

### **Task Committee on Experimental Uncertainty and Measurement Errors in Hydraulic Engineering: An Update**

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#### *Abstract*

As part of their long range goals for disseminating information on measurement techniques, instrumentation, and experimentation in the field of hydraulics, the Technical Committee on Hydraulic Measurements and Experimentation formed the Task Committee on Experimental Uncertainty and Measurement Errors in Hydraulic Engineering in January 2003. The overall mission of this Task Committee is to provide information and guidance on the current practices used for describing and quantifying measurement errors and experimental uncertainty in hydraulic engineering and experimental hydraulics. The final goal of the Task Committee on Experimental Uncertainty and Measurement Errors in Hydraulic Engineering is to produce a report on the subject that will cover: (1) sources of error in hydraulic measurements, (2) types of experimental uncertainty, (3) procedures for quantifying error and uncertainty, and (4) special practical applications that range from uncertainty analysis for planning an experiment to estimating uncertainty in flow monitoring at gaging sites and hydraulic structures. Currently, the Task Committee has adopted the first order variance estimation method outlined by Coleman and Steele (1999) as the basic methodology to follow when assessing the uncertainty in hydraulic measurements. In addition, the Task Committee has begun to develop its report on uncertainty in hydraulic engineering. This paper is intended as an update on the Task Committee's overall progress.

## *Introduction*

Over the past 20 years, increasing emphasis has been placed on uncertainty analyses in experimental work. Consequently, many professional journals, such as the *Journal of Heat Transfer, Journal of Fluids Engineering,* and all of the journals from the American Institute of Aeronautics and Astronautics (AIAA), have adopted policies requiring some type of uncertainty analysis for all of their articles. Some of these policies do not specify the methodology to be used, but simply state that an uncertainty analysis needs to be performed. This is the case of the policy of AIAA that succinctly reads:

*The AIAA journals will not accept for publication any paper reporting (1) numerical solutions of an engineering problem that fails to adequately address accuracy of the computed results or (2) experimental results unless the accuracy of the data is adequately presented.*

The policies of some journals are more specific. For example the *Journal of Fluids Engineering* provides guidelines on the presentation of experimental data indicating that the precision limit, the bias limit, the total uncertainty, and a description of the methodology for estimating the uncertainty should be included. The *Journal of Fluids Engineering* also indicates that it is preferred that authors include the precision limit and the bias limits of each variable and parameter along with a statement comparing the precision scatter from repeated measurements with the expected scatter based on the uncertainty analysis. The guidelines include an example illustrating the propagation of uncertainty from various error sources and emphasize the importance of accounting for correlated and uncorrelated error sources, referring authors to ANSI/ASME PTC 19.1 and Coleman and Steele (1999) for a detailed discussion on the subject of the estimation of uncertainties from correlated error sources.

Currently, the American Society of Civil Engineers (ASCE) does not have a policy requiring an uncertainty analysis for any of its journals. There are a few publications dealing with uncertainty analyses in the civil engineering field (e.g., Ang and Tang 1975, Yen and Tung 1993, and Harr 1997). However, these works are focused mostly on reliability analyses pertinent to the design and safety of hydraulic structures rather than on uncertainty analyses for hydraulic measurements. Thus, as part of ASCE's long range goals for disseminating information on measurement techniques, instrumentation, and experimentation in the field of hydraulics, the Technical Committee on Hydraulic Measurements and Experimentation formed the Task Committee on Experimental Uncertainty and Measurement Errors in Hydraulic Engineering in January 2003.

The mission of this Task Committee is to provide information and guidance on practices for describing and quantifying measurement errors and experimental uncertainty in field and laboratory hydraulic measurements. The final goal of the Task Committee on Experimental Uncertainty and Measurement Errors in Hydraulic Engineering is to produce a report on the subject that will cover: (1) sources of error in hydraulic measurements, (2) types of experimental uncertainty, (3) procedures for quantifying error and uncertainty, and (4) special practical applications that range from uncertainty analysis for planning an experiment to estimating uncertainty in

flow monitoring at gaging sites and hydraulic structures. This report is designed to assist hydraulic engineers performing uncertainty analyses on their experimental results. It is also the first step toward the development of an ASCE policy for reporting experimental uncertainty in the *Journal of Hydraulic Engineering* .

## *Why is an Uncertainty Analysis Important?*

Every measurement has some error and imprecision associated with it regardless of how carefully the measurement was made. Generally and collectively this error and imprecision is called the *uncertainty*. The end results of analyses based on experimental measurements also contain uncertainty. Because uncertainty affects the usefulness of measurements and experimental results and decisions that might be made using them, it is important for engineers to be able to quantify uncertainty. In fact, an inadequate consideration of experimental error can seriously compromise the final design of large, complex systems like aircraft (AIAA 1999). Kline (1985a) outlines 12 reasons why an uncertainty analysis should be performed. Some of these reasons are discussed in the next few paragraphs.

