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Abstract

Changes in monetary policy are typically implemented gradually, an empirical observation known as interest-rate smoothing. We propose the explanation that time-non-separable preferences may render interest-rate smoothing optimal. We find that when consumers have "catching-up-with-the-Joneses" preferences, optimal monetary policy reacts gradually to shocks to prevent inefficiently fast adjustments in consumption. We also extend our basic model to investigate the effects of capital formation and nominal rigidities on the dynamics of optimal monetary policy. Optimal policy responses continue to be gradual in the presence of capital and sticky prices, with a size and speed that are in line with empirical findings for the U.S. economy. Our results emphasize that gradualism in monetary policy may be needed simply to guide the economy on an optimally smooth path.

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Keywords: optimal monetary policy, habit persistence, catching up with the Joneses

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Characterizing the nature of optimal monetary policy both in the long run and over the course of business cycles continues to be a fundamental question in macroeconomics. The gradual manner in which monetary policy is usually implemented has received attention in both academic and policy debates of late, including in mid-2004 as the Federal Reserve was on the verge of raising its target for the federal funds rate. Rather than a case for gradualism being made solely on the basis of uncertainty on the part of policy-makers about the state of the economy, which is an often-cited reason for a cautious approach to monetary policy, much of the case seemed to rest on inducing a smooth transition for both the real economy and financial markets. This paper provides a justification for the latter motivation.

By gradualism in monetary policy, often referred to as interest-rate smoothing, we mean that following an exogenous shock that requires policy to respond, the peak response does not occur immediately, but rather with a time lag. Sack (2000) shows that the Greenspan-era Fed's peak response does not occur until up to 15 months following some types of shocks, and Sack and Wieland (2000) and Dennis (2003) provide further evidence along the same lines.[1] The literature has taken the position that such gradualism reflects optimal policy-setting, and the challenge is to understand why such persistence is optimal. We take this position here as well.

We offer the explanation that catching-up-with-the-Joneses preferences renders interest-rate smoothing optimal. Under catching-up-with-the-Joneses preferences, each consumer's utility depends not only on the level of his own consumption, but also on the level of lagged aggregate consumption. Thus, preferences display an externality as well as time-non-separability. For the optimal policy-maker, who internalizes the consumption externality, this latter feature of preferences acts as internal habit persistence.[2] Time-non-separability

[1] Empirical Taylor-style monetary policy rules also suggest, through ubiquitously large positive and significant estimated coefficients on lagged interest rates, that there is a great deal of persistence in monetary policy during the Greenspan era. See, for example, Clarida, Gali, and Gertler (2000), Rudebusch (2002), and English, Nelson, and Sack (2003). Estimated coefficients on the lagged interest rate are typically in the range 0.40 to 0.80. However, Taylor rules, typically specified with inflation as an explanatory variable, will not be a good way to consider the conduct of optimal policy in our model because inflation will track the nominal interest rate perfectly.

[2] As such, we will sometimes refer to our model as displaying external habit persistence, which is used synonymously in the literature with the term catching-up-with-the-Joneses. A further point about terminology: "catching up with the Joneses" has come to mean something different from "keeping up with the Joneses:" the latter refers to a contemporaneous externality, while the former refers to a lagged externality. We use "keeping up" in our title because it is the more common idiom, but it is the catching-up specification

in preferences has been shown in several recent studies to be important in understanding gradual responses of real variables to monetary policy shocks, so our explanation here of the dynamics of optimal policy is closely related.[3] The intuition for the interest-rate smoothing result in our model is the following. With the lagged consumption externality, private agents choose to adjust consumption in response to shocks suboptimally quickly if policy is held fixed. Because the lagged consumption externality acts as internal habit persistence from the perspective of the optimal policy-maker, the policy-maker chooses a smoother path of consumption than would be chosen by the private sector. In order to support this smoother path of consumption, a smoothly varying nominal interest rate is required.

Consumption externalities and time-non-separability in preferences have also proven useful recently in explaining macroeconomic puzzles in asset pricing (for example, Abel (1990) and Campbell and Cochrane (1999)) and international economics (Uribe (2002)). The implications of consumption externalities for fiscal policy have recently been studied by Ljungqvist and Uhlig (2000) and Abel (forthcoming). We continue building on this recent tradition of investigating the macroeconomic consequences of such preferences.

Of further interest is that the approach we employ is based on the Ramsey formulation of a dynamic stochastic general equilibrium model, with no prior assumptions about the targets of monetary policy. Recent studies that also use this approach include Schmitt-Grohe and Uribe (2004), Siu (2004), and King and Wolman (1999). To our knowledge, though, we are the first to solve a Ramsey-type optimal policy problem with time-non-separable preferences.

The issue of interest-rate smoothing has attracted attention in recent years, especially since Clarida, Gali, and Gertler (1999, p. 1702) identified it as an important outstanding issue in monetary policy. Woodford (2003a and 2003b) has recently argued that infusing monetary policy with gradualism is welfare-enhancing through its effects on private sector expectations of future policy and hence its effects on long-term interest rates. Our explanation of optimal interest-rate smoothing is different from his in two respects: we do not restrict ourselves apriori to a particular class of monetary reaction functions — we instead solve the full Ramsey problem — and, more fundamentally, it is preferences that make such policy optimal in our model. As mentioned above, another strand of the literature shows that gradualism is optimal in the presence of various types of uncertainty facing by policy-makers.

that we use throughout.

[3]Recent studies that argue that time-non-separability in preferences is important to understanding dynamics in monetary economies are Fuhrer (2000), Christiano et al (2001), Altig et al (2002), and Boivin and Giannoni (2003).

Our paper demonstrates another possibly important motivation for gradualism.

The rest of our work is organized as follows. We first present the simplest version of our model, which demonstrates the basic reason why external habit persistence makes interest-rate smoothing optimal. In Section 1, we describe this basic model, which features the consumption externality, perfectly competitive product markets, and no physical capital. Here we find two central results — that the Friedman Rule is suboptimal in the presence of catching-up-with-the-Joneses and optimal monetary policy responds gradually to technology shocks. In Section 2, we study a more empirically-relevant model by adding capital formation and nominal rigidities, the key features of RBC models and New Keynesian models, respectively.[4] In this full model, the main result of optimal interest-rate smoothing remains, and the size and speed of smoothing continues to match fairly well the empirical findings of Sack (2000). Thus, our results are obtained in both models with flexible prices and sticky prices, showing that intertemporal linkages caused by dynamic price-setting need not be a key factor in explaining monetary policy inertia. In addition, this full model does quite well in matching key business cycle statistics of the U.S. economy. Section 3 concludes.

1 An Economy with Consumption Externalities

In this section, we describe the simplest version of our cash-in-advance model, an economy that features a consumption externality, no physical capital, and perfectly competitive product markets.

1.1 Households

Household preferences are represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, C_{1t-1}, C_{2t-1}, n_t), \tag{1}$$

where c_{1t} denotes the household's consumption of cash goods in period t, c_{2t} denotes consumption of credit goods in period t, n_t denotes labor supply in period t, and C_{1t-1} and C_{2t-1} denote, respectively, lagged aggregate consumption of cash goods and credit goods. Instantaneous utility is strictly increasing in c_{1t} and c_{2t} and strictly decreasing in C_{1t-1}, C_{2t-1},

[4] A companion paper, Chugh (2004), studies the separate effects of capital formation, monopolistic competition, and sticky prices on optimal interest-rate smoothing in the presence of catching-up-with-the-Joneses preferences.

and n_t. While cash goods and credit goods appear separately as arguments to the utility function, we will define the consumption externality to accrue to an aggregate of the two types of consumption, thus avoiding the somewhat awkward assumption that externalities may be of potentially different magnitudes for the two kinds of consumption goods.[5]

The household faces a flow budget constraint

$$c_{1t} + c_{2t} + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} = w_t n_t + \frac{M_t}{P_t} + R_t \frac{B_t}{P_t} + t_t, \tag{2}$$

where P_t is the aggregate price level, w_t is the real wage, M_t is money holdings chosen in period $t-1$ and carried into period t, and B_t is nominal bond holdings chosen in period $t-1$ and carried into period t. The gross nominal interest rate on bonds is R_t, and t_t is a real lump-sum tax levied by the government. This lump-sum tax finances monetary policy. As usual, the cash good and credit good are technologically identical and so have a common price.[6] The household also faces the cash-in-advance constraint

$$c_{1t} \leq \frac{M_t}{P_t}, \tag{3}$$

which is the source of money demand. Thus, we explicitly consider the costs associated with holding money, and, as we will show, optimal policy in our model operates through exactly these money demand frictions. In this regard, our analysis of optimal monetary policy is in keeping with the spirit of Friedman (1969), which emphasizes the opportunity costs of holding money, and is complementary to analyses conducted in models that abstract from money demand.

