

Conceptual Knowledge & Skills

Task Group
Progress Report
January 11, 2007
New Orleans, LA

- Panelists Contributing
 - Francis (Skip) Fennell, Task Group Chair
 - Larry Faulkner
 - Liping Ma
 - Wilfried Schmid
 - Tyrrell Flawn, Staff
- Other Contributors
 - Hung-Hsi Wu
 - Joan Ferrini-Mundy
 - Sandra Stotsky
 - Outside Reviewers - several

- The following topical lists were derived through careful analysis of state curricular standards in this country and also included review of the American Diploma Project Benchmarks and K-8 Benchmarks (Achieve, 2004, 2006), the intended math curricula for Japan, Korea, Flemish-Belgium, Singapore, Chinese Taipei, the work of William Schmidt with TIMSS and beyond (2002) and the recent work of the National Council of Teachers of Mathematics (2006).

- With the following topical list for levels PreK-8, it is important to provide the instructional time and emphasis needed for students to develop concepts, solve problems, and compute.

Essential Concepts and Skills

Prekindergarten through Grade Eight

Number and Operations

Understand whole numbers, including concepts of correspondence, counting, cardinality, and comparison;
Develop place value concepts and an understanding of multiple decompositions of numbers;
Represent, compare, and order whole numbers and join and separate sets;
Understand addition and subtraction and strategies for basic addition facts and related subtraction facts;
Develop automaticity of addition facts and related subtraction facts and fluency with multidigit addition and subtraction.
Understand multiplication and division, and strategies for basic multiplication facts and related division facts;
Develop automaticity of multiplication facts and related division facts and fluency with the multiplication and division of whole numbers;
Understand decimals as special classes of fractions, fraction and decimal equivalence and connections between fractions, decimals and common percents;
Develop an understanding of and fluency with operations on all rational numbers (i.e. addition, subtraction, multiplication and division of fractions and decimals, negative integers).
Define ratio and rate in terms of multiplication and division
Develop an understanding of and apply proportionality, including similarity.

Algebra

Write, interpret, and use mathematical expressions and equations
Analyze and represent linear functions and solve linear equations and systems of linear equations.

Geometry and Measurement

Identify and describe shapes and spatial relationships and compose and decompose geometric shapes;
Identify measurable attributes and compare and order objects by using these attributes;
Develop an understanding of linear measurement and facility in measuring lengths.
Describe two-dimensional shapes and analyze their properties, including perimeter and area and understand and use these formulas;
Describe three-dimensional shapes and analyze their properties, including volume and surface area and understand and use these formulas
Analyze two- and three-dimensional space and figures by using distance and angle.

Data Analysis and Probability

Analyze and summarize data sets using descriptive statistics (mean, median, mode, and range)
Use theoretical and experimental probability

- The task group members and the Panel at large judge that the National Council of Teachers of Mathematics is on sound footing with its recent publication of the *Curriculum Focal Points* (NCTM, 2006). The Panel's final report may articulate grade-by-grade expectations. If so, the *Curriculum Focal Points* and other documents supporting grade-by-grade expectations will be seriously considered.

- While there is agreement to the sequence of particular concepts and skills in PreK-8 mathematics (e.g. whole numbers), this is not true with algebra. Therefore the following list of essentials should not be considered a linear sequencing of these topics.

- As with PreK-8 mathematics, it is important to provide the instructional time and emphasis needed for students to develop concepts, solve problems, and compute as they learn algebra. As an example, understanding in algebra would include understanding the relationship between factoring quadratic polynomials, completing the square and the quadratic formula.

Elements of Algebra

Symbols and expressions

- Geometric sequences and series
- Polynomial expressions
- Rational expressions
- Radical expressions
- Arithmetic and geometric sequences and series

Linear Relations

- Fundamental relationships between linear equations and the graphical representations of such equations
- Solving problems with linear equations
- Linear inequalities and their graphs – to include compound inequalities
- Graphing and solving systems of simultaneous linear equations

Quadratic Relations

- Factors and factoring of trinomials with integer coefficients.
- Factors and factoring of polynomials
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions

- Quadratic functions – solve problems involving quadratic functions
- Fundamental relationships between quadratic functions and their graphs
- Polynomial functions (know graphs of basic functions)
- Simple nonlinear functions (e.g. square and cube root functions; absolute value)
- Rational exponents and exponential functions
- Logarithmic functions
- Rational functions
- Trigonometric functions
- Fitting simple mathematical models to data

Polynomials

- Roots and factorization
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's triangle)
- Mathematical induction and the binomial theorem

Combinatorics and finite probability

Connections between algebra and other areas (e.g. linear functions and best fit in statistics; similarity relationships and distance in geometry)

Next Steps...