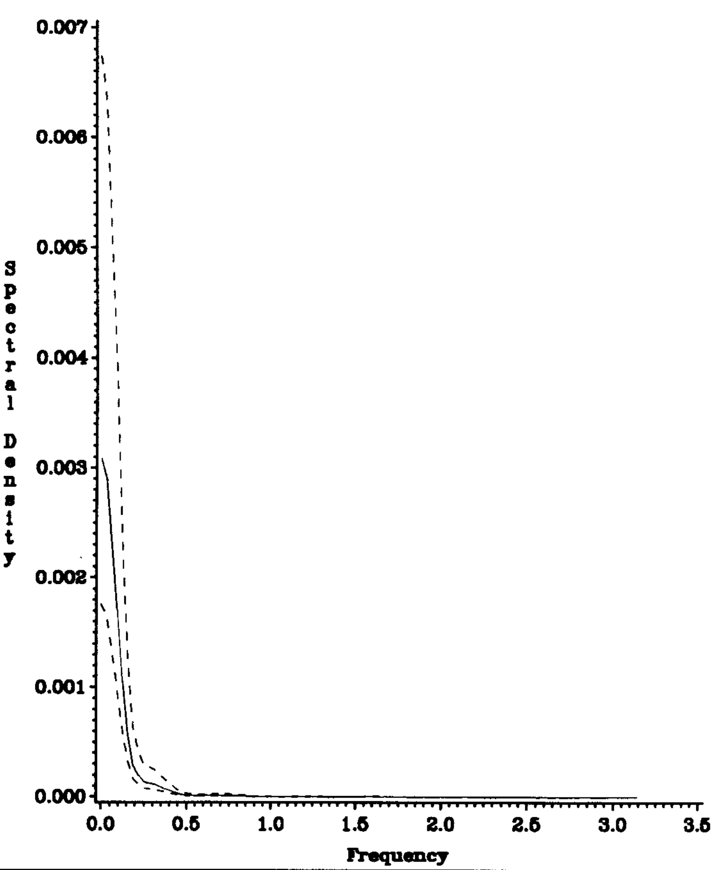

Figure 10.
Growth Rate of Real State and Local Consumption and Investment
1947:2 to 1997:1

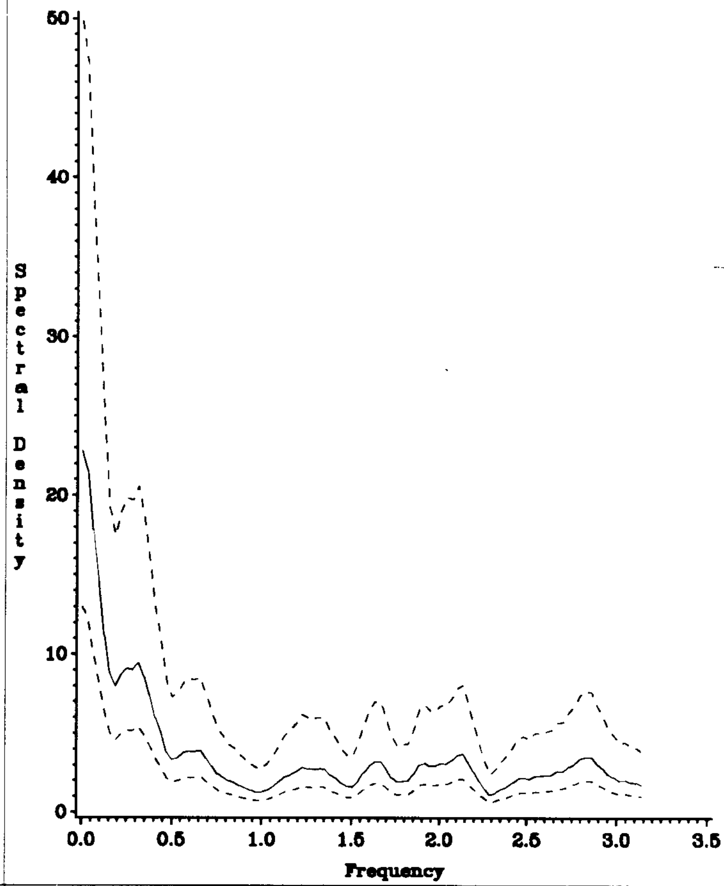

Figure 11.
Growth Rate of Real State and Local Consumption and Investment
and the Change in the Unemployment Rate
1948:2 to 1997:1

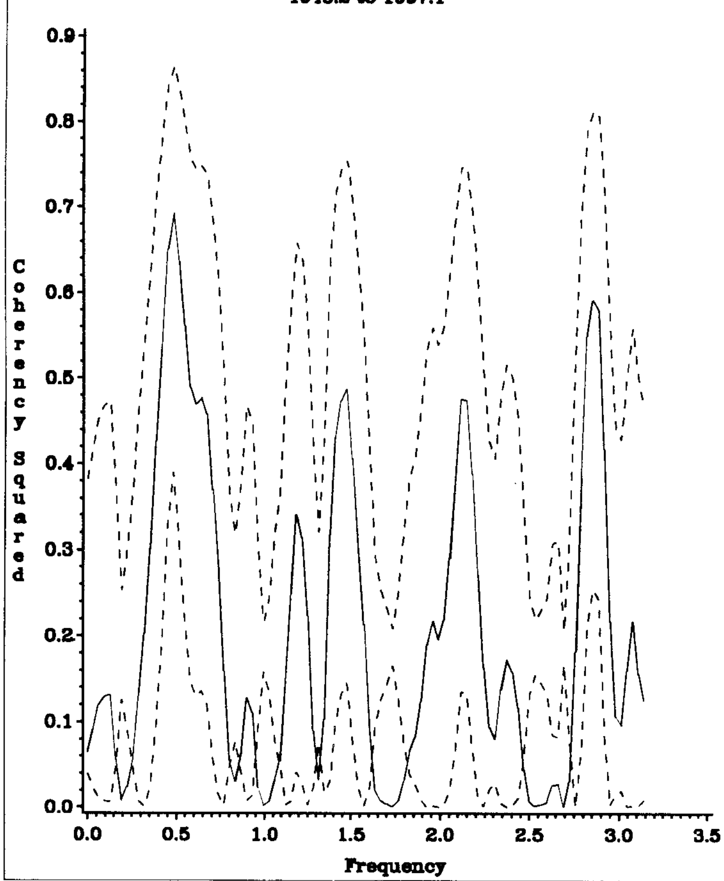

Figure 12.
Growth Rate of Real Nondefense Consumption and Investment
1947:2 to 1997:1

