Impact of Soil Moisture Aggregation on Surface Energy Flux Prediction During SGP'97

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[1] The nonlinear nature of the point-scale soil moisture-transpiration relationship, combined with the highly heterogeneous spatial structure of soil moisture fields and the limited horizontal resolution of spaceborne microwave radiometers, has generated considerable interest in the soil moisture aggregation problem. Specifically, understanding the impact of aggregating soil moisture information up to regional grid-scales ($\sim 10^3$ km^2) on the grid-scale prediction of surface water and energy fluxes within the land surface component of a weather prediction model. Using data from the 1997 Southern Great Plains Hydrology Experiment (SGP'97), a conceptual link is presented between the impact of soil moisture aggregation on surface energy flux prediction and the spatial scaling properties of soil moisture fields. *INDEX TERMS:* 1833 Hydrology: Hydroclimatology, 1866 Hydrology: Soil moisture, 1878 Hydrology: Water/energy interactions

1. Introduction

[2] A critical issue in efforts to improve the modeling of land surface water and energy balance processes is the discontinuity between the scale at which physical relationships between forcing variables and surface fluxes are valid and the resolution at which forcing data will be available in the foreseeable future. This incompatibility appears especially severe when assimilating soil moisture data derived from spaceborne passive microwave remote sensing into land surface models. At best, next-generation spaceborne sensors will have horizontal resolutions of 10^2 to 30^2 km^2 [*Jackson et al.*, 1999]. While the relationship between soil moisture and transpiration is relatively well understood at the point-scale, the nonlinear nature of the relationship, in combination with the vast array of scales at which soil moisture exhibits spatial variability, suggests that it is inappropriate to apply these point-scale relationships at spatial scales equivalent to a microwave remote sensing footprint.

[3] Recognition of this incompatibility has led to considerable interest concerning the "soil moisture aggregation effect." That is, in the presence of subgrid-scale heterogeneity, what is the effect of aggregating soil moisture up to grid-scales near 10^3 km^2 on the prediction of grid-scale water and energy fluxes. The problem is well studied in terms of grid-scale evapotranspiration [*Wetzel and Chang*, 1988; *Famiglietti and Wood*, 1995]. Since the impact of aggregation is determined by the magnitude of subgrid variability, the scaling properties of soil moisture fields have also received considerable attention. Most studies of multi-scale soil moisture spatial variability have utilized simple- or multi-scaling concepts to convey the lack of any characteristic length scale within soil moisture fields [*Rodriguez-Iturbe et al.*, 1996; *Hu et al.*, 1998].

[4] Using a common model of the nonlinear soil moisture-transpiration relationship and data from the 1997 Southern Great Plains Field Experiment (SGP'97), this paper will formalize the conceptual link between the soil moisture aggregation effect and the spatial scaling properties of soil moisture fields. Two separate questions will be addressed. First, how is the impact of degrading the resolution of soil moisture information from the field-scale ($\sim$1 squ. km) to a regional-scale ($\sim 10^3$ km^2) linked to the spatial scaling properties of soil moisture? Second, based on the spatial statistics of soil moisture fields sampled during SGP'97, at what grid-scales are extreme assumptions of zero versus large subgrid-scale soil moisture variability appropriate for grid-scale surface energy flux prediction?

2. SGP'97 data sets

[5] One of the few data sets capable of providing multi-scale surface (5 cm) soil moisture data over an extended period of time was produced during the 1997 Southern Great Plains Hydrology Experiment (SGP'97). The experiment was a NASA-funded field campaign run from June 16 to July 17, 1997 within central Oklahoma. The two SGP'97 data sets of interest here are the soil impedance probe and gravimetric soil moisture measurements. Gravimetric sampling was designed to measure mean field-scale soil moisture and carried out on 23 fields within the 603-km^2 Little Washita Basin. Impedance probe measurements were designed to estimate subfield-scale soil moisture variability and made along fixed grids set up on three fields in the basin. The impedance probe sampling configuration for each field consisted of 49 nodal locations spaced 100 m apart in a square 7 by 7 grid. Figure 1 shows a map of the study location and illustrates the scales considered in the analysis.