The household maximizes (1) subject to (2), (3), and a sufficiently large negative bound on bond holdings by choosing decision rules for c_{1t}, c_{2t}, n_t, M_{t+1}, and B_{t+1}, taking as given the prices w_t and R_t as well as lagged aggregate consumption C_{1t-1} and C_{2t-1}. The household observes the log-technology realization z_t, which is the only source of uncertainty in the economy, before making decisions. Associating the Lagrange multiplier ϕ_t with the budget constraint and λ_t with the cash-in-advance constraint, the first-order conditions for the household problem are

$$\frac{\partial u_t}{\partial c_{1t}} = \phi_t + \lambda_t, \tag{4}$$

[5] An assumption made particularly awkward by the usual interpretation, which we follow, that cash goods and credit goods are physically indistinguishable.

[6] Technically, they have a common price both because of the unit marginal rate of transformation and because both credit sales and cash sales in period t result in cash available for spending at the beginning of period $t+1$. See Lucas and Stokey (1987, p. 493-494) for more discussion.

$$\frac{\partial u_t}{\partial c_{2t}} = \phi_t, \tag{5}$$

$$-\frac{\partial u_t}{\partial n_t} = \phi_t w_t, \tag{6}$$

$$\frac{\phi_t}{P_t} = \beta E_t \left[\frac{\phi_{t+1} + \lambda_{t+1}}{P_{t+1}} \right] \tag{7}$$

$$\frac{\phi_t}{P_t} = \beta R_{t+1} E_t \left[\frac{\phi_{t+1}}{P_{t+1}} \right]. \tag{8}$$

These optimality conditions have the standard interpretation. Condition (4) states that the marginal utility of cash good consumption is equated to its marginal cost, which is the sum of the marginal utility of wealth and the marginal utility of (real) money due to the cash-in-advance constraint. Condition (5) states that the marginal utility of credit good consumption is equated to the marginal utility of income. Condition (6) states that the marginal utility of leisure is equated to the marginal utility of the wage. Condition (7) states that marginal utility of current income is equated to discounted marginal utility of next-period cash good consumption, and condition (8) states that marginal utility of current income is equated to discounted marginal utility of next-period income. In condition (8), the nominal return R_{t+1} received at time $t+1$ appears outside the expectation operator because it is known at time t.

Finally, note that combining the first-order conditions on money holdings and bond holdings and using the optimality conditions for cash consumption and credit consumption gives

$$R_t = \frac{\partial u_t / \partial c_{1t}}{\partial u_t / \partial c_{2t}}, \tag{9}$$

a standard result in cash-credit economies that the gross nominal interest rate equals the marginal rate of substitution between cash goods and credit goods.

1.2 Firms

A representative firm produces the output good by hiring labor in a perfectly-competitive labor market. The firm's production function is linear in labor,

$$f(n_t) = e^{z_t} n_t. \tag{10}$$

Maximization of profits implies the marginal product of labor is equated to the real wage.

1.3 Government

The government has a simple role: it simply puts nominal money into or takes nominal money out of circulation through lump-sum transfers to households. The government budget constraint is

$$t_t = \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t}. \tag{11}$$

There is no government spending that requires financing, hence no fiscal constraints on the monetary authority's plans. By eliminating fiscal policy, our work is comparable to King and Wolman (1999), but different from Chari et al (1991), Schmitt-Grohe and Uribe (2004), and Siu (2004). In the latter studies, monetary policy at least partially responds to fiscal developments, but we abstract from fiscal policy to isolate principles of monetary policy.

1.4 Competitive Monetary Equilibrium

A competitive monetary equilibrium for this economy is stochastic processes for c_{1t}, c_{2t}, n_t, M_{t+1}, B_{t+1}, w_t, and R_{t+1} such that the household maximizes utility with the cash-in-advance constraint holding with equality, the firm maximizes profit, the labor market clears, the bond market clears every period, so that

$$B_t = 0, \tag{12}$$

the government budget constraint (11) is satisfied, and the resource constraint

$$c_{1t} + c_{2t} = n_t \tag{13}$$

is satisfied. In addition, if bonds earn a gross nominal return of less than one, the household can make unbounded profits by buying money and selling bonds, which implies that in a monetary equilibrium $R_t \geq 1$, which, using condition (9), means that

$$\frac{\partial u_t}{\partial c_{1t}} \geq \frac{\partial u_t}{\partial c_{2t}} \tag{14}$$

must hold as well.

1.5 Optimal Allocations

To consider optimal policy, we construct a Ramsey-type allocation problem. The constraints that competitive monetary equilibrium impose, beyond the resource constraint, on the policy-maker are presented in the following Proposition, the proof of which appears in more general form in Appendix A.

Proposition 1. *The consumption and labor allocations in a competitive monetary equilibrium satisfy the resource constraint (13), the no-arbitrage condition (14), the implementability constraint*

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{\partial u}{\partial c_{2t}} c_{1t} + \frac{\partial u}{\partial c_{2t}} c_{2t} + \frac{\partial u}{\partial n_t} n_t \right] = 0, \tag{15}$$

and the labor-market clearing condition

$$-\frac{\partial u_t / \partial n_t}{\partial u_t / \partial c_{2t}} - e^{z_t} = 0. \tag{16}$$

In addition, allocations that satisfy (13), (14), (15), and (16) can be supported as a competitive monetary equilibrium.

Notice that it is the marginal utility of credit good consumption, rather than the marginal utility of cash good consumption as in the implementability constraint of Chari and Kehoe's (1999) monetary model or Siu's (2004) model, that multiplies cash good consumption. This difference arises due to the lump-sum manner, rather than open market operations, through which monetary policy is conducted in our model.

The constraint (16) is needed to ensure labor-market clearing because the government is assumed to not have any instruments that drive a wedge between the wage and the marginal product of labor. In the absence of this constraint, an implicit time-varying labor tax would correct the consumption externality, eliminating the need for monetary policy. To the extent that monetary policy can respond to business cycle events more quickly than labor taxes, we think specifying the policy problem this was is realistic.[7] The term $(\partial u_t / \partial n_t)/(\partial u_t / \partial c_{2t})$ is the wage expressed in terms of allocations using the household first-order conditions (5) and (6).

The Ramsey government internalizes the consumption externality and chooses allocations for consumption and labor. That is, in the Ramsey problem, we set $C_{1t-1} = c_{1t-1}$ and $C_{2t-1} = c_{2t-1}$ directly in the utility function before optimizing. The Ramsey problem is thus to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, c_{1t-1}, c_{2t-1}, n_t), \tag{17}$$

subject to the resource constraint (13), the no-arbitrage condition (14), the implementability constraint (15), and the labor-market clearing constraint (16). We conjecture, and then verify numerically when we conduct simulations of the model, that in the presence of the

[7]See Chari and Kehoe (1999, p. 1680) for more discussion on this.

consumption externality, the no-arbitrage condition will always be slack, so we proceed by dropping this constraint from the Ramsey problem. For convenience, define the functions

$$H(c_{1t}, c_{2t}, c_{1t-1}, c_{2t-1}, n_t) \equiv \left[\frac{\partial u_t}{\partial c_{2t}} c_{1t} + \frac{\partial u_t}{\partial c_{2t}} c_{2t} + \frac{\partial u_t}{\partial n_t} n_t\right] \tag{18}$$

and

$$W(c_{1t}, c_{2t}, c_{1t-1}, c_{2t-1}, n_t) \equiv -\frac{\partial u_t / \partial n_t}{\partial u_t / \partial c_{2t}} - e^{z_t}. \tag{19}$$