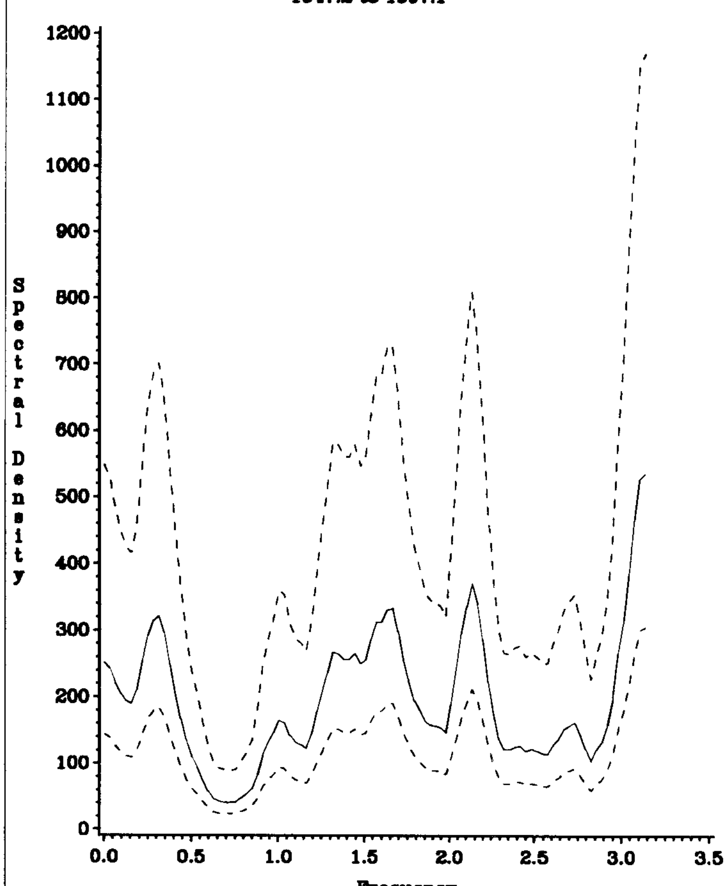

Figure 13.
Nominal Nondefense Consumption and Investment
(Percent of Nominal GDP)
1947:2 to 1997:1

