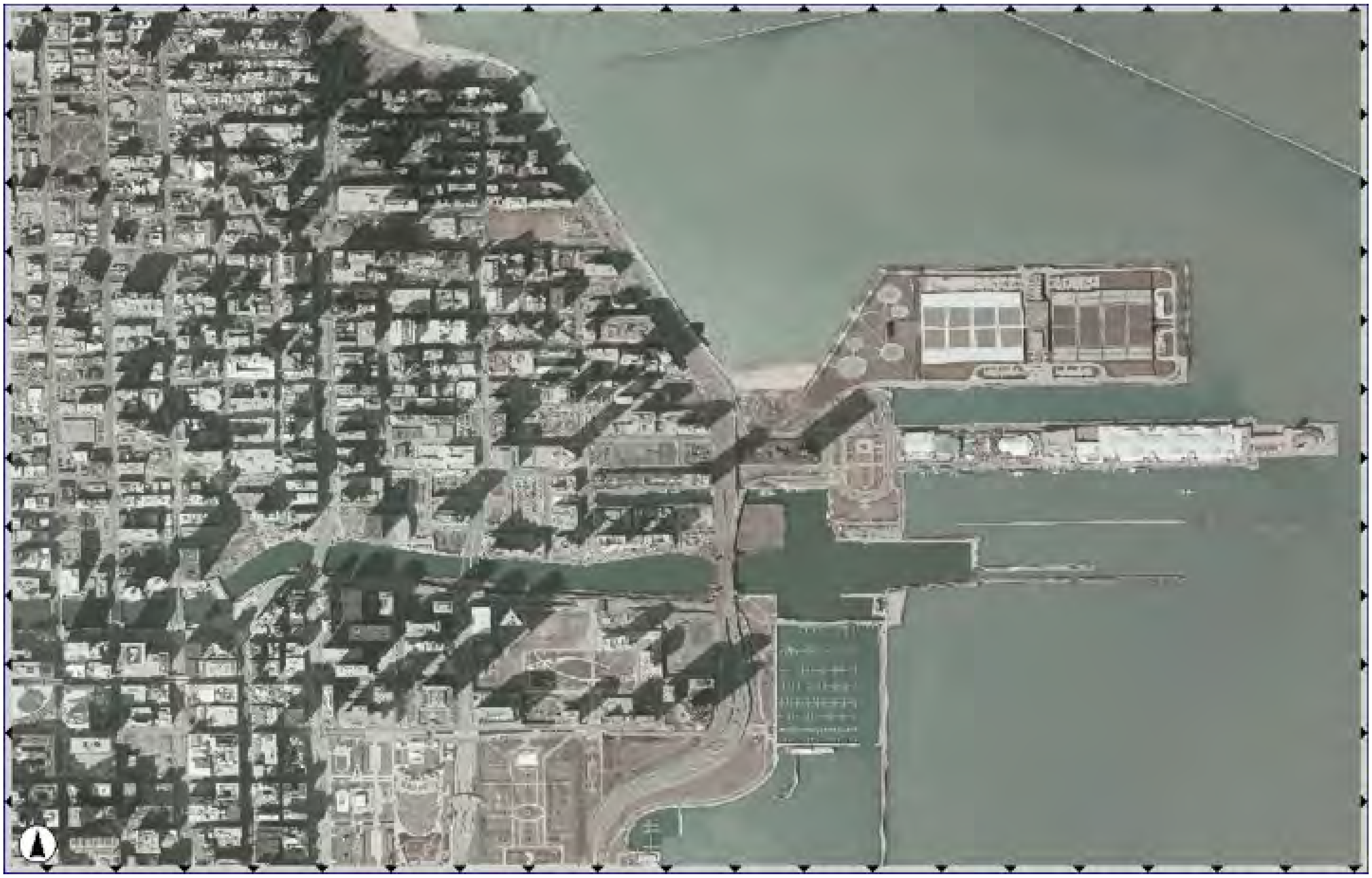
Prepared in cooperation with the U.S. Army Corps of Engineers, Chicago District

Computation and Error Analysis of Discharge for the Lake Michigan Diversion Project in Illinois: 1997-99 Water Years



Scientific Investigations Report 2006-5018

U.S. Department of the Interior
U.S. Geological Survey

Computation and Error Analysis of Discharge for the Lake Michigan Diversion Project in Illinois: 1997-99 Water Years

By James J. Duncker, Thomas M. Over, and Juan A. Gonzalez

Prepared in cooperation with the U.S. Army Corps of Engineers, Chicago District

Scientific Investigations Report 2006–5018

U.S. Department of the Interior
U.S. Geological Survey

U.S. Department of the Interior
Dirk Kempthorne, Secretary

U.S. Geological Survey
P. Patrick Leahy, Acting Director

U.S. Geological Survey, Reston, Virginia: 2006

For product and ordering information:
World Wide Web: http://www.usgs.gov/pubprod
Telephone: 1-888-ASK-USGS

For more information on the USGS--the Federal source for science about the Earth, its natural and living resources, natural hazards, and the environment:
World Wide Web: http://www.usgs.gov
Telephone: 1-888-ASK-USGS

Any use of trade, product, or firm names is for descriptive purposes only and does not imply endorsement by the U.S. Government.

Although this report is in the public domain, permission must be secured from the individual copyright owners to reproduce any copyrighted materials contained within this report.

Suggested citation:
Duncker, J.J., Over, T.M., and Gonzalez, J.A., 2006, Computation and error analysis of discharge for the Lake Michigan Diversion Project in Illinois: 1997–99 Water Years: U.S. Geological Survey Scientific Investigations Report 2006-5018, 70 p.

Cover image: U.S. Geological Survey High Resolution Orthoimagery for the Chicago, IL Urban Area, 2005. The image shown is a screen capture from the Seamless Data Distribution Web site at http://seamless.usgs.gov/

Contents

Figures

Tables

CONVERSION FACTORS, VERTICAL DATUM, AND ABBREVIATIONS

Multiply	By	To obtain
	Length	
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
	Area	
square foot (ft^2)	929.0	square centimeter (cm^2)
square foot (ft^2)	0.09290	square meter (m^2)
square mile (mi^2)	2.590	square kilometer (km^2)
	Flow rate	
foot per second (ft/s)	0.3048	meter per second (m/s)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)

Temperature in degrees Celsius (°C) may be converted to degrees Fahrenheit (°F) as follows:

$°F=(1.8\times°C)+32$

Vertical coordinate information is referenced to the National Geodetic Vertical Datum of 1929 (NGVD 29) and to the Chicago City Datum (CCD). The Chicago City Datum (CCD) is 579.48 feet above NGVD 29.

Abbreviations used in this report:

AVM	acoustic velocity meter
ADCP	acoustic Doppler current profiler
WY	water year
#	number
<	less than
>	greater than

Computation and Error Analysis of Discharge for the Lake Michigan Diversion Project in Illinois: 1997-99 Water Years

By James J. Duncker, Thomas M. Over, and Juan A. Gonzalez

Abstract

Acoustic velocity meters (AVM's) and acoustic Doppler current profilers (ADCP's) were used to measure streamflow at four streamflow-gaging stations in the Chicago River system. The streamflow data were used to compute discharge and to determine the uncertainty in the computed annual mean discharge at each station for the Lake Michigan Diversion Project in Illinois. Descriptions of the instrumentation at each station, stage-area and index-velocity ratings, and methods utilized for computing discharge and estimating missing record are given. Daily mean and annual mean discharges were computed for each station for 1997-99 water years (WY's). A water year is defined as the 12-month period from October 1 through September 30. The water year is designated by the calender year in which it ends and which includes 9 of the 12 months. Thus, the year ending September 30, 1999, is called the 1999 water year.

A first-order error analysis was applied to acoustic velocity meter (AVM) data, stage-area, and index-velocity ratings at each streamflow-gaging station. The error analysis results indicate that the uncertainty is sensitive to the value of uncertainty associated with acoustic Doppler current profiler (ADCP) discharge measurement data. At the Chicago River at Columbus Drive at Chicago, Illinois station for the 1997-99 WY's, the uncertainty, expressed as a standard deviation of the average annual discharge, ranged from 13 to 18 cubic feet per second (ft^3/s) when ADCP uncertainty was not included, whereas total uncertainty ranged from 55 to 69 ft^3/s when ADCP uncertainty was included. At the Chicago Sanitary and Ship Canal at Romeoville, Illinois station for the 1997-99 WY's, the uncertainty ranged from 18 to 20 ft^3/s when ADCP uncertainty was not included, whereas it ranged from 64 to 68 ft^3/s when it was included. At the Calumet River below O'Brien Lock and Dam at Chicago, Illinois station for the 1997-99 WY's, the uncertainty ranged from 13 to 22 ft^3/s when ADCP uncertainty was not included, whereas it ranged from 35 to 53 ft^3/s when it was included. At the North Shore Channel at Wilmette, Illinois station for the 1997-99 WY's, when the record was entirely estimated, the uncertainty ranged from 8 to 12 ft^3/s when the ADCP uncertainty was not included, and from 16 to 17 ft^3/s when it was included. For the 2000 WY, the estimated uncertainty was 8.6 ft^3/s when ADCP uncertainty is not included and 12.5 ft^3/s when ADCP uncertainty was included.

Introduction

The State of Illinois directly diverts water from Lake Michigan into the Chicago River system at three locations in the Chicago vicinity. Lake Michigan water is a shared natural resource among the Great Lakes States (Illinois Indiana, Michigan, Minnesota, New York, Ohio, Pennsylvania, and Wisconsin) and Canada. According to a U.S. Supreme Court decree (Wisconsin v. Illinois 1980), the State of Illinois is limited in the amount of Lake Michigan water that it is allowed to divert (an annual mean discharge of 3,200 ft^3/s). The U.S. Geological Survey (USGS) collects streamflow data at four streamflow-gaging stations in the Chicago River system that are part of an overall Lake Michigan diversion accounting system. These stations and their USGS identifiers are listed below and their locations are shown in figure 1:

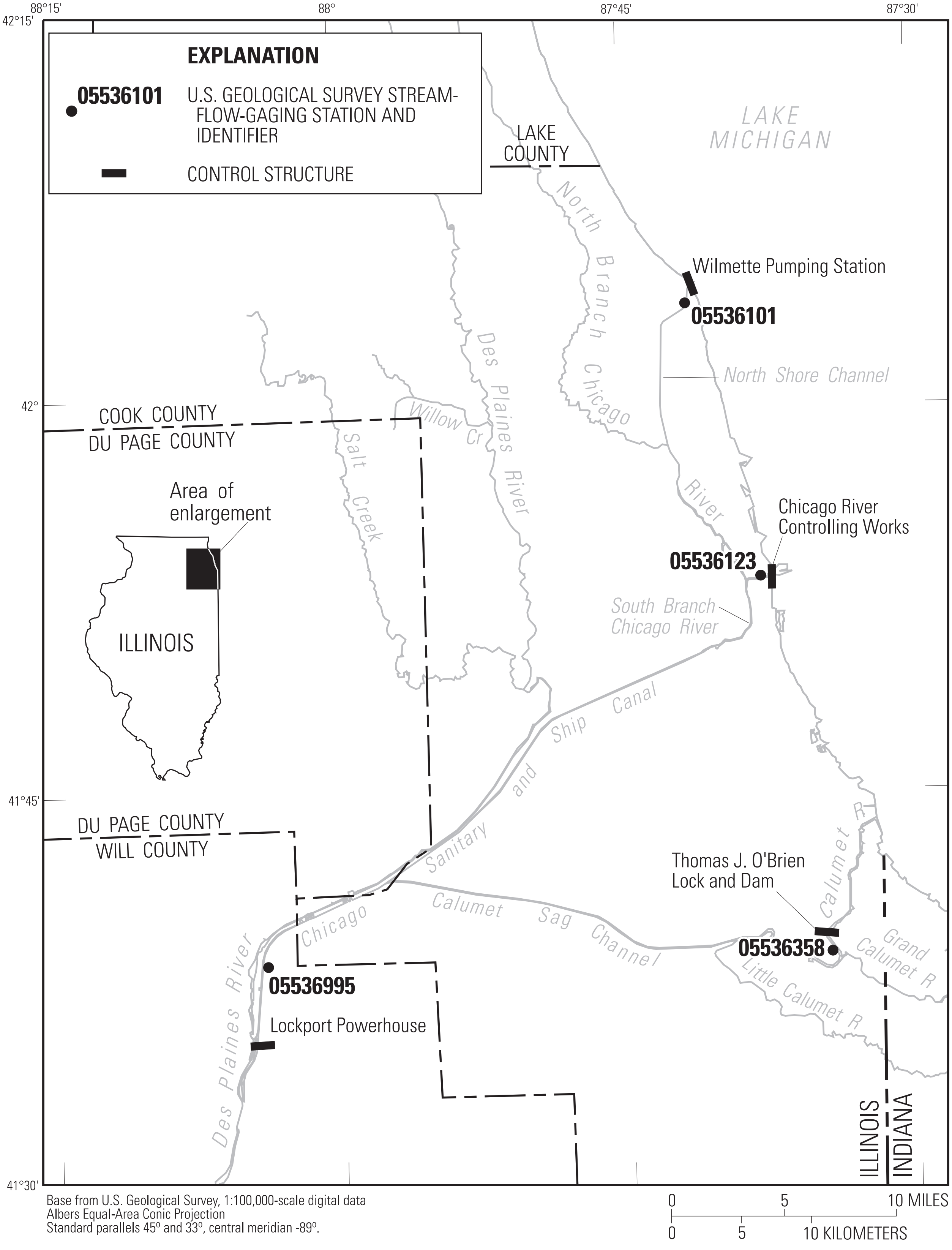
(1) Chicago Sanitary and Ship Canal at Romeoville, Illinois (05536995)
(2) Chicago River at Columbus Drive at Chicago, Illinois (05536123)
(3) Calumet River below O'Brien Lock and Dam at Chicago, Illinois (05536358)
(4) North Shore Channel at Wilmette, Illinois (05536101)

The U.S. Army Corps of Engineers, Chicago District (Corps) is tasked with responsibility for the overall accounting of Lake Michigan diversions. The Corps utilizes an accounting system that summarizes all of the withdrawals and reports an annual diversion. The overall importance of the Lake Michigan diversion requires an estimate of the accuracy of the mean annual discharge. The primary station used by the Corps for the Lake Michigan diversion accounting system is located at the Chicago Sanitary and Ship Canal at Romeoville, Illinois (fig. 1). Streamflow data have been collected at the Romeoville station since October 1984.

To measure direct diversions more accurately, the USGS, in cooperation with U.S. Army Corps of Engineers, Chicago District, established three additional streamflow-gaging stations in 1996 in the Chicago River system in close proximity to the Lake Michigan lakefront (fig. 1). These stations are located at the Chicago River at Columbus Drive at Chicago, Illinois; at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois; and on the North Shore Channel at Wilmette, Illinois. These three stations, located in close proximity to lakefront-control structures, are collectively referred to as the lakefront accounting system. The U.S. Army Corps of Engineers, Chicago District was charged with the task of evaluating the accuracy of the lakefront accounting system against the traditional system of computing the Lake Michigan diversion using the station at the Chicago Sanitary and Ship Canal at Romeoville, Illinois. To accomplish the above, the USGS and the Corps began a cooperative investigation in 1996.

Acoustic velocity meters (AVM's) are utilized at all four stations to measure discharge. AVM's are required because of the complex site hydraulics characterized by unsteady-flow conditions, backwater, and low velocities. AVM's transmit sound waves across the channel at a known angle to the flow direction. The difference between the upstream and downstream travel times for the sound wave is a function of the water velocity. (For additional information on AVM's and ADCP's see the USGS Hydroacoustics Web page at *http://hydroacoustics.usgs.gov/*). Index-velocity ratings are developed to relate the AVM measured velocity to the mean channel velocity as calculated from acoustic Doppler current profiler (ADCP) discharge measurements. Stage is measured at each site using either an AVM uplooker transducer, a float-driven shaft encoder within a stilling well, or a pressure transducer. Bathymetric surveys relate stage to cross-sectional area of the channel. Velocity and stage data are recorded using electronic dataloggers at 5-minute intervals at each station.

Discharge is computed at each station utilizing a multistep process. A stage-area rating is utilized to convert the stage data to a cross-sectional area. The velocity measured with the AVM then is converted to a mean channel velocity using the index-velocity rating. Discharge is computed by multiplying the cross-sectional area by the mean channel velocity. This process is repeated for each 5-minute gage-height and average velocity reading. A daily mean discharge is computed by averaging the 5-minute unit values throughout the day. Daily mean discharges are tabulated throughout the water year (WY); a water year is the 12-month period from October 1 through September 30. The water year is designated by the calendar year in which it ends and which includes



Base from U.S. Geological Survey, 1:100,000-scale digital data
Albers Equal-Area Conic Projection
Standard parallels 45° and 33°, central meridian -89°.

Figure 1. Location of U.S. Geological Survey (USGS) streamflow-gaging stations within the Lake Michigan Diversion Project study area, Illinois.

9 of the 12 months of that calendar year. For example, the year ending September 30, 1999, is called the 1999 water year.

This report describes the methods used to compute discharge at four streamflow-gaging stations in the Chicago River system that are used in the Lake Michigan Diversion Project in Illinois. The report also describes for each station (1) the AVM instrumentation, (2) stage-area and index-velocity ratings, (3) the methods used to compute discharge, and (4) the methods used to estimate missing record. A method is derived using first-order error analysis for computing the total uncertainty of the discharge estimates at 5-minute to annual time scales and this method is applied to the discharges at the four stations.

Descriptions of Streamflow-Gaging Stations and Methods for Computing Discharge and Estimating Missing Record

The following sections describe the instrumentation utilized at each streamflow-gaging station and the methods used for computing discharge and estimating missing record.

Chicago Sanitary and Ship Canal at Romeoville, Illinois

The streamflow-gaging station (05536995) at the Chicago Sanitary and Ship Canal at Romeoville, Illinois (fig.1) was established on October 1, 1984. The station consists of an electronic datalogger with a pressure transducer and an AVM housed within a concrete-block building on the

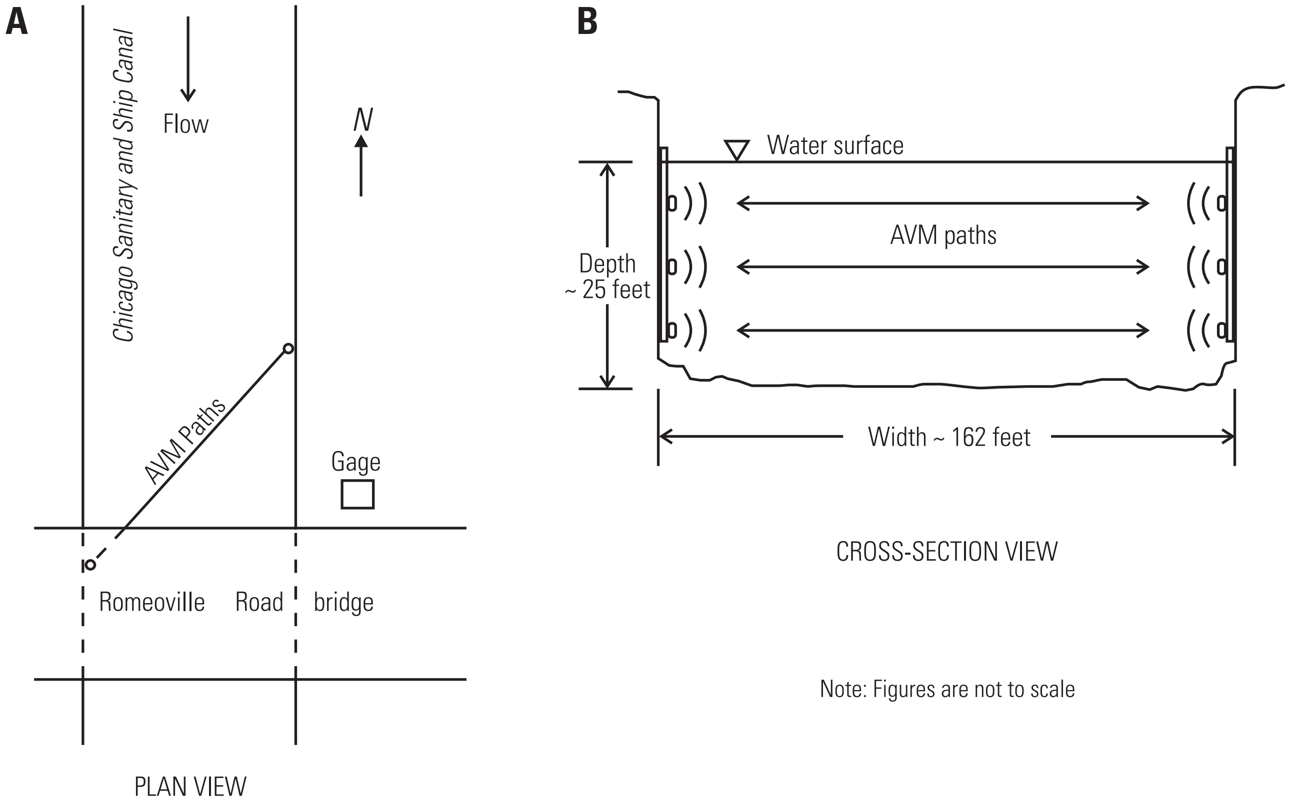


Figure 2. (A) Plan view and (B) cross-sectional view of the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station at the Chicago Sanitary and Ship Canal at Romeoville, Illinois.

east side of the Chicago Sanitary and Ship Canal and beneath the Romeoville Road bridge (fig. 2). Auxiliary stage and velocity meters provide back-up data in the case of instrument failures.

The AVM at the Romeoville streamflow-gaging station is configured to measure velocity on three paths that are set at fixed elevations (12.28 ft, 16.53 ft , and 19.54 ft referenced to the gage datum of 551.89 ft, NGVD 1929) in the channel (fig. 2). The three path velocities are averaged to compute a mean channel velocity for the AVM for the last 5 minutes of each 15-minute interval. A regression between each of the velocity paths and the mean channel velocity was developed to allow for computation of the mean channel velocity if one or two of the paths were not working.

Stage data are referenced to a datum of 551.89 ft above NGVD 1929. Station levels were resurveyed in 1995, 1998, and 2000. A bathymetric survey of the channel was completed in 1984 to determine the stage-area rating (fig. 3) and resurveyed in 1997 to determine any changes in the channel bottom.

ADCP discharge measurements are made during the water year to define the index-velocity rating (fig. 4). The discharge measured using the ADCP was divided by the rated cross-sectional area from the stage-area rating to determine the ADCP mean channel velocity. The index-velocity rating was developed by regression of the AVM mean channel velocity and ADCP mean channel velocity assuming a zero intercept. Flow in the Chicago Sanitary and Ship Canal is regulated by control structures owned and operated by the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) at the upper and lower reaches of the Chicago River system and flow is affected further by three large wastewater-treatment plant discharges. During the period of record (1985-2003 WY's) for the Romeoville streamflow-gaging station, the daily mean discharge has ranged from 915 to 19,466 ft^3/s. The USGS daily mean discharges for the Romeoville streamflow-gaging station for the 1997-99 WY's are listed in tables B.1-B.3 (in appendix B). The USGS daily mean discharges for the streamflow-gaging station at the Chicago Sanitary and Ship Canal at Romeoville and the MWRDGC flow records for Lockport are shown in shown in figures 5-7.

The procedures for estimating missing daily mean discharge in the Chicago Sanitary and Ship Canal at Romeoville are based upon methods developed by Melching and Oberg (1993) using the discharge estimates made by the MWRDGC at the Lockport powerhouse, lock, and controlling

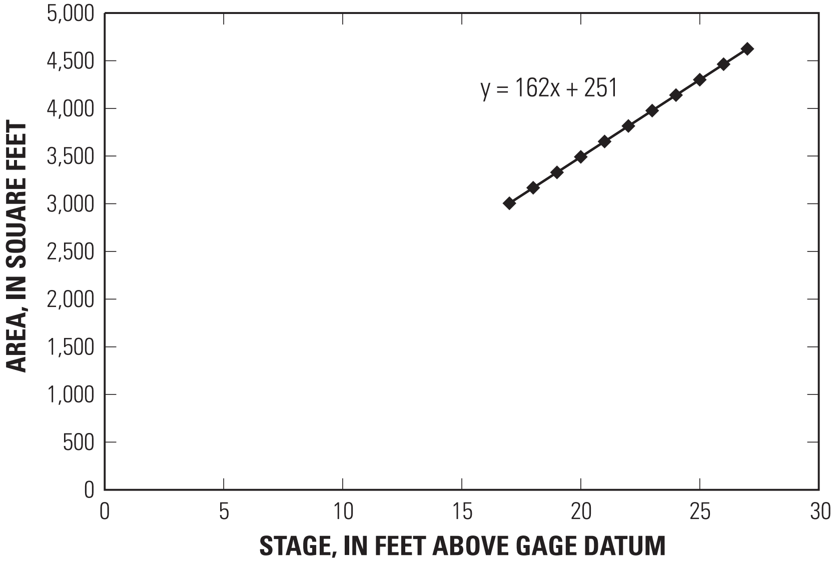


Figure 3. Stage–area rating for the U.S. Geological Survey streamflow-gaging station at the Chicago Sanitary and Ship Canal at Romeoville, Illinois, as determined by bathymetric survey.

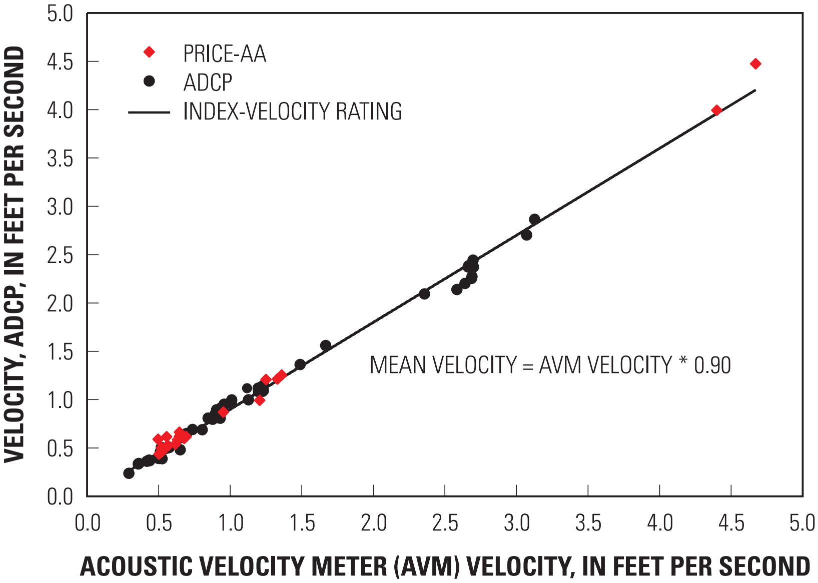


Figure 4. Index-velocity rating for the U.S. Geological Survey streamflow-gaging station at the Chicago Sanitary and Ship Canal at Romeoville, Illinois, as determined from Price-AA current meter and acoustic Doppler current profiler (ADCP) discharge measurements.

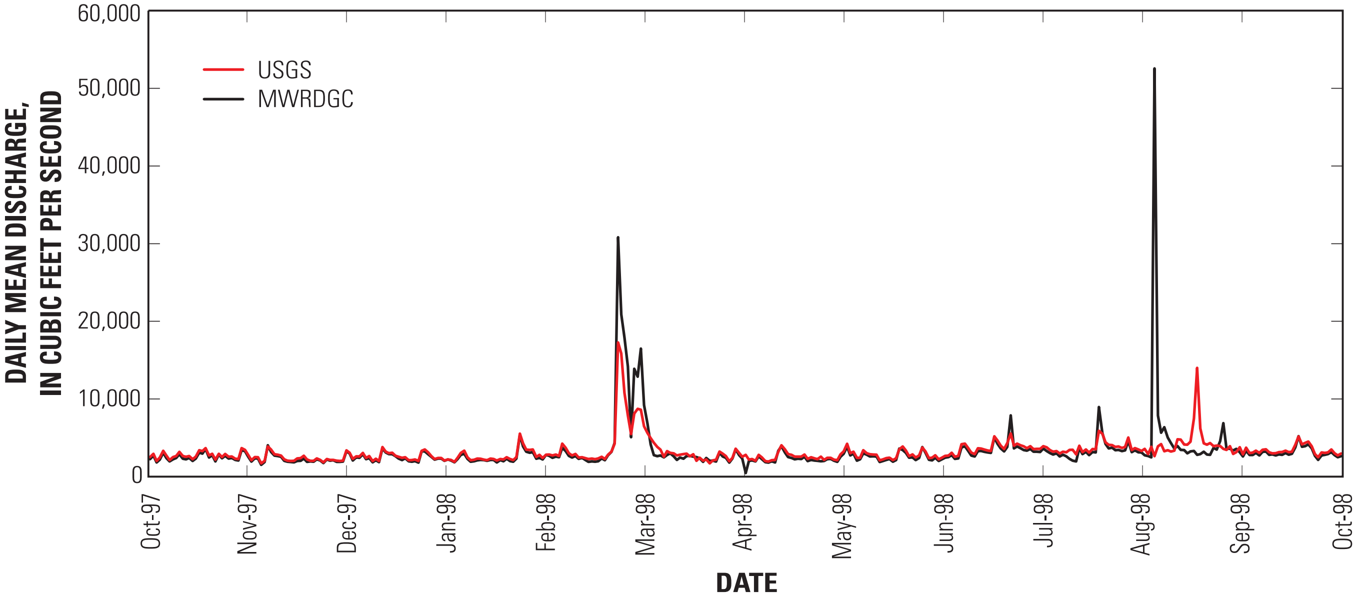


Figure 5. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station at the Chicago Sanitary and Ship Canal at Romeoville, Illinois and the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) Lockport record, October 1, 1996, through September 30, 1997.

