Chapter 3

Noncohesive Sediment Transport

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by
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3.1 Introduction

Engineers, geologists, and river morphologists have studied the subject of sediment transport for centuries. Different approaches have been used for the development of sediment transport functions or formulas. These formulas have been used for solving engineering and environmental problems. Results obtained from different approaches often differ drastically from each other and from observations in the field. Some of the basic concepts, their limits of application, and the interrelationships among them have become clear to us only in recent years. Many of the complex aspects of sediment transport are yet to be understood, and they remain among the challenging subjects for future studies.

The mechanics of sediment transport for cohesive and noncohesive materials are different. Issues relating to cohesive sediment transport will be addressed in chapter 4. This chapter addresses noncohesive sediment transport only. This chapter starts with a review of the basic concepts and approaches used in the derivation of incipient motion criteria and sediment transport functions or formulas. Evaluations and comparisons of some of the commonly used criteria and transport functions give readers general guidance on the selection of proper functions under different flow and sediment conditions. Some of the materials summarized in this chapter can be found in the book *Sediment Transport Theory and Practice* (Yang, 1996). Most noncohesive sediment transport formulas were developed for sediment transport in clear water under equilibrium conditions. Understanding sediment transport in sediment-laden flows with a high concentration of wash load is necessary for solving practical engineering problems. The need to consider nonequilibrium sediment transport in a sediment routing model is also addressed in this chapter.

3.2 Incipient Motion

Incipient motion is important in the study of sediment transport, channel degradation, and stable channel design. Due to the stochastic nature of sediment movement along an alluvial bed, it is difficult to define precisely at what flow condition a sediment particle will begin to move. Consequently, it depends more or less on an investigator's definition of incipient motion. They use terms such as "initial motion," "several grain moving," "weak movement," and "critical movement." In spite of these differences in definition, significant progress has been made on the study of incipient motion, both theoretically and experimentally.

Figure 3.1 shows the forces acting on a spherical sediment particle at the bottom of an open channel. For most natural rivers, the channel slopes are small enough that the component of gravitational force in the direction of flow can be neglected compared with other forces acting on a spherical sediment particle. The forces to be considered are the drag force F_D, lift force F_L, submerged weight W_S, and resistance force F_R. A sediment particle is at a state of incipient motion when one of the following conditions is satisfied:

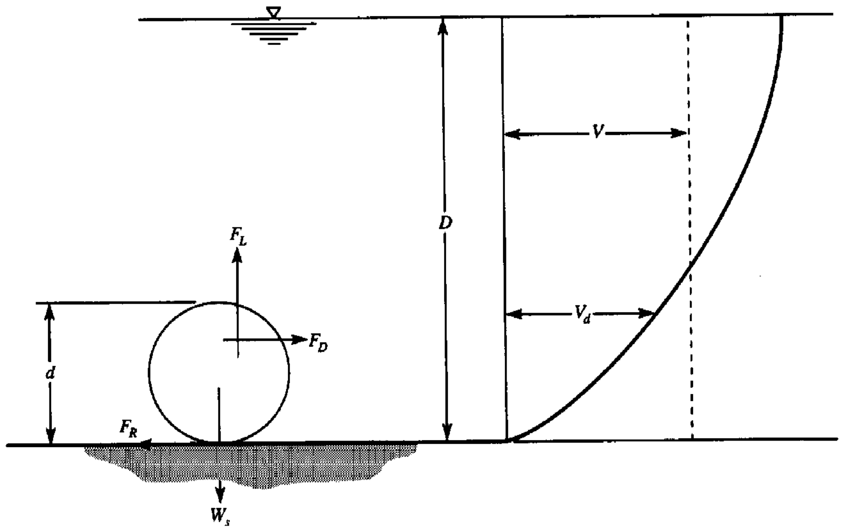


Figure 3.1. Diagram of forces acting on a sediment particle in open channel flow (Yang, 1973).

$$F_L = W_s \tag{3.1}$$

$$F_D = F_R \tag{3.2}$$

$$M_O = M_R \tag{3.3}$$

where M_O = overturning moment due to F_D and F_L, and
M_R = resisting moment due to F_L and W_s.

Most incipient motion criteria are derived from either a shear stress or a velocity approach.

3.2.1 Shear Stress Approach

One of the most prominent and widely used incipient motion criteria is the Shields diagram (1936) based on shear stress. Shields assumed that the factors in the determination of incipient motion are the shear stress τ, the difference in density between sediment and fluid ρ_s - ρ_f, the diameter of the particle d, the kinematic viscosity v, and the gravitational acceleration g. These five quantities can be grouped into two dimensionless quantities, namely,

$$d\frac{\left(\tau_c/\rho_f\right)^{1/2}}{v} = \frac{dU_*}{v} \tag{3.4}$$

and

$$\frac{\tau_c}{d\left(\rho_s - \rho_f\right)g} = \frac{\tau_c}{d\gamma\left[\left(\rho_s/\rho_f\right)-1\right]} \tag{3.5}$$

where ρ_s and ρ_f = densities of sediment and fluid, respectively,
γ = specific weight of water,
U_* = shear velocity, and
τ_c = critical shear stress at initial motion.

The relationship between these two parameters is then determined experimentally. Figure 3.2 shows the experimental results obtained by Shields and other investigators at incipient motion. At points above the curve, the particle will move. At points below the curve, the flow is unable to move the particle. It should be pointed out that Shields did not fit a curve to the data but showed a band of considerable width. Rouse (1939) first proposed the curve shown in Figure 3.2. Although engineers have used the Shields diagram widely as a criterion for incipient motion, dissatisfactions can be found in the literature. Yang (1973) pointed out the following factors and suggested that the Shields' diagram may not be the most desirable criterion for incipient motion.

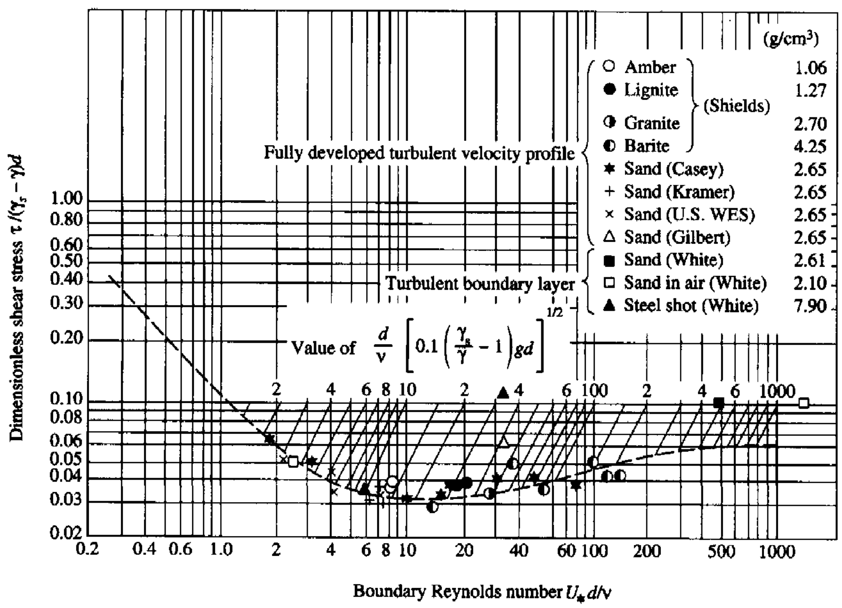


Figure 3.2. Shields diagram for incipient motion (Vanoni, 1975).

- The justification for selecting shear stress instead of average flow velocity is based on the existence of a universal velocity distribution law that facilitates computation of the shear stress from shear velocity and fluid density. Theoretically, water depth does not appear to be related directly to the shear stress calculation, while the main velocity is a function of water depth. However, in common practice, the shear stress is replaced by the average shear stress or tractive force $\tau = \gamma DS$, where γ is the specific weight of water, D is the water depth, and S is the energy slope. In this case, the average shear stress depends on the water depth.

- Although by assuming the existence of a universal velocity distribution law, the shear velocity or shear stress is a measure of the intensity of turbulent fluctuations, our present knowledge of turbulence is limited mainly to laboratory studies.

- Shields derived his criterion for incipient motion by using the concept of a laminar sublayer, according to which the laminar sublayer should not have any effect on the velocity distribution when the shear velocity Reynolds number is greater than 70. However, the Shields diagram clearly indicates that his dimensionless critical shear stress still varies with shear velocity Reynolds number when the latter is greater than 70.

- Shields extends his curve to a straight line when the shear velocity Reynolds number is less than three. This means that when the sediment particle is very small, the critical tractive force is independent of sediment size (Liu, 1958). However, White (1940) showed that for a small shear velocity Reynolds number, the critical tractive force is proportional to the sediment size.

- It is not appropriate to use both shear stress τ and shear velocity U_* in the Shields diagram as dependent and independent variables because they are interchangeable by $U_* = (\tau/\rho)^{1/2}$, where ρ is the fluid density. Consequently, the critical shear stress cannot be determined directly from Shields' diagram; it must be determined through trial and error.

- Shields simplified the problem by neglecting the lift force and considering only the drag force. The lift force cannot be neglected, especially at high shear velocity Reynolds numbers.

- Because the rate of sediment transport cannot be uniquely determined by shear stress (Brooks, 1955; Yang, 1972), it is questionable whether critical shear stress should be used as the criterion for incipient motion of sediment transport.

One of the objections to the use of the Shields diagram is that the dependent variables appear in both ordinate and abscissa parameters. Depending on the nature of the problem, the dependent variable can be critical shear stress or grain size. The American Society of Civil Engineers Task Committee on the Preparation of a Sediment manual (Vanoni, 1977) uses a third parameter

$$\frac{d}{\nu}\left[0.1\left(\frac{\gamma_s}{\gamma}-1\right)gd\right]^{1/2}$$

as shown in Figure 3.2. The use of this parameter enables us to determine its intersection with the Shields diagram and its corresponding values of shear stress. The basic relationship shown in Figure 3.2 has been tested and modified by different investigators. Figure 3.3 shows the results summarized by Govers (1987) in accordance with a modified Shields diagram suggested by Yalin and Karahan (1979).

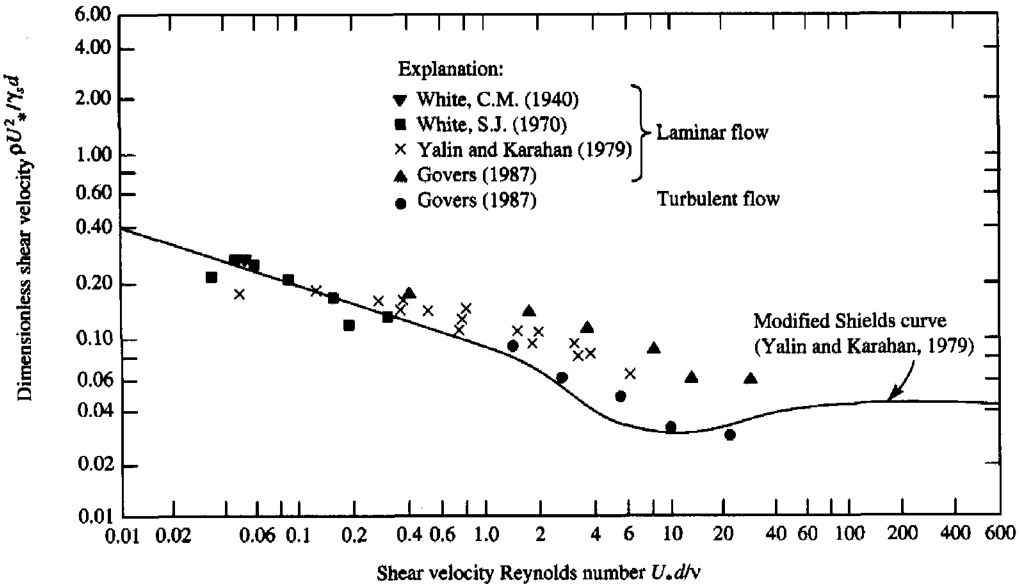


Figure 3.3. Modified Shields diagram (Govers, 1987).

Bureau of Reclamation (1987) developed some stable channel design criteria based on the critical shear stress required to move sediment particles in channels under different flow and sediment conditions. The critical tractive force can be expressed by:

$$\tau_c = \gamma DS \tag{3.6}$$

where τ_c = critical tractive force or shear stress (in lb/ft^2 or g/m^2),
γ = specific weight of water (= 62.4 lb/ft^3 or 1 ton/m^3), and
D = mean flow depth (in ft or m).

Figure 3.4 shows the relationship between critical tractive force and mean sediment diameter for stable channel design recommended by Bureau of Reclamation (1977).

Lane (1953) developed stable channel design curves for trapezoidal channels with different typical side slopes. These curves are based on maximum allowable tractive force and are shown in Figure 3.5. Figure 3.5(a) is for the channel sides, and Figure 3.5(b) is for the channel bottom. Figure 3.5 indicates that the maximum shear stress is about equal to γDS and $0.75\gamma DS$ for the bottom and the sides of the channel, respectively. Lane's study also shows that shear stress is zero at the corners.

The shear stress acting on the channel side at incipient motion is:

$$\tau_w = W_s \cos\theta \tan\phi \left(1 - \frac{\tan^2\theta}{\tan^2\phi}\right)^{1/2} \tag{3.7}$$

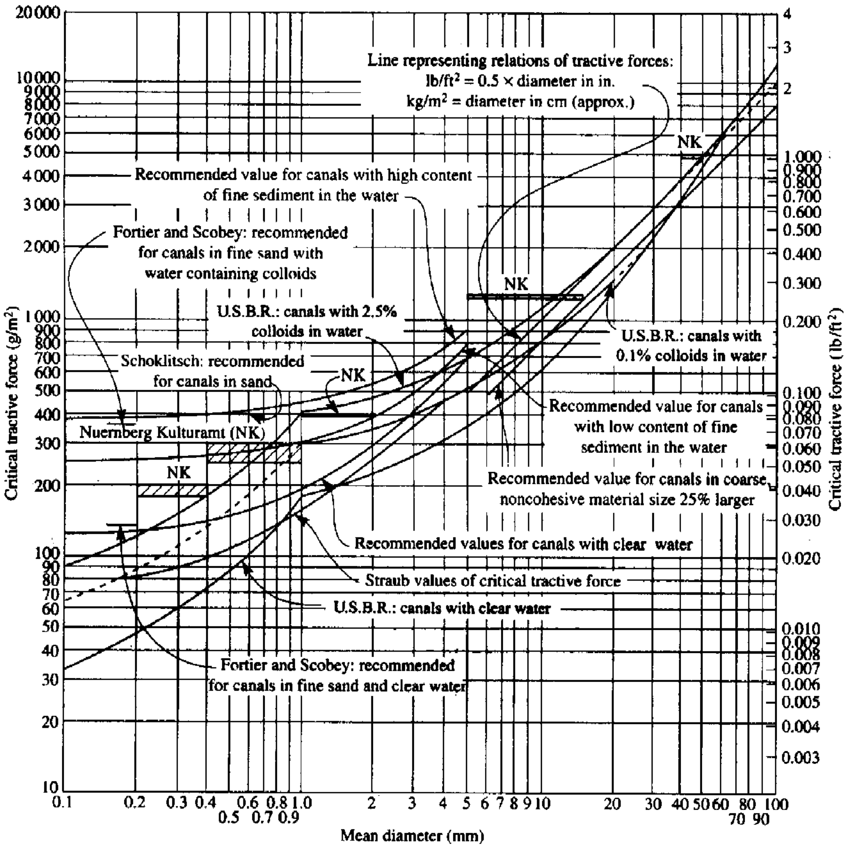


Figure 3.4. Tractive force versus transportable sediment size (Bureau of Reclamation, 1987).

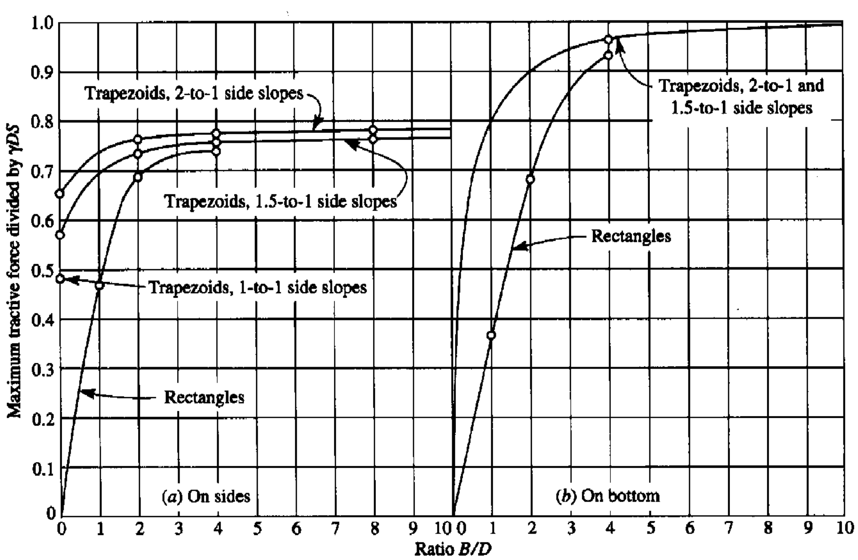


Figure 3.5. Maximum shear stress in a channel (Lane, 1953).

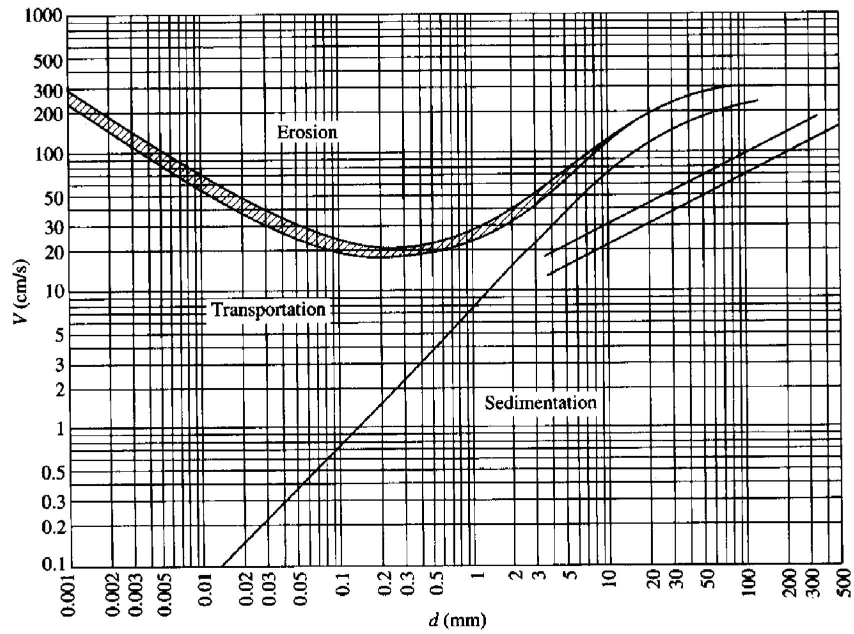


Figure 3.6. Erosion-deposition criteria for uniform particles (Hjulstrom, 1935).

At the bottom of a channel, θ = 0, and equation (3.7) becomes:

$$\tau_b = W_s \tan\phi \tag{3.8}$$

The ratio of limiting tractive forces acting on the channel side and channel bottom is:

$$K = \frac{\tau_w}{\tau_b} = \cos\theta\left(1 - \frac{\tan^2\theta}{\tan^2\phi}\right)^{1/2} \tag{3.9}$$

For stable channel design, the value of τ_b can be obtained from the Shields diagram as shown in Figure 3.2, or from Figure 3.4 for channels of different materials.

3.2.2 Velocity Approach

Fortier and Scobey (1926) made an extensive field survey of maximum permissible values of mean velocity in canals. Table 3.1 shows their permissible velocities for canals of different materials. Hjulstrom (1935) made detailed analyses of the movement of uniform materials on the bottom of channels. Figure 3.6 gives the relationship between sediment size and average flow velocity for erosion, transportation, and sedimentation. The American Society of Civil Engineers Sedimentation Task Committee (Vanoni, 1977) suggested the use of Figure 3.7 for stable channel design.

Table 3.1 Permissible canal velocities (Fortier and Scobey, 1926)

	Velocity* (ft/s)		
Original material excavated for canal (1)	Clear water, no detritus (2)	Water transporting colloidal silts (3)	Water transporting noncolloidal silts, sands, gravels, or rock fragments (4)
Fine sand (noncolloidal)	1.50	2.50	1.50
Sandy loam (noncolloidal)	1.75	2.50	2.00
Silt loam (noncolloidal)	2.00	3.00	2.00
Alluvial silts when noncolloidal	2.00	3.50	2.00
Ordinary firm loam	2.50	3.50	2.25
Volcanic ash	2.50	3.50	2.00
Fine gravel	2.50	5.00	3.75
Stiff clay (very colloidal)	3.75	5.00	3.00
Graded, loam to cobbles, when noncolloidal	3.75	5.00	5.00
Alluvial silts when colloidal	3.75	5.00	3.00
Graded, silt to cobbles, when colloidal	4.00	5.50	5.00
Coarse gravel (noncolloidal)	4.00	6.00	6.50
Cobbles and shingles	5.00	5.50	6.50
Shales and hard pans	6.00	6.00	5.00

* For channels with depth of 3 ft or less after aging

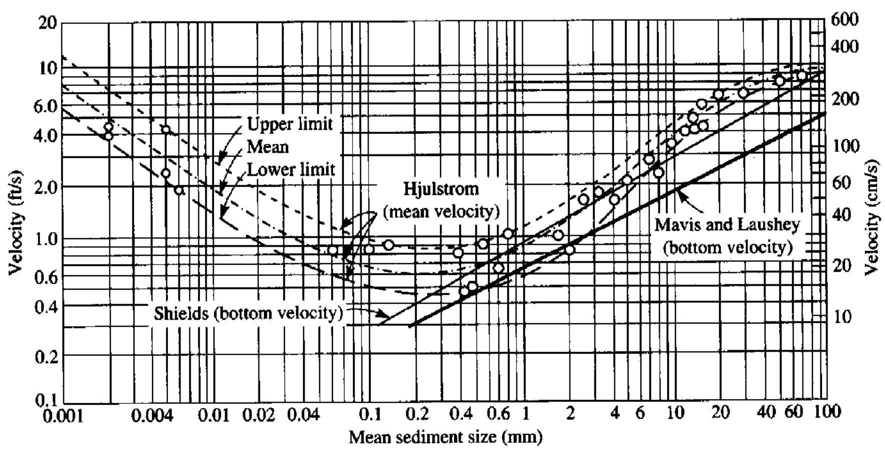


Figure 3.7. Critical water velocities for quartz sediment as a function of mean grain size (Vanoni, 1977).

Yang (1973) applied some basic theories in fluid mechanics to develop his incipient motion criteria. At incipient motion, the resistance force F_R in Figure 3.1 should be balanced by the drag force F_D.

It can be shown that:

$$F_D = \frac{\pi d^3}{6\psi_1}(\rho_s - \rho) g \left(\frac{V}{\omega}\right)^2 \left[\frac{B}{5.75\left[\log(D/d) - 1\right] + B}\right]^2 \tag{3.10}$$

The lift force acting on the particle can be obtained as:

$$F_L = \frac{\pi d^3}{6\psi_1\psi_2}(\rho_s - \rho) g \left(\frac{V}{\omega}\right)^2 \left[\frac{B}{5.75\left[\log(D/d) - 1\right] + B}\right]^2 \tag{3.11}$$

The submerged weight of the particle is:

$$W_S = \frac{\pi d^3}{6}(\rho_S - \rho) g \tag{3.12}$$

The resistant force:

$$\begin{aligned} F_D &= \psi_3 (W_S - F_L) \\ &= \frac{\psi_3 \pi d^3}{6}\left(\rho_S - \rho\right) g \left\{1 - \frac{1}{\psi_1\psi_2}\left(\frac{V}{\omega}\right)^2 \left[\frac{B}{5.75\left[\log(D/d) - 1\right] + B}\right]^2\right\} \end{aligned} \tag{3.13}$$

where ψ_1, ψ_2, ψ_3 = coefficients,
ρ, ρ_S = density of water and sediment, respectively,
D = average flow depth,
D = sediment particle diameter,
ω = sediment particle fall velocity,
V = average flow velocity, and
B = roughness function.

Assume that the incipient motion occurs when $F_D = F_R$. From equations (3.10) and (3.13):

$$\frac{V_{cr}}{\omega} = \left[\frac{5.75\left[\log(D/d) - 1\right]}{B} + 1\right]\left(\frac{\psi_1\psi_2\psi_3}{\psi_2 + \psi_3}\right)^{1/2} \tag{3.14}$$

where V_{cr} = average critical velocity at incipient motion, and
V_{cr}/ω = dimensionless critical velocity.

In the hydraulically smooth regime, B is a function of only the shear velocity Reynolds number $U_* d/\nu$, that is,

$$B = 5.5 + 5.75 \log\frac{U_* d}{\nu}, \quad 0 < \frac{U_* d}{\nu} < 5 \tag{3.15}$$

where U_* = shear velocity, and
ν = kinematic viscosity of water.

Then equation (3.14) becomes:

$$\frac{V_{cr}}{\omega}=\left[\frac{\log(D/d)-1}{\log(U_*d/v)+0.956}+1\right]\left(\frac{\psi_1\psi_2\psi_3}{\psi_2+\psi_3}\right)^{1/2} \tag{3.16}$$

which is a hyperbola on a semilog plot between V_{cr}/ω and U_*d/v. The relative roughness d/D should not have any significant influence on the shape of this hyperbola in the hydraulically smooth regime. In the completely rough regime, the laminar friction contribution can be neglected, and B is a function of only the relative roughness d/D, that is:

$$B=8.5, \qquad \frac{U_*d}{v}>70 \tag{3.17}$$

Then equation (3.14) becomes:

$$\frac{V_{cr}}{\omega}=\left[\frac{\log(D/d)-1}{1.48}+1\right]\left(\frac{\psi_1\psi_2\psi_3}{\psi_2+\psi_3}\right)^{1/2} \tag{3.18}$$

Equation (3.18) indicates that in the completely rough regime, the plot of V_{cr}/ω against U_*d/v is a straight horizontal line. The position of this horizontal line depends on the value of the relative roughness, ψ_1, ψ_2, and ψ_3.

In the transition regime with the shear velocity Reynolds number between 5 and 70, protrusions extend partly outside the laminar sublayer. Both the laminar friction and turbulent friction contributions should be considered. In this case, B deviates gradually from equation (3.15) with increasing U_*d/v. It is reasonable to expect that, basically, equation (3.16) is still valid, but with the relative roughness d/D playing an increasingly important role as U_*d/v increases.

Yang (1973) used laboratory data collected by different investigators for the determination of coefficients in equations (3.16) and (3.18). The incipient motion criteria thus obtained are:

$$\frac{V_{cr}}{\omega}=\frac{2.5}{\log(U_*d/v)-0.06}+0.66, \qquad 1.2<\frac{U_*d}{v}<70 \tag{3.19}$$

and

$$\frac{V_{cr}}{\omega}=2.05, \qquad 70\leq\frac{U_*d}{v} \tag{3.20}$$

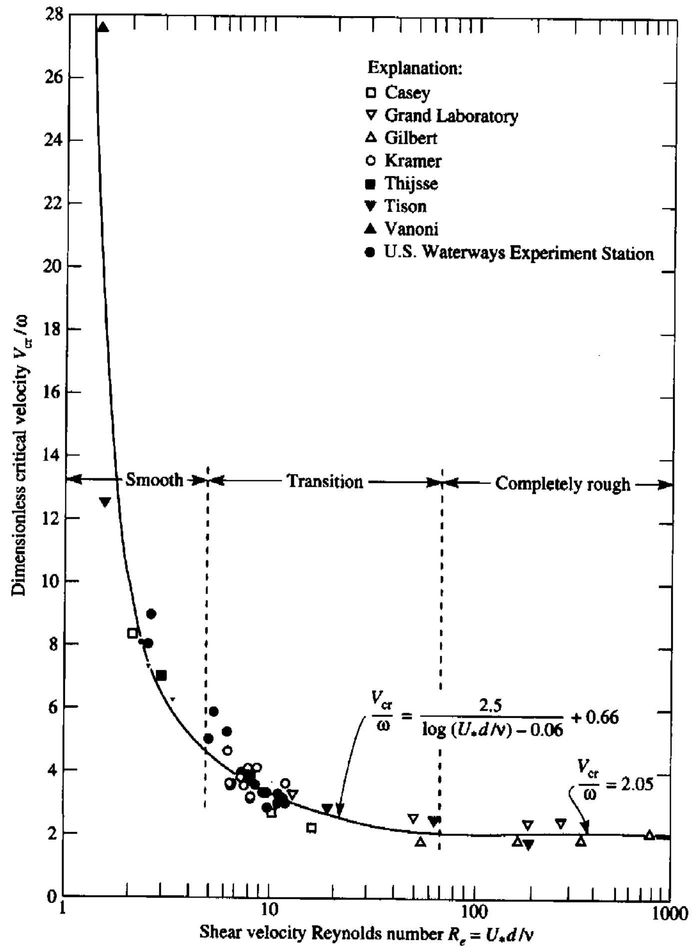


Figure 3.8. Relationship between dimensionless critical average velocity and Reynolds number (Yang, 1973).

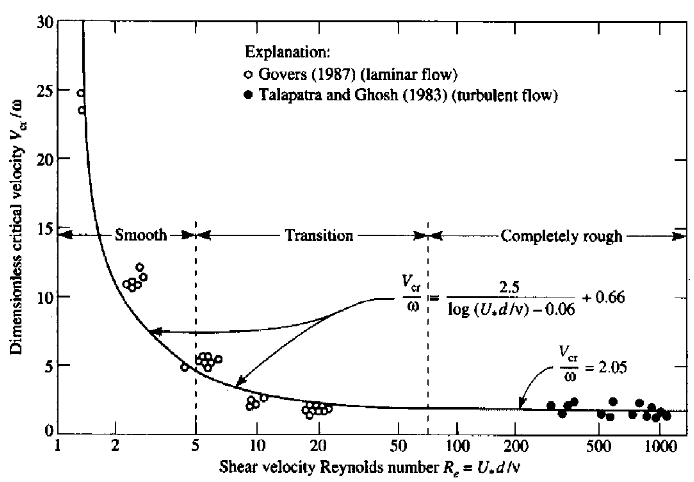


Figure 3.9. Verification of Yang's incipient motion criteria (Yang, 1996, 2003).

Equation (3.19) indicates that the relationship between dimensionless critical average flow velocity and Reynolds number follows a hyperbola when the Reynolds number is less than 70. When the Reynolds number is greater than 70, V_{cr}/ω becomes a constant, as shown in equation (3.20). Figure 3.8 shows comparisons between equations (3.19), (3.20), and laboratory data. Figure 3.9 summarizes independent laboratory verification of Yang's criteria by Govers (1987) and Talapatra and Ghosh (1983).

3.3 Sediment Transport Functions

The basic approaches used in the derivation of sediment transport functions or formulas are the regime, regression, probabilistic, and deterministic approaches. The basic assumptions, their limits of applications, and the theoretical basis of the above approaches and some of the more recent approaches based on the power concept are summarized herein.

3.3.1 Regime Approach

A regime channel is an alluvial channel in dynamic equilibrium without noticeable long-term aggradations, degradation, or change of channel geometry and profile. Some site-specific quantitative relationships exist among sediment transport rates or concentration, hydraulic parameters, and channel geometry parameters. The so-called "regime theory" or "regime equations" are empirical results based on long-term observations of stable canals in India and Pakistan. Blench (1969) summarizes the range of regime channel data as shown in Table 3.2. The regime equations obtained from the regime concept are mainly obtained from the regression analysis of regime canal data.

Different sets of regime equations have been proposed by different investigators, such as those by Blench (1969), Kennedy (1895), and Lacy (1929). According to Blench, applications of regime equations have the following limitations:

- Steady discharge.
- Steady bed-sediment discharges of too small an amount to appear explicitly in the equations.
- Duned sand bed with the particle size distribution natural in the sense of following log-normal distribution.
- Suspended load insufficient to affect the equations.
- Steep, cohesive sides that are erodible or depositable from suspension and behave as hydraulically smooth.
- Straightness in the plan, so that the smoothed, duned bed is level across the cross-section.
- Uniform section and slope.

- Constant water viscosity.
- Range of important parameters as shown in Table 3.2 or in whatever extrapolated range permits the same phase of flow.

Table 3.2 Regime canal data range (after Blench, 1969)

Particle size d, mm	0.10–0.60
Silt grading	log probability
Concentration per 10^5	0 to about 3
Suspended load	0–1%
Water temperature	50–86 °F
Channel sides material	clay, smooth
Width-depth ratio, B/D	4–30
V^2/D, ft/s^2	0.5–1.5
VB/v	10^6–10^8
Water discharge, Q, ft^3/s	1–10,000
Bed form	dunes
D/d	> 1,000

Specifically, the equations are unlikely to apply if the width-depth ratio falls below about 5, or the depth below about 400 millimeters.

The channel-forming discharge, or the dominant discharge, and sediment load or silt factors are the two most important factors to be considered in regime equations. The regime equations are useful engineering tools for stable canal design, especially for those in Pakistan and India. However, they have been subject to criticism for their lack of rational and physical rigor. No regime equations are given in this chapter. Readers who are interested in the application of regime equations should study the conditions under which these empirical equations were obtained. Applications of regime equations to conditions outside of the range of data used in deriving them could lead to erroneous results.

The concept of "regime" is similar to the concepts of "dynamic equilibrium" and "hydraulic geometry." Lacy's (1929) regime equation describing the relationships among channel slope S, water discharge Q, and silt factor f_s for sediment transport is:

$$S = 0.0005423 \frac{f_s^{5/3}}{Q^{1/6}} \tag{3.21}$$

Leopold and Maddock's (1953) hydraulic geometry relationships are:

$$W = aQ^b \tag{3.22}$$

$$D = cQ^f \tag{3.23}$$

$$V = kQ^m \tag{3.24}$$

where W = channel width,
D = channel depth,
V = average flow velocity,
Q = water discharge, and
a, b, c, f, k, m = site-specific constants.

Yang, et al. (1981) applied the unit stream power theory for sediment transport (Yang, 1973), the theory of minimum unit stream power (Yang, 1971, 1976; Yang and Song, 1979, 1986), and the hydraulic geometry relationships shown in equations (3.22) through (3.24) to derive the relationship between Q and S. They also assumed that:

$$S = iQ^j \tag{3.25}$$

where i, j = constants.

The theoretically derived j value is $-2/11$, which is very close to the empirical value of $-1/6$ shown in equation (3.21).

3.3.2 Regression Approach

Some researchers believe that sediment transport is such a complex phenomenon that no single hydraulic parameter or combination of parameters can be found to describe sediment transport rate under all conditions. Instead of trying to find a dominant variable that can determine the rate of sediment transport, they recommend the use of regressions based on laboratory and field data. The parameters used in these regression equations may or may not have any physical meaning relating to the mechanics of sediment transport.

Shen and Hung (1972) proposed the following regression equation based on 587 sets of laboratory data in the sand size range:

$$\log C_t = -107{,}404.45938164 + 324{,}214.74734085Y - 326{,}309.58908739Y^2 + 109{,}503.87232539Y^3 \tag{3.26}$$

where Y = $(VS^{0.57}/\omega^{0.32})^{0.00750189}$,
C_t = total sediment concentration in ppm by weight, and
ω = average fall velocity of sediment particles.