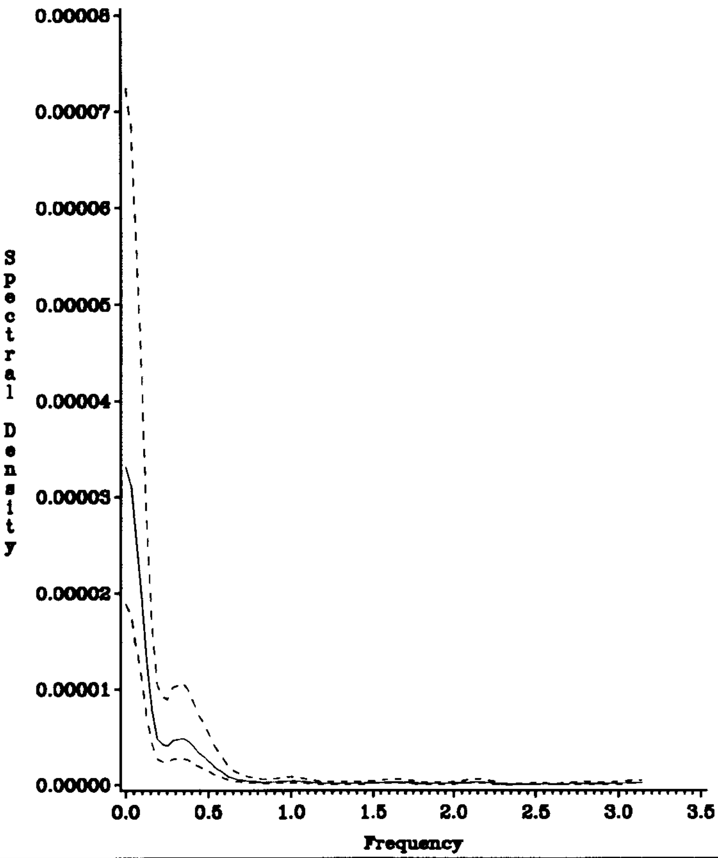

Figure 14.
Growth Rate of Real Nondefense Services Compensation
1947:2 to 1997:1

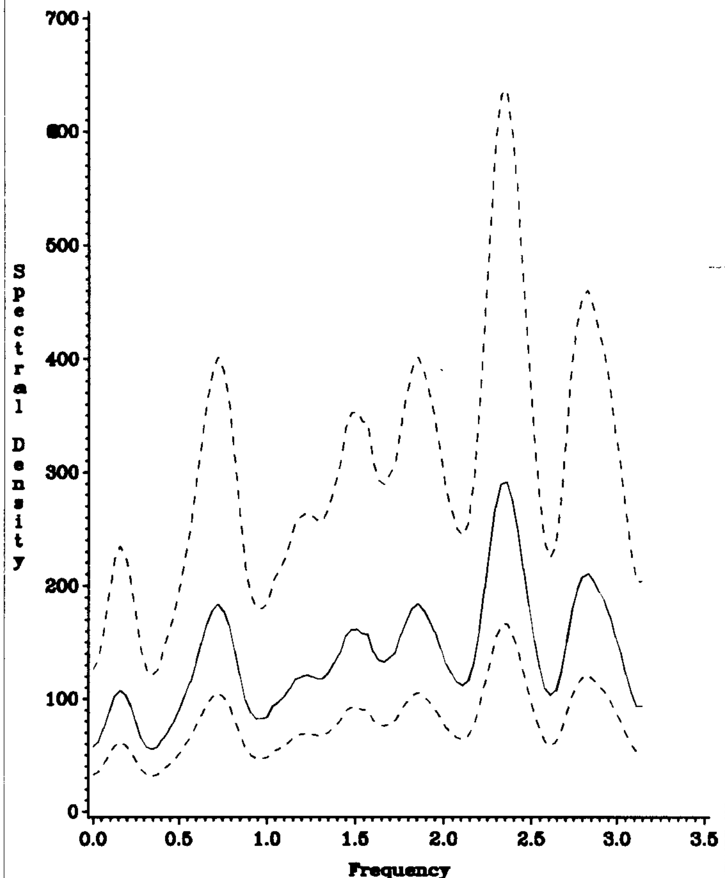

Figure 15.
Growth Rate of Real Nondefense Services — Other
1947:2 to 1997:1

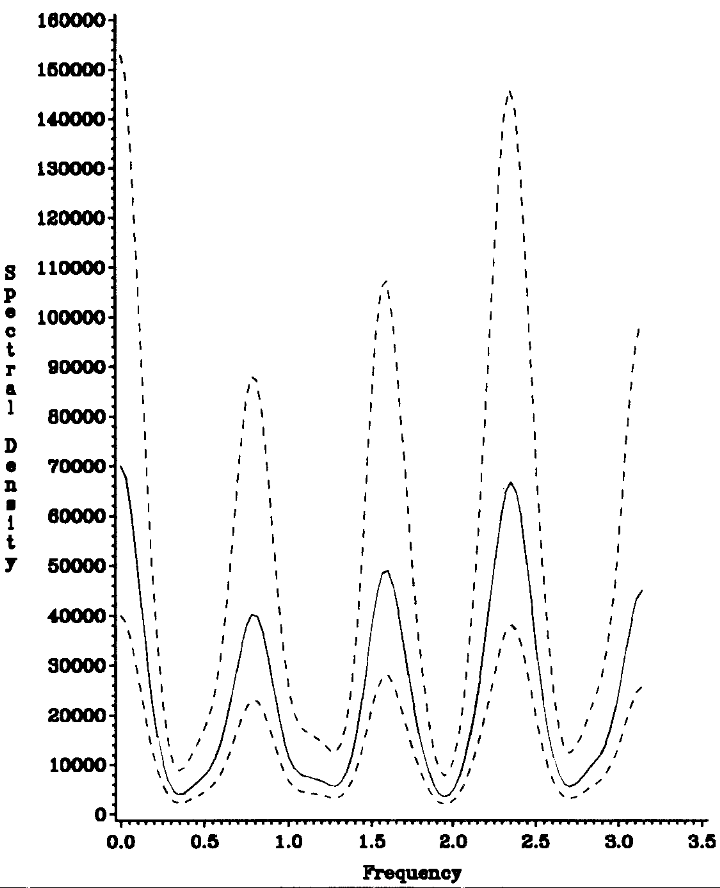

Figure 16.
Unemployment Rate
1959:1 to 1997:2

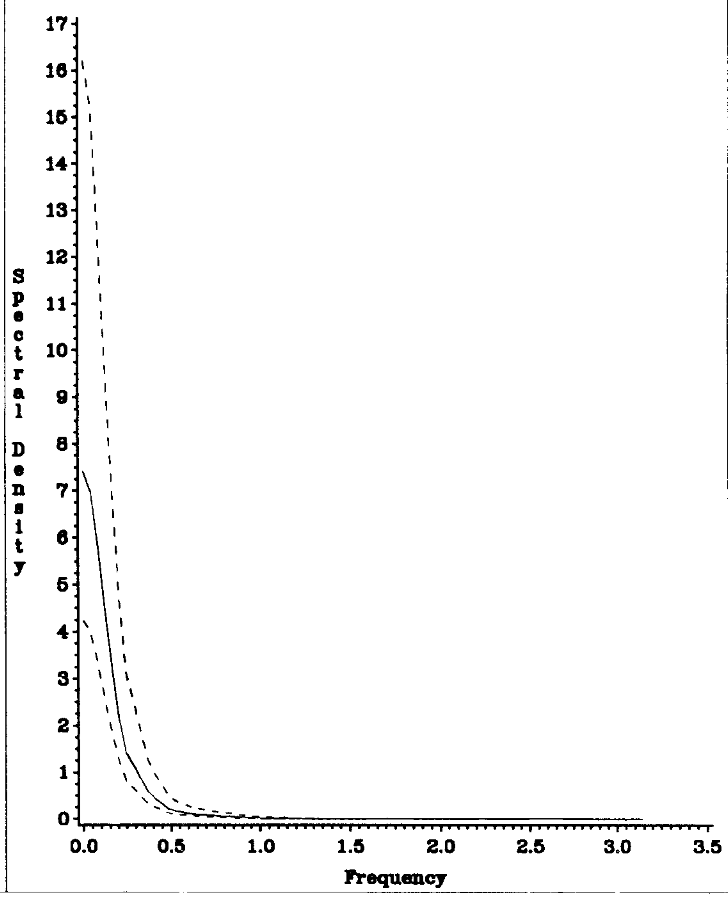

Figure 17.
Change in the Unemployment Rate
1959:2 to 1997:1

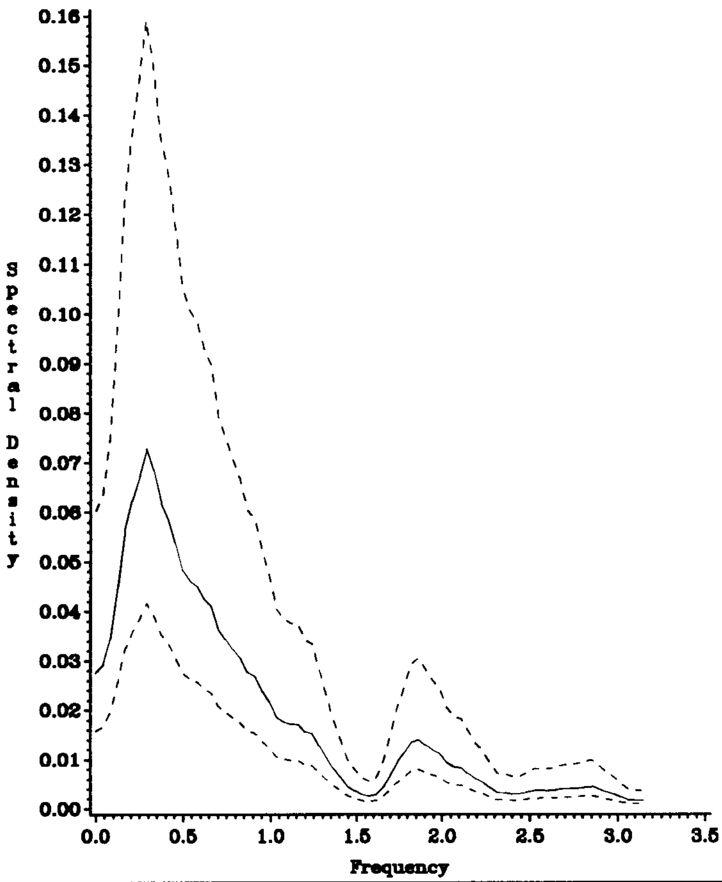

Figure 18.
Unemployment Insurance