An uncertainty analysis is a vital part of planning an experiment. It can be used before an experiment has even begun to determine if the end result will have the required accuracy. If there is no hope that the experiment can deliver an end product with the required accuracy, then there is no value in performing the experiment. The uncertainty analysis saves the time and money that would be wasted performing the useless experiment. In addition, an uncertainty analysis can identify the instruments or measurements that most influence the accuracy of the final result. Effort can then be concentrated on improving the accuracy of these vital instruments instead of on instruments that do not affect the end result significantly. Also, an uncertainty analysis can be a powerful tool used to locate sources of trouble in a misbehaving experiment (Moffat 1988).

An uncertainty analysis can also be used to help make decisions about policy. For example, if a proposed water conservation practice claims that it will save 5% of the annual volume of water that passes through a certain site and the annual volume can only be determined to within 10%, then the impact of the proposed water conservation practice cannot be measured by simply comparing it to the annual volume of water used, since it will be lost in the uncertainty of the accumulated volume estimate. With this knowledge, policymakers might seek other ways to evaluate the impact of the conservation practice, and they can avoid dismissing the practice simply because measurements of annual volume suggest that it is ineffective.

An uncertainty analysis can also provide a large financial savings. For example, Baker (2001) reports that an error in the energy content of natural gas of just 0.01  $MJ/m<sup>3</sup>$  (typically representing a 0.02 to 0.05% uncertainty) can have an economic impact that easily exceeds one billion dollars because of the large amounts of natural gas traded worldwide.

More recently, environmental compliance requires accurate estimates of total loads of nutrients and pollutants in water released into sensitive environmental preserves.

Total loads are typically computed as the product of concentrations of nutrients and pollutants in field samples and the estimated total flows. As a result, estimating the uncertainty of the flow estimates is a prerequisite for estimating the uncertainty of the total loads estimates. More importantly, failure to comply with environmental regulations may result in costly lawsuits and fines to those holding environmental permits.

### *Uncertainty Analyses in Hydraulic Engineering*

Hydraulic measurements are essential in fields as diverse as hydraulic structures and hydromachinery, environmental fluid mechanics, erosion and sedimentation, irrigation and drainage, and hydrology (Ogden 1996). Consequently, hydraulic engineers must make a wide variety of measurements using a large spectrum of measurement devices. Ogden (1996) presents a table of several of the parameters that hydraulic engineers must measure as well as some of the possible measurement devices that can be used to make the measurements. A modified version of Ogden's (1996) table appears in [Table 1](#page-4-0), and this list is by no means an inclusive list of all the possible measurement parameters or measurement devices. As can be seen from [Table 1](#page-4-0), the number of measurement devices that a hydraulic engineer may encounter is daunting.

# *Which Methodology Should Be Used?*

There are a wide variety of methodologies that can be used to perform an uncertainty analysis. The methodology used will determine how different sources of uncertainty are propagated to obtain the uncertainty in an experimental end result. For example, an uncertainty analysis can be performed using the first order variance estimation method (Coleman and Steele 1999), the most probable point method (Du and Chen 2000), Rosenblueth's point estimation method (Rosenblueth 1981), integral transformation techniques (Kendall et al. 1998), or many other techniques. In addition, variations can even occur within a given methodology. For example, Coleman and Steele (1999) discuss six (6) different variations of the first order variance estimation method (which is the most popular uncertainty estimation method). These first order variance estimation methods differ in how they classify uncertainty, how they combine the random and systematic uncertainty components, what underlying statistical distribution is used to describe the random uncertainty components, etc. The plethora of available methods makes it daunting to choose just one, but Kline (1985b) reminds us that ultimately it is more important to do an (any!) uncertainty analysis than to use a particular methodology.

A further difficulty is the fact that all methods require estimates of the uncertainties associated with individual measurements. Unfortunately, Coleman and Steele (1999) observe that there is a universal human reluctance to estimate these uncertainties, perhaps out of fear of using the wrong values. One should keep in mind that the purpose of uncertainty analysis is to learn more about the experimental result, set attainable experimental performance goals, and identify barriers that may need to be

<span id="page-4-0"></span>overcome. An uncertainty analysis should not be an onerous process, and it should be recognized that even when estimates of uncertainty are themselves uncertain, insight can still be gained.