Associating the Lagrange multiplier ρ_t to the resource constraint, the multiplier ξ to the implementability constraint, and the multiplier ω_t to the wage constraint, the first-order conditions for the Ramsey problem with respect to cash-good consumption, credit-good consumption, and labor are

$$\frac{\partial u_t}{\partial c_{1t}} + \xi \frac{\partial H_t}{\partial c_{1t}} + \omega_t \frac{\partial W_t}{\partial c_{1t}} + \beta E_t \left[\frac{\partial u_{t+1}}{\partial c_{1t}} + \xi \frac{\partial H_{t+1}}{\partial c_{1t}} + \omega_{t+1} \frac{\partial W_{t+1}}{\partial c_{1t}}\right] = \rho_t, \tag{20}$$

$$\frac{\partial u_t}{\partial c_{2t}} + \xi \frac{\partial H_t}{\partial c_{2t}} + \omega_t \frac{\partial W_t}{\partial c_{1t}} + \beta E_t \left[\frac{\partial u_{t+1}}{\partial c_{2t}} + \xi \frac{\partial H_{t+1}}{\partial c_{2t}} + \omega_{t+1} \frac{\partial W_{t+1}}{\partial c_{2t}}\right] = \rho_t, \tag{21}$$

$$\frac{\partial u_t}{\partial n_t} + \rho_t f_n(n_t) + \xi \frac{\partial H_t}{\partial n_t} + \omega_t \frac{\partial W_t}{\partial n_t} = 0. \tag{22}$$

In the Ramsey government's optimality conditions on c_{1t} and c_{2t}, the $\partial u_{t+1}/\partial c_{it}$, $\partial H_{t+1}/\partial c_{it}$, and $\partial W_{t+1}/\partial c_{it}$, $i = 1, 2$, terms appear because the policy-maker internalizes the consumption externality. In the absence of the consumption externality ($\alpha = 0$), it is a well-known result that the Friedman Rule of a zero net nominal interest rate is Pareto optimal — for example, see Chari and Kehoe (1999). If the consumption externality exists ($\alpha > 0$), we cannot obtain an analytical characterization of the optimal nominal interest rate and hence must resort to numerical methods. This is the task to which we turn next.

1.6 Calibration

We first describe how we calibrate the model and then present the quantitative results. The period utility function is

$$\ln(c_t - \alpha c_{t-1}) - \frac{\zeta}{1+\nu} n_t^{1+\nu}, \tag{23}$$

in which α parameterizes the strength of the consumption externality and the consumption aggregate is

$$c_t = \left[(1-\sigma) c_{1t}^{v} + \sigma c_{2t}^{v}\right]^{1/v}. \tag{24}$$

Parameter	β	ν	ζ	σ	υ	ρ	σ_ϵ	α
Value	0.99	1.7	20-50	0.62	0.79	0.95	0.007	0.40

Table 1: Calibration of the model with consumption externality, perfectly competitive product markets, and no physical capital. The time unit is one quarter.

We use Siu's (2004) estimates of the parameters of this aggregator, which are $\sigma = 0.62$ and $\upsilon = 0.79$. The parameter ν is set to 1.7 to match Hall's (1997) estimate of the elasticity of marginal disutility of work, and the parameter ζ is chosen so that in the steady-state of the optimal allocation households devote one-third of their time to work. The parameter ζ thus varies with the rest of the calibration. To calibrate a quarterly model, we set the subjective discount factor $\beta = 0.99$ to match an annual steady-state real interest rate of four percent.

The stochastic process $\{z_t\}$ follows the AR(1) process

$$z_t = \rho z_{t-1} + \epsilon_t, \tag{25}$$

where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. Following Cooley and Prescott (1995), we choose $\rho = 0.95$ and $\sigma_\epsilon = 0.007$, standard values in the RBC literature.

We calibrate the externality parameter α so that in the optimal steady-state the nominal interest rate matches the three-month Treasury bill rate during the period January 1988-September 2003, which was 4.6 percent.[8] Given the rest of our calibration, matching this nominal interest rate requires $\alpha = 0.40$. This parameter value is a bit lower than but in line with the estimate 0.60 by Ravn, Schmitt-Grohe, and Uribe (2003).[9] Table 1 summarizes the calibration of this version of our model.

[8]We choose this sample period because the empirical Taylor Rule literature has noted that estimated monetary policy reaction functions during this period differ statistically from those estimated using pre-Greenspan periods or longer periods that include the Greenspan era. See, for example, Taylor (1999, p. 330). This finding suggests that monetary policy has been conducted fundamentally differently in the past 17 years than in the pre-Greenspan era.

[9]Ravn, Schmitt-Grohe, and Uribe (2003) model consumption externalities in a way fairly closer to our way and estimate a parameter which is comparable to our parameter α of 0.60. They also allow the subutility function over the quasi-difference in consumption $c_t - \alpha c_{t-1}$ to have coefficient of relative risk aversion σ, whereas we have a coefficent of relative risk aversion of 1 due to our log specification. However, their estimate is $\sigma = 1.5$, so their results seem to be fairly comparable to our specification. Also note that the empirical works of Ferson and Constantinides (1991) and Fuhrer (2000), using macroeconomic data, and Dynan (2000), using microeconomic data, both assume internal habit persistence and so are not directly relevant here.

α	0	0.1	0.2	0.3	0.4	0.5	0.6
R	1	1.0090	1.0194	1.0315	1.0459	1.0630	1.0841

Table 2: Optimal steady-state annual gross nominal interest rate for various degrees of consumption externality in the economy without physical capital. The parameter α measures the intensity of the consumption externality.

1.7 Quantitative Results

We now characterize numerically the dynamic properties of the optimal allocations in this version of our model. Our solution method is standard log-linear techniques — in particular, we use Uhlig's (1999) method of undetermined coefficients. We log-linearize the policy-maker's first-order conditions (20), (21), and (22), and the resource constraint (13), around the deterministic steady-state of these conditions.[10] Table 2 presents the optimal steady-state nominal interest rate for several values of α. As soon as the consumption externality rises above zero, the Friedman Rule of a zero net nominal interest rate is longer optimal, which is our first central result.

With $\alpha > 0$, the policy-maker chooses an allocation in which the marginal utilities of the two goods are not equal even though the two goods are technologically identical. The economic reason for the result is straightforward. In the absence of any other tax instruments, the nominal interest rate can be used to tax overall consumption through its effect on cash good consumption. The consumption externality, if uncorrected, causes the consumption index to be too high relative to the Ramsey government's choice, thus taxing consumption via a strictly positive net nominal interest rate brings down the index. Unregulated consumption is not too far above its optimum for small values of α because the externality is mild, so the inflation tax rate — controlled via the nominal interest rate — does not need to be very large. As α rises, however, the externality problem is more severe and pushes unregulated consumption even further above its optimum, requiring a larger tax. Monetary policy in our model thus operates through the costs of holding money, in contrast to the often "cashless" environments of IS-LM-style models in which this channel does not exist. This "fiscal" use of monetary policy is similar in idea to the finding of Ljungqvist and Uhlig (2000) that a

[10]We do not prove that a Ramsey steady-state exists, or even that a solution to the Ramsey problem exists. Furthermore, as is also a common problem in the Ramsey literature, we cannot guarantee that even if a Ramsey steady-state exists that the decision rules imply convergence to it. We do find numerically a steady-state of the Ramsey first-order conditions.

labor tax corrects a consumption externality. However, our focus is on the implications for optimal interest-rate smoothing, so we now consider the dynamics of the model.

For all dynamic results, here and in Section 2, the economy is assumed to begin in the deterministic steady-state associated with the optimal allocation. Considering an impulse response analysis illustrates the basic idea behind the optimal interest-rate smoothing result in the model. Figure 1 presents twenty-quarter impulse responses for consumption of each type of good, output, the optimal nominal interest rate, the inflation rate, and the money stock under the optimal policy in response to a one standard deviation positive shock to technology. The peak response of a 39-basis-point tightening of the nominal interest rate occurs after four quarters. This magnitude and speed of policy adjustment is in line with that found by Sack (2000) and Dennis (2003) for the Greenspan-era Fed. Using a VAR, each finds that the magnitude of the peak response of the nominal interest rate is approximately 20 basis points following a one standard deviation supply shock, which we interpret as the technology shock in our model.[11] Sack (2000) finds that the peak response occurs after four quarters, while Dennis (2003) finds that the peak response is reached after three quarters. The prediction of our model is thus in line with empirical evidence.

The intuition for why interest-rate smoothing is optimal is as described in the introduction, and we now elaborate a bit further. In response to a shock, the policy-maker would like consumption to adjust more gradually than the private sector would choose on its own because no individual takes into account the fact that the level of his current consumption, holding all else constant, negatively affects all others (including himself) in the future. A way to think about this is that utility depends not only positively on the level of consumption but also negatively on the growth rate of consumption. This latter aspect, consumers disliking large swings in consumption, gives rise to the terminology "habit persistence" — consumers are assumed to become accustomed to their level of consumption and dislike changes. However, this concern for consumption growth rates is not taken into account by the individual with catching-up-with-the-Joneses preferences, while it is taken into account by the policy-maker. This time-non-separability of preferences which is internalized by the policy-maker drives the main result of optimal interest-rate smoothing both in the basic model and the richer model of Section 2, as well as other versions that we consider in a companion paper, Chugh (2004).

Figure 1 shows that output and consumption of each type of good also reach their peak

[11] Sack (2000) uses the Federal Funds rate and Dennis (2003) uses the three-month Treasury bill rate.

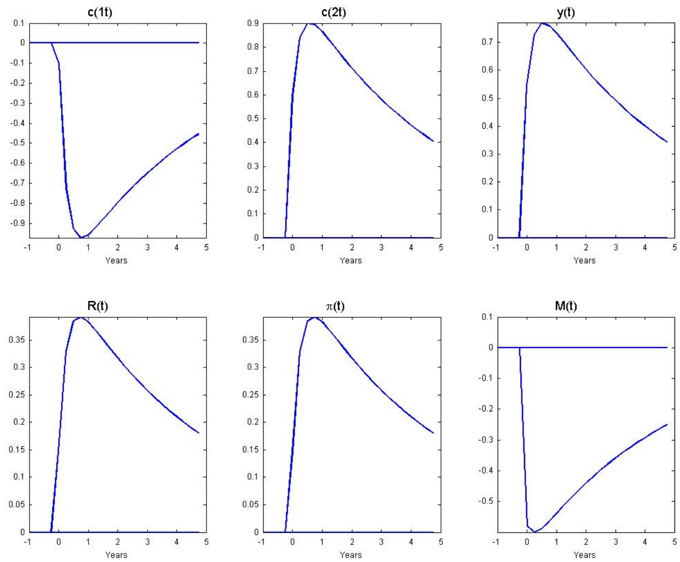

Figure 1: Twenty-quarter impulse responses to a one standard deviation positive technology shock in the economy with no physical capital and perfectly competitive product markets. Variables are measured in percent deviation from their steady-state values. The externality parameter is $\alpha = 0.40$, and the steady-state policy is $R = 1.046$.

responses gradually. Because the nominal interest rate is the opportunity cost of holding money, an increase in the nominal interest rate leads to a decrease in consumption of cash goods. Overall, however, consumption, and hence output, rise due to the positive technology shock, in line with standard RBC predictions. In particular, the substitution effect and income effect both work in the same direction so total consumption, the sum of cash consumption and credit consumption, unambiguously rises. Under the optimal policy, inflation tracks the nominal interest perfectly because the real interest rate is constant, thus inflation rises gradually as well.

Next, Table 3 presents key business cycle moments for $\alpha = 0$ and $\alpha = 0.40$. To compute these business cycle statistics, we conduct 1,000 simulations, each of length 100 periods, of the model and report the median first and second moments. As mentioned in Section 1.5, in principle we must impose a constraint on the Ramsey problem that ensures that allocations can be supported as a monetary equilibrium. In solving for the decision rules and conducting these simulations, we drop this no-arbitrage constraint and numerically check during the simulations that the no-arbitrage constraint never binds. The results we report are for a standard deviation of the productivity shock of $\sigma_\epsilon = 0.007$, which turns out to be small

Variable	Mean	S.D. %	Aut. corr.	Corr(x,y)		Mean	S.D. %	Aut. corr.	Corr(x,y)
	$\alpha=0$					$\alpha=0.40$			
R	0	—	—	—		4.60	0.44	0.87	0.96
π	-4	—	—	—		0.41	0.44	0.87	0.96
y	0.33	1.29	0.67	1		0.33	1.52	0.82	1
n	0.33	0.00	0.67	1		0.33	0.27	0.50	0.38

Table 3: Business cycle properties of the economy with no physical capital and perfectly competitive product markets for $\alpha = 0$ and $\alpha = 0.40$. R and π are reported in annualized percentage points, while y and n are reported in levels.

enough that the no-arbitrage constraint is always slack during the simulations. If we increase the volatility to $\sigma_\epsilon = 0.035$, we still find that the no-arbitrage constraint is always slack during the simulations.[12] Only when we increase the volatility to $\sigma_\epsilon = 0.07$, ten times our benchmark calibration, we do encounter simulations in which the zero lower bound on the nominal interest rate is reached. Thus, only when the volatility of the technology process is very large does the no-arbitrage constraint become a computational issue. With our standard calibration of $\sigma_\epsilon = 0.007$, this is not an issue, so we seem numerically justified dropping the no-arbitrage constraint from the Ramsey problem.

As Table 3 shows, in our model with the consumption externality both the nominal interest rate and the inflation rate π are procyclical, consistent with empirical evidence. However, the model's contemporaneous correlations of the nominal interest rate and inflation with output are stronger than those in data — for example, Stock and Watson (1999) report that the contemporaneous correlation of the the three-month Treasury bill with output is 0.41 and the contemporaneous correlation of the GDP deflator-measured inflation rate with output is 0.15. The nominal interest rate in our model essentially tracks consumption because its role is to mitigate the consumption externality. Because consumption equals output in a model without investment, the nominal interest rate in the model tracks output, explaining the model's near-perfect correlations between the nominal interest-rate and output and between inflation and output.

The procyclicality of the nominal interest rate in our model at least stands in contrast to countercyclical nominal interest rates often found in technology-shock driven models,

[12] With $\alpha = 0.4$ and $\sigma_\epsilon = 0.035$, the steady-state nominal interest rate is 4.6 percent, and the lowest nominal interest rate ever realized during our simulations was 1.2 percent.

such as Altig et al (2002), but our model must be extended to better quantitatively match the data. With this motivation, as well as in the interest of building a model that can explain key business cycle facts in addition to being suited for an investigation of optimal monetary policy, we turn in Section 2 to the task of extending the model to include both capital formation and nominal rigidities, which are the key features of RBC models and New Keynesian models, respectively. We incorporate these elements because both seem to be important in explaining observed business cycle dynamics and so may reasonably be suspected to have an effect on optimal monetary policy over the business cycle. As we will show, the quantitative predictions regarding interest-rate smoothing of this full model with external habit, capital, and sticky prices continue to match U.S. data quite well. Our companion paper, Chugh (2004), studies versions of the model that are intermediate between the basic model of this section and the full model of the next section.

2 Capital and Sticky Prices

Capital formation and nominal rigidities lie at the heart of RBC models and New Keynesian models, respectively. Most macroeconomic models that include capital formation assume prices are flexible, while most New Keynesian models that feature sluggish price adjustment abstract from capital formation. Both features are likely important in explaining observed business cycle phenomena, however, and bringing these ideas together in macro models is an understudied area of research. The general sense in the literature has been that capital, because it is little correlated with GDP at business cycle frequencies, plays an unimportant role in determining policy over the business cycle and therefore can be safely abstracted from. We know of no work, however, that conclusively shows that capital formation is indeed unimportant for the consideration of optimal monetary policy. Here, we bring together these features along with external habit persistence to study in a richer, and likely more empirically relevant, framework the issue of interest-rate smoothing.

We introduce monopolistic competition and nominal rigidities in standard New Keynesian fashion, by segmenting product markets into final goods and differentiated intermediate goods. We sketch how the model is modified to accommodate these elements, show how the optimal policy problem is affected, and then present results.

2.1 Final Goods Producers

Cash goods, credit goods, and investment goods are physically indistinguishable — they are all final goods. These final goods are produced in a competitive sector according to a Dixit-Stiglitz CES aggregator

$$y_t = \left[\int_0^1 y(i)^{1/\varepsilon} di \right]^{\varepsilon}, \tag{26}$$

where $\varepsilon/(\varepsilon - 1)$ is the elasticity of substitution between any two intermediate goods. Here, i indexes the differentiated intermediate goods. Final goods production requires only the differentiated intermediate goods. Profit maximization by final goods producers gives rise to demand functions for each intermediate good i

$$y_{i,t} = \left[\frac{P_t}{P_{i,t}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} y_t, \tag{27}$$

where aggregate demand y_t and the aggregate price level P_t are both taken as given by each intermediate good producer, and $P_{i,t}$ denotes the nominal price of intermediate good i.

2.2 Intermediate Goods Producers

Each differentiated good i is produced by firm i using a constant-returns-to-scale technology

$$y_{i,t} = e^{z_t} f(k_{i,t}, n_{i,t}) \tag{28}$$

by the corresponding intermediate goods producer, and the technology shock z_t is common to all intermediate goods producers.

We employ Taylor staggered price-setting to introduce nominal rigidities. Once set, an intermediate firm's nominal price cannot be changed for T periods. The profit maximization problem of each intermediate goods firm j is thus a dynamic one,

$$\max E_t \sum_{s=0}^{T-1} \beta^s \left(P_{s+1,t+s} - \Upsilon_{t+s} \right) y_{s+1,t+s}, \tag{29}$$

where $P_{s+1,t+s}$ denotes the nominal price of intermediate good j s periods after it was set, Υ_t denotes nominal marginal cost of intermediate goods producer j in period t, and T denotes the number of periods for which the nominal price an intermediate firm sets is fixed.

Throughout our analysis, we restrict attention to the case $T = 2$. Imposing $T = 2$ and using the demand function (27) to substitute for $y_{s+1,t+s}$, the first-order condition of (29) with respect to $P_{1,t}$ is

$$\left(1 - \varepsilon \left[\frac{P_t}{P_{1,t}} \right] \upsilon_t \right) y_{1,t} + \beta E_t \left(\frac{P_t}{P_{t+1}} - \varepsilon \left[\frac{P_t}{P_{1,t}} \right] \upsilon_{t+1} \right) y_{2,t+1} = 0, \tag{30}$$

where $\upsilon_t \equiv \Upsilon_t / P_t$ denotes real marginal cost and P_t is the nominal price of the final good — equivalently, P_t is the aggregate price level of the economy. This condition states that the present value of marginal profits from a price change is zero. Gross inflation is $\pi_{t+1} \equiv P_{t+1}/P_t$, and note that $P_{1,t} = P_{2,t+1}$ due to the nominal rigidity. With these points in mind, the preceding expression becomes

$$\left(1 - \varepsilon \left[\frac{P_t}{P_{1,t}}\right] \upsilon_t\right) y_{1,t} + E_t \frac{\beta}{\pi_{t+1}} \left(1 - \varepsilon \left[\frac{P_{t+1}}{P_{2,t+1}}\right] \upsilon_{t+1}\right) y_{2,t+1} = 0, \tag{31}$$

an expression we manipulate further in Section 2.4 when we present the optimal policy problem.

We consider only symmetric equilibria in which all intermediate producers in a given pricing cohort make the same decisions, thus subscripts other than time subscripts denote only to which pricing cohort a firm belongs. Given the demand it faces, firm j then purchases capital and labor in spot markets to minimize cost. Cost minimization implies that

$$\frac{f_n(k_{j,t}, n_{j,t})}{f_k(k_{j,t}, n_{j,t})} = \frac{w_t}{r_t}. \tag{32}$$

Because the technology is constant returns to scale, capital-labor ratios are equated across intermediate goods producers. Finally, a consequence of cost-minimization and constant capital-labor ratios is that marginal cost can be expressed in terms of only aggregates,

$$\upsilon_t = -\frac{1}{1-\theta} \frac{\partial u_t / \partial n_t}{\partial u_t / \partial c_{2t}} \left(\frac{n_t}{k_t}\right)^{\theta}. \tag{33}$$

With two-period Taylor price-setting, and with symmetry within each pricing cohort, the final goods aggregator can be written as

$$y_t = \left[\frac{1}{2}\left(e^{z_t} f(k_{1,t}, n_{1,t})\right)^{1/\varepsilon} + \frac{1}{2}\left(e^{z_t} f(k_{2,t}, n_{2,t})\right)^{1/\varepsilon}\right]^{\varepsilon}, \tag{34}$$

which we will denote by $y_t = y(k_{1,t}, k_{2,t}, n_{1,t}, n_{2,t})$. Using the first-order condition (30) and demand function (27), gross inflation can be expressed as

$$\pi_t = \left(\frac{y_{1,t}}{y_{2,t}}\right)^{\frac{1-\varepsilon}{\varepsilon}}, \tag{35}$$

and profits of intermediate firm i can be expressed as

$$pr_{i,t} = (1 - \upsilon_t) y_{i,t}, \tag{36}$$

both of which are in terms of only allocations and will also be used in constructing the constraints of the optimal policy problem in Section 2.4.

2.3 Households

The household problem is a straightforward extension of that described in Section 1.1. The household chooses aggregate labor and aggregate capital, along with consumption of cash and credit goods as well as asset holdings to maximize (1) subject to the cash-in-advance constraint (3), a sufficiently large negative bound on bond holdings, and the flow budget constraint

$$c_{1t} + c_{2t} + k_{t+1} + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} = w_t n_t + (1 - \delta + r_t)k_t + \frac{M_t}{P_t} + R_t \frac{B_t}{P_t} + t_t + pr_t, \qquad (37)$$

where r_t denotes the rental rate of capital, δ is the depreciation rate of capital, and pr_t denotes profits of the intermediate goods producers which the household receives lump-sum.

Associating the Lagrange multiplier ϕ_t with the budget constraint and λ_t with the cash-in-advance constraint, the first-order conditions for the household problem are the same as expressions (4)- (8), with the added optimality condition on capital holdings

$$\phi_t = \beta E_t[\phi_{t+1}(1 - \delta + r_{t+1})], \qquad (38)$$

which states that the marginal utility of current-period credit good consumption is equated to discounted marginal utility of credit-good consumption generated from savings carried into next period. Finally, just as in the basic model of Section 1, the household optimality conditions imply condition (9), so the channel through which the nominal interest rate affects the economy — the marginal rate of substitution between cash goods and credit goods — is the same as before.

2.4 Optimal Allocations

We now describe the optimal policy problem for this model. The main difference with the basic model of Section 1 is an extra constraint that ensures that the nominal rigidity in product markets is respected. The policy-maker maximizes the representative household's lifetime discounted expected utility subject to the resource constraint

$$c_{1t} + c_{2t} + k_{t+1} - (1 - \delta)k_t = y(k_{1,t}, k_{2,t}, n_{1,t}, n_{2,t}), \qquad (39)$$

the household implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{\partial u}{\partial c_{2t}} c_{1t} + \frac{\partial u}{\partial c_{2t}} c_{2t} + \frac{\partial u_t}{\partial n_t} n_t - \frac{\partial u_t}{\partial c_{2t}} \left(\frac{1}{2} pr_{1,t} + \frac{1}{2} pr_{2,t} \right) \right] = \frac{\partial u_0}{\partial c_{20}} (1 - \delta + f_k(k_0, n_0))k_0, \qquad (40)$$

the sticky-price firm constraint

$$\left(1-\varepsilon\left[\frac{y_{1,t}}{y_t}\right]^{\frac{\varepsilon-1}{\varepsilon}}\upsilon_t\right)y_{1,t}+\beta E_t\left(\frac{y_{1,t+1}}{y_{2,t+1}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(1-\varepsilon\left[\frac{y_{2,t+1}}{y_{t+1}}\right]^{\frac{\varepsilon-1}{\varepsilon}}\upsilon_{t+1}\right)y_{2,t+1}=0, \tag{41}$$

constraints that ensure that marginal products equal their respective factor prices, which are

$$W(c_{1t},c_{2t},c_{1t-1},c_{2t-1},n_t)\equiv-\frac{\partial u_t/\partial n_t}{\partial u_t/\partial c_{2t}}-f_n(k_t,n_t)=0 \tag{42}$$

and

$$L(c_{1t},c_{2t},c_{1t-1},c_{2t-1},c_{1t-2},c_{2t-2},n_t,k_t)\equiv\frac{\partial u_{t-1}/\partial c_{2t-1}}{\beta(\partial u_t/\partial c_{2t})}-1+\delta-f_k(k_t,n_t)=0, \tag{43}$$

the aggregate capital condition

$$k_t=\frac{1}{2}k_{1,t}+\frac{1}{2}k_{2,t}, \tag{44}$$

and the aggregate labor condition

$$n_t=\frac{1}{2}n_{1,t}+\frac{1}{2}n_{2,t}. \tag{45}$$

The household implementability constraint (40) is an extension of that in the basic model, with profit terms arising because the policy-maker optimally redistributes them back to households. Because in our dynamic analysis we assume the economy starts in the Ramsey steady-state, the initial capital stock is that implied by the steady-state of the Ramsey optimality conditions for $t>0$. Thus, we adopt the "timeless" approach to policy, discussed by Woodford (2003b, p. 538-540), in which features of the economy associated with only the start-up period are treated as non-generic events.[13] With this timeless perspective, all decision rules are time-invariant. Hence we restrict attention to the first-order conditions for $t>0$.

As in the basic model of Section 1, constraints are required to ensure factor markets clear because we assume the government has no tax instruments besides the nominal interest rate. The constraints (42) and (43) ensure labor market clearing and capital market clearing, respectively. Notice that in these constraints we use the aggregates k_t and n_t in the marginal product terms — we can do so because capital-labor ratios are equated across firms, as discussed above.

[13]This "timeless" issue does not arise in the model without capital. In the model without capital, there is no asymmetry between the decision rules for $t=0$ and the decision rules for $t>0$ because there are no initial assets.

The sticky-price firm constraint is simply the optimal pricing constraint (31) with prices substituted out using the demand function (27) and gross inflation substituted out using (35). In the first term of the sticky-price constraint, the marginal cost term υ_t depends on the type-1 firm's capital-labor ratio, and in the second term of the sticky price constraint, the marginal cost term υ_{t+1} depends on the type-2 firm's capital-labor ratio. As noted by (33), though, these capital-labor ratios are common. So we will use marginal cost as a function of the aggregate capital-labor ratio, and doing so ensures that capital-labor ratios are always equated.

Finally, as discussed in Section 1.7, we do not impose the no-arbitrage constraint that guarantees the chosen allocations can be supported as a monetary equilibrium because in our simulations the zero-lower bound on the nominal interest rate was never reached.

In time period t, the policy-maker chooses c_{1t}, c_{2t}, n_t, $n_{1,t}$, $n_{2,t}$, k_{t+1}, $k_{1,t}$, and $k_{2,t}$ to maximize the representative household's lifetime discounted utility. Note that while the policy-maker chooses aggregate capital k_{t+1} to carry into the next period, the choice about how to divide the current capital stock k_t among the two types of firms is a static one.

For brevity, we do not present the first-order conditions of the optimal policy problem, except to note that they now include both forward-looking terms and backward-looking terms. The former arise because of the time-non-separability of preferences, capital formation, and the nominal rigidity, while the latter arise because of the nominal rigidity. We again proceed by log-linearizing these first-order conditions around their deterministic steady-state and present the results in the next section.

2.5 Quantitative Results

With the exception of the externality parameter α, we use the same parameter values presented in Table 1. We also now have a few new parameter values to choose. The production function is assumed to be Cobb-Douglas, $f(k_t, n_t) = k_t^{\theta} n_t^{1-\theta}$, with capital's share of output $\theta = 0.36$ to match the annual capital to output ratio of 2.8 observed in the U.S. The quarterly depreciation rate of capital is set to $\delta = 0.02$. We set the static gross markup of intermediate goods producers at $\varepsilon = 1.1$, which is consistent with the empirical evidence of Basu and Fernald (1997) and is a commonly-used value in the New Keynesian literature. Finally, we use the same strategy discussed in Section 1.6 to calibrate a value for α. Matching an optimal steady-state annual nominal interest rate of 4.6 percent here requires $\alpha = 0.58$.

First, we discuss the steady-state inflation rate in the absence of habit (i.e., with $\alpha = 0$).

α	π	R
0	0.997	1.038
0.20	0.999	1.040
0.40	1.002	1.043
0.58	1.007	1.046

Table 4: Optimal steady-state annual gross inflation rate and gross nominal interest rate for various degrees of consumption externality in the economy with physical capital and sticky prices.

A result in many sticky-price models is that zero inflation is optimal because of the resource misallocation cost that would result from any non-zero level of inflation. In our model, zero inflation is not optimal even in the absence of habit persistence. The reason is that there is an explicit cost of holding money in our model, which we model through the cash-in-advance constraint, in contrast to the often "cashless" environments studied in many monetary policy models. In the presence of an explicit cost of holding money but absent nominal rigidities or any other market imperfections, a well-known result is that the optimal monetary policy is the Friedman Rule — deflate at the rate of time preference. With both nominal rigidities and costs of holding money, a natural conjecture is that, absent habit persistence, the optimal inflation rate would be between the Friedman deflation and price stability. This is in fact what we find, as Table 4 shows. Table 4 shows that with $\alpha = 0$, the optimal steady-state gross annual inflation rate is $\pi = 0.997$ with an associated optimal gross annual nominal interest rate of $R = 1.038$. Optimal monetary policy balances the opportunity cost of holding money against the resource misallocation cost arising from non-zero inflation, which is consistent with those studies, such as Khan, King, and Wolman (2003), that do employ both nominal rigidities and costs of holding money.

Next, we characterize the dynamic optimal monetary policy response. As before, we conduct an impulse response analysis to a one-time, one-standard-deviation positive shock to z_t. Figure 2 shows that the peak response of the nominal interest rate is a tightening of 96 basis points, which occurs seven quarters after the shock. The time lag with which the peak response occurs is longer than in the basic model, a result driven by the presence of capital formation. Jermann (1998) and Lettau and Uhlig (2000) show that in the presence of both capital and habits, consumption is excessively smooth compared to data and compared to RBC models without habit, and our result seems closely related. As we discussed in Section 1.7, the nominal interest rate essentially tracks consumption in our model because it

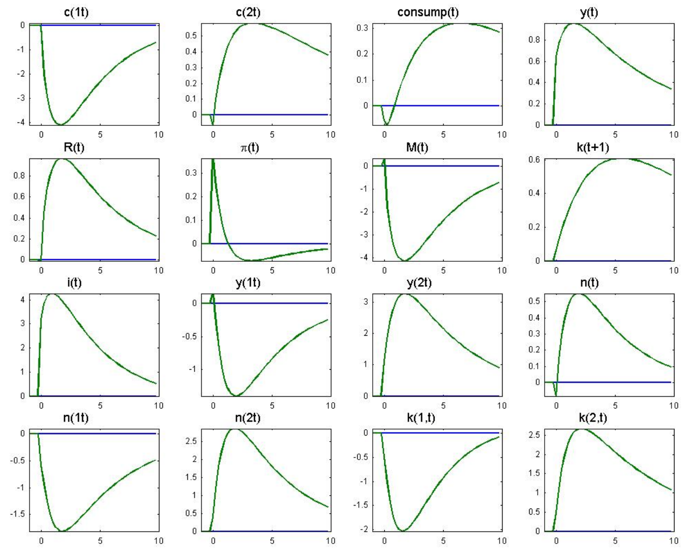

Figure 2: Forty-quarter impulse responses to a one standard deviation positive technology shock in the economy with physical capital and sticky prices. Variables are measured in percent deviation from their steady-state values. The externality parameter is $\alpha = 0.58$, the steady-state gross markup is $\varepsilon = 1.1$, and the steady-state policy is $R = 1.046$.

supports the optimal consumption path. The slow optimal response of consumption in the presence of capital is being supported by a slow optimal response of the nominal interest rate. Nevertheless, the seven-quarter lag of the peak response is not grossly out of line with the three- or four-quarter lag Sack (2000) and Dennis (2003) find. The peak response of optimal policy in this model is higher than in the basic model due to the stronger degree of habit persistence required to calibrate the steady-state. Comparison of Figure 2 with Figure 3, which uses $\alpha = 0.40$ — our calibrated externality parameter in the basic model of Section 1.6 — illustrates the role of the larger α in driving the peak response of policy higher. With $\alpha = 0.40$, the steady-state optimal policy is $R = 1.043$ and the peak response is a tightening of 46 basis points, which occurs six quarters after the shock.