1959:1 to 1997:2

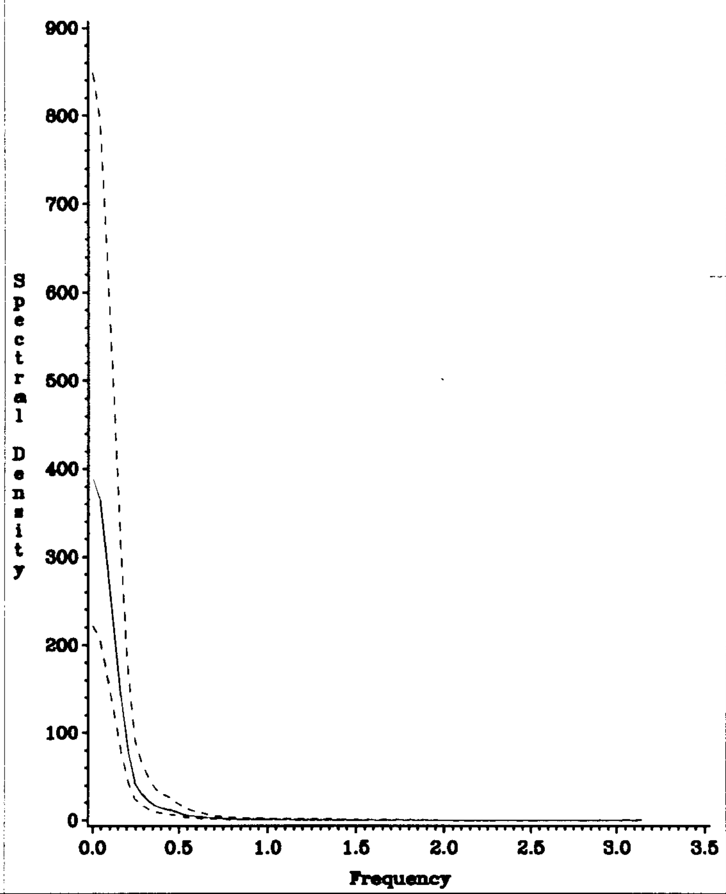

Figure 19.
Unemployment Insurance
(Percent of Nominal GDP)
1959:1 to 1997:1

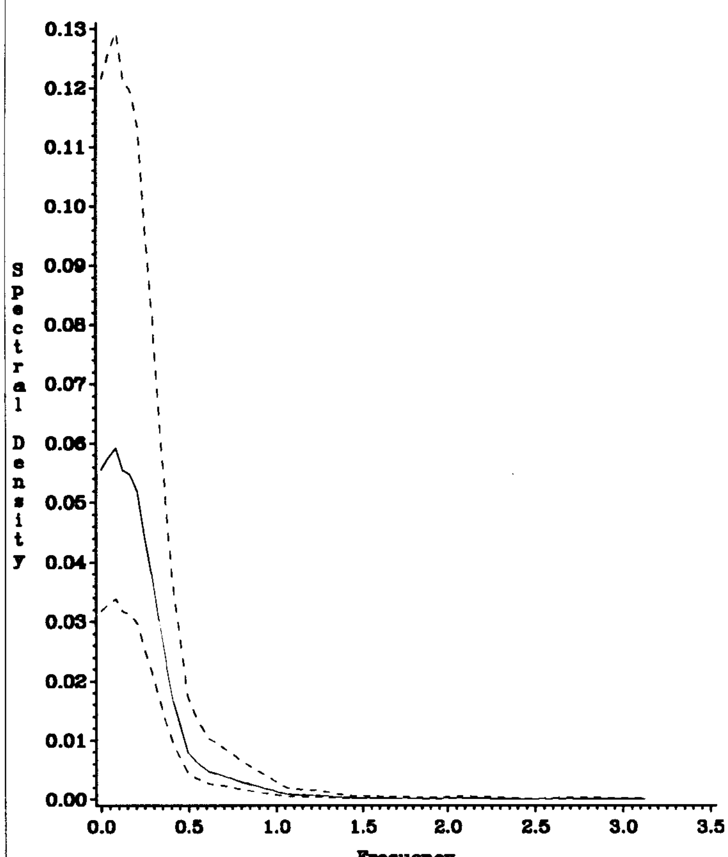

Figure 20.
Growth Rate of Unemployment Insurance
1959:2 to 1997:1

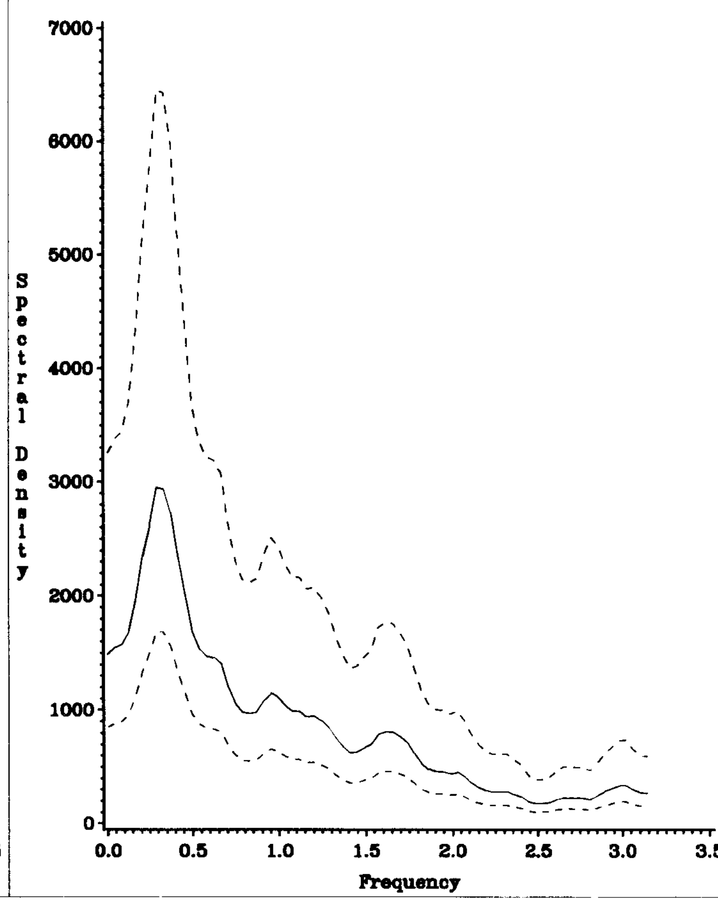

Figure 21.
Unemployment Insurance
and the Unemployment Rate
1959:1 to 1997:2

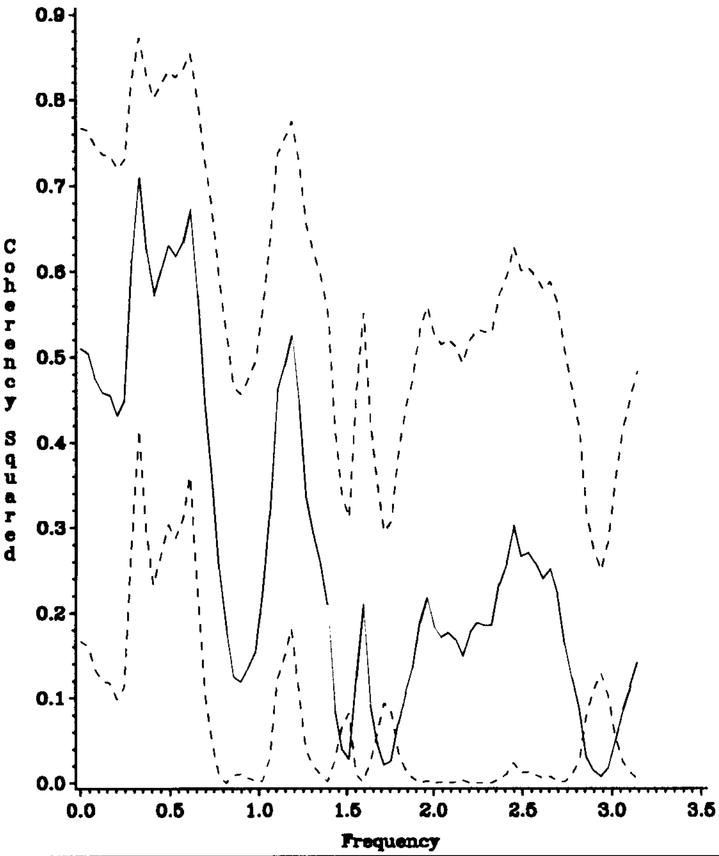

Figure 22.
Growth Rate of Unemployment Insurance
and the Change in the Unemployment Rate
1959:2 to 1997:1