3. Role of multi-scale variability

[6] Figure 1 demonstrates the three spatial scales of interest for soil moisture (θ) in this analysis: the point-scale (θ'), the field-scale (θ), and the regional-scale ($\bar{\theta}$). The field-scale soil moisture-transpiration relationship is represented by $E(\theta)$. For an individual field, the error associated with using a regionally averaged soil moisture value to calculate transpiration is given by a Taylor's series expansion of $E(\theta)$ around $\bar{\theta}$. The expectation of such an expansion for all field-scale locations within a region describes the impact on regional-scale transpiration of aggregating soil moisture from the field- to regional-scale:

$$\overline{E(\theta)} = E(\bar{\theta}) + \sum_{i=2}^{\infty} \frac{1}{i!} \overline{(\theta - \bar{\theta})^i} \left[\frac{\delta^i E(\theta)}{(\delta\theta)^i} \right]_{\bar{\theta}}. \tag{1}$$

Taking only the first two terms of the expansion yields:

$$\overline{E(\theta)} - E(\bar{\theta}) \approx \frac{1}{2} \sigma_{\theta}^2 \left[\frac{\delta^2 E(\theta)}{(\delta\theta)^2} \right]_{\bar{\theta}} \tag{2}$$

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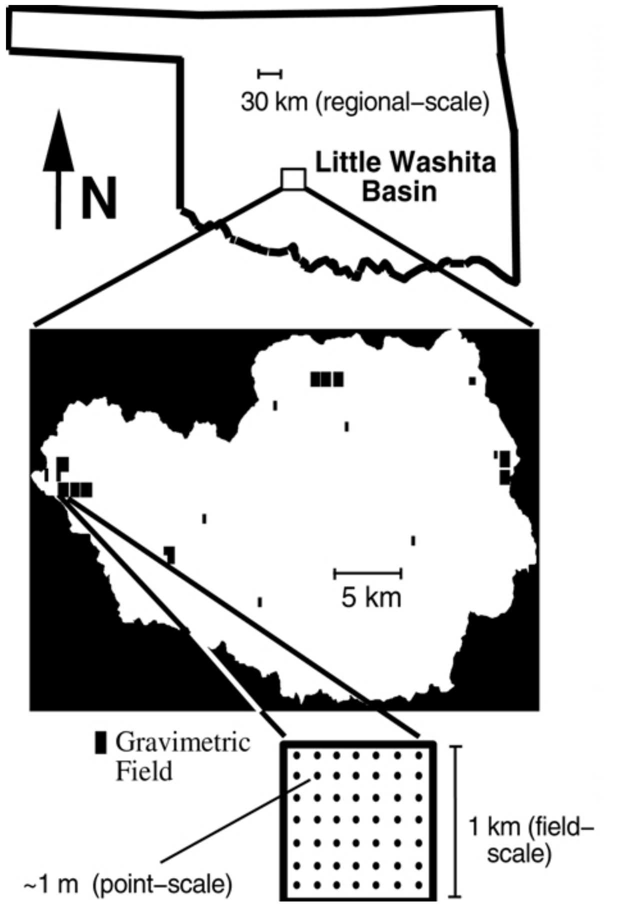

Figure 1. Location of Little Washita Basin within the state of Oklahoma and illustration of spatial scales considered in the analysis.

where $\sigma_\theta{}^2$ is the field-scale soil moisture variance within a given region.

[7] Many land surface schemes follow *Wetzel and Chang* [1987] and model point-scale evapotranspiration (E') as the minimum of a potential (E_P) and threshold evapotranspiration (E_T'):

$$E' = \mathrm{Min}\left[E_P, E_T'\right] \tag{3}$$

where the dependence of E_T' on θ' is expressed in a simplified linear form:

$$E_T' = \begin{cases} 0 & \theta' < \theta_w \\ (E_T)_{max} \cdot \left(\frac{\theta' - \theta_w}{\theta_c - \theta_w}\right) & \theta_w < \theta' < \theta_c \\ (E_T)_{max} & \theta' > \theta_c. \end{cases} \tag{4}$$

The threshold evapotranspiration is defined as the maximum evapotranspiration the soil-plant system is capable of sustaining for a given level of soil moisture. Potential evapotranspiration follows the definition of "unstressed evapotranspiration" given by *Federer* [1979] as the rate at which well-watered vegetation will transpire. Combining (3) and (4) and assuming $E_P \leq (E_T)_{max}$ yields the following E' parameterization:

$$E_T' = \begin{cases} 0 & \theta' < \theta_w \\ E_P \cdot \left(\frac{\theta' - \theta_w}{\theta_c^* - \theta_w}\right) & \theta_w < \theta' < \theta_c^* \\ E_P & \theta' > \theta_c^*. \end{cases} \tag{5}$$

where:

$$\theta_c^* = \theta_w + \frac{E_P \cdot (\theta_c - \theta_w)}{(E_T)_{max}}. \tag{6}$$