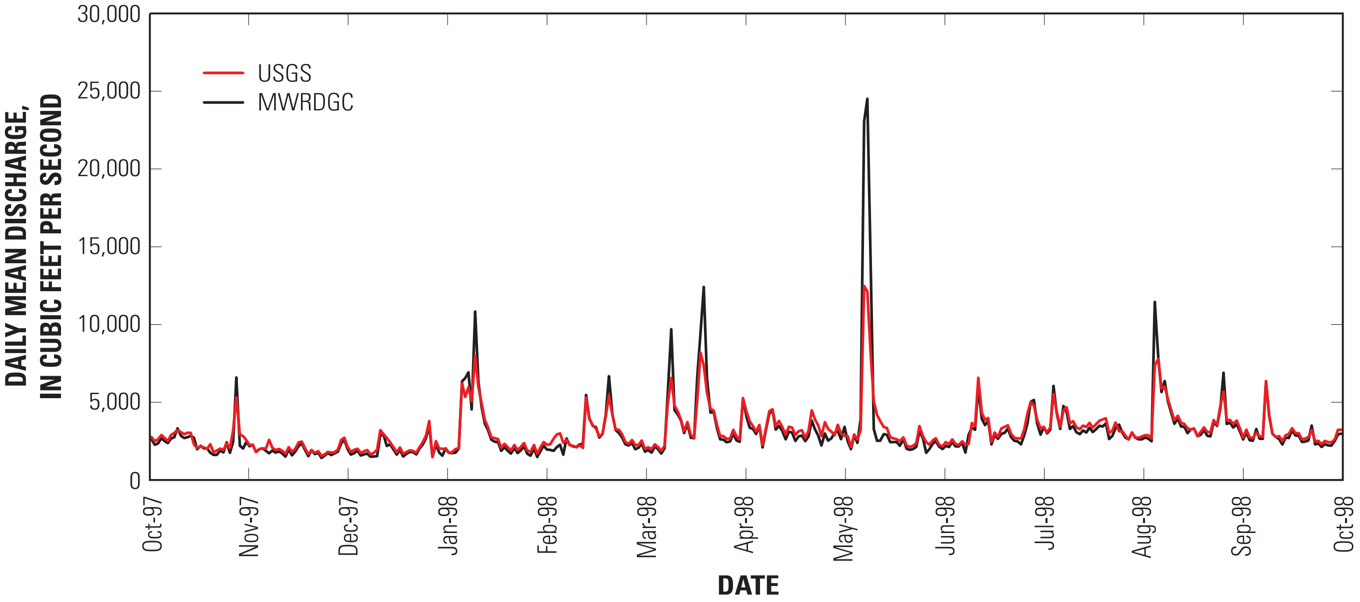


Figure 6. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station (measured and estimated) at the Chicago Sanitary and Ship Canal at Romeoville, Illinois and at Lockport from Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record, October 1, 1997, through September 30, 1998.

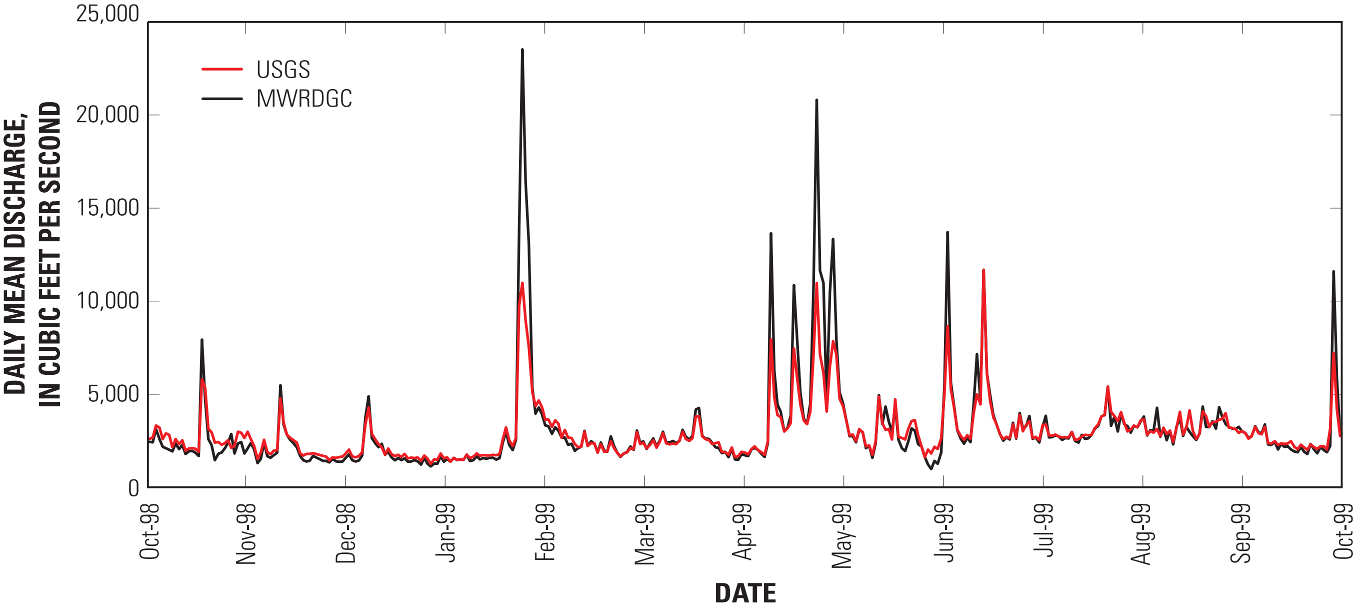


Figure 7. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station (measured and estimated) at the Chicago Sanitary and Ship Canal at Romeoville, Illinois and at Lockport from Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record, October 1, 1998, through September 30, 1999.

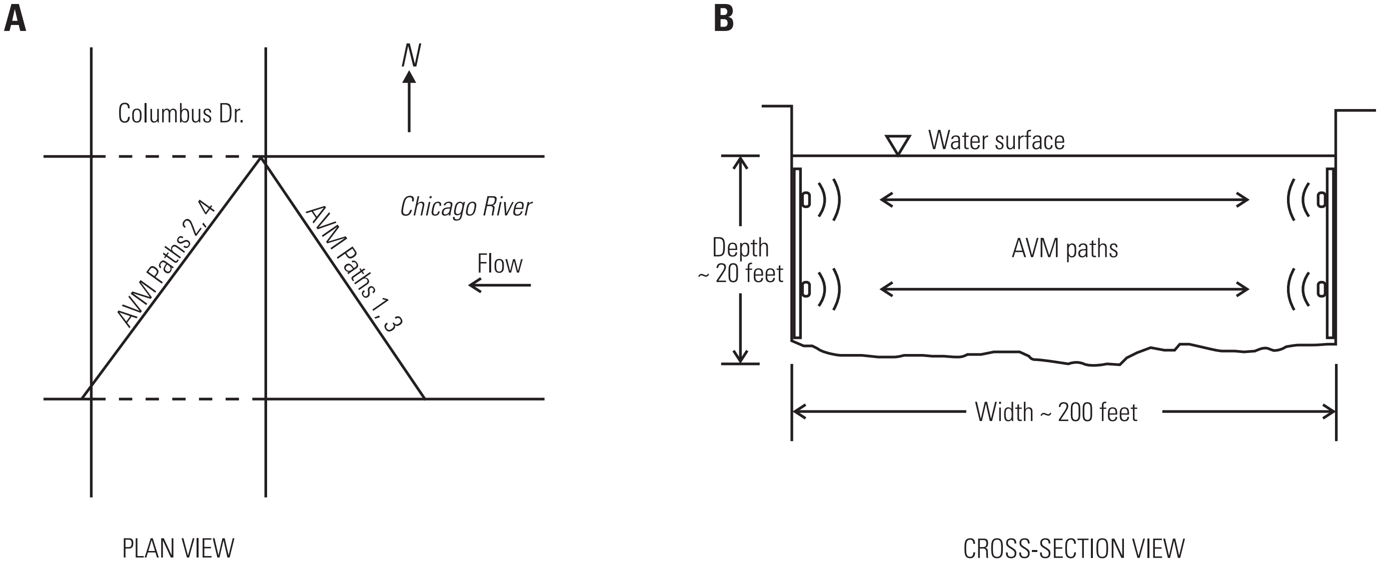
works. Melching and Oberg (1993) defined regression equations for the relation between daily flows from the various outlet components—turbines, lockage, leakage, powerhouse sluice gates, and controlling works at Lockport—and the USGS streamflow-gaging station at Romeoville. The MWRDGC discharge estimates and these regression equations are used to estimate discharge at the Romeoville streamflow-gaging station when the AVM is not operational. The regression equations defined by Melching and Oberg (1993) utilized turbine flows that the MWRDGC calculated based upon theoretical ratings for the turbines. In 1994, the MWRDGC installed AVM's in the turbine forebays and changed their method for computing turbine discharges from the theoretical turbine ratings to discharges calculated with turbine AVM data (James Vey, Metropolitan Water Reclamation District of Greater Chicago, oral commun., 2004). The change in methodology for computing the turbine flows necessitated a change in the methods where the MWRDGC flow data are used to estimate missing USGS data at the Romeoville streamflow-gaging station, as the flows computed using the theoretical turbine ratings did not match the turbine AVM flows. When the turbine flows reported by MWRDGC are measured using the turbine AVM's, the reported flow through the turbines is used directly, with the regression equations applied to the other outlet components (sluice gate and controlling works) where applicable.

Chicago River at Columbus Drive at Chicago, Illinois

The Chicago River at Columbus Drive at Chicago streamflow-gaging station (05536123) (fig. 1) was established on December 2, 1996. The gage consists of an electronic datalogger with a pressure transducer and an AVM housed within the Columbus Avenue bridge on the south side of the Chicago River.

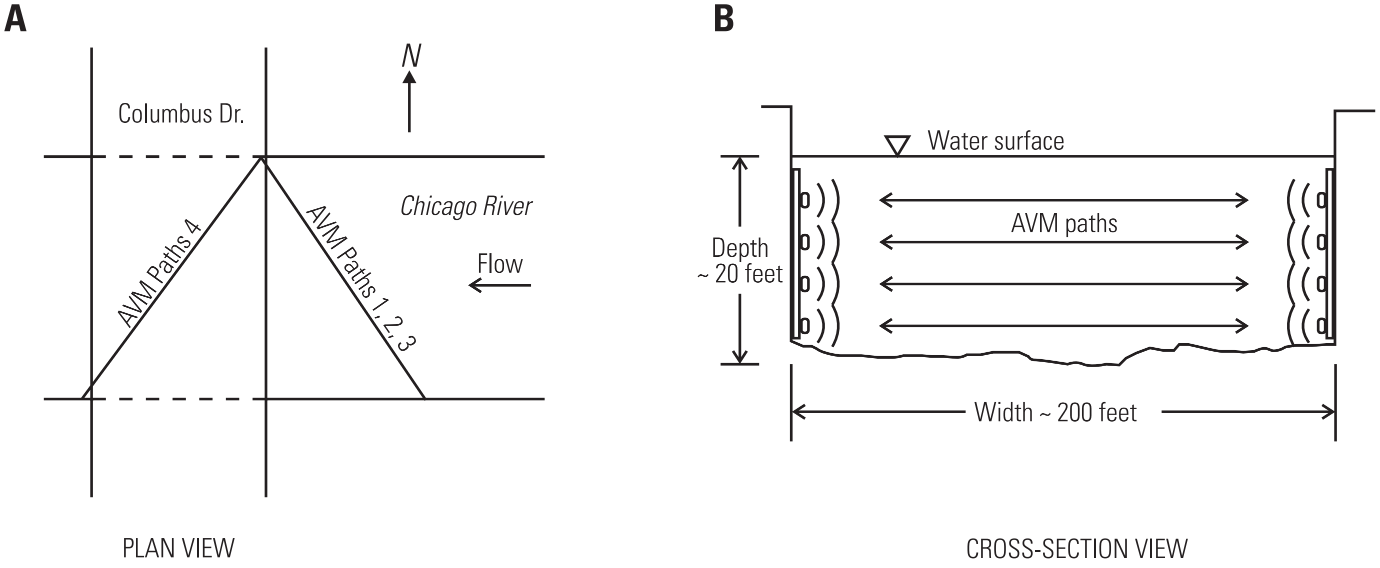
The AVM gaging station at Columbus Drive is configured to measure velocity on four paths that are set at fixed elevations (referenced to Chicago City Datum, CCD) in the channel. Stage data are referenced to the Chicago City Datum (CCD), which is an elevation of 579.48 ft NGVD 1929, through a survey completed on October 24, 1996. The gaging station initially was installed with two AVM's with a duplicate set of two transducers at fixed elevations of –8.00 ft CCD (571.48 ft NGVD 29) and –15.00 ft CCD (554.48 ft NGVD 29) (fig. 8a). This configuration was selected to provide a back-up set of AVM velocity data in case of AVM instrument failure. Later ADCP discharge measurements at this site indicated complex flow hydraulics that required more detailed information on the vertical distribution of channel velocity. In August 1998, the AVM transducer configuration was modified such that the duplicate set of transducers were relocated to provide more detailed information about the vertical velocity distribution (fig. 8b). In the new configuration , the four transducers are set at fixed elevations of approximately –8.00 ft, –10.00 ft, –15.00 ft , and –17.00 ft CCD. A bathymetric survey of the channel was completed on March 26, 1997, to determine the stage-area rating (fig. 9).

ADCP discharge measurements are made during the water year to define the index-velocity rating (fig. 10). The index-velocity rating for path 3 of the AVM was used to compute discharge for the 1997 and 1998 water years. Daily mean discharges for the 1999 WY were computed using the path 4 velocity data and the path 4 index-velocity rating. The change in primary paths for the discharge computations was made because of an irregular shift in the velocity data for path 3 of the AVM in the 1999 WY. The exact cause of the irregular shift in the velocity data is not known, but may have been the result of mixing water temperatures at the elevation of path 3. Velocity data from the AVM path 4 during this period do not indicate the same irregular shift and were used in the discharge computations for the entire 1999 WY.



Note: Figures are not to scale

Figure 8a. (A) Plan and (B) cross-sectional views of the of the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois, showing the AVM configuration used from December 1996 through August 1998.



Note: Figures are not to scale

Figure 8b. (A) Plan and (B) cross-sectional views of the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois, and the reconfiguration of the AVM paths, August 1998-2005.

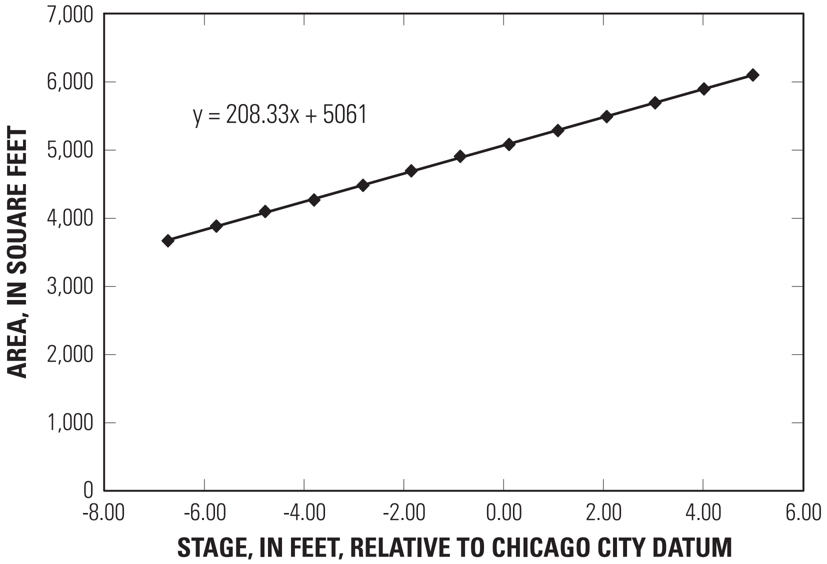


Figure 9. Stage–area rating for the U.S. Geological Survey streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois, as determined by bathymetric survey.

The procedure for estimating missing daily mean discharge at the Chicago River at Columbus Drive streamflow-gaging station for the 1997-99 WY's was chosen based on the assessment of the following three approaches:

(1) A relation between discharge at Columbus Drive and stage difference between Lake Michigan and the Chicago Harbor at the MWRDGC Chicago River Controlling Works (CRCW).

(2) Hydrologic univariate time-series modeling of the autoregressive, integrated, moving average (ARIMA), using the data record prior to each non-operating period for the AVM's.

(3) Algebraic relations between the flow records independently collected by the MWRDGC at the CRCW and the USGS at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station.

According to the assessment of all these approaches, the third approach provides the most reliable and accurate estimates of missing discharge records at the Chicago River at Columbus Drive at Chicago streamflow-gaging station. Period-wise regression equations that relate the discharge record reported by the MWRDGC for the CRCW to the State of Illinois (Illinois Department of Natural Resources form LMO-6) and the USGS discharge records for the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station estimated from the AVM data are used in this approach. A brief description of the chosen approach is given in the following paragraphs.

The daily mean discharge record collected by the USGS at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station between December 2, 1996, and September 30, 1997, and the MWRDGC discharge at the CRCW reported in LMO-6 for the 1997 WY are shown in figure 11. During the 1997 WY, the AVM at the Columbus Drive streamflow-gaging station only failed to collect velocity data on July 18-19, 1997; however, because the AVM at this station started operating on December 2, 1996, in order to complete the discharge record of the 1997 WY, the daily mean discharges from October 1 to December 1, 1996, had to be indirectly estimated.

The trends of the discharge records for the CRCW and the Columbus Drive streamflow-gaging station are similar, particularly during high-flow periods (fig. 11). The daily discharge data from the MWRDGC record for the CRCW are plotted with the corresponding USGS data collected at

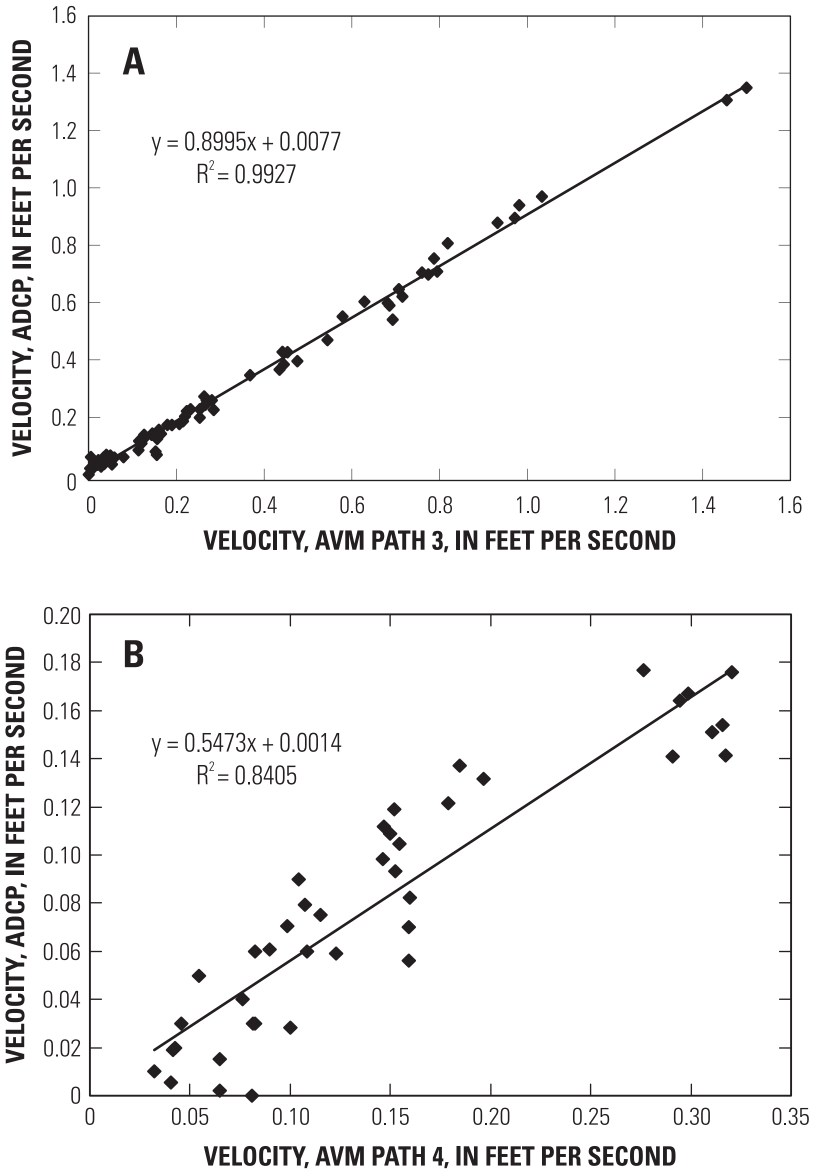


Figure 10. Index-velocity ratings for the acoustic velocity meter (AVM) paths A) 3 and B) 4 of the U.S. Geological Survey streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois, as determined by acoustic Doppler current profiler (ADCP) discharge measurements.

the Columbus Drive station for the 1997 WY in figure 12. Whereas MWRDGC high-flow data (discharges greater than about 50 ft^3/s) are well correlated with the USGS data, low-flow data are not correlated. When low-flow data are removed (flows less than 50 and 200 ft^3/s at the CRCW and Columbus Drive, respectively), the remaining high-flow data are well correlated (coefficient of determination, $R^2 = 0.91$). Furthermore, the data are equally scattered about the regression line throughout the WY (fig. 13), indicating that the best-fit curve can be used to estimate missing data during high-flow periods. The equation used to estimate high-flow data missing from December 2, 1996, to September 30, 1997, is

$$Q_{USGS} = 1.24\ Q_{MWRDGC} + 252.9\ , \qquad (1)$$

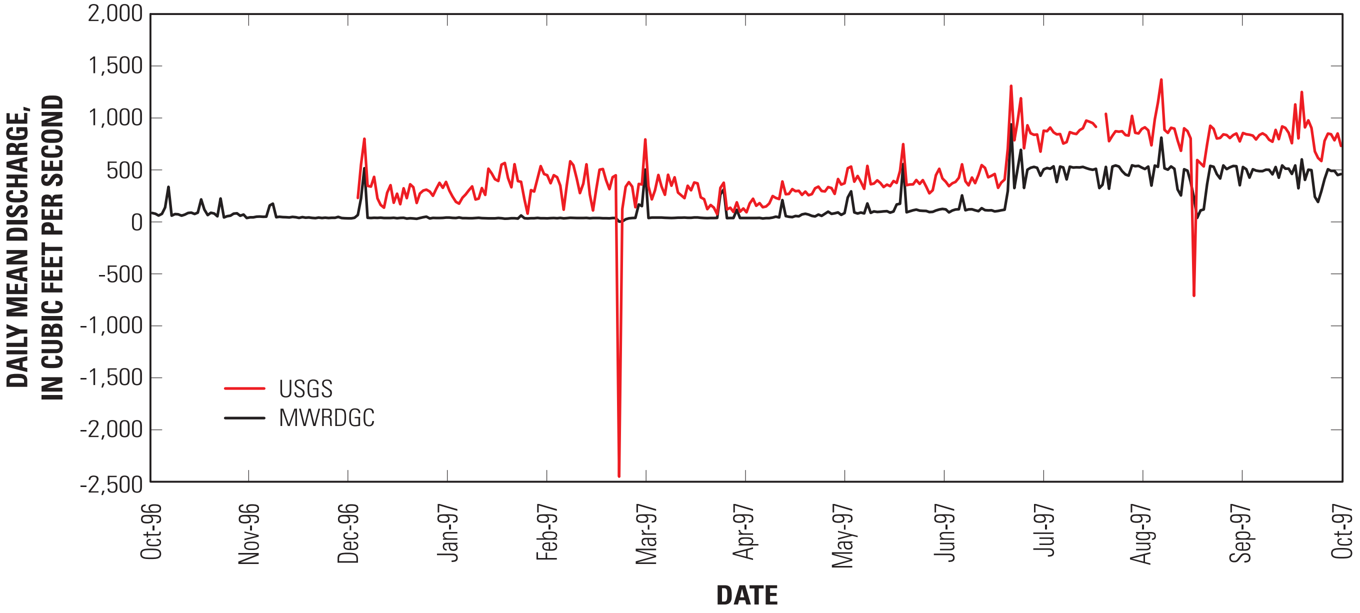


Figure 11. Daily mean discharges at the U.S. Geological Survey (USGS) streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois from December 2, 1996, to September 30, 1997, and discharges measured for the Chicago River Controlling Works (CRCW) as reported by the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC). Negative discharges indicate flow reversals during large storms. Storms in February and August 1997 resulted in large negative discharges.

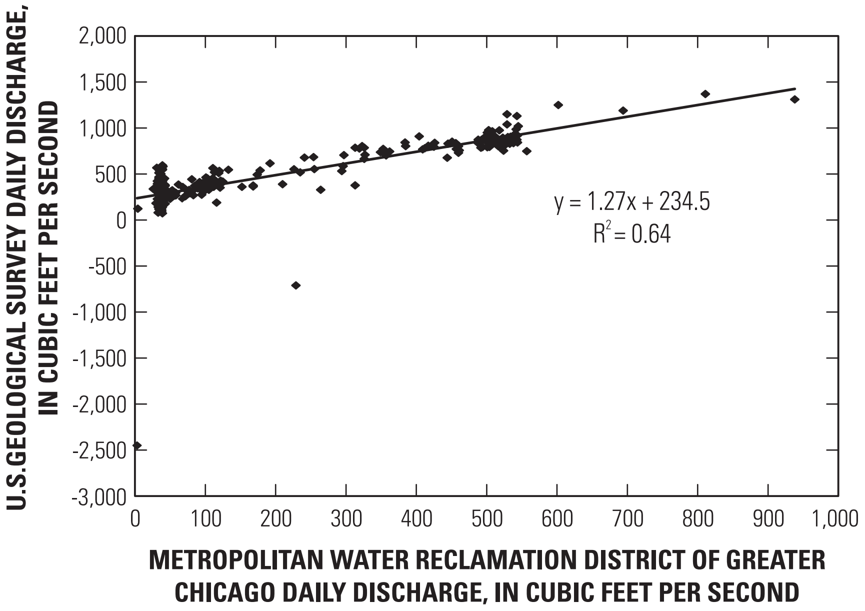


Figure 12. Regression between daily mean discharges measured at the U.S. Geological Survey (USGS) streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois from December 2, 1996, to September 30, 1997 and the daily mean discharges for the Chicago River Controlling Works (CRCW) as reported by the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC). Negative discharges indicate flow reversals during large storms.

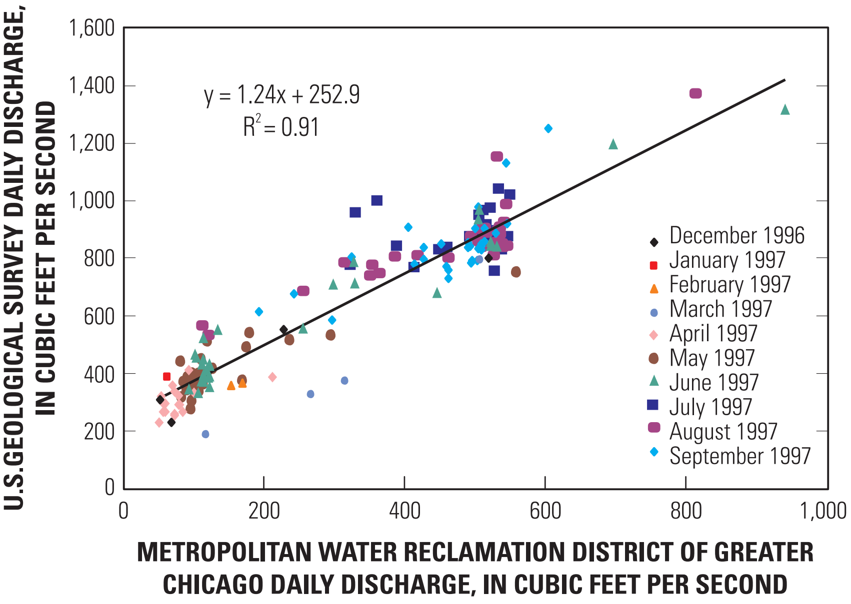


Figure 13. Daily mean discharges at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station by the U.S. Geological Survey (USGS) in water year 1997 and for Chicago River Controlling Works as reported by the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) (data points with MWRDGC flows greater than or equal to 50 cubic feet per second and USGS flows greater than or equal to 200 cubic feet per second only are shown).

where Q_{USGS} is the daily mean discharge computed by the USGS, Q_{MWRDGC} is the daily mean discharge as reported by the MWRDGC, and the coefficient of determination, R^2 , of equation 1 is 0.91.

Equation 1 was derived based on high-flow data collected from December 2, 1996, to September 30, 1997, only; therefore, equation 1 is not applicable for estimating the missing data for October and November 1996. Instead, these missing data were estimated using monthly equations derived from the daily flow data reported by the MWRDGC and recorded by the USGS in October and November 1997. These data together with the respective best-fit curves are shown in figures 14 and 15, respectively. The corresponding equations are

$$Q_{USGS} = 41.7\, Q_{MWRDGC}^{0.491} \,, \tag{2}$$

$$Q_{USGS} = 369.4\, Ln\, (Q_{MWRDGC}) - 1201 \,, \tag{3}$$

with R^2 values of 0.92 and 0.71, respectively.

Periods of missing data during the 1998-99 WY's were estimated based upon regressions derived from the daily flow data reported by the MWRDGC for adjacent periods where both the USGS and MWRDGC stations were operating. The corresponding equations are

$$Q_{USGS} = 1.176 Q_{LMO\text{-}6} + 138.24 \quad \text{(for MWRDGC discretionary flow periods)}, \tag{4}$$

$$Q_{USGS} = 1.775 Q_{LMO\text{-}6} - 2.265 \quad \text{(for MWRDGC nondiscretionary flow periods)}, \tag{5}$$

with R^2 values of 0.90 and 0.24, respectively.

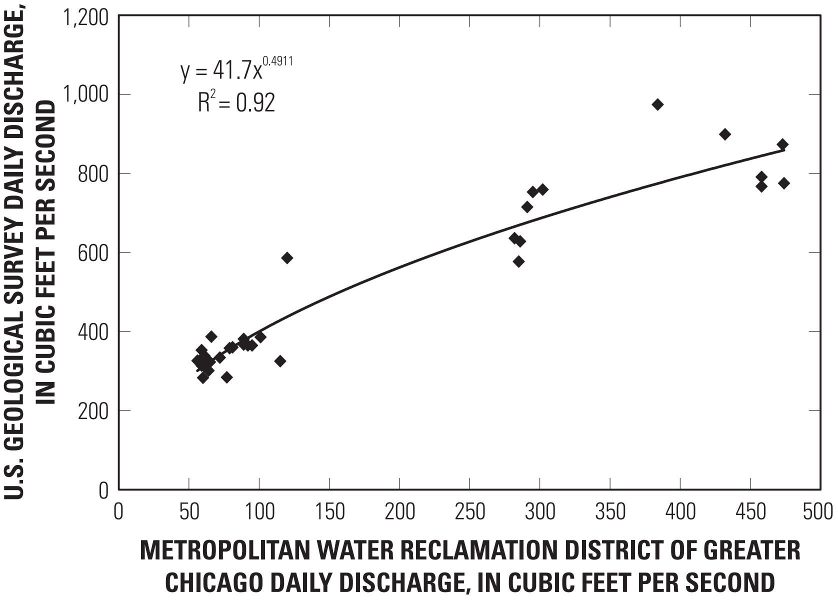


Figure 14. Daily mean discharges at the U.S. Geological Survey streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois in October 1997 and for the Chicago River Controlling Works as reported by the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC).