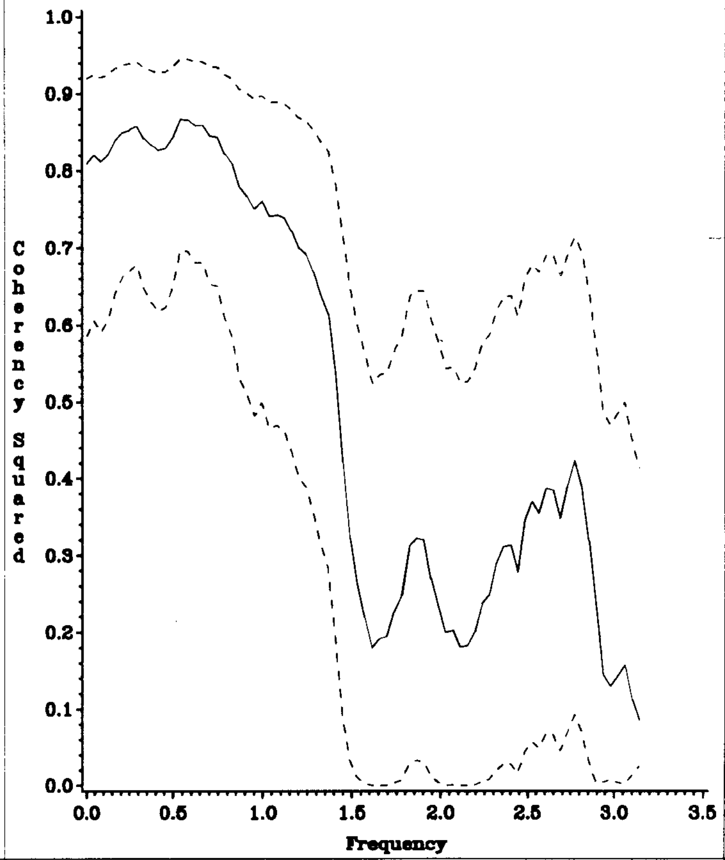

Figure 23.
Unemployment Insurance as a Percent of Nominal GDP
and the Unemployment Rate
1959:1 to 1997:1

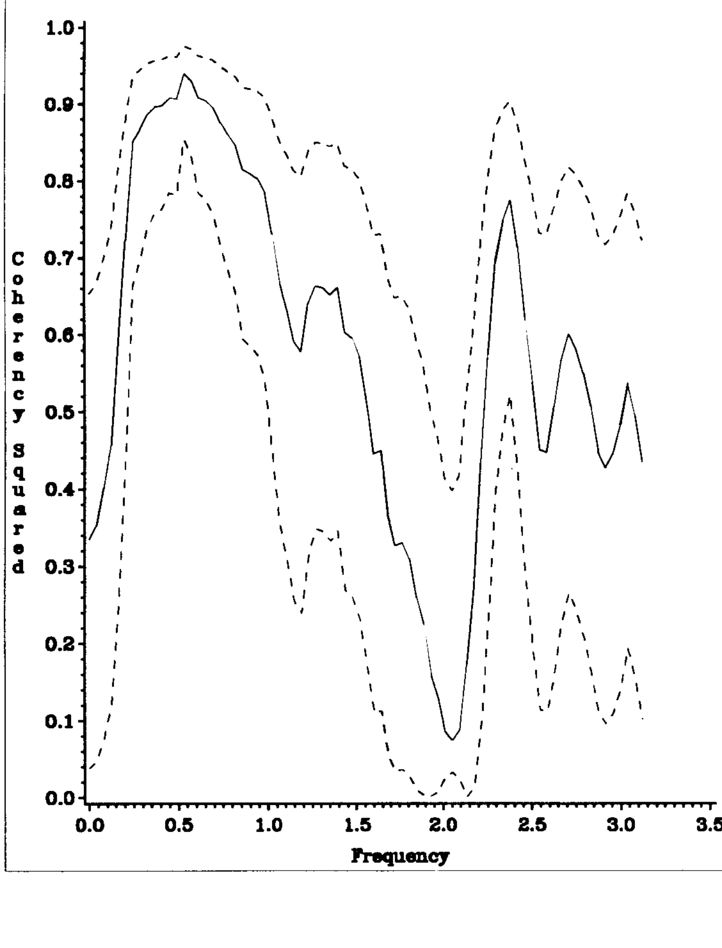

Figure 24.
Growth Rate of OASDI
1959:2 to 1997:1

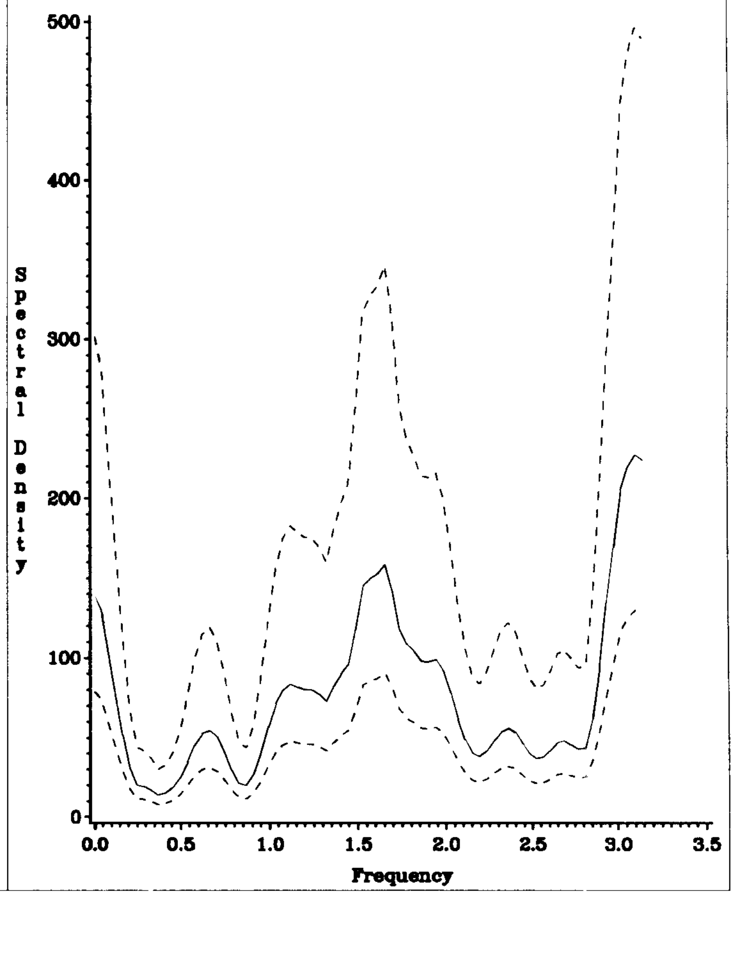

Figure 25.
Growth Rate of OASDI and the
Change in the Unemployment Rate
1959:2 to 1997:1