<span id="page-5-0"></span>Although there are other methods that can be used to perform an uncertainty analysis, the Task Committee felt is was necessary to recommend one method to help simplify the issues surrounding uncertainty analyses. Thus, the methodology that the Task Committee recommends for use in performing an uncertainty analysis is the first order variance estimation method outlined by Coleman and Steele (1999). This methodology can be applied in both a straightforward and comprehensive manner and is applicable to almost every measurement situation. In addition, this methodology was also adopted as AIAA's standard for uncertainty analyses associated with wind tunnel experiments (AIAA 1999).

### *Coleman and Steele's (1999) First Order Variance Estimation Method*

A quick outline of the first order variance estimation procedure recommended by Coleman and Steele (1999) is presented in this section. For more details on the derivation of Coleman and Steele's method, please see Appendix B of their book (1999). First, consider a general case in which an experimental result, *r*, is computed from *J* measured variables  $X_{1...J}$ , so that the data reduction equation is:

$$
r = r(X_1, X_2, \dots, X_J) \tag{1}
$$

Each measured variable  $X_l$  through  $X_l$  has both a systematic uncertainty (i.e., bias) and a random uncertainty. The uncertainty in the experimental result is calculated by taking the first order Taylor series expansion of the data reduction equation. This expression is then substituted into an expression for the variance of the underlying parent distribution. After simplifying, the uncertainty in the experimental result, *Ur*, is given by:

$$
U_r^2 = \sum_{i=1}^J \left(\frac{\partial r}{\partial X_i}\right)^2 U_{X_i}^2 + 2 \sum_{i=1}^{J-1} \sum_{k=i+1}^J \left(\frac{\partial r}{\partial X_i}\right) \left(\frac{\partial r}{\partial X_k}\right) U_{X_k}
$$
(2)

where  $U_{X_i}$  is the uncertainty in the variable  $X_i$  and  $U_{X_i}$  is the covariance between the uncertainties in variable  $X_i$  and variable  $X_k$ . Eq. (2) is the most general expression for the uncertainty using the first order variance method (Coleman and Steele 1999). For this work, it was assumed that the measured variables are independent and, thus, the covariance terms will be zero. This assumption simplifies eq. ( 2 ) to the following:

$$
U_r^2 = \left(\frac{\partial r}{\partial X_1}\right)^2 U_{X_1}^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 U_{X_2}^2 + \dots + \left(\frac{\partial r}{\partial X_J}\right)^2 U_{X_J}^2
$$
 (3)

When applying the uncertainty propagation equation, the individual uncertainties should all be expressed with the same odds (e.g., at 95% confidence).

<span id="page-6-0"></span>**Non-Dimensional Forms.** Two non-dimensional forms of eq. [\( 3 \)](#page-5-0) are useful in performing an uncertainty analysis. Dividing each term by  $r^2$  and multiplying the terms on the right-hand side by  $(X_i/X_i)^2$  yields:

$$
\left(\frac{U_r}{r}\right)^2 = \left(\frac{X_1}{r}\frac{\partial r}{\partial X_1}\right)^2 \left(\frac{U_{X_1}}{X_1}\right)^2 + \left(\frac{X_2}{r}\frac{\partial r}{\partial X_2}\right)^2 \left(\frac{U_{X_2}}{X_2}\right)^2 + \dots + \left(\frac{X_J}{r}\frac{\partial r}{\partial X_J}\right)^2 \left(\frac{U_{X_J}}{X_J}\right)^2
$$
 (4)

In this equation,  $U_r/r$  is the relative uncertainty in the result and the factors  $U_{x_i}/X_i$  are the relative uncertainties of each variable. The factors in parentheses that multiply the relative uncertainties of the variables are called *uncertainty magnification factors* (UMFs). They indicate the influence of uncertainty in a particular variable on the uncertainty in the result. When the UMF is greater than 1, uncertainty in a variable is magnified as it propagates through the data reduction equation; if less than 1, the uncertainty in the variable is reduced. The UMF depends on the value of a variable relative to the result and the manner in which it is incorporated into the data reduction equation, but it is independent of the actual uncertainty in the variable. Since the UMFs are always squared when inserted into eq. ( 4 ), only their absolute values are important.