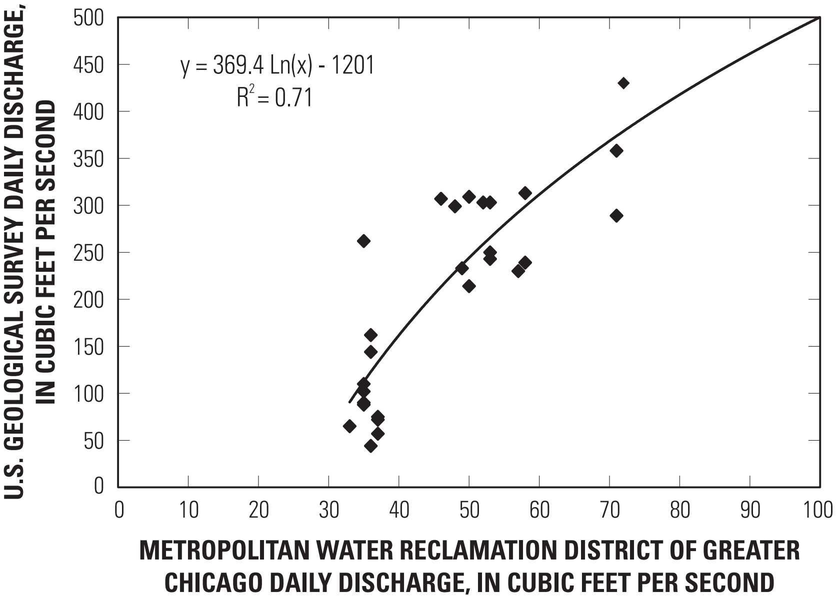


Figure 15. Daily mean discharges at the U.S. Geological Survey streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois in November 1997 and for the Chicago River Controlling Works as reported by the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC).

The USGS daily mean discharge record for the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station for the 1997-99 WY's, completed using equations 1, 2, and 3, are listed in tables B.4-B.6 (appendix B). Daily mean discharge hydrographs for the USGS streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois and the MWRDGC flow record at the CRCW for the 1997-99 WY's are shown in figures 16-18.

Calumet River below O'Brien Lock and Dam at Chicago, Illinois

The Calumet River below O'Brien Lock and Dam streamflow-gaging station (05536358) (fig. 1) was established on October 1, 1996. The station consists of an electronic datalogger with a shaft encoder operated over a float within the U.S. Army Corps of Engineers (Corps) stilling well, an AVM, and a cell-phone telemetry system within an aluminum instrument shelter. A 10 ft meteorological tower supports an anemometer and a tipping-bucket raingage. Electricity (110 volts AC) is available to run the equipment.

The AVM at the Calumet River below O'Brien Lock and Dam is configured to measure velocity on two paths that are located approximately 800-1,000 ft downstream of the riverside lock gates (fig. 19). The AVM velocity paths measure line velocities along paths between the riverside lock guidewall and a pile cluster near the left bank. Using this configuration, the AVM computes velocity for an upstream velocity path (path 1) and a downstream velocity path (path 2). At the initial gage installation, velocity was computed for each AVM path averaged at 15-minute intervals. On July 13, 1998, the AVM was reprogrammed to compute velocity averaged at 5-minute intervals in order to provide more detailed information on the variability of flow over the shorter time interval. Daily mean discharges for each velocity path are averaged to determine the daily mean discharge for the gaging station.

The stilling-well shaft encoder at the gaging station provided a satisfactory stage record for the 1997-99 WY period. Stage data are referenced to Chicago City datum (579.48 ft NGVD 29)

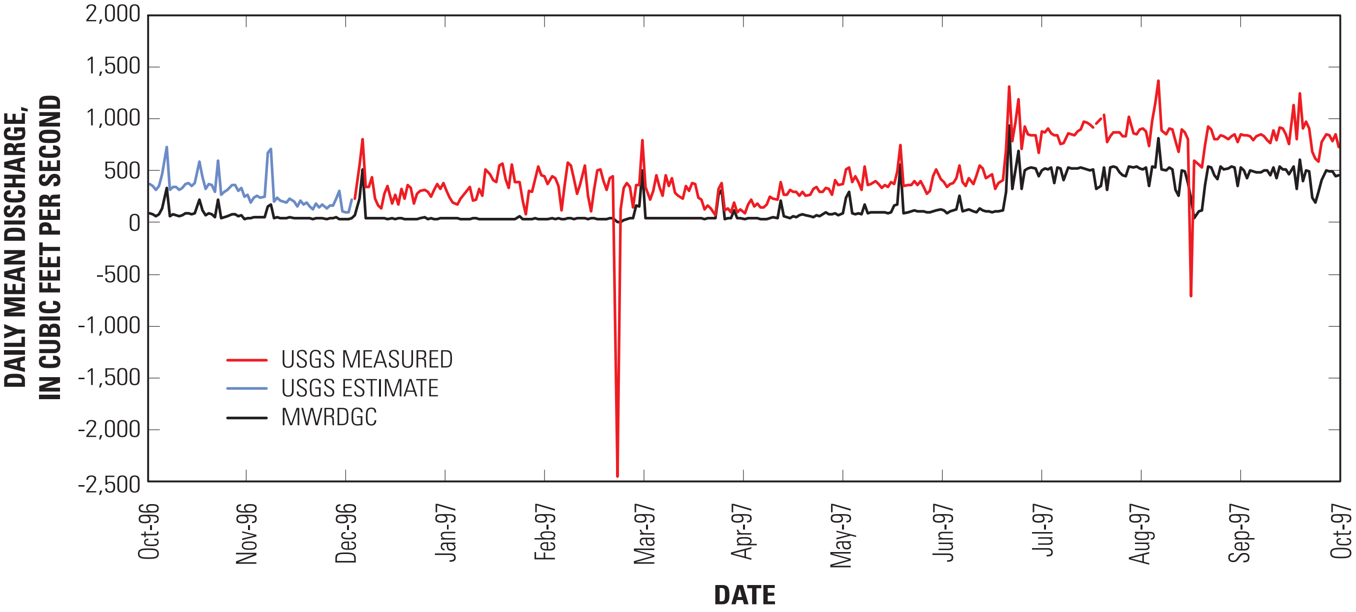


Figure 16. Daily mean discharges (measured and estimated) at the U.S. Geological Survey (USGS) streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois, October 1, 1996, through September 30, 1997, and for the Chicago River Controlling Works as reported by the the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC). Negative discharges indicate flow reversals during large storms. Storms in February and August 1997 resulted in large negative discharges.

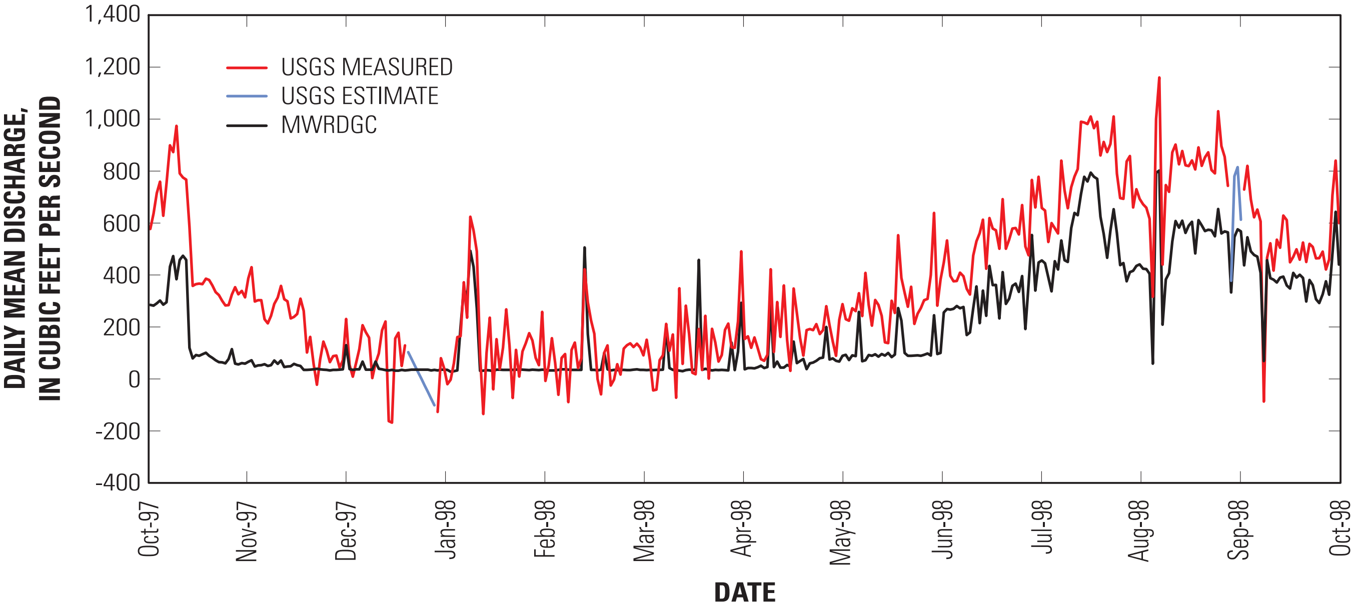


Figure 17. Daily mean discharges (measured and estimated) at the U.S. Geological Survey (USGS) streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois, October 1, 1997, through September 30, 1998, and for the Chicago River Controlling Works as reported by the the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC). Negative discharges indicate flow reversals during large storms.

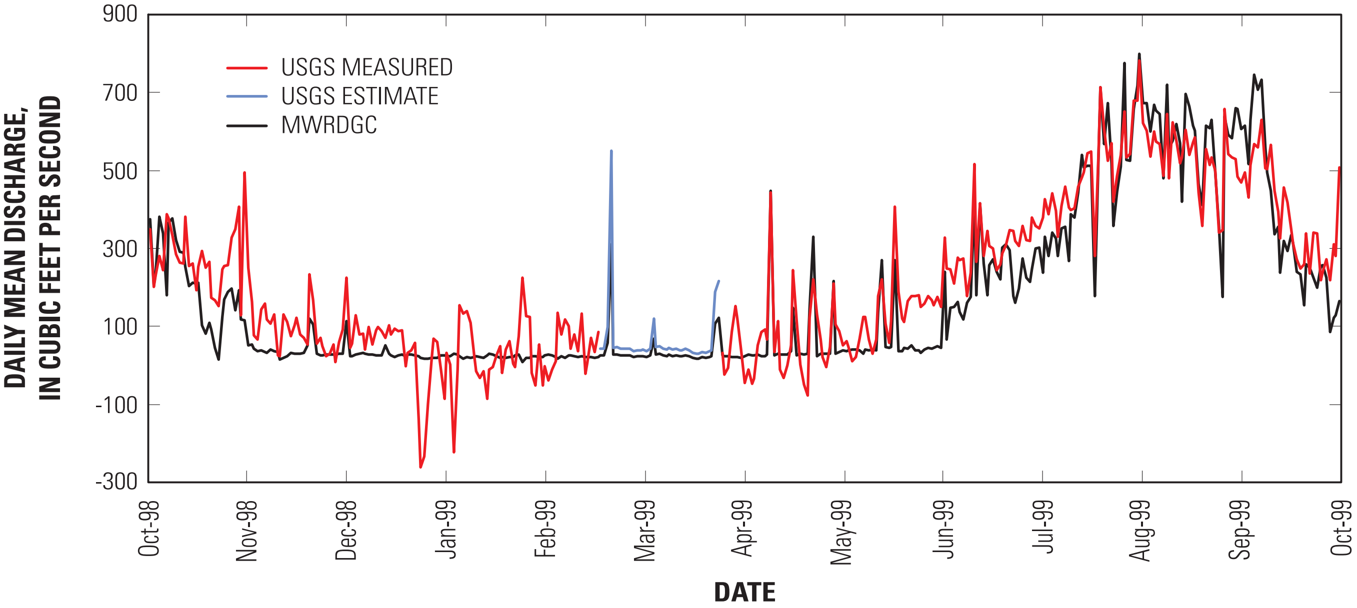


Figure 18. Daily mean discharges (measured and estimated) at the U.S. Geological Survey (USGS) streamflow-gaging station at the Chicago River at Columbus Drive at Chicago, Illinois, October 1,1998, through September 30, 1999, and for the Chicago River Controlling Works as reported by the the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC). Negative discharges indicate flow reversals during large storms.

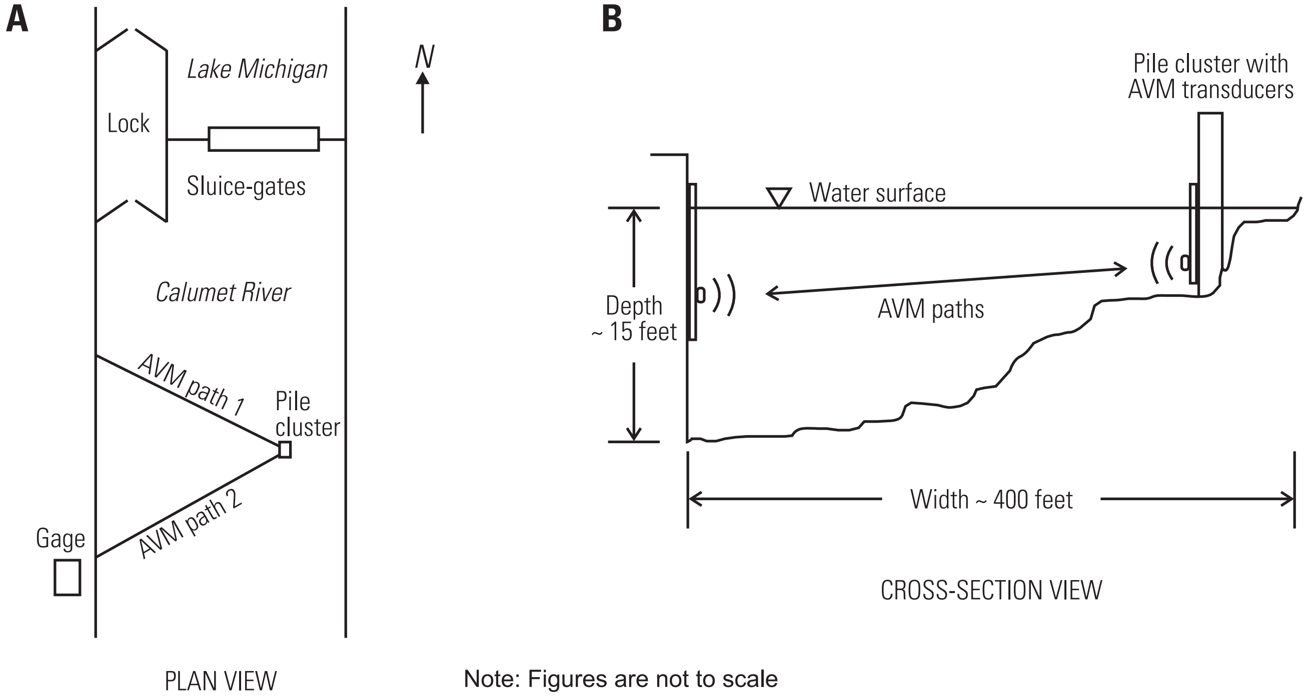


Figure 19. (A) Plan and (B) cross-sectional views of the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois.

through a survey completed on October 24, 1996. A bathymetric survey of the channel was completed on March 24, 1997, to determine the stage-area rating (fig. 20).

Satisfactory data were collected with the AVM throughout the 1997-99 WY period except as follows: November 15-December 6, 1996, and January 14-July 17, 1997. The extended period of missing record from January 14 to July 17 was the result of major damage to the piling supporting the AVM transducers because of either barge traffic or ice. Thirteen series of ADCP discharge

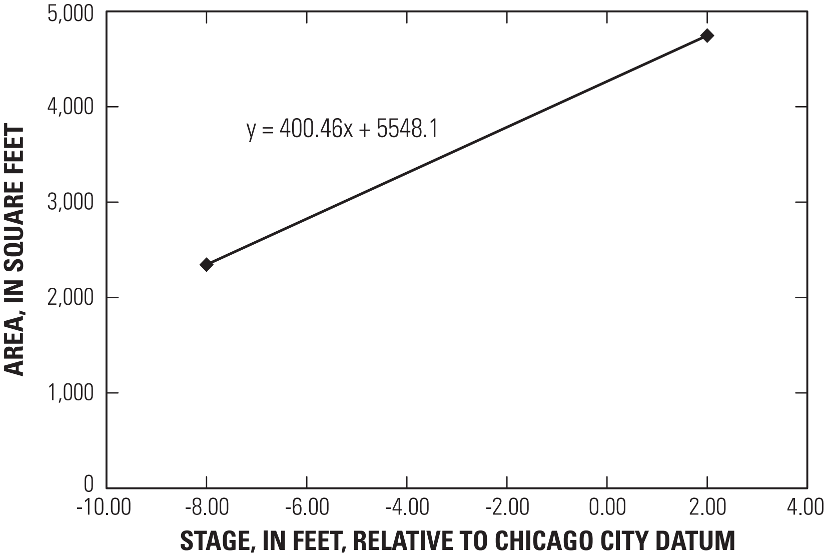


Figure 20. Stage–area rating for the U.S. Geological Survey streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois.

measurements (261 individual transects) were made during the 1997-99 WY period to define the index-velocity rating (fig. 21). These measurements ranged from –3,180 to 2,881 ft^3/s. Measurement numbers 64-67 were made during a reverse flow event, when the flow direction was reversed (towards Lake Michigan).

The daily mean discharge during the 1997-99 WY period ranged from –769 to 1,069 ft^3/s. The minimum daily mean discharge during the water year occurred during a storm on Feb. 21, 1997. The daily mean discharge during the 1997-99 WY period ranged from –769 to 1,069 ft^3/s. The minimum daily mean discharge during the water year was measured during a storm on Feb. 21, 1997, when the Calumet River was allowed to backflow (reverse flow) into Lake Michigan. A second storm-related backflow on Aug. 20, 1997, resulted in a daily mean discharge of –301 ft^3/s. The maximum daily mean discharge of 1,069 ft^3/s occurred on June 21, 1997. The USGS daily

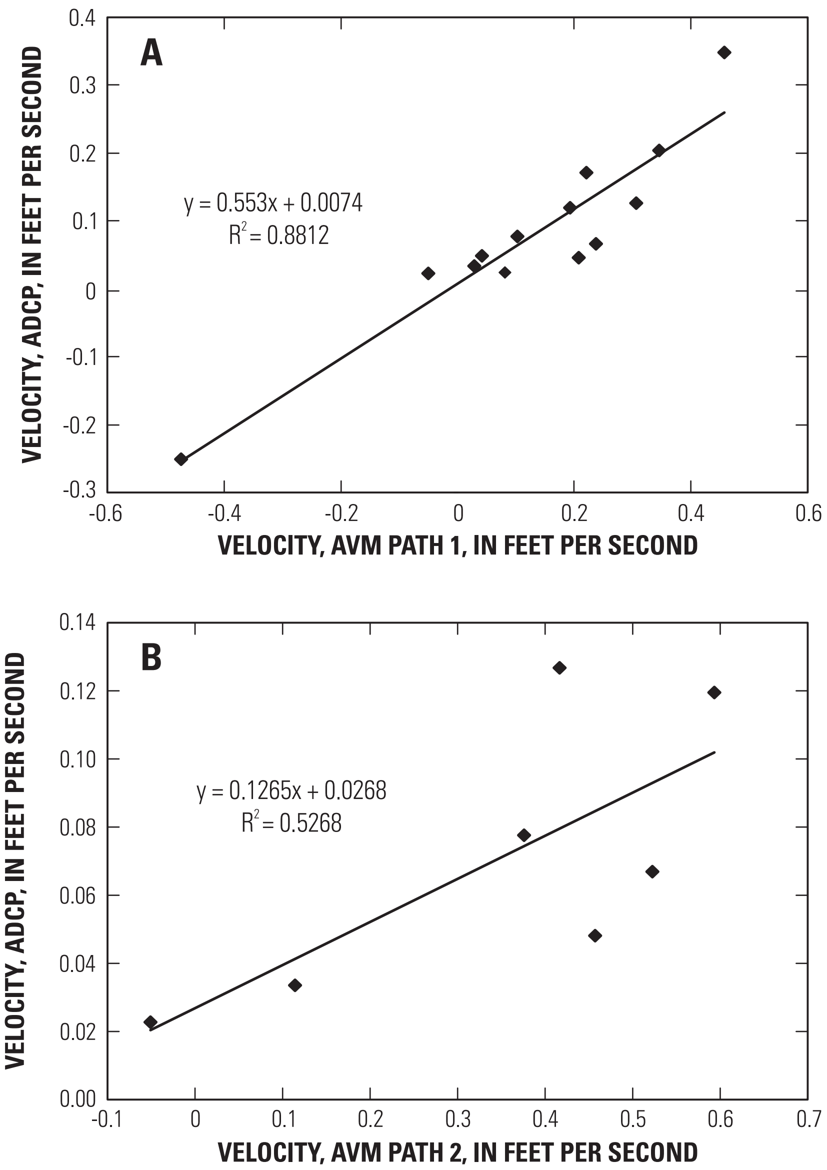


Figure 21. Index-velocity ratings for the U.S. Geological Survey acoustic velocity meter (AVM) paths A) 1 and B) 2 of the streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois. Negative velocities indicate flow reversals during large storms.

mean discharge record for the O'Brien Lock and Dam station for the 1997-99 WY's is listed in tables 2.7-2.9 (appendix B).

The procedure used for estimating missing flows in the Calumet River below O'Brien Lock and Dam for days on which the AVM was inoperative was developed by postulating that, similar to what was observed at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station, the daily mean flow records at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois streamflow-gaging station by MWRDGC and USGS are correlated. Available data from 1997-99 WY's appear to support this postulate, particularly the data points representing days with navigational make-up or discretionary flows (fig. 22).

Topography in this reach of river and the operational variability of the structures at the lock and dam result in different flow patterns. These flow patterns also are affected by the magnitude of the total flow and the relative value of the flow components. Because of these flow conditions, flows at Calumet River below O'Brien Lock and Dam at Chicago, Illinois streamflow-gaging station cannot be estimated by indexing the mean flow with only one AVM path. To better estimate flows at this station, the mean flow is indexed with the two AVM paths. The layout of these AVM paths is congruent with both the observed flow patterns and the shape of the channel cross section. In estimating flows from the AVM data, it was necessary to assess whether the mean flow is more accurately indexed by a specific AVM path. After careful comparison of different weighting factors for averaging the AVM data from the two paths, it was determined that the best correlation between the MWRDGC data and the USGS AVM data is obtained when each path is given the same weight. The MWRDGC data from days with navigation and/or discretionary flows in 1997 and 1998 WY's and the USGS data estimated as the weighted average of the data from AVM path 1 and AVM path 2 are shown in figure 22. The MWRDGC data and the USGS data seem better correlated when the USGS flows are estimated as the average of the flow estimates from each AVM, rather than when estimated giving different weight to the data from each AVM path. In addition, giving the same weight to the data from each AVM path also makes the data from each

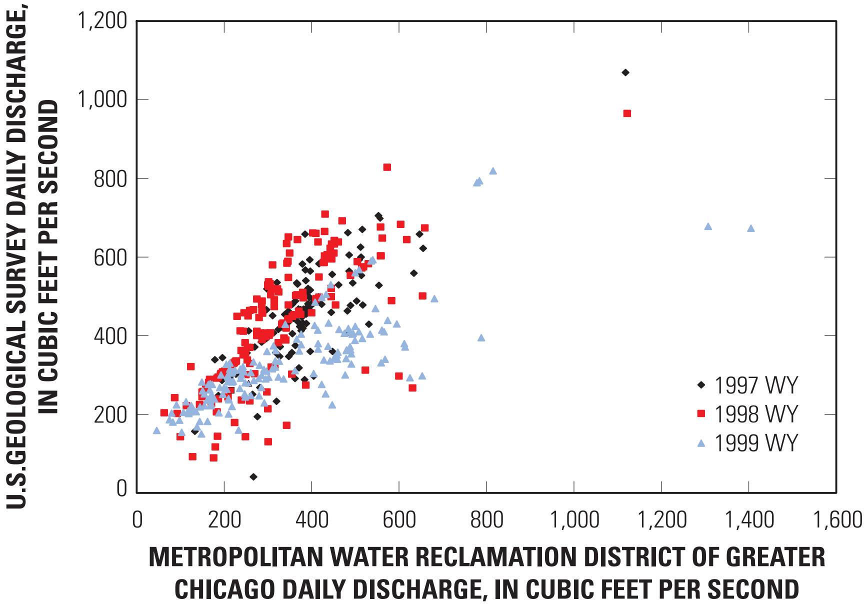


Figure 22. Daily mean discharge for the U.S. Geological Survey streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois, and flows reported by the Metropolitan Water Reclamation District of Greater Chicago for days with navigation or discretionary flows for the 1997-99 water years (WY's).

year more homogeneous, thus, reducing bias. The equations for estimating missing record for the Calumet River below O'Brien Lock and Dam at Chicago, Illinois streamflow-gaging station were derived from these data based on linear regression (fig. 22). The resulting equation is

$$Q_{USGS} = 0.822\ Q_{MWRDGC} + 149.2\ , \qquad (6)$$

with a coefficient of determination $R^2 = 0.49$.

Equation 6 was used to estimate missing data for days with navigation and/or discretionary flows only at the streamflow-gaging station. Navigation and/or discretionary flows are diversion flows through the control structures to maintain channel depths needed for navigation or flows used at the discretion of MWRDGC to maintain water quality in the Chicago River system. For days with neither navigation nor discretionary flows, missing USGS data were assumed equal to those reported by MWRDGC. The USGS daily mean discharge record for the Calumet River below O'Brien Lock and Dam at Chicago, Illinois streamflow-gaging station for the 1997-99 WY's are listed in tables B.7-B.9 (appendix B). Daily mean discharge hydrographs for the USGS streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois and the MWRDGC flow record for the 1997-99 WY's are shown in figures 23-25.

North Shore Channel at Wilmette, Illinois

The North Shore Channel at Wilmette streamflow-gaging station (05536101) was established on September 7, 1999. The station consists of an electronic datalogger with an AVM, acoustic stage transducer, and telephone telemetry system within an aluminum instrument shelter. Electric power (110 volts AC) is available to run the equipment.

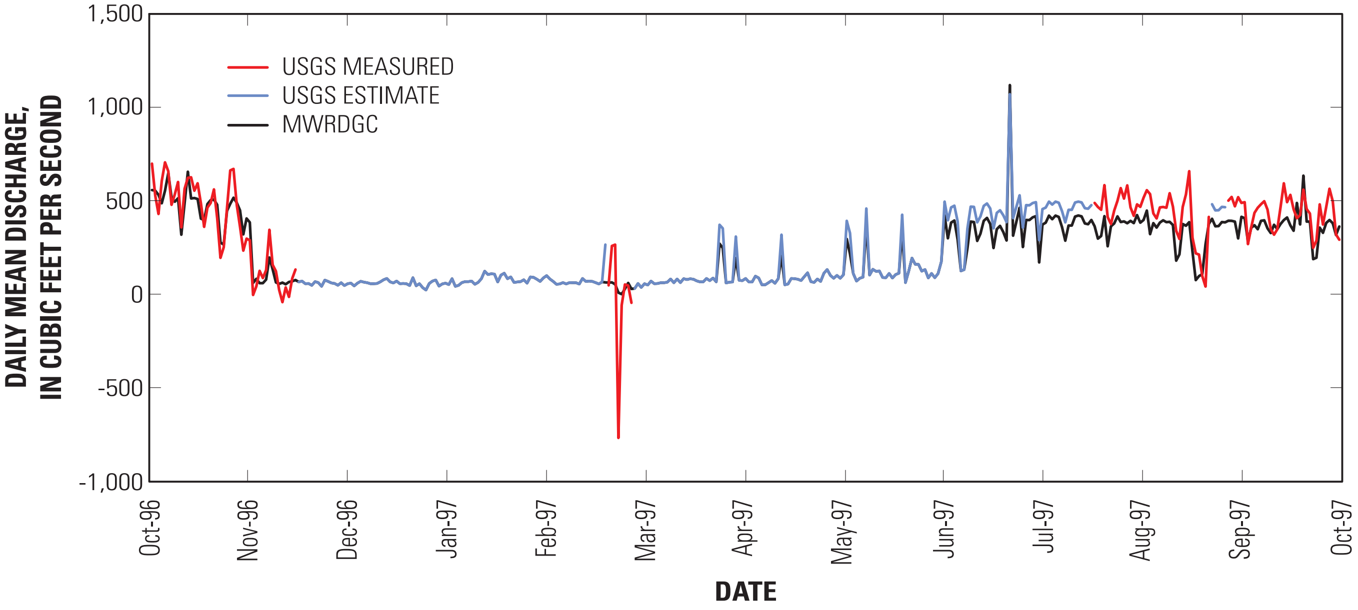


Figure 23. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois and from the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record for the 1997 water year. Negative discharges indicate flow reversals during large storms. A storm in March 1997 resulted in large negative discharges.