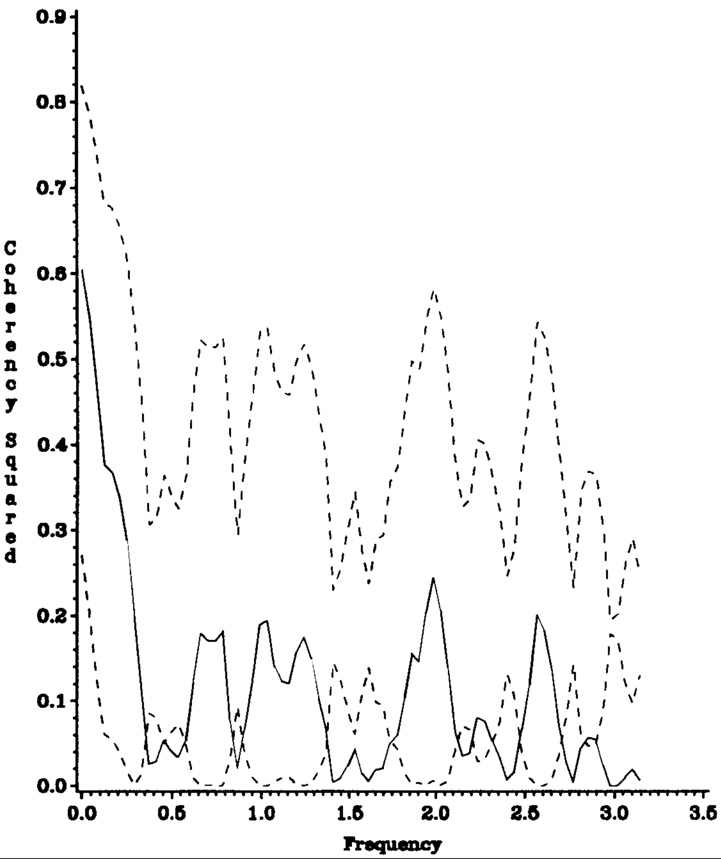

Figure 26.
Growth Rate of Medicare
1967:4 to 1997:1

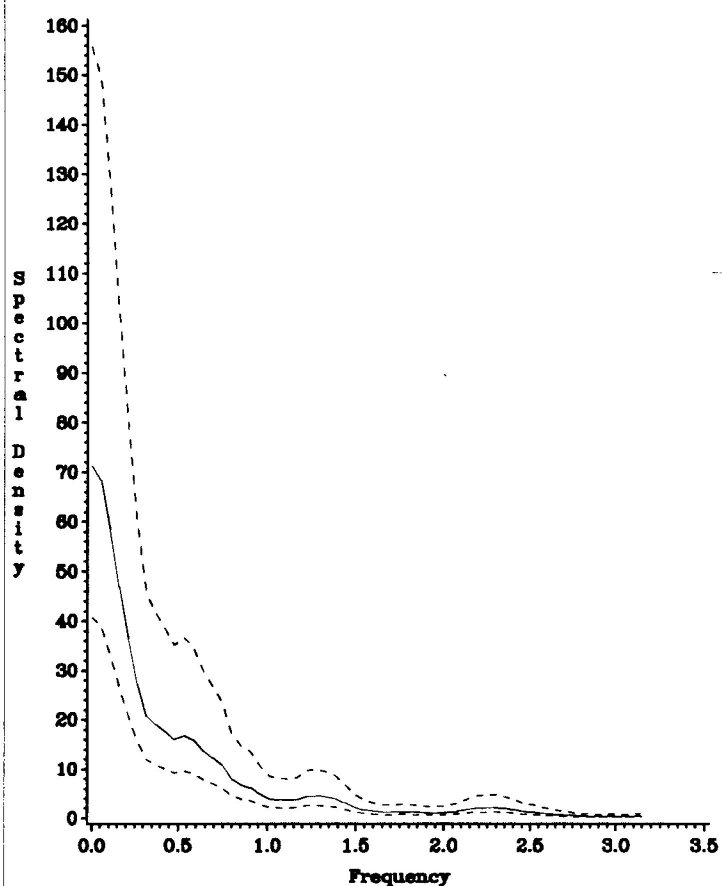

Figure 27.
Growth Rate of Medicare and the
Change in the Unemployment Rate
1966:4 to 1997:1

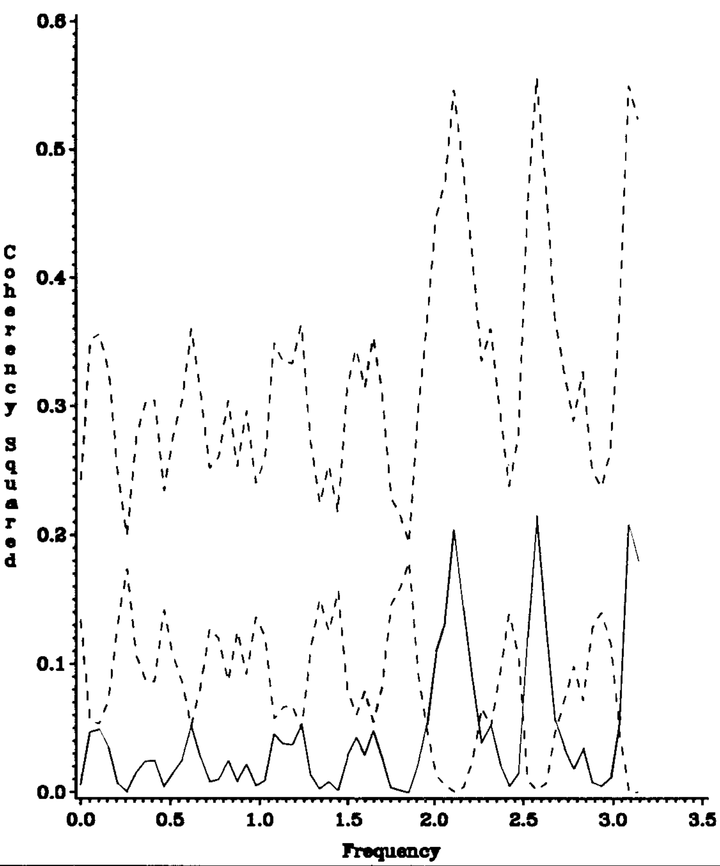

Figure 28.
Growth Rate of Medicare and
Relative Medical Services Inflation
1967:4 to 1997:1