[8] The $E'(\theta')$ relationship described in (5) is only strictly valid at a point-scale. Scaling the relationship up to the field-scale requires that it be integrated over all subfield-scale soil moisture levels:

$$E(\theta) = \int_0^\infty E'(\theta') f(\theta') d\theta' \tag{7}$$

where $f(\theta')$ is the histogram of point-scale soil moisture measurements within a given field. The choice of a normal probability distribution for soil moisture histograms at these scales is suggested by *Bell et al.* [1980]. Assuming f to represent a normal distribution $N(\theta, \sigma_{\theta'}^2)$, where $\sigma_{\theta'}^2$ is taken to be the point-scale variability of soil moisture field around the field scale mean θ, Figure 2 shows $E(\theta)$ curves corresponding to various levels of $\sigma_{\theta'}^2$. Combining (7) with (5) and taking a second-derivative with respect to θ gives:

$$\left[\frac{\delta^2 E(\theta)}{(\delta\theta)^2}\right]_{\bar{\theta}} = \frac{E_P\left(-\exp\left[-\frac{(\theta_c^* - \bar{\theta})^2}{2\sigma_{\theta'}^2}\right] + \exp\left[-\frac{(\theta_w - \bar{\theta})^2}{2\sigma_{\theta'}^2}\right]\right)}{\sigma_{\theta'}\sqrt{2\pi}\left(\theta_c^* - \theta_w\right).} \tag{8}$$

Inserting this expression into (2) yields:

$$\overline{E(\theta)} - E(\bar{\theta}) \approx \frac{\sigma_\theta^2 E_P\left(-\exp\left[-\frac{(\theta_c^* - \bar{\theta})^2}{2\sigma_{\theta'}^2}\right] + \exp\left[-\frac{(\theta_w - \bar{\theta})^2}{2\sigma_{\theta'}^2}\right]\right)}{2\sigma_{\theta'}\sqrt{2\pi}\left(\theta_c^* - \theta_w\right)}. \tag{9}$$

Setting $\bar{\theta}$ equal to either θ_w or θ_c^*, and assuming $\sigma_{\theta'}^2$ is small relative to $(\theta_c^* - \theta_w)^2$, provides a measure of the maximum

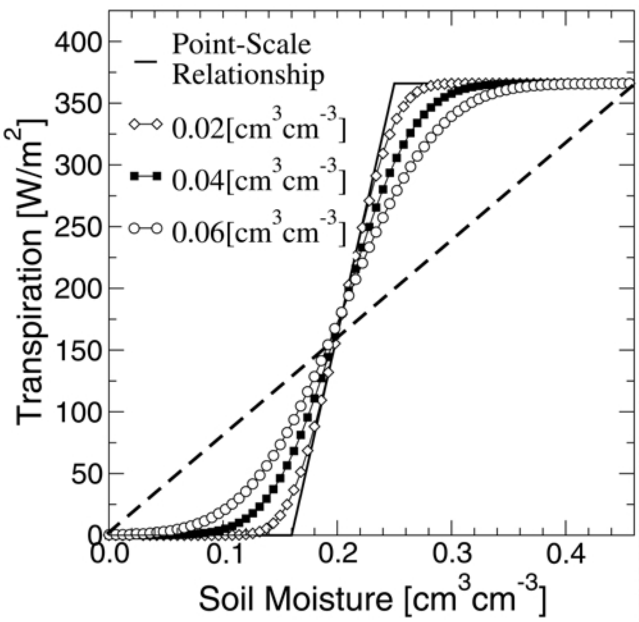

Figure 2. Appropriate field-scale transpiration-soil moisture relationships for various levels of subgrid soil moisture variability $\sigma_{\theta'}$ [$\mathrm{cm}^3_{\mathrm{water}}\ \mathrm{cm}^{-3}_{\mathrm{soil}}$]. Also shown is the extreme case of large $\sigma_{\theta'}$ — a linear relationship between zero and θ_{sat}.

absolute error associated with aggregation of soil moisture within a region:

$$\mathrm{Max}\left[\left|\overline{E(\theta)} - E(\bar{\theta})\right|\right] \approx \left|\frac{\sigma_{\bar{\theta}}^2 E_P}{\sigma_{\theta'}(\theta_c^* - \theta_w)2\sqrt{2\pi}}\right|. \tag{10}$$

Equation (10) demonstrates that the effect of degrading soil moisture resolution from the field- to the regional-scale does not depend strictly on levels of field-scale variability, but rather is proportional to the dimensionless ratio $\sigma_{\bar{\theta}}^2/\sigma_{\theta'}(\theta_c^* - \theta_w)$. Increasing the subfield variability $\sigma_{\theta'}$ linearizes the field-scale $E(\theta)$ relationship (see Figure 2) and reduces the aggregation impact associated with averaging field-scale values up to the regional-scale.

[9] Assuming the regional-scale to be the entire 603-km^2 Little Washita Basin, Figure 3 evaluates (9) for the basin during SGP'97. Values for $\overline{E(\theta)} - E(\bar{\theta})$ range between -100 and 75 Wm^{-2}. Impedence probe measurements are used to estimate average point-scale soil moisture variability around each field-scale mean ($\sigma_{\theta'}^2$) and gravimetric data to estimate field-scale variability around the regional-scale mean ($\sigma_{\bar{\theta}}^2$). Using impedance probe data from SGP'97, *Famiglietti et al.* [1999] argues that a Beta distribution is a better fit for subfield-scale soil moisture histograms than a normal distribution. To accommodate this suggestion, values derived from a numerical evaluation of (2) assuming a Beta distribution shape for *f* in (7) are also plotted on Figure 3. Soil type was taken to be silty loam/loam and vegetation to be rangeland grass and agricultural crops. E_P was taken as 450 Wm^{-2} and $(E_T)_{max}$ as 750 Wm^{-2} based on values quoted for various agricultural crops in *Choudhury and Idso* [1985]. Values for θ_w and θ_c were chosen to be consistent with the observed soil texture within the basin and *Jacquemin and Noilhan* [1990].

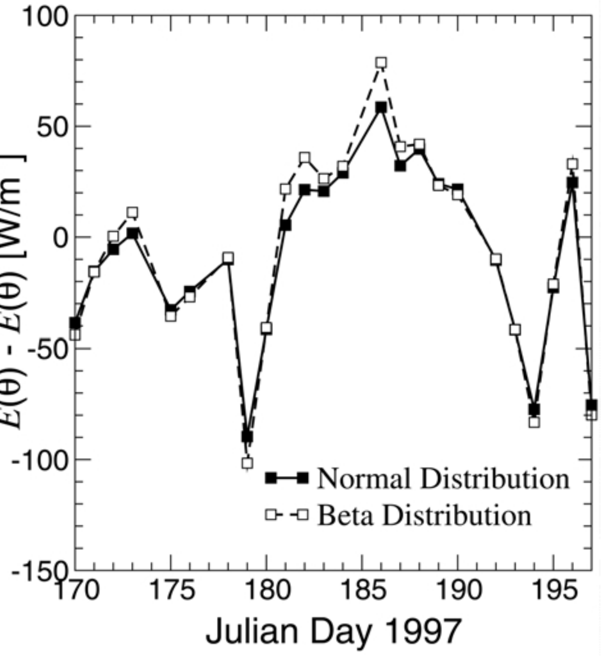

Figure 3. Following (9), errors in transpiration (E) associated with the aggregation of soil moisture from the field- to regional-scale within the Little Washita Basin during SGP'97.

4. Grid-scale modeling strategies

[10] Let $\sigma_{\theta'}^2(\lambda)$ be the point-scale soil moisture variability contained within an arbitrary grid-scale λ. In the absence of any information regarding the magnitude of $\sigma_{\theta'}^2(\lambda)$, grid-scale models must make one of two contrasting assumptions. The first option is to assume $\sigma_{\theta'}^2(\lambda)$ is zero and apply a point-scale model at the grid-scale. The error associated with this assumption is:

$$\varepsilon_1 = \int_0^\infty \mathrm{E}'(\theta') f(\theta, \sigma_{\theta'}^2(\lambda)) d\theta' - \mathrm{E}'(\bar{\theta}). \tag{11}$$

The second option is to assume $\sigma_{\theta'}^2(\lambda)$ is large enough such that the appropriate field-scale $E(\theta)$ relationship is effectively linearized between zero and saturation (θ_{sat}) (see Figure 2). The error associated with this approach is:

$$\varepsilon_2 = \int_0^\infty E'(\theta') f(\theta, \sigma_{\theta'}^2(\lambda)) d\theta' - E_P \frac{\bar{\theta}}{\theta_{sat}}. \tag{12}$$

For low levels of $\sigma_{\theta'}^2(\lambda)$, ε_2 is the larger error term. As $\sigma_{\theta'}^2(\lambda)$ rises, assuming zero subgrid variability becomes steadily less appropriate. Consequently, ε_1 rises and ε_2 falls until a second regime of large $\sigma_{\theta'}^2(\lambda)$ is entered where ε_1 is the dominate error term. A critical level of subgrid variability ($\hat{\sigma}_{\theta'crit}$) exists where both errors are equal.

[11] Assuming $\bar{\theta}$ equal to θ_c^* (the point of greatest concavity and thus largest aggregation impact), small $\sigma_{\theta'}^2$, and a normal probability distribution for *f*, ε_1 can be approximated as:

$$\varepsilon_1 \approx \frac{-E_P \sigma_{\theta'}}{\sqrt{2\pi}(\theta_c^* - \theta_w)}, \tag{13}$$

and ε_2 as:

$$\varepsilon_2 \approx \frac{E_P(\theta_{sat} - \theta_c^*)}{\theta_{sat}} - \frac{E_P \sigma_{\theta'}}{\sqrt{2\pi}(\theta_c^* - \theta_w)}. \tag{14}$$

The subgrid soil moisture variability at which the absolute value of both terms will be equal is:

$$\sigma_{\theta' crit} \approx \sqrt{\frac{\pi}{2}} \frac{(\theta_{sat} - \theta_c^*)(\theta_c^* - \theta_w)}{\theta_{sat}}. \tag{15}$$

Inserting values of θ_{sat}, θ_c^*, and θ_w used in Figure 3 yields a $\sigma_{\theta' crit}$ estimate of 0.067 cm$^3_{\mathrm{water}}$ cm$^{-3}_{\mathrm{soil}}$.

[12] As first described by *Reynolds* [1974], values of $\sigma_{\theta'}^2(\lambda)$ rise monotonically with scale λ; therefore, this critical level of variability is uniquely associated with a grid-scale λ_{crit}. Using soil and vegetation parameters typical for the Little Washita Basin, and a numerical evaluation of (11) and (12) assuming *f* represents a Beta distribution, the root-mean-squared magnitudes of ε_1 and ε_2 were evaluated at the point-, field- ($\sim$1 squ. km), and regional-scale ($\sim 10^3$ km^2) during SGP'97.

[13] Results demonstrate that the assumption of zero (large) subgrid variability becomes steadily less (more) appropriate as the grid-scale is coarsened. Root-mean-squared (RMS) values for ε_1 of 0.0, 58, and 82 Wm^{-2} are found at grid-scales corresponding to the point-, field-, and regional-scale respectively. Conversely, RMS values for ε_2 are 164, 108, and 77 Wm^{-2} along the same series of grid-scales. This suggests an assumption of zero subgrid variability is more accurate than an assumption of large subgrid variability at the field-scale ($\sim$1 squ. km) but not the regional-scale ($\sim 10^3$ km^2). Qualitatively similar results are found when *f* is assumed to represent a normal distribution.

5. Summary and conclusions

[14] This paper uses SGP'97 soil moisture data to draw a conceptual link between the spatial scaling of soil moisture fields and pertinent questions surrounding the impact of soil moisture

aggregation on efforts to estimate the surface energy balance at regional scales.

[15] Equation (10) demonstrates that the impact of degrading soil moisture from the field- to the regional-scale on regional-scale energy flux prediction is directly proportional to the variance of field-scale soil moisture within the region and inversely proportional to the average standard deviation of point-scale soil moisture variability within each field. Consequently, the role of soil moisture spatial heterogeneity with respect to the soil moisture aggregation question varies according to the length scales at which this heterogeneity is expressed and how the scaling properties of soil moisture fields evolve with time [*Crow and Wood*, 1999].

[16] Evaluation of (11) and (12) demonstrate the role of subgrid-scale variability in determining the optimal strategy for grid-scale transpiration prediction and the manner in which this strategy varies with grid size. During SGP'97 it is shown that the transition between an assumption of zero and large subgrid variability should occur between the field- ($\sim$1 squ. km) and regional-scale ($\sim 10^3$ km^2). This transition scale corresponds to the grid-scale at which subgrid levels of variability exceed the threshold described in (15).

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