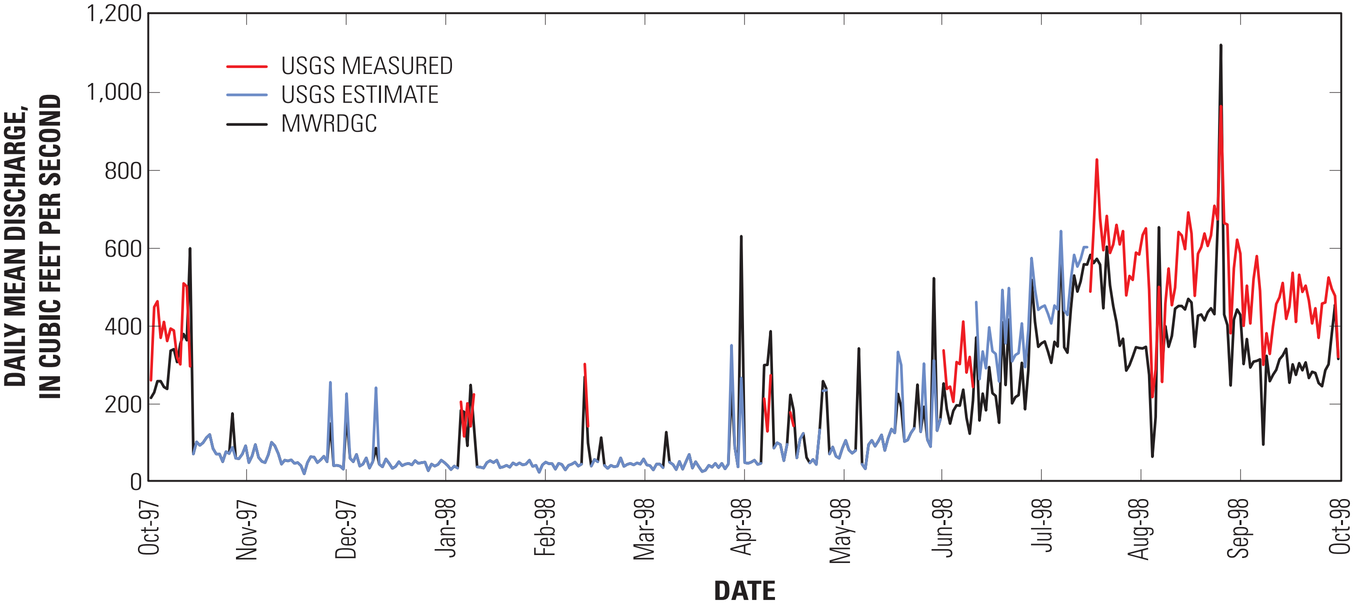


Figure 24. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois and from the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record for the 1998 water year.

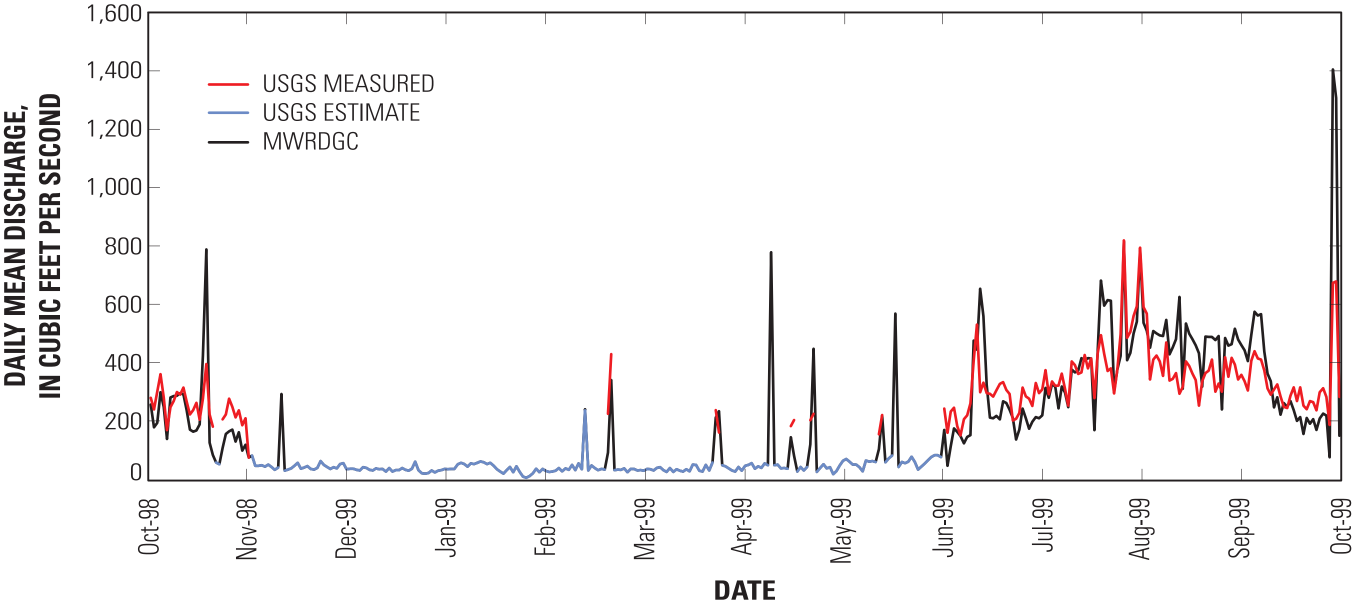


Figure 25. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station at the Calumet River below O'Brien Lock and Dam at Chicago, Illinois and from the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record for the 1999 water year.

The AVM at the North Shore Channel at Wilmette is configured to measure velocity on a single path. Stage data that is set at a fixed elevation of approximately –4.00 ft (CCD 571.48 ft NGVD 29) in the channel (fig. 26).

Because the USGS station was established after the 1999 WY, discharge records for the entire 1997-99 WY's were estimated based upon a correlation between USGS and MWRDGC daily discharge data for the 2000 WY (fig. 27) as

$$Q_{USGS} = 0.9596\ Q_{MWRDGC} + 0.5914\ . \qquad (7)$$

The acoustic stage transducer at the gaging station provided a satisfactory stage record for the period from September 7, 1999, to September 30, 2000, except as follows: October 14, October 27-November 17, 1999, March 29, April 1-4, and April 12-17, 2000. Stage data are referenced to Chicago City datum (579.48 ft NGVD 29) through a survey completed on September 6, 2000. A bathymetric survey of the channel was completed on December 7, 1999, to determine the stage-area rating (fig. 28).

The AVM provided satisfactory record throughout the water year except during April 12-17, 2000. Ten series of ADCP discharge measurements (241 individual transects) were made during the 2000 WY to define the index-velocity rating (fig. 29). These measurements ranged from -22.5 to 179 ft^3/s.

The estimated daily mean discharge during the 1997-99 WY's ranged from 0.59 to 245 ft^3/s and is listed in tables B.10-B.12 (appendix B). The minimum daily mean discharge during the 1997-99 WY period was measured on many days during the period. The maximum daily mean discharge of 245 ft^3/s was measured on September 10, 1997.

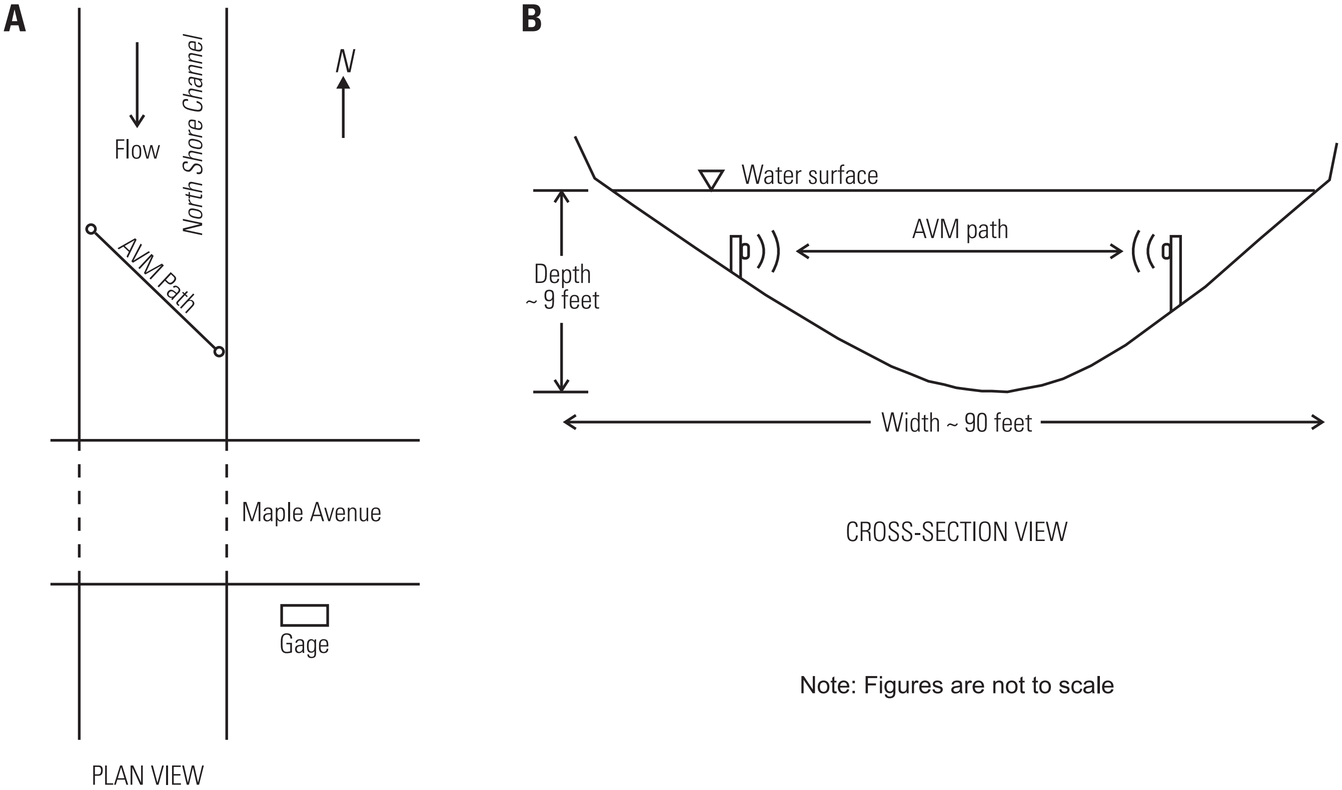


Figure 26. (A) Plan and (B) cross-sectional views of the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station on the North Shore Channel at Wilmette, Illinois.

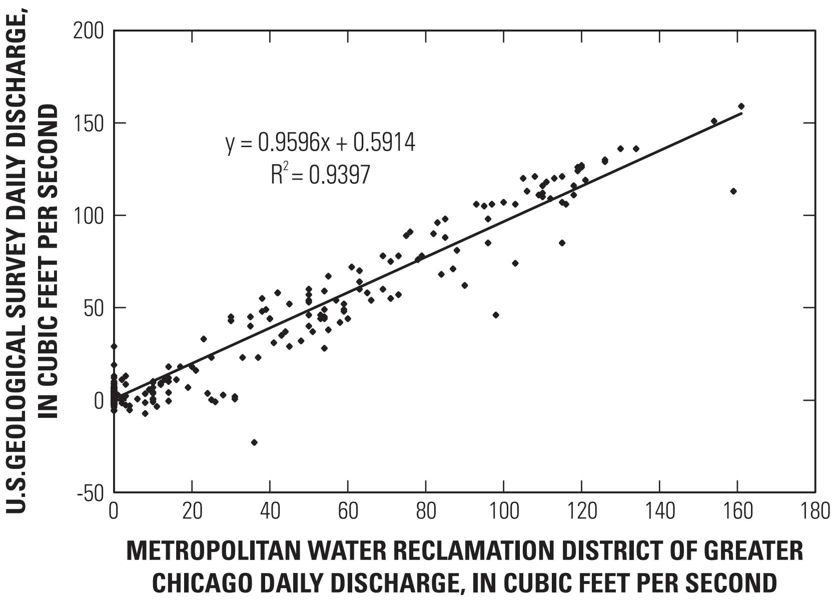


Figure 27. Correlation between the U.S. Geological Survey and Metropolitan Water Reclamation District of Greater Chicago daily mean discharge at the North Shore Channel at Wilmette, Illinois acoustic velocity meter (AVM) streamflow-gaging station for the 2000 water year.

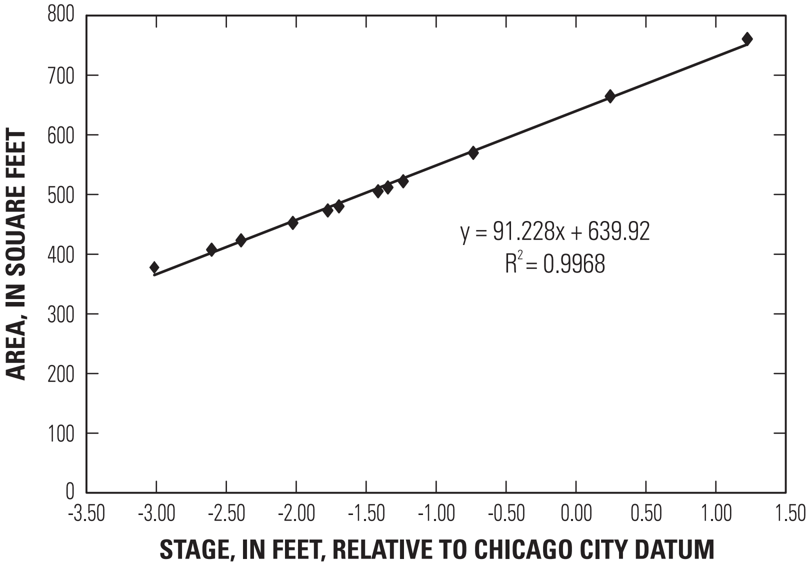


Figure 28. Stage-area rating for the U.S. Geological Survey streamflow-gaging station on the North Shore Channel at Wilmette, Illinois, as determined by bathymetric survey.

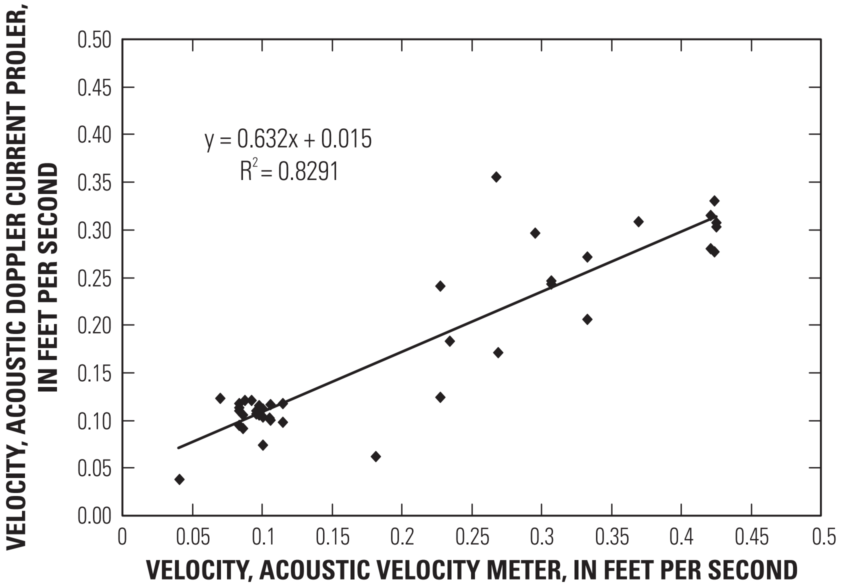


Figure 29. Index-velocity rating for the U.S. Geological Survey streamflow-gaging station on the North Shore channel at Wilmette, Illinois.

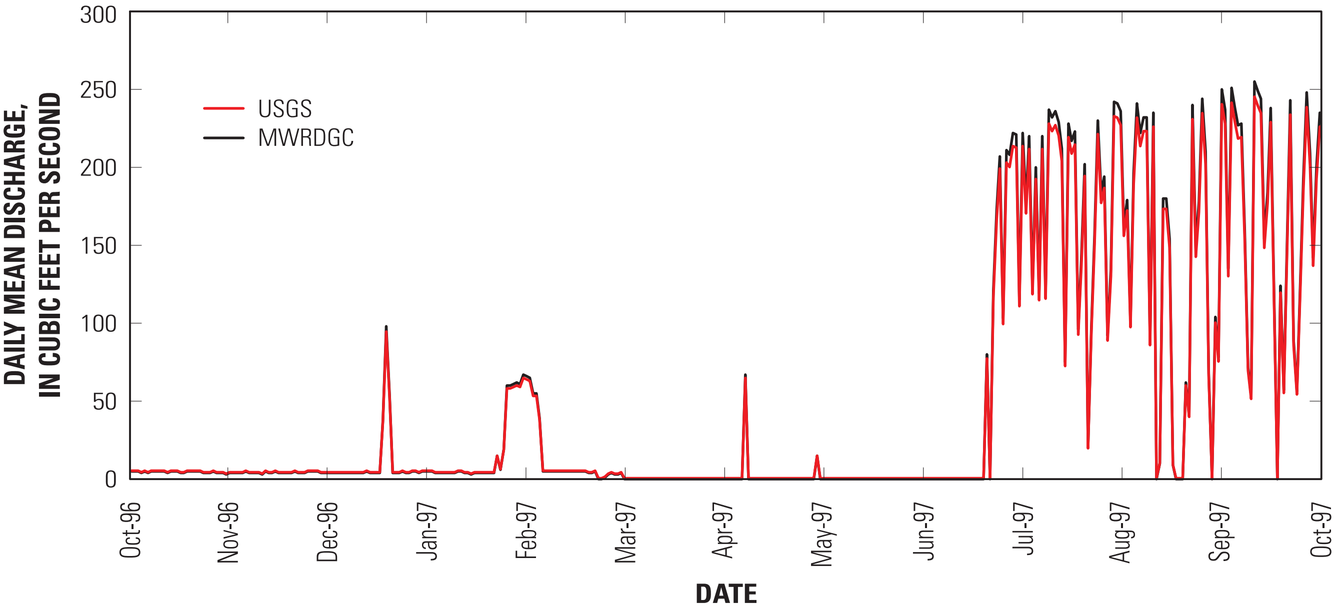


Figure 30. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station on the North Shore Channel at Wilmette, Illinois and from the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record for the 1997 water year.

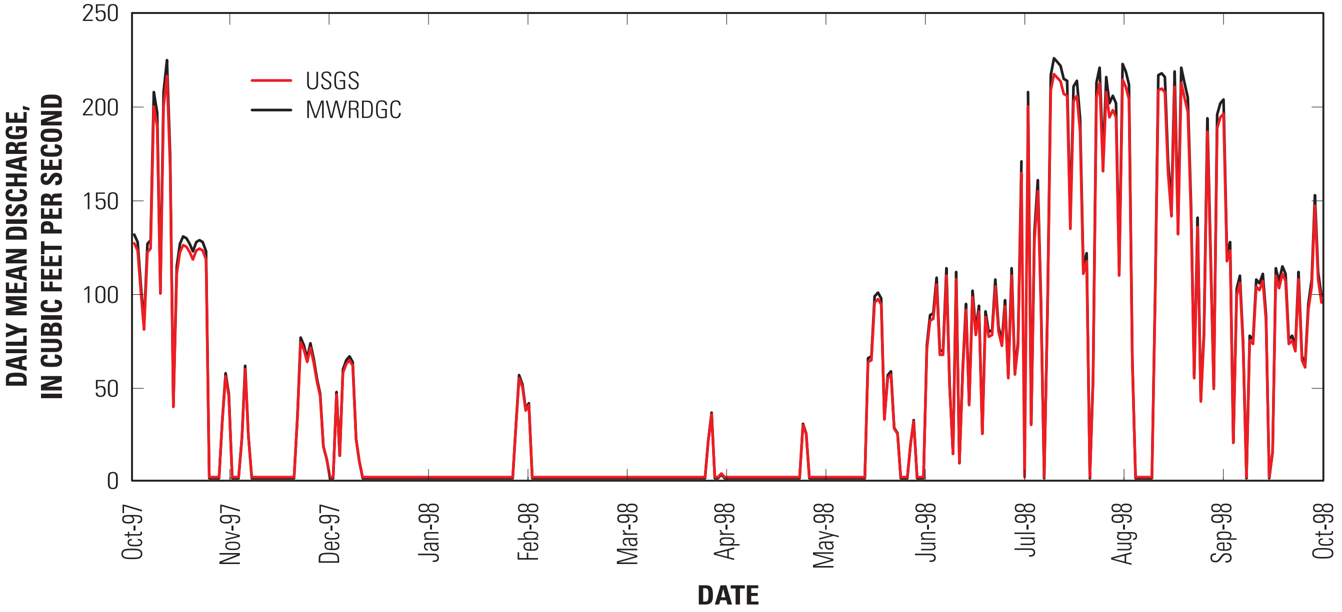


Figure 31. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station on the North Shore Channel at Wilmette, Illinois and from the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record for the 1998 water year.

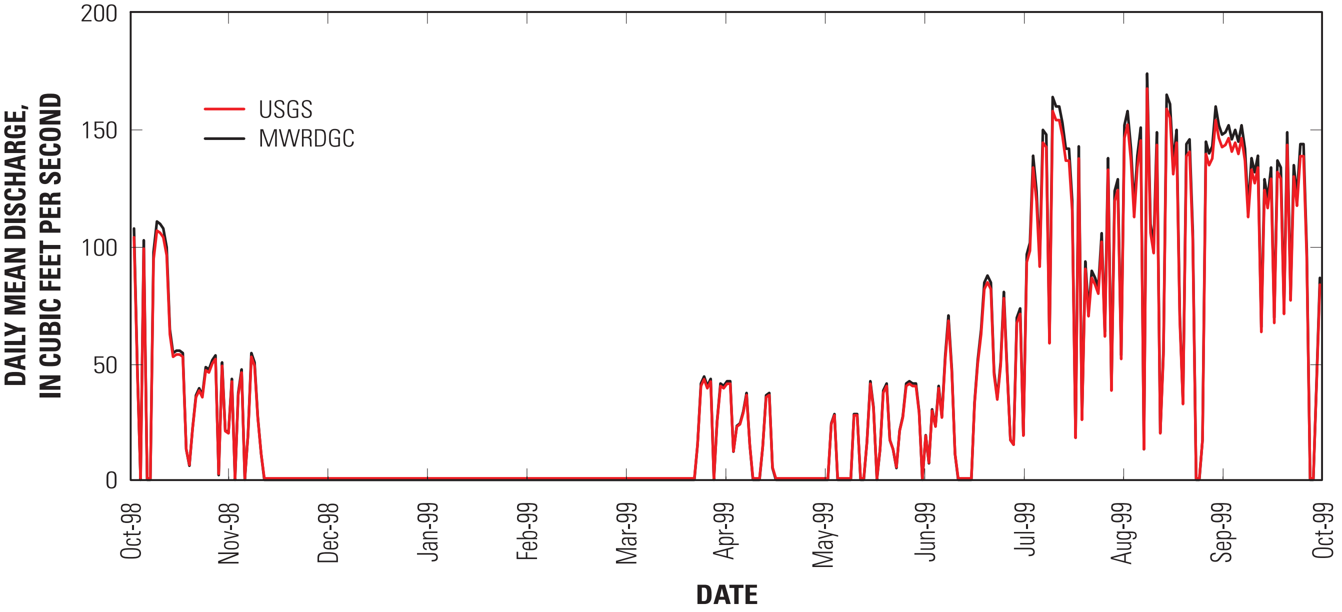


Figure 32. Daily mean discharges for the U.S. Geological Survey (USGS) streamflow-gaging station on the North Shore Channel at Wilmette, Illinois and from the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) record for the 1999 water year.

Error Analysis of Discharge Computations from Acoustic Velocity Meter Measurements

In this section, a method for computing the total error of the discharge estimates at 5-minute to annual time scales is derived and applied to the discharges at the four streamflow-gaging stations on the Chicago River system (fig. 1). An introduction to the method, called first-order error analysis, in the context of its application to streamflow discharge uncertainty estimation is presented first. Uncertainty estimates for velocity and cross-sectional area are required for the first-order error analysis. A method for computing the uncertainty of the cross-sectional area is presented next; then a method for computing the uncertainty of the velocity is presented using the statistical properties of the index-velocity rating (IVR) between a single AVM path and ADCP measurements. This rating completes the development of the basic method for error analysis of unit discharge measurements when the AVM is operational. Extension to uncertainty estimation for discharge averaged over time intervals longer than the measurement period, such as a year, is required for Lake Michigan Diversion Accounting and is presented next. The method for computation of velocity uncertainty then is extended to the case of velocity computed as the average of the velocities computed from two AVM paths with separate index-velocity ratings, as at the Calumet River below O’Brien Lock and Dam at Chicago, Illinois streamflow-gaging station. This section is followed by the final section of the presentation of the methods. This section contains a description of the method for computing uncertainty on days when AVM measurements are completely or mostly missing and the discharges are estimated from other data sources. The presentation of the method is followed by application of the method to the four streamflow-gaging stations. The presentation of the application of the method begins with computation of the stage-area, index-velocity, and uncertainty parameters used for each station. Finally, the resulting discharge uncertainty estimates are presented and described.

First-Order Error Analysis of Discharge

The total uncertainty of flow measurements traditionally has been estimated as the square root of the summation of the squares of the total uncertainties from different sources (for example, Carter and Anderson, 1963; Simpson and Oltman, 1993). The International Organization for Standardization (1992) recommends estimating the total uncertainty of measurements from AVM’s with a method based on a different approach. In this method, the total uncertainty of AVM measurements is estimated as the square root of the sums of the squares of the uncertainties of the contributing sources weighted with prescribed factors; however, the values of these factors prescribed in the standard are not clearly justified. More advanced methods for estimating total uncertainty gradually are being incorporated into present-day (2005) engineering practice. One such method is the first-order-variance method (also known as the first-order error analysis) (Ang and Tang, 1984; Tung and Yen, 1993; ANSI/ASME, 1998; Muste and Stern, 2000).

The first-order variance method has two fundamental components: (1) the measured or estimated value of some quantity X is considered to be the sum of a fixed and true but unknown value X' and an independent, mean zero error term ε_X; that is, $X = X' + \varepsilon_X$; and (2) the estimation error of a quantity Y that is a function of one or more measured or estimated quantities arises because of the error of estimation of the variables from which it is computed, and is computed according to a first-order approximation to the complete variance formula. Formally, for a quantity $Y = g\left(X_1, X_2, \ldots, X_n\right)$, in the first-order variance method, the uncertainty of Y is the first-order approximation to the variance of *Y*,

$$\sigma_Y^2 \cong \sum_{i=1}^{n} \sum_{j=1}^{n} \left. \frac{\partial g}{\partial X_i} \right|_{\mu} \left. \frac{\partial g}{\partial X_j} \right|_{\mu} \sigma_{X_i,X_j} \tag{8}$$

(see, for example, Benjamin and Cornell (1970, p. 184)), where $\left. \frac{\partial g}{\partial X_i} \right|_{\mu}$ indicates the evaluation of the partial derivative at the mean μ and σ_{X_i,X_j} indicates the covariance of X_i and X_j. As the mean μ (the true value) is not known, the partial derivatives are, in practice, evaluated at the observations, which have the same mean as the true values. Equation 8 may be derived from a multi-dimensional Taylor series expansion (Shenk, 1979) of $Y = g\left(X_1, X_2, \ldots, X_n\right)$. In the case that the variables $X_1, X_2, \ldots, X_n$ are mutually statistically independent, equation 8 reduces to the form in which the first-order variance method usually is expressed, as

$$\sigma_Y^2 \cong \sum_{i=1}^{n} \left(\left. \frac{\partial g}{\partial X_i} \right|_{\mu} \right)^2 \sigma_{X_i}^2 . \tag{9}$$

In the case of the discharge measurements considered here, the dependent variable is the discharge *Q*, and the independent variables are the velocity *V* and the cross-sectional area *A*. As separate measurement instruments are used to obtain velocity and area, *V* and *A* usually can be taken to be independent. Therefore, the simpler (as compared to equation 8) equation 9 may be used, and the basis of the analysis here is the expression

$$\sigma_Q^2 \approx \left(\left. \frac{\partial Q}{\partial V} \right|_{\mu} \right)^2 \sigma_V^2 + \left(\left. \frac{\partial Q}{\partial A} \right|_{\mu} \right)^2 \sigma_A^2 = A^2 \sigma_V^2 + V^2 \sigma_A^2 \cdot \tag{10}$$

In order to evaluate equation 10, estimates of σ_A^2 and σ_V^2 clearly are needed.

Uncertainty in Cross-Sectional Area

In much of the Chicago River system, because the channels have nearly vertical straight walls in the range of observed stages, the stage-area rating is well-described by the linear equation

$$A(h) = a + bh . \tag{11}$$

In general, the uncertainty in cross-sectional area would include the effects of uncertainty in the parameters describing the stage-area rating (*a* and *b*) because of surveying errors and variation in the cross section over the channel reach covered by the acoustic path of the AVM. However, for this study, these errors are eliminated because the mean channel velocity used in the IVR originally is measured as a discharge and is converted to a velocity by dividing by the estimated *A*. Therefore, when the IVR is used to compute the mean channel velocity from the AVM velocity, followed by multiplication by *A* to obtain *Q*, any systematic error in *A* is accounted for. Biases in *Q* may remain but result because of biases in mean channel velocity, which is accounted for separately.

The only remaining error in A is in the actual measurement of the stage, h. Applying first-order error analysis to the expression for area (equation 11) under the assumption that a and b are nonrandom results in the relation

$$\sigma_A^2 = b^2\sigma_h^2. \tag{12}$$

Uncertainty in Velocity

The uncertainty in estimating the mean velocity in a stream from AVM line-velocity measurements is the most difficult to assess of all the contributing uncertainties in estimating discharge. This difficulty particularly is true at streamflow-gaging stations where the flow structure has characteristics that depend highly upon the mean flow, or where the mean flow is affected by the operation of nearby hydraulic structures. Patino and Ockerman (1997) discussed the various factors affecting the flow structure in open channels and how they affect the relation between the acoustic AVM line velocity and the mean streamwise velocity. In particular, they discussed the relevance of three important sources of uncertainty in estimating mean velocity from AVM line velocity, namely: (a) the vertical distribution of the streamwise velocity; (b) temperature and density gradients; and (c) the variability of flow patterns in the channel. All these sources potentially can affect the flow structure in the channel independently or combined. It is apparent that calibrating the relation between mean velocity and AVM line velocity at sites with complex and variable flow structures based on theoretical velocity distributions as suggested by Laenen (1985) is a crude approximation. Thus, the IVR often is established from concurrent AVM line-velocity measurements and reference mean flow measured with current meters or ADCP's. Moreover, at sites where the range of stage varies appreciably over time, more accurate IVR's are developed as a function of the stage (Laenen, 1985). At more complex sites, such as those with tide-affected flows, ratings include the backwater effects because of ebb, flood, and slack tide conditions. These kinds of ratings typically include a loop and some attempts have been made to represent them mathematically through a Gaussian function (Simpson and Bland, 2000).