Before equation (3.26) was finally adopted by Shen and Hung, they performed a sensitivity analysis on the importance of different variables to the rate of sediment transport. Because laboratory data have limited range of variation of water depth, the sensitivity analysis indicated that the rate of sediment transport was not sensitive to changes in water depth. Consequently, water depth was eliminated from consideration. The dimensionally nonhomogeneous parameters used and the lack of ability to reflect the effect of depth change limit the application of equation (3.26) to laboratory flumes and small rivers with particles in the sand size range.

Karim and Kennedy (1990) used nonlinear, multiple-regression analyses to derive relations between flow velocity, sediment discharge, bed-form geometry, and friction factor of alluvial rivers. They used a total of 339 sets of river data and 608 sets of flume data in the analyses. The sediment discharge and velocity relationships adopted by them have the following general forms:

$$\log \frac{q_s}{\left(1.65 g d_{50}^3\right)^{1/2}} = A_0 + A_{ijk} \sum_i \sum_j \sum_k \log X_i \log X_j \log X_k \tag{3.27}$$

$$\log \frac{V}{\left(1.65 g d_{50}^3\right)^{1/2}} = B_0 + B_{pqr} \sum_p \sum_q \sum_r \log X_p \log X_q \log X_r \tag{3.28}$$

where q_s = volumetric total sediment discharge per unit width,
g = gravitational acceleration,
d_{50} = median bed-material particle diameter,
V = mean velocity,
A_0, A_{ijk}, B_0, and B_{pqr} = constants determined from regression analyses, and
X_i, X_j, X_k, X_p, X_q, and X_r = nondimensional independent variables.

The uncoupled relations recommended by Karim and Kennedy are:

$$\begin{aligned} \log \frac{q_s}{(1.65 g d_{50}^3)^{1/2}} = & -2.279 + 2.972 \log \frac{V}{(1.65 g d_{50})^{1/2}} \\ & + 1.060 \log \frac{V}{(1.65 g d_{50})^{1/2}} \log \frac{U_* - U_{*c}}{(1.65 g d_{50})^{1/2}} \\ & + 0.299 \log \frac{D}{d_{50}} \log \frac{U_* - U_{*c}}{(1.65 g d_{50})^{1/2}} \end{aligned} \tag{3.29}$$

and

$$\frac{V}{(1.65 g d_{50})^{1/2}} = 2.822 \left[\frac{q}{(1.65 g d_{50}^3)^{1/2}} \right]^{0.376} S^{0.310} \tag{3.30}$$

where q = water discharge per unit width,
S = energy slope,
V = average flow velocity,
U_* = bed shear velocity = $(gDS)^{1/2}$,
U_{*c} = Shields' value of critical shear velocity at incipient motion, and
D = water depth.

Equation (3.30) can be used for flows well above the incipient sediment motion. If it is necessary to take into account the bed configuration changes in the development of a friction or velocity predictor, equation (3.30) should be replaced by:

$$\frac{V}{(1.65gd_{50})^{1/2}} = 9.82\left[\frac{q}{(1.65gd_{50}^3)^{1/2}}\right]^{0.216}\left(\frac{f}{f_0}\right)^{-0.164} \tag{3.31}$$

where f = the Darcy-Weisbach friction factor.

The grain roughness factor f_0 can be expressed as:

$$f_0 = \frac{8}{\left[6.25 + 2.5\ln\left(D/2.5d_{50}\right)\right]^2} \tag{3.32}$$

The friction factor ratio f/f_0 in equation (3.31) can be computed as:

$$\frac{f}{f_o} = 1.20 + 8.92\left[0.08 + 2.24\left(\frac{\theta}{3}\right) - 18.13\left(\frac{\theta}{3}\right)^2 + 70.90\left(\frac{\theta}{3}\right)^3 - 88.33\left(\frac{\theta}{3}\right)^4\right] \quad \text{for } \theta \le 1.5 \tag{3.33a}$$

$$\frac{f}{f_o} = 1.20 \quad \text{for } \theta > 1.5 \tag{3.33b}$$

where

$$\theta = \frac{\tau_0}{1.65\gamma d_{50}} = \frac{DS}{1.65d_{50}} \tag{3.34}$$

and γ = specific weight of water.

Equations (3.29), (3.31), and (3.33) constitute a set of coupled sediment discharge friction, and bed-form relations. Yang (1996) summarized the interaction scheme for solving equations (3.29), (3.31), and (3.33) for a set of known values of q, S, and d_{50}.

A regression equation may give fairly accurate results for engineering purposes if the equation is applied to conditions similar to those from where the equation was derived. Application of a regression equation outside the range of data used for deriving the regression equation should be carried out with caution. In general, regression equations without a theoretical basis and without using dimensionless parameters should not be used for predicting sediment transport rate or concentration in natural rivers.

3.3.3 Probabilistic Approach

Einstein (1950) pioneered sediment transport studies from the probabilistic approach. He assumed that the beginning and ceasing of sediment motion can be expressed in terms of probability. He also assumed that the movement of bedload is a series of steps followed by rest periods. The average step length is 100 times the particle diameter. Einstein used the hiding correction factor and lifting correction factor to better match theoretical results with observed laboratory data.

In spite of the sophisticated theories used, the Einstein bedload transport function is not a popular one for engineering applications. This is partially due to the complex computational procedures required. However, the probabilistic approach developed by Einstein has been used as a theoretical basis for developing other transport functions, such as the method proposed by Toffaleti (1969).

Based on the mode of transport, total sediment load consists of bedload and suspended load. Total load can also be divided into measured and unmeasured load. The original Einstein function has been modified by others for the estimation of unmeasured load. The original Einstein function is a predictive function for sediment transport. The "modified Einstein method" is not a predictive function. The method can be used to estimate bedload or unmeasured load based on measured suspended load for the estimation of total load or total bed-material load. The method proposed by Colby and Hembree (1955) is one of the most commonly used modified Einstein methods for the computation of total bed-material load.

Application of the original Einstein method and the modified Einstein method is labor intensive. Unless necessary, these methods are not commonly used for solving engineering problems or used in a computer model for routing sediment. Yang (1996) provided detailed explanations of these methods with step-by-step computation examples for engineers to follow.

3.3.4 Deterministic Approach

The basic assumption in a deterministic approach is the existence of one-to-one relationship between independent and dependent variables. Conventional, dominant, independent variables used in sediment transport studies are water discharge, average flow velocity, shear stress, and energy or water surface slope. More recently, the use of stream power and unit stream power have gained increasing acceptance as important parameters for the determination of sediment transport rate or concentration. Other independent parameters used in sediment transport functions are sediment particle diameter, water temperature, or kinematic viscosity. The accuracy of a deterministic sediment transport formula depends on the generality and validity of the assumption of whether a unique relationship between dependent and independent variables exists. Deterministic sediment transport formulas can be expressed by one of the following forms:

$$q_s = A_1(Q - Q_c)^{B_1} \tag{3.35}$$

$$q_s = A_2(V - V_c)^{B_2} \tag{3.36}$$

$$q_s = A_3(S - S_c)^{B_3} \tag{3.37}$$

$$q_s = A_4(\tau - \tau_c)^{B_4} \tag{3.38}$$

$$q_s = A_5(\tau V - \tau_c V_c)^{B_5} \quad (3.39)$$

$$q_s = A_6(VS - V_c S_c)^{B_6} \quad (3.40)$$

where q_s = sediment discharge per unit width of channel,
Q = water discharge,
V = average flow velocity,
S = energy or water surface slope,
τ = shear stress,
τV = stream power per unit bed area,
VS = unit stream power,
$A_1, A_2, A_3, A_4, A_5, A_6, B_1, B_2, B_3, B_4, B_5, B_6$ = parameters related to flow and sediment conditions, and
c = subscript denoting the critical condition at incipient motion.

Yang (1972, 1983) used laboratory data collected by Guy et al. (1966) from a laboratory flume with 0.93-mm sand, as shown in Figure 3.10, as an example to examine the validity of these assumptions.

Figure 3.10(a) shows the relationship between the total sediment discharge and water discharge. For a given value of Q, two different values of q_t can be obtained. Field data obtained by Leopold and Maddock (1953) also indicate similar results. Some of Gilbert's (1914) data indicate that no correlation exists at all between water discharge and sediment discharge. Apparently, different sediment discharges can be transported by the same water discharge, and a given sediment discharge can be transported by different water discharges. The same sets of data shown in Figure 3.10(a) are plotted in Figure 3.10(b) to show the relationship between total sediment discharge and average velocity. Although q_t increases steadily with increasing V, it is apparent that for approximately the same value of V, the value of q_t can differ considerably, owing to the steepness of the curve. Some of Gilbert's (1914) data also indicate that the correlations between q_t and V are very weak. Figure 3.10(c) indicates that different amounts of total sediment discharges can be obtained at the same slope, and different slopes can also produce the same sediment discharge. Figure 3.10(d) shows that a fairly well-defined correlation exists between total sediment discharge and shear stress when total sediment discharge is in the middle range of the curve. For either higher or lower sediment discharge, the curve becomes vertical, which means that for the same shear stress, numerous values of sediment discharge can be obtained.

It is apparent from Figure 3-10(a-d) that more than one value of total sediment discharge can be obtained for the same value of water discharge, velocity, slope, or shear stress. The validity of the assumption that total sediment discharge of a given particle size could be determined from water discharge, velocity, slope, or shear stress is questionable.

Because of the basic weakness of these assumptions, the generality of an equation derived from one of these assumptions is also questionable. When the same sets of data are plotted on Figure 3.10(e), with stream power as the independent variable, the correlation improves. Further improvement can be made by using unit stream power as the dominant variable, as shown in Figure 3.10(f). This close correlation exists in spite of the presence of different bed forms, such as plane bed, dune, transition, and standing wave.

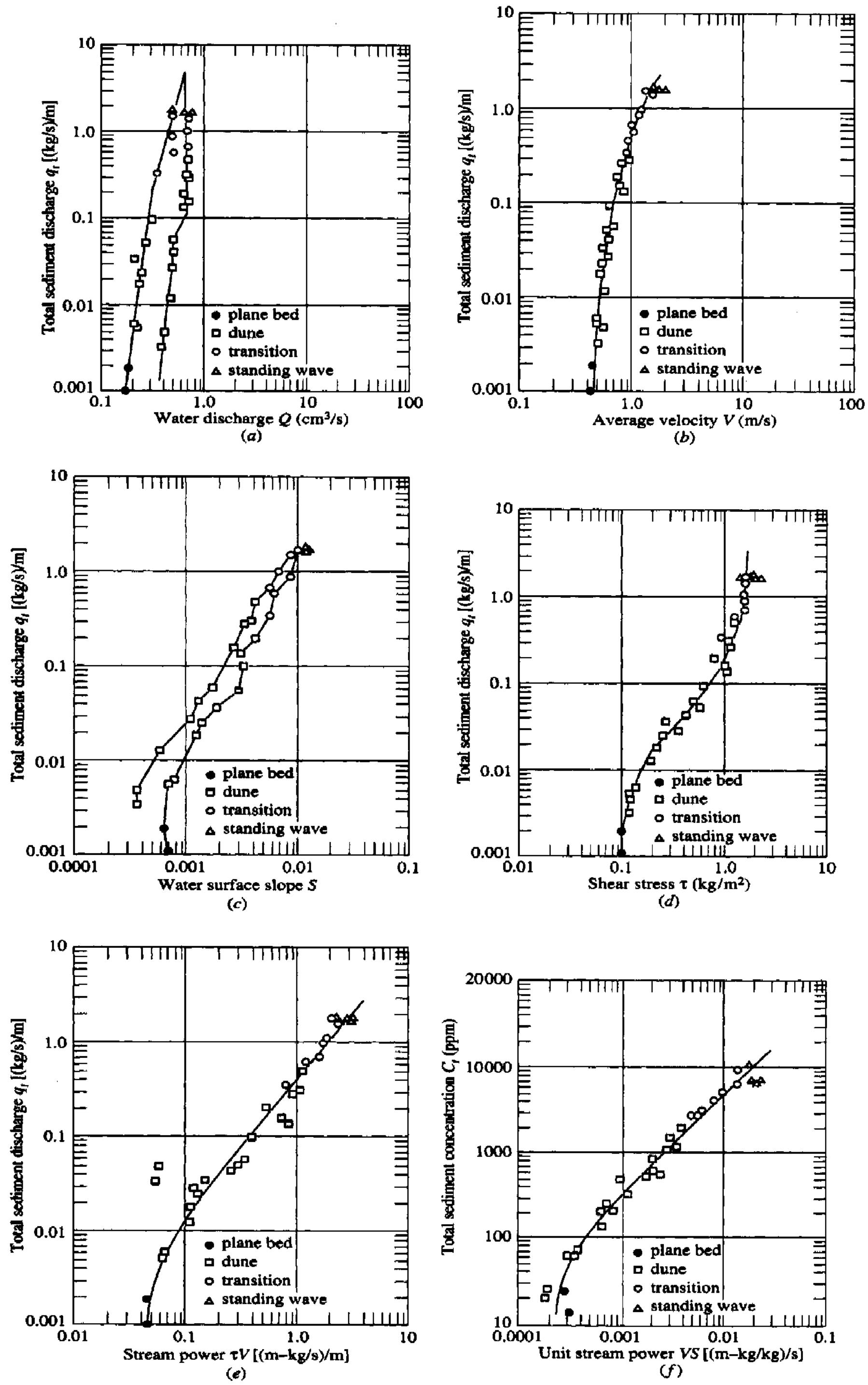


Figure 3.10. Relationships between total sediment discharge and (a) water discharge, (b) velocity, (c) slope, (d) shear stress, (e) stream power, and (f) unit stream power, for 0.93-mm sand in an 8-ft wide flume (Yang, 1972, 1983).

The close relationship between total sediment concentration and unit stream power exists not only in straight channels but also in those channels that are in the process of changing their patterns from straight to meandering, and to braided channels, as shown in Figure 3.11 (Yang, 1977). Schumm and Khan (1972) collected these data.

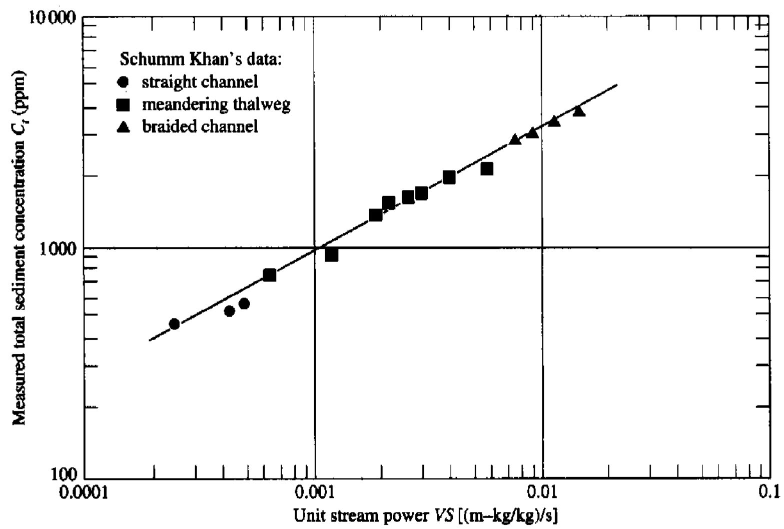


Figure 3.11. Relationship between total concentration and unit stream power during process of channel pattern development from straight to meandering, and to braided (Yang, 1977).

Vanoni (1978), among others, has confirmed the fact that unit stream power dominates sediment discharge or concentration. It is apparent from the results in Figure 3.12 that sediment concentration cannot be determined from relative roughness D/d_{50} and Froude number Fr. However, when the same data are plotted in Figure 3.13 using dimensionless unit stream power VS/ω as the dominant variable, the improvement is apparent.

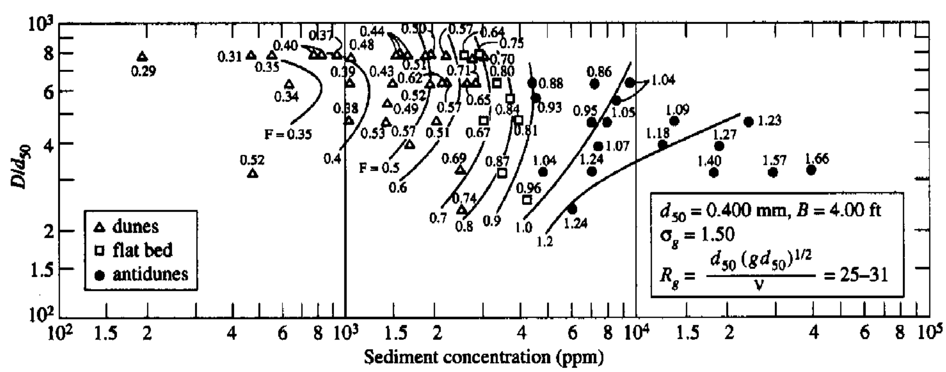


Figure 3.12. Plot of Stein's (1965) data as sediment discharge concentration against Froude number Fr (indicated by the number next to each data point) and the ratio of flow depth D to bed-sediment size d_{50} (Vanoni, 1978).

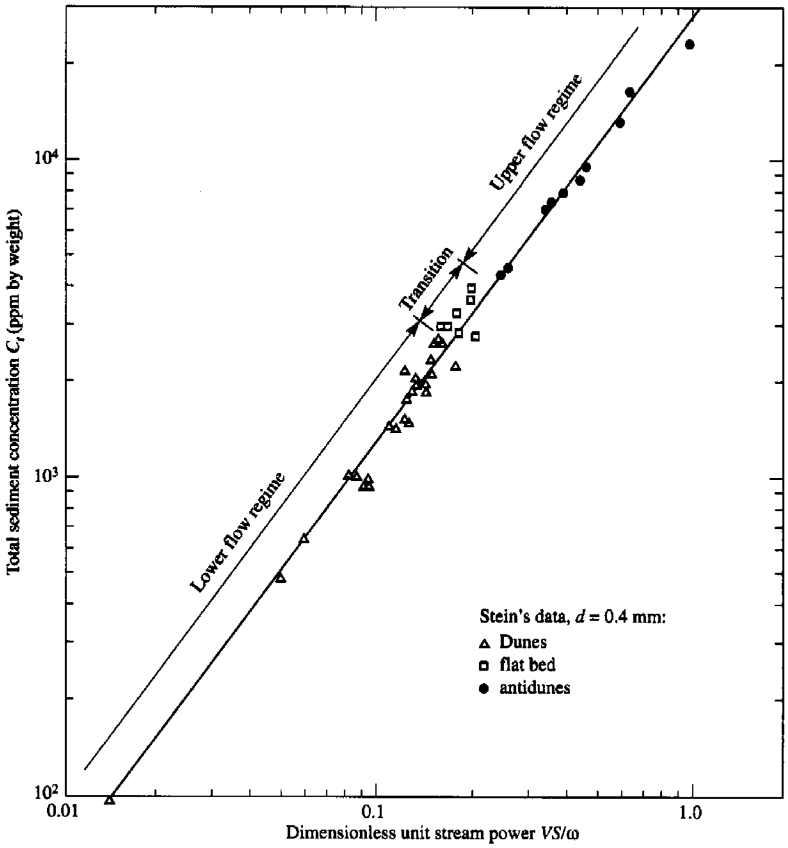


Figure 3.13. Relationship between sediment concentration and dimensionless unit stream power (Yang and Kong, 1991).

Many investigators believe that shear stress τ or stream power τV would be more suitable for the study of coarse material or bedload movement, because these parameters represent the force or power acting along the bed. Yang and Molinas (1982) have shown theoretically that bedload and suspended load, as well as total load, are directly related to unit stream power.

Yang (1983, 1984) used Meyer-Peter and Müller's (1948) gravel data to verify the theoretical finding that bedload can be more accurately determined by unit stream power than by shear stress or stream power. Figure 3.14 shows the loop effect when shear stress or stream power is used as the dominant variable. Gilbert's (1914) data (Figure 3.15) indicate that a family of curves exists between gravel concentration and shear stress or stream power, with water discharge as the third parameter. These results indicate that bedload may not be determined by using shear stress, stream power, or water discharge as the dominant variable. In each case, more than one value of gravel concentration can be obtained at a given value of shear stress, stream power, or water discharge. However, the well-defined strong correlation between gravel concentration and dimensionless unit stream power VS/ω shown in Figures 3.14 and 3.15 is apparent.

It can be concluded that, of all the parameters used in the determination of sediment transport rate, stream power and unit stream power have the strongest correlation with sediment transport rate or concentration. Based on the theoretical derivations and measured data, unit stream power *VS* or dimensionless unit stream power *VS/ω* are preferable to other parameters for the determination of sediment transport rate or concentration. The lack of well-defined strong correlation between sediment load or concentration and a dominant variable selected for the development of a sediment transport equation may be the fundamental reason for discrepancies between computed and measured results under different flow and sediment conditions.

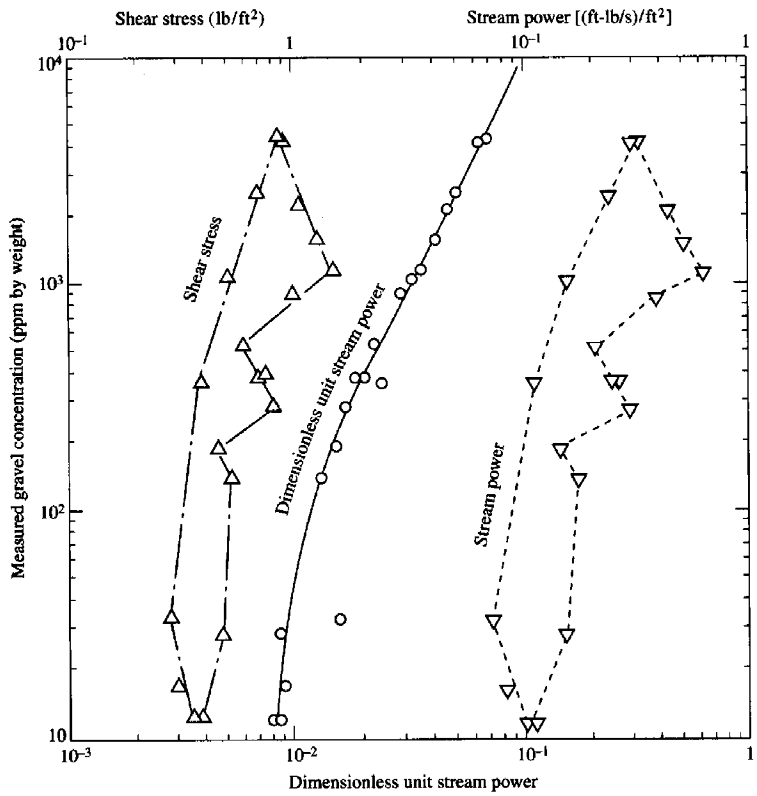


Figure 3.14. Relationship between dimensionless unit stream power, stream power, shear stress, and 5.12-mm gravel concentration measure by Meyer-Peter and Müller (Yang, 1984).

Yang (1996) summarized more detailed explanations and derivations. Due to the importance of stream power, unit stream power, and other power approaches to the determination of sediment transport rate or concentration, more detailed analyses will be made in the following sections.

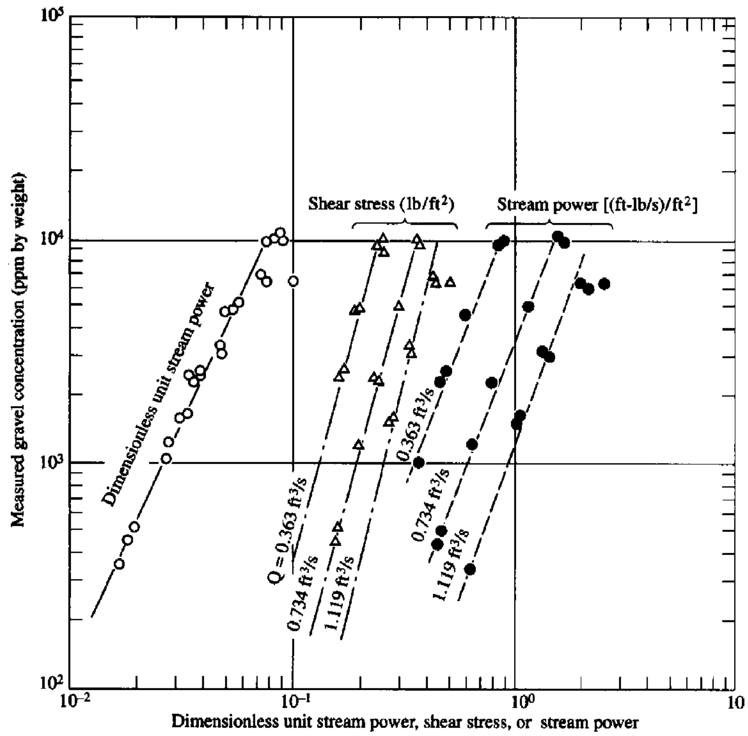


Figure 3.15. Relationship between dimensionless unit stream power, shear stress, stream power, and 4.94-mm gravel concentration measured by Gilbert from a 0.2-m flume (Yang, 1983, 1984).

3.3.5 Stream Power Approach

Bagnold (1966) introduced the stream power concept for sediment transport based on general physics. Engelund and Hansen (1972), and Ackers and White (1973) later used the concept as the theoretical basis for developing their sediment transport functions (Yang, 2002). These transport functions are summarized herein.

3.3.5.1 Bagnold's Approach

From general physics, the rate of energy used in transporting materials should be related to the rate of materials being transported. Bagnold (1966) defined stream power τV as the power per unit bed area which can be used to transport sediment. Bagnold's basic relationship is:

$$\frac{\gamma_s - \gamma}{\gamma} q_{bw} \tan \alpha = \tau V e_b \tag{3.41}$$

where
γ_s and γ = specific weights of sediment and water, respectively,
q_{bw} = bedload transport rate by weight per unit channel width,
$\tan \alpha$ = ratio of tangential to normal shear force,

τ = shear force acting along the bed,
V = average flow velocity, and
e_b = efficiency coefficient.

In equation (3.41), the values of e_b and tan α were given by Bagnold in two separate figures. The rate of work needed in transporting the suspended load is:

$$\phi_s = \frac{\gamma_s - \gamma}{\gamma} q_{sw} \frac{\omega}{\bar{u}_s} \tag{3.42}$$

where q_{sw} = suspended load discharge in dry weight per unit time and width,
$\bar{u}_s$ = mean transport velocity of suspended load, and
ω = fall velocity of suspended sediment.

The rate of energy available for transporting the suspended load is:

$$\phi_s' = \tau V (1 - e_b) \tag{3.43}$$

Based on general physics, the rate of work being done should be related to the power available times the efficiency of the system; that is:

$$\frac{\gamma_s - \gamma}{\gamma} q_{sw} \frac{\omega}{\bar{u}_s} = \tau V (1 - e_b) e_s \tag{3.44}$$

where e_s = suspended load transport efficiency coefficient.

Equation (3.44) can be rearranged as:

$$\frac{\gamma_s - \gamma}{\gamma} q_{sw} = (1 - e_b) e_s \frac{\bar{u}_s}{\omega} \tau V \tag{3.45}$$

Assuming $\bar{u}_s = V$, Bagnold found $(1 - e_b)e_s = 0.01$ from flume data. Thus, the suspended load can be computed by:

$$\frac{\gamma_s - \gamma}{\gamma} q_{sw} = 0.01 \tau V^2 / \omega \tag{3.46}$$

The total load in dry weight per unit time and unit width is the sum of bedload and suspended load; that is, from equations (3.41) and (3.46):

$$q_t = q_{bw} + q_{sw} = \frac{\gamma}{\gamma_s - \gamma} \tau V \left(\frac{e_b}{\tan \alpha} + 0.01 \frac{V}{\omega} \right) \tag{3.47}$$

where q_t = total load [in (lb/s)/ft].

3.3.5.2 Engelund and Hansen's Approach

Engelund and Hansen (1972) applied Bagnold's stream power concept and the similarity principle to obtain a sediment transport function:

$$f'\phi = 0.1\theta^{5/2} \tag{3.48}$$

with

$$f' = \frac{2gSD}{V^2} \tag{3.49}$$

$$\phi = \frac{q_t}{\gamma_s}\left[\left(\frac{\gamma_s - \gamma}{\gamma}\right)gd^3\right]^{-1/2} \tag{3.50}$$

$$\theta = \frac{\tau}{(\gamma_s - \gamma)d} \tag{3.51}$$

where g = gravitational acceleration,
S = energy slope,
V = average flow velocity,
q_t = total sediment discharge by weight per unit width,
γ_s and γ = specific weights of sediment and water, respectively,
d = median particle diameter, and
τ = shear stress along the bed.

Strictly speaking, equation (3.48) should be applied to those flows with dune beds in accordance with the similarity principle. However, Engelund and Hansen found that it can be applied to the dune bed and the upper flow regime with particle size greater than 0.15 mm without serious deviation from the theory. Yang (2002) made step-by-step theoretical derivations to show that the basic form of Engelund and Hansen's transport function can be obtained from Bagnold's stream power concept. Yang (1966) also provided a numerical example on the application of Engelund and Hansen's transport function.

3.3.5.3 Ackers and White's Approach

Ackers and White (1973) applied dimensional analysis to express mobility and sediment transport rate in terms of some dimensionless parameters. Their mobility number for sediment transport is:

$$F_{gr} = U_*^n\left[gd\left(\frac{\gamma_s}{\gamma} - 1\right)\right]^{-1/2}\left[\frac{V}{\sqrt{32}\log(\alpha D/d)}\right]^{1-n} \tag{3.52}$$

where U_* = shear velocity,
n = transition exponent, depending on sediment size,
α = coefficient in rough turbulent equation (= 10),
d = sediment particle size, and
D = water depth.

They also expressed the sediment size by a dimensionless grain diameter:

$$d_{gr} = d\left[\frac{g\left(\gamma_s/\gamma-1\right)}{\nu^2}\right]^{1/3} \tag{3.53}$$

where ν = kinematic viscosity.

A general dimensionless sediment transport function can then be expressed as:

$$G_{gr} = f\left(F_{gr}, d_{gr}\right) \tag{3.54}$$

with

$$G_{gr} = \frac{XD}{d\gamma_s/\gamma}\left(\frac{U_*}{V}\right)^n \tag{3.55}$$

where X = rate of sediment transport in terms of mass flow per unit mass flow rate; i.e., concentration by weight of fluid flux.

The generalized dimensionless sediment transport function can also be expressed as:

$$G_{gr} = C\left(\frac{F_{gr}}{A}-1\right)^m \tag{3.56}$$

Ackers and White (1973) determined the values of A, C, m, and n based on best-fit curves of laboratory data with sediment size greater than 0.04 mm and Froude number less than 0.8. For the transition zone with $1 < d_{gr} \leq 60$,

$$n = 1.00 - 0.56 \log d_{gr} \tag{3.57}$$

$$A = 0.23 d_{gr}^{-1/2} + 0.14 \tag{3.58}$$

For coarse sediment, $d_{gr} > 60$:

$$n = 0.00 \tag{3.59}$$

$$A = 0.17 \tag{3.60}$$

$$m = 1.50 \tag{3.61}$$

$$C = 0.025 \tag{3.62}$$

For the transition zone:

$$m = \frac{9.66}{d_{gr}} + 1.34 \tag{3.63}$$

$$\log C = 2.86 \log d_{gr} - (\log d_{gr})^2 - 3.53 \tag{3.64}$$

The procedure for the computation of sediment transport rate using Ackers and White's approach is summarized as follows:

1. Determine the value of d_{gr} from known values of d, g, γ_s/γ, and v in equation (3.53).

2. Determine values of n, A, m, and C associated with the derived d_{gr} value from equations (3.57) through (3.64).

3. Compute the value of the particle mobility F_{gr} from equation (3.52).

4. Determine the value of G_{gr} from equation (3.56), which represents a graphical version of the new sediment transport function.

5. Convert G_{gr} to sediment flux X, in ppm by weight of fluid flux, using equation (3.55).

Although it is not apparent from the above procedures, Yang (2002) provided step-by-step derivations to show that Ackers and White's basic transport function can be derived from Bagnold's stream power concept.

The original Ackers and White formula is known to overpredict transport rates for fine sediments (smaller than 0.2 mm) and for relatively coarse sediments. To correct that tendency, a revised form of the coefficients was published in 1990 (HR Wallingford, 1990). Table 3.3 gives the comparison between the original and revised coefficients.

Reclamation's computer models GSTARS 2.1 (Yang and Simões, 2000) and GSTARS3 (Yang and Simões, 2002) allow users to select either the 1973 or the 1990 values in their application of the Ackers and White sediment transport function.

Table 3.3. Coefficients for the 1973 and 1990 versions of the Ackers and White transport function

1973	1990
$1 < d_{gr} \le 60$ $A = 0.23 d_{gr}^{-1/2} + 0.14$	$A = 0.23 d_{gr}^{-1/2} + 0.14$
$\log C = -3.53 + 2.86 \log d_{gr} - (\log d_{gr})^2$	$\log C = -3.46 + 2.79 \log d_{gr} - 0.98 (\log d_{gr})^2$
$m = 9.66\, d_{gr}^{-1} + 1.34$	$m = 6.83\, d_{gr}^{-1} + 1.67$
$n = 1.00 - 0.56 \log d_{gr}$	$n = 1.00 - 0.56 \log d_{gr}$
$d_{gr} > 60$ $A = 0.17$	$A = 0.17$
$C = 0.025$	$C = 0.025$
$m = 1.50$	$m = 1.78$
$n = 0$	$n = 0$

3.3.6 Unit Stream Power Approach

The rate of energy per unit weight of water available for transporting water and sediment in an open channel with reach length x and total drop of Y is:

$$\frac{dY}{dt} = \frac{dx}{dt}\frac{dY}{dx} = VS \tag{3.65}$$

where V = average flow velocity, and
S = energy or water surface slope.

Yang (1972) defines unit stream power as the velocity-slope product shown in equation (3.65). The rate of work being done by a unit weight of water in transporting sediment must be directly related to the rate of work available to a unit weight of water. Thus, total sediment concentration or total bed-material load must be directly related to unit stream power. While Bagnold (1966) emphasized the power applies to a unit bed area, Yang (1972, 1973) emphasized the power available per unit weight of water to transport sediments.

To determine total sediment concentration, Yang (1973) considered a relation between the relevant variables of the form

$$\Phi(C_t, VS, U_*, \nu, \omega, d) = 0 \tag{3.66}$$

where C_t = total sediment concentration, with wash load excluded (in ppm by weight):
VS = unit stream power,
U_* = shear velocity,
V = kinematic viscosity,
ω = fall velocity of sediment, and
d = median particle diameter.

Using Buckingham's π theorem and the analysis of laboratory data, C_t in equation (3.66) can be expressed in the following dimensionless form:

$$\log C_t = I + J \log\left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right) \tag{3.67}$$

where $V_{cr}S/\omega$ = critical dimensionless unit stream power at incipient motion.

I and J in equation (3.67) are dimensionless parameters reflecting the flow and sediment characteristics, that is:

$$I = a_1 + a_2 \log\frac{\omega d}{v} + a_3 \log\frac{U_*}{\omega} \tag{3.68}$$

$$J = b_1 + b_2 \log\frac{\omega d}{v} + b_3 \log\frac{U_*}{\omega} \tag{3.69}$$

where $a_1, a_2, a_3, b_1, b_2, b_3$ = coefficients.

Yang (1973) used 463 sets of laboratory data for the determination of coefficients in equations (3.68) and (3.69). The dimensionless unit stream power equation for sand transport thus obtained is:

$$\log C_{ts} = 5.435 - 0.286 \log\frac{\omega d}{v} - 0.457 \log\frac{U_*}{\omega} + \left(1.799 - 0.409 \log\frac{\omega d}{\omega} - 0.314 \log\frac{U_*}{\omega}\right)\log\left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right) \tag{3.70}$$

where C_{ts} = total sand concentration in ppm by weight.

The critical dimensionless unit stream power $V_{cr}S/\omega$ is the product of dimensionless critical velocity $V_{cr}S/\omega$ shown in equations (3.19) and (3.20) and the energy slope S. Yang and Molinas (1982) made a step-by-step derivation to show that sediment concentration is indeed directly related to unit stream power, based on basic theories in fluid mechanics and turbulence. They showed that the vertical sediment concentration distribution is directly related to the vertical distribution of turbulence energy production rate; that is:

$$\frac{\bar{C}}{\bar{C}_a} = \left[\frac{\tau_{xy} d\bar{U}_x/dy}{\left(\tau_{xy} d\bar{U}_x/dy\right)_{y=a}}\right]^{Z_1} \tag{3.71}$$

where $\bar{C}, \bar{C}_a$ = time-averaged sediment concentration at a given cross-section and at a depth a above the bed, respectively,

τ_{xy} = turbulence shear stress,

dU_x/dy = velocity gradient,

$\tau_{xy}\, dU_x/dy$ = turbulence energy production rate,

Z_1 = $\omega/k\beta U_*$,

ω = sediment particle fall velocity,

β = coefficient,

k = von Karman constant, and

U_* = shear velocity.

Figure 3.16 shows comparisons between measured and theoretical results from equation (3.71). This confirmation is independent from the selection of reference elevation a.

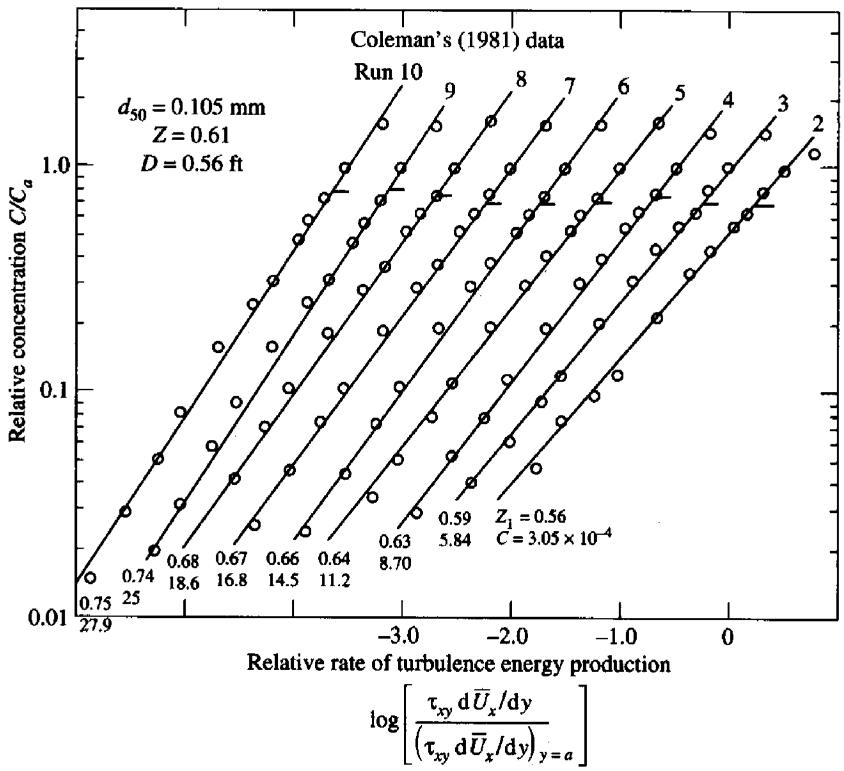


Figure 3.16. Comparison between theoretical and measured suspended sediment concentration distributions (Yang, 1985).

For sediment concentration higher than about 100 ppm by weight, the need to include incipient motion criteria in a sediment transport equation decreases. Yang (1979) introduced the following dimensionless unit stream power equation for sand transport with concentration higher than 100 ppm:

$$\log C_{ts} = 5.165 - 0.153 \log \frac{\omega d}{v} - 0.297 \log \frac{U_*}{\omega} + \left(1.780 - 0.360 \log \frac{\omega d}{v} - 0.480 \log \frac{U_*}{\omega}\right) \log \frac{VS}{\omega} \tag{3.72}$$

Yang (1984) extended his dimensionless unit stream power equation for sand transport to gravel transport by calibrating the coefficients in equations (3.68) and (3.69) with gravel data. The gravel equation thus obtained is:

$$\log C_{tg} = 6.681 - 0.633 \log \frac{\omega d}{v} - 4.816 \log \frac{U_*}{\omega} + \left(2.784 - 0.305 \log \frac{\omega d}{v} - 0.282 \log \frac{U_*}{\omega}\right) \log \left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right) \tag{3.73}$$

where C_{tg} = total gravel concentration in ppm by weight.

The incipient motion criteria given in equations (3.19) and (3.20) should be used for equation (3.73).

Most of the sediment transport equations were developed for sediment transport in rivers where the effect of fine or wash load on fall velocity, viscosity, and relative density can be ignored. The Yellow River in China is known for its high sediment concentration and wash load. The relationship between fall velocity of sediment in clear water and that of a sediment-laden flow of the Yellow River is:

$$\omega_m = \omega \left(1 - C_v\right)^{7.0} \tag{3.74}$$

where ω and ω_m = sediment particle fall velocities in clear water and in sediment-laden flow, respectively, and

C_v = suspended sediment concentration by volume, including wash load.

The kinematic viscosity of the sediment-laden Yellow River is:

$$v_m = \frac{\rho}{\rho_m} - e^{5.06 C_v} v \tag{3.75}$$

where ρ and ρ_m = specific densities of water and sediment-laden flow, respectively, and:

$$\rho_m = \rho + \left(\rho_s - \rho\right) C_v \tag{3.76}$$

where ρ_s = specific density of sediment particles.

If sediments are transported in a sediment-laden flow with high concentrations of fine materials, it can be shown that:

$$C_v = (1 - e_b)\frac{\gamma_m}{\gamma_s - \gamma_m}\left(\frac{VS}{\omega_m}\right) \tag{3.77}$$

where γ and γ_m = specific weights of sediment and sediment-laden flow, respectively, and
e_b = efficient coefficient for bedload.

It can be seen from equation (3.77) that when the unit stream power concept is applied to the estimation of sediment transport in sediment-laden flows, a modified dimensionless unit stream power $[\gamma_s/(\gamma_s - \gamma_m)]VS/\omega_m$ should be used. The modified Yang's unit stream power formula (Yang et al., 1996) for a sediment-laden river, such as the Yellow River, becomes:

$$\log C_{ts} = 5.165 - 0.153\log\frac{\omega_m d}{\nu_m} - 0.297\log\frac{U_*}{\omega_m} + \left(1.780 - 0.360\log\frac{\omega_m d}{\nu_m} - 0.480\log\frac{U_*}{\omega_m}\right)\log\left(\frac{\gamma_m}{\gamma_s - \gamma_m}\frac{VS}{\omega_m}\right) \tag{3.78}$$

It should be noted that the coefficients in equation (3.78) are identical to those in equation (3.72). However, the values of fall velocity, kinematic viscosity, and relative specific weight are modified for sediment transport in sediment-laden flows with high concentrations of fine suspended materials.

It has been the conventional assumption that wash load depends on supply and is not a function of the hydraulic characteristics of a river. Yang (1966) demonstrated that the conjunctive use of equations (3.72) and (3.78) can determine not only bed-material load but also wash load in a sediment-laden river. Yang and Simões (2005) made a systematic and thorough analysis of 1,160 sets of data collected from 9 gauging stations along the Middle and Lower Yellow River. They confirmed that the method suggested by Yang (1996) can be used to compute wash load, bed-material load, and total load in the Yellow River with accuracy.

3.3.7 Power Balance Approach

Pacheco-Ceballos (1989) derived a sediment transport function based on power balance between total power available and total power expenditure in a stream; that is:

$$P = P_l + P_s + P_b + P_2 \tag{3.79}$$

where P = total power available per unit channel width,
P_l = power expenditure per unit width to overcome resistance to flow,
P_s = power expenditure per unit width to transport suspended load,
P_b = power expenditure per unit width to transport bedload, and
P_2 = power expenditure per unit width by minor or other causes which will not be considered hereinafter.

According to Bagnold (1966):

$$P = \tau_0 V = \rho g D S V \tag{3.80}$$

where ρ = density of water,
g = gravitational acceleration, and
D = average depth of flow.

According to Einstein and Chien (1952):

$$P_s = (\rho_s - \rho) g \frac{Q_s \omega}{BV} \tag{3.81}$$

where ρ_s = density of sediment,
Q_s = suspended load,
ω = fall velocity of sediment, and
B = channel width.

Accounting to the power concept and balance of acting force,

$$P_b = g Q_b \frac{\rho_s - \rho}{B} \tan\phi \tag{3.82}$$

where: Q_s = bedload, and
$\tan\phi$ = angle of repose of sediments.

If it is assumed that a certain portion of the available power is used to overcome resistance to flow, then:

$$P_1 = K_0 P = K_0 \rho g S Q / B \tag{3.83}$$

where K_0 = proportionality factor, and
Q = water discharge.

Substituting equations (3.80) through (3.83) into equation (3.79) yields:

$$K = \frac{V Q_b \tan\phi + \omega Q_s}{QVS} \tag{3.84}$$

where

$$K = \frac{(1 - K_0)\rho}{\rho_s - \rho} \tag{3.85}$$

The total sediment concentration can be expressed in the following general form:

$$C_t = \frac{KVS}{K''V\tan\phi + (1 - K'')\omega} = K'VS \tag{3.86}$$

where K'' = ratio between bedload and total load,
K' = parameter,
C_t = total sediment concentration, and
VS = Yang's unit stream power.

When K'' = 1, equation (3.86) becomes a bedload equation; that is:

$$C_b = \frac{KVS}{V\tan\phi} \tag{3.87}$$

When K'' = 0, equation (3.86) becomes a suspended-load equation, that is:

$$C_s = \frac{KVS}{\omega} \tag{3.88}$$

Thus, the analytical derivation by Pacheco-Ceballos (1989) based on power balance shows that bedload, suspended-load, and total-load concentrations are all functions of unit stream power. It should be pointed out that K is not a constant. The K value given by Pacheco-Ceballos is:

$$K = \frac{\rho_m}{\Delta\rho D b_f}\left(\frac{aV_b}{V} + ea_s\right) \tag{3.89}$$

where ρ_m = density of water and sediment mixture,
$\Delta\rho$ = $(\rho_s - \rho)/\rho$,
a and a_s = thicknesses of bed layer and suspended layer, respectively,
e = dimensionless coefficient,
D = average depth of flow,
b_f = bed form shape factor, and
V_b = bottom velocity.

3.3.8 Gravitational Power Approach

Velikanov (1954) derived his transport function from the gravitational power theory. He divided the rate of energy dissipation for sediment transport into two parts. These are the power required to overcome flow resistance and the power required to keep sediment particles in suspension against the gravitational force. Velikanov's basic relationship can be expressed as:

$$\rho g(1 - C_{vy})V_yS = \rho V_y \frac{d\left[(1 - C_{vy})\overline{u_x u_y}\right]}{dy} + g(\rho_s - \rho)C_{vy}(1 - C_{vy})\omega \tag{3.90}$$

(I) (II) (III)

where C_{vy} = time-averaged sediment concentration at a distance y above the bed (in % by volume),
V_y = time averaged flow velocity at a distance y above the bed,
u_x and u_y = fluctuating parts of velocity in the x and y directions, respectively,
ρ_s and ρ = densities of sediment and water, respectively, and
g = gravitational acceleration.

Equation (3.90) has the following physical meaning:

(I) = effective power available per unit volume of flowing water,

(II) = rate of energy dissipation per unit volume of flow to overcome resistance, and

(III) = rate of energy dissipation per unit volume of flow to keep sediment particles in suspension.

Assuming that the sediment concentration is small, integration of equation (3.90) over the depth of flow, D, yields:

$$gVSD = \frac{f_0 V^3}{8} + \frac{\rho_s - \rho}{\rho} gD\omega C_v \tag{3.91}$$

where C_v = average sediment concentration by volume.

Equation (3.91) shows that sediment concentration by volume is a function of unit stream power.

The Darcy-Weisbach resistance coefficients with and without sediment can be expressed, respectively, as:

$$f = \frac{8gDS}{V^2} \text{ for } C_v \neq 0 \tag{3.92}$$

$$f_0 = \frac{8gDS_0}{V^2} \text{ for } C_v = 0 \tag{3.93}$$

where S and S_0 = energy slopes with and without sediment, respectively, and
C_v = time-averaged sediment concentration (in % by volume).

It can be shown that Velikanov's equation can be expressed in the following general form:

$$C_v = K\frac{V^3}{gD\omega} \tag{3.94}$$

where K = a coefficient to be determined from measured data.

Several Chinese researchers have used Velikanov's gravitational power theory as the theoretical basis for the derivation of sediment transport equations. For example, Dou (1974) suggested that the rate of energy dissipation used by flowing water to keep sediment particles in suspension should be equal to that used by sediment particles in suspension, and proposed the following equation:

$$C_t = K_2 \frac{V^3}{gD\omega} \tag{3.95}$$

where K_2 = a variable to be determined, and
C_t = total sediment concentration.

Zhang (1959) assumed that the rate of energy dissipation used in keeping sediment particles in suspension should come from turbulence instead of the effective power available from the flow. He also considered the damping effect and believed that the existence of suspended sediment particles could reduce the strength of turbulence. Zhang's equation for sediment transport is:

$$C_t = K_3 \left(\frac{V^3}{gR\omega} \right)^m \tag{3.96}$$

where K_3 and m = parameters related to sediment concentration, and
R = hydraulic radius.

Yang (1996) gave a detailed comparison of transport functions based on gravitational and unit stream power approaches.

3.4 Other Commonly Used Sediment Transport Functions

Engineers have used sediment transport functions, formulas, or equations obtained from different approaches described in section 3.3 for solving engineering and river morphological problems. In addition to those proposed by Bagnold (1966), Ackers and White (1973), Engelund and Hansen (1967), and by Yang (1973, 1979, 1984) described previously, other commonly used transport formulas are summarized herein. Yang (1996) has published more detailed descriptions of the commonly used formulas, their theoretical basis, and their limits of application. Stevens and Yang (1984) published computer programs for 13 commonly used sediment transport formulas for PC application. They are given in Yang's book (1996, 2003).

3.4.1 Schoklitsch Bedload Formula

Schoklitsch (1934) developed a bedload formula based mainly on Gilbert's (1914) flume data with median sediment sizes ranging from 0.3 to 5mm. The Schoklitsch formula for unigranular material is:

$$G_s = \frac{86.7}{\sqrt{D}} S^{3/2} (Q - Wq_0) \tag{3.97}$$

where:

$$q_0 = \frac{0.00532D}{S^{4/3}} \tag{3.98}$$

where G_s = the bedload discharge, in lb/s,
D = the mean grain diameter, in in.,
S = the energy gradient, in ft per ft,
Q = the water discharge in ft^3/s,
W = the width, in ft, and
q_0 = the critical discharge, in ft^3/s per ft of width.

The formula can be applied to mixtures by summing the computed bedload discharges for all size fractions. The discharge for each size fraction is computed using the mean diameter and the fraction of the sediment in the sized fraction. Converting the equation for use with mixtures and changing the grain diameter from inches to feet and the bedload discharge from pounds to pounds per foot of width gives:

$$g_s = \sum_{i=1}^{n} i_b \frac{25}{\sqrt{D_{si}}} S^{3/2} \left(q - q_0\right) \tag{3.99}$$

where:

$$q_0 = \frac{0.0638D_{si}}{S^{4/3}} \tag{3.100}$$

where g_s = the bedload discharge, in lb/s per ft of width,
i_b = the fraction, by weight, of bed material in a given size fraction,
D_{si} = the mean grain diameter, in ft, of sediment in size fraction *i,*
Q = the water discharge, in ft^3/s per ft of width,
q_0 = the critical discharge, in ft^3/s per ft of width, for sediment of diameter D_{si}, and,
n = the number of size fractions in the bed-material mixture.

3.4.2 Kalinske Bedload Formula

The formula developed by Kalinske (1947) for computing bedload discharge of unigranular material is based on the continuity equation, which states that the bedload discharge is equal to the product of the average velocity of the particles in motion, the weight of each particle, and the number of particles. The average particle velocity is related to the ratio of the critical shear to the total shear. The formula is:

$$g_s = \sum_{i=1}^{n} U_* \gamma_s D_{si} P_i 7.3 \left(\frac{\overline{U_g}}{\overline{U}} \right) \tag{3.101}$$

where:

$$U_* = \frac{\sqrt{\tau_0}}{\rho} \tag{3.102}$$

$$\frac{\bar{U}_g}{\bar{U}} = f\left(\frac{\tau_{ci}}{\tau_0}\right) \tag{3.103}$$

$$\tau_{ci} = 12D_{si} \tag{3.104}$$

$$P_i = \frac{0.35}{m}\left(\frac{i_b}{D_{si}}\right) \tag{3.105}$$

where g_s = the bedload discharge in lb/s per ft of width,
n = the number of size fractions in the bed-material mixture,
U_* = the shear velocity in ft/s,
g_s = the specific weight of the sediment in lb/ft^3,
D_{si} = the mean grain diameter in ft of sediment in size fraction i,
P_i = the proportion of the bed area occupied by the particles in size fraction i,
$\bar{U}_g$ = the average velocity, in ft/s, of particles in size fraction i,
$\bar{U}$ = the mean velocity of flow, in ft/s, at the grain level,
τ_0 = the total shear at the bed, in lb/ft^2, which equals 62.4dS,
d = the mean depth in ft,
S = the energy gradient in ft per ft,
ρ = the density of water in slugs per ft^3,
f = denotes function of,
τ_{ci} = the critical tractive force in lb/ft^2,
m = the summation of values of i_b/D_{si} for all size fractions in the bed-material mixture, and
i_b = the fraction, by weight, of bed material in a given size fraction.

Using the values of 165.36 for γ_s and 1.94 for ρ, the formula is:

$$g_s = 25.28\sqrt{\tau_0}\sum_{i=1}^{n}\tau_{ci}\frac{\frac{i_b}{D_{si}}}{m}\left(\frac{\bar{U}_g}{\bar{U}}\right) \tag{3.106}$$

Figure 3.17 shows values of τ_{ci}/τ_0.

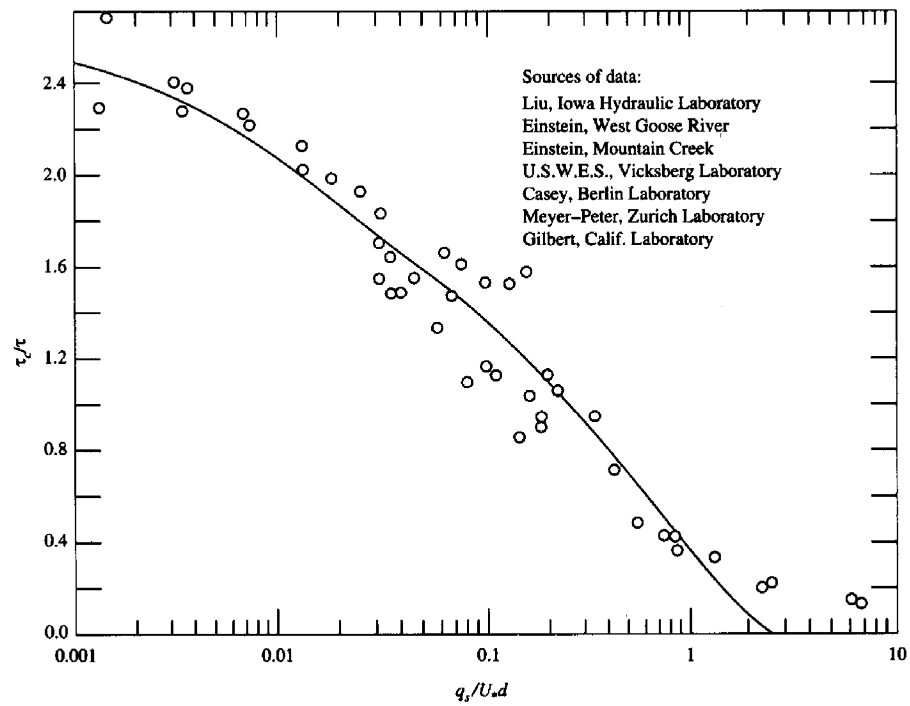


Figure 3.17. Kalinske's bed-load equation (Kalinske, 1947).

3.4.3 Meyer-Peter and Müller Formula

Meyer-Peter and Müller (1948) developed an empirical formula for the bedload discharge in natural streams. The original form of the formula in metric units for a rectangular channel is:

$$\gamma \frac{Q_s}{Q}\left(\frac{K_s}{K_r}\right)^{3/2} dS = 0.047\gamma_s' D_m + 0.25\left(\frac{\gamma}{g}\right)^{1/3} g_s^{2/3} \tag{3.107}$$

in which:

$$D_m = \sum_{i=1}^{n} D_{si} i_b \tag{3.108}$$

where γ = the specific weight of water and equals 1 t/m^3,

Q_s = that part of the water discharge apportioned to the bed in l/s,

Q = the total water discharge in L/s,

K_s = Strickler's coefficient of bed roughness, equal to 1 divided by Manning's roughness coefficient n_s,

K_r = the coefficient of particle roughness, equal to $26/D_{90}^{1/6}$,

D_{90} = the particle size, in m, for which 90% of the bed mixture is finer,

d = the mean depth in m,

S = the energy gradient in m per m,

γ_s = the specific weight of sediment underwater, equal to 1.65 t/m^3 for quartz,
D_m = the effective diameter of bed-material mixture in m,
g = the acceleration of gravity, equal to 9.815 m/s^2,
g_s = the bedload discharge measured underwater in t/s per m of width,
n = the number of size fractions in the bed material,
D_{si} = the mean grain diameter, in m, of the sediment in size fraction i, and
i_b = the fraction, by weight, of bed material in a given size fraction.

Converting the formula to English units gives:

$$g_s = \left[0.368 \frac{Q_s}{Q} \left(\frac{D_{90}^{1/6}}{n_s} \right)^{3/2} dS - 0.0698 D_m \right]^{3/2} \quad (3.109)$$

where g_s = the bedload discharge for dry weight, in lb/s per ft of width,
Q, Q_s = sediment and water discharges, respectively, in ft^3/s,
D_{90}, D_m = sediment particle diameter at which 90% of the material, by weight, is finer and mean particle diameter, respectively,
d = water depth in ft, and
n_s = Manning's roughness value for the bed of the stream.

3.4.4 Rottner Bedload Formula

Rottner (1959) developed an equation to express bedload discharge in terms of the flow parameters based on dimensional considerations and empirical coefficients. Rottner applied a regression analysis to determine the effect of a relative roughness parameter D_{90}/d. Rottner's equation is dimensionally homogenous, so that it can be presented directly in English units:

$$g_s = \gamma_s \left[\left(S_g - 1 \right) g d^3 \right]^{1/2} \left\{ \frac{V}{\sqrt{\left(S_g - 1 \right) g d}} \left[0.667 \left(\frac{D_{50}}{d} \right)^{2/3} - 0.14 \right] - 0.778 \left(\frac{D_{50}}{d} \right) \right\} \quad (3.110)$$

where g_s = the bedload discharge in lb/ft of width,
γ_s = the specific weight of sediment in lb/ft^3,
S_g = the specific gravity of the sediment,
g = the acceleration of gravity in ft/s^2,
d = the mean depth in ft,
V = the mean velocity in ft/s, and
D_{50} = the particle size, in ft, at which 50% of the bed material by weight is finer.

In this derivation, wall and bed form effects were excluded. Rottner stated that his equation may not be applicable when small quantities of bed material are being moved.

3.4.5 Einstein Bedload Formula

The bedload function developed by Einstein (1950) is derived from the concept of probabilities of particle motion. Due to the complexity of the bedload function, a description of the procedure will not be presented here. Interested readers should refer to Einstein's original paper or the summary published by Yang (1996).

3.4.6 Laursen Bed-Material Load Formula

The equation developed by Laursen (1958) to compute the mean concentration of bed-material discharge is based on empirical relations:

$$\bar{C} = \sum_{i=1}^{n} i_b \left(\frac{D_{si}}{d}\right)^{7/6} \left(\frac{\tau_o}{\tau_c} - 1\right) f\left(\frac{U_*}{\omega_i}\right) \quad (3.111)$$

where:

$$\tau_0' = \frac{\rho V^2}{58}\left(\frac{D_{50}}{d}\right)^{1/3} \quad (3.112)$$

$$\tau_c = Y_c \rho g (S_g - 1) D_{si} \quad (3.113)$$

where $\bar{C}$ = the concentration of bed-material discharge in % by weight,
n = the number of size fractions in the bed material,
i_b = the fraction, by weight, of bed material in a given size fraction,
D_{si} = the mean grain diameter, in ft, of the sediment in size fraction i,
d = the mean depth in ft,
τ_0' = Laursen's bed shear stress due to grain resistance,
τ_c = critical shear stress for particles of a size fraction,
f = denotes function of,
U_* = the shear velocity in ft/s,
ω_i = the fall velocity, in ft/s, of sediment particles of diameter D_{si},
ρ = the density of water in slugs per ft^3,
V = the mean velocity in ft/s,
D_{50} = the particle size, in ft, at which 50% of the bed material, by weight, is finer,
Y_c = a coefficient relating critical tractive force to sediment size,
g = acceleration of gravity in ft/s^2, and
S_g = the specific gravity of sediment.

The density ρ has been introduced into the original τ'_0 equation presented by Laursen so that the equation is dimensionally homogeneous, and Laursen's coefficient has been changed accordingly. Substituting for τ'_0 and τ_c in equation (3.111) and converting $\bar{C}$ to C gives:

$$C = 10^4 \sum_{i=1}^{n} i_b \left(\frac{D_{si}}{d} \right)^{7/6} \left[\frac{V^2}{58 Y_c D_{si} (S_g - 1) g d} \left(\frac{D_{50}}{d} \right)^{1/3} - 1 \right] f\left(\frac{U_*}{\omega_i} \right) \tag{3.114}$$

where C = the concentration of bed-material discharge, in parts per million by weight.

Figure 3.18 shows values of $f(U_* / \omega_i)$.

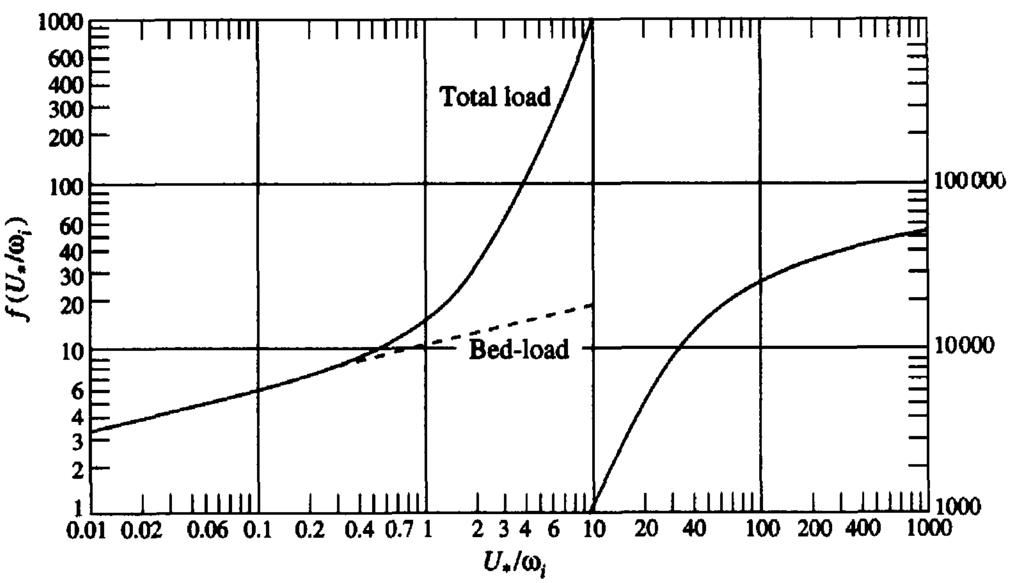


Figure 3.18 Function $f(U_*/\omega_i)$ in Laursen's approach (Laursen, 1958).

3.4.7 Colby Bed-Material Load Formula

Colby (1964) presented a graphical method to determine the discharge of sand-size bed material that ranged from 0.1 to 0.8 mm. The bed-material discharge g_s, in lb/s/ft of width, at a water temperature of 15.6 degrees Celsius (°C) (Colby's 1964 fig. 6) is:

$$g_s = A(V - V_c)^B 0.672 \tag{3.115}$$

where:

$$V_c = 0.4673 d^{0.1} D_{50}^{0.33} \tag{3.116}$$

where V = the mean velocity in ft/s,
V_c = the critical velocity in ft/s,
D = the mean depth in ft,
d_{50} = the practical size, in mm, at which 50% of the bed material by weight is finer,
A = a coefficient, and
B = an exponent.

Colby developed his graphical solutions for total load mainly from laboratory and field data using Einstein's (1950) bedload function as a guide. His graphical solutions are shown in Figures 3.19 and 3.20. The required information in Colby's approach comprises the mean flow velocity V, average depth D, median particle diameter d_{50}, water temperature T, and fine sediment concentration C_f. The total load can be computed by the following procedure:

Step 1: with the given V and d_{50}, determine the uncorrected sediment discharge q_{ti} for the two depths shown in Figure 3.19 that are larger and smaller than the given depth D, respectively.

Step 2: interpolate the correct sediment discharge q_{ti} for the given depth D on a logarithmic scale of depth versus q_{ti}.

Step 3: with the given depth D, median particle size d_{50}, temperature T, and fine sediment concentration C_f, determine the correction factors k_1, k_2, and k_3 from Figure 3.20.

Step 4: the total sediment discharge (in ton/day/ft of channel width), corrected for the effect of water temperature, fine suspended sediment, and sediment size, is:

$$q_t = [1 + (k_1 k_2 - 1)0.01 k_3] q_{ti} \tag{3.117}$$

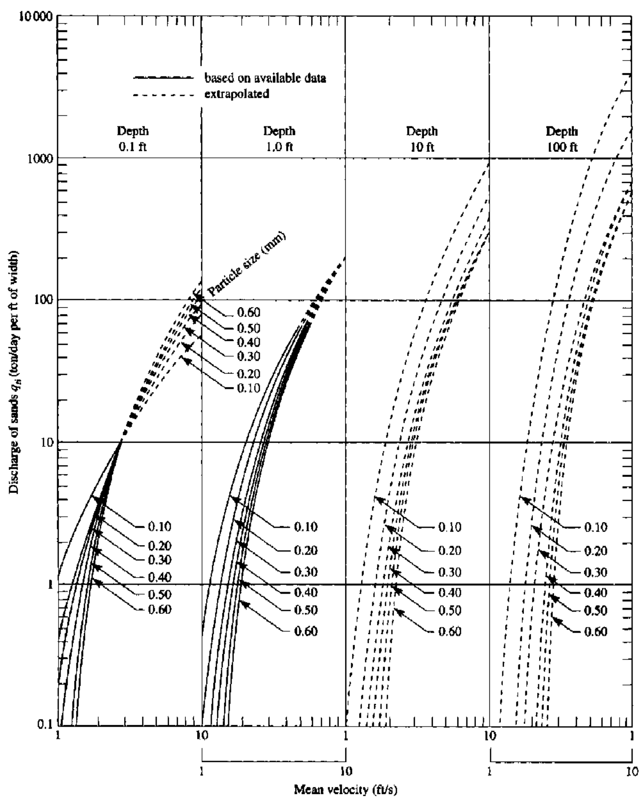


Figure 3.19 Relationship of discharge of sands to mean velocity for six median sizes of bed sands, four depths of flow, and a water temperature of 60 °F (Colby, 1964).

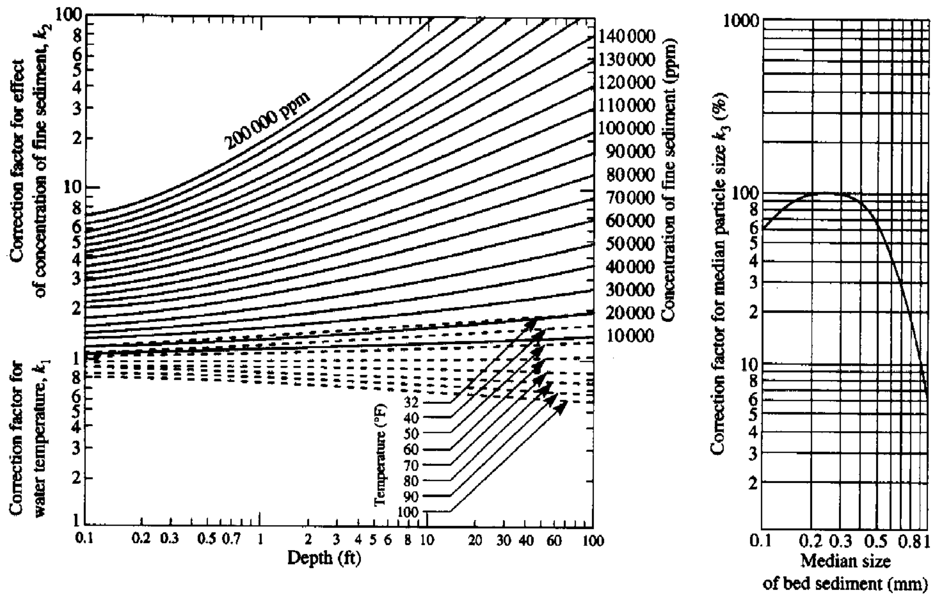


Figure 3.20. Approximate effect of water temperature and concentration of fine sediment on the relationship of discharge of sands to mean velocity (Colby, 1964).

From Figure 3.20, $k_1 = 1$ for $T = 60$ °F, $k_2 = 1$ where the effect of fine sediment can be neglected, and $k_3 = 100$ when the median particle size is in the range of 0.2 to 0.3 mm. Because of the range of data used in the determination of the rating curves shown in Figures 3.19 and 3.20, Colby's approach should not be applied to rivers with median sediment diameter greater than 0.6 mm and depth greater than 3 m.

3.4.8 Einstein Bed-Material Load Formula

Einstein (1950) presented a method to combine his computed bedload discharges with a computed suspended bed-material discharge to yield the total bed-material discharge. A complete description of the complex procedure will not be presented here. Interested readers should follow the original Einstein paper or the summary made by Yang (1996) to apply the Einstein bed-material formula.

3.4.9 Toffaleti Formula

The procedure to determine bed-material discharge developed by Toffaleti (1968) is based on the concepts of Einstein (1950) with three modifications:

1. Velocity distribution in the vertical is obtained from an expression different from that used by Einstein;

2. Several of Einstein's correction factors are adjusted and combined; and

3. The height of the zone of bedload transport is changed from Einstein's two grain diameters.

Toffaleti defines his bed-material discharge as total river sand discharge, even though he defines the range of bed-size material from 0.062 to 16 mm. The complex procedures in the Toffaleti formula will not be presented here. Interested readers should follow the original Toffaleti procedures or the summary by Yang (1996) to apply the procedures.

3.5 Fall Velocity

Sediment particle fall velocity is one of the important parameters used in most sediment transport functions or formulas. Depending on the sediment transport functions used and sediment particle size in a particular study, different methods have been developed for the computation of sediment particle fall velocity. Some of the commonly used methods for fall velocity computation are summarized herein.