The second non-dimensional form is obtained by dividing by  $U_r^2$ , which produces:

$$
1 = \left(\frac{\partial r}{\partial X_1}\right)^2 \left(\frac{U_{X_1}}{U_r}\right)^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 \left(\frac{U_{X_2}}{U_r}\right)^2 + \dots + \left(\frac{\partial r}{\partial X_J}\right)^2 \left(\frac{U_{X_J}}{U_r}\right)^2 \tag{5}
$$

Each term on the right-hand side of eq. ( 5 ) gives the fractional contribution of the squared uncertainty in a given variable to the squared uncertainty in the result. In percentage terms we can define *uncertainty percentage contributions* (UPCs) as

$$
\text{UPC}_{i} = \left(\frac{\partial r}{\partial X_{i}}\right)^{2} \left(\frac{U_{X_{i}}}{U_{r}}\right)^{2} \times 100 = \left(\frac{X_{i}}{r}\frac{\partial r}{\partial X_{i}}\right)^{2} \left(\frac{U_{X_{i}}}{X_{i}}\right)^{2} \frac{1}{\left(U_{r}/r\right)^{2}} \times 100
$$
 (6)

The UPCs include the effects of both the UMF and the uncertainty of the variable, so they are useful in the late planning phase and early design phase when measurement equipment and methods are being selected and measurement uncertainties can be estimated.

Note that the relative uncertainty of the estimated quantity will not be a constant but will vary within the range of the independent variables. Because the uncertainty of measuring extreme quantities is larger than that of measuring typical or average quantities for which the measuring devices are designed, the relative uncertainty will increase towards the upper and lower range of the measured value.

**Special Case.** The most useful form of the uncertainty propagation equation for performing an uncertainty analysis is eq. ( 4 ), in which the squares of the relative uncertainties are related through the UMFs. In a great many cases, eq. ( 4 ) can be further simplified. When the data reduction equation is of the form

$$
r = kX_1^a X_2^b X_3^c \cdots \tag{7}
$$

<span id="page-7-0"></span>with *a*, *b*, *c*, and *k* being constants, applying eq. (4) produces a simplified equation for this special case:

$$
\frac{U_r^2}{r^2} = a^2 \left(\frac{U_{X_1}}{X_1}\right)^2 + b^2 \left(\frac{U_{X_2}}{X_2}\right)^2 + c^2 \left(\frac{U_{X_3}}{X_3}\right)^2 + \cdots
$$
 (8)

In such a case, the UMFs are the exponents and the uncertainty propagation equation can be written down by simple inspection. One must keep several things in mind when considering the use of this special form of the uncertainty propagation equation. First, the user must solve for the experimental result before applying the equation. Second, the *Xi*s must be directly measured variables, so an equation of the form  $R = a(\cos(\theta))$  is not in the proper form if  $\theta$  is measured directly. Note that it would be in the proper form if  $cos(\theta)$  were measured directly. Also, a data reduction equation of the form  $Q = C_d A \sqrt{2g(h_2 - h_1)}$  is acceptable if  $h_2 - h_1$  is measured directly, but not if  $h_2$  and  $h_1$  are measured separately.

**Example for the Special Case.** Consider the case of flow measurement over a fully contracted, sharp-crested rectangular weir. To determine the discharge over the weir, the following equation can be used:

$$
Q = CLh_1^{1.5} \tag{9}
$$

The variables that will be measured are the weir length, *L*, and the upstream head,  $h_1$ . Each of these measurements will have an associated uncertainty. The value of the discharge coefficient, *C*, is an empirical constant which will also have some uncertainty associated with it. Before determining how to measure the weir length and upstream head, a general uncertainty analysis can be used to gain an understanding of the relationship between the measurement uncertainties and the uncertainty in the result. Applying eq. [\( 4 \)](#page-6-0), the general uncertainty expression is:

$$
\left(\frac{U_{Q}}{Q}\right)^{2} = \left(\frac{C}{Q}\frac{\partial Q}{\partial C}\right)^{2} \left(\frac{U_{C}}{C}\right)^{2} + \left(\frac{L}{Q}\frac{\partial Q}{\partial L}\right)^{2} \left(\frac{U_{L}}{L}\right)^{2} + \left(\frac{h_{1}}{Q}\frac{\partial Q}{\partial h_{1}}\right)^{2} \left(\frac{U_{h_{1}}}{h_{1}}\right)^{2}
$$
(10)

The UMFs are:

$$
UMF_c = \frac{C}{Q} \frac{\partial Q}{\partial C} = \frac{C}{Q} L h_1^{1.5} = \frac{CLh_1^{1.5}}{Q} = 1
$$
 (11)

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$$
UMF_{L} = \frac{L}{Q} \frac{\partial Q}{\partial L} = \frac{L}{Q} Ch_{1}^{1.5} = \frac{CLh_{1}^{1.5}}{Q} = 1
$$
 (12)

$$
UMF_{h_1} = \frac{h_1}{Q} \frac{\partial Q}{\partial h_1} = \frac{h_1}{Q} (1.5CLh_1^{0.5}) = \frac{1.5CLh_1^{1.5}}{Q} = 1.5
$$
 (13)

Substituting the UMFs back into eq. [\( 10 \)](#page-7-0) yields:

$$
\left(\frac{U_Q}{Q}\right)^2 = \left(\frac{U_C}{C}\right)^2 + \left(\frac{U_L}{L}\right)^2 + (1.5)^2 \left(\frac{U_{h_1}}{h_1}\right)^2\tag{14}
$$

This equation could have been written down by inspection (i.e., from eqs. ( 7 ) and ( 8 )) since the data reduction equation was in the special case form discussed previously. Notice that the uncertainties in head measurement are magnified in the result due to the exponent of 1.5.

Now suppose that the weir length is 2 m and the head is 0.3 m. How accurately can the discharge be measured if the relative uncertainty in *C* is 5%, the weir length is measured with an uncertainty of 2 mm, and the head is measured with an uncertainty of 3 mm? Inserting the values of the variables and their uncertainties yields:

$$
\left(\frac{U_{\varrho}}{Q}\right)^2 = (0.05)^2 + \left(\frac{0.002}{2}\right)^2 + (1.5)^2 \left(\frac{0.003}{0.3}\right)^2
$$

$$
= 0.0025 + 0.000001 + 0.000225
$$

$$
= 0.00273
$$

$$
\frac{U_{\varrho}}{Q} = \sqrt{0.00273} = 0.052
$$

Thus, the relative uncertainty in discharge measurement is 5.2%, with the primary sources of uncertainty being the discharge coefficient and the measurement of the upstream head. It must be pointed out that in the most general case the uncertainties of the individual parameters (e.g., the discharge coefficient) may vary depending on the flow condition, so the relative uncertainty of the discharge measurement will vary over the measurement range as a function of both the values of the measured variables and the changes in their uncertainties. These uncertainties will typically increase towards its upper and lower ends.

# *Status of the Task Committee*

At this point, the Task Committee has performed a thorough literature review of uncertainty analyses and has compiled an annotated bibliography on the subject. This bibliography is available online at:

[http://www.usbr.gov/pmts/hydraulics\\_lab/professional/uncertainty/annotated\\_bibliography.html](http://www.usbr.gov/pmts/hydraulics_lab/professional/uncertainty/annotated_bibliography.html)

After preparing the annotated bibliography, the Task Committee decided to select the uncertainty analysis procedure presented by Coleman and Steele (1999), which is essentially the one adopted by the AIAA as the recommended method for performing uncertainty analyses. Other methods of uncertainty analysis are acceptable; however, the Task Committee recommends the use of Coleman and Steele's (1999) first order variance estimation technique. This method is already in widespread use, is easy to understand, and can be used in a wide variety of applications.

One of the main goals of the Task Committee is to produce a report that discusses the Task Committee's recommended method of performing an uncertainty analysis as it pertains to hydraulic engineering. Since it would be unreasonable to address uncertainty analyses of all of the hydraulic measurements that appear in [Table 1](#page-4-0), the Task Committee decided to develop detailed uncertainty analysis examples of selected hydraulic measurements. These detailed application examples will be written by either Task Committee members or other volunteers. The Task Committee has developed the following outline for the final report:

- 1) Chapter 1 Introduction
	- a) Definitions
	- b) Objectives
	- c) Scope
	- d) Impacts
- 2) Chapter 2 Basic Theory of Uncertainty Analysis
	- a) Rationale for using the first order variance estimation method
	- b) Outline of theory
- 3) Chapter 3 Planning an Experiment
	- a) Identify sources of error
	- b) Money savings examples
- 4) Chapter 4 Estimating Uncertainty in Measured Variables
	- a) Known distributions
	- b) Unknown distributions
	- c) Curve fitting
	- d) Sampling
- 5) Chapter 5 Applications
	- a) Broad-crested weirs or flumes
	- b) Slope-area method
	- c) Acoustic Doppler Velocimeters
	- d) Current-meter discharge measurements
	- e) Stage-discharge relations
	- f) Flow estimates at culverts using acoustic index-velocity meters

At this point, first drafts have been written for the first four chapters of the report. These chapters are currently being internally reviewed by the Task Committee. Some initial work has been done on some of the applications listed in the fifth chapter. A first draft of this chapter is expected to be completed in the near future.

**Future Work.** The Task Committee plans to continue to work on the final report. This includes finishing the review of the first four chapters and completing a draft version of the fifth chapter. Any volunteers who wish to assist in this process would be greatly appreciated.

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