In both Figure 2 and Figure 3, we see that factor use and hence production by the firm currently setting price, the type-1 firm, falls while factor use and hence production by the firm not currently setting price, the type-2 firm, rises. This result is similar to that described by Woodford (2003b, Chapter 3) — when there is an asymmetry across sectors, substitutability allows the possibility that one sector's output rises and another sector's falls in response to a common shock. Here, the asymmetry across the two pricing cohorts is that

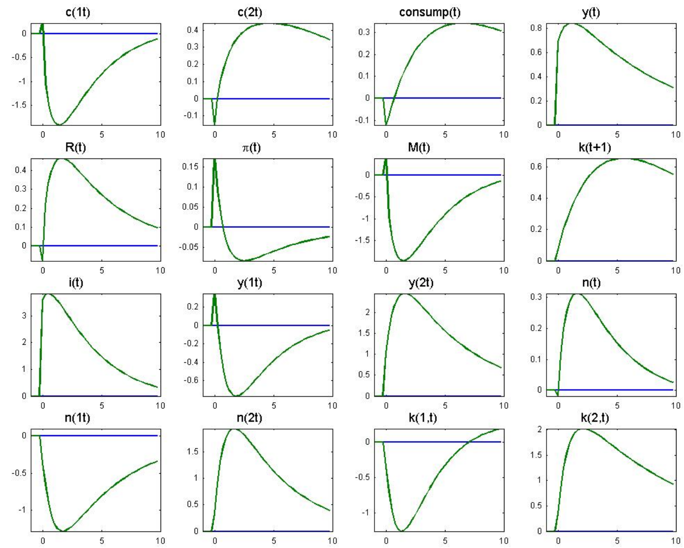

Figure 3: Forty-quarter impulse responses to a one standard deviation positive technology shock in the economy with physical capital and sticky prices. Variables are measured in percent deviation from their steady-state values. The externality parameter is $\alpha = 0.40$, the steady-state gross markup is $\varepsilon = 1.1$, and the steady-state policy is $R = 1.043$.

in steady-state they produce different quantities — if inflation is strictly positive, the price set by price-adjusting firms is higher than the price of non-adjusters and so they produce less. Under both the parameter values $\alpha = 0.40$ and $\alpha = 0.58$, this is exactly the case, so the optimal policy-maker takes advantage of this and assigns non-adjusting firms larger output following the shock. Also interesting to note is that in both Figure 2 and Figure 3 we see that investment has a hump-shaped profile, a feature of business cycles that Christiano, Eichenbaum, and Evans (2001) call an important one for business cycle models to replicate. They replicate this feature by introducing adjustment costs of capital, while we obtain this result without such costs.

Table 5 presents business cycle moments. Comparing these with the moments reported by King and Rebelo (1999, p. 938) for the U.S. economy, we see that our model performs reasonably well. Regarding real variables, it performs best on first-order autocorrelations and less so on contemporaneous correlations with output. The main reason for this is that with strong habit persistence, consumption optimally moves very slowly, so its correlation with output is reduced, leaving investment to drive more of the fluctuations in output. In our models, the correlation of investment with output is higher than found in data and the

Variable	Mean	S.D. %	Aut. corr.	Corr(x, y)		Mean	S.D. %	Aut. corr.	Corr(x, y)
	$\alpha = 0.40$					$\alpha = 0.58$			
R	4.30	0.56	0.81	0.64		4.60	1.03	0.89	0.78
π	0.19	0.23	0.03	0.28		0.41	0.43	0.58	0.44
y	1.26	1.03	0.79	1		1.26	1.07	0.82	1
n	0.33	0.37	0.89	0.66		0.33	0.60	0.88	0.64
c	0.99	0.26	0.81	0.43		0.99	0.21	0.91	0.40
i	0.27	5.20	0.76	0.91		0.27	5.25	0.81	0.99

Table 5: Business cycle properties of the economy with physical capital and sticky prices for $\alpha = 0.40$, and $\alpha = 0.58$. R and π are reported in annualized percentage points, while y, n, c (which is the sum of c_1 and c_2), and i are reported in levels.

correlation of consumption with output is much lower than found in data. Regarding nominal variables, the contemporaneous correlation of inflation with output is in line with that found by Stock and Watson (1999, p. 32) for the U.S. economy, while that of the nominal interest rate is a bit higher than they find.

Thus, optimal interest-rate smoothing of empirically plausible size and speed emerges in our model with habits, capital accumulation, and sticky prices. The time lag of the peak response is a bit longer than empirically observed but not grossly so. We conjecture that the policy response can be sped up by the introduction of adjustment costs of capital, just as Chugh (2004) does in a model with capital accumulation but flexible prices. However, this does not seem to be a first-order concern in this case. Furthermore, the business cycle properties are in line with empirical facts, providing further encouragement to researchers to study models with both capital and sticky prices.

3 Conclusion

The result that gradualism in monetary policy may optimally arise due to concern by policy-makers to move the economy along a smooth path is robust to many features of the economic environment, as the results here and in Chugh (2004) show. We believe this motivation for interest-rate smoothing is one that the literature on monetary policy has largely ignored. The existing literature has focused on policy-makers' uncertainty about data and/or parameters of the economy as the main driving force behind gradualism. While uncertainty may be one reason behind this approach to policy, we have shown that concern for a smooth path for the economy may be another important reason. In fact, our models match fairly well empirically-observed sizes and speeds of policy responses despite full certainty on the part of policy-makers about parameters and shocks.

In addition to addressing the issue of optimal interest-rate smoothing, we aimed to build a model which may be useful in explaining both optimal monetary policy and key business cycle facts. Capital formation and nominal rigidities are widely viewed as two important sources of persistence in the economy. Understanding the properties of optimal monetary policy in a model that features both thus seems crucial to better guide policy, but relatively little work has been done in this area to date.

On a broader note, the study of the nature of optimal monetary policy in the presence of habit persistence, both internal and external, is receiving growing attention. For example, Amato and Laubach (2004) have begun exploring this issue as well, albeit in a somewhat different class of models.[14] Schmitt-Grohe and Uribe (2004) note that they intend to characterize optimal monetary and fiscal policy in a business cycle model along the lines of Christiano, Eichenbaum, and Evans (2001), which includes, among other features, habit persistence. Our work here makes a contribution to the beginning of this field.

Consumption externalities and the often closely-related idea of habit persistence are not yet widely-accepted features of the representative agent's preferences in macroeconomic models, but they have proven useful recently in explaining macroeconomic puzzles in the monetary transmission mechanism, asset pricing, and international economics. We have found here that such preferences also explain gradualism in monetary policy. The study of optimal monetary policy can thus be conducted in macroeconomic models that are able to address these other issues as well. Even more generally, we believe it worthwhile to study the consequences for optimal macroeconomic policy of other richer forms of preferences as well, as Backus, Routledge, and Zin (2004) suggest.

[14] They consider internal habit, whereas we study external habit. Furthermore, they focus attention on the use of monetary feedback rules, whereas we solve a Ramsey-type optimal taxation problem. Nonetheless, we view our work as complementary to theirs.

A Ramsey Problem — Flexible Price Models

Here we present the Ramsey problem for the most general flexible-price version of our model, one that features consumption externalities, physical capital, and monopolistic competition. We specialize to the specific models considered here and in Chugh (2004) as we proceed.

We wish to prove that any competitive monetary equilibrium (CME) satisfies the resource constraint

$$c_{1t} + c_{2t} + k_{t+1} = f(k_t, n_t) + (1-\delta)k_t, \tag{46}$$

the no-arbitrage constraint

$$\frac{\partial u_t}{\partial c_{1t}} - \frac{\partial u_t}{\partial c_{2t}} \geq 0, \tag{47}$$

the labor-market clearing condition

$$w_t = \frac{f_n(k_t, n_t)}{\varepsilon}, \tag{48}$$

the rental-market clearing condition

$$r_t = \frac{f_k(k_t, n_t)}{\varepsilon}, \tag{49}$$

and the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{\partial u_t}{\partial c_{2t}} c_{1t} + \frac{\partial u_t}{\partial c_{2t}} c_{2t} + \frac{\partial u_t}{\partial n_t} n_t - \frac{\partial u_t}{\partial c_{2t}} \left(1 - \frac{1}{\varepsilon}\right) e^{z_t} f(k_t, n_t) \right] = \frac{\partial u_0}{\partial c_{20}} \left(1 - \delta + \frac{f_k(k_0, n_0)}{\varepsilon}\right) k_0, \tag{50}$$

and also that any allocation that satisfies (46), (47), (48), (49), and (50) can be supported as a CME.

First we prove that a CME satisfies (46), (47), (48), (49), and (50). Any equilibrium must satisfy the resource constraint (46). Because we restrict attention to monetary equilibria in which the net nominal interest rate must be non-negative, (47) must also hold. The conditions (48) and (49) must hold because the government does not have access to instruments that tax labor or capital, thus these conditions ensure factor-market clearing.

To show that a CME must also satisfy (50), we first present again the household problem. Begin with the household budget constraint

$$c_{1t} + c_{2t} + k_{t+1} + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} = w_t n_t + (1 - \delta + r_t)k_t + \frac{M_t}{P_t} + R_t \frac{B_t}{P_t} + t_t + pr_t \tag{51}$$

and the cash-in-advance constraint holding with equality

$$c_{1t} = \frac{M_t}{P_t}. \tag{52}$$

Recall the household's first-order conditions for this problem are

$$\frac{\partial u_t}{\partial c_{1t}} = \phi_t + \lambda_t \tag{53}$$

$$\frac{\partial u_t}{\partial c_{2t}} = \phi_t \tag{54}$$

$$-\frac{\partial u_t}{\partial n_t} = \phi_t w_t \tag{55}$$

$$\phi_t = \beta E_t[\phi_{t+1}(1-\delta+r_{t+1})] \tag{56}$$

$$\frac{\phi_t}{P_t} = \beta E_t\left[\frac{\phi_{t+1}+\lambda_{t+1}}{P_{t+1}}\right] \tag{57}$$

$$\frac{\phi_t}{P_t} = \beta R_{t+1} E_t\left[\frac{\phi_{t+1}}{P_{t+1}}\right], \tag{58}$$

where ϕ_t is the Lagrange multiplier on the budget constraint and λ_t is the Lagrange multiplier on the cash-in-advance constraint. The strategy is standard in the literature — namely, the elimination of prices and policies from a lifetime version of the household budget constraint. Begin by using the government budget constraint, which must hold in equilibrium, $t_t = \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t}$, as well as the bond market clearing condition, $B_t = 0$, to reduce the household flow budget constraint to

$$c_{1t} + c_{2t} + k_{t+1} = w_t n_t + (1-\delta+r_t)k_t + pr_t. \tag{59}$$

Next, multiply by $\beta^t \phi_t$ and sum over t to get

$$\sum_{t=0}^{\infty}\beta^t\phi_t c_{1t} + \sum_{t=0}^{\infty}\beta^t\phi_t c_{2t} + \sum_{t=0}^{\infty}\beta^t\phi_t k_{t+1} = \sum_{t=0}^{\infty}\beta^t\phi_t w_t n_t + \sum_{t=0}^{\infty}\beta^t\phi_t(1-\delta+r_t)k_t + \sum_{t=0}^{\infty}\beta^t\phi_t pr_t. \tag{60}$$

Into the left-hand-side of this expression, substitute for ϕ_t using the household's first-order conditions on capital holdings to obtain

$$\begin{aligned}&\sum_{t=0}^{\infty}\beta^t\phi_t c_{1t} + \sum_{t=0}^{\infty}\beta^t\phi_t c_{2t} + \sum_{t=0}^{\infty}\beta^{t+1}\phi_{t+1}(1-\delta+r_{t+1})k_{t+1}\\ &\quad= \sum_{t=0}^{\infty}\beta^t\phi_t w_t n_t + \sum_{t=0}^{\infty}\beta^t\phi_t(1-\delta+r_t)k_t + \sum_{t=0}^{\infty}\beta^t\phi_t pr_t.\end{aligned} \tag{61}$$

Eliminating common summations, we have

$$\sum_{t=0}^{\infty}\beta^t\phi_t c_{1t} + \sum_{t=0}^{\infty}\beta^t\phi_t c_{2t} - \sum_{t=0}^{\infty}\beta^t\phi_t w_t n_t - \sum_{t=0}^{\infty}\beta^t\phi_t pr_t = \phi_0(1-\delta+r_0)k_0, \tag{62}$$

Then use the household's first-order conditions (54) and(55) to substitute for terms ϕ_t and $\phi_t w_t$, and use the facts that equilibrium profits are given by $pr_t = (1-1/\varepsilon)y_t$ (derived in Section 2.2) and the real interest rate is given by (49) to arrive at the implementability constraint (50). From this general flexible-price implementability constraint, expressions (15) is easily recovered. To obtain the implementability constraint for the model with physical capital and perfectly competitive product markets studied in Chugh (2004), set $\varepsilon = 1$. To obtain the implementability constraint for the model with monopolistic competition but without physical capital also studied in Chugh (2004), set $k_0 = 0$ and $f(k_t, n_t) = n_t$. To obtain (15) for the model with neither monopolistic competition nor physical capital, set $k_0 = 0$, $f(k_t, n_t) = n_t$, and $\varepsilon = 1$.

We now show that any allocation of consumption, capital, and labor that satisfies (46), (47), (48), (49), and (50) can be supported as a CME. To establish this, we need to construct prices, policies, money holdings, and bond holdings.

First multiply the household budget constraint (51) by $\beta^t \phi_t$, and sum over time following period r to get

$$\sum_{t=r+1}^{\infty} \beta^t \phi_t (c_{1t} + c_{2t} - w_t n_t) + \sum_{t=r+1}^{\infty} \beta^t \phi_t k_{t+1} + \sum_{t=r+1}^{\infty} \beta^t \phi_t \frac{M_{t+1}}{P_t} + \sum_{t=r+1}^{\infty} \beta^t \phi_t \frac{B_{t+1}}{P_t} =$$
$$\sum_{t=r+1}^{\infty} \beta^t \phi_t (1 - \delta + r_t) k_t + \sum_{t=r+1}^{\infty} \beta^t \phi_t \frac{M_t}{P_t} + \sum_{t=r+1}^{\infty} \beta^t \phi_t R_t \frac{B_t}{P_t} + \sum_{t=r+1}^{\infty} \beta^t \phi_t (t_t + pr_t). \tag{63}$$

Real money holdings are constructed using the cash-in-advance constraint holding with equality,

$$\frac{M_t}{P_t} = c_{1t}. \tag{64}$$

Given this sequence money holdings, construct government lump-sum transfers according to (11). Using the government budget constraint, we can reduce the last expression to

$$\sum_{t=r+1}^{\infty} \beta^t \phi_t (c_{1t} + c_{2t} - w_t n_t) + \sum_{t=r+1}^{\infty} \beta^t \phi_t k_{t+1} + \sum_{t=r+1}^{\infty} \beta^t \phi_t \frac{B_{t+1}}{P_t} =$$
$$\sum_{t=r+1}^{\infty} \beta^t \phi_t (1 - \delta + r_t) k_t + \sum_{t=r+1}^{\infty} \beta^t \phi_t R_t \frac{B_t}{P_t} + \sum_{t=r+1}^{\infty} \beta^t \phi_t pr_t. \tag{65}$$

The nominal interest rate is determined by the marginal rate of substitution between cash goods and credit goods

$$\frac{\partial u_t / \partial c_{1t}}{\partial u_t / \partial c_{2t}} = R_t. \tag{66}$$

Construct the real wage from (54) and (55), and construct the real interest rate from (56).

The last remaining item to consider is bond holdings. The previous summation can be used to obtain an expression for bond holdings at time $r + 1$, which depends, as in Chari and Kehoe (1999), on future allocations. It is difficult to see from the resulting expression, however, that bond holdings are zero, which they must be because they are privately-issued bonds. An easier way to understand why the bond market clears is to construct the household budget constraint from the resource constraint. Because factor markets clear and the technology is constant-returns-to-scale, we have

$$c_{1t} + c_{2t} + k_{t+1} = w_t n_t + (1 - \delta + r_t) k_t. \tag{67}$$

With money holdings and government lump-sum transfers constructed as above, we can write this as

$$c_{1t} + c_{2t} + k_{t+1} + \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} = w_t n_t + (1 - \delta + r_t) k_t + t_t. \tag{68}$$

The only way in which (68) differs from the household budget constraint (51) is that bond-holdings are absent. By imposing the initial condition $B_0 = 0$, it must be that the entire path of bond holdings is $B_t = 0$. Or, in other words, the bond market clears by Walras' Law.

B Ramsey Problem — Sticky Price Models

The Ramsey problem for the model with staggered price-setting is a straightforward extension of the Ramsey problem for the model with flexible prices described in Appendix A. The only difference is in the equilibrium expression for aggregate profits. With symmetry in each pricing cohort,

$$pr_t = \frac{1}{2} pr_{1,t} + \frac{1}{2} pr_{2,t}, \tag{69}$$

and the additional constraint

$$\left(1 - \varepsilon \left[\frac{y_{1,t}}{y_t}\right]^{\frac{\varepsilon-1}{\varepsilon}} \left(-\frac{(\partial u_t/\partial n_t)/(\partial u_t/\partial c_{2t})}{e^{z_t}}\right)\right) y_{1,t} +$$
$$E_t \frac{\partial u_t/\partial c_{2t}}{\partial u_{t+1}/\partial c_{1t+1}} \left(1 - \varepsilon \left[\frac{y_{2,t+1}}{y_{t+1}}\right]^{\frac{\varepsilon-1}{\varepsilon}} \left(-\frac{(\partial u_{t+1}/\partial n_{t+1})/(\partial u_{t+1}/\partial c_{2t+1})}{e^{z_{t+1}}}\right)\right) y_{2,t+1} = 0 \tag{70}$$

ensures that the Ramsey allocation respects optimal price-setting by intermediate goods producers. Money holdings, prices, and policies are constructed using the household first-order conditions as described in Appendix A.

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