When Toffaleti's equation is used, Rubey's (1933) formula should be employed; that is:

$$\omega_s = F\sqrt{dg(G - 1)} \tag{3.118}$$

where

$$F = \left[\frac{2}{3} + \frac{36\nu^2}{gd^3(G-1)}\right]^{1/2} - \left[\frac{36\nu^2}{gd^3(G-1)}\right]^{1/2} \tag{3.119}$$

for particles with diameter, d, between 0.0625 mm and 1 mm, and where F = 0.79 for particles greater than 1 mm. In the above equations, ω_s = fall velocity of sediments; g = acceleration due to gravity; G = specific gravity of sediment = 2.65; and ν = kinematic viscosity of water. The viscosity of water is computed from the water temperature, T, using the following expression:

$$\nu = \frac{1.792 \times 10^{-6}}{1.0 + 0.0337T + 0.000221T^2} \tag{3.120}$$

with T in degrees Centigrade and ν in m^2/s.

When any of the other sediment transport formulas are used, the values recommended by the U.S. Interagency Committee on Water Resources Subcommittee on Sedimentation (1957) are used (Figure 3.21). Yang and Simões (2002) use a value for the Corey shape factor of SF = 0.7, for natural sand in their computer model GSTARS3, where:

$$SF = \frac{c}{\sqrt{ab}} \tag{3.121}$$

where a, b, and c = the length of the longest, the intermediate, and the shortest mutually perpendicular axes of the particle, respectively.

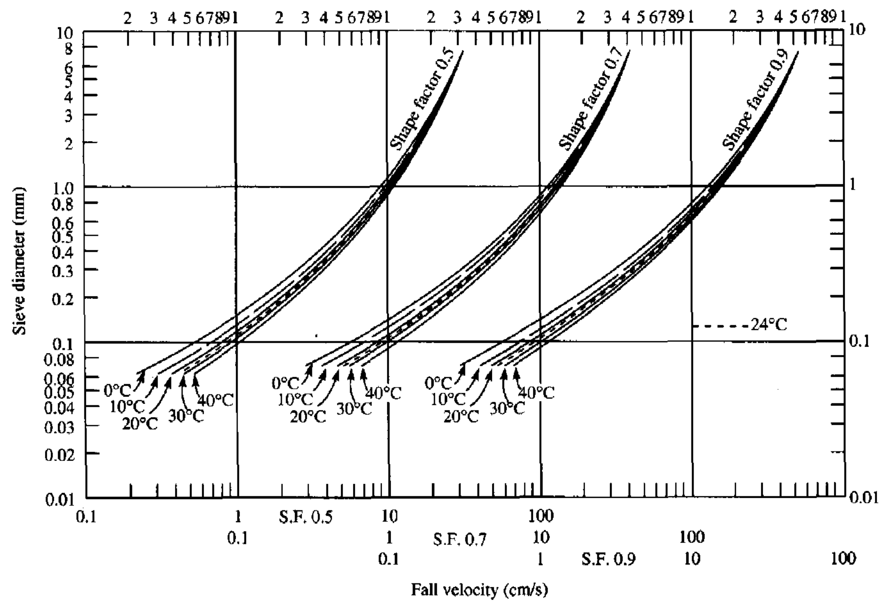


Figure 3.21.—Relation between particle sieve diameter and its fall velocity according to the U.S. Interagency Committee on Water Resources Subcommittee on Sedimentation (1957).

Yang and Simões (2002) also used the following approximations for the computation of fall velocities. For particles with diameters greater than 10 mm, which are above the range given in Figure 3.21, the following formula is used:

$$\omega_s = 1.1\sqrt{(G-1)gd} \tag{3.122}$$

For particles in the silt and clay size ranges, namely with diameters between 1 and 62.5 μm, the sediment fall velocities are computed from the following equations:

unhindered settling:

$$\omega_s = \frac{(G-1)gd^2}{18\nu} \text{ for } C < C_1 \tag{3.123}$$

flocculation range:

$$\omega_s = MC^N \text{ for } C_1 \le C \le C_2 \tag{3.124}$$

hindered settling:

$$\omega_s = \omega_0 (1 - kC)^l \text{ for } C > C_2 \tag{3.125}$$

where ω_0 is found by equations (3.124) and (3.125) at $C = C_2$, that is:

$$\omega_0 = \frac{MC_2^N}{(1 - kC_2)^l} \qquad (3.126)$$

and k, l, M, and N are site-specific constants supplied by the user; figure 3.22 shows fall velocities in flocculation range for different natural conditions. The expression $\omega_s = 1.0C^{1.0}$ represents the average values with ω_s in mm/s and C in kg/m^3.

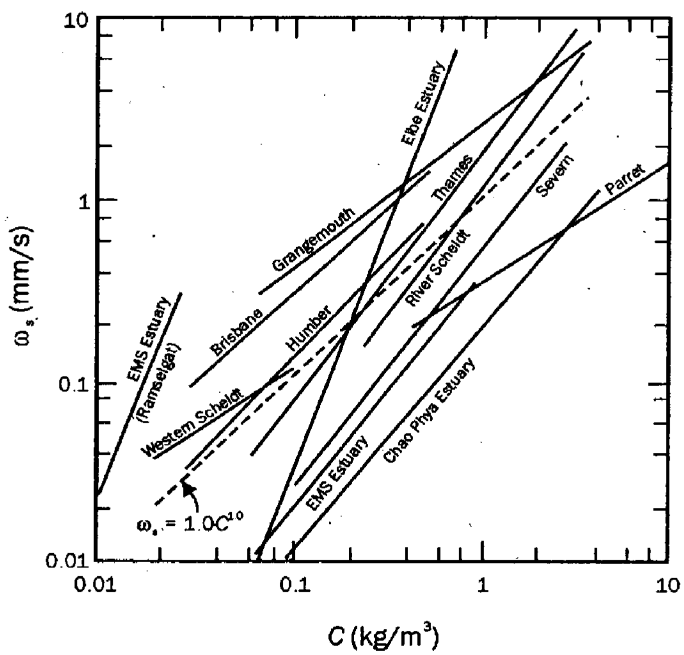


Figure 3.22. Variability of the parameters M and N of eq. (3.124) for several well known rivers and estuaries (Yang and Simões, 2002).

3.6 Resistance to Flow

For a steady, uniform, open channel flow of constant width W without sediment, the water depth D and velocity V can be determined for a given discharge Q and channel slope S by using the continuity equation:

$$Q = WDV \qquad (3.127)$$

and a friction equation, such as the Darcy-Weisback formula:

$$V = \sqrt{\frac{8gRS}{f}} \qquad (3.128)$$

where V = average flow velocity,
g = gravitational acceleration,

R = hydraulic radius,
S = water surface or energy slope, and
f = Darcy-Weisbach friction factor.

For fluid hydraulics with sediment transport, the total roughness for resistance to flow consists of two parts. If equation (3.128) is used:

$$f = f' + f'' \tag{3.129}$$

where f' = Darcy-Weisbach friction factor due to grain roughness, and
f'' = Darcy-Weisbach friction factor due to form roughness on the existence of bed forms.

Figure 3.23 is based on the data collected by Guy et al. (1966) in a laboratory flume with 0.19-mm sand. Figure 3.23 shows that f' is a constant, but the f'' value depends on the bed form, such as plane bed, ripple, dune, transition, antidune, and chute-pool. Although empirical methods exist for the determinations of bed forms, no consistent result can be obtained from empirical methods. Consequently, the Darcy-Weisbach friction factor f or the Manning's coefficient n cannot be assumed as a given constant in an alluvial channel with sediment. Assume that sediment concentration can be determined by the following function:

$$C_t = \Phi\,(V, S, D, d, v, \omega) \tag{3.130}$$

where C_t = total sediment concentration, in parts per million by weight,
d = median sieve diameter of bed material,
v = kinematic viscosity of water, and
ω = terminal fall velocity of sediment.

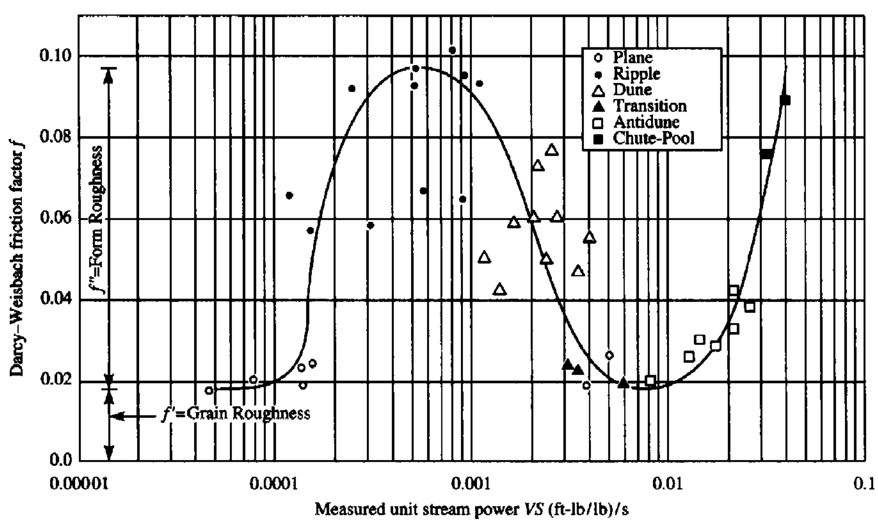


Figure 3.23 Variation of friction factor with bed form and measured unit stream power (Yang, 1996).

Because the f value of an alluvial channel cannot be predicted with confidence, we have equations (3.127) and (3.130) with three unknown, namely V, D, and S. Thus, fluvial hydraulics is still basically indeterminate despite the significant progress made in the past decades.

Due to the site-specific nature of empirical methods, they will not be introduced here. The following sections will introduce only analytical methods for the determination of resistance to flow or the roughness coefficient, or the determination of V, S, D without prior knowledge of the roughness coefficient.

3.6.1 Einstein's Method

Einstein (1950) expressed the resistance due to grain roughness by:

$$\frac{V}{U'_*} = 5.75 \log\left(12.27\frac{R'}{k_s}x\right) \tag{3.131}$$

where
- U'_* = shear velocity due to skin friction or grain roughness = $(gR'S)^{1/2}$,
- R' = hydraulic radius due to skin friction,
- k_s = equivalent grain roughness = d_{65},
- x = a function of k_s/δ, and
- δ = boundary layer thickness, which can be expressed as:

$$\delta = \frac{11.6\nu}{U'_*} \tag{3.132}$$

where ν = kinematic viscosity.

Figure 3.24 shows the relationship between x and k_s/δ suggested by Einstein (1950). With the given values of V, d_{65}, and x determined from Figure 3.24, equation (3.131) can be used to compute the value of R'. Einstein (1950) suggested that:

$$\frac{V}{U''_*} = \phi(\psi') \tag{3.133}$$

where

$$\psi' = \frac{\gamma_s - \gamma}{\gamma}\frac{d_{35}}{SR'} \tag{3.134}$$

The functional relationship between V/U''_* and ψ' was determined from field data by Einstein and Barbarossa (1952) as shown in Figure 3.25.

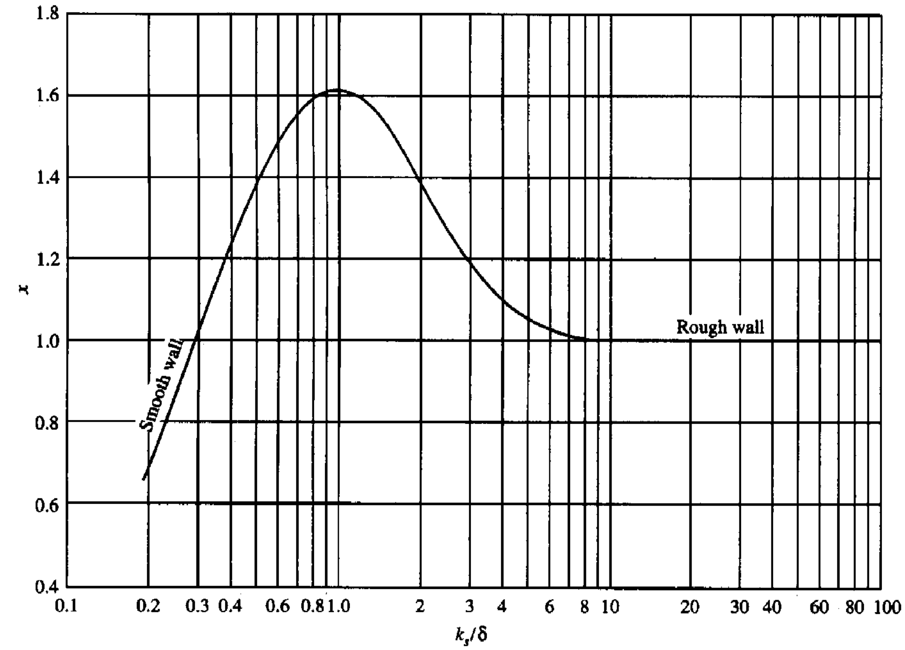


Figure 3.24. Correction factor in the logarithmic velocity distribution (Einstein, 1950).

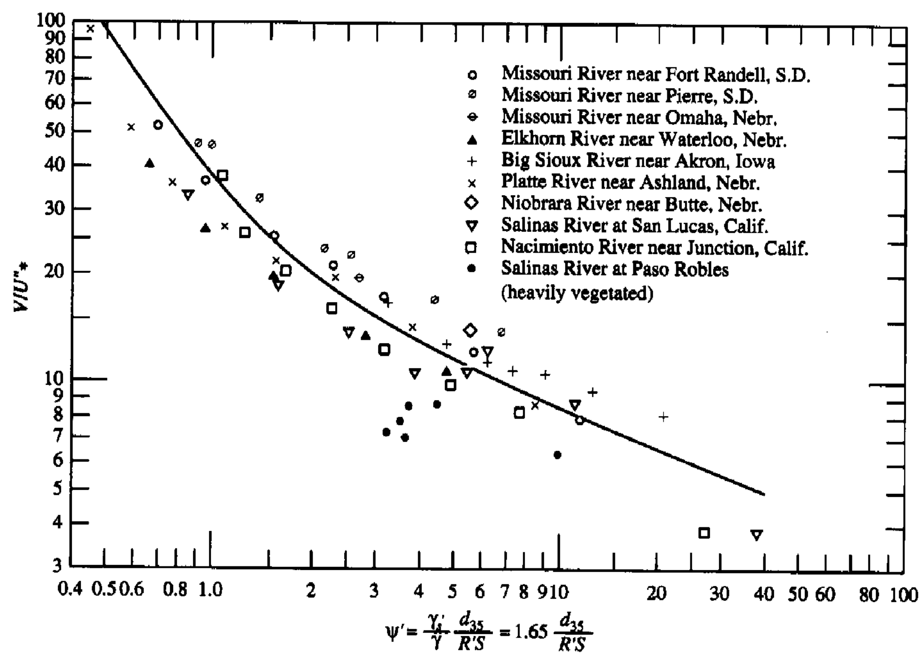


Figure 3.25. Friction loss due to channel irregularities as a function of sediment transport rate (Einstein and Barbarossa, 1952).

Einstein and Barbarossa (1952) suggested the following procedures for the computation of total hydraulic radius due to grain and form roughness when the water discharge is given, or vice versa.

Case A. Determine R with given Q

Step 1: Assume a value of R'.

Step 2: Apply equation (3.131) and Figure 3.24 to determine V.

Step 3. Compute ψ' using equation (3.134) and the corresponding value of V/U''_* from Figure 3.25.

Step 4: Compute U'_* and the corresponding value of R''.

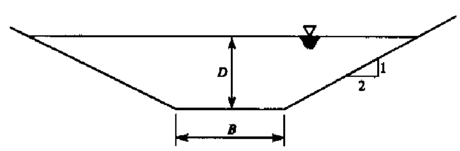
Step 5: Compute $R = R' + R''$ and the corresponding channel cross-sectional area A.

Step 6: Verify using the continuity equation $Q = VA$. If the computed Q agrees with the given Q, the problem is solved. Otherwise, assume another value of R' and repeat the procedure until agreement is reached between the computed and the given Q.

Case B. Determine Q with given R. The five first steps are identical to those for case A. After the R value has been computed, it is compared with the given value of R. If these values agree, the problem is solved, and $Q = VA$. If not, the computation procedures will be repeated by assuming different values of R' until the computed R agrees with the given R. Yang (1996) gave the following examples, using Einstein's method.

Example 3.1 Given the following data, determine the flow depth D for the channel shown using the Einstein procedures:

$Q = 40$ m^3/s, $B = 5$ m
$\nu = 10^{-6}$ m^2/s, $S = 0.0008$
Specific gravity of sand = 2.65
$d_{35} = 0.3$ mm, $d_{65} = 0.9$ mm



Solution:
(a) Assume R'.
(b) Determine velocity from equation (3.12):

$$V = 5.75 U'_* \log\left(12.27 \frac{R'}{k_s} x\right)$$

The equivalent sand roughness k_s may be taken as equal to $d_{65} = 0.0009$ m, and shear velocity U'_* is

$$U'_* = (gR'S)^{1/2} = 0.089(R')^{1/2}$$

The correction factor x is a function of k_s/δ, and may be read from Figure 3.24. The laminar sublayer thickness δ can be estimated from equation (3.132); that is,

$$\delta = 11.6\frac{v}{U'_*} = \frac{11.6(10^{-6})}{0.089(R')^{1/2}} = \frac{1.31\times10^{-4}}{(R')^{1/2}}$$

so

$$\frac{k_s}{\delta} = \frac{0.0009(R')^{1/2}}{1.31\times10^{-4}} = 6.87(R')^{1/2}$$

Substituting for U'_* and k_s, the velocity can be estimated from:

$$V = 0.509(R')^{1/2}\log(13.633R')^{1/2}$$

(c) Compute ψ' from equation (3.134),

$$\psi' = (2.65-1)\frac{d_{35}}{SR'} = 1.65\frac{0.0003}{0.0008R'} = \frac{0.619}{R}$$

and determine V/U'_* from Figure 3.25.

(d) Compute U'_* and R'' from:

$$U''_* = \left(\frac{V}{U''_*}\right)^{-1} V$$

$$R'' = \frac{(U''_*)^2}{gS} = \frac{(U''_*)^2}{0.0078}$$

(e) Determine $R = R' + R''$ and the corresponding depth D and area A.

(f) Determine $Q = AV$, and reiterate if necessary.

The determination of depth and area from the hydraulic radius may be facilitated by developing curves relating these variables. The relations may be expressed as:

$$A = 5D + 2D^2$$

$$R = \frac{5D + 2D^2}{5 + 4.47D}$$

Assuming values of *D*, the relationship between *D*, *A*, and *R* can be computed from the above two equations as follows:

D	A	R
0.6	3.72	0.484
0.8	5.28	0.616
1.0	7.00	0.737
1.2	8.88	0.857
1.5	12.00	1.025
2.0	18.00	1.290

The following is a tabulation of the solution procedure:

R' (m)	$\frac{k_s}{\delta}$	x	V (m/s)	ψ'	$\frac{V}{U'_*}$	U'_* (m/s)	R'' (m)	R (m)	A (m^2)	Q (m^3/s)
0.50	4.86	1.06	1.39	1.24	31	0.045	0.26	0.76	7.0	9.7
0.20	3.07	1.18	0.798	3.10	15	0.053	0.36	0.56	4.5	3.6
1.00	6.87	1.02	2.11	0.619	75	0.028	0.10	1.10	14.0	29.5
1.20	7.53	1.01	2.35	0.516	97	0.024	0.08	1.28	18.0	42.3
1.15	7.37	1.01	2.29	0.538	90	0.025	0.08	1.23	16.5	37.8
1.17	7.43	1.01	2.32	0.529	93	0.025	0.08	1.25	17.0	39.4
1.18	7.46	1.01	2.33	0.525	94	0.025	0.08	1.26	17.5	40.8

For $Q = 40$ m^3/s, $R = 1.254$ m
The corresponding water depth is $D = 1.93$ m.

Example 3.2 Use the fluid and sediment properties given in example 3.1 and the flow depth determined there; compute the water discharge using the Einstein procedure.

Solution: Use the same procedure as outlined for example 3.1, but reiterate until the computed *R* agrees with the actual *R*; then determine the discharge $Q = AV$.

The following is a tabulation of the solution procedure:

R' (m)	$\frac{k_s}{\delta}$	x	V (m/s)	ψ'	$\frac{V}{U'_*}$	U'_* (m/s)	R'' (m)	R (m)
1.17	7.43	1.01	2.32	0.529	93	0.025	0.08	1.25
1.18	7.46	1.01	2.33	0.525	94	0.025	0.08	1.26

For $R = 1.254$ m, $V = 93.4 \times 0.025 = 2.335$ m/s
Channel cross-sectional area: $A = 5(1.93) + 2(1.93)^2 = 17.10$ m^2
Discharge: $Q = 17.10\ (2.335) = 39.9$ m^3/s ≈ 40 m^3/s

3.6.2 Engelund and Hansen's Method

Engelund and Hansen (1966) expressed the energy loss or frictional slope due to bed form as:

$$S'' = \frac{\Delta H''}{L} = \frac{q^2}{2gL}\left(\frac{1}{D - \frac{1}{2}A_m} - \frac{1}{D + \frac{1}{2}A_m}\right) = \frac{V^2}{2gL}\left(\frac{A_m}{D}\right)^2 \tag{3.135}$$

where $\Delta H''$ = frictional loss due to bed forms of wave length L,
q = flow discharge per unit width,
D = mean depth, and
A_m = amplitude of sand waves.

The total shear stress can also be expressed as:

$$\tau = \gamma R (S' + S'') \tag{3.136}$$

or

$$\frac{\tau}{\gamma R} = \frac{\tau'}{\gamma R} + S'' \tag{3.137}$$

Substituting equation (3.135) for S'' into equation (3.137) and assuming $R = D$ for a wide open channel,

$$\frac{\tau}{\gamma D} = \frac{\tau'}{\gamma D} + \frac{V^2}{2gL}\left(\frac{A_m}{D}\right)^2 \tag{3.138}$$

Let

$$\theta = \frac{DS}{[(\rho_s/\rho) - 1]d} \tag{3.139}$$

$$\theta' = \frac{D'S}{[(\rho_s/\rho) - 1]d} \tag{3.140}$$

and

$$\theta'' = \frac{1}{2}F_r^2 \frac{A_m^2}{[(\rho_s/\rho) - 1]dL} \tag{3.141}$$

where ρ_s and ρ = densities of sediment and water, respectively,
D and D' = water depth and corresponding depth due to grain roughness, respectively,
d = sediment particle size, and
Fr = Froude number = $V/(gD)^{1/2}$

From equations (3.139), (3.140), and (3.141):

$$\theta = \theta' + \theta'' \tag{3.142}$$

This relation was proposed by Engelund and Hansen (1967). For narrow channels, D and D' should be replaced by R and R' in equations (3.138) to (3.140). Figure 3.26 shows the relationship between θ and θ' for different bed forms. For the upper flow region, it can be assumed that form drag is not associated with the flow and $\theta = \theta'$. Figure 3.26 can be applied to the determination of a stage-discharge relationship by the following procedures:

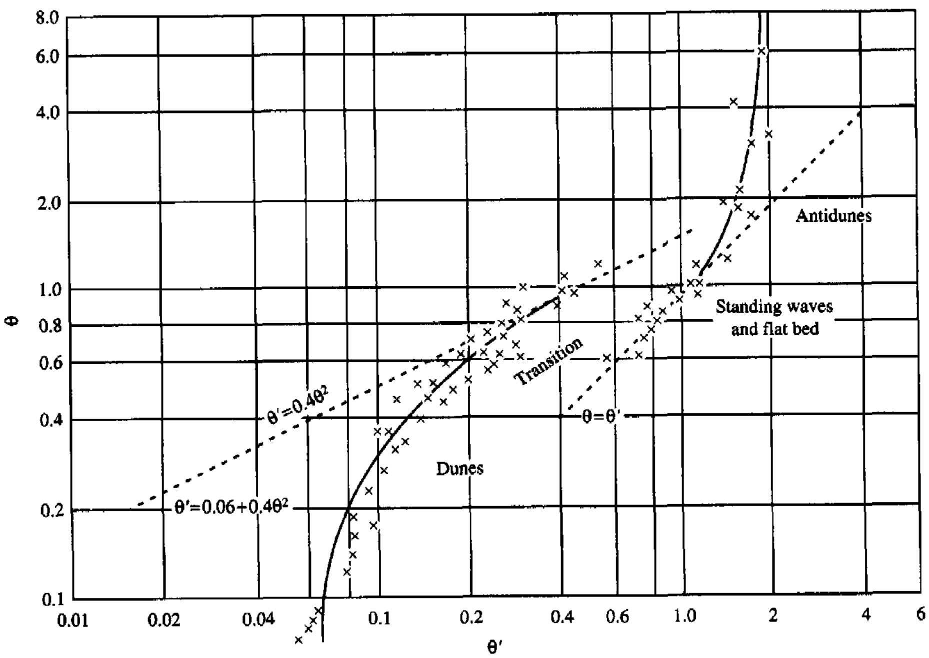


Figure 3.26. Flow resistance relationship (Engelund and Hansen, 1967).

Step 1: Determine S and D from a field survey of slope and channel cross-section.

Step 2: Compute θ from equation (3.139) for the given sediment size d.

Step 3: Determine θ' from Figure (3.26) with θ from step 2.

Step 4: Compute D' from equation (3.140).

Step 5: Compute V from equation (3.141).

Step 6: Determine the channel cross-sectional area A corresponding to the D value selected in step 1.

Step 7: Compute $Q = AV$. The stage-discharge relationship can be determined by selecting different D values and repeating the process.

Yang (1996) gave the following example using Engelund and Hansen's method.

Example 3.3 For the fluid and sediment properties and channel cross-section given in example 3.1, obtain the stage-discharge relationship using the procedure proposed by Engelund and Hansen.

Solution:

(a) Assume a depth of flow D.

(b) Compute θ for given R, S, and d from equation (3.139).

$$\theta = \frac{RS}{(\rho_s / \rho - 1)d}$$

For this analysis, the slope will be assumed equal to S_o (uniform flow), and the sediment size d will be assumed equal to:

$$d = ½(d_{35} + d_{65}) = ½(0.3 + 0.9) = 0.6 \text{ mm}$$

The hydraulic radius R may be determined from the assumed depth as:

$$R = \frac{5D + 2D^2}{5 + 2D\sqrt{5}}$$

Substituting:

$$\theta = \frac{0.0008(5D + 2D^2)}{1.65(0.0006)(5 + 2D\sqrt{5})} = 0.808\frac{5D + 2D^2}{5 + 2D\sqrt{5}}$$

(c) Determine θ' from Figure 3.26.

(d) Compute R' from:

$$R' = \frac{\theta'(\rho_s / \rho - 1)d}{S} = \frac{\theta'(1.65)(0.0006)}{0.0008} = 1.24\theta'$$

(e) Compute the velocity V from equation (3.131).

$$V = 5.75U'_* \log\left(12.27\frac{R'}{k_s}x\right)$$

The shear velocity $U'_* = (gR'S)^{1/2} = [9.81(0.0008)R']^{1/2} = 0.089(R')^{1/2}$. The equivalent sand roughness k_s may be taken as equal to $d_{65} = 0.9$ mm, and the correction factor x may be determined from Figure 3.24. A necessary parameter for the use of Figure 3.24 is k_s/δ, which can be computed from equation (3.132)

$$\frac{k_s}{\delta} = \frac{k_s U'_*}{11.6\nu} = \frac{0.0009(0.089)(R')^{1/2}}{11.6(10^{-6})} = 6.87(R')^{1/2}$$

(f) Compute the cross-sectional area A from

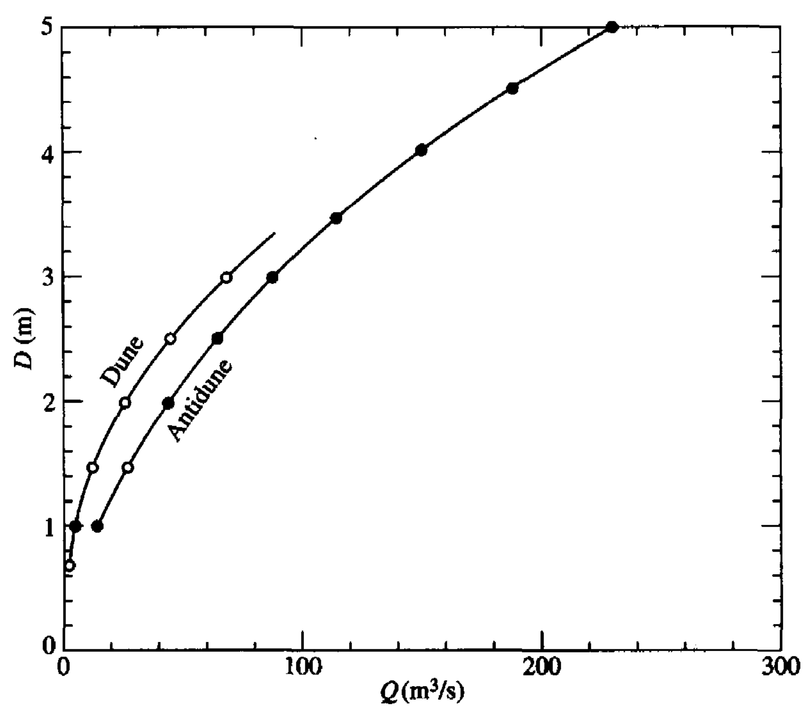
$$A = 5D + 2D^2$$

(g) Determine the discharge Q by continuity as

$$Q = AV$$

This procedure can be repeated for various values of D. Computations are shown in the table below.

The stage-discharge relationship for example 3.3 is shown below:



D (m)	R (m)	θ	θ'	R' (m)	k_s/δ	x	U' (m/s)	V (m/s)	Q (m^3/s)
0.5	0.415	0.335	0.12	0.15	2.7	1.22	0.034	0.663	2.0
1.0	0.739	0.597	0.18	0.22	3.2	1.05	0.042	0.845	5.9
			(0.59)	(0.73)	(5.9)	(1.02)	(0.076)	(1.76)	(12.3)
1.5	1.02	0.828	0.28	0.35	4.0	1.10	0.052	1.10	13.2
			(0.80)	(0.99)	(6.8)	(1.01)	(0.088)	(2.09)	(25.1)
2.0	1.29	1.04	0.50	0.62	5.4	1.02	0.070	1.58	28.4
			(1.0)	(1.24)	(7.7)	(1.00)	(0.099)	(2.41)	(43.4)
2.5	1.55	1.25	0.66	0.82	6.2	1.00	0.080	1.86	46.5
			(1.2)	(1.49)	(8.4)	(1.00)	(0.108)	(2.68)	(67.0)
3.0	1.79	1.45	0.87	1.08	7.1	1.00	0.092	2.20	72.6
			(1.25)	(1.55)	(8.6)	(1.00)	(0.110)	(2.74)	(90.4)
3.5	2.03	1.64	(1.3)	(1.61)	8.7	1.00	0.112	2.80	118
4.0	2.27	1.84	(1.37)	(1.70)	9.0	1.00	0.116	2.91	151
4.5	2.51	2.03	(1.43)	(1.77)	9.1	1.00	0.118	2.97	187
5.0	2.74	2.21	(1.5)	(1.86)	9.4	1.00	0.121	3.06	230

Values in parenthesis are for the upper flow regime or antidune.

3.6.3 Yang's Method

The theory of minimum rate of energy dissipation (Yang, 1976; Yang and Song, 1979, 1984) states that when a dynamic system reaches its equilibrium condition, its rate of energy dissipation is a minimum. The minimum value depends on the constraints applied to the system. For a uniform flow of a given channel width, where the rate of energy dissipation due to sediment transport can be neglected, the rate of energy dissipation per unit weight of water is:

$$\frac{dY}{dt} = \frac{dx}{dt}\frac{dY}{dx} = VS = \text{unit stream power} \tag{3.143}$$

where Y = potential energy per unit weight of water.

Thus, the theory of minimum unit stream power requires that:

$$VS = V_m S_m = \text{a minimum} \tag{3.144}$$

subject to the given constraints of carrying a given amount of water discharge Q and sediment concentration C_t of a given size d. The subscript m denotes the value obtained with minimum unit stream power. Utilization of equation (3.144) in conjunction with equations (3.127) and (3.130) can give a solution for the three unknowns, V, D, and S, without any knowledge of the total roughness. The procedures by Yang (1973) for the determination of Manning's coefficient based on his dimensionless unit stream power formula (Yang, 1973) are as follows.

Step 1: Assume a value of the depth D.

Step 2: For the values of Q, C_{ts}, W, d, ω, and v, solve equations (3.127) and (3.70) for V and S.

Step 3: Compute the unit stream power as the product of V and S.

Step 4: Select another D and repeat the steps.

Step 5: Compare all the computed VS values and select the one with minimum value as the solution in accordance with equation (3.144).

Step 6: Once VS has been determined, the corresponding values of V, S, and D can be computed from equations (3.127) and (3.70). Manning's coefficient can be computed from Manning's formula without any knowledge of the bed form.

Figure 3.27 shows an example of the relationship between generated unit stream power $V_i S_i$ and water depth D_i. The minimum unit stream power $V_m S_m$ determined is in close agreement with the measured unit stream power VS. Figure 3.28 shows examples of comparisons between measured and computed results from the above procedure. The subscript m in Figure 3.27 denotes the value obtained using equation (3.144). In the above procedures, it is assumed that sediment transport equations used are accurate in predicting the total bed-material concentration. If the measured concentration is significantly different from the computed one, the agreement may not be as good as those shown in Figure 3.27.

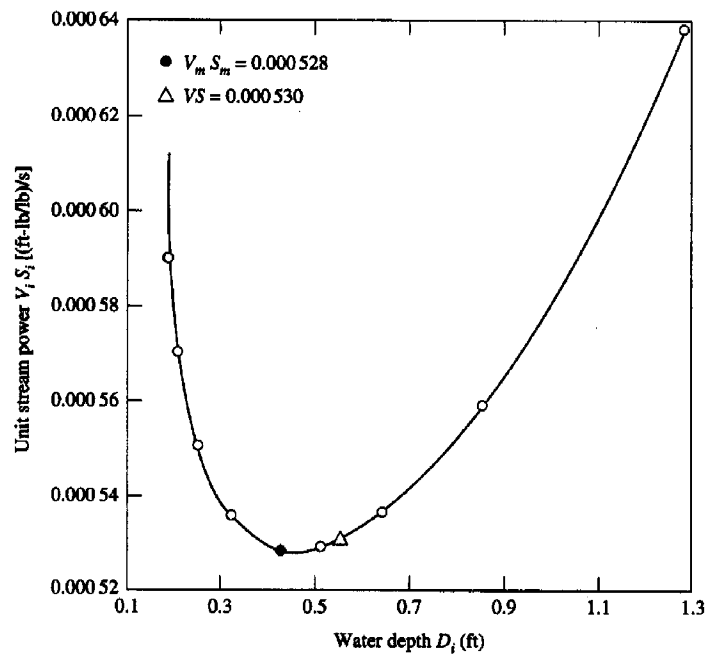


Figure 3.27. Relationship between unit stream power and water depth with 0.19-mm sand in a laboratory flume (Yang, 1976).

Parker (1977), in his discussion of Yang's paper (1976), compared resistance relationships obtained from the theory of minimum unit stream power and those from extensive actual data fitting. Figure 3.28 shows Parker's comparison. These results suggest that the theory of minimum unit stream power can provide a simple theoretical tool for the determination of roughness of alluvial channels, at least for the lower flow region, where the sediment transport rate is not too high, and the rate of energy dissipation due to sediment transport can be neglected. As the sediment concentration or the Froude number increases, the rate of energy dissipation can no longer be neglected, and the accuracy of Yang's method decreases.

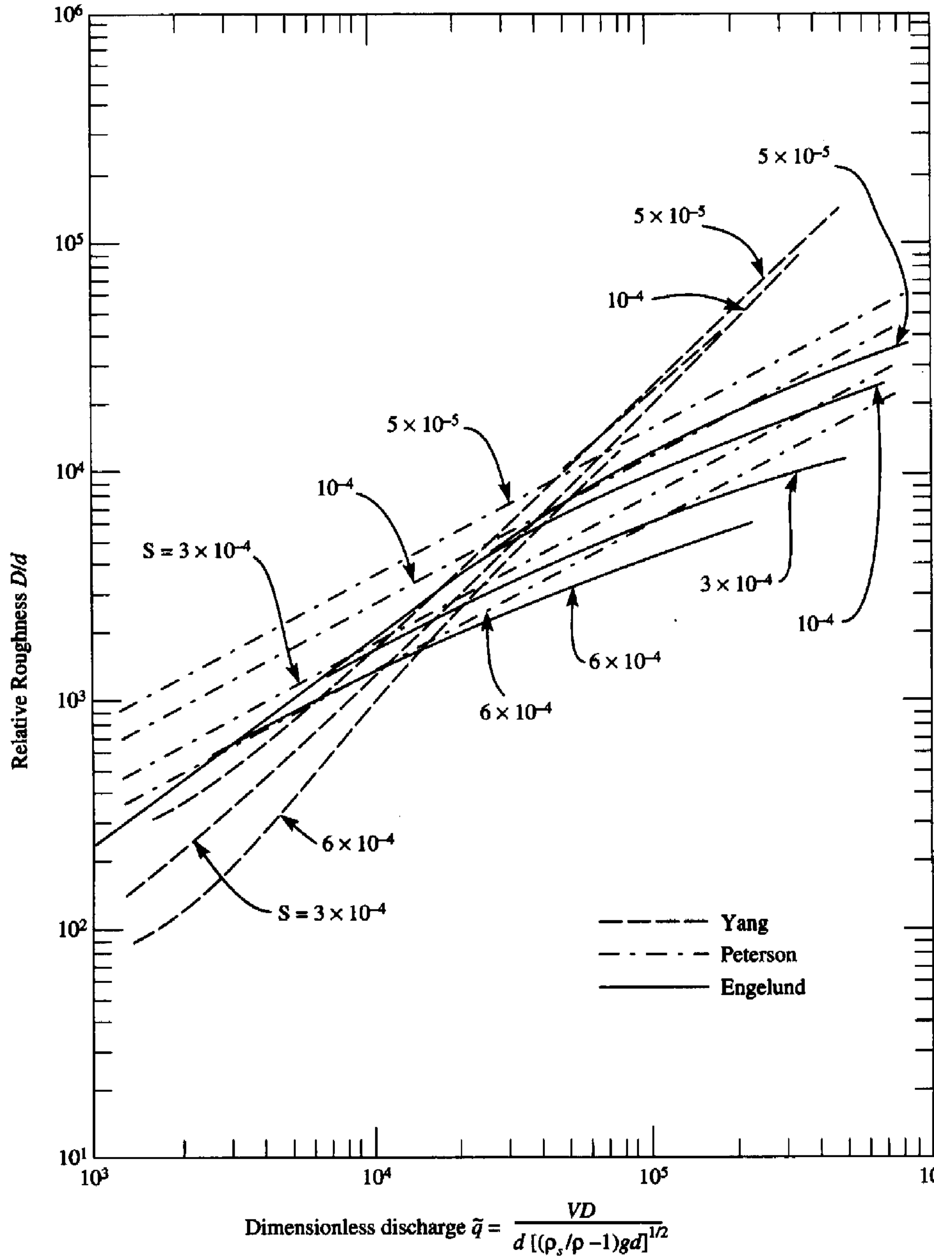


Figure 3.28. Comparisons between relative roughness determined from the theory of minimum unit stream power and those obtained by Peterson and Engelund (Parker, 1977).

Other sediment transport formulas can also be used in Yang's method as long as the formula can accurately estimate sediment load or concentration at the study site. The following example is given to illustrate the application of this method (Yang, 1996):

Example 3.4 The following data were collected from Rio Grande River Section F with width of 370 ft near Bernalillo, New Mexico.

$$d_{50} = 0.31 \text{ mm} \quad V = 3.2 \text{ ft/s} \quad D = 2.41 \text{ ft} \quad S = 0.00076 \quad T = 21.1\ ^\circ\text{C}$$

Determine Manning's roughness coefficient using the minimum unit stream power theory and Yang's (1973) unit stream power equation.

Solution: The computed sediment concentration from equation (3.70) is 517 ppm by weight. The following table summarizes the minimum unit stream power computation:

D_i (ft)	V_i (ft/s)	S_i	$V_i S_i$ [(ft-lb/lb)/s]
3.51	2.2	0.001114	0.002451
3.08	2.5	0.000977	0.002443
2.75	2.8	0.000870	0.002435
2.49	3.1	0.000784	0.002431
2.27	3.4	0.000715	0.002430 (min)
2.08	3.7	0.000657	0.002432
1.93	4.0	0.000608	0.002433
1.79	4.3	0.000566	0.002434
1.71	4.5	0.000541	0.002435

The minimum unit stream power $V_m S_m$ = 0.002430 (ft-lb/lb)/s, which is close to the measured unit stream power VS = 0.002432 (ft-lb/lb)/s. The corresponding values of depth, velocity, and slope are:

$$D_m = 2.27 \text{ ft} \quad V_m = 3.4 \text{ ft/s} \quad S_m = 0.000715$$

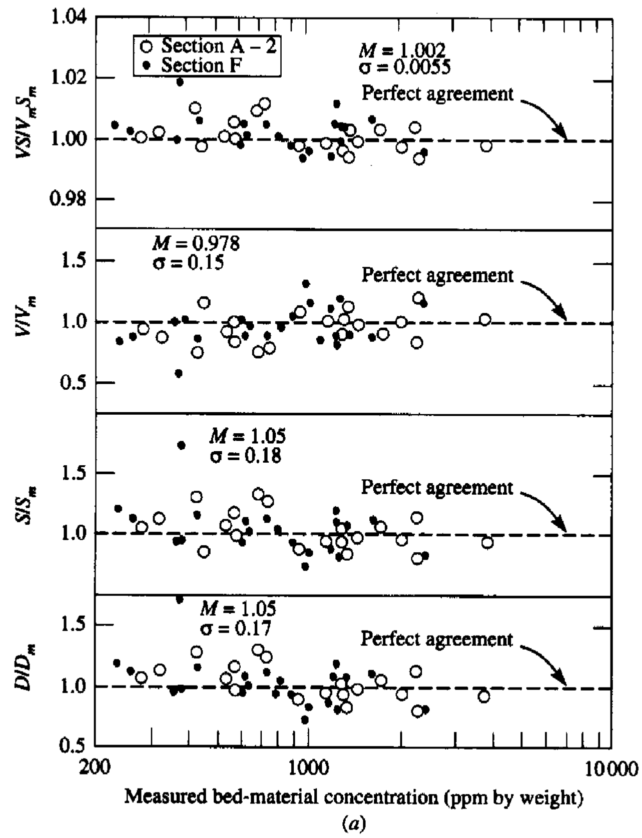
Manning's roughness coefficient with minimum unit stream power is:

$$n_m = \frac{1.49}{V_m} D_m^{2/3} S_m^{1/2} = \frac{1.49}{3.4}(2.27)^{2/3}(0.000715)^{1/2} = 0.0203$$

The actual n value based on the measured V, S, and D is:

$$n = \frac{1.49}{3.2}(2.41)^{2/3}(0.00076)^{1/2} = 0.0231$$

Figure 3.29 summarizes the comparisons between computed values based on Yang's methods and measurements from two river stations of the Rio Grande.



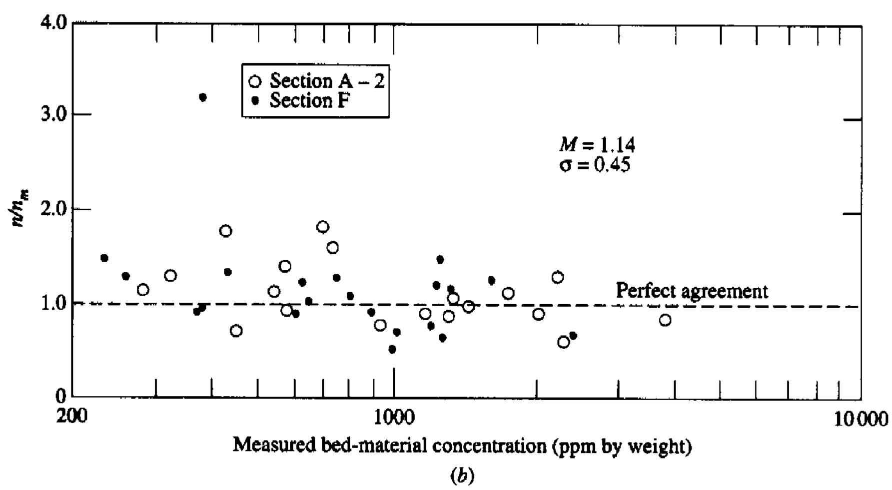


Figure 3.29. Comparisons between measured data from the Rio Grande and computed values from the theory of minimum unit stream power: (a) hydraulic parameters; (b) Manning's roughness coefficient (Yang and Song, 1979).

3.7 Nonequilibrium Sediment Transport

Most of the sediment transport functions were derived under the equilibrium condition with no scour nor deposition. The computed sediment load or concentration in a river from a sediment transport function is the river's sediment-carrying capacity.

When the wash load or concentration of fine material is high, a transport function should be modified by taking the effects of wash load into consideration before its application. An example of this type of modification is the modified dimensionless unit stream power formula proposed by Yang et al. (1996) as shown in equation (3.78). When a sediment transport function is used in a computer model for sediment routing, we also assume equilibrium sediment transport. Under this condition, if a river's sediment-carrying capacity determined from a sediment transport function is different from the sediment supply rate from upstream, scour or deposition would occur instantaneously. This assumption is valid for sand or coarse materials. For fine materials, the concept of nonequilibrium sediment transport should be applied. Based on the analytical solution of the convection-diffusion equation, Han (1980) proposed the following equation for the determination of nonequilibrium sediment transport rate:

$$C_i = C_{t,i} + \left(C_{i-1} - C_{t,i-1}\right)\exp\left\{-\frac{\alpha\omega_s\Delta x}{q}\right\} + \left(C_{t,i-1} - C_{t,i}\right)\left(\frac{q}{\alpha\omega_s\Delta x}\right)\left[1 - \exp\left\{-\frac{\alpha\omega_s\Delta x}{q}\right\}\right] \tag{3.145}$$

where C = sediment concentration,
C_t = sediment-carrying capacity, computed from an equilibrium sediment transport function,
q = discharge of flow per unit width,
Δx = reach length,
ω_s = sediment fall velocity,
i = cross-section index (increasing from upstream to downstream), and
α = a dimensionless parameter.

Equation (3.145) is employed for each of the particle size fractions in the cohesiveless range; that is, with diameter greater than 62.5μm. The parameter α is a recovery factor. Han and He (1990) recommended a value of 0.25 for deposition and 2.0 for entrainment. Although equation (3.145) was derived for suspended load, its application to bed-material load is reasonable. Yang and Simões (2002) gave more detailed analysis on the use of equation (3.145) for sediment routing.

3.8 Comparison and Selection of Sediment Transport Formulas

The selection of appropriate sediment transport formulas under different flow and sediment conditions are important to sediment transport and river morphologic studies. Computed sediment load or concentration from different sediment transport formulas can give vastly different results from each other and from field measurement. Consequently, engineers must compare the accuracies and limits of application of different formulas before their final selection. Comparisons of accuracies of sediment formulas were published by Schulits and Hill (1968), White et al. (1975), Yang (1976, 1979, 1984, 1996), Alonso (1980), Brownlie (1981), Yang and Molinas (1982), ASCE (1982), Vetter

(1989), German Association for Water and Land Improvement (1990), Yang and Wan (1991), and Yang and Huang (2001). The comparisons were made directly based on measured results or indirectly based on simulated results of a computer model.

3.8.1 Direct Comparisons with Measurements

Vanoni (1975) compared the computed sediment discharges from different equations with the measured results from natural rivers. Yang (1977) replotted his comparisons. The total measured sediment load does not include wash load. Figures 3.30 and 3.31 show these comparisons. With the exception of Yang's (1973) unit stream power equation, the results in Figures 3.30 and 3.31 are obtained directly from Vanoni's (1971) comparisons.

Figure 3.30 shows a comparison between computed and measured results by Colby and Hembree (1955) from the Niobrara River near Cody, Nebraska. Among the 14 equations, computed results from Yang's (1973) unit stream power equation give the best agreement with measurements. Colby's, Lauren's and, Toffaleti's equations and Einstein's bedload function can all provide reasonable estimates of the total sediment discharge form the Niobrara River. Figure 3.31 shows that Yang's (1973) unit stream power equation is the only one that can provide a close estimate of the total sediment discharge in Mountain Creek. The Schoklitsch equation ranks second in accuracy in this case.

White, Milli, and Crabe (1975) reviewed and compared sediment transport theories. They reviewed and compared most of the available equations at that time, with the exception of Yang's (1973) and Shen and Hung's (1972) equations. Their comparison was based on over 1,000 flume experiments and 260 field measurements. They excluded data with Froude numbers greater than 0.8. They used two dimensionless parameters for comparison purposes: the dimensionless particle size D_{gr} and the discrepancy ratio. The latter is defined as the ratio between calculated and measured sediment loads. D_{gr} is defined as:

$$D_{gr} = \left[\frac{g(\rho_s / \rho - 1)}{\nu^2} \right]^{1/3} d \tag{3.146}$$

where g = gravitational acceleration,
ρ_s and ρ = densities of sediment and water, respectively,
ν = kinematic viscosity of water, and,
d = particle diameter.

Comparisons made by White et al. (1975) indicated that Ackers and White's (1973) equation is the most accurate, followed by Engelund and Hansen's (1972), Rottner's (1959), Einstein's (1950), Bishop, Simons, and Richardson's (1965), Toffaleti's (1969), Bagnold's (1966), and Meyer-Peter and Müller's (1948) equations.

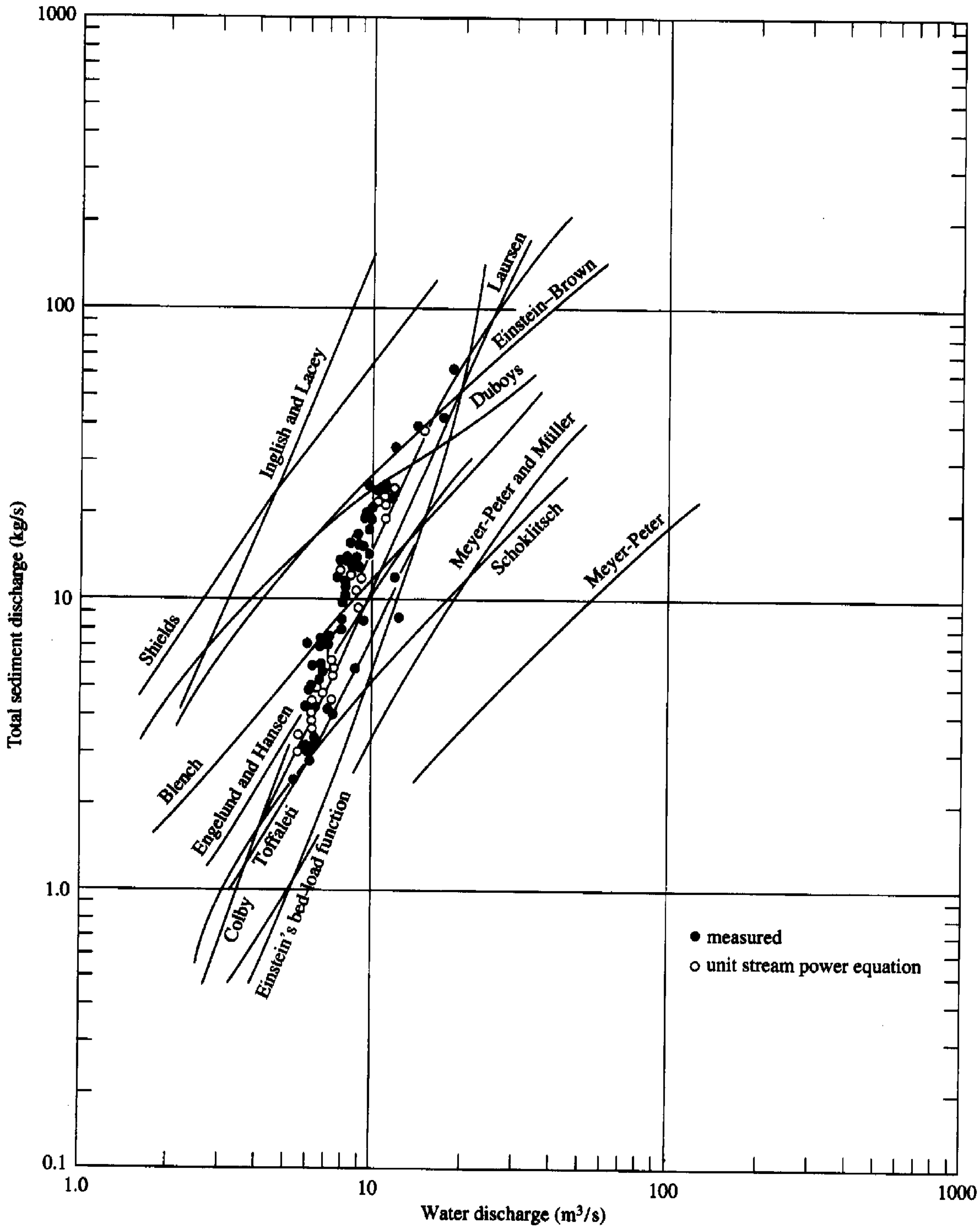


Figure 3.30. Comparison between measured total sediment discharge of the Niobrara River near Cody, Nebraska, and computed results of various equations (Yang, 1977).

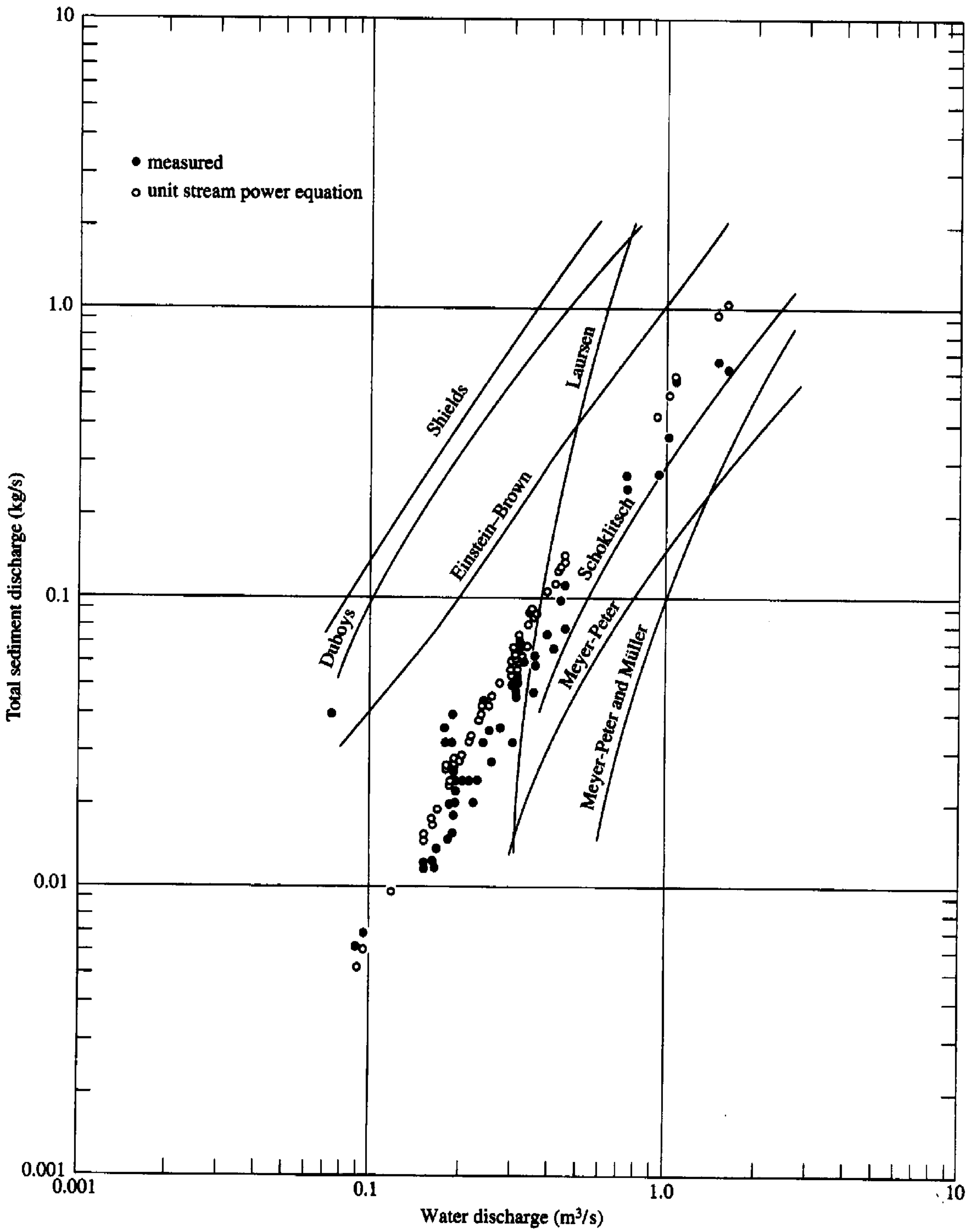


Figure 3.31. Comparison between measured total sediment discharge of Mountain Creek at Greenville, South Carolina, and computed results of various equations (Yang, 1977).

Yang (1976) made a similar analysis of 1,247 sets of laboratory and river data and discussed the results of White et al. (1975). Because the data used for comparison by Yang and by White et al. are basically the same, Table 3.4 combines the comparisons to give a relative rating of different sediment transport equations.

Table 3.4. Summary of accuracies of different equations (Yang, 1976)

Equation	Percent of data with discrepancy ratio between ½ and 2
Yang (1973)	91
Shen and Hung* (1972)	85
Ackers and White (1973)	68
Engelund and Hansen (1972)	63
Rottner (1959)	56
Einstein (1950)	46
Bishop et al. (1965)	39
Toffaleti (1969)	37
Bagnold (1966)	22
Meyer-Peter and Müller (1948)	10

* Should not be applied to large rivers

Alonso (1980) and Alonso et al. (1982) made systematic and detailed evaluations of sediment transport equations. The equations they evaluated cover wide ranges of sediment size, from very fine to very coarse. Among the 31 transport equations initially considered by Alonso (1980), only 8 received detailed comparison and evaluation. Some of the equations were not included for detailed evaluation by Alonso because they have not received extensive application. Others, such as Toffaleti's (1969) and the modified Einstein (Hubbell and Matejke, 1955) methods, are too complicated or require knowledge of the concentration of the measured suspended load and, therefore, not suitable for hydrologic or engineering simulation. Table 3.5 shows the results of the comparison by Alonso (1980) for sand transport. The MPME method, as shown in Table 3.5, estimates the total load by adding the bedload predicted by Meyer-Peter and Müller (1948) formulas to the suspended load computed by Einstein's (1950) procedures.

Alonso limited his comparisons of field data to those where the total bed-material load can be measured by special facilities. Thus, uncertainties in the unmeasured load do not exist. Table 3.5 indicates that Yang's (1973) equation has an average error of 1 percent for both field and flume data. When the depth-particle diameter ratio D/d is less then 70, the flow is shallow, and surface wave effects become important. In this range, most sediment formulas may fail because they do not account for interactions with free surface waves.

Table 3.6 provides a summary rating of selected sediment transport formulas by the American Society of Civil Engineers (ASCE, 1982). The German Association for Water and Land Improvement (1990) published similar ratings.

Table 3.5. Analysis of discrepancy ratio distributions of different transport formulas (Alonso, 1980)

Formula	Number of tests	Ratio between predicted and measured load				Percentage of tests with ratio between ½ and 2
		Mean	95% confidence limits of the mean		Standard deviation	
Field data						
Ackers and White (1973)	40	1.27	1.05	1.48	0.68	87.8
Engelund and Hansen (1972)	40	1.46	1.28	1.64	0.56	82.9
Laursen (1958)	40	0.65	0.49	0.80	0.48	56.1
MPME**	40	0.83	0.50	1.15	1.02	58.5
Yang (1973)	40	1.01	0.89	1.13	0.39	92.7
Bagnold (1966)	40	0.39	0.31	0.47	0.26	32.0
Meyer-Peter and Müller (1948)	40	0.24	0.22	0.27	0.09	0
Yalin (1963)	40	2.59	2.08	3.11	1.62	46.3
Flume data with $D/d \geq 70$						
Ackers and White (1973)	177	1.34	1.24	1.54	1.29	73.0
Engelund and Hansen (1972)	177	0.73	0.63	0.83	0.68	51.1
Laursen (1958)	177	0.81	0.73	0.88	0.51	71.4
MPME**	177	3.11	2.95	3.52	2.75	42.1
Yang (1973)	177	0.99	0.93	1.08	0.60	79.8
Bagnold (1966)	177	0.85	0.81	1.22	2.50	20.8
Meyer-Peter and Müller (1948)	177	0.40	0.39	0.47	0.49	18.5
Yalin (1963)	177	1.62	1.38	2.23	4.08	32.6
Flume data with $D/d \leq 70$						
Ackers and White (1973)	48	1.12	0.93	1.28	0.52	89.6
Engelund and Hansen (1972)	48	0.75	0.59	0.90	0.50	66.7
Laursen (1958)	48	1.04	0.76	1.32	0.99	79.2
MPME**	48	1.34	1.04	1.64	1.04	66.7
Yang (1973)	48	0.90	0.79	1.05	0.51	85.4
Bagnold (1966)	48	1.53	1.46	1.87	1.14	45.8
Meyer-Peter and Müller (1948)	48	1.03	1.00	1.27	0.83	72.9
Yalin (1963)	48	1.92	1.45	2.41	1.65	64.6

* MPME = Meyer-Peter and Müller's (1948) formula for bedload and Einstein's (1950) formula for suspended load.

Table 3.6. Summary of rating of selected sediment transport formulas (ASCE, 1982)

Formula number (1)	Reference (2)	Type (3)	Comments (4)
1	Ackers and White (1973)	Total load	rank* = 3
2	Engelund and Hansen (1967)	Total load	rank = 4
3	Laursen (1958)	Total load	rank = 2
4	MPME	Total load	rank = 6
5	Yang (1973)	Total load	rank = 1, best overall predictions
6	Bagnold (1966)	Bedload	rank = 5
7	Meyer-Peter and Müller (1948)	Bedload	rank = 7
8	Yalin (1963)	Bedload	rank = 8

* Based on mean discrepancy ratio (calculated over observed transport rate) from 40 tests using field data and 165 tests using flume data

Direct comparisons between measured and computed results from different sediment transport equations indicate that, on the average, Yang's (1973) dimensionless unit stream power equation is more accurate than others for sediment transport in the sand size range. Figure 3.32 shows a summary comparison between measured bed-material discharge from six river stations and computed results from Yang's (1973) equation.

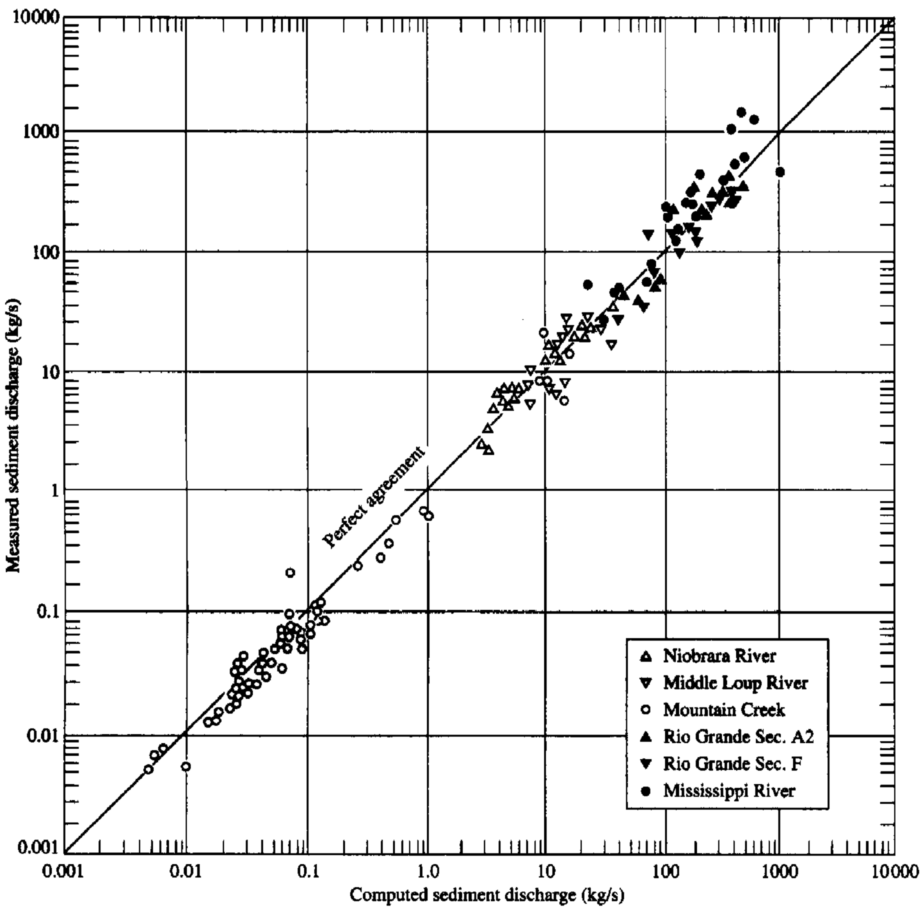


Figure 3.32. Comparison between measured total bed-material discharge from six river stations and computed results from Yang's (1973) equation (Yang, 1979, 1980).

The results shown in Table 3.7 indicate that the average mean discrepancy ratio of Yang's (1973) equation for 1,247 sets of laboratory and river data is 1.03. This means that, on the average, Yang's (1973) equation has an error of 3 percent. Figure 3.33 shows that the distributions of discrepancy ratio of Yang's (1973) equation for both laboratory and river data follow normal distributions. This means that no systematic error exists in Yang's (1973) equation. The reason that computed loads for natural rivers are generally higher than measurements is that Yang's (1973) equation includes loads in the unmeasured zone, while for most natural rivers, loads in the unmeasured zone are not included in the measurements.

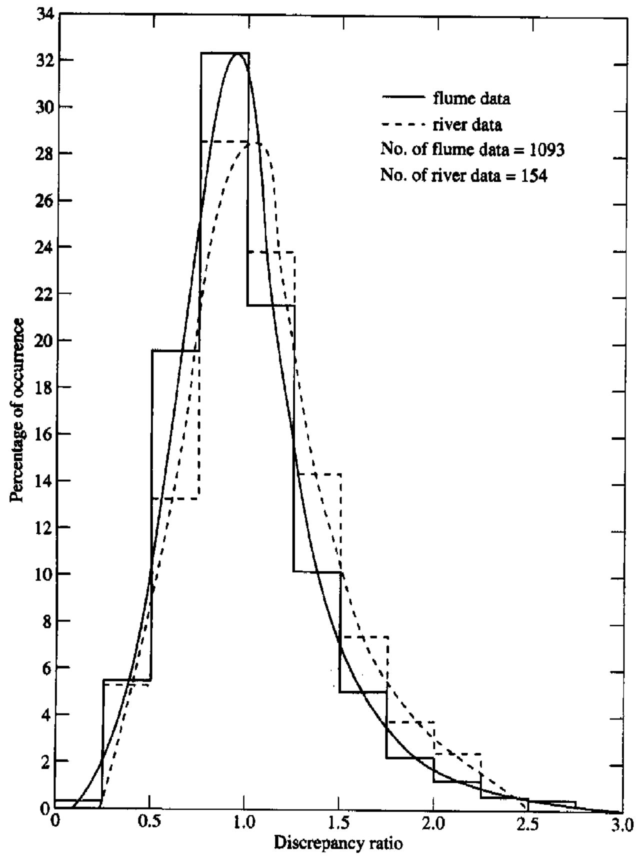


Figure 3.33. Distribution of discrepancy ratio of Yang's (1973) equation (Yang, 1977).

Table 3.7. Summary of accuracy of Yang's (1973) equation (Yang, 1977)

	Discrepancy ratio							
	Max. (1)	Mean (2)	Min. (3)	0.75–1.25 (4)	0.5–1.50 (5)	0.25–1.75 (6)	0.5–2.0 (7)	No. of data sets (8)
Sand in laboratory flumes	2.05	1.02	0.57	54%	84%	94%	91%	1,093
Sand in rivers	1.92	1.08	0.47	53%	80%	93%	92%	154
All data	2.03	1.03	0.56	54%	83%	94%	91%	1,247

Most of the comparisons of accuracy of equations were made for data collected in the sand size range. For coarser materials, sediments mainly travel as bedload or in the unmeasured zone. No reliable instrument can be used to measure bedload in most natural rivers under normal conditions. Thus, comparisons can be made only for laboratory flume data, where bedload can be measured by special equipment. Figure 3.34 shows an example of a comparison of four equations. It indicates that equations of Yang (1984), Engelund and Hansen (1972), Ackers and White (1973), and Meyer-Peter and Müller (1948) are all reasonably accurate for Gilbert's (1914) 7.0- mm gravel data collected from a laboratory flume. However, with the exception of Yang's (1984) gravel equation, the agreement between measured (Cassie, 1935) and computed results shown in Figure 3.35 is poor. This is due to the lack of generality of the assumptions used in the development of these equations, as explained in section 3.3.4.

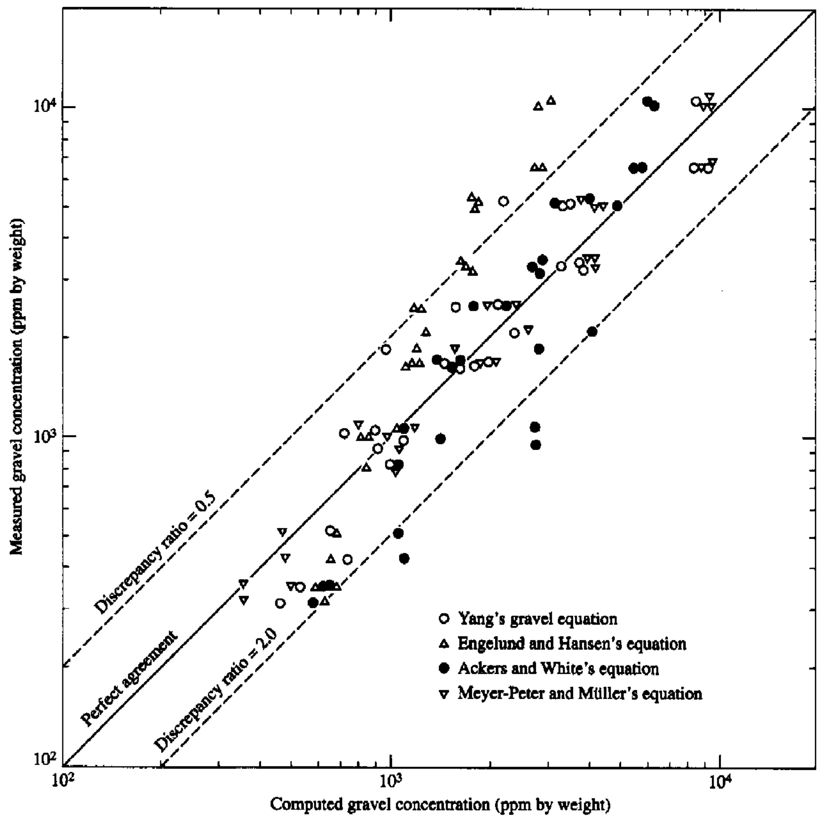


Figure 3.34. Comparison between 7.01-mm gravel concentration measured by Gilbert (1914) and results computed using different equations (Yang, 1984).

Among deterministic sediment transport equations, the modified Yang's (1996) unit stream power equation (3.78) is the one that can be applied to flows with high concentration of wash load. Yang et al. (1996) compared the computed results from equation (3.78) and 580 sets of measured data from 9 gauging stations along the Middle and Lower Yellow River. Their comparisons have an averaged discrepancy ratio of 1.0034 and a standard deviation of 1.6692. Figure 3.36 shows their comparisons. The slope of the Middle and Lower Yellow River is very flat. The flatter the slope, the higher the percentage error of measurement that can be caused by water surface fluctuation.

Figure 3.37 shows a comparison between the computed and measured results from the Yellow River, excluding 112 sets of data with slope less than 0.0001 from a total of 580 sets of data. The improvement shown in Figure 3.37 over that in Figure 3.36 is apparent. Thus, in a comparison with field data, the possibility of having measurement errors should not be overlooked.

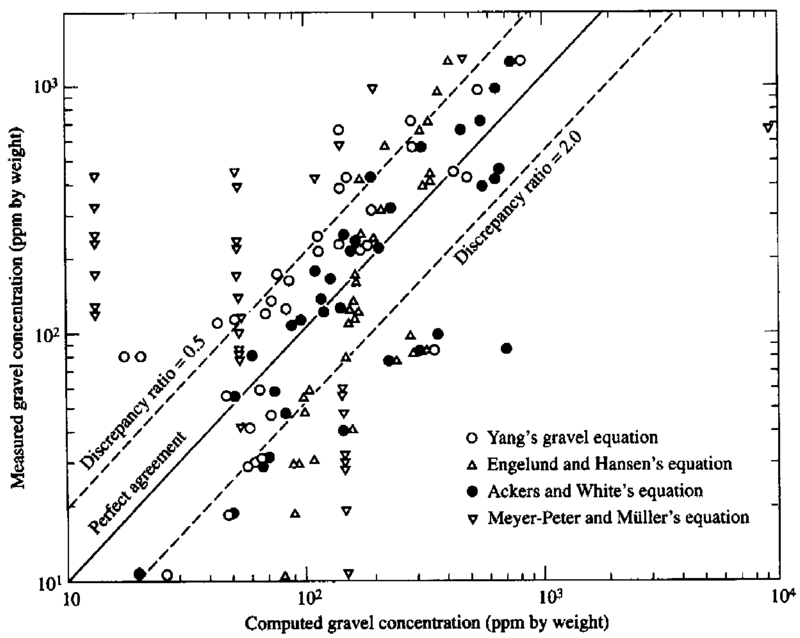


Figure 3.35. Comparison between 2.46-mm gravel concentration measured by Cassie (1935) and results computed using different equations (Yang, 1984).

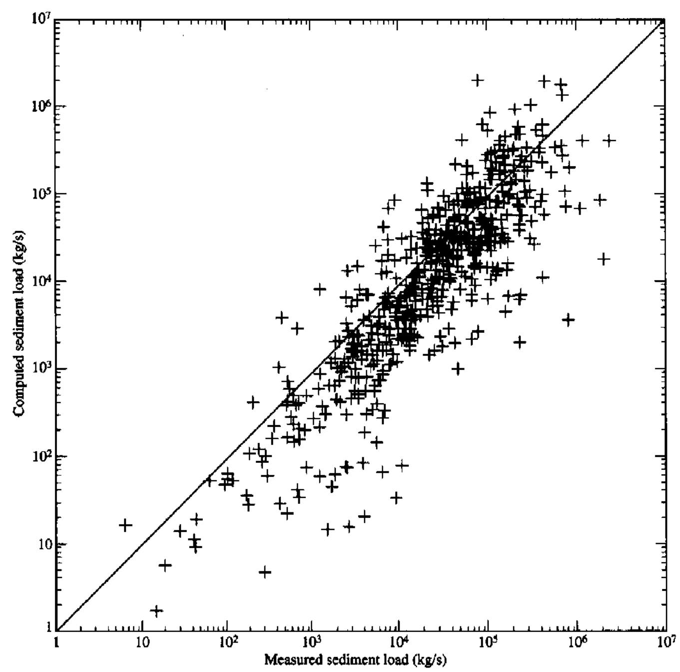


Figure 3.36. Comparison between computed and measured results based on the modified Yang's unit stream power formula, equation (3.78), and measurements from the Yellow River with sediment diameter larger than 0.01 mm.

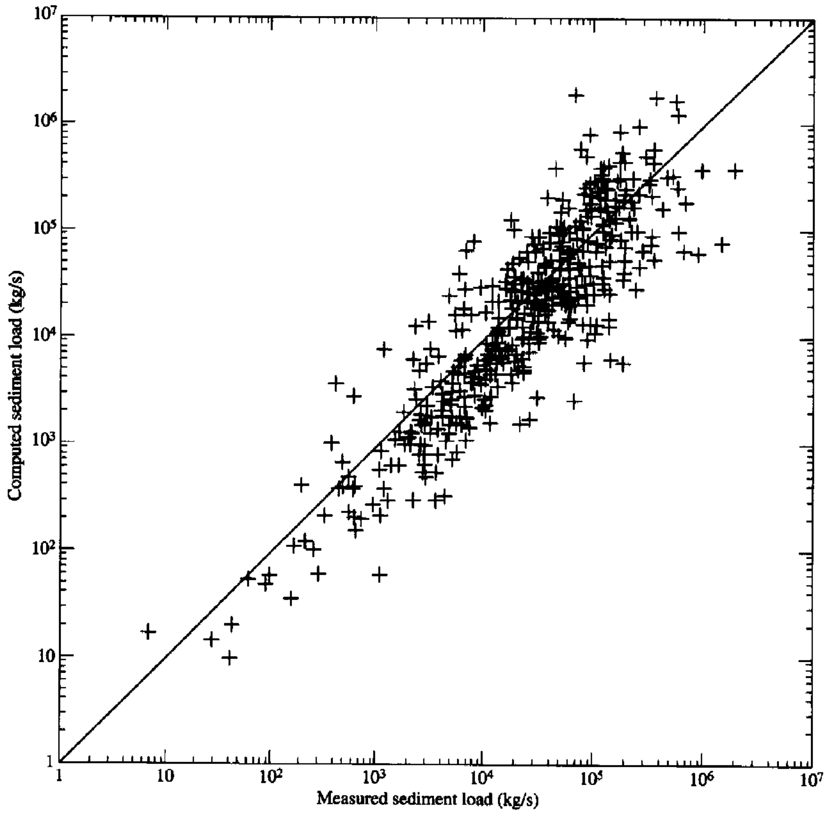


Figure 3.37. Comparison between computed and measured results based on the modified Yang's unit stream power formula, equation (3.78), and measurements from the Yellow River with sediment diameter larger than 0.01 mm and slope greater than 0.0001.

Equation (3.70) was developed as a predictive equation for sand transport. Figure 3.38 indicates that equation (3.70) can be used to predict sediment transport rate in the clay-size range if the effective diameter of clay aggregate is used. The scattering shown in Figure 3.38 was mainly due to the fact that different numbers of fine particles are bunched together to form clay aggregate of different effective diameters. Moore and Burch (1986) applied equation (3.70) in conjunction with the theory of minimum unit stream power for the determination of surface and rill erosion rate. Figure 3.39 indicates that equation (3.70) can accurately predict surface and rill erosion rate, especially if soil particles are in the ballistic dispersion mode when most sediment particles are being eroded (see Chapter 2, Erosion and Reservoir Sedimentation).

3.8.2 Comparison by Size Fraction

Not all sediment particles move at the same rate under a given flow condition when the particle sizes are not uniform. Yang and Wan (1991) made detailed comparisons of formulas based on size fraction.

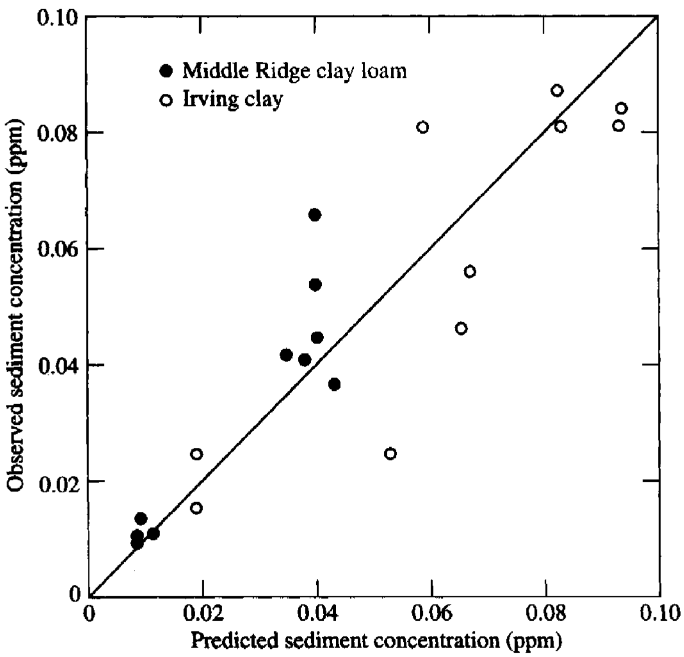


Figure 3.38. Comparison between observed and predicted clay concentrations from Yang's unit stream power (Moore and Burch, 1986).

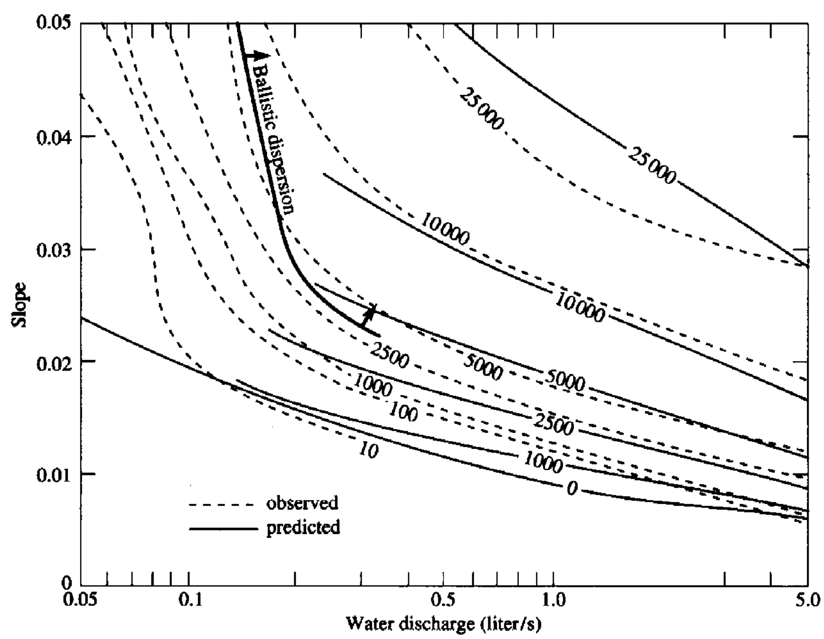


Figure 3.39. Comparison between observed and predicted sediment concentrations in ppm, by weight, from Yang's unit stream power equation with a plane bed composed of 0.43-mm sand (Moore and Burch, 1986).

They defined the discrepancy ratio γ_1 as the ratio between the median particle diameter d_c in transportation, computed by a formula, and actually measured particle diameter d_m in transportation; that is,

$$\gamma_1 = \frac{d_c}{d_m} \tag{3.147}$$

They also defined the discrepancy ratio γ_2 as the ratio between the median particle diameter d_c in transportation, computed by a formula, and d_{50} of the original bed materials on the alluvial bed; that is,

$$\gamma_2 = \frac{d_c}{d_{50}} \tag{3.148}$$

Most sediment transport equations were originally developed for fairly uniform bed materials. When they are applied to nonuniform materials, the total sediment concentration can be computed by size fraction (Yang, 1988):

$$C = \sum_{i=1}^{j} p_i C_i \tag{3.149}$$

where: j = total number of size fractions in the computation,
p_i = percentage of material available in size i, and
C and C_i = total concentration and concentration for size i computed from an equation, respectively.

The discrepancy ratio γ_1 should give an indication of the average accuracy of a formula in predicting the size distribution of bed materials in transportation. The discrepancy ratio γ_2 should give us an indication of the reasonableness of a formula in predicting the effect of the sorting process or the reduction of average particle size in the transport process. The results shown in Table 3.8 indicate that Yang's (1973) fraction formula has the best overall discrepancy ratio of 0.95. These results also show that the γ_1 value for Yang's (1973) fraction formula is not very sensitive to variations in Froude number. They suggest that Yang's fraction formula can be used with accuracy to predict size distribution of bed materials in transportation. This study also indicates that the median sizes of bed materials in transportation predicted by Laursen (1958) and Toffaleti (1968) are too small, while those predicted by Einstein (1950) are too large.

Table 3.9 indicates that, with the exception of Einstein's formula, bed materials in transportation computed by Laursen (1958), Yang (1973) by size fraction, and Toffaleti (1968) are finer than the original bed materials on the bed, which is consistent with the sorting phenomena. This sorting process explains why bed-material size should decrease in the downstream direction. The measured γ_2 value based on Yang's (1973) fraction formula changes very little, and Table 3.9 shows an average value of 0.77. The γ_2 values of Einstein's (1950) formula are greater than unity for all flow conditions, which means that the materials in transportation computed using Einstein's formula are coarser than the original bed materials, and the bed-material size would increase in the downstream direction, which is not reasonable. Yang and Wan's results suggest that Einstein's hiding and lifting factors may overcorrect the effect of nonuniformity of bed-material size on transport of graded bed materials. Einstein's assumption, that the average step length of 100 particle diameters implies that larger particles would have longer step length, is also erroneous.

Table 3.8. Comparison between computed and measured bed-material sizes in transportation (Yang and Wan, 1991)

Formula	Mean	Discrepancy ratio γ_1				Number of data sets
		Percentage of data in the range			Standard deviation	
		0.75–1.25	0.5–1.5	0.25–1.75		
F_r = 0.20–0.30						
Laursen	0.86	79	100	100	0.15	19
Yang (fraction)	1.09	79	100	100	0.18	19
Einstein	2.64	0	0	0	0.84	19
Toffaleti	0.77	42	100	100	0.19	19
F_r = 0.30–0.50						
Laursen	0.82	60	97	100	0.19	117
Yang (fraction)	1.03	85	93	100	0.21	117
Einstein	1.94	22	43	50	0.86	117
Toffaleti	0.61	32	59	85	0.28	117
F_r = 0.50–1.00						
Laursen	0.78	60	88	95	0.23	86
Yang (fraction)	0.91	80	92	99	0.25	86
Einstein	1.41	55	77	84	0.64	86
Toffaleti	0.55	22	51	83	0.27	86
F_r = 1.00–2.00						
Laursen	0.73	48	89	100	0.16	83
Yang (fraction)	0.85	66	99	100	0.18	83
Einstein	0.99	86	99	100	0.18	83
Toffaleti	0.53	22	47	94	0.23	83
All data						
Laursen	0.79	58	92	99	0.19	305
Yang (fraction)	0.95	78	95	100	0.21	305
Einstein	1.58	47	65	70	0.61	305
Toffaleti	0.58	27	65	87	0.26	305

Table 3.9. Comparison between computed and bed-material size in transportation and measured original bed-material size (Yang and Wan, 1991)

Formula	Discrepancy ratio γ_2					Number of data sets
	Mean	Percentage of data in the range			Standard deviation	
		0.75–1.25	0.5–1.5	0.25–1.75		
F_r = 0.20–0.30						
Laursen	0.66	26	84	100	0.14	19
Yang (fraction)	0.81	89	100	100	0.08	19
Einstein	1.98	0	26	37	0.54	19
Toffaleti	0.59	26	74	100	0.16	19
Measured value	0.77	58	95	100	0.10	19
F_r = 0.30–0.50						
Laursen	0.61	17	87	91	0.16	117
Yang (fraction)	0.76	80	91	91	0.18	117
Einstein	1.34	51	70	87	0.39	117
Toffaleti	0.43	8	40	79	0.20	117
Measured value	0.77	56	91	96	0.17	117
F_r = 0.50–1.00						
Laursen	0.63	37	79	91	0.19	86
Yang (fraction)	0.73	70	88	93	0.20	86
Einstein	1.06	90	97	100	0.17	86
Toffaleti	0.43	13	40	78	0.21	86
Measured value	0.77	79	91	99	0.17	86
F_r = 1.00–2.00						
Laursen	0.65	17	93	100	0.10	83
Yang (fraction)	0.76	61	100	100	0.08	83
Einstein	0.90	80	100	100	0.14	83
Toffaleti	0.46	5	36	94	0.15	83
Measured value	0.77	70	98	100	0.17	83
All data						
Laursen	0.63	23	86	94	0.15	305
Yang (fraction)	0.75	73	93	95	0.15	305
Einstein	1.18	67	83	91	0.27	305
Toffaleti	0.45	10	41	84	0.19	305
Measured value	0.77	66	93	98	0.17	305

3.8.3 Computer Model Simulation Comparison

Computer models have been increasingly used to predict or simulate the scour and deposition procedures of a river due to artificial or natural causes. The simulated results are sensitive to the selection of sediment transport equations used in the computer model. Therefore, the agreement between the measured and simulated results from a sediment transport equation is an indication of the accuracy of that equation. One of the most commonly used one-dimensional sediment routing models is HEC-6, developed by the U.S. Army Corps of Engineers (1977, 1993).

The U.S. Army Corps of Engineers (1982) applied the HEC-6 model to the study of scour and deposition process along several rivers due to engineering constructions. The sediment transport equations included in HEC-6 that were selected by the Los Angeles District of the Corps for comparison included those of Yang (1973, 1984), Toffaleti (1969), Laursen (1958), and DuBoys (1879). After a thorough comparison of all the transport equations available in HEC-6, Yang's (1973) equation was selected.

The Los Angeles District gave the following reasons:

> This function was selected because of (1) previous successful application in sediment studies performed on similar streams in southern California by the Los Angeles District, (2) the conclusions reported in a study conducted by the U.S. Department of Agriculture, and (3) comparison with the results from other transport functions applied in this study.

Before the Corps finally selected Yang's (1973) equation, sensitivity tests of the results using different transport functions in HEC-6 were made. These tests reached the following conclusions:

> Of the four functions applied, the Toffaleti transport capacity was found to be much less than the others. The result has reasonably small changes in computed bed elevations. The Duboys equation produced trends opposite from those predicted in the preliminary analysis indicated in table 1 (U.S. Army Corps of Engineers, 1982). Likewise, the Laursen function produced trends in the middle reach that were opposite from those predicted, and moreover, indicated unreasonably high deposition in the downstream reach. By contrast, the Yang equation produced trends that agreed well with the preliminary analysis throughout the study reach with the exception of the very downstream end, as was previously discussed (due to the lack of reliable estimation of Manning's *n* value). Thus, even though the computed changes in bed elevation were found to be very sensitive to different functions, the Yang equation clearly yielded the most reasonable results of the four functions incorporated into the HEC-6 program. For this reason and for the reasons discussed previously, it was concluded that the Yang function is the most appropriate to use in simulating sediment transport in the San Luis Rey River.

Figures 3.40 and 3.41 are two examples of comparisons made by the Corps of Engineers, Los Angeles District. These data are in the sand-size range. The comparisons indicate that generally good correlation between the observed and reconstituted bed profiles was obtained from the HEC-6 model using Yang's (1973) equation.

The HEC-6 computer model is a one-dimensional model for water and sediment routing. The bed elevation adjustment is parallel to the original bed without any variation in the lateral direction. The Bureau of Reclamation's GSTARS (Molinas and Yang, 1986) is a generalized stream tube model for alluvial river simulation. GSTARS can simulate the hydraulic conditions in a semi-two-dimensional manner, and channel geometry change in a semi-three-dimensional manner. Figure 3.42 shows a three-dimensional plot of the variation of computed scour pattern at the Mississippi River Lock and Dam No. 26 replacement site. Yang's 1973 sand formula and his 1984 gravel formula were used in the GSTARS simulation. Figure 3.43 shows the comparison between measured and computed results based on GSTARS. Because GSTARS cannot simulate secondary flow and eddies, a simplified assumption of a straight line extension of the cofferdam, as shown in Figure 3.43(b), was adopted. Despite this simplification, Figure 3.43 shows that the scour patterns predicted by GSTARS using Yang's sand and gravel formulas agree very well with measured results.

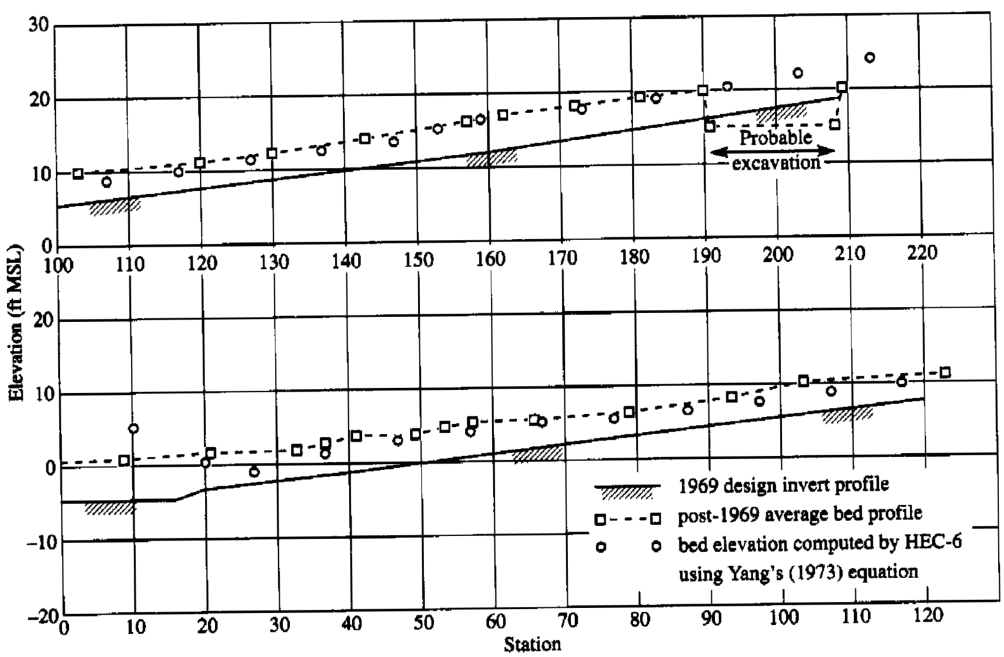


Figure 3.40. Reconstituted bed profiles of the Lower Santa Ana River after the 1969 flood, using Yang's (1973) equation (U.S. Army Corps of Engineers, 1982).

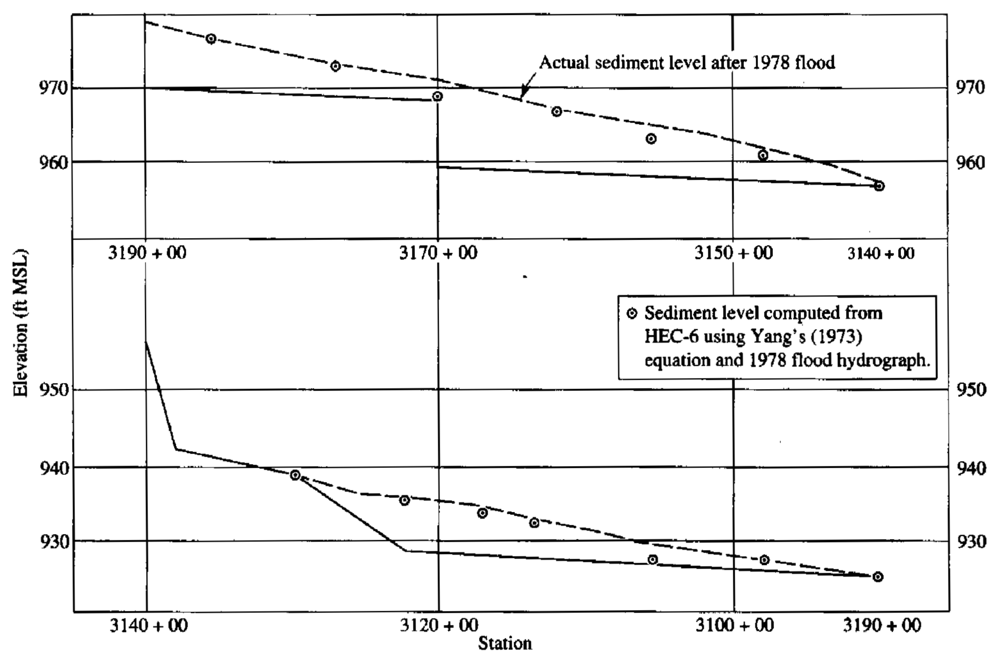


Figure 3.41. Reconstituted bed profiles of the Upper Santa Ana River after the 1978 flood, using Yang's (1973) equation (U.S. Army Corps of Engineers, 1982).

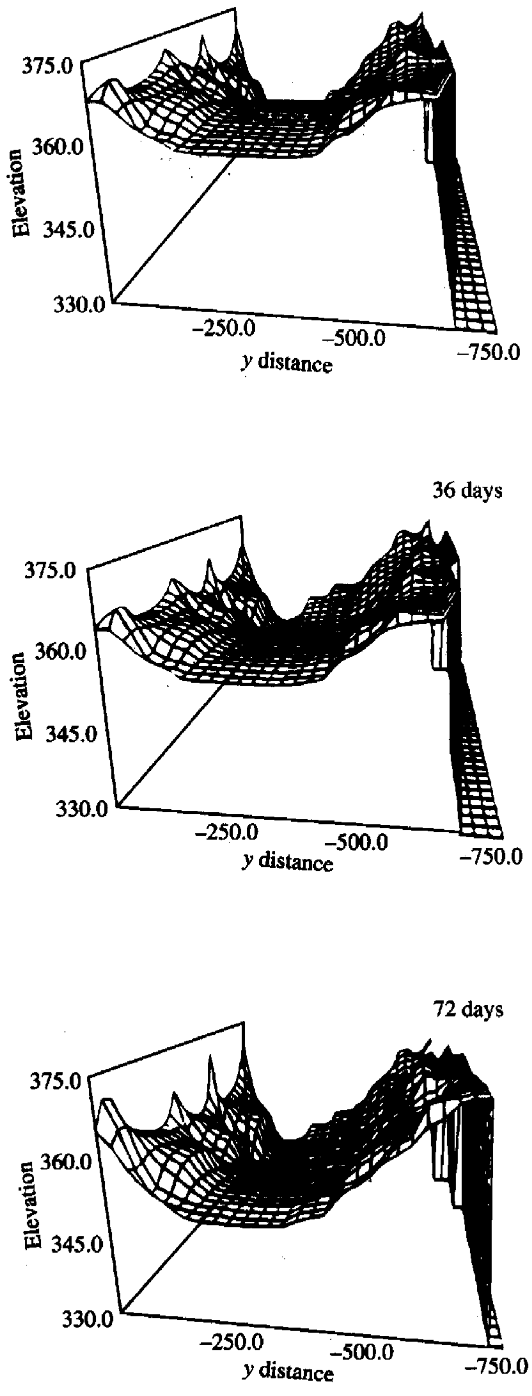


Figure 3.42. Three-dimensional plot of the variation of computed scour pattern at the Mississippi River Lock and Dam No. 26 replacement site (Yang et al., 1989).

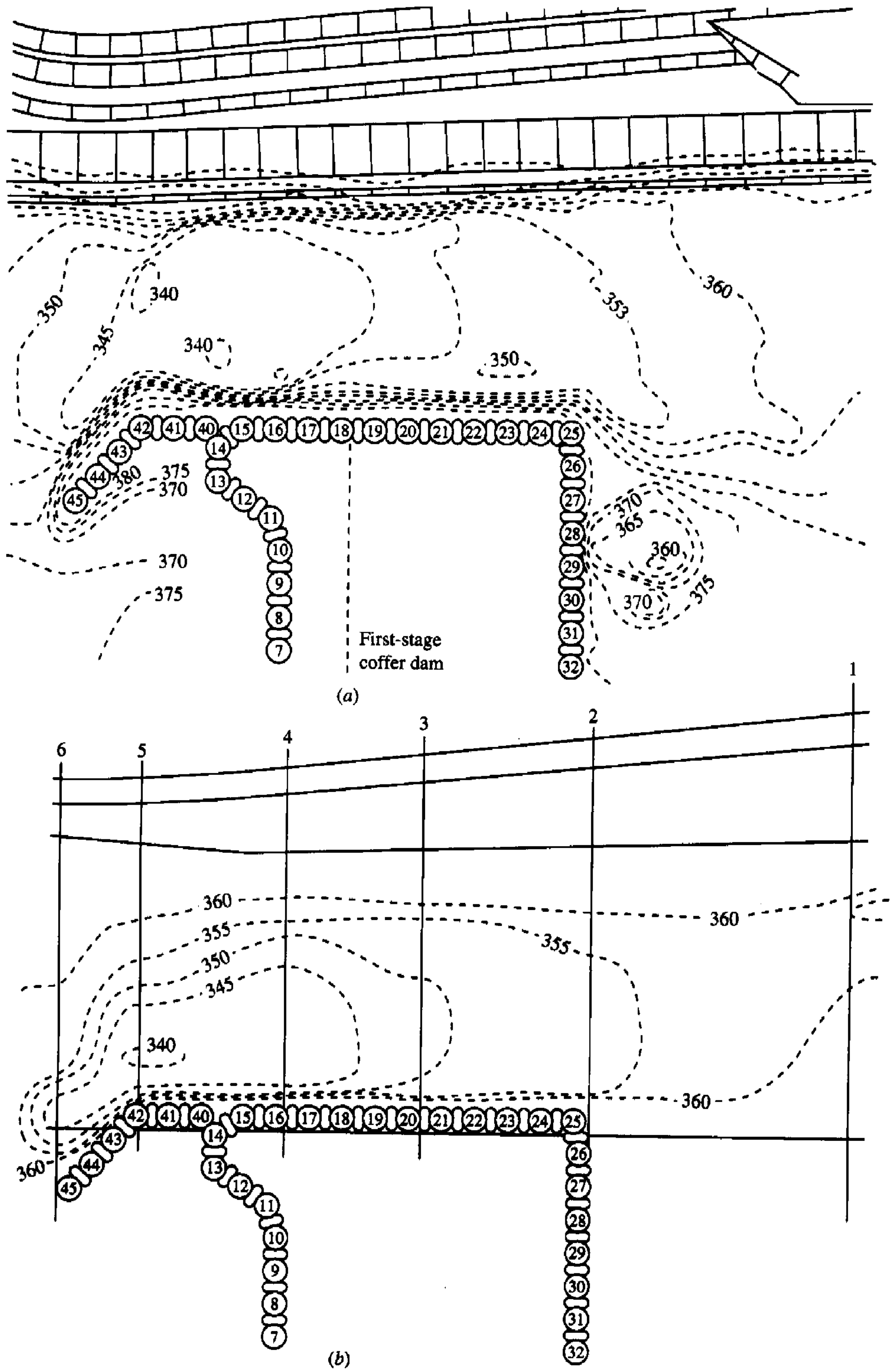


Figure 3.43. Scour pattern (a) measured and (b) computed, based on the flow condition of April 1,1982, at the Mississippi River Lock and Dam No. 26 replacement site (Yang et al., 1989).

The GSTARS computer model series has evolved through different revised and improved versions since its original release in 1986. They are GSTARS 2.0 (Yang et al., 1998), GSTARS 2.1 (Yang and Simões, 2000), and GSTARS3 (Yang and Simões, 2002). Information on these programs can be found by accessing website: http://www.usbr.gov/pmts/sediment. One of the important features of all the GSTARS models is the ability to simulate and predict channel width adjustments based on the theory of minimum energy dissipation rate (Yang, 1976; Yang and Song, 1979, 1984) or its simplified version of minimum stream power. Figure 3.44 compares the measured and predicted channel cross-sectional change of the unlined emergency spillway downstream from Lake Mescalero in New Mexico. The computation was based on Yang's sand (1973) and gravel (1984) formulas using GSTARS 2.1. It is apparent that the use of the optimization option based on the theory of minimum stream power can more accurately predict and simulate channel geometry changes. It is also apparent that the accuracy of simulated results depends not only on the selection of a sediment transport formula, but also on the capability and limits of application of the computer model used in the simulation.

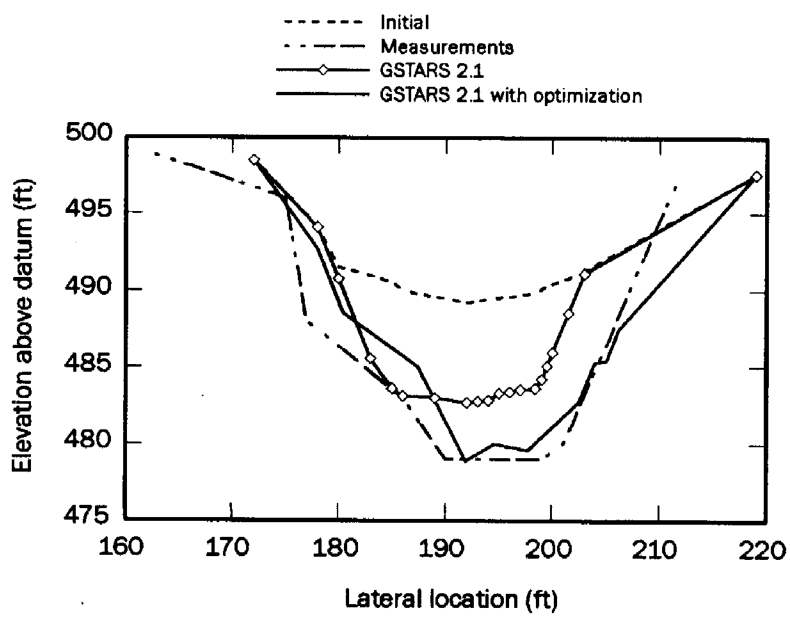


Figure 3.44. Comparison of results produced by GSTARS 2.1 and survey data for runs with and without width changes due to stream power minimization (Yang and Simões, 1998).

It is difficult to determine the accuracy and applicability of a bedload or gravel transport formula directly when it is applied to a natural river. This is because of the limitation of existing sampling methods. Chang (1991, 1994) developed a method for selecting a gravel transport formula based on the measured changes in stream morphology instead of site-specific gravel transport data.

The measured scour at the Highway No. 32 bridge crossing Stony Creek in Glen County, California ,is 77.6 m^2 in cross-sectional area. The simulated values based on Meyer-Peter and Müller's (1948), Parker's (1990), Yang's (1984), and Engelund and Hansen's formulas are 58.5, 79.9, 75.2, and 143.1 m^2, respectively. The measured deposition at station 46200 is 150 m^2 in cross-sectional area.

The simulated values based on Meyer-Peter and Müller's, Parker's, Yang's, and Engelund and Hansen's formulas are 63, 155, 149, and 273 m^2, respectively. These results indicate that the gravel formulas of Yang and Parker can accurately predict the scour and deposition process. Engelund and Hansen's formula produced a higher transport rate, while Meyer-Peter and Müller's produced a lower transport rate than the measurements.

Although the depositions simulated using Parker's formula and Yang's (1984) formula are similar, Yang's showed a more uniform distribution of deposition along the channel and correlated better with measurement (Chang, 1991). For this reason, Chang (1991, 1994) adopted Yang's formula for the Stony Creek morphological study. Figures 3.45 and 3.46 show examples of Chang's simulation results using Yang's (1984) formula.

3.8.4 Selection of Sediment Transport Formulas

The ranking of the accuracy of formulas in the published comparisons is not consistent, mainly because they were based on different sets of data. Some of the comparisons are not strictly valid, because data outside of the range of application recommended by the authors of the formulas were used in the comparison. Although no lack of data for comparison exists, the accuracies of data, especially field data, may be questionable.

Yang and Huang (2001) published a comprehensive comparison of 13 sediment transport formulas to determine their limits of application. Published, reliable data by different authors were used to give unbiased comparisons. Different amounts of data were used for different formulas because only the data within the applicable range of a formula are used to test its accuracy. Dimensionless parameters were used to determine the sensitivities of formulas to these parameters.

Stevens and Yang (1989) published FORTRAN and BASIC computer programs for 13 commonly used sediment transport formulas in river engineering. Yang's 1996 book, *Sediment Transport Theory and Practice*, includes the complete source codes in both FORTRAN and BASIC and a floppy diskette of the programs. The 13 formulas are those proposed by Schoklitsch (1934), Kalinske (1947), Meyer-Peter and Müller (1948), Einstein (1950) for bedload, Einstein (1950) for bed-material load, Laursen (1958), Rottner (1959), Engelund and Hansen (1967), Toffaleti (1968), Ackers and White (1973), Yang (1973) for sand transport with incipient motion criteria, Yang (1979) for sand transport without incipient motion criteria, and Yang (1984) for gravel transport. Yang and Huang (2001) selected these formulas, because the computer program used in comparison is readily available to the public. Many of these formulas have been incorporated in sediment transport models, such as the U.S. Army Corps of Engineers' HEC-6 computer model, Scour and Deposition in Rivers and Reservoirs (1993), and the Bureau of Reclamation's Generalized Stream Tube Model for Alluvial River Simulation (GSTARS) by Molinas and Yang (1986) and its revised and improved versions of GSTARS2 (Yang, et al., 1998), GSTARS 2.1 (Yang and Simões, 2000), and GSTARS3 (Yang and Simões, 2002).

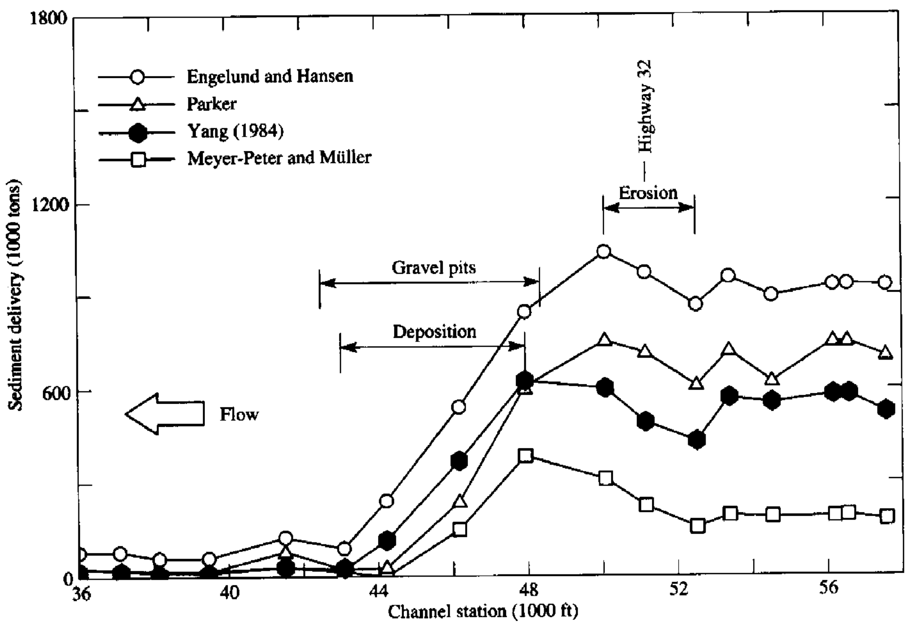


Figure 3.45. Spatial variations of the Stony Creek sediment delivery by the 1978 flood based on four sediment-transport formulas (Chang, 1994).

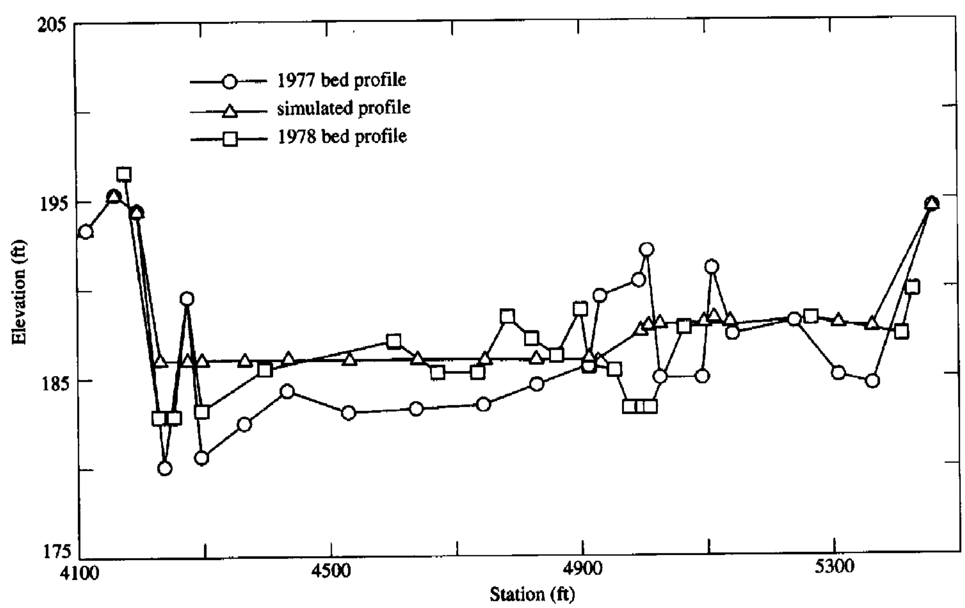


Figure 3.46. Measured cross-sectional changes at Stony Creek section 52400 and those simulated based on Yang's (1984) formula (courtesy of Chang).

3.8.4.1 Dimensionless Parameters

The accuracy of a sediment transport formula may vary with varying flow and sediment conditions. To determine the sensitivities of a transport formula to varying flow and sediment conditions, Yang and Huang (2001) selected seven dimensionless parameters for comparison. They are dimensionless particle diameter, relative depth, Froude number, relative shear stress, dimensionless unit stream power, sediment concentration, and discrepancy ratio.

Different transport formulas were developed for sediment transport in different size ranges. The dimensionless particle diameter used in the comparisons is defined as:

$$D_* = d\left[\frac{\gamma_s - \gamma}{\gamma} g / v^2\right]^{1/3} \tag{3.150}$$

where: d = sediment particle diameter,
γ_s, γ = specific weight of sediment and water, respectively,
g = gravitational acceleration, and
v = kinetic viscosity of water.

The relative depth is defined as the ratio between average water depth D and sediment particle diameter d. The inverse of relative depth is the relative roughness, which has been considered by many investigators as an important parameter for the determination of sediment transport rate and resistance to flow. One major difference between laboratory and river data is that the former have a much smaller value of relative depth. If the relative depth is small, say less than 50, the water surface wave and the size of bed form may affect accuracy of measurements.

Froude number is one of the most important parameters for open channel flow studies. Most sediment transport formulas were developed for subcritical flows.

Relative shear velocity is defined as the ratio between shear velocity U_* and sediment particle fall velocity ω. Many researchers consider U_*/ω as an index of flow intensity for sediment transport. For example, Julien (1995) believes that there is no sediment movement if $U_*/\omega < 0.2$; sediment transport is in the form of bedload if $0.2 < U_*/\omega < 0.4$; sediment transport is in the form of both bedload and suspended load if $0.4 < U_*/\omega < 2.5$; sediment transport is in the form of suspended load if $U_*/\omega > 2.5$.

Yang (1973) defined the dimensionless unit stream power as VS/ω, where V = cross-sectional average flow velocity; S = energy or water surface slope; and ω = sediment particle fall velocity. Yang (1973, 1996) considered VS/ω the most important parameter for the determination of sediment concentration or sediment transport rate.

Sediment concentration is defined as the ratio between sediment transport rate and water discharge by weight.

Discrepancy ratio is defined as the ratio between computed sediment concentration and measured sediment concentration; that is,

$$R = C_C/C_m \quad (3.151)$$

where C_C = computed sediment concentration in parts per million by weight, and
C_m = measured total bed-material concentration in parts per million by weight.

The average discrepancy ratio is defined as:

$$\overline{R} = \frac{\sum_{i=1}^{j} R_i}{j} \quad (3.152)$$

where i = data set number, and
j = total number of data used in the comparison.

3.8.4.2 Data Analysis

A total of more than 6,200 sets of sediment transport and hydraulic data were available to Yang and Huang (2001) for preliminary comparison and analysis. One of the difficulties in the selection of data for final comparison and analysis is the determination of accuracies of data published by different investigators. The following criteria were used to eliminate data of questionable accuracy:

- Only those data published by an investigator with more than 50 percent in a range of discrepancy ratio between 0.5 and 2, based on two or more of the 13 formulas, were included. Data with less than 10 sets were excluded. A total of 3,391 sets of data met this requirement. These data were compiled by Yang (2001).

- To avoid the uncertainties related to incipient motion, measured sediment concentrations less than 10 ppm, by weight, were excluded.

- Most of the laboratory data were fairly uniform in size. The median particle diameter was used for all sediment transport formula computations. The gradation coefficient is defined as:

$$\sigma = \frac{1}{2}\left(\frac{d_{84.1}}{d_{50}} + \frac{d_{50}}{d_{15.9}}\right) \quad (3.153)$$

where $d_{15.9}$, d_{50}, $d_{84.1}$ = sediment particle size corresponding to 15.9%, 50%, and 84.1% finer, respectively.

Data with $\sigma \geq 2.0$ were excluded from further analysis.

- To avoid the inclusion of wash load, data with median particle diameters of less than 0.0625 mm were excluded.

All the laboratory data had to be collected under steady equilibrium conditions. Natural river sediment and hydraulic data had to be collected within a day, and flow conditions had to be fairly steady to ensure a close relationship between sediment and flow conditions for a given set of river data.

Based on the above criteria, a total of 3,225 sets of laboratory data and 166 sets of river data were selected for final analysis and comparison. Table 3.10 summarizes these data.

Some of the transport formulas were intended for sand transport and some for gravel transport. The second step of comparison was to determine the range of application of sediment particle size based on discrepancy ratio for each formula. Table 3.11 shows the results. Based on the results shown in Table 3.11, Table 3.12 gives the ranges of application of the 13 formulas. Yang and Huang (2001) used only those data within the range of application of each formula as shown in Table 3.12 for further comparison and analysis.

Table 3.13 summarizes the sensitivity of the accuracy of formulas as a function of relative depth. The relatively large variations of discrepancy ratio for 13 formulas with $4 < D/d < 50$ suggest that the influences of water surface wave and bed form may be significant. If we exclude the data with $4 < D/d < 50$, Yang's 1979 sand formula is least sensitive to the variation of relative depth, followed by Yang's 1973 sand formula, and Yang's 1984 gravel formula. The Rottner formula and the Kalinske formula are the most sensitive. The Ackers and White formula has a tendency to overestimate sediment concentration with increasing flow depth, while the Engelund and Hansen formula has the reverse tendency.

Table 3.14 and Figure 3.47 summarize the sensitivity of the accuracy of formulas as a function of Froude number. The Rottner formula is most sensitive to the variation of Froude number, followed by Einstein's bedload and bed-material load formulas and the Kalinske formula. Yang's 1979 and 1973 sand formulas are least sensitive to the variation of Froude number. Table 3.14 shows that Yang's 1973, 1979, and 1984 formulas can be applied to subcritical, supercritical, and transitional flow regimes, while other formulas should be applied to subcritical flow only. Table 3.15 summarizes the sensitivity of the accuracy of formulas as a function of relative shear velocity. The Rottner and Kalinske formulas are most sensitive to the variation of relative shear velocity. Yang's 1973, 1979, and 1984 formulas are least sensitive to the variation of relative shear velocity.

Yang considered the dimensionless unit stream power to be the most important parameter in his 1973, 1979, and 1984 formulas. Table 3.16 shows that Yang's three formulas consistently and reliably predict sediment concentration or transport rates. The formulas by Ackers and White and by Engelund and Hansen also can give accurate estimation of sediment concentration or load for a wide range of dimensionless unit stream power. The least reliable ones are the Rottner, Kalinske, and Einstein's bedload and bed-material load formulas. While the Kalinske and Laursen formulas consistently overestimate sediment concentration and transport rate, the Meyer-Peter and Müller formula consistently underestimates sediment concentration and transport rate.

Table 3.17 and Figure 3.48 summarize the accuracies of transport equations as a function of measured sediment concentration. Accuracy apparently increases for all formulas when the measured sediment concentration is greater than 100 ppm by weight. This may be related to the fact that it is more difficult to measure accurately when the concentration is low. If we limit our comparisons with concentration greater than 100 ppm by weight, the most accurate formulas are those proposed by Yang in 1973, 1979, and 1984. The Ackers and White and the Engelund and Hansen formulas can also give reasonable estimates. The least accurate ones are the Kalinske, Rottner, Einstein bedload and bed-material load, Taffoletti, and the Meyer-Peter and Müller formulas.

The difference between Yang's 1973 and 1979 formulas is that the 1973 formula includes incipient motion criteria, while the 1979 formula does not have incipient motion criteria. Consequently, the 1973 formula should be used where measured total bed-material concentration is less than 100 ppm by weight. The 1979 formula should give slightly more accurate results at high concentrations because the uncertainty and the importance of incipient motion criteria decrease with increasing sediment concentration. Tables 3.18 and 3.19 and Figure 3.49 summarize the comparison between Yang's 1973 and 1979 formulas. It is apparent that the 1973 formula should be used where total bed-material concentration is less than 100 ppm by weight, while the 1979 formula is slightly more accurate where the concentration is greater than 100 ppm by weight.

The Meyer-Peter and Müller and the 1984 Yang formulas should be used for bed materials in the very coarse sand to coarse gravel range. Figure 3.50 shows that the 1984 Yang formula gives more reasonable prediction than the Meyer-Peter and Müller formula.

Table 3.19 summarizes the recommended ranges of application and the accuracy of 13 formulas. It is apparent that formulas based on energy dissipation rate either directly or indirectly, such as those by Yang, Ackers and White, and Engelund and Hansen, outperform those based on other approaches. The Einstein transport functions were based on probability concepts. In spite of the sophisticated theories and the complicated computational procedures used, Einstein's bedload and bed-material transport formulas are less accurate than others for engineering applications. This is mainly due to the lack of generality of Einstein's assumptions, such as step length, hiding factor, and lifting factor (Yang and Wan, 1991). Einstein's formulas should not be used in any computer model if sediment routing based on size fractions is performed. Yang and Wan (1991) pointed out that if computation is based on size fraction using Einstein's formulas, sediment in transportation would be coarser than the original bed-material gradation, and coarser materials would be transported further in the downstream direction at a higher rate than the finer materials.

The Rottner formula is a regression equation without much theoretical basis. The results shown in Table 3.19 indicate that the Rottner formula is less reliable than others based on discrepancy ratio. Formulas purely based on regression analysis should not be applied to places other than where the data were used in the original regression analyses.

Table 3.19 also indicates that the classical approach based on shear stress, such as the Kalinske and the Meyer-Peter and Müller formulas, is less accurate than those based on the energy dissipation rate theories used by Yang directly and by Ackers and White, and Engelund and Hansen indirectly. Yang's approach was based on his unit stream power theory, while Ackers and White and Engelund and Hansen applied Bagnold's (1966) stream power concept to obtain their transport functions (Yang, 1996, 2002).

Table 3.10. Summary of basic data (Yang and Huang, 2001)

Author	D_l	s	D/d	Fr	U_l/ω	VS/ω	C	N
Ansely (1963)	5.83	1.33	58.9-157.0	2.301-3.362	2.042-3.446	1.0312-2.2163	29576-198664	26
Chyn (1935)	19.5-21.0	1.23-1.58	59.4-106.0	0.514-0.764	0.261-0.440	0.0043-0.0152	123-751	22
MacDougal (1933)	16.5-31.5	1.29-1.71	29.6-190.3	0.433-0.799	0.218-0.507	0.0038-0.0212	123-1237	74
USACE (1935)	4.50-12.5	1.31-1.94	46.6-1021	0.253-0.735	0.208-3.260	0.0043-0.00786	10-833	279
USACE (1936)	23.8	1.44	30.5-206.9	0.324-0.674	0.206-0.506	0.0032-0.0102	16-379	101
Sato et al. (1958)	26.0-114.5	1.00	60.2-421.1	0.189-0.754	0.210-0.626	0.0010-0.0115	10-500	219
Casey (1935)	61.5	1.16	11.0-89.1	0.425-0.880	0.179-0.286	0.0034-0.0173	10-960	36
Meyer-Peter and Müller (1948)	130.3-716.3	1.00	11.1-47.7	0.623-1.414	0.222-0.440	0.0092-0.0787	10-7000	51
Graf and Suszka (1987)	307.5-587.5	1.23-1.24	3.99-20.9	0.772-1.264	0.205-0.293	0.0114-0.0552	12-2910	101
Song et al. (1998)	307.5	1.37	6.84-17.1	0.698-0.991	0.227-0.288	0.0113-0.0316	11-2519	48
							Total	3225
(b) River data								
Colby and Hembree (1955)	7.08	1.76	1465-2036	0.304-0.535	1.763-3.294	0.0205-0.0716	392-2220	25
Hubbell and Matejka (1959)	4.50-6.00	1.58-2.54	1365-2019	0.326-0.723	2.165-4.425	0.0263-0.0919	632-2440	15
Nordin (1964)	4.75-9.75	1.44-1.89	1107-5045	0.258-0.735	1.055-3.607	0.0112-0.0591	260-3787	42
Jordan (1965)	4.75-19.5	1.43-1.98	9735-45078	0.100-0.158	0.710-4.579	0.0005-0.0064	13.1-226	23
Einstein (1944)	25.0	1.84	61.0-399.3	0.394-0.497	0.251-0.710	0.0047-0.0106	40-664	61
							Total	166

Total number of laboratory and river data = 3,391

Note: D_l = dimensionless diameter; σ = gradation; D/d = relative depth; Fr = Froude number; U_l/ω = ratio of shear velocity to fall velocity; VS/ω = dimensionless unit stream power; C = Concentration (ppm by weight); N = number of data set

Table 3.10. Summary of basic data (Yang and Huang, 2001)

Author	D_l	s	D/d	Fr	U_l/ω	VS/ω	C	N
(a) Laboratory data								
Gilbert (1914)	7.63-175.3	1.06-1.34	5.74-295.9	0.292-3.540	0.240-1.998	0.0057-0.6628	77-35340	886
Guy et al. (1996)	4.75-30.0	1.25-1.67	109.2-1701	0.220-1.698	0.235-7.236	0.0014-0.6533	10-50000	272
Willis et al. (1972)	2.50	1.30	1036-3780	0.218-1.005	4.217-10.427	0.0167-0.3810	87-19400	96
Willis (1979)	13.5	1.12	191.9-276.6	0.272-1.155	0.437-1.276	0.0035-0.1248	15-6670	32
Willis (1983a)	13.8	1.60	698.3-2810	0.163-0.643	0.776-2.392	0.0024-0.0693	27-4620	42
Willis (1983b)	13.8	1.60	310.0-642.9	0.284-1.159	0.395-1.533	0.0021-0.1603	61-6180	27
Barton and Lin (1955)	4.50	1.26	508.0-1321	0.161-0.872	1.428-3.428	0.0119-0.1141	19-3776	28
Stein (1965)	10.0	1.50	228.6-777.2	0.243-1.664	0.747-2.467	0.0045-0.3118	93-39293	57
Nordin (1976)	6.25	1.44	951.0-3438	0.222-1.128	1.308-3.722	0.0041-0.2744	18-17200	45
Foley (1975)	7.25	1.37	102.0-162.9	0.656-1.375	0.953-1.554	0.0393-0.2193	845-11693	12
Taylor (1971)	5.70	1.52	346.2-701.8	0.278-0.988	1.106-2.653	0.0111-0.1146	14-2270	13
Williams (1970)	33.8	1.20	20.1-164.8	0.343-3.504	0.216-1.490	0.0020-0.5207	10-34575	175
Kennedy (1961)	5.83-13.7	1.14-1.47	41.1-465.7	0.499-1.964	0.639-4.137	0.0355-0.7779	490-58500	41
Brooks (1957)	2.20-3.63	1.11-1.17	325.8-983.7	0.274-0.799	2.545-8.507	0.0425-0.2759	190-5300	21
Vanoni and Brooks (1957)	3.43	1.38	527.3-1230	0.252-0.810	2.061-4.377	0.0078-0.1613	37-3000	14
Nomicos (1956)	3.80	1.76	483.3-508.7	0.287-0.956	2.246-3.755	0.0323-0.2136	300-5600	12
Laursen (1958)	2.75	1.20	692.7-2757	0.243-0.863	4.440-6.626	0.0224-0.1580	140-5150	16
Davis (1971)	3.75	1.17	508.0-2032	0.190-0.623	2.083-3.844	0.0073-0.1024	11-1760	70
Pratt (1970)	12.0	1.11	159.4-956.5	0.210-0.502	0.407-1.074	0.0016-0.0195	12-560	29
Singh(1960)	15.5	1.16	23.6-329.4	0.313-1.244	0.269-0.954	0.0041-0.1355	19-9200	286
Znamenskaya (1963)	20.0	1.60	62.5-254.9	0.422-1.213	0.298-0.862	0.0055-0.0478	126-3000	26
Straub (1954)	4.78	1.40	218.6-1232	0.399-1.299	1.800-2.626	0.0222-0.2788	423-12600	18
Krishnappan and Engel (1988)	30.0	1.00	118.1-137.9	0.459-0.765	0.283-0.745	0.0040-0.0451	88-2087	15
Wang et al. (1998)	2.78	1.94	845.8-1229	0.329-1.128	6.894-13.716	0.1045-0.9641	13750-118180	35

Table 3.11. Applicability test of formulas according to dimensionless diameter D_j (all data) (Yang and Huang, 2001)

Author of formula	D_j = 1.56-6.25 (d = 0.0625-0.25mm)			D_j = 6.25-20.0 (d = 0.25-0.8mm)			D_j = 20.0-50.0 (d = 0.8-2.0mm)			D_j = 50.0-720.0 (d = 2.0-28.8mm)			N_T
	R			R			R			R			
	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	
Ackers and White (1973)	1.31	77%	505	1.06	95%	1700	1.07	89%	491	1.26	74%	535	3231
Einstein (1950)	0.23	30%	505	1.38	52%	1703	1.77	53%	523	2.45	25%	553	3284
Einstein (1950)	0.55	46%	505	1.42	64%	1703	1.83	52%	523	2.49	21%	553	3284
Engelund and Hansen (1967)	0.87	82%	505	1.22	88%	1703	1.31	83%	523	1.63	72%	553	3284
Kalinske (1947)	1.23	49%	505	1.88	33%	1703	3.62	9%	523	5.84	4%	553	3284
Laursen (1958)	1.26	82%	495	1.29	85%	1690	1.48	67%	491	2.11	43%	473	3149
Meyer-Peter and Müller (1948)	0.16	11%	502	0.61	60%	1617	0.44	36%	374	0.58	63%	308	2801
Rottner (1959)	0.63	58%	505	1.84	47%	1703	3.77	11%	523	8.34	3%	553	3284
Schoklitsch (1934)	0.43	39%	488	0.82	83%	1242	1.25	73%	224	1.31	85%	284	2238
Toffaleti (1968)	0.21	26%	505	0.38	35%	1703	0.79	54%	523	1.68	48%	553	3284
Yang (sand) (1973)	1.06	90%	505	1.04	93%	1703	1.24	86%	523	9.86	6%	528	3259
Yang (sand) (1979)	0.99	94%	505	1.01	96%	1703	1.21	85%	523	8.85	7%	528	3259
Yang (gravel) (1984)	0.03	1%	505	0.29	24%	1703	0.66	53%	523	0.89	81%	528	3259

Note: R = discrepancy ratio; $\overline{R}$ = average discrepancy ratio; N = number of data sets; N_T = total number of data

Table 3.12. Range of application of median sediment particle size (Yang and Huang, 2001)

Author of formula	Median particle diameter (mm)
Ackers and White (1973)	0.065–32 (coarse silt–coarse gravel)
Einstein Bedload (1950)	0.25–32 (medium sand–coarse gravel)
Einstein Bed material (1950)	0.0625–32 (coarse silt–coarse gravel)
Engelund and Hansen (1967)	0.0625–32 (coarse silt–coarse gravel)
Kalinske (1947)	0.0625–2 (coarse silt–coarse sand)
Laursen (1958)	0.0625–2 (coarse silt–coarse sand)
Meyer-Peter and Müller (1948)	2.0–32 (very coarse sand–coarse gravel)
Rottner (1959)	0.0625–2 (coarse silt–very coarse sand)
Schoklitsch (1934)	0.25–32 (median sand–very coarse gravel)
Toffaleti (1968)	0.25–32 (median sand–coarse gravel)
Yang (sand) (1973)	0.0625–2.0 (coarse silt–very coarse sand)
Yang (sand) (1979)	0.0625–2.0 (coarse silt–very coarse sand)
Yang (gravel) (1984)	2.0–32 (very coarse sand–coarse gravel)

Table 3.13. Applicability test of formulas according to relative depth *D/d* (using applicable data) (Yang and Huang, 2001)

Author of formula	D/d = 4.0–50			D/d = 50–200			D/d = 200–1000			D/d = 1000–50,000			N_T
	R			R			R			R		N	
	$\overline{R}$	0.5–2.0	N	$\overline{R}$	0.5–2.0	N	$\overline{R}$	0.5–2.0	N	$\overline{R}$	0.5–2.0		
Ackers and White (1973)	1.27	75%	589	1.08	94%	1561	1.05	90%	646	1.28	79%	436	3232
Einstein (bedload) (1950)	2.10	32%	624	1.66	52%	1521	1.46	50%	448	0.76	46%	186	2779
Einstein (bed material) (1950)	2.17	31%	624	1.60	52%	1577	1.41	62%	647	0.55	68%	436	3284
Engelund and Hansen (1967)	1.68	73%	624	1.23	85%	1577	1.17	91%	647	0.82	83%	436	3284
Kalinske (1947)	3.76	11%	289	2.20	28%	1385	1.65	38%	621	1.28	46%	436	2731
Laursen (1958)	1.74	68%	266	1.31	81%	1356	1.23	84%	618	1.22	86%	436	2676
Meyer-Peter and Müller (1948)	0.63	71%	136	0.52	55%	150	0.68	68%	22	-	-	0	308
Rottner (1959)	4.46	9%	289	2.06	33%	1385	1.57	59%	621	0.70	69%	436	2731
Schoklitsch (1934)	1.25	81%	237	1.02	86%	931	0.74	80%	401	0.71	68%	181	1750
Toffaleti (1968)	1.56	49%	624	0.52	42%	1521	0.37	32%	448	0.32	30%	186	2779
Yang (sand) (1973)	1.24	86%	289	1.10	90%	1385	1.06	93%	621	1.02	95%	436	2731
Yang (sand) (1979)	1.25	85%	289	1.04	93%	1385	1.01	96%	621	1.00	97%	436	2731
Yang (gravel) (1984)	0.83	79%	264	0.96	83%	238	0.82	84%	26	-	-	0	528

Note: R = discrepancy ratio; $\overline{R}$ = average discrepancy ratio; N = number of data sets; N_T = total number of data

Table 3.14. Applicability test of formulas according to Froude number *Fr* (using applicable data) (Yang and Huang, 2001)

Author of formula	*Fr* = 0.10–0.40			*Fr* = 0.40–0.80			*Fr* = 0.80–1.20			*Fr* = 1.20–3.60			N_T
	R			R			R			R		N	
	$\overline{R}$	0.5–2.0	N	$\overline{R}$	0.5–2.0	N	$\overline{R}$	0.5–2.0	N	$\overline{R}$	0.5–2.0		
Ackers and White (1973)	1.09	88%	641	1.08	94%	1349	1.11	84%	644	1.33	78%	597	3231
Einstein (bedload) (1950)	1.12	62%	421	2.23	42%	1237	1.93	49%	564	0.56	44%	557	2779
Einstein (bed material) (1950)	0.88	47%	647	1.90	50%	1387	2.22	49%	653	0.63	66%	597	3284
Engelund and Hansen (1967)	1.61	80%	647	1.27	83%	1387	1.14	87%	653	0.93	85%	597	3284
Kalinske (1947)	1.44	44%	639	1.91	35%	1162	2.34	24%	424	3.13	13%	506	2731
Laursen (1958)	1.39	69%	611	1.33	84%	1138	1.32	88%	421	1.21	84%	506	2676
Meyer-Peter and Müller (1948)	-	-	-	0.68	72%	94	0.54	60%	174	0.52	55%	40	308
Rottner (1959)	0.51	31%	659	3.25	39%	1142	2.48	44%	424	0.64	62%	506	2731
Schoklitsch (1934)	1.29	80%	47	1.16	85%	611	0.87	82%	537	0.78	79%	555	1750
Toffaleti (1968)	0.34	32%	421	0.55	40%	1237	0.76	47%	564	1.32	45%	557	2779
Yang (sand) (1973)	1.18	88%	659	1.07	91%	1142	1.04	92%	424	1.02	95%	506	2731
Yang (sand) (1979)	1.14	90%	659	1.03	93%	1142	1.01	96%	424	0.99	97%	506	2731
Yang (gravel) (1984)	0.74	75%	8	0.86	79%	216	0.91	82%	263	0.94	87%	41	528

Note: R = discrepancy ratio; $\overline{R}$ = average discrepancy ratio; N = number of data sets; N_T = total number of data

Table 3.15. Applicability test of formulas according to relative shear velocity U_* /ω (using applicable data) (Yang and Huang, 2001)

Author of formula	U_* /ω = 0.18-0.40			U_* /ω = 0.40-1.00			U_* /ω = 1.00-2.50			U_* /ω = 2.50-15.00			N_T
	R			R			R			R			
	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	
Ackers and White (1973)	1.30	80%	1030	0.97	96%	1237	1.06	90%	552	1.32	80%	412	3231
Einstein (bedload) (1950)	2.00	35%	1081	1.42	58%	1229	1.53	43%	461	0.65	45%	28	2799
Einstein (bed material) (1950)	2.07	33%	1081	1.47	57%	1239	1.33	74%	552	0.57	58%	412	3284
Engelund and Hansen (1967)	1.64	76%	1081	1.08	89%	1239	1.11	86%	552	0.92	84%	412	3284
Kalinske (1947)	3.38	11%	640	1.97	31%	1127	1.51	40%	552	1.21	52%	412	2731
Laursen (1958)	1.49	74%	601	1.28	82%	1115	1.26	84%	548	1.25	85%	412	2676
Meyer-Peter and Müller (1948)	0.60	65%	212	0.55	60%	93	-	-	-	-	-	-	305
Rottner (1959)	3.54	13%	640	2.20	44%	1127	0.82	73%	552	0.55	41%	412	2731
Schoklitsch (1934)	1.27	82%	372	0.91	85%	910	0.80	77%	441	0.63	65%	27	1750
Toffaleti (1968)	1.12	49%	1081	0.48	38%	1229	0.39	30%	461	0.30	28%	28	2799
Yang (sand) (1973)	1.17	88%	640	1.06	92%	1127	1.05	93%	552	1.01	94%	412	2731
Yang (sand) (1979)	1.14	90%	640	1.03	94%	1127	1.01	96%	552	0.98	95%	412	2731
Yang (gravel) (1984)	0.88	80%	386	0.92	84%	142	-	-	-	-	-	-	528

Note: R = discrepancy ratio; $\overline{R}$ = average discrepancy ratio; N = number of data sets; N_T = total number of data

Table 3.16. Applicability test of formulas according to dimensionless unit stream power VS/ω (using applicable data) (Yang and Huang, 2001)

Author of formula	VS/ω = 0.0005-0.10			VS/ω = 0.10-0.02			VS/ω = 0.05-0.10			VS/ω = 0.10-2.50			N_T
	R			R			R			R			
	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	
Ackers and White (1973)	1.18	87%	847	1.09	90%	1141	1.02	94%	505	1.23	81%	738	3231
Einstein (bedload) (1950)	2.21	43%	897	1.69	43%	1105	1.39	60%	361	0.67	54%	416	2779
Einstein (bed material) (1950)	2.22	42%	897	1.70	49%	1144	1.38	67%	505	0.54	59%	738	3284
Engelund and Hansen (1967)	1.57	73%	897	1.23	87%	1144	1.18	90%	505	0.94	87%	738	3284
Kalinske (1947)	3.63	11%	513	2.15	29%	986	1.57	36%	494	1.30	46%	738	2731
Laursen (1958)	1.65	72%	476	1.25	83%	971	1.27	82%	491	1.23	84%	738	2676
Meyer-Peter and Müller (1948)	0.63	68%	176	0.53	59%	121	0.46	37%	8	-	-	-	305
Rottner (1959)	4.18	11%	513	2.17	41%	986	1.44	62%	494	0.58	52%	738	2731
Schoklitsch (1934)	1.29	83%	121	1.09	87%	904	0.82	82%	314	0.66	71%	411	1750
Toffaleti (1968)	1.31	47%	897	0.50	40%	1105	0.37	37%	361	0.31	32%	416	2779
Yang (sand) (1973)	1.21	85%	513	1.08	91%	986	1.05	92%	494	1.02	95%	738	2731
Yang (sand) (1979)	1.22	84%	513	1.02	96%	986	1.01	95%	494	0.98	97%	738	2731
Yang (gravel) (1984)	0.85	78%	334	0.96	86%	1891	0.92	91%	11	-	-	-	2236

Note: R = discrepancy ratio; $\overline{R}$ = average discrepancy ratio; N = number of data sets; N_T = total number of data

Table 3.17. Applicability test of formulas according to sediment concentration C (using applicable data) (Yang and Huang, 2001)

Author of formula	C = 10.0-100 ppm			C = 100-1000 ppm			C = 1000-10,000 ppm			C = 10,000-20,000 ppm			N_T
	R			R			R			R			
	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	$\overline{R}$	0.5-2.0	N	
Ackers and White (1973)	1.22	78%	480	1.14	87%	1211	1.05	94%	1152	1.26	85%	388	3231
Einstein (bedload) (1950)	2.39	28%	505	1.49	47%	1185	1.58	54%	993	0.77	57%	116	2779
Einstein (bed material) (1950)	2.44	24%	521	1.51	55%	1223	1.49	61%	1152	0.50	54%	388	3284
Engelund and Hansen (1967)	1.55	74%	521	1.39	84%	1223	1.09	88%	1152	0.88	82%	388	3284
Kalinske (1947)	4.72	7%	204	2.28	28%	1079	1.71	36%	1060	1.24	41%	388	2731
Laursen (1958)	1.86	71%	178	1.34	80%	1052	1.20	84%	1058	1.34	81%	388	2676
Meyer-Peter and Müller (1948)	0.66	73%	77	0.59	64%	109	0.53	57%	112	0.57	61%	7	305
Rottner (1959)	4.25	12%	204	2.19	35%	1079	1.82	46%	1060	0.68	67%	388	2731
Schoklitsch (1934)	1.29	81%	96	1.11	85%	662	0.84	83%	878	0.66	58%	114	1750
Toffaleti (1968)	1.49	49%	505	0.66	42%	1185	0.42	37%	993	0.32	28%	116	2799
Yang (sand) (1973)	1.28	85%	204	1.09	89%	1079	1.06	93%	1060	1.02	95%	388	2731
Yang (sand) (1979)	1.30	83%	204	1.05	92%	1079	1.01	96%	1060	0.99	97%	388	2731
Yang (gravel) (1984)	0.78	76%	203	0.91	83%	181	1.03	87%	137	0.91	86%	7	528

Note: R = discrepancy ratio; $\overline{R}$ = average discrepancy ratio; N = number of data sets; N_T = total number of data

Table 3.18. Comparison of equations of Yang (1973) and Yang (1979) for sand transport (Yang and Huang, 2001)

Author of formula	*C* = 10.0–40.0 ppm			*C* = 40.0–70.0 ppm			*C* = 70.0–100.0 ppm		
	Discrepancy ratio		No. of data sets	Discrepancy ratio		No. of data sets	Discrepancy ratio		No. of data sets
	Mean	0.5–2.0		Mean	0.5–2.0		Mean	0.5–2.0	
Yang (1973)	1.46	80%	37	1.30	84%	58	1.21	87%	109
Yang (1979)	1.52	73%	37	1.33	80%	58	1.21	88%	109
	C = 100–1000 ppm			*C* = 1000–10,000			*C* = 10,000–200,000		
Yang (1973)	1.09	89%	1079	1.06	93%	1060	1.02	95%	388
Yang (1979)	1.09	92%	1079	1.01	96%	1060	0.99	97%	388

Table 3.19. Summary of comparison of accuracy of formulas in their applicable ranges (Yang and Huang, 2001)

Author of formula	Discrepancy ratio		No. of data sets
	Mean	Data between 0.5 and 2.0	
For coarse silt to very coarse sand, d_{50} = 0.0625–2 mm			
Yang (1979)	1.04	94%	2731
Yang (1973)	1.08	91%	2731
Ackers and White (1973)	1.11	90%	2696
Engelund and Hansen (1967)	1.17	93%	2731
Laursen (1958)	1.32	81%	2676
Einstein (bed material) (1950)	1.34	58%	2731
Rottner (1959)	1.99	42%	2731
Kalinske (1947)	2.09	31%	2731
For medium sand to coarse gravel, d_{50} = 0.25–32 mm			
Schoklitsch (1934)	0.85	82%	1750
Toffaleti (1968)	0.72	41%	2779
Einstein (bedload) (1950)	1.67	47%	2779
For very coarse sand to coarse gravel, d_{50} = 2–32 mm			
Yang (1984)	0.89	81%	528
Meyer-Peter and Müller (1948)	0.58	63%	308
For coarse silt to coarse gravel, d_{50} = 0.0625–32 mm			
Yang (1979) and Yang (1984)	1.02	91%	3259
Yang (1973) and Yang (1984)	1.05	89%	3259
Ackers and White (1973)	1.13	88%	3231
Engelund and Hansen (1967)	1.25	84%	3284
Einstein (bed-material) (1950)	1.53	52%	3284

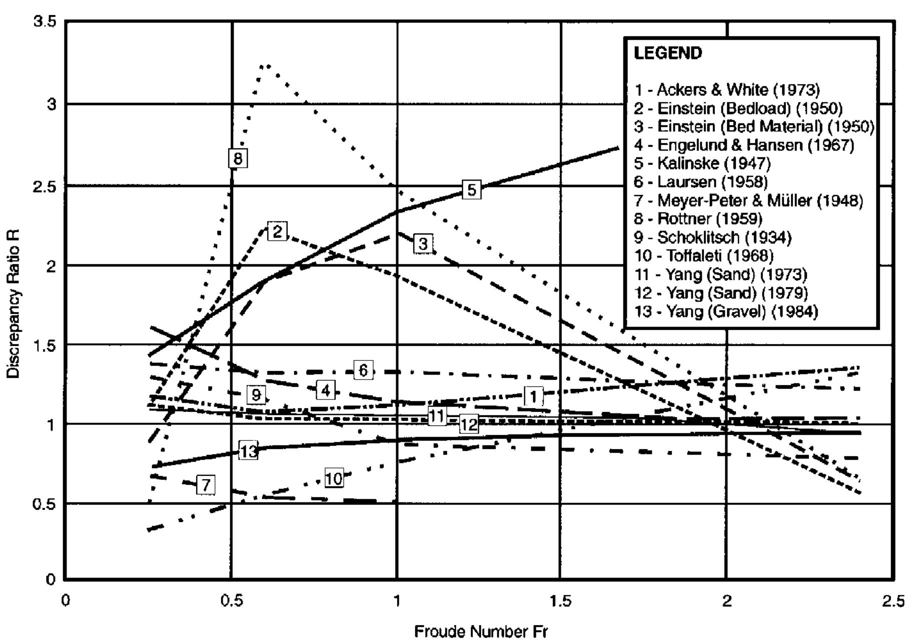


Figure 3.47. Comparison of discrepancy ratio based on Froude number.

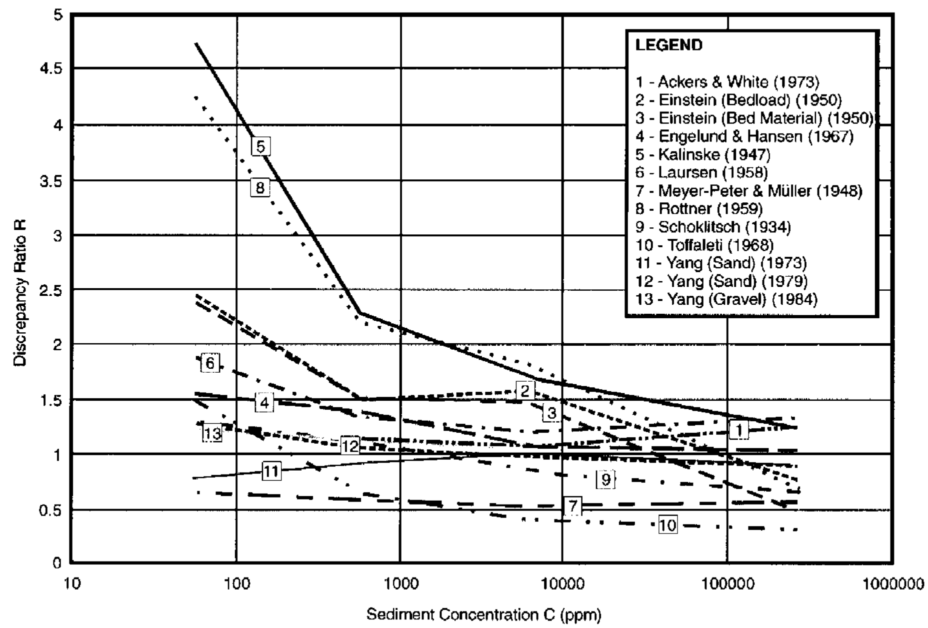


Figure 3.48. Comparison of discrepancy ratio based on concentration.

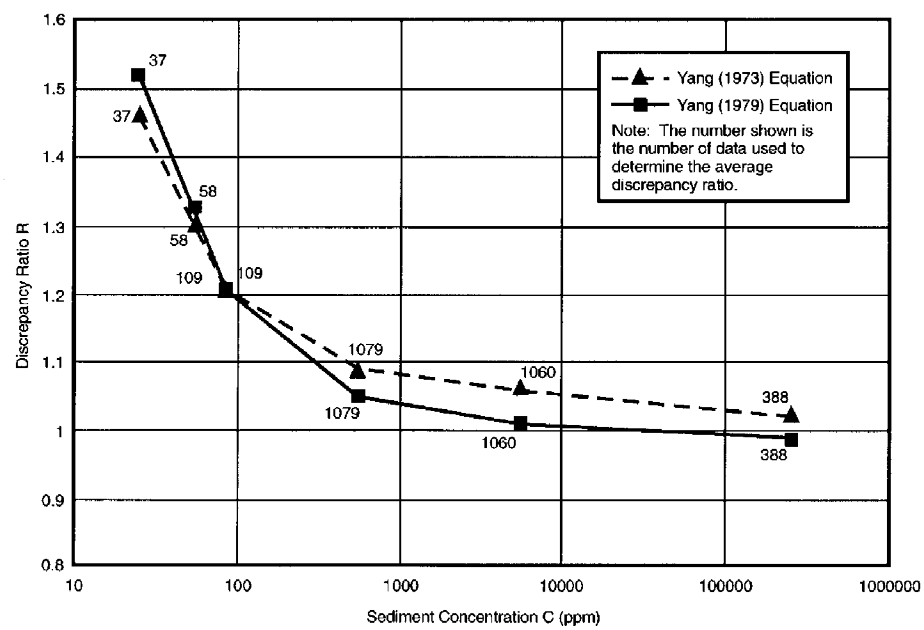


Figure 3.49. Comparison of equations of Yang (1973) and Yang (1979) for sand transport.

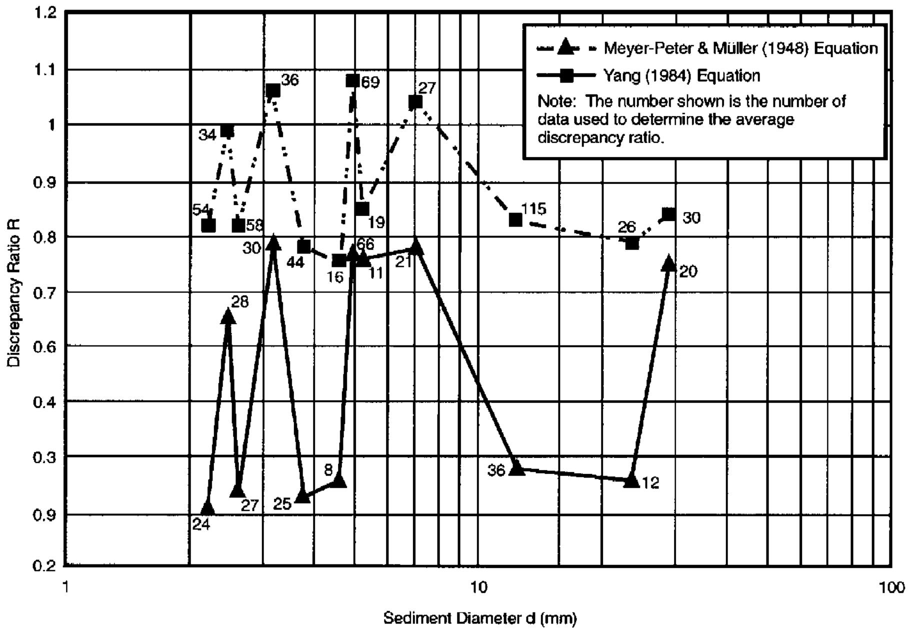


Figure 3.50. Comparison of equations of Meyer-Peter and Müller (1948) and Yang (1984) for gravel transport.

Most of the river sediment transport studies involve sediments in the coarse silt to coarse gravel size range. Table 3.19 indicates that the priority of selection should be Yang (1979) for $d_{50} < 2$ mm plus Yang (1984) for $d_{50} > 2$ mm, followed by Yang (1973) for $d_{50} < 2$ mm plus Yang (1984) for $d_{50} > 2$ mm, and then followed by Ackers and White (1973) and Engelund and Hansen (1967). If the local conditions on the range of variations of dimensionless particle diameter, relative depth, Froude number, relative shear velocity, dimensionless unit stream power, and measured bed-material load concentration are available, Tables 3.11 to 3.19 should be used as references to finalize the selection of the most appropriate formula for engineers to use.

The analyses by Yang and Huang (2001) reached the following conclusions:

- Sediment transport formulas based on energy dissipation rate or the power concept are more accurate than those based on other concepts. Yang's (1973, 1979, 1984) formulas were derived directly from the unit stream power theory, while the formulas by Engelund and Hansen (1967) and by Ackers and White (1973) were obtained indirectly from Bagnold's (1966) stream power concept.

- Among the 13 formulas compared, Yang's 1973, 1979, and 1984 formulas are the most robust, and their accuracies are least sensitive to the variation of relative depth, Froude number, dimensionless shear velocity, dimensionless unit stream power, and sediment concentration.

- With the exception of Yang's (1973, 1979, and 1984) dimensionless unit stream power formulas and Engelund and Hansen's (1967) formula, the application of other sediment transport formulas should be limited to subcritical flows.

- Engineers should use Table 3.19 as a reference for the preliminary selection of appropriate formulas for different size ranges of sediment particle diameter. Tables 3.13 to 3.17 should be used to determine whether a formula is suitable for a given range of dimensionless parameters before the final selection of formula is made.

- Yang's 1973 and 1979 sand transport formulas have about the same degree of accuracy. However, the 1973 formula with incipient motion criteria is slightly more accurate when the sand concentration is less than 100 ppm, while the 1979 formula without incipient motion criteria is slightly more accurate for concentrations higher than 100 ppm.

- The Einstein bed-material load (1950) and bedload (1950) formulas and those by Toffaleti (1958) and Meyer-Peter and Müller are not as accurate as those formulas based on the power approach. Some engineers use the Meyer-Peter and Müller formula for bedload, and the Einstein bed-material or Toffaleti formula for suspended load for the estimate of total load. This kind of combined use may not be justified from a theoretical point of view nor from the accuracies of these equations based on the results shown in this chapter.

3.8.4.3 Procedures for Selecting Sediment Transport Formulas

No perfect assumption exists that can be used to derive a sediment transport formula. However, the generalities of these assumptions do differ. Based on the majority of published data, it appears that unit stream power dominates the rate of sediment transport or sediment concentration more than any other variable. Even if perfect assumptions could be found and used in the derivation of a formula, the coefficients in the formula would still have to be determined by comparing the mathematical model and measured data. Thus, the applicability of a formula depends not only on the assumptions and theories used in its derivation, but also on the range of data used for the determination of the coefficients in the formula. Sediment discharge in natural rivers depends not only on the independent variables mentioned in previous sections, but also on the gradation and shape factor of sediment, the percentage of bed surface covered by coarse material, the availability of bed material for transport, variations in the hydrologic cycle, the rate of supply of fine material or wash load, the water temperature, the channel pattern and bed configuration, the strength of turbulence, etc. Because of the tremendous uncertainties involved in estimating sediment discharge at different flow and sediment conditions under different hydrologic, geologic, and climatic constraints, it is extremely difficult, if not impossible, to recommend one formula for engineers and geologists to use in the field under all circumstances (Yang, 1996). The following procedures are based on the recommendations made by Yang (1977, 1980, 1996) with minor modifications.

Step 1: Determine the kind of field data available or measurable within the time, budget, and staffing limits.

Step 2: Examine all the formulas and select those with measured values of independent variables determined from step 1.

Step 3: Compare the field situation and the limitations of formulas selected in step 2. If more than one formula can be used, calculate the rate of sediment transport by these formulas and compare the results.

Step 4: Decide which formulas can best agree with the measured sediment load, and use these to estimate the rate of sediment transport at those flow conditions when actual measurements are not possible.

Step 5: In the absence of measured sediment load for comparison, the following formulas or procedures should be considered:

- Use Meyer-Peter and Müller's formula when the bed material is coarser than 5 mm;
- Use Einstein's bedload transport function when bedload is a significant portion of the total load;
- Use Toffaleti's formula for large sand-bed rivers;
- Use Colby's formula for rivers with depth less than 10 ft;
- Use Shen and Hung's regression formula for laboratory flumes and very small rivers;
- Use Karim and Kennedy's regression formula for natural rivers with a wide range of variations of flow and sediment conditions;
- Use Yang's (1973) formula for sand transport in laboratory flumes and natural rivers;
- Use Yang's (1979) formula for sand transport when the critical unit stream power at incipient motion can be neglected;
- Use Yang's (1984) or Parker's (1990) gravel formulas for bedload or gravel transport;
- Use the modified Yang (1996) formula for nonequilibrium, high-concentration flows when wash load or concentration of fine material is high;
- Use Ackers and White's or Engelund and Hansen's formula for the subcritical flow condition in the lower flow regime;
- Use Yang's formulas (1973, 1979, 1984) for subcritical, transition, and supercritical flow conditions in the lower and upper flow regimes;
- Use Laursen's formula for laboratory flumes and shallow rivers with fine sand or coarse silt;
- Use Meyer-Peter and Müller's formula for bedload and the modified Einstein's formula for suspended load to obtain total load;
- A regime or regression formula can be applied to a river only if the flow and sediment conditions are similar to those from where the formula was derived;
- Select a formula according to its degree of accuracy, shown in Table 3.6;
- Based on the analyses of Yang and Huang (2001), select a formula that is most accurate under the given range of flow and sediment conditions.

Step 6: When none of the existing sediment transport formulas can give satisfactory results, use the existing data collected from a river station and plot sediment load or concentration against water discharge, velocity, slope, depth, shear stress, stream power, unit stream power or dimensionless unit stream power, and Velikanov's parameter. The least scattered curve without systematic deviation from a one-to-one correlation between dependent and independent variables should be selected as the sediment rating curve for the station.

3.9 Summary

This chapter comprehensively reviews and evaluates basic approaches and theories used in the determination of noncohesive sediment transport rate or concentration. The basic approaches used for the development of sediment transport functions or formulas are the regime, regression, probabilistic, and deterministic approaches. The concept that the rate of sediment transport should be directly related to the rate of energy dissipation rate in transporting sediment has gained increasing acceptance in recent years. Formulas derived from the power approach are those based on stream power (Bagnold, Engelund and Hansen, and Ackers and White), unit stream power (Yang), power balance (Pacheco-Ceballos), and gravitational power (Velikanov, Dou, and Zhang). Comparisons between measured results and computed results from different formulas indicate that, on the average, formulas derived from the power approach, especially the unit stream power approach, can more accurately predict sediment transport rate than formulas derived from other approaches.

Due to the complexity of flow and sediment conditions of natural rivers, recommendations are made for engineers to select appropriate formulas under different flow and sediment conditions. Sediment particle fall velocity and resistance to flow are two of the important parameters used in sediment transport and fluvial hydraulic computations. This chapter compares and evaluates different methods used for fall velocity computation and the estimation of resistance to flow or roughness coefficient for alluvial channels. This chapter also addresses the need to consider nonequilibrium sediment transport and the impact of wash load on sediment transport.

3.10 References

Ackers, P., and W.R. White (1973). "Sediment Transport: New Approach and Analysis," *Journal of the Hydraulics Division*, ASCE, vol. 99, no. HY11, pp. 2041-2060.

Alonso, C.V. (1980). "Selecting a Formula to Estimate Sediment Transport Capacity in Nonvegetated Channels," *CREAMS A Field Scale Model for Chemicals, Runoff, and Erosion from Agricultural Management System*, edited by W.G. Knisel, U.S.D.A. Conservation Research Report no. 26, Chapter 5, pp. 426-439.

Alonso, C.V., W.H. Neibling, and G.R. Foster (1982). "Estimating Sediment Transport Capacity in Watershed Modeling," *Transactions of the ASCE*, vol.24, no.5, pp. 1211-1220 and 1226.

American Society of Civil Engineers (ASCE), Task Committee on Relations Between Morphology of Small Stream and Sediment Yield (1982). "Relationships Between Morphology of Small Streams and Sediment Yield," *Journal of the Hydraulics Division*, ASCE, vol. 108, no. HY11, pp. 1328-2365.

Bagnold, R.A. (1966). *An Approach to the Sediment Transport Problem from General Physics*, U.S. Geological Survey Professional Paper 422-J.

Bishop, A.A., D.B. Simons, and E.V. Richardson (1965). "A Total Bed-Material Transport," *Journal of the Hydraulics Division*, ASCE, vol. 91, no. HY2, pp. 175-191.

Blench, T. (1969). Mobile-Bed Fluvialogy, A Regime Theory Treatment of Canals and Rivers for Engineers and Hydrologists, The University of Alberta Press, Edmonton, Alberta, Canada.

Brooks, N.H. (1955). "Mechanics of Streams with Moveable Beds of Fine Sand," *Proceedings of the ASCE*, vol. 81, no. 668, pp. 668-1 through 668-28.

Brownlie, W.R. (1981). "Prediction of Flow Depth and Sediment Discharge in Open Channels," W.M. Keck Laboratory of Hydraulics and Water Resources, Report no. KH-R-43A, California Institute of Technology, Pasadena, California.

Bureau of Reclamation (1987). *Design of Small Dams*, Denver, Colorado.

Cassie, H.J. (1935). *Über Geschiebebewegung*, Preuss. Versuchsanst für Wasserbau und Schifibau, Berlin, Mitt., vol. 19 (translation on file at the U.S. Soil Conservation Service, Washington, D.C.).

Chang, H.H. (1991). *Test and Calibration on Fluvial-12 Model Using Data from Stony Creek*, Report prepared for the California Department of Transportation.

Chang, H.H. (1994). "Selection of Gravel-Transport Formula for Stream Modeling," *Journal of Hydraulic Engineering*, ASCE, vol. 120, no. 5, pp. 646-651.

Colby, B.R. (1964). "Practical Computations of Bed-Material Discharge," *Journal of the Hydraulics Division*, ASCE, vol. 90, no. HY2.

Colby, B.R., and C.H. Hembree (1955). *Computation of Total Sediment Discharge, Niobrara River Near Cody, Nebraska*, U.S. Geological Survey Water Supply Paper 1357.

Dou, G. (1974). "Similarity Theory and Its Application to the Design of Total Sediment Transport Model," *Research Bulletin of Nanjing Hydraulic Research Institute*, Nanjing, China (in Chinese).

DuBoys, M.P. (1879). "LeRhône et les Rivères à lit affouillable," *Annales de Ponts et Chausées*, Sel 5, vol. 18, pp. 141-195.

Einstein, H.A. (1950). *The Bedload Function for Sediment Transport in Open Channel Flow*, U.S. Department of Agriculture Soil Conservation Technical Bulletin No. 1026.

Einstein, H.A., and N.L. Barbarossa (1952). "River Channel Roughness," *Transactions of the ASCE*, vol. 117, pp. 1121-1132.

Einstein, H.A., and N. Chien (1952). *Second Approximation to the Solution of Suspended Load Theory*, Institute of Engineering Research, University of California, Issue 2, Series 47.

Engelund, F., and E. Hansen (1972). *A Monograph on Sediment Transport in Alluvial Streams*, Teknisk Forlag, Copenhagen.

Fortier, S., and F.C. Scobey (1926). "Permissible Canal Velocities," *Transactions of the ASCE*, vol.89.

German Association for Water and Land Improvement (1990). *Sediment Transport in Open Channels—Calculation Procedures for the Engineering Practice*, Bulletin no. 17, Verlag Paul Parey, Hamburg and Berlin, Germany.

Gilbert, K.G. (1914). *The Transportation of Debris by Running Waters*, U.S. Geological Survey Professional Paper 86.

Govers, G. (1987). "Initiation of Motion in Overland Flow," *Sedimentology*, no. 34, pp. 1157-1164.

Guy, H.P., D.B. Simons, and E.V. Richardson (1966). *Summary of Alluvial Channel Data from Flume Experiments, 1956-1961*," U.S. Geological Survey Professional Paper 462-1.

Han, Q. (1980). "A Study on the Non-Equilibrium Transport of Suspended Load," *Proceedings of the International Symposium on River Sedimentation*, Beijing, China, pp. 793-802 (in Chinese).

Han, Q., and M. He (1990). "A Mathematical Model for Reservoir Sedimentation and Fluvial Processes," *International Journal of Sediment Research*, vol. 5, no. 2, pp. 43-84.

Hjulstrom, F. (1935). "The Morphological Activity of Rivers as Illustrated by River Fyris," *Bulletin of the Geological Institute*, Uppsala, vol. 25, ch. 3.

HR Wallingford (1990). Sediment Transport, the Ackers and White Theory Revised, Report SR237, England.

Julien, P.Y. (1995). *Erosion and Sedimentation*, Cambridge, University Press, Cambridge, UK.

Kalinske, A.A. (1947). "Movement of Sediment as Bedload in Rivers," *American Geophysical Union Transactions*, vol. 28, pp. 615-620.

Karim, M.F., and J.F. Kennedy (1990). "Means of Coupled Velocity and Sediment-Discharge Relationships for Rivers," *Journal of Hydraulic Engineering*, ASCE, vol. 116, no. 8, pp. 973-996.

Kennedy, R.G. (1895). "Prevention of Silting in Irrigation Canals," *Institute of Civil Engineers*, Proceeding Paper no. 2826.

Lacey, G. (1929). "Stable Channel in Alluvium," *Institute of Civil Engineers*, Proceeding Paper no. 4736.

Lane, E.W. (1953). "Progress Report on Studies on the Design of Stable Channels of the Bureau of Reclamation," *Proceedings of ASCE*, vol. 79.

Laursen, E.M. (1958). "The Total Sediment Load of Streams," *Journal of the Hydraulics Division*, ASCE, vol. 84, no. HY1, 1530-1 through 1530-36.

Leopold, L.B., and T. Maddock, Jr. (1953). *The Hydraulic Geometry of Stream Channels and Some Physiographic Implications*, U.S. Geological Survey Professional Paper 252.

Liu, H.K. (1958). "Closure to Mechanics of Sediment-Ripple Formation," *Journal of the Hydraulics Division*, ASCE, vol. 84, no. HY5, pp. 1832-10 through 1932-12.

Meyer-Peter, E., and R. Müller (1948). "Formulas for Bedload Transport," *Proceedings*, the Second Meeting of the International Association for Hydraulic Structures Research, Stockholm.

Molinas, A., and C.T. Yang (1986). Computer Program User's Manual for GSTARS (Generalized Stream Tube Model for Alluvial River Simulation), U.S. Bureau of Reclamation, Denver, Colorado.

Pacheco-Ceballos, P. (1989). "Transport of Sediments: Analytical Solution," *Journal of Hydraulic Research*, vol. 27, no. 4, pp. 501-518.

Parker, G. (1977). "Discussion of 'Minimum Unit Stream Power and Fluvial Hydraulics,' by C.T. Yang," *Journal of the Hydraulics Division*, ASCE vol. 103, no. HY7, pp. 811-816.

Parker, G. (1990). "Surface-Based Bedload Transport Relation for Gravel Rivers, *Journal of Hydraulics Research*, vol. 28, no. 4, pp. 501-518.

Rottner, J. (1959). "A Formula for Bedload Transportation," *LaHouille Blanche*, vol. 14, no. 3, pp. 285-307.

Rouse, H. (1939). *An Analysis of Sediment Transportation in the Light of Fluid Turbulence*, Soil Conservation Service Report no. SCS-TP-25, U.S. Department of Agriculture, Washington, D.C.

Rubey, W. (1933). "Settling Velocities of Gravel, Sand, and Silt Particles," *American Journal of Science*, vol. 25.

Schoklitsch, A. (1934). "Der Geschiebetrieb und die Geschiebefracht," *Wasserkraft und Wasserwirtschaft*, vol. 29, no. 4, pp. 37-43.

Schulits, S., and R.D. Hill, Jr. (1968). *Bedload Formulas*, Pennsylvania State University, College of Engineering, State College, Pennsylvania.

Shen, H.W., and C.S. Hung (1972). "An Engineering Approach to Total Bed-Material Load by Regression Analysis," *Proceedings of the Sedimentation Symposium*, ch. 14, pp. 14-1 through 14-17.

Shields, A. (1936). *Application of Similarity Principles and Turbulence Research to Bedload Movement*, California Institute of Technology, Pasadena (translated from German).

Shumm, S.A., and H.R. Khan (1972). "Experimental Study of Channel Patterns," *Geological Society of America*, Bulletin no. 83, p. 407.

Stein, R.A. (1965). "Laboratory Studies of Total Load and Apparent Bedload," *Journal of Geophysical Research*, vol. 70, no. 8, pp. 1831-1842.

Stevens, H.H., and C.T. Yang (1989). *Computer Programs for 13 Commonly Used Sediment Transport Formulas*, U.S. Geological Survey Water Resources Investigation Report 89-4026.

Talapatra, S.L., and S.N. Ghosh (1983). "Incipient Motion Criteria for Flow Over a Mobile Bed Sill," *Proceedings of the Second International Symposium on River Sedimentation*, Nanjing, China, pp. 459-471.

Toffaleti, F.B. (1969). "Definitive Computations of Sand Discharge in Rivers," *Journal of the Hydraulics Division*, ASCE, vol. 95, no. HY1, pp. 225-246.

U.S. Army Corps of Engineers (1977, 1993). *Generalized Computer Program, HEC-6, Scour and Deposition in Rivers and Reservoirs, Users' Manual*, the Hydrologic Engineering Center, Davis, California (March 1977; revised August 1993).

U.S. Army Corps of Engineers (1982). General Design Review Conference, Los Angeles District, Los Angeles, California.

U.S. Bureau of Reclamation (1987). *Design of Small Dams*. Denver, Colorado.

U.S. Interagency Committee on Water Resources, Subcommittee on Sedimentation (1957). *Some Fundamentals of Particle Size Analysis*, Report no. 12.

Vanoni, V.A., ed. (1975). *Sedimentation Engineering*, ASCE Task Committee for the Preparation of the Manual on Sedimentation of the Sedimentation Committee of the Hydraulics Division (reprinted 1977).

Vanoni, V.A. (1978). "Predicting Sediment Discharge in Alluvial Channels," *Water Supply and Management*, Pergamon Press, Oxford, pp. 399-417.

Velikanov, M.A. (1954). "Gravitational Theory of Sediment Transport," *Journal of Science of the Soviet Union Geophysics*, vol. 4 (in Russian).

Vetter, M. (1989). *Total Sediment Transport in Open Channels*, Report no. 26, Institute of Hydrology, University of the German Federal Army, Munich, Germany (translated from *Gesamttransport van Sedimenten in Offenen Gerinnen* into English by the U.S. Bureau of Reclamation, Denver, Colorado).

White, C.M. (1940). "The Equilibrium of Grains on the Bed of an Alluvial Channel," *Proceedings of the Royal Society of London, Series A*, vol. 174, pp. 332-338.

White, W.R., H. Milli, and A.D. Crabe (1975). "Sediment Transport Theories: A Review," *Proceedings of the Institute of Civil Engineers*, London, Part 2, no. 59, pp. 265-292.

Yalin, MS., and E. Karahan (1979). "Inception of Sediment Transport," *Journal of the Hydraulics Division*, ASCE, vol. 105, no. HY11, pp. 1433-1443.

Yang, C.T. (1971). "Potential Energy and River Morphology," *Water Resources Research,* vol. 7, no. 2, pp. 312-322.

Yang, C.T. (1972). "Unit Stream Power and Sediment Transport," *Journal of the Hydraulic Division*, ASCE, vol. 18, no. HY10, pp. 1805-1826.

Yang, C.T. (1973). "Incipient Motion and Sediment Transport," *Journal of the Hydraulic Division*, ASCE, vol. 99, no. HY10, pp. 1679-1704.

Yang, C.T. (1976). "Minimum Unit Stream Power and Fluvial Hydraulics," *Journal of the Hydraulics Division*, ASCE, vol. 102, no. HY7, pp. 919-934.

Yang, C.T. (1976). "Discussion of Sediment Transport Theories—A Review, by W.R. White, H. Milli, and A.D. Crabble," *Institute of Civil Engineering*, Part 2, vol. 61, pp. 803-810.

Yang, C.T. (1979). "The Movement of Sediment in Rivers," *Geophysical Survey 3*, D. Reidel, Dordrecht, pp. 39-68.

Yang, C.T. (1983). "Rate of Energy Dissipation and River Sedimentation," *Proceedings of the 2nd International Symposium on River Sedimentation*, Nanjing, China, pp. 575-585.

Yang, C.T. (1984). "Unit Stream Power Equation for Gravel," *Journal of Hydraulic Engineering, ASCE*, vol. 110, no. 12, pp. 1783-1797.

Yang, C.T. (1985). "Mechanics of Suspended Sediment Transport," *Proceedings of Euromech 192: Transport of Suspended Solids in Open Channels*, ed. W. Bechteler, Institute of Hydromechanics, University of the Armed Forces, Munich/Nuremberg, Germany, pp. 87-91.

Yang, C.T. (1996). *Sediment Transport Theory and Practice*, The McGraw-Hill Companies, Inc., New York (reprint by Krieger Publishing Company, Malabar, Florida, 2003).

Yang, C.T. (2002). "Sediment Transport and Stream Power," *International Journal of Sediment Research*, vol. 17, no. 1, Beijing, China, pp. 31-38.

Yang, C.T., and C. Huang (2001). "Applicability of Sediment Transport Formulas," *International Journal of Sediment Research*, vol. 16, no. 3, Beijing, China, pp. 335-343.

Yang, C.T., and A. Molinas (1982). "Sediment Transport and Unit Stream Power Function," *Journal of the Hydraulics Division*, ASCE, vol. 108, no. HY6, pp. 776-793.

Yang, C.T., and F.J.M. Simões (2000). *User's Manual for GSTARS 2.1 (Generalized Sediment Transport Model for Alluvial River Simulation Version 2.1)*, U.S. Bureau of Reclamation, Technical Service Center, Denver, Colorado.

Yang, C.T., and F.J.M. Simões (2002). *User's Manual for GSTARS3 (Generalized Sediment Transport Model for Alluvial River Simulation Version 3.0)*, U.S. Bureau of Reclamation, Technical Service Center, Denver, Colorado.

Yang, C.T., and F.J.M. Simões (2005). "Wash Load and Bed-Material Load Transport in the Yellow River," *Journal of Hydraulic Engineering*, ASCE, vol. 131, no. 5, pp. 413-418.

Yang, C.T., and C.C.S. Song (1979). "Theory of Minimum Rate of Energy Dissipation," *Journal of the Hydraulics Division*, ASCE, vol. 105, no. HY7, pp. 769-784.

Yang, C.T., and C.C.S. Song (1986). "Theory of Minimum Energy and Energy Dissipation Rate," *Encyclopedia of Fluid Mechanics*, Chapter 11, edited by N.P. Cheremisinoff, Gulf Publishing Company, Houston, Texas.

Yang, C.T., and S. Wan (1991). "Comparison of Selected Bed-Material Load Formulas," *Journal of Hydraulics Engineering*, ASCE, vol. 117, no. 8, pp. 973-989.

Yang, C.T., A. Molinas, and B. Wu (1996). "Sediment Transport in the Yellow River," *Journal of Hydraulic Engineering*, ASCE, vol. 122, no. 5, pp. 237-244.

Yang, C.T., C.C.S. Song, and M.J. Woldenberg (1981). "Hydraulic Geometry and Minimum Rate of Energy Dissipation," *Water Resources Research*, vol. 17, no. 4, pp. 1014-1018.

Yang, C.T., A. Molinas, and C.C.S. Song (1989). "GSTARS – Generalized Stream Tube Model for Alluvial River Simulation," *Twelve Selected Computer Stream Sedimentation Model Developed in the United States* (U.S. Interagency Subcommittee Report on Sedimentation), S.S. Fan (editor), Federal Energy Regulatory Commission, Washington DC, pp. 148-178.

Yang, C.T., M.A. Treviño, and F.J.M. Simões (1998). *User's Manual for GSTARS 2.0 (Generalized Stream Tube Model for Alluvial River Simulation Version 2.0)*, U.S. Bureau of Reclamation, Technical Service Center, Denver, Colorado.

Zhang, R. (1959). "A Study of the Sediment Transport Capacity of the Middle and Lower Yangtze River," *Journal of Sediment Research*, Beijing, China, vol. 4, no. 2 (in Chinese).