At the gaging stations in the Chicago River system considered here, it is observed, as discussed previously, that the IVR's are linear and do not obviously depend on variables other than the AVM line velocity. Letting $V_{ADCP}(t)$ denote the ADCP velocity at time t and $V_L(t)$ the AVM line velocity at time t, the IVR then is given by $V_{ADCP}(t) = \alpha + \beta V_L(t)$. When relating AVM line velocity to the true mean velocity, additional slope and intercept terms are added to simulate the effect of unknown ADCP biases, that is, $V_{bias}(t) = \gamma + \delta V_L(t)$. Combining these assumptions and observations, the velocity equation becomes

$$V_t = V_t' + \varepsilon_V(t) = V_{ADCP}(t) + V_{bias}(t) + \varepsilon_V(t) = \alpha + \gamma + (\beta + \delta)V_L(t) + \varepsilon_V(t). \tag{13}$$

The parameters of equation 13 (α, β, γ, and δ) are, in general, random variables. How their distributions may be estimated is discussed later in this report.

Using the standard formula for the variance of a sum (see Benjamin and Cornell, (1970, p. 168)), the variance of V_t is, therefore, given as

$$Var(V_t) = \sigma_\alpha^2 + \sigma_\gamma^2 + 2Cov(\alpha,\gamma) + V_L^2(t)\left[\sigma_\beta^2 + \sigma_\delta^2 + 2Cov(\beta,\delta)\right] + \\ 2V_L(t)\left[Cov(\alpha,\beta) + Cov(\alpha,\delta) + Cov(\beta,\gamma) + Cov(\delta,\gamma)\right] + \sigma_{\varepsilon_V}^2, \quad (14)$$

using the independence of the error term $\varepsilon_V(t)$ from the other terms in the equation. These terms are not necessarily independent in time, however. Notice that AVM line velocity in equation 14 is taken as a fixed (non-random) observation, whereas, in reality, AVM measurements are subject to measurement error. This result is appropriate because the same statistical population of AVM values is used in the development of the IVR and in the prediction of velocity where the IVR is used. An alternative method to the one taken here is to estimate the AVM measurement error independently (by a first-order error analysis of the AVM measurement process) and incorporate this result into the computation of velocity uncertainty with a more complex regression method where measurement error is accounted for (see, for example, Fuller, 1987). This type of method is required when the true relation between the explanatory and predicted variables is needed. In the present case, all that is required is the prediction, so the more complex method is not needed. However, a method for first-order error analysis of AVM measurements is presented and was applied to two of the gaging stations (Chicago River at Columbus Drive at Chicago, Illinois and Chicago Sanitary and Ship Canal at Romeoville, Illinois) described in appendix A as a first step to this more complete analysis.

Some of the covariance terms in equation 14 may be determined as follows and the equation simplified. It may be shown using the so-called "normal equations" for the parameters of a linear regression (see Benjamin and Cornell (1970, p. 430)) that if a random constant γ is added to each y-value, the regression intercept (here given as α) will be increased by the same amount. Therefore, γ and α have unit correlation and $Cov(\alpha,\gamma) = \sigma_\alpha\sigma_\gamma$. Similarly, it may be shown that γ and β are uncorrelated and, therefore, $Cov(\beta,\gamma) = 0$. The application of a random constant factor δ to each y-value multiplies each linear regression parameter by the same factor; therefore, both δ and α, and δ and β have unit correlation, and $Cov(\alpha,\delta) = \sigma_\alpha\sigma_\delta$ and $Cov(\beta,\delta) = \sigma_\beta\sigma_\delta$. Further, δ and γ are taken to be uncorrelated by assumption. The coefficients α and β are not independent because the coefficients are computed together from the same data by linear regression; their covariance may be estimated from the input data to the IVR regression by applying the standard formula (see Mood and Graybill (1963, p. 333)).

Therefore, equation 14 may be simplified as

$$Var(V_t) = \sigma_\alpha^2 + \sigma_\gamma^2 + 2\sigma_\alpha\sigma_\gamma + V_L^2(t)\left[\sigma_\beta^2 + \sigma_\delta^2 + 2\sigma_\beta\sigma_\delta\right] + \\ 2V_L(t)\left[Cov(\alpha,\beta) + \sigma_\alpha\sigma_\delta\right] + \sigma_{\varepsilon_V}^2. \quad (15)$$

The uncertainty of the unit discharge at a given time *t*, $\sigma_{Q_t}^2 = Var(Q_t)$; thus, the uncertainty can be computed using equations 10, 12, and 15.

Uncertainty in Time-Averaged Discharge

The real quantity of interest here is not the uncertainty in the unit flow at a given time, $\sigma_{Q_t}^2 = Var(Q_t)$, but the uncertainty of the total or average flow over some time period such as a year, where the variance of the total flow is $\mathbf{V} = Var\left(\sum_{t=1}^{n} Q_t\right)$ and the variance of the average flow is $\mathbf{A} = \mathbf{V}/n^2 = Var\left(\frac{1}{n}\sum_{t=1}^{n} Q_t\right)$. In particular, $\mathbf{V}$ is given by

$$\mathbf{V} = Var\left(\sum_{t=1}^{n} Q_t\right) = \sum_{t=1}^{n} \sigma_{Q_t}^2 + 2\sum_{t=1}^{n-1}\sum_{s=t}^{n} Cov\left(Q_t, Q_s\right). \tag{16}$$

The first term on the right-hand side of equation 16 is the sum of the unit discharge uncertainties, $\sigma_{Q_t}^2$, at each time *t*. The expression for $\sigma_{Q_t}^2$ is given above (equation 10), allowing computation of this term. The second term consists of the sum of the covariance between the unit discharge at each pair of times *t* and *s*, and includes contributions from any uncertainty that is in common between the discharges at those times. The dominant common uncertainties between discharges at different times are the uncertainties in the index-velocity regression and the ADCP bias errors. A derivation of an expression for the discharge covariance is given below.

The covariance of flows at times *t* and *s*, $Cov\left(Q_t, Q_s\right)$, may be computed as

$$Cov\left(Q_t, Q_s\right) = E\left[Q_t Q_s\right] - E\left[Q_t\right] E\left[Q_s\right]. \tag{17}$$

To compute the second term in equation 17, it may be shown that

$$E\left[Q_t\right] = E\left[A_t V_t\right] = E\left[\left(A_t' + \varepsilon_A(t)\right)\left(V_t' + \varepsilon_V(t)\right)\right] = E\left[A_t' V_t'\right] = A_t' V_t' , \tag{18}$$

because $\varepsilon_A(t)$ and $\varepsilon_V(t)$ for all *t* are independent and have zero mean. An analogous expression holds for $E\left[Q_s\right]$. Therefore, $E\left[Q_t\right] E\left[Q_s\right]$ is given by $E\left[Q_t\right] E\left[Q_s\right] = A_t' V_t' A_s V_s'$.

For the first term in equation 17, $E\left[Q_t Q_s\right]$ may be derived as

$$\begin{aligned} E\left[Q_t Q_s\right] = {} & E\left[\left(A_t' + \varepsilon_A(t)\right)\left(V_t' + \varepsilon_V(t)\right)\left(A_s' + \varepsilon_A(s)\right)\left(V_s' + \varepsilon_V(s)\right)\right] = \\ & E\left[A_t' V_t' A_s' V_s'\right] + E\left[V_t' V_s'\right] E\left[\varepsilon_A(t)\varepsilon_A(s)\right] + E\left[A_t' A_s'\right] E\left[\varepsilon_V(t)\varepsilon_V(s)\right] + \\ & E[\varepsilon_A(t)\varepsilon_A(s)]\ E\ [\varepsilon_V(t)\varepsilon_V(s)]\ , \end{aligned} \tag{19}$$

where the last three terms on the right-hand side are retained because of possible serial correlation in $\varepsilon_A(t)$ and $\varepsilon_V(t)$. A model for this serial correlation will be discussed later. All the other terms drop out because $\varepsilon_A(t)$ and $\varepsilon_V(t)$ have zero mean and are independent of each other.

To simplify algebraic terms, the next step in the analysis is to assume that the area *A* varies little, as is generally true here, particularly for the lakefront AVM streamflow-gaging stations. Therefore, $E\left[A_t' A_s'\right]$ and $A_t A_s$ may be approximated as $\left(\bar{A}\right)^2$, where $\bar{A}$ is the average area. Then, it follows that

$$\begin{aligned} Cov\left(Q_t, Q_s\right) \cong {} & \left(\bar{A}\right)^2 E\left[V_t' V_s'\right] + E\left[V_t' V_s'\right] E\left[\varepsilon_A(t)\varepsilon_A(s)\right] + \left(\bar{A}\right)^2 E\left[\varepsilon_V(t)\varepsilon_V(s)\right] + \\ & E[\varepsilon_A(t)\varepsilon_A(s)]\ E\ [\varepsilon_V(t)\varepsilon_V(s)] - \left(\bar{A}\right)^2 V_t'\, V_s'. \end{aligned} \tag{20}$$

Using the same assumptions used to obtain equation 15, the first and last terms on the right-hand side of equation 20 can be combined into

$$\left(\bar{A}\right)^2\left(E\left[V_t'V_s'\right]-V_t'V_s'\right)\cong$$
$$\left(\bar{A}\right)^2\left[\sigma_\alpha^2+2\sigma_\alpha\sigma_\gamma+\sigma_\gamma^2+\left(V_L(s)+V_L(t)\right)\left(Cov(\alpha,\beta)+\sigma_\alpha\sigma_\delta\right)+V_L(s)V_L(t)\left(\sigma_\beta^2+2\sigma_\beta\sigma_\delta+\sigma_\delta^2\right)\right] \tag{21}$$

Thus, the covariance can be written as

$$Cov\left(Q_t,Q_s\right)\cong$$
$$\left(\bar{A}\right)^2\left[\sigma_\alpha^2+2\sigma_\alpha\sigma_\gamma+\sigma_\gamma^2+\left(V_L(s)+V_L(t)\right)\left(Cov(\alpha,\beta)+\sigma_\alpha\sigma_\delta\right)+V_L(s)V_L(t)\left(\sigma_\beta^2+2\sigma_\beta\sigma_\delta+\sigma_\delta^2\right)\right]$$
$$+\left(\bar{A}\right)^2E\left[\varepsilon_V(t)\varepsilon_V(s)\right]+E\left[V_t'V_s'\right]E\left[\varepsilon_A(t)\varepsilon_A(s)\right]+E\left[\varepsilon_A(t)\varepsilon_A(s)\right]E\left[\varepsilon_V(t)\varepsilon_V(s)\right]\ . \tag{22}$$

The contribution to the discharge covariance of the autocorrelation of the area errors is assumed to be negligible. This assumption allows simplification of equation 22 to

$$Cov\left(Q_t,Q_s\right)\cong$$
$$\left(\bar{A}\right)^2\left[\sigma_\alpha^2+2\sigma_\alpha\sigma_\gamma+\sigma_\gamma^2+\left(V_L(s)+V_L(t)\right)\left(Cov(\alpha,\beta)+\sigma_\alpha\sigma_\delta\right)+V_L(s)V_L(t)\left(\sigma_\beta^2+2\sigma_\beta\sigma_\delta+\sigma_\delta^2\right)\right]$$
$$+\left(\bar{A}\right)^2E\left[\varepsilon_V(t)\varepsilon_V(s)\right]. \tag{23}$$

Once a serial correlation model for the error term is assumed and values of the statistical properties of the rating curve coefficients and the ADCP uncertainty parameters are estimated, the variance of the total flow $\mathbf{V}$ can be computed using equations 10, 12, 15, 16, and 23. A further, computationally convenient, approximation to the covariance is provided by assuming the velocities are constant and equal to the mean AVM line velocity $\overline{V_L}$, which results in

$$Cov\left(Q_t,Q_s\right)\cong$$
$$\left(\bar{A}\right)^2\left[\sigma_\alpha^2+2\sigma_\alpha\sigma_\gamma+\sigma_\gamma^2+2\overline{V_L}\left(Cov(\alpha,\beta)+\sigma_\alpha\sigma_\delta\right)+\left(\overline{V_L}\right)^2\left(\sigma_\beta^2+2\sigma_\beta\sigma_\delta+\sigma_\delta^2\right)\right]$$
$$+\left(\bar{A}\right)^2E\left[\varepsilon_V(t)\varepsilon_V(s)\right]. \tag{24}$$

Uncertainty of Discharge Using Two Averaged Acoustic Velocity Meter Paths

As described earlier in the section “Description of Streamflow-Gaging Stations and Methods for Computing Discharge and Estimating Missing Record”, the method of discharge calculation at the Calumet River below O’Brien Lock and Dam at Chicago, Illinois streamflow-gaging station is different than the generic single-path method that forms the basis of the uncertainty computation described above. Because of the difficulty of measurement at this site, when both AVM paths are operating, each path is used to compute a separate discharge estimate. These two estimates then are averaged to obtain a final discharge value. During periods when only one AVM path is operating, the uncertainty estimation at this site reduces to the single-path method described above. Multiple AVM paths also are used at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station (fig. 1), but separate discharge values are not computed from each path;

rather, a weighted-average AVM velocity is computed from the multiple paths, which reduces to the single-path method.

In the case of the Calumet River below O'Brien Lock and Dam at Chicago, Illinois streamflow-gaging station (fig. 1), the error model becomes

$$Q = \left(\tfrac{1}{2}\right)\left(A' + \varepsilon_A\right)\left(V_1' + \varepsilon_{V_1} + V_2' + \varepsilon_{V_2}\right), \tag{25}$$

where Q, A' , and ε_A are defined as before (with the time subscript suppressed for simplicity because all time-varying quantities are contemporaneous), V_1' and V_2' are the velocities for AVM paths 1 and 2, respectively (where $V_i' = \alpha_i + \gamma + \left(\beta_i + \delta\right)V_{L_i}$, for $i = 1,2$), and ε_{V_1} and ε_{V_2} are the velocity errors in AVM paths 1 and 2, respectively. Equation 16 still may be used to compute the variance of the total discharge. The variance and covariance terms to be used in equation 16 are

$$\begin{aligned}Var\left(Q\right) = {} & \left(\bar{A}/2\right)^2\left[Var\left(V_1'\right) + \sigma^2_{\varepsilon_{V_1}} + Var\left(V_2'\right) + \sigma^2_{\varepsilon_{V_2}} + 2Cov\left(V_1', V_2'\right)\right], \\ & + \left(\tfrac{1}{2}\right)^2\left(E\left[V_1' + \varepsilon_{V_1} + V_2' + \varepsilon_{V_2}\right]\right)^2\left[Var\left(A'\right) + \sigma^2_A\right]\end{aligned} \tag{26}$$

where

$$\begin{aligned}& Var\left(V_i'\right) = \sigma^2_{\alpha_i} + V^2_{L_i}\left(\sigma^2_{\beta_i} + \sigma^2_\delta + 2\sigma_{\beta_i}\sigma_\delta\right) + \sigma^2_\gamma + 2V_{L_i}\left[Cov\left(\alpha_i, \beta_i\right) + \sigma_{\alpha_i}\sigma_\delta\right] + 2\sigma_{\alpha_i}\sigma_\gamma \text{ , } i = 1,2 \text{ ,} \\ & Cov\left(V_1', V_2'\right) = Cov\left(\alpha_1, \alpha_2\right) + \sigma_{\alpha_1}\sigma_\gamma + \sigma_{\alpha_2}\sigma_\gamma + \sigma^2_\gamma + V_{L_1}\left[Cov\left(\beta_1, \alpha_2\right) + \sigma_{\alpha_2}\sigma_\delta\right] + V_{L_2}\left[Cov\left(\beta_2, \alpha_1\right) + \sigma_{\alpha_1}\sigma_\delta\right] \\ & + V_{L_1}V_{L_2}\left[Cov\left(\beta_1, \beta_2\right) + \sigma_{\beta_1}\sigma_\delta + \sigma_{\beta_2}\sigma_\delta + \sigma^2_\delta\right], \\ & \left(E\left[V_1' + \varepsilon_{V_1} + V_2' + \varepsilon_{V_2}\right]\right)^2 = \left(E\left[\alpha_1\right] + E\left[\beta_1\right]V_{L_1} + E\left[\alpha_2\right] + E\left[\beta_2\right]V_{L_2}\right)^2 = \left(\alpha_1 + \beta_1 V_{L_1} + \alpha_2 + \beta_2 V_{L_2}\right)^2\end{aligned}$$

and

$$Var\left(A'\right) = 0,$$

where again the time subscripts are suppressed because all quantities are contemporaneous. Under the constant area approximation used above, neglecting the autocorrelation of ε_{V_1} , ε_{V_2} , and ε_A , and assuming, as above, the independence of δ and γ, the covariance is

$$\begin{aligned}Cov\left(Q_t, Q_s\right) \cong {} & \left(\bar{A}/2\right)^2\left[4\left(\sigma^2_\gamma + \sigma_{\alpha_1}\sigma_\gamma + \sigma_{\alpha_2}\sigma_\gamma\right) + 2Cov\left(\alpha_1, \alpha_2\right) + \sigma^2_{\alpha_1} + \sigma^2_{\alpha_2}\right] \\ & + \left(\bar{A}/2\right)^2 V_{L_1}(t)V_{L_1}(s)\left[\sigma^2_{\beta_1} + \sigma^2_\delta + 2\sigma_{\beta_1}\sigma_\delta\right] + V_{L_2}(t)V_{L_2}(s)\left[\sigma^2_{\beta_2} + \sigma^2_\delta + 2\sigma_{\beta_2}\sigma_\delta\right] \\ & + \left(\bar{A}/2\right)^2\left[V_{L_1}(t)V_{L_2}(s) + V_{L_2}(t)V_{L_1}(s)\right]\left[Cov\left(\beta_1, \beta_2\right) + \sigma_{\beta_1}\sigma_\delta + \sigma_{\beta_2}\sigma_\delta + \sigma^2_\delta\right] \\ & + \left(\bar{A}/2\right)^2\left[V_{L_1}(t) + V_{L_1}(s)\right]\left[Cov\left(\alpha_1, \beta_1\right) + \sigma_{\alpha_1}\sigma_\delta + Cov\left(\alpha_2, \beta_1\right) + \sigma_{\alpha_2}\sigma_\delta\right] \\ & + \left(\bar{A}/2\right)^2\left[V_{L_2}(t) + V_{L_2}(s)\right]\left[Cov\left(\alpha_2, \beta_2\right) + \sigma_{\alpha_2}\sigma_\delta + Cov\left(\alpha_1, \beta_2\right) + \sigma_{\alpha_1}\sigma_\delta\right].\end{aligned} \tag{27}$$

The average velocity approximation (analogous to equation 24) to this covariance relation is

$$\begin{aligned} Cov\left(Q_t, Q_s\right) \cong & \left(\bar{A}/2\right)^2 \left[4\left(\sigma_\gamma^2 + \sigma_{\alpha_1}\sigma_\gamma + \sigma_{\alpha_2}\sigma_\gamma\right) + 2Cov\left(\alpha_1, \alpha_2\right) + \sigma_{\alpha_1}^2 + \sigma_{\alpha_2}^2\right] \\ & + \left(\bar{A}/2\right)^2 \left(\overline{V}_{L_1}\right)^2 \left[\sigma_{\beta_1}^2 + \sigma_\delta^2 + 2\sigma_{\beta_1}\sigma_\delta\right] + \left(\overline{V}_{L_2}\right)^2 \left[\sigma_{\beta_2}^2 + \sigma_\delta^2 + 2\sigma_{\beta_2}\sigma_\delta\right] \\ & + \left(\bar{A}/2\right)^2 2\overline{V}_{L_1}\ \overline{V}_{L_2} \left[Cov\left(\beta_1, \beta_2\right) + \sigma_{\beta_1}\sigma_\delta + \sigma_{\beta_2}\sigma_\delta + \sigma_\delta^2\right. \\ & + \left(\bar{A}/2\right)^2 2\overline{V}_{L_1} \left[Cov\left(\alpha_1, \beta_1\right) + \sigma_{\alpha_1}\sigma_\delta + Cov\left(\alpha_2, \beta_1\right) + \sigma_{\alpha_2}\sigma_\delta\right] \\ & + \left(\bar{A}/2\right)^2 2\overline{V}_{L_2} \left[Cov\left(\alpha_2, \beta_2\right) + \sigma_{\alpha_2}\sigma_\delta + Cov\left(\alpha_1, \beta_2\right) + \sigma_{\alpha_1}\sigma_\delta\right] \ . \end{aligned} \qquad (28)$$

Uncertainty of Discharge on Days with Estimated Flows

In most cases, on days when the AVM record at a given site was missing, the flows were estimated by a regression performed at the daily time scale between daily flows computed at the streamflow-gaging station when it was operational and the flow estimates provided by the MWRDGC. Often, these regressions were stratified by flow condition, such as at the Chicago Sanitary and Ship Canal at Romeoville, Illinois which has separate regressions for flows at Lockport only from turbine, lockage, and leakage as opposed to when there also is sluice-gate or controlling-works flows (Melching and Oberg, 1993).

When regression between the daily measured discharge and the MWRDGC-reported discharge was used to predict the daily average discharge, the uncertainty of that discharge consists of a sum of two components. One component accounts for the uncertainty of the prediction of the measured daily average flow and is computed from the statistical properties of the regression by a method similar to how the velocity uncertainty is obtained from the AVM measurements using the uncertainty parameters of the IVR (equations 13-15 without ADCP uncertainty parameters). The other component consists of an estimate of the uncertainty of the daily measured discharge predicted by the daily regression.

For the first uncertainty component, the uncertainty arising from the daily regressions, the error model is the standard linear regression model given as

$$Q_t' = a' + b'Q_t + \varepsilon_Q\left(t\right), \qquad (29)$$

where Q_t' is the USGS mean daily discharge (known when performing the regression and to be estimated, otherwise) on day *t,* and Q_t is the MWRDGC flow, also on day *t*, which is used as the estimator. The contribution of this component to the variance of the total period being estimated in this way again is computed using equation 16. The quantities in this application of equation 16 may be shown to be

$$Var\left(Q_t'\right) = \sigma_{a'}^2 + \sigma_{b'}^2 Q_t^2 + 2Cov\left(a', b'\right)Q_t + \sigma_{\varepsilon_Q}^2 \qquad (30)$$

and

$$Cov\left(Q_t', Q_s'\right) = \sigma_{a'}^2 + \left(Q_t + Q_s\right)Cov\left(a', b'\right) + Q_t Q_s \sigma_{b'}^2 . \qquad (31)$$

At the Columbus Drive streamflow-gaging station (fig. 1), there are periods when non-linear functions of discharge were found to provide more satisfactory relations between USGS and MWRDGC daily discharges. These relations required slight modifications of the above methodology. The two non-linear functions used are the log-log (power-law) model

$$\ln Q_t' = a' + b' \ln Q_t + \varepsilon_{\ln Q}, \tag{32}$$

and the log-linear (exponential) model

$$Q_t' = a' + b' \ln Q_t + \varepsilon_Q. \tag{33}$$

In the log-linear model, the random variables are in the same functional relation as in the linear formula, so the variance and covariance formulas from the linear case may be used after substituting $\ln Q_t$ for Q_t. Therefore,

$$Var\left(Q_t'\right) = \sigma_{a'}^2 + \sigma_{b'}^2 \left(\ln Q_t\right)^2 + 2Cov\left(a', b'\right) \ln Q_t + \sigma_{\varepsilon_Q}^2, \tag{34}$$

and

$$Cov\left(Q_t', Q_s'\right) = \sigma_{a'}^2 + \left(\ln Q_t + \ln Q_s\right) Cov\left(a', b'\right) + \ln Q_t \ln Q_s \sigma_{b'}^2. \tag{35}$$

In the power-law model, the variance and covariance terms of the USGS daily discharge needed for equation 16 may be computed according to first-order approximations. For the variance, if $Y = g\left(X_1, X_2, \ldots, X_n\right)$, then $Var\left(Y\right) \cong \sum_{i=1}^{n} \sum_{j=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_m \left.\frac{\partial g}{\partial X_j}\right|_m Cov\left(X_i, X_j\right)$ (see Benjamin and Cornell (1970, p. 184)), where $X|_m$ means *X* evaluated at its mean. Note that here $Y = Q_t'$ and $\left(X_1, X_2, X_3\right) = \left(a', b', \varepsilon_{\ln Q}\right)$. Applying this formula to the power-law model yields

$$Var\left(Q_t'\right) \cong \left(e^{E[a']} Q_t^{E[b']}\right)^2 \left[\sigma_{a'}^2 + \left(\ln E\left[b'\right]\right)^2 \sigma_{b'}^2 + \sigma_{\varepsilon_{\ln Q}}^2 + 2 \ln E\left[b'\right] Cov\left(a', b'\right)\right]. \tag{36}$$

For the covariance, the first-order approximation is $Cov\left(Y_1, Y_2\right) \cong \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left.\frac{\partial g_1}{\partial X_i}\right|_m \left.\frac{\partial g_2}{\partial X_j}\right|_m Cov\left(X_i, X_j\right)$, where $Y_1 = g_1\left(X_1, X_2, \ldots, X_n\right)$ and $Y_2 = g_2\left(X_1, X_2, \ldots, X_n\right)$ (see Benjamin and Cornell (1970, p. 185)). Applying this approximation in the present case yields

$$Cov\left(Q_t', Q_s'\right) \cong \frac{1}{2} \left(Q_t Q_s\right)^{E[b']} e^{2E[a']} \left[\sigma_{a'}^2 + Cov\left(a', b'\right)\left(\ln Q_s + \ln Q_t\right) + \ln Q_s \ln Q_t \sigma_{b'}^2\right] \cdot \tag{37}$$

The second component of the discharge uncertainty during periods when flows are estimated using regressions between measured and MWRDGC-reported discharges is computed by applying equation 16 (using equations 10, 12, and 15 to compute the variance of the daily discharge and equation 23, without serial correlation in velocity errors, to compute its covariance) at the daily time scale to sum up the estimated discharge uncertainty for the period. To implement this method, it is necessary to make assumptions on the cross-sectional area and AVM velocity. The cross-sectional area is assumed to be given by its annual average $\overline{A}$. The AVM velocity $V_L(t)$ is obtained by solving the equation $Q' = \overline{A}\left(\alpha + \beta V_L(t)\right)$ for $V_L(t)$, obtaining $V_L(t) = \left(Q'\big/\overline{A} - \alpha\right)\big/\beta$, where Q' is the estimated daily average discharge, $\overline{A}$ is again the annual average cross-sectional area, and α and β are the appropriate IVR regression parameters.

There are two other methods also used to estimate mean daily discharges, which are applied when regressions between measurements and reported MWRDGC values are unavailable. One method is to use the MWRDGC-reported values without modification. In this method, some uncertainty on the MWRDGC discharges must be assumed. The method used here in this case is to assume some regression uncertainty parameters (see table 3) and to compute the uncertainty as if there had been a regression between measured and MWRDGC-reported values that resulted in a slope of one and an intercept of zero, except that because there is no measured discharge involved, the second component of the uncertainty is not included.

The other method for estimating discharges other than daily regressions is applied only at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station (fig. 1). During some periods at this station, missing daily discharges were estimated by linear interpolation of the daily discharges immediately preceding and following the missing period. Therefore, $Q_t = \dfrac{Q_b - Q_a}{t_b - t_a}\left(t - t_a\right) + Q_a$, where Q_t is the estimated daily flow on day *t*, Q_a and Q_b are the measured flows immediately preceding and following the missing period, respectively, and t_a and t_b are the days immediately preceding and following the missing period, respectively. The method used here to compute the variance of this interpolated period is a sum of the linearly interpolated standard deviation of the values before and after the missing period, σ_{Q_a} and σ_{Q_b}, denoted here as $\sum_t \hat{\sigma}_t^2$, and the difference of the linearly interpolated value from the nearest known value (Q_a or Q_b), denoted here as $\sum_t \sigma_t^2$.

These two quantities are computed as

$$\sum\nolimits_t \sigma_t^2 = \sum_{t=t_a+1}^{\left[(t_b-t_a)/2\right]} \left(\frac{Q_b - Q_a}{t_b - t_a}\left(t - t_a\right)\right)^2 + \sum_{t=\left[(t_b-t_a)/2\right]+1}^{t_b-1} \left(\frac{Q_b - Q_a}{t_b - t_a}\left(t - t_a\right) + Q_a - Q_b\right)^2, \qquad (38)$$

where $[t]$ denotes the greatest integer not exceeding *t* and

$$\sum\nolimits_t \hat{\sigma}_t^2 = \sum_{t=t_a+1}^{t_b-1} \left(\frac{\sigma_{Q_b} - \sigma_{Q_a}}{t_b - t_a}\left(t - t_a\right) + \sigma_{Q_a}\right)^2. \qquad (39)$$

The total discharge variance of the interpolated period then is given by

$$Var\left(\sum\nolimits_{t=t_a+1}^{t=t_b+1} Q_t\right) = \sum \sigma_t^2 + \sum \hat{\sigma}_t^2.$$

Combining Uncertainty Results at Different Time Scales

An equation for computing the total variance of the average discharge using discharges recorded at different time scales is required in order to combine the computed discharges at the unit value time scale that are obtained when the streamflow-gaging stations are operating normally with the estimates of daily discharge that are required during periods with missing record. Such an equation may be derived as follows. For two time scales $(\Delta t)_1$ and $(\Delta t)_2$, the combined average discharge $\bar{Q}$ may be computed as

$$\bar{Q} = \frac{Total\,Flow\,Volume}{Total\,Time\,Elapsed} = \frac{(\Delta t)_1 \sum_{i=1}^{I_1} Q_i' + (\Delta t)_2 \sum_{i=1}^{I_2} Q_i''}{I_1 (\Delta t)_1 + I_2 (\Delta t)_2}, \qquad (40)$$

where Q_i' and Q_i'' are the *i*th discharges at time scales 1 and 2, respectively, and I_1 and I_2 are the number of discharges at time scales 1 and 2, respectively. Therefore, the variance of the combined average discharge may be computed as

$$Var\left(\bar{Q}\right) = \frac{Var\left(\sum_{i=1}^{I_1} Q_i' + \sum_{i=1}^{I_2} Q_i''\right)}{\left(I_1 (\Delta t)_1 + I_2 (\Delta t)_2\right)^2} = \frac{\left[(\Delta t)_1\right]^2 Var\left(\sum_{i=1}^{I_1} Q_i'\right) + \left[(\Delta t)_2\right]^2 Var\left(\sum_{i=1}^{I_2} Q_i''\right)}{\left(I_1 (\Delta t)_1 + I_2 (\Delta t)_2\right)^2}, \qquad (41)$$

where the variance of the summed discharges at the diferent time scales ($Var\left(\sum_{i=1}^{I_1} Q_i'\right)$ and $Var\left(\sum_{i=1}^{I_2} Q_i''\right)$) may be obtained according to the methods described previously (see equation 16).

Application of First-Order Error Analysis

In this section, the analysis methodology described in previous section, “First-Order Error Analysis of Discharge”, is applied to data from the four stations analyzed in this report. This section has two subsections. In the first subsection, the parameters required for the analysis are presented and discussed. In the second subsection, the resulting estimates of uncertainty of average annual discharge are presented and discussed.

Uncertainty Parameters

The stage-area ratings and the associated uncertainty parameters at each streamflow-gaging station are listed in table 1. The uncertainty parameters are based on a statistical analysis of the parameters from three stage-area ratings obtained from three surveyed cross sections at different locations along the AVM paths at each streamflow-gaging station. The area error is computed using equation 12, where σ_h is obtained by assuming a round-off error in stage measurements that is uniform over the range ± 0.005 ft, giving $\sigma_h = 0.00289$ ft.

The IVR’s and associated uncertainty parameters are listed in table 2. These ratings and parameters all were obtained, as explained earlier, from least-squares linear regressions of mean channel velocity measurements (the y-values) against concurrent AVM velocity measurements (the x-values). The mean channel velocities were obtained by dividing ADCP discharge measurements by the area obtained by applying the stage-area rating to the concurrent stage measurement.

As discussed previously, at the Chicago Sanitary and Ship Canal at Romeoville streamflow-gaging station, where four AVM paths are used, the IVR was developed using an average of all four paths weighted according to the path depth (table 2). The IVR's of subsets of the four paths were not substantially different, so the same IVR and uncertainty parameters were used regardless of the number of AVM paths operating at a given time.

The parameters of the methods used to estimate flows on days with missing record are given in table 3. As mentioned in the subsection "Uncertainty of Discharge on Days with Estimated Flows", most were derived from regressions of daily discharges when the streamflow-gaging station was operating normally (Q_{USGS}) and the daily discharge estimated by the MWRDGC and reported on the LMO-6 form (Q_{LMO6}). The uncertainty of the regression parameters then could be obtained under standard regression results. At the Chicago Sanitary and Ship Canal at Romeoville streamflow-gaging station, during periods when MWRDGC data were used without modification, the uncertainty parameters were assumed to be the same as those associated with the regression at that station. At the Calumet River below O'Brien Lock & Dam at Chicago, Illinois streamflow-gaging station, during periods when MWRDGC data were used without modification, the uncertainty parameters were chosen by judgment based on the methods used to compute the MWRDGC and their day-to-day variation.

Table 1. Stage-area ratings and uncertainty parameters for U.S. Geological Survey streamflow-gaging stations on the Chicago River system used in the application of first-order error analysis for the Lake Michigan Diversion Project in Illinois.

[ft^2, square feet; ft, feet; A, area; ε_A, area error]

Streamflow-Gaging Station (fig. 1)	Intercept a (ft^2)	Slope b (ft)	Area Error $\sigma_A = \sigma_{\varepsilon_A}$ (ft^2)
Columbus Drive (05536123)	5,061	208.3	0.602
Romeoville (05536995)	251	162.0	.468
Wilmette (05536101)	640	91.2	.264
O'Brien (05536358)	5,548	400.5	1.157

Table 2. Index-velocity ratings and uncertainty parameters for the U.S. Geological Survey streamflow-gaging stations on the Chicago River system used in the application of first-order error analysis for the Lake Michigan Diversion Project in Illinois.

[ft/s, feet per second; S.E., standard error of estimate; Corr, correlation; IVR, index-velocity rating; ε_V, velocity error; R^2, coefficient of determination; WY, water year; ----, no data]

Station and Date of Record	Intercept α (ft/s)	Slope β	Intercept S.E. σ_α (ft/s)	Slope S.E. σ_β	Corr(α , β)	IVR S.E. σ_{ε_V} (ft/s)	R^2	Number of Observations
Columbus Dr./ WY 97-98 (Path 3)	0.0096	0.894	0.00363	0.00836	-0.695	0.0383	0.982	215
Columbus Dr./ WY 99 (Path 4)	.00138	.547	.00652	.0382	- .857	.0215	.841	41
Romeoville	----	.90	----	.00506	----	.0672	.993	82
Wilmette / 10-1-99 to 7-14-00	.0152	.556	.0255	.175	- .998	.00478	.591	9
Wilmette / 7-14-00 to 9-30-00	.0452	.632	.0110	.0465	- .837	.0381	.829	40
O'Brien / Path 1	.0074	.553	.0157	.0612	- .926	.0487	.881	13
O'Brien / Path 2	.0268	.127	.0219	.0537	- .850	.0304	.527	7

Autocorrelation of errors $\varepsilon_V(t)$ and $\varepsilon_A(t)$, and the choice of the ADCP uncertainty parameters (δ and γ) are two additional parameter choices. With respect to autocorrelation in area errors, $\varepsilon_A(t)$, because area fluctuations were a small part of the error budget, it was assumed that the effect of autocorrelation in area errors is negligible. With respect to autocorrelation in velocity errors $\varepsilon_V(t)$, an analysis of the sequences of concurrent ADCP-AVM measurements at Columbus Drive was performed. Overall, the analysis was inconclusive with respect to the presence of autocorrelation in the velocity errors, defined here as $V_{ADCP}(t) - V_L(t)$. However, some of the measurement sets show a substantial serial correlation with an approximately exponential decay as a function of time (indicating an autoregressive lag-one (AR(1)) model) with the lag-one autocorrelation coefficient $\rho(1)$ of about 0.5. Therefore, to be conservative in the calculations, this error term initially was included. However, the results of uncertainty computations with and without this term were found to be negligibly different.

Table 3. Methods and parameters of estimated discharge for the U.S. Geological Survey streamflow-gaging stations on the Chicago River system used in the application of first-order error analysis for the Lake Michigan Diversion Project in Illinois.

[AVM, acoustic velocity meter; S.E., standard error; ft^3/s, cubic feet per second; ----, not applicable; Q_{USGS}, U.S. Geological Survey estimate of mean daily discharge; Q_{LM06}, Metropolitan Water Reclamation District of Greater Chicago Form LMO-6 estimate of mean daily discharge; $Q_{TLL\text{-}AVM}$, turbine, lockage, and leakage discharge measured by acoustic velocity meter; Q_{SG} , sluice-gate discharge; Q_{CW} , controlling works discharge]

Station and date of record	Equation (Discharge, Q ,in ft^3/s)	Intercept S.E.	Slope S.E.	Slope-Intercept Correlation	S.E.
Columbus Dr., Oct. 1-31, 1996	$\ln Q_{USGS} = 0.491 \ln Q_{LM06} + 3.73$	0.139 ln(ft^3/s)	0.0278	-0.986	0.121 ln(ft^3/s)
Columbus Dr., Nov. 1 -Dec. 1, 1996	$Q_{USGS} = 369.4 \ln Q_{LM06} - 1{,}201$	177 ft^3/s	46.2 ft^3/s/ln(ft^3/s)	-.998	59.1 ft^3/s
Columbus Dr., Jul. 18-19, 1997	$Q_{USGS} = 1.186 Q_{LM06} + 277.02$	16.05 ft^3/s	.0380	-.903	82.8 ft^3/s
Columbus Dr., Dec. 19-27, 1997	$Q_{USGS} = 0.9128 Q_{LM06} + 54.06$	10.90 ft^3/s	.0958	-.574	98.6 ft^3/s
Columbus Dr., Aug. 29-30, 1998	$Q_{USGS} = 1.176 Q_{LM06} + 138.24$	11.52 ft^3/s	.0297	-.842	83.5 ft^3/s
Columbus Dr., Feb. 16 to Mar. 24, 1999	$Q_{USGS} = 1.775 Q_{LM06} - 2.265$	6.40 ft^3/s	.0283	-.530	61.4 ft^3/s
Romeoville / TLL only; before Lockport turbine AVMs (Melching and Oberg, 1993, Table 6)	$Q_{USGS} = 1.127 Q_{LM06} + 75.48$	15.59 ft^3/s	.00523	-.962	155 ft^3/s
Romeoville / Straight LMO6	$Q_{USGS} = Q_{LM06}$	15.59 ft^3/s	.00523	-.962	155 ft^3/s
Romeoville / TLL-AVM + SG (Melching and Oberg, 1993, equation 11, modified)	$Q_{USGS} = Q_{TLL\text{-}AVM} + .6842 Q_{SG} + 219.7$	72 ft^3/s	.0204 (TLL-AVM) .0187 (SG)	.0	295.9 ft^3/s
Romeoville / TLL-AVM + SG + CW (Melching and Oberg, 1993, equation 12, modified)	$Q_{USGS} = Q_{TLL\text{-}AVM} + .4361 Q_{SG} + .3228 Q_{CW} + 1086$	342 ft^3/s	.314 (TLL-AVM) .0467 (SG) .0763 (CW)	.0	1,245 ft^3/s
Wilmette	$Q_{USGS} = 0.9596 Q_{LM06} + 0.59$	.703 ft^3/s	.0141	-.579	9.91 ft^3/s
O'Brien / days with navigation makeup or discretionary flow	$Q_{USGS} = 0.822 Q_{LM06} + 149.2$	21.34 ft^3/s	.0561	-.941	107.45 ft^3/s
O'Brien / other missing days	$Q_{USGS} = Q_{LM06}$	10.0 ft^3/s	.0	----	10.0 ft^3/s

The results of the uncertainty analysis are sensitive to the value of the parameters δ and γ being used to simulate the ADCP uncertainty. Unfortunately, it is difficult to determine values for these parameters. Field tests using similar procedures and instrumentation as used here, but work on free-flowing rivers and streams by Morlock (1996) and Mueller (2003), show differences in discharge between ADCP and Price AA current-meter measurements and discharges obtained from the station rating curves of 1-7 percent. This result should imply similar differences in velocity. However, such comparisons do not identify whether measurements made with the ADCP or Price AA current meter are in error, and the comparisons do not characterize the differences over a range of flows or velocities at a given site, as is needed here. Preliminary computations (Blair Brumley and Joel Gast, RD Instruments, written commun., 2003) of expected discharge and velocity errors also have been provided by RD Instruments, the manufacturer of the ADCP instruments typically used at the streamflow-gaging stations on the Chicago River system. These computations predict errors of about 0.4 percent of velocity plus 0.0016 ft/s on a bin-by-bin basis. These estimates do not include errors arising from estimating discharge in unmeasured portions of the channel, which would indicate that they are low. Also, these estimates do not address how consistent these errors would be between bins in a given cross section or between transects. Many of these predicted errors could average out when multiple measurements across the whole cross section are considered. In light of these uncertainties, values of $\sigma_\delta = 0.01$ (a 1-percent error when the index velocity slope is 1.0) and $\sigma_\gamma = 0.01$ ft/s were considered reasonable and conservative. These values can give an indication of the effect of ADCP uncertainties on the complete error budget.

Annual Average Discharge and Uncertainty

Annual average discharge and uncertainty of annual average discharge estimates for each gaging station computed with the methods described previously are presented in tables 4-11. The annual average discharge results are given in tables 4, 6, 8, and 10, and the uncertainty in annual average discharge results in tables 5, 7, 9, and 11. Except for Wilmette (tables 8 and 9), all results cover WY's 1997-99. At Wilmette, WY 2000 is added because WY's 1997-99 are completely estimated. The uncertainty results are presented as standard deviations of the average discharge; that is, $\mathbf{A}^{1/2}$, where, as previously described, **A** is the variance of the average discharge, the variance is computed as $\mathbf{A} = \mathbf{V} / n^2$, where **V** is the variance of the summed unit discharges and n is the number of unit discharges. **V** is computed using equation 16. If **Q** is used to denote the annual average discharge, then the one standard-deviation range around the annual average discharge would be $\mathbf{Q}$ +/- $\mathbf{A}^{1/2}$. Note that the annual average discharge values in tables 4, 6, 8, and 10 may not agree exactly with the values in appendix B, which were computed using ADAPS. These differences occur because of differences with respect to rounding and interpolating missing periods shorter than a day between ADAPS and the computer programs used to perform the error analysis computations.

The first point that may be observed about these results is that the uncertainty arising from the sum of covariance terms, given by

$$\frac{1}{n}\sqrt{2\sum_{t=1}^{n-1}\sum_{s=t+1}^{n} Cov\left(Q_t, Q_s\right)}, \tag{42}$$

dominates the total uncertainty at the annual time scale. As mentioned previously, this result is not because of autocorrelation in the velocity or area errors but because of the uncertainty of the index

Table 4. Annual average discharges at the U.S. Geological Survey acoustic velocity meter streamflow-gaging station on the Chicago River at Columbus Drive at Chicago, Illinois.

[ft^3/s, cubic feet per second]

Water Year	Average Discharge on Non-Estimated Days (ft^3/s)	Average Discharge on Estimated Days (ft^3/s)	Number of Estimated Days	Average Discharge on All Days Combined (ft^3/s)
1997	495.9	304.4	64	462.3
1998	363.8	462.8	11	366.8
1999	220.5	66.3	37	204.9

Table 5. Uncertainties in annual average discharge for the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station on the Chicago River at Columbus Drive at Chicago, Illinois.

[σ_δ , the standard deviation of the slope of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; σ_γ , the standard deviation of the intercept of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; ft/s, feet per second; ft^3/s, cubic feet per second]

Water Year	σ_δ	σ_γ (ft/s)	Uncertainty Arising from Sum of Variance on Non-Estimated Days (ft^3/s)	Uncertainty Arising from Covariance on Non-Estimated Days (ft^3/s)	Total Uncertainty on Non-Estimated Days (ft^3/s)	Total Uncertainty on Estimated Days (ft^3/s)	Total Uncertainty on All Days Combined (ft^3/s)
1997	0.0	0.0	0.61	14.49	14.50	29.65	13.04
	.01	.0	.62	21.25	21.26	30.61	18.34
	.0	.01	.65	63.43	63.44	68.50	53.67
	.01	.01	.65	65.31	65.31	68.92	55.20
1998	.0	.0	.57	15.20	15.21	65.31	14.88
	.01	.0	.57	19.67	19.67	65.44	19.18
	.0	.01	.60	63.65	63.65	89.92	61.79
	.01	.01	.60	64.86	64.87	90.02	62.97
1999	.0	.0	.34	19.48	19.48	35.84	17.88
	.01	.0	.35	27.41	27.41	36.37	24.91
	.0	.01	.41	74.05	74.05	79.93	67.04
	.01	.01	.42	76.52	76.52	80.17	69.24

velocity regression parameters, and, when present, the ADCP uncertainty parameters δ and γ. This uncertainty can be understood as follows. If the IVR between the mean channel and AVM velocities is inaccurate and it is applied as a kind of calibration procedure to the AVM velocities, the velocity at every time step will be inaccurate, and in the same direction. The sum of variance component, on the other hand, may dominate at short time scales (days and weeks), but these errors are “random” errors that average out over a long period such as a year. A practical lesson that follows from this observation is that reducing the uncertainty of the IVR by making more ADCP measurements is the best way to reduce the uncertainty of the annual average discharge.

The second point that may be observed is the sensitivity of the uncertainty to the presence of ADCP uncertainty. At all streamflow-gaging stations, when σ_δ and σ_γ are both given their assumed non-zero values (0.01 and 0.01 ft/s, respectively), the estimated uncertainty is from about 1.5 to more than 4.0 times as great as when both σ_δ and σ_γ are zero. Except at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station, the sensitivity is greater, for

Table 6. Annual average discharges at the U.S. Geological Survey acoustic velocity meter streamflow-gaging station on the Chicago Sanitary and Ship Canal at Romeoville, Illinois.

[ft^3/s, cubic feet per second]

Water Year	Average Discharge on Non-Estimated Days (ft^3/s)	Average Discharge on Estimated Days (ft^3/s)	Number of Estimated Days	Average Discharge on All Days Combined (ft^3/s)
1997	3,227	2,698	11	3,211
1998	3,125	3,012	11	3,121
1999	2,926	2,922	17	2,926

Table 7. Uncertainties in annual average discharge for the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station on the Chicago Sanitary and Ship Canal at Romeoville, Illinois.

[σ_δ , the standard deviation of the slope of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; σ_γ , the standard deviation of the intercept of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; ft/s, feet per second; ft^3/s, cubic feet per second]

Water Year	σ_δ	σ_γ (ft/s)	Uncertainty Arising from Sum of Variance on Non-Estimated Days (ft^3/s)	Uncertainty Arising from Covariance on Non-Estimated Days (ft^3/s)	Total Uncertainty on Non-Estimated Days (ft^3/s)	Total Uncertainty on Estimated Days (ft^3/s)	Total Uncertainty on All Days Combined (ft^3/s)
1997	0.0	0.0	1.60	18.44	18.51	71.04	17.95
	.01	.0	1.63	54.53	54.55	71.37	52.95
	.0	.01	1.61	47.39	47.41	72.99	46.04
	.01	.01	1.64	69.85	69.87	73.12	67.80
1998	.0	.0	1.60	17.88	17.91	46.93	17.33
	.01	.0	1.63	52.84	52.86	46.93	51.29
	.0	.01	1.62	47.24	47.26	46.93	45.86
	.01	.01	1.64	68.58	68.60	46.93	66.55
1999	.0	.0	1.61	16.82	16.90	271.6	20.48
	.01	.0	1.64	49.66	49.69	271.8	49.03
	.0	.01	1.63	46.81	46.84	272.0	46.42
	.01	.01	1.65	66.14	66.16	272.2	64.34

the values of σ_δ and σ_γ chosen, to the intercept uncertainty σ_γ . The Romeoville streamflow-gaging station is an exception to this pattern; presumably this occurs because velocities measured at this station are, on average, much larger than at the other stations, making the uncertainty in the slope relatively more important than uncertainty in the intercept.

The range of discharge uncertainties for each streamflow-gaging station are combined in table 12. The minimum (maximum) uncertainty value is the minimum (maximum) over all WY's and all choices of values of the ADCP bias parameters (δ and γ), including zero. The minimum value in all cases arises when the ADCP bias parameters are taken to be zero, and the maximum value when the bias parameters are given their maximum assumed values ($\delta = 0.01$ and $\gamma = 0.01$ ft/s). The range of uncertainty values given in table 12 is large. This range arises because of the large uncertainty in the accuracy of ADCP measurements, which is reflected in the uncertainty in the values of the ADCP bias parameters.

Table 8. Annual average discharges at the U.S. Geological Survey acoustic velocity meter streamflow-gaging station on the North Shore Channel at Wilmette, Illinois.

[ft^3/s, cubic feet per second; LMO-6, Illinois Department of Natural Resources monthly report form for Lake Michigan diversion flows; ----, no data]

Water Year	Average Discharge on Non-Estimated Days (ft^3/s)	Average Discharge on Estimated Days (ft^3/s)	Number of Estimated Days	Average Discharge on All Days Combined (ft^3/s)	Average Discharge when All Days are Treated as Estimated (ft^3/s)	Average of LMO-6 Discharge on All Days Combined (ft^3/s)
1997	----	47.7	365	47.7	47.7	49.1
1998	----	50.0	365	50.0	50.0	51.5
1999	----	38.0	365	38.0	38.0	39.0
2000	27.85	14.2	43	26.3	28.7	29.3

Table 9. Uncertainties in annual average discharge for the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station on the North Shore Channel at Wilmette, Illinois.

[σ_δ , the standard deviation of the slope of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; σ_γ , the standard deviation of the intercept of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; ft/s, feet per second; ft^3/s, cubic feet per second; ----, no data]

Water Year	σ_δ	σ_γ (ft/s)	Uncertainty Arising from Sum of Variance on Non-Estimated Days (ft^3/s)	Uncertainty Arising from Covariance on Non-Estimated Days (ft^3/s)	Total Uncertainty on Non-Estimated Days (ft^3/s)	Total Uncertainty on Estimated Days (ft^3/s)	Total Uncertainty on All Days Combined (ft^3/s)
1997	0.0	0.0	----	----	----	11.71	11.71
	.01	.0	----	----	----	11.83	11.83
	.0	.01	----	----	----	17.51	17.51
	.01	.01	----	----	----	17.59	17.59
1998	.0	.0	----	----	----	8.24	8.24
	.01	.0	----	----	----	8.64	8.64
	.0	.01	----	----	----	15.40	15.40
	.01	.01	----	----	----	15.62	15.62
1999	.0	.0	----	----	----	11.31	11.31
	.01	.0	----	----	----	11.45	11.45
	.0	.01	----	----	----	17.24	17.24
	.01	.01	----	----	----	17.33	17.33
2000	.0	.0	0.054	9.65	9.65	11.17	8.62
	.01	.0	.055	9.74	9.74	11.32	8.70
	.0	.01	.067	13.89	13.89	17.15	12.42
	.01	.01	.068	13.95	13.95	17.25	12.47

Table 10. Annual average discharges at the U.S. Geological Survey acoustic velocity meter streamflow-gaging station on the Calumet River below O'Brien Lock & Dam at Chicago, Illinois.

[ft^3/s, cubic feet per second]

Water Year	Average Discharge on Non-Estimated Days (ft^3/s)	Average Discharge on Estimated Days (ft^3/s)	Number of Estimated Days	Average Discharge on All Days Combined (ft^3/s)
1997	242.2	164.2	240	190.9
1998	330.1	119.2	241	190.8
1999	303.3	70.5	213	167.4

Table 11. Uncertainties in annual average discharge for the U.S. Geological Survey acoustic velocity meter (AVM) streamflow-gaging station on the Calumet River below O'Brien Lock and Dam at Chicago, Illinois.

[σ_δ , the standard deviation of the slope of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; σ_γ , the standard deviation of the intercept of the hypothetical line relating acoustic Doppler current profiler velocity bias and the AVM line velocity; ft/s, feet per second; ft^3/s, cubic feet per second]

Water Year	σ_δ	σ_γ (ft/s)	Uncertainty Arising from Sum of Variance on Non-Estimated Days (ft^3/s)	Uncertainty Arising from Covariance on Non-Estimated Days (ft^3/s)	Total Uncertainty on Non-Estimated Days (ft^3/s)	Total Uncertainty on Estimated Days (ft^3/s)	Total Uncertainty on All Days Combined (ft^3/s)
1997	0.0	0.0	1.55	58.81	58.83	11.50	21.52
	.01	.0	1.56	62.08	62.10	14.75	23.37
	.0	.01	1.83	115.2	115.2	27.93	43.51
	.01	.01	1.84	116.9	116.9	29.42	44.46
1998	.0	.0	1.08	31.08	31.10	11.35	12.95
	.01	.0	1.12	52.97	52.98	12.81	19.89
	.0	.01	1.31	83.19	83.20	21.00	31.48
	.01	.01	1.34	93.59	93.60	21.82	34.91
1999	.0	.0	.71	38.48	38.48	9.89	17.03
	.01	.0	.75	66.26	66.26	10.30	28.24
	.0	.01	.88	112.9	112.9	11.55	47.49
	.01	.01	.91	125.1	125.1	11.91	52.56

Table 12. Summary of uncertainties in annual average discharge at the U.S. Geological Survey acoustic velocity meter streamflow-gaging stations on the Chicago River system, Illinois.

[ft^3/s, cubic feet per second]

Streamflow-Gaging Station (fig. 1)	Mininum Uncertainty (ft^3/s)	Maximum Uncertainty (ft^3/s)
Columbus Drive (05536123)	13.04	69.24
Romeoville (05536995)	17.33	67.80
Wilmette (05536101)	8.24	17.59
O'Brien (05536358)	12.95	52.56

Summary

The State of Illinois diverts water from Lake Michigan into the Chicago River system at three locations in the Chicago vicinity. A U. S. Supreme Court decree limits the diversion to an annual mean discharge of 3,200 cubic feet per second (ft^3/s). An accurate computation of discharge and an assessment of the uncertainty in the discharge data are required for the Lake Michigan Diversion Project. As part of this project, the U.S. Geological Survey (USGS), in cooperation with the U.S. Army Corps of Engineers-Chicago District, computed discharge and analyzed the error to evaluate the uncertainty in the discharge computations for Water Years (WY's) 97-99.

Daily mean discharges were computed at four acoustic velocity meter (AVM) streamflow-gaging stations on the Chicago River system for a 3-year period (1997-99 WY's). The four stations were:

Chicago Sanitary and Ship Canal at Romeoville, Illinois (05536995)
Chicago River at Columbus Drive at Chicago, Illinois (05536123)
Calumet River below O'Brien Lock and Dam at Chicago, Illinois (05536358)
North Shore Channel at Wilmette, Illinois (05536101)

The first station listed, the Chicago Sanitary and Ship Canal at Romeoville, Illinois has been the primary station used for the Lake Michigan Diversion Project since 1984. The following three stations: Chicago River at Columbus Drive at Chicago, Illinois; Calumet River below O'Brien Lock and Dam at Chicago, Illinois; and the North Shore Channel at Wilmette, Illinois make up an alternative accounting system for the Lake Michigan Diversion Project and are collectively termed the lakefront accounting streamflow-gaging stations.

The stage-area and index-velocity ratings (IVR's) used in the discharge computations for each streamflow-gaging station were calculated. Whereas the same general index-velocity method to compute discharge was followed at all four streamflow-gaging stations, the methodology applied varied slightly at each station depending on site-specific conditions and complexities in the flow.

A methodology for error analysis of the discharge computations was developed using first-order error analysis, with the statistical properties of the IVR's used in the velocity error estimate, and including terms simulating possible acoustic Doppler current profiler (ADCP) velocity bias. Variants of the basic method were developed to handle days with estimated discharges. The methodology was applied to each station to obtain the uncertainty of the annual discharge.

The results of the uncertainty analysis are sensitive to the value of the ADCP uncertainty; however, it is difficult to determine the ADCP uncertainty. Although field tests by other researchers compare discharge measurements made with ADCP's to those made with Price AA current meters, they do not establish which meter is in error. Preliminary computations by the ADCP manufacturer indicate small errors in the velocity measurement in the measured portion of the channel cross section. However, the errors associated with the estimated discharge in the unmeasured part of the cross section are not accounted for in the manufacturer's computations. Because of these uncertainties, a conservative estimate of 1-percent ADCP error was used to indicate the effect of ADCP uncertainty on the complete error budget in terms of flows.

Discharges during the period of study varied widely because, in part, of the regulation of flows in the Chicago River system by the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC). Large negative discharges (backflow conditions) were recorded at the streamflow-gaging stations on the Chicago River at Columbus Drive at Chicago and the Calumet River below O'Brien Lock and Dam at Chicago. The large negative discharges were associated with storms and the regulation of flow by the MWRDGC to prevent flooding.

Missing streamflow data at the USGS streamflow-gaging stations were estimated using three methods, depending on the station and the length of missing record. The estimation methods utilized were based upon 1) detailed flow records at the MWRDGC structures and regressions developed by previous investigations, 2) short-term (a few days) regressions between the mean daily flows at the USGS stations and the MWRDGC structures, and 3) long-term (weeks to months) regressions between the mean daily flows at the USGS stations and the MWRDGC structures.

Results indicated that for the Chicago River at Columbus Drive at Chicago, Illinois station for the 1997-99 WY's, the total uncertainty, expressed as a standard deviation of the average annual discharge, ranged from 13 to 18 ft^3/s when ADCP uncertainty was not included, whereas total uncertainty ranged from 55 to 69 ft^3/s when ADCP uncertainty was included. At the Chicago Sanitary and Ship Canal at Romeoville, Illinois station for the 1997-99 WY's, the uncertainty ranged from 18 to 20 ft^3/s when ADCP uncertainty was not included, whereas it ranged from 64 to 68 ft^3/s when it was included. At the Calumet River below O'Brien Lock and Dam at Chicago, Illinois station for the 1997-99 WY's, the uncertainty ranged from 13 to 22 ft^3/s when ADCP uncertainty was not included, whereas it ranged from 35 to 53 ft^3/s when it was included. At the North Shore Channel at Wilmette, Illinois station for the 1997-99 WY's, when the record was entirely estimated, the uncertainty ranged from 8 to 12 ft^3/s when the ADCP uncertainty was not included, and from 16 to 17 ft^3/s when it was included. For the 2000 WY, the estimated uncertainty was 8.6 ft^3/s when ADCP uncertainty is not included and 12.5 ft^3/s when ADCP uncertainty was included.

The results for the uncertainty in the average annual flow computation for the entire study period (1997-99 WY's), including the estimated days, indicate that the uncertainty from the sum of the covariance terms dominates the total uncertainty. This total uncertainty results from the uncertainty of the IVR parameters and the uncertainty associated with the ADCP. This result also indicates that reducing the IVR uncertainty by making more ADCP measurements will reduce the uncertainty of the annual average discharge.

Four years (1997-2000 WY's) of flow record (3 years of MWRDGC record and 1 year of USGS streamflow-gaging station record) were used for the error analysis for the North Shore Channel at Wilmette station because this station was not installed until the end of the 1999 WY. As such, flow records for the 1997-99 WY's were estimated completely. Results of the analysis indicated a small estimate of uncertainty for the estimated periods relative to that for the nonestimated periods. This was because of the small amount of scatter in the regression between the USGS data and the MWRDGC data used to compute the estimated periods, indicating substantially smaller uncertainties than the index-velocity regression. Because the index-velocity regression was used to compute the USGS daily mean discharges in the USGS-MWRDGC regression, there is the possibility that the averaging of the noise in the unit-value time scale to generate the daily time scale gives a more accurate, more certain way to estimate daily mean discharge.

Acknowledgments

The authors are grateful to Thomas Fogarty, U.S. Army Corps of Engineers, for his long-term support and assistance in the Lake Michigan Diversion Project. Richard Lanyon, Metropolitan Water Reclamation District of Greater Chicago provided data, valuable field coordination, and historical information on the Chicago River system. Daniel Injerd, Illinois Department of Natural Resources, provided data and support for this study. Brent Troutman, U.S. Geological Survey, and Arthur Schmidt, University of Illinois both provided valuable guidance on methods of error analysis. Charles S. Melching, Marquette University, and T. Y. Su, U.S. Army Corps of Engineers, provided technical report reviews.

Literature Cited

Wisconsin versus Illinois 449 U.S. 48, p. 557-560 (1980).

Ang, A. H-S, and Tang, W.H., 1984. Probability Concepts in Engineering Planning and Design, Volume II-Decision, Risk, and Reliability: John Wiley and Sons, New York, 562 p.

American National Standards Institute/American Society of Mechanical Engineers, 1998, New York, New York.

Benjamin, J. R., and Cornell, C. A., 1970, Probability, Statistics, and Decision for Civil Engineers: McGraw-Hill, New York, 684 p.

Carter, R.W., and Anderson, I.E., 1963, Accuracy of Current Meter Measurements: *Journal of the Hydraulics Division*, ASCE, 89(4), 105-115.

Fuller, W. A., 1987, Measurement Error Models: John Wiley and Sons, New York, 440 p.

International Organization for Standardization (ISO), 1992, Measurement of liquid flow in open channels—Measurement of discharge by the ultrasonic (acoustic) method: International Standard ISO 6416, Geneva, Switzerland, 28 p.

Laenen, A., 1985, Acoustic Velocity Meter Systems: Techniques of Water-Resources Investigations of the U.S. Geological Survey, book 3, chapter A17, 38 p.

Melching, C. S., and Oberg, K. A., 1993, Comparison, Analysis, and Estimation of Discharge Data from Two Acoustic Velocity Meters on the Chicago Sanitary and Ship Canal at Romeoville, Illinois: U. S. Geological Survey Water-Resources Investigations Report 93-4048, 61 p.

Mood, A. M., and Graybill, F. A., 1963, Introduction to the Theory of Statistics, Second Edition: McGraw-Hill, New York, 443 p.

Morlock, S.E., 1996, Evaluation of Acoustic Doppler Current Profiler Measurements of River Discharge: U.S. Geological Survey Water-Resources Investigations Report 95-4218, 37 p.

Muste, M., and Stern, F., 2000, Proposed Uncertainty Assessment Methodology for Hydraulic and Water Resources Engineering: Proceedings Joint Conference of Water Resources Engineering, Planning and Management, American Society of Civil Engineers, Minneapolis, Minn.

Patino, E., and Ockerman, D.,1997, Computation of Mean Velocity in Open Channels Using Acoustic Velocity Meters: U.S. Geological Survey Open-File Report 97-220, 12 p.

Shenk, A., 1979, Calculus and Analytical Geometry, Second Edition: Glenview, Illinois: Scott Foresman and Company, 998 p.

Simpson, M.R., and Bland, R., 2000, Methods for Accurate Estimation of Net Discharge in a Tidal Channel: Journal of Oceanic Engineering, IEEE, 25(4), 437-445.

Simpson, M.R., and Oltman, R.N., 1993, Discharge-Measurement System Using an Acoustic Doppler Current Profiler with Applications to Large Rivers and Estuaries: U.S. Geological Survey Water-Supply Paper 2395, 32 p.

Tung, Y-K, and Yen, B.C., 1993, Some Recent Progress in Uncertainty Analysis for Hydraulic Design: In Reliability and Uncertainty Analyses in Hydraulic Designs, (B.C. Yen and Y-K Tung, eds.) p. 17-34, American Society of Civil Engineers, New York, N.Y.

APPENDIX A—Error Analysis of Acoustic Velocity Meter Measurements on the Chicago River System in Illinois

In this appendix, a method, based on first-order error analysis, is developed to compute the uncertainty of acoustic velocity meter (AVM) velocity measurements as they are made at the streamflow-gaging stations on the Chicago River system. The method then is applied to estimate AVM velocity uncertainty at two streamflow-gaging stations:

Chicago River at Columbus Drive at Chicago, Illinois
Chicago Sanitary and Ship Canal at Romeoville, Illinois

Whereas discharge uncertainty is not computed by this method, these results are presented as a contribution to the understanding and possible improvement of the accuracy of AVM measurements at these stations.

Mathematical Model for First-Order Error Analysis of Acoustic Velocity Meter Velocity

The line velocity, v_L , measured with an AVM is determined by (for a derivation see Laenen and Smith, 1983)

$$v_L = \frac{B}{2\cos\theta}\left(\frac{1}{t_2} - \frac{1}{t_1}\right), \tag{A.1}$$

where B is the length of the acoustic path; θ is the angle between the streamwise direction and the acoustic path; and t_1 and t_2 are travel times of the acoustic signal in the upstream and downstream directions between the two acoustic transducers, respectively. Based on this equation for the computation of line velocity, total measurement variance can be derived using first-order error analysis as

$$\sigma_{v_L}^2 = \left(\frac{\partial v_L}{\partial B}\right)^2 \sigma_B^2 + \left(\frac{\partial v_L}{\partial \cos\theta}\right)^2 \sigma_{\cos\theta}^2 + \left(\frac{\partial v_L}{\partial t_1}\right)^2 \sigma_{t_1}^2 + \left(\frac{\partial v_L}{\partial t_2}\right)^2 \sigma_{t_2}^2 , \tag{A.2}$$

assuming errors in B, θ, t_1, and t_2 are mutually independent. The partial derivatives of the line-velocity function defined by equation A.2 with respect to each of the independent random variables are given by

$$\frac{\partial v_L}{\partial B} = \frac{1}{2\cos\theta}\left(\frac{1}{t_2} - \frac{1}{t_1}\right), \tag{A.3}$$

$$\frac{\partial v_L}{\partial \cos\theta} = \frac{-B}{2\cos^2\theta}\left(\frac{1}{t_2} - \frac{1}{t_1}\right), \tag{A.4}$$

$$\frac{\partial v_L}{\partial t_1} = \frac{B}{2\cos\theta}\left(\frac{1}{t_1^2}\right), \tag{A.5}$$

and

$$\frac{\partial v_L}{\partial t_2} = \frac{-B}{2\cos\theta}\left(\frac{1}{t_2^2}\right). \tag{A.6}$$

After placing equations A.3-A.6 into equation A.2, dividing through by v_L, and simplifying, the following equation is obtained

$$\Omega_{v_L}^2 = \Omega_B^2 + \Omega_{\cos\theta}^2 + \left(\frac{t_1 t_2}{t_1 - t_2}\right)^2 \left(\frac{\Omega_{t_1}^2}{t_1^2} + \frac{\Omega_{t_2}^2}{t_2^2}\right), \tag{A.7}$$

where $\Omega = {}^{\sigma}\!/_{\mu}$ represents the coefficent of variation (CV), and the subscripts denote the independent random variables.

Application to the Chicago River at Columbus Drive

In this section, the first-order error analysis method previously developed for estimating the uncertainty of the AVM velocity measurements is applied to the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station. This method is applied for path 3 that was used to compute discharge for the 1997-98 water years (WY's). The uncertainty of each of the three sources (acoustic path length, angle of the acoustic path, and travel time of the acoustic signal) identified as the main contributors to the total uncertainty of AVM line-velocity measurements is determined in the following subsections.

Uncertainty in the Length of the Acoustic Path

The distance between transducers at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station estimated from survey data is 236.44 feet (ft). These survey data were collected using a total station with a specifed accuracy of ± 0.05 ft. The actual path followed by the acoustic signal as it travels between the transducers, however, can be deflected as it propagates across the stream because of ray bending. Ray bending is the result of density stratification of the water in the stream. Density stratification can be induced by one or more factors, including temperature gradients, salinity gradients, and sediment concentration gradients. Data collected by the U.S. Geological Survey (USGS) in the Chicago River in March 1998 indicate that temperature gradients of 0.3 °C per meter and specific conductance gradients of 100 micromohs per meter are not uncommon during low-flow periods in the winter season. According to charts in Leanen (1985), for the distance between the acoustic transducers at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station and the temperature and specific-conductance gradients observed in the Chicago River during the winter season, the deflection of the acoustic beam can be on the order of 8 ft because of a temperature gradient and on the order of 0.1 ft because of a salinity gradient. Based on these estimates, it seems reasonable to expect that the path of the acoustic beam is subject to mean deflections in the winter season on the order of 8 ft. Further, by assuming that the deflection errors have an upper triangular distribution and that the surveying distance errors have a symmetrical triangular distribution—see, for example, Ang and Tang (1984) for the expressions for the CV's of typical probability density functions—the CV of the deflected path, Ω_{BD}^2, and the CV of the distance between transducers, Ω_{BT}^2, can be estimated as

$$\Omega_{BD}^2 = \frac{1}{2}\frac{B_{D_u}^2 + B_{D_m}^2 + B_{D_l}^2 - B_{D_u}B_{D_m} - B_{D_u}B_{D_l} - B_{D_m}B_{D_l}}{\left(B_{D_u} + B_{D_m} + B_{D_l}\right)^2} = 6.49 \times 10^{-5}, \tag{A.8}$$

where B_{D_u} = 236.44 + 8.15 ft is the upper limit of the upper triangular distribution of path length in the presence of possible deflection, σ_A^2 = 236.44 ft is the mode of distribution, and B_{D_l} = 236.44-0.05 ft is the lower limit of the distribution; and

$$\Omega_{BT}^2 \approx \left(\frac{1}{\sqrt{6}} \frac{B_{T_u} - B_{T_l}}{B_{T_u} + B_{T_l}} \right)^2 = \left(\frac{1}{\sqrt{6}} \frac{(236.44 + 0.05) - (236.44 - 0.05)}{(236.44 + 0.05) + (236.44 - 0.05)} \right)^2 = 7.45 \times 10^{-9}, \qquad \text{(A.9)}$$

where B_{T_u} = 236.44 + 8.15 ft is the upper limit of the upper triangular distribution of path length in the presence of possible deflection, and B_{T_l} = 236.44-0.05 ft is the lower limit of the distribution. These CV's can be used to estimate the total uncertainty during periods with and without density gradients, respectively, for path 3 during the 1997-98 WY's.

Uncertainty in the Angle between the Acoustic Path and the Streamflow

In determining the total uncertainty of line-velocity measurements with AVMs, it is the uncertainty in the cosine of the angle between the acoustic path and the streamwise direction ($\cos\theta$) that is relevant. Moreover, because the angle θ is estimated from distances surveyed at the site, it can be shown that the CV of $\cos\theta$ can be expressed as a function of the CV of the length of the acoustic path as

$$\Omega_{\cos\theta}^2 = \left(1 + \frac{1}{\cos^2\theta} \right) \Omega_B^2 \approx 4.57 \Omega_B^2, \qquad \text{(A.10)}$$

using $\theta = 58.07^\circ$. In this expression, Ω_B represents the CV of the distance between the acoustic transducers because of survey uncertainty.

Uncertainty in the Travel Time of the Acoustic Signal

Typically, because of practical reasons, the time series of the times of travel of the acoustic signal (t_1, t_2) as it propagates upstream and downstream between the transducers is not kept as part of the streamflow-gaging data at AVM sites. Only line-velocity data, as estimated with equation A.1 using the transit time between transducers measured by the AVM, are mantained in the record. Thus, in the present analysis, equation A.1 is used to estimate the ratio $\left(\frac{t_1 t_2}{t_1 - t_2} \right)$, which yields

$$\left(\frac{t_1 t_2}{t_1 - t_2} \right) = \frac{B}{2 v_L \cos\theta}. \qquad \text{(A.11)}$$

The travel times of the acoustic signals in the upstream and downstream directions between the transducers (for example, figures 8a and 8b) are of the same order of magnitude and so are their CV's; thus, $t_1 \approx t_2 = t,$and $\Omega_{t_1} \approx \Omega_{t_2} = \Omega_t$. Further, the travel time can be estimated as the ratio of the path length and the velocity of sound, that is, $t = B/C$. Based on these approximations and replacing the third term of equation A.3 with equation A.11, the following equation is obtained:

$$\left(\frac{t_1 t_2}{t_1 - t_2}\right)^2 \left(\frac{\Omega_{t_1}^2}{t_1^2} + \frac{\Omega_{t_2}^2}{t_2^2}\right) = \left(\frac{C}{2v_L \cos\theta}\right)^2 \left(2\Omega_t^2\right). \tag{A.12}$$

According to the ISO 6416 standard (International Organization for Standardization, 1992), the accuracy in timing the travel of the acoustic signal between transducers with current technology can be reduced to less than 30 nanoseconds. The speed of sound in water is typically about 4,925 feet per second (ft/s). Thus, the value of Ω_t^2 at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station can be estimated as

$$\Omega_t^2 = \left(\frac{30 \times 10^{-9}}{B/C}\right)^2 = \left(\frac{30 \times 10^{-9}}{236.44/4,925}\right)^2 = 3.90 \times 10^{-13}. \tag{A.13}$$

Total Uncertainty of Acoustic Velocity Meter Line-Velocity Measurements

The total uncertainty of line velocity measurements in the Chicago River at Columbus Drive in terms of the CV, accounting for the uncertainty from all the contributing sources based on first-order error analysis, is a function of the magnitude of the line velocity and velocity of sound in water as expressed in equations A.14 and A.15 as

$$\Omega_{v_L}^2 = \left(1 + \frac{1}{\cos^2\theta}\right)\Omega_{BT}^2 + \Omega_{BP}^2 + \left(\frac{C}{2v_L \cos\theta}\right)^2 2\Omega_t^2, \tag{A.14}$$

or to avoid dividing through by zero when v_L equals zero

$$\sigma_{v_L}^2 = v_L^2\left[\left(1 + \frac{1}{\cos^2\theta}\right)\Omega_{BT}^2 + \Omega_{BP}^2\right] + \left(\frac{C}{2\cos\theta}\right)2\Omega_t^2, \tag{A.15}$$

where Ω_{BP}^2 is the CV of the acoustic path length, which can take the value of Ω_{BD}^2, when a density gradient is present, or Ω_{BT}^2, when there is no density gradient.

Using equation A.10 and inserting the typical value of the speed of sound in water (4,925 ft/s) and the flow angle $\theta = 58.067°$, equation A.14 becomes

$$\Omega_{v_L}^2 = 4.57\Omega_{BT}^2 + \Omega_{BP}^2 + \left(\frac{4,925}{2v_L \cos(58.067)}\right)^2 \left(2 \times 3.90 \times 10^{-13}\right). \tag{A.16}$$

Further, by replacing the estimates of Ω_{BP} as described above for flows with and without density gradients from equations A.8 and A.9, the respective CVs can be computed as

$$\Omega^2_{v_{L-DG}} = 4.57 \times 7.45 \times 10^{-9} + 6.23 \times 10^{-5} + \left(\frac{4,925}{2v_L \cos(58.067)} \right)^2 7.80 \times 10^{-13}$$
$$= 6.23 \times 10^{-5} + \frac{1.69 \times 10^{-5}}{v_L^2} \qquad \text{(A.17)}$$

and

$$\Omega^2_{v_L} = 4.57 \times 7.45 \times 10^{-9} + 7.45 \times 10^{-9} + \left(\frac{4,925}{2v_L \cos(58.067)} \right)^2 7.80 \times 10^{-13}$$
$$= 4.15 \times 10^{-8} + \frac{1.69 \times 10^{-5}}{v_L^2}. \qquad \text{(A.18)}$$

The respective variances are

$$\sigma^2_{v_{L-DG}} = 6.23 \times 10^{-5} v_L^2 + 1.69 \times 10^{-5} \text{ ft}^2/\text{s}^2 \text{ (feet-squared per second-squared)} \qquad \text{(A.19)}$$

and

$$\sigma^2_{v_L} = 4.15 \times 10^{-8} v_L^2 + 1.69 \times 10^{-5} \text{ ft}^2/\text{s}^2. \qquad \text{(A.20)}$$

In the presence of a density gradient, the dependence of the CV on line velocity is weak relative to the dependence when there is no density gradient (equations A.17-A.20 and figures A.1 and A.2). There is no observable dependence of the uncertainty on line velocity results when there is no density gradient (figs. A.1-A.2); rather there is a constant value of about 0.0041 ft/s. It should be noted that density gradients are most likely when velocities are small ($v_L < 0.05$ ft/s).

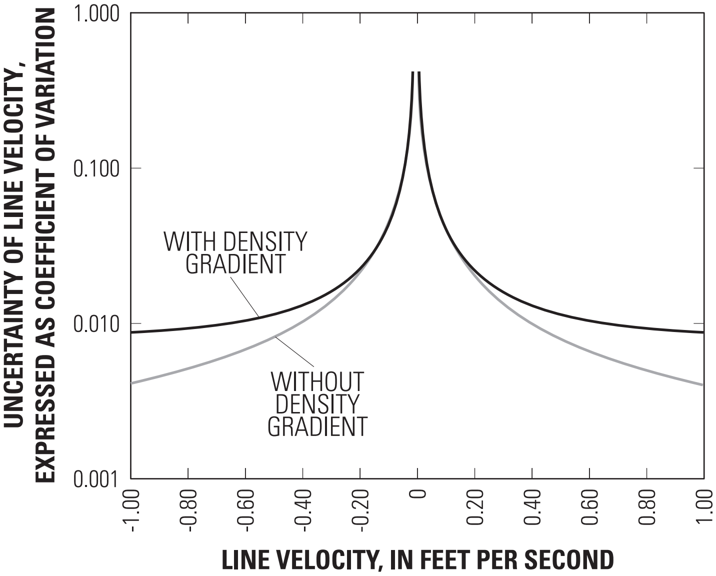


Figure A.1. Uncertainty of line velocity at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station expressed as coefficient of variation. Negative line velocities indicate reverse-flow (negative discharge) conditions. The value when line velocity is zero is a singularity and is omitted.

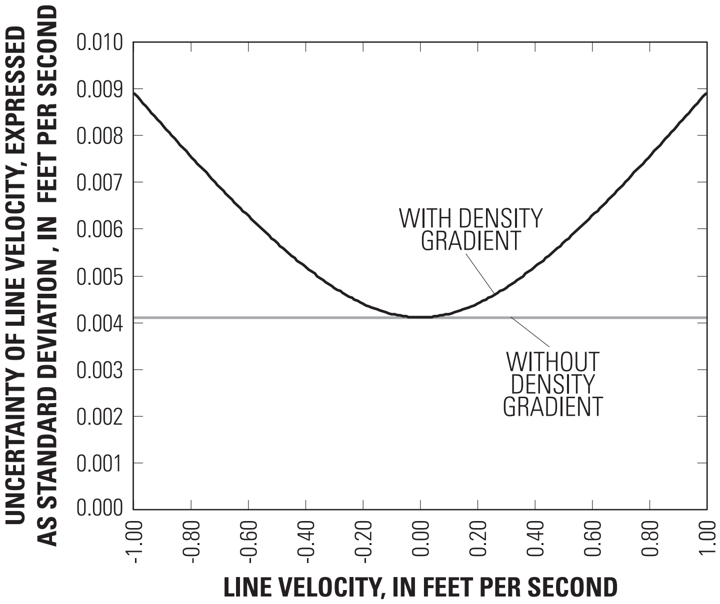


Figure A.2. Uncertainty of line velocity at Chicago Sanitary and Ship Canal at Romeoville streamflow-gaging station expressed as standard deviation. Negative line velocities indicate reverse-flow (negative discharge) conditions.

Application to the Chicago Sanitary and Ship Canal at Romeoville

For purposes of comparison, first-order error analysis was used to estimate the uncertainty of discharge estimates at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station, as well as for the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station. The application is similar at the two stations; the following items show the additional assumptions and calculations required:

1. Path-Length Uncertainty: It is assumed that at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station that there was no ray bending because of density gradients, so uncertainty in path length is due only to uncertainty in surveyed distance, which is 239.0 ft. Thus,

$$\Omega_{BT}^2 = \left(\frac{1}{\sqrt{6}} \frac{B_{T_u} - B_{T_l}}{B_{T_u} + B_{T_l}} \right)^2 = \left(\frac{1}{\sqrt{6}} \frac{(239.0 + 0.05) - (239.0 - 0.05)}{(239.0 + 0.05) + (239.0 - 0.05)} \right)^2 = 7.291 \times 10^{-9}, \qquad \text{(A.21)}$$

which is about the same as the value at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station (compare to equation A.9).

2. Flow-Angle Uncertainty:

$$\Omega_{\cos\theta}^2 = \Omega_B^2 \left(1 + \frac{1}{\cos^2 \theta} \right) = \Omega_B^2 \left(1 + \frac{1}{\cos^2 44.5^\circ} \right) = 2.97 \Omega_B^2. \qquad \text{(A.22)}$$

3. Flow-Time Uncertainty: These quantities are computed in terms of previously determined parameters as

$$\Omega_t^2 = \left(\frac{30 \times 10^{-9}}{239/4,925} \right)^2 = 3.82 \times 10^{-13} \tag{A.23}$$

using equation A.13 and

$$\left(\frac{C}{2v_L \cos\theta} \right)^2 2\Omega_t^2 = \left(\frac{4,925}{2v_L \cos(44.5^\circ)} \right)^2 \left(2 \times 3.82 \times 10^{-13} \right) \tag{A.24}$$

using equation A.12.

Using equation A.14, the CV of the line velocity may be computed as

$$\begin{aligned} \Omega_{v_L}^2 &= 2.97 \times 7.29 \times 10^{-9} + 7.29 \times 10^{-9} + \left(\frac{4,925}{2v_L \cos(44.5)} \right)^2 \left(2 \times 2.82 \times 10^{-13} \right) \\ &= 2.89 \times 10^{-8} + \frac{9.11 \times 10^{-6}}{v_L^2}, \end{aligned} \tag{A.25}$$

and the variance as

$$\sigma_{v_L}^2 = 2.89 \times 10^{-8} v_L^2 + 9.11 \times 10^{-6} \text{ ft}^2/\text{s}^2. \tag{A.26}$$

It may be observed by comparing equations A.24 and A.25 with equations A.18 and A.20 that, because of differences in path length and flow angle, even without consideration of density gradients, the computed line velocity uncertainty at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station is less than that at the Chicago River at Columbus Drive at Chicago, Illinois streamflow-gaging station. The dependence of the uncertainty of line velocity on line velocity, where uncertainty is expressed as CV and standard deviation, is shown in figures A.2 and A.3, respectively. The uncertainty in line velocity at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station, expressed as standard deviation, is approximately constant with a value of about 0.0030 ft/s.

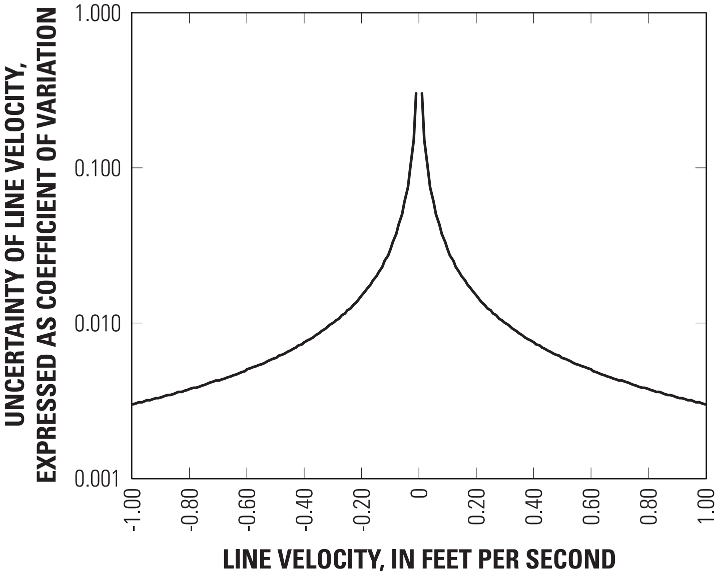


Figure A.3. Uncertainty of acoustic velocity meter (AVM) line velocity at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station expressed as coefficient of variation. Negative line velocities indicate reverse-flow (negative discharge) conditions. The value when line velocity is zero is a singularity and is omitted.

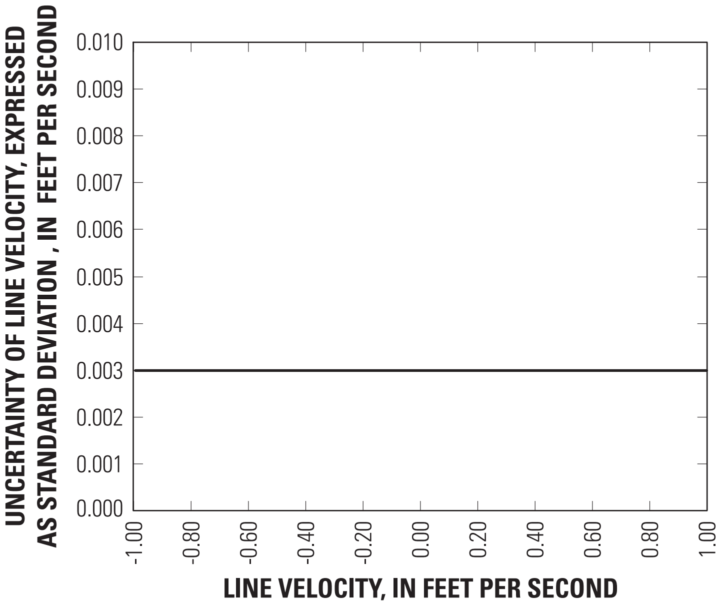


Figure A.4. Uncertainty of acoustic velocity meter (AVM) line velocity at the Chicago Sanitary and Ship Canal at Romeoville, Illinois streamflow-gaging station expressed as standard deviation. Negative line velocities indicate reverse-flow (negative discharge) conditions.

Literature Cited

Ang, A. H-S, and Tang, W.H., 1984, Probability Concepts in Engineering Planning and Design, Volume II-Decision, Risk, and Reliability: John Wiley and Sons. New York, p. 382-392.

International Organization for Standardization (ISO), 1992, Measurement of liquid flow in open channels—Measurement of discharge by the ultrasonic (acoustic) method: International Standard ISO 6416, Geneva, Switzerland, 28 p.

Laenen, A., 1985, Acoustic Velocity Meter Systems: Techniques of Water-Resources Investigations of the U.S. Geological Survey, book 3, chapter A17, 38 p.

Laenen, A., and Smith, W., 1983, Acoustic systems for the measurement of streamflow: U.S. Geological Survey Water-Supply Paper 2213, 26 p.

APPENDIX B—Daily Mean Discharge Tables for the Four Streamflow-Gaging Stations on the Chicago River System in Illinois

Table B.1. Daily mean discharges for the Chicago Sanitary and Ship Canal at Romeoville, Illinois for the 1997 water year.

05536995 Chicago Sanitary and Ship Canal at Romeoville, Il
1997 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	2513	2108	3078	2156	2738	6489	2802	3245	2682	3915	3578	3725
2	2936	2526	2282	1908	2855	5613	2188	4224	2712	e3282	2911	3030
3	2027	2509	2609	2450	2651	4959	2267	3011	3087	3417	3856	3102
4	2456	1705	2588	3050	4243	4364	e2075	3217	2558	3215	2637	3348
5	3319	2040	3062	3346	3707	3825	2814	2421	2680	3286	3912	3065
6	2705	3887	2408	2493	3011	3476	2479	2432	4184	2980	4189	3472
7	2132	3496	2657	2115	2725	2558	1999	3413	4258	3217	3262	3524
8	2512	2886	2003	2172	2925	3282	1949	3017	3690	3141	3412	3142
9	2664	2819	2299	2329	2461	3064	2116	2888	3100	3436	3230	3000
10	3197	2706	1994	2308	2502	2997	e1836	2851	2911	3455	3343	3012
11	2647	2232	3812	2202	2381	2735	3202	2826	3648	2857	4806	3146
12	2565	2057	3226	2122	2265	2822	4036	2094	3615	3982	4765	3183
13	2654	2020	3012	2337	2319	2911	3512	2225	3487	3163	4130	3349
14	2254	2033	3084	2278	2250	2968	e2517	2376	3369	3495	4119	3158
15	2571	e2330	2700	2087	2350	2612	2809	2450	3301	3106	4497	3218
16	3412	e2370	2566	2264	2558	2876	2579	2135	5200	3559	7502	4118
17	3284	e2680	2432	2215	2266	2039	2581	2375	4652	3527	13997	5171
18	3670	e2200	2508	2505	2836	2435	2576	3524	4001	5911	6211	4149
19	2723	2033	2124	2266	3232	2052	2845	3890	3556	5523	4281	4341
20	2952	2045	2136	e2140	e4258	2206	2280	3361	4336	4426	4115	4522
21	2184	2342	2209	2520	17281	1720	e2156	2641	5566	4095	4319	3908
22	2952	2164	2017	5542	15860	2089	2364	2857	3976	4053	e3409	2902
23	2523	1849	3323	4284	10795	2181	2251	2453	4253	3826	e3479	2502
24	2927	2265	3501	3490	7865	3238	2585	2736	4037	3895	3951	3121
25	2554	2149	3121	3453	5481	2964	2157	3752	3903	3687	3585	3058
26	2597	2150	2645	3507	8113	2648	2415	3245	3656	3837	3440	3108
27	2303	2054	2277	2535	8739	1976	2430	2386	3922	5033	e3418	3498
28	2277	2019	2402	2818	8627	2421	2200	2443	3542	3427	2920	3025
29	3675	2067	2409	2406		3596	2108	2757	3581	3683	3156	2739
30	3437	3378	2075	2837		3054	2860	2236	3570	3496	3792	2986
31	2786		2206	2842		2565		2435		3318	2865	

Mean annual discharge = 3,211

Table B.2. Daily mean discharges for the Chicago Sanitary and Ship Canal at Romeoville, Illinois for the 1998 water year.

05536995 Chicago Sanitary and Ship Canal at Romeoville, Il
1998 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	2762	e2266	1815	1794	2655	1997	4390	2841	2436	3407	2885	3236
2	2497	e1805	1938	1984	2928	2107	3783	2814	2301	3059	2870	2744
3	2599	e1991	2011	2039	2999	1993	3471	2111	2618	3373	2680	2688
4	2897	2024	1741	6237	2398	2289	3123	2927	2312	5528	7362	3036
5	2698	2021	1838	5338	2576	2112	3538	2603	2250	4386	7786	2822
6	2516	2586	1926	5966	2243	1877	2239	3206	2491	3414	5889	2757
7	2900	2077	1647	5067	2200	2200	3361	12470	2270	4475	6013	e6357
8	3025	1929	1743	7993	2106	4880	4422	12116	2323	4678	5207	e4127
9	3211	1998	1951	5990	2240	6564	4553	8185	3662	3644	4513	e2974
10	3074	1870	3208	4937	2057	4786	3466	5062	3471	3779	3839	e2757
11	2952	1684	2943	3863	5296	4413	3805	4184	6574	3392	4130	2817
12	3036	2118	2709	3374	3921	3897	3427	3795	4341	3272	3673	2496
13	3032	1814	2435	2751	3500	3127	2922	3407	3867	3503	3600	2878
14	2286	2004	2138	2689	3305	3732	3423	3341	3983	3402	3328	2958
15	2066	2429	1776	2639	2809	2893	3360	2752	2459	3674	e3250	3332
16	2167	2472	2089	2067	2940	2683	2893	2672	3050	3365	e3311	3007
17	2097	2035	1692	2317	4069	5209	3134	2625	2766	3585	e2817	3013
18	1999	1671	1798	2117	5414	8180	3204	2496	3319	3808	3089	2624
19	2297	1929	1905	1891	4162	7369	2720	2739	3416	3894	3386	2659
20	1774	1742	1871	2253	3247	5599	3150	2230	3521	3972	3092	2758
21	1893	1851	1755	1935	3229	4568	4475	2128	2983	3011	3061	3217
22	2024	1549	2118	2123	2947	4464	3896	2201	2730	3173	3854	2475
23	1937	1642	2434	2379	2462	3565	3521	2360	2677	3698	3542	2522
24	2369	1817	2861	1896	2322	2823	2820	3466	2663	3130	4765	2302
25	2050	1764	3791	1808	2571	2825	3716	2825	3257	3146	5682	2512
26	3187	1783	e1480	2246	2252	2646	3281	2437	4277	2839	3846	2435
27	5309	1956	2500	1682	2241	2669	3112	2271	5007	2602	3868	2418
28	2930	2574	2022	2090	2539	3223	2869	2539	4775	3058	3598	2613
29	2826	2714	2014	2441		2802	3549	2686	3880	2736	3821	3223
30	2557	2150	1993	2272		2603	2976	2272	3317	2714	3326	3223
31	2245		1739	2287		5264		2159		2805	2727	

Mean annual discharge = 3,121

Table B.3. Daily mean discharges for the Chicago Sanitary and Ship Canal at Romeoville, Illinois for the 1999 water year.

05536995 Chicago Sanitary and Ship Canal at Romeoville, Il
1999 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	2601	2586	2019	e1372	3242	2393	1861	4340	5008	3335	3647	2908
2	2672	2224	1651	e1576	3588	2088	1768	3589	8652	3364	2817	2608
3	3319	1525	1604	e1465	3423	2274	2050	2792	5277	2788	3013	2751
4	3204	1866	1675	e1503	e2663	2523	2198	2829	4260	2764	2932	3246
5	2592	2548	1886	e1471	3078	2124	1995	2475	3098	2830	3130	2909
6	2894	1897	3377	e1747	2668	2441	1892	3128	2747	2719	2739	2847
7	2817	1778	4277	1569	2643	2915	1746	2953	2554	2690	3199	3243
8	2148	1957	2875	1646	2328	2360	2451	2198	2801	2697	2872	2386
9	2592	2000	2564	1813	2147	2300	7942	2248	2596	2677	3033	2340
10	2308	4763	2313	1697	2190	2353	4840	1745	4003	2973	2445	2469
11	2530	3473	2154	1730	2993	2312	3880	2448	4985	2545	3249	2276
12	1970	2819	1749	1707	2214	2498	3769	4865	4451	2590	4051	2361
13	2080	2661	2080	1713	2377	2986	2999	3459	e7743	2733	2828	2325
14	2100	2536	1785	1745	2352	2590	3156	3081	e5308	2815	3197	2350
15	2067	2399	1669	1722	1851	2492	3435	3137	4633	2801	4125	2482
16	1913	1832	1739	1793	2314	2636	7452	2567	3757	e2803	3038	2168
17	5795	1699	1634	2568	1912	3799	5888	4723	3373	3164	2595	2047
18	5339	1787	1787	3214	1924	3814	4452	2714	e2838	3377	2870	2287
19	3132	1789	1553	2538	2469	2742	3578	2650	2591	e4805	4087	2144
20	2954	1829	1587	2215	2095	2584	3386	2565	2710	3868	3832	2008
21	2398	1785	1569	2635	1844	2533	4052	3160	2629	e2246	3251	2309
22	2428	1731	1598	9745	1654	2315	7106	3554	3373	4051	3496	2150
23	2287	1683	1414	10981	1804	2366	10973	3609	2660	3828	3514	2070
24	2368	1645	1703	8973	1869	2415	7139	2807	3882	3562	3630	2202
25	2529	1437	1545	7550	2100	1894	6144	e2187	3010	4041	3680	2207
26	2122	1603	1263	5118	2036	1893	4071	e1618	3358	3463	3981	2033
27	2228	1575	1502	4354	2961	1747	6490	2020	3549	3002	3206	3153
28	2979	1625	1481	4662	2321	2134	7852	1815	2637	3102	3185	7202
29	2928	1644	1815	4302		1614	7123	2171	2711	e3350	3127	4173
30	2650	1812	1555	3628		1667	4759	2051	2537	e3208	2919	2762
31	2985		1605	3641		1899		2631		3599	3023	

Mean annual discharge = 2,927

Table B.4. Daily mean discharges for the Chicago River at Columbus Drive at Chicago, Illinois for the 1997 water year.

05536123 Chicago River at Columbus Drive at Chicago, Il
1997 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	e371	e197	e102	244	450	794	93	372	386	878	910	843
2	e354	e244	e220	193	430	342	178	517	344	873	881	839
3	e314	e259	231	176	349	217	223	533	372	908	737	830
4	e352	e237	552	233	117	304	155	387	385	862	986	793
5	e470	e251	800	264	358	452	181	442	425	842	1150	836
6	e726	e669	347	306	583	341	141	386	554	846	1370	852
7	e311	e709	340	342	546	261	149	318	391	754	886	833
8	e347	e205	434	216	416	454	178	539	351	768	856	791
9	e343	e237	226	224	277	348	249	362	427	864	906	771
10	e311	e213	164	326	368	429	231	369	373	852	897	885
11	e336	e205	137	260	554	282	220	400	453	846	785	796
12	e371	e188	281	512	263	257	389	374	547	883	684	919
13	e378	e229	354	466	110	228	266	335	520	901	901	905
14	e349	e205	184	411	320	348	270	359	429	975	874	856
15	e378	e152	272	394	502	304	316	344	444	964	804	757
16	e585	e213	170	551	509	379	295	394	462	950	-711	1130
17	e437	e171	323	567	401	371	300	375	328	915	595	804
18	e326	e188	228	423	315	236	260	492	378	e665	564	1250
19	e376	e152	362	333	430	219	292	749	408	e702	534	909
20	e361	e123	334	555	449	123	257	354	706	1040	745	977
21	e295	e180	182	393	-2540	163	267	360	1310	777	925	904
22	e595	e143	273	386	124	138	323	361	785	829	895	678
23	e264	e171	302	215	338	74	340	403	963	873	802	616
24	e298	e133	311	80	383	328	297	366	1190	868	808	586
25	e311	e162	292	306	340	377	293	402	709	873	842	780
26	e358	e162	251	292	141	119	335	340	930	836	836	850
27	e365	e229	319	418	365	137	326	275	853	830	807	843
28	e309	e133	376	537	360	96	269	305	838	1020	836	785
29	e338	e112	328	460		189	413	450	841	857	851	851
30	e245	e102	385	450		98	359	511	676	851	774	731
31	e267		301	372		129		418		894	855	

Mean annual discharge = 461

Table B.5. Daily mean discharges for the Chicago River at Columbus Drive at Chicago, Illinois for the 1998 water year.

05536123 Chicago River at Columbus Drive at Chicago, Il
1998 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	577	430	54	-3	157	91	154	288	448	658	672	729
2	636	298	9	60	62	151	164	230	533	648	659	820
3	715	303	61	161	-61	72	120	224	397	527	616	691
4	759	303	115	165	81	-44	160	272	376	600	316	622
5	628	230	207	372	96	-41	119	261	377	580	1000	652
6	753	214	180	236	-89	73	76	330	409	560	1160	606
7	899	243	159	624	112	91	70	242	395	840	440	-87
8	873	289	4	571	140	212	93	408	347	726	746	466
9	974	313	61	491	42	109	422	285	325	657	720	522
10	791	358	98	94	79	171	104	205	475	738	872	417
11	775	307	187	-135	422	-72	296	304	530	779	902	536
12	767	299	221	101	295	349	162	285	561	808	826	505
13	586	233	-162	236	225	58	360	243	613	990	877	629
14	358	239	-168	-40	174	282	150	141	424	987	822	611
15	365	250	156	153	0	173	31	138	619	981	819	449
16	367	309	178	38	-59	24	348	301	579	1010	841	470
17	365	262	50	120	101	18	254	254	572	965	806	499
18	386	102	129	268	129	192	157	553	501	990	891	459
19	381	162	e85	129	-25	35	90	389	692	860	820	525
20	360	57	e87	-73	-2	243	188	332	501	912	853	450
21	334	-22	e87	112	57	2	192	278	536	873	872	520
22	323	75	e87	9	17	193	188	355	579	904	804	508
23	301	144	e87	106	117	141	211	212	581	1010	791	464
24	283	110	e87	137	130	67	173	251	556	790	1030	465
25	284	65	e87	176	137	92	188	273	669	696	896	490
26	325	88	e84	148	123	189	280	303	510	693	855	421
27	353	90	e86	81	136	213	207	308	476	837	743	460
28	326	44	-127	53	121	122	144	400	766	858	e277	707
29	339	72	80	258		120	89	639	660	660	e780	840
30	314	231	41	-7		221	228	282	778	730	e815	600
31	387		-20	51		491		383		693	e738	

Mean annual discharge = 367

Table B.6. Daily mean discharges for the Chicago River at Columbus Drive at Chicago, Illinois for the 1999 water year.

05536123 Chicago River at Columbus Drive at Chicago, Il
1999 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	350	98	25	26	29	e40	-44	26	328	379	622	494
2	202	77	26	32	24	e37	-22	46	248	426	603	432
3	232	66	30	30	20	e44	-36	37	246	388	536	514
4	281	144	33	26	25	e120	-34	-28	210	442	591	568
5	243	149	35	20	22	e47	51	29	277	396	574	559
6	388	111	32	23	27	e49	78	85	271	329	576	630
7	377	103	30	22	29	e42	96	112	274	409	486	505
8	333	121	31	24	25	e40	69	125	190	458	644	503
9	284	17	27	26	23	e46	443	-12	239	405	480	566
10	264	-6	29	23	25	e40	35	-28	516	399	624	448
11	262	111	29	22	24	e42	113	6	265	402	577	395
12	381	12	54	29	23	e39	5	106	466	459	519	327
13	256	-20	32	33	23	e39	-25	220	304	482	534	457
14	262	96	29	30	21	e42	6	70	352	494	604	417
15	192	106	24	25	24	e39	72	38	307	544	540	350
16	254	73	29	21	e42	e31	243	90	300	549	572	312
17	293	70	30	24	e44	e30	116	376	244	282	585	272
18	251	51	26	21	e99	e30	-3	83	259	548	443	249
19	265	191	30	23	e550	e35	-45	95	287	714	357	259
20	175	166	30	23	e46	e31	-77	95	310	607	555	338
21	168	56	29	27	e47	e37	121	165	347	525	514	236
22	152	70	26	30	e44	e39	220	177	345	571	533	341
23	246	52	21	11	e42	e188	130	177	320	420	498	339
24	256	24	20	21	e42	e216	67	179	306	487	341	219
25	258	-2	19	22	e42	27	21	149	358	540	348	241
26	327	60	22	25	e37	25	10	159	322	651	657	273
27	349	-5	21	25	e39	25	46	178	320	535	541	218
28	408	78	22	24	e39	20	209	167	379	542	533	311
29	129	114	27	23		152	105	154	358	680	530	280
30	474	221	27	27		117	81	175	351	678	484	508
31	242		25	30		-6		149		783	470	

Mean annual discharge = 205

Table B.7. Daily mean discharges for the Calumet River at O'Brien Lock and Dam at Chicago, Illinois for the 1997 water year.

05536357 Calumet River at O'Brien Lock and Dam at Chicago, IL
1997 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	444	65	e59	e83	e68	e56	e84	e104	e495	e455	321	338
2	358	91	e44	e42	e53	e49	e65	e392	e395	e463	317	198
3	306	122	e59	e47	e55	e70	e65	e327	e464	e495	219	261
4	376	105	e69	e62	e62	e56	e96	e110	e473	e479	294	265
5	432	130	e64	e67	e55	e57	e87	e70	e389	e495	265	209
6	385	196	e61	e67	e62	e61	e50	e85	e124	e489	280	272
7	323	116	e56	e70	e61	e61	e50	e91	e131	e444	294	330
8	344	117	e56	e53	e61	e64	e61	e458	e351	e384	289	236
9	377	94	e57	e63	e54	e81	e73	e102	e468	e451	299	179
10	231	62	e66	e83	e84	e61	e58	e133	e467	e451	280	207
11	353	88	e77	e123	e68	e81	e83	e122	e384	e490	226	244
12	385	83	e84	e105	e70	e62	e318	e124	e416	e495	197	295
13	386	132	e66	e109	e68	e83	e50	e89	e472	e491	325	295
14	365	138	e60	e107	e62	e78	e54	e88	e485	e460	278	250
15	373	e65	e70	e65	e55	e82	e84	e111	e458	e457	321	254
16	289	e70	e56	e102	e65	e80	e83	e86	e352	e476	141	272
17	242	e56	e57	e112	252	e71	e80	e105	e435	279	93	252
18	306	e57	e56	e83	47	e66	e73	e112	e449	249	143	244
19	309	e47	e46	e92	265	e66	e94	e425	e422	285	32	257
20	362	e67	e88	e63	243	e84	e114	e62	e385	343	331	306
21	272	e62	e45	e65	-844	e72	e67	e123	e1069	238	260	276
22	162	e42	e56	e67	-38	e86	e63	e193	e407	290	e481	155
23	193	e75	e35	e77	52	e66	e81	e160	e467	314	e448	202
24	310	e68	e22	e59	46	e371	e68	e159	e529	334	e449	279
25	381	e60	e57	e92	-46	e352	e112	e123	e356	292	e467	262
26	417	e57	e70	e90	e30	e60	e132	e132	e476	315	e465	298
27	343	e48	e76	e80	e57	e64	e100	e87	e475	302	320	307
28	269	e61	e43	e68	e36	e65	e85	e111	e487	330	341	325
29	157	e46	e55	e87		e308	e100	e88	e492	285	283	206
30	205	e56	e59	e99		e74	e85	e110	e289	307	240	254
31	209		e51	e81		e72		e174		298	332	

Mean annual discharge = 191

Table B.8. Daily mean discharges for the Calumet River at O’Brien Lock and Dam at Chicago, Illinois for the 1998 water year.

05536357 Calumet River at O’Brien Lock and Dam at Chicago, IL
1998 Water Year
Daily mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	192	e67	e61	e32	e47	e59	e50	e87	333	e449	459	325
2	184	e96	e52	e41	e33	e44	e48	e107	198	e453	463	380
3	198	e63	e71	e36	e48	e42	e51	e82	210	e432	368	318
4	188	e53	e41	180	e45	e31	e56	e74	210	e407	203	390
5	220	e50	e46	106	e31	e46	e45	e81	302	e453	247	423
6	184	e70	e62	223	e43	e46	e48	167	258	e443	383	373
7	242	e102	e36	148	e46	e37	243	e46	318	e644	246	265
8	228	e93	e52	179	e51	93	68	e34	291	e441	362	295
9	199	e73	203	e39	e41	e51	369	e96	302	e429	413	254
10	200	e46	e46	e38	e46	e45	e87	e107	301	e520	348	306
11	274	e56	e39	e36	258	e32	e101	e92	e462	e583	376	352
12	271	e54	e59	e50	119	e53	e96	e105	e264	e553	453	364
13	190	e57	e48	e55	e41	e33	e55	e121	e335	e572	459	384
14	e72	e48	e35	e50	e58	e52	e97	e81	e292	e603	434	328
15	e103	e50	e40	e56	e52	e71	178	e112	e397	e603	494	345
16	e94	e42	e52	e37	206	e36	151	e136	e336	485	461	389
17	e101	e21	e42	e39	e43	e52	e62	e126	e329	448	363	322
18	e114	e50	e46	e43	e35	e39	e110	e334	e258	561	425	392
19	e122	e65	e48	e38	e43	e27	e125	e300	e493	481	433	369
20	e86	e64	e44	e49	e39	e30	228	e104	e356	431	458	383
21	e72	e50	e55	e44	e41	e43	e49	e108	e498	482	443	355
22	e72	e57	e48	e49	e62	e38	e58	e126	e310	424	462	313
23	e52	e66	e50	e44	e40	e48	e45	e138	e326	447	507	328
24	e78	e52	e51	e46	e45	e37	e134	296	e331	465	484	296
25	e73	e273	e29	e56	e48	e48	236	e129	e407	449	660	348
26	182	e42	e46	e40	e44	e34	227	299	e295	466	478	364
27	e61	e43	e41	e43	e49	e46	e72	e109	e390	366	478	391
28	e60	e41	e45	e25	e46	e351	e90	e91	e575	394	314	370
29	e72	e33	e56	e44		e90	e66	307	e494	381	409	369
30	e93	e227	e50	e51		e39	e61	e132	e442	430	448	270
31	e50		e42	e47		262		e161		418	434	

Mean annual discharge = 190

Table B.9. Daily mean discharges for the Calumet River at O'Brien Lock and Dam at Chicago, Illinois for the 1999 water year.

05536357 Calumet River at O'Brien Lock and Dam at Chicago, IL
1999 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	279	e81	e36	e35	e26	e29	e46	e63	242	308	e590	337
2	239	e46	e36	e35	e29	e35	e49	e69	159	375	e568	304
3	300	e46	e32	e51	e38	e34	e55	e60	232	291	343	407
4	360	e48	e30	e56	e27	e30	e36	e50	245	335	411	439
5	280	e43	e41	e52	e37	e39	e44	e50	182	321	425	414
6	168	e50	e39	e43	e32	e41	e39	e41	150	321	404	411
7	247	e42	e32	e54	e49	e31	e54	e27	206	363	355	373
8	268	e32	e30	e52	e31	e25	e48	e64	219	310	469	311
9	299	e40	e37	e56	e54	e38	e789	e60	260	252	338	290
10	291	224	e34	e61	e34	e25	e48	e62	406	404	345	308
11	315	e29	e35	e58	232	e34	e50	e59	530	391	364	325
12	267	e33	e26	e52	e30	e29	e37	150	298	361	293	284
13	222	e37	e38	e56	e47	e26	e38	215	331	367	316	248
14	237	e46	e26	e46	e38	e34	e36	e58	295	427	404	244
15	262	e56	e31	e34	e31	e27	177	e71	293	381	388	287
16	204	e36	e31	e26	e35	e50	199	e81	283	414	363	315
17	276	e40	e38	e19	e33	e49	e26	335	305	279	340	268
18	395	e45	e32	e31	e224	e29	e42	e42	327	432	252	315
19	222	e35	e29	e42	e429	e26	e30	e59	333	494	337	253
20	180	e33	e36	e24	e31	e49	e42	e55	307	430	364	240
21	e57	e39	e60	e44	e33	e30	197	e60	292	371	373	268
22	e51	e62	e31	e25	e32	e57	219	e77	201	381	410	264
23	205	e49	e20	e9	e37	233	e25	e59	207	294	300	236
24	222	e42	e19	e5	e24	156	e39	e33	227	377	328	299
25	276	e35	e21	e11	e35	e48	e51	e43	314	e497	301	312
26	249	e41	e30	e20	e35	e44	e37	e53	284	e819	419	282
27	212	e37	e23	e36	e30	e37	e41	e65	277	e485	351	186
28	236	e52	e29	e24	e31	e30	e17	e76	251	e505	416	673
29	185	e55	e31	e35		e26	e28	e82	330	e560	393	678
30	209	e34	e35	e28		e42	e47	e82	295	e593	342	282
31	e74		e34	e25		e29		e76		e794	358	

Mean annual discharge = 167

Table B.10. Estimated daily mean discharges for the North Shore Channel at Wilmette, Illinois for the 1997 water year.

05536101 North Shore Channel at Wilmette, IL
1997 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	e5.4	e4.4	e4.4	e5.4	e53	e0.6	e0.6	e0.6	e0.6	e214	e156	e228
2	e5.4	e4.4	e4.4	e4.4	e53	e0.6	e0.6	e0.6	e0.6	e170	e172	e130
3	e5.4	e4.4	e4.4	e4.4	e38	e0.6	e0.6	e0.6	e0.6	e212	e98	e242
4	e4.4	e4.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e119	e189	e229
5	e5.4	e5.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e192	e232	e218
6	e4.4	e4.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e115	e214	e219
7	e5.4	e4.4	e4.4	e4.4	e5.4	e0.6	e65	e0.6	e0.6	e212	e223	e154
8	e5.4	e4.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e116	e223	e70
9	e5.4	e4.4	e4.4	e5.4	e5.4	e0.6	e0.6	e0.6	e0.6	e228	e86	e52
10	e5.4	e3.5	e4.4	e5.4	e5.4	e0.6	e0.6	e0.6	e0.6	e223	e226	e245
11	e5.4	e5.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e227	e0.6	e240
12	e4.4	e4.4	e5.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e220	e10.2	e235
13	e5.4	e4.4	e4.4	e3.5	e5.4	e0.6	e0.6	e0.6	e0.6	e204	e173	e148
14	e5.4	e5.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e73	e173	e177
15	e5.4	e4.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e219	e148	e229
16	e4.4	e4.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e209	e9.2	e115
17	e4.4	e4.4	e37	e4.4	e5.4	e0.6	e0.6	e0.6	e0.6	e215	e0.6	e0.6
18	e5.4	e4.4	e95	e4.4	e4.4	e0.6	e0.6	e0.6	e0.6	e93	e0.6	e120
19	e5.4	e4.4	e52	e4.4	e4.4	e0.6	e0.6	e0.6	e0.6	e137	e0.6	e55.3
20	e5.4	e5.4	e4.4	e4.4	e5.4	e0.6	e0.6	e0.6	e77	e194	e60	e148
21	e5.4	e4.4	e4.4	e15	e0.6	e0.6	e0.6	e0.6	e0.6	e20	e40	e234
22	e5.4	e4.4	e4.4	e6.3	e0.6	e0.6	e0.6	e0.6	e117	e94	e231	e86
23	e4.4	e4.4	e5.4	e19	e1.6	e0.6	e0.6	e0.6	e167	e149	e143	e54
24	e4.4	e5.4	e4.4	e58	e3.5	e0.6	e0.6	e0.6	e199	e221	e171	e120
25	e4.4	e5.4	e4.4	e58	e4.4	e0.6	e0.6	e0.6	e99	e177	e235	e186
26	e5.4	e5.4	e5.4	e59	e3.5	e0.6	e0.6	e0.6	e203	e187	e202	e239
27	e4.4	e5.4	e5.4	e60	e3.5	e0.6	e0.6	e0.6	e200	e89	e66	e201
28	e4.4	e4.4	e4.4	e59	e4.4	e0.6	e0.6	e0.6	e214	e130	e0.6	e137
29	e4.4	e4.4	e5.4	e65		e0.6	e15	e0.6	e213	e233	e100	e192
30	e3.5	e4.4	e5.4	e64		e0.6	e0.6	e0.6	e111	e232	e75	e226
31	e4.4		e5.4	e63		e0.6		e0.6		e227	e240	

Mean annual discharge = 48

Table B.11. Estimated daily mean discharges for the North Shore Channel at Wilmette, Illinois for the 1998 water year.

05536101 North Shore Channel at Wilmette, IL
1998 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	e125	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e69	e0.6	e209	e116
2	e122	e0.6	e45	e0.6	e0.6	e0.6	e0.6	e0.6	e84	e198	e202	e122
3	e99	e22	e12	e0.6	e0.6	e0.6	e0.6	e0.6	e85	e28	e61	e19
4	e79	e58	e56	e0.6	e0.6	e0.6	e0.6	e0.6	e103	e126	e0.6	e98
5	e121	e23	e61	e0.6	e0.6	e0.6	e0.6	e0.6	e66	e153	e0.6	e104
6	e122	e0.6	e63	e0.6	e0.6	e0.6	e0.6	e0.6	e66	e91	e0.6	e71
7	e198	e0.6	e60	e0.6	e0.6	e0.6	e0.6	e0.6	e108	e0.6	e0.6	e0.6
8	e188	e0.6	e21	e0.6	e0.6	e0.6	e0.6	e0.6	e49	e90	e0.6	e74
9	e98	e0.6	e9	e0.6	e0.6	e0.6	e0.6	e0.6	e13	e207	e0.6	e72
10	e198	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e106	e216	e94	e102
11	e215	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e8	e214	e207	e100
12	e166	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e57	e212	e208	e105
13	e38	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e90	e205	e206	e84
14	e109	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e62	e39	e204	e163	e0.6
15	e121	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e63	e97	e133	e140	e14
16	e124	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e94	e77	e201	e209	e108
17	e123	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e96	e89	e204	e130	e101
18	e121	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e93	e24	e185	e211	e109
19	e117	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e31	e86	e109	e203	e105
20	e122	e32	e0.6	e0.6	e0.6	e0.6	e0.6	e53	e75	e116	e195	e72
21	e122	e73	e0.6	e0.6	e0.6	e0.6	e0.6	e55	e76	e0.6	e141	e74
22	e122	e69	e0.6	e0.6	e0.6	e0.6	e0.6	e26	e102	e51	e53	e68
23	e117	e62	e0.6	e0.6	e0.6	e0.6	e0.6	e24	e78	e203	e134	e106
24	e0.6	e70	e0.6	e0.6	e0.6	e0.6	e28	e0.6	e71	e211	e41	e63
25	e0.6	e62	e0.6	e0.6	e0.6	e0.6	e24	e0.6	e92	e164	e75	e59
26	e0.6	e51	e0.6	e27	e0.6	e20	e0.6	e0.6	e53	e206	e185	e90
27	e0.6	e44	e0.6	e53	e0.6	e34	e0.6	e18	e108	e193	e117	e102
28	e31	e17	e0.6	e49	e0.6	e0.6	e0.6	e30	e55	e196	e48	e145
29	e54	e10	e0.6	e36		e0.6	e0.6	e0.6	e71	e193	e187	e106
30	e44	e0.6	e0.6	e39		e2.5	e0.6	e0.6	e163	e108	e193	e94
31	e0.6		e0.6	e0.6		e0.6		e0.6		e213	e194	

Mean annual discharge = 50

Table B.12. Estimated daily mean discharges for the North Shore Channel at Wilmette, Illinois for the 1999 water year.

05536101 North Shore Channel at Wilmette, IL
1999 Water Year
Daily Mean Discharge (cubic feet per second); e, estimated value

Day	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
1	e103	e0.6	e0.6	e0.6	e0.6	e0.6	e41	e0.6	e19	e19	e145	e143
2	e46	e36	e0.6	e0.6	e0.6	e0.6	e41	e0.6	e7	e93	e151	e145
3	e0.6	e46	e0.6	e0.6	e0.6	e0.6	e12	e24	e29	e98	e136	e140
4	e98	e0.6	e0.6	e0.6	e0.6	e0.6	e23	e27	e23	e133	e112	e144
5	e0.6	e19	e0.6	e0.6	e0.6	e0.6	e24	e0.6	e39	e119	e133	e139
6	e0.6	e52	e0.6	e0.6	e0.6	e0.6	e28	e0.6	e26	e91	e145	e145
7	e94	e49	e0.6	e0.6	e0.6	e0.6	e36	e0.6	e51	e144	e13	e136
8	e106	e26	e0.6	e0.6	e0.6	e0.6	e15	e0.6	e68	e142	e167	e112
9	e105	e11	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e46	e58	e105	e132
10	e103	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e27	e11	e157	e97	e126
11	e96	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e27	e0.6	e153	e143	e133
12	e62	e0.6	e0.6	e0.6	e0.6	e0.6	e14	e0.6	e0.6	e153	e20	e63
13	e52	e0.6	e0.6	e0.6	e0.6	e0.6	e35	e0.6	e0.6	e146	e53	e123
14	e53	e0.6	e0.6	e0.6	e0.6	e0.6	e36	e16	e0.6	e136	e158	e116
15	e53	e0.6	e0.6	e0.6	e0.6	e0.6	e5	e41	e0.6	e136	e154	e128
16	e52	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e30	e32	e115	e130	e67
17	e13	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e50	e18	e144	e131
18	e6	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e13	e62	e137	e68	e128
19	e22	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e37	e81	e26	e32	e71
20	e35	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e40	e84	e90	e138	e143
21	e38	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e17	e81	e70	e140	e76
22	e35	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e13	e45	e86	e102	e129
23	e47	e0.6	e0.6	e0.6	e0.6	e15	e0.6	e5	e34	e83	e0.6	e117
24	e46	e0.6	e0.6	e0.6	e0.6	e40	e0.6	e21	e49	e79	e0.6	e138
25	e50	e0.6	e0.6	e0.6	e0.6	e43	e0.6	e26	e77	e101	e17	e138
26	e51	e0.6	e0.6	e0.6	e0.6	e39	e0.6	e40	e47	e61	e139	e95
27	e3	e0.6	e0.6	e0.6	e0.6	e42	e0.6	e41	e17	e132	e134	e0.6
28	e49	e0.6	e0.6	e0.6	e0.6	e0.6	e0.6	e40	e15	e38	e137	e0.6
29	e21	e0.6	e0.6	e0.6		e25	e0.6	e40	e67	e119	e153	e40
30	e20	e0.6	e0.6	e0.6		e40	e0.6	e28	e71	e123	e145	e83
31	e42		e0.6	e0.6		e39		e0.6		e51		

Mean annual discharge = 38

Literature Cited

Laenen, A., and Smith, W., 1983, Acoustic systems for the measurement of streamflow: U.S. Geological Survey Water-Supply Paper 2213, 26 p.

Duncker and others—**Computation and Error Analysis of Discharge for the Lake Michigan Diversion Project in Illinois:**
1997-99 Water Years—Scientific Investigations Report 2006-5018