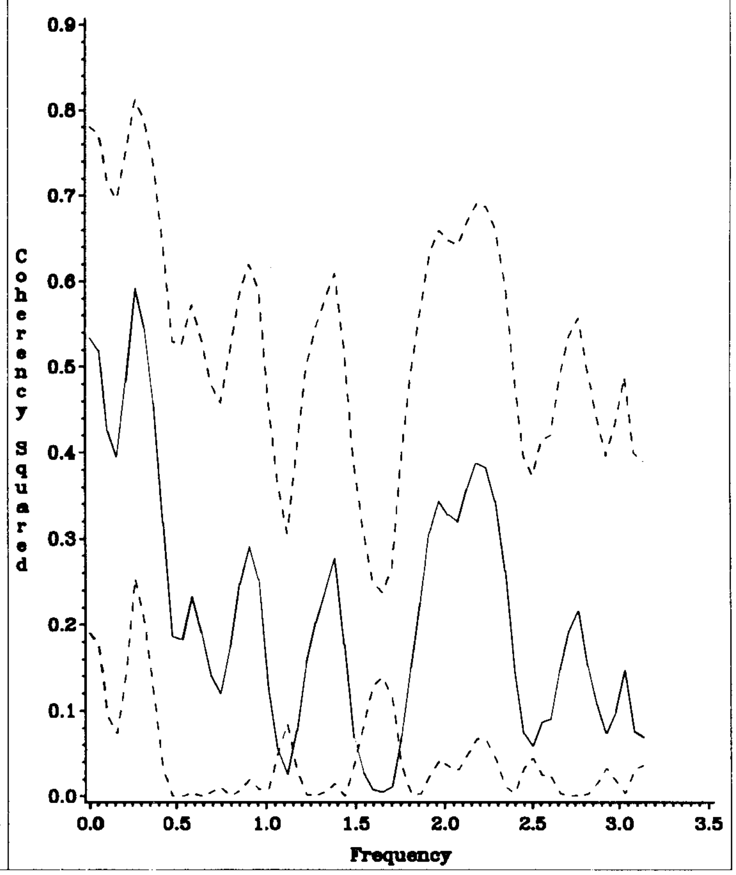

Figure 29.
Growth Rate of Foodstamps
1963:2 to 1997:1

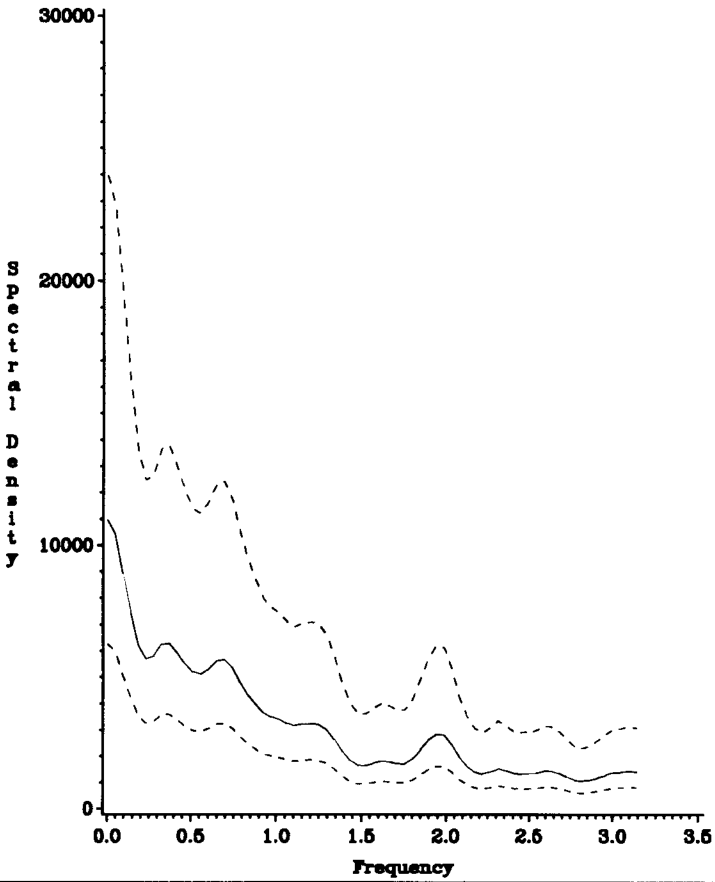

Figure 30.
Growth Rate of Foodstamps and
the Change in the Unemployment Rate
1961:4 to 1997:1

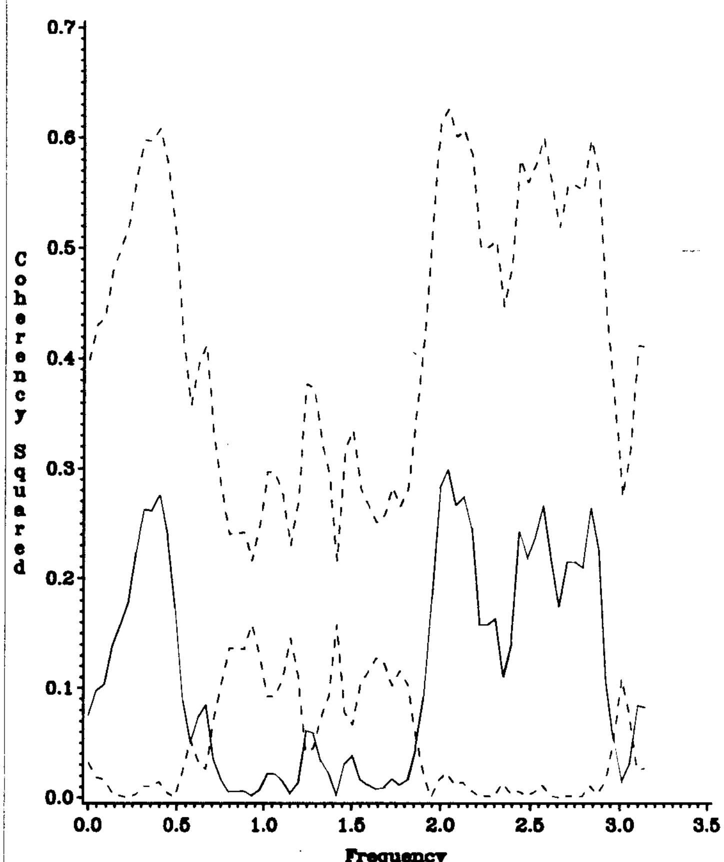

Figure 31.
Growth Rate of Medicaid
1975:2 to 1997:1

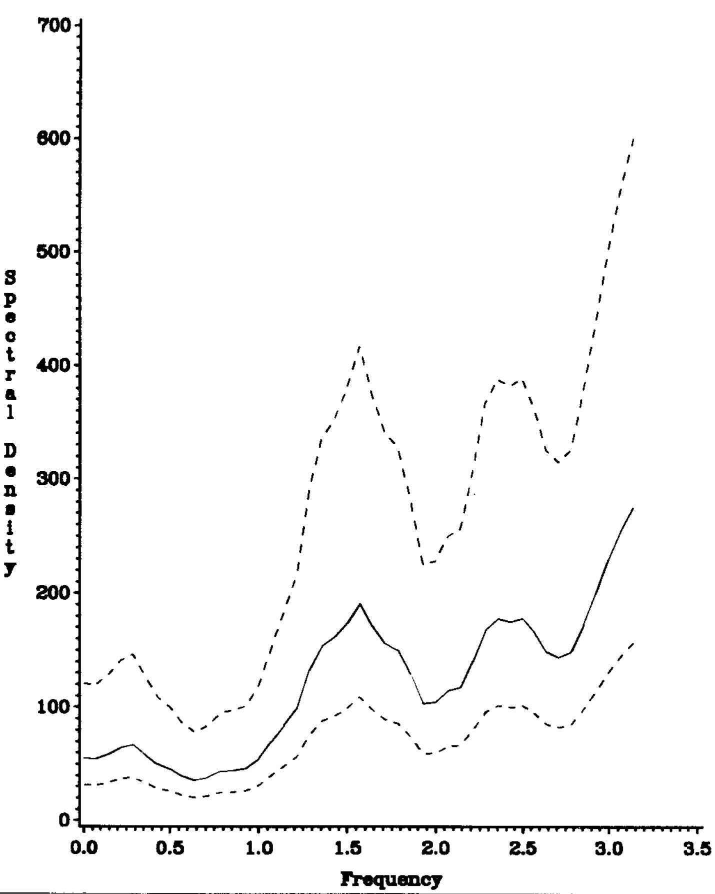

Figure 32.
Growth Rate of Medicaid and the
Change in the Unemployment Rate
1975:2 to 1997:1

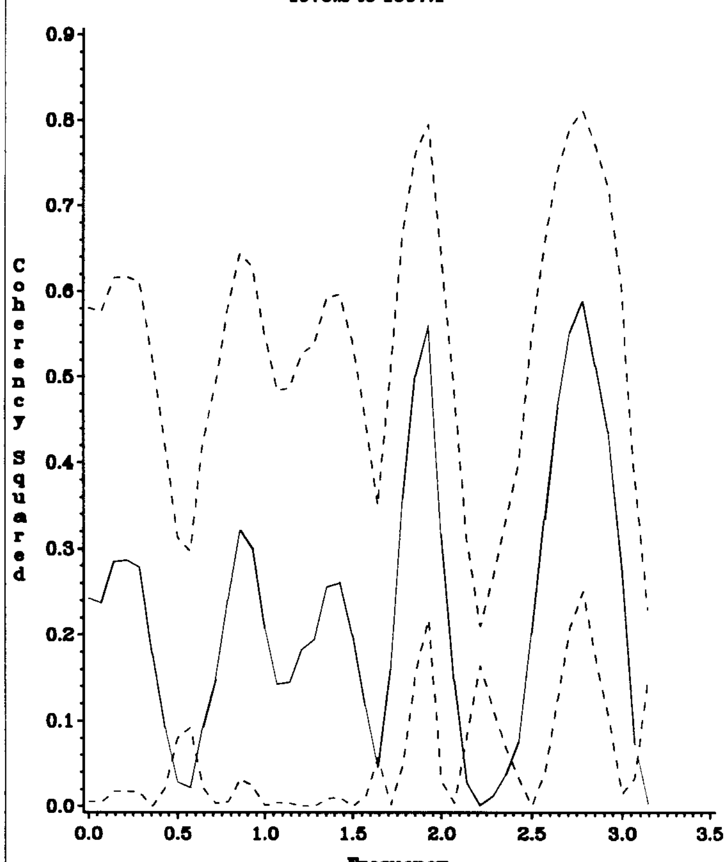

Figure 36.

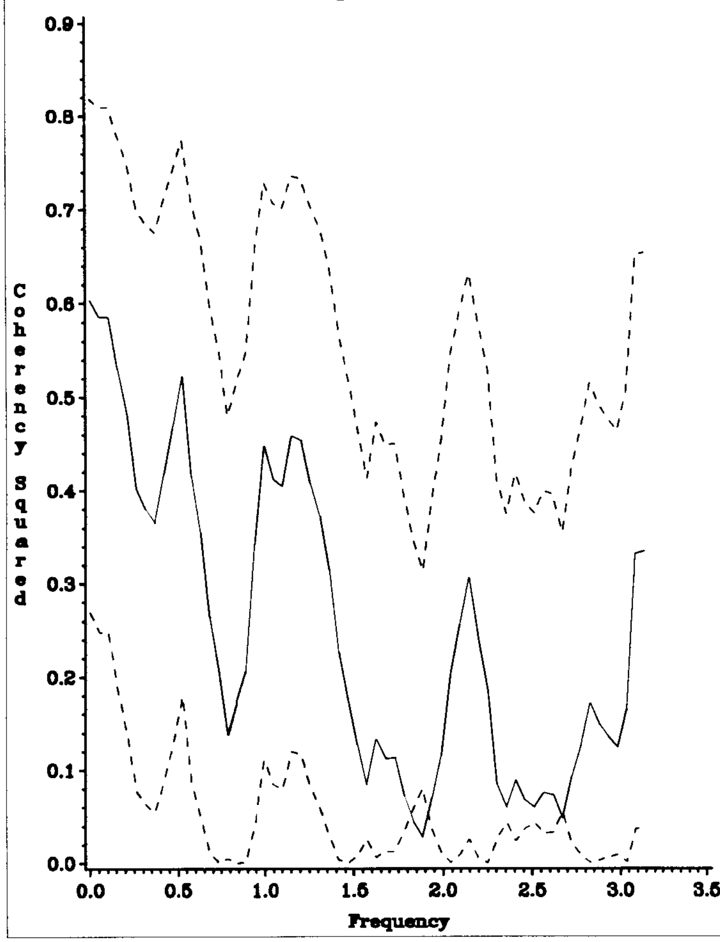

Figure 37.

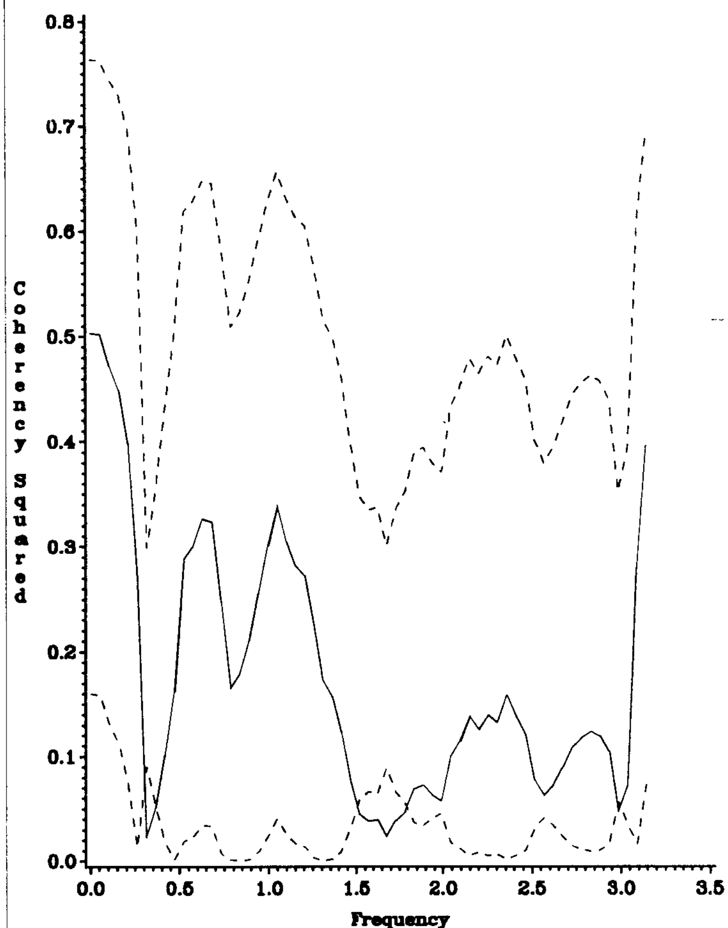

Figure 36.
Change in Real PCE Health ex Medicare as a Percent of Total Pop. and the Change in Real Medicare as a Percent of Pop. 65 and over 1967:2 to 1997:1

Figure 37.
Change in Nominal PCE Health ex Medicare as a Percent of Total Pop. and the Change in Nominal Medicare as a Percent of Pop. 65 and over 1967:2 to 1997:1