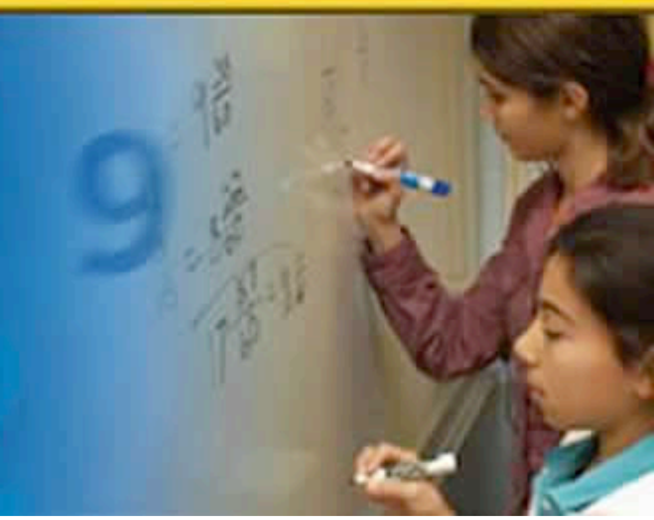
National Mathematics Advisory Panel

Learning Processes Task Group

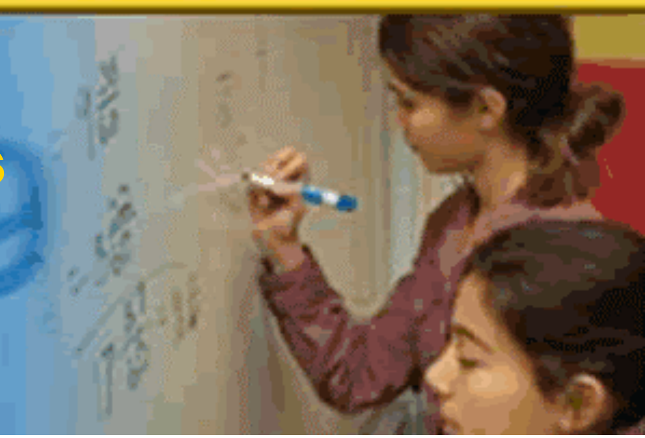
St. Louis Meeting

Progress Report

September 7, 2007



Learning Processes Task Group



Contributing Members

- Dave Geary
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- Valerie Reyna
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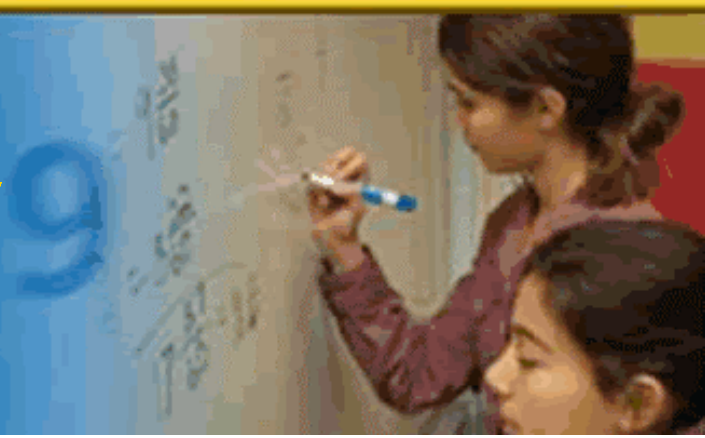
- Jennifer Graban, staff

Areas Reviewed

- General principles: from cognitive processes to learning outcomes
- Social, motivational, and affective influences on learning
- Mathematical knowledge children bring to school
- Mathematical learning and cognition in:
  - Whole number arithmetic
  - Fractions, decimals, and ratios
  - Estimation
  - Geometry
  - Algebra
- Individual and group differences: Race, Ethnicity, SES, Gender, Learning Disability, and Gifted students
- Brain Science and Math Learning

Methodology



- **Conclusions of the Task Group are based primarily on studies that test explicit hypothesis about the mechanisms promoting the learning of mathematics.**
- **The evidence regarded as strongest for this purpose is that which shows convergent results across procedures and study types. Conclusions are qualified when the evidence is not strong, and suggestions for research that will strengthen the ability to draw conclusions, are provided.**
- **The multiple approaches, procedures, and study types reviewed and assessed with regard to convergent results include the following:**
  - Observation (e.g., quantitative codes of children's problem solving)
  - Verbal report (e.g., of problem solving approaches)
  - Reaction time and error patterns
  - Priming and implicit measures
  - Experimental manipulation of process mechanisms (e.g., random assignment to dual task, or practice conditions)
  - Computer simulations of learning and cognition
  - Studies using brain imaging and related technologies
  - Large-scale longitudinal studies
  - International comparisons of math achievement
  - Process-oriented intervention studies

Methodology

<u>Procedures</u>

Literature search and study inclusion

- Literature searches were based on key terms linking mathematical content and learning and cognitive processes.
    - The first search focused on core peer-reviewed learning, cognition, and developmental journal.
    - A second search supplemented the first and included other empirical journals indexed in PsychInfo and the Web of Science.

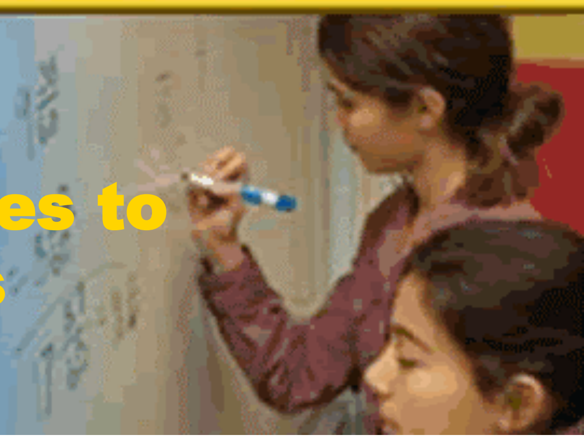
Criteria for inclusion:

- Published in English
- Participants are age 3 years to young adult.
- Published in a peer reviewed empirical journal or a review of empirical research in books or annual reviews
- Experimental, quasi-experimental or correlational methods

General Principles: From Cognitive Processes to Learning Outcomes

- There is a great deal of scientific knowledge on learning and cognition that could be applied to improve student achievement, but is not being used in the nation’s classrooms.
- Basic research on the factors that promote learning provides an essential grounding for the development and evaluation of effective educational practices.
- Inherent limits in working-memory capacity can impede proficient performance in mathematics. Practice can offset this limitation by achieving automaticity, which frees up working memory resources.

General Principles: From Cognitive Processes to Learning Outcomes



- The learning of facts, algorithms, and concepts are inter-related. Conceptual knowledge aids in the choice of algorithms; practice of algorithms can provide a context for making inferences about concepts; committing facts to long-term memory allows attention to be focused on more complex problem features.

- Conceptual understanding promotes transfer of learning to new problems and better long-term retention.

- Higher order thinking and problem solving, which presume acquisition of basic skills, are not only necessary for entry into the scientific and technical workforce, but are also becoming increasingly important for achieving success in other kinds of occupations.

- The mathematical knowledge that children from both low- and middle-income families bring to school influences their learning for many years thereafter, probably throughout their education.

General Principles: From Cognitive Processes to Learning Outcomes

- Several effective programs have been developed to improve the mathematical knowledge of preschoolers and kindergartners, especially those from at-risk backgrounds.

- Many children and adults in the U.S. do not solve simple arithmetic problems as fast and efficiently as their peers in other nations, because they have not practiced these problems frequently enough.

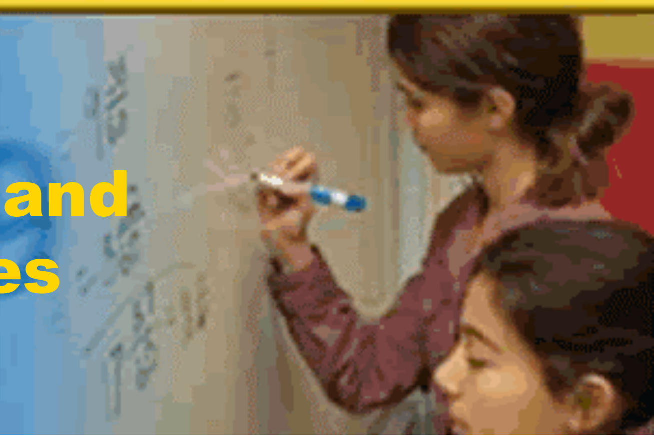
- The learning of algorithms to solve complex arithmetic problems is influenced by working memory, conceptual knowledge, degree of mastery of basic facts, and practice. Learning is most effective when practice using algorithms is combined with instruction on related concepts.

Social, Motivational and Affective Influences

Vygotsky's socio-cultural perspective

- The socio-cultural perspective of Vygotsky has been influential in education.
  - It treats learning as a social induction process through which learners become increasingly able to function independently through the tutelage of more knowledgeable peers and adults.

- Due to a shortage of controlled experiments, the usefulness of this approach for improving math learning is difficult to evaluate at this time.

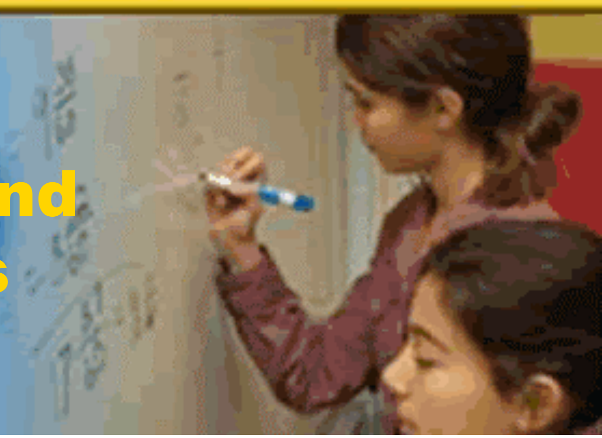


Social, Motivational and Affective Influences

- Among the factors that can influence mathematical performance above and beyond mathematical competence are:

  Self-regulation
  The ability to set goals, plan, monitor, and evaluate progress is correlated with mathematics achievement.

  Mathematics anxiety;
  Anxiety about mathematics performance lowers test scores. There are interventions that significantly reduce anxiety and improve test scores.



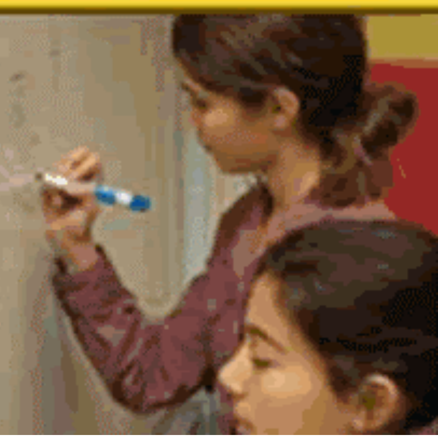
Social, Motivational and Affective Influences

<u>Intrinsic and extrinsic motivation</u>

- Young children's intrinsic motivation to learn (desire to learn for its own sake) is positively correlated with academic outcomes in mathematics.
- However, intrinsic motivation declines across grades, especially in mathematics and the sciences as material becomes increasingly complex.
- The educational environment can influence students' intrinsic motivation to learn in later grades.

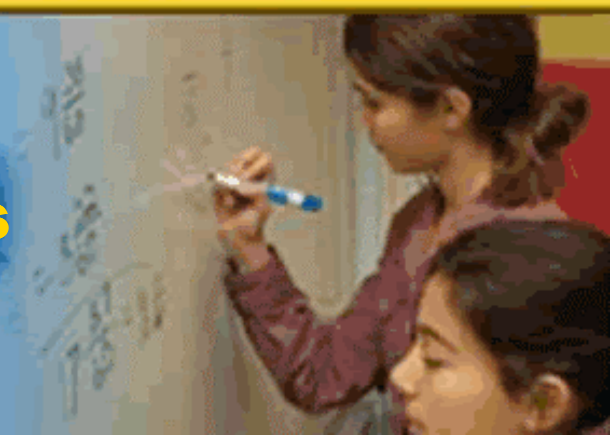
<u>Beliefs about effort versus ability</u>

- Relative to children in nations with high mathematics achievement, children in the U.S. tend to attribute mathematical achievement more to ability than effort.
- Experimental studies have demonstrated that children's beliefs about the relative importance of effort and ability can be changed, and that increased emphasis on the importance of effort is related to improved mathematics grades.



What Children Bring to School

- Most children start school with a fair amount of numerical knowledge.
- The mathematical knowledge that children from both low- and middle-income families bring to school influences their learning and mathematics achievement for many years thereafter, probably throughout their education.
- The numerical knowledge of children from low-income backgrounds lags behind even before they start school.
- Promising instructional programs exist for increasing low income preschoolers' numerical knowledge.
- Studies that evaluate the effectiveness of the scaled-up application of these programs is recommended.



Mathematical Areas

- For each of the areas reviewed, the Task Group makes recommendations organized around:
  - Classroom practices or research needed to facilitate practice
  - Training of teachers and future researchers
  - Curriculum, including content and textbooks
  - Basic and applied research

- The Task Group cannot review all of the basic findings in these areas or all of the corresponding recommendations here. The Task Group highlights core points.

- For all of the areas a pipeline of research must be funded that extends from the basic science of learning to field studies in classrooms.

Whole Number Arithmetic

- Cognitive studies indicate that many children do not master whole number arithmetic.
  - In comparison to children in many other nations, it takes U.S. children many more years to become fast and efficient at solving basic arithmetic problems.
  - They frequently make errors when using standard algorithms; error patterns suggest poor conceptual knowledge (e.g., of base-10).
  - By the end of elementary school, the majority of children do not appear to understand many basic concepts, including the distributive property and the inverse relation between division and multiplication.

Whole Number Arithmetic

- The research base for core arithmetical procedures and concepts that are crucial for learning algebra such as division algorithms and the distributive property is inadequate.

- Few curricula in the United States provide sufficient practice and a strong conceptual context for this practice. Studies of how to best organize this practice and with well-defined outcomes are needed to guide curriculum development.

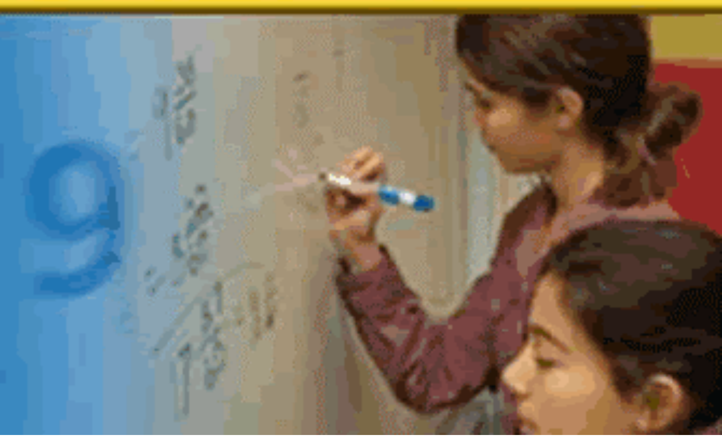
- Priorities include:
  - expanding the research base on children's learning of core concepts
  - promoting better understanding of the reciprocal relation between procedural and conceptual learning
  - development of mechanisms that facilitate the translation of basic research into knowledge usable in the classroom (e.g., for formative assessments)

Fractions

- Fractions are formally introduced in elementary school, and yet remain difficult for many adults
  - 27% of 8th graders could not correctly shade 1/3 of a rectangle (NAEP, 2005)
  - 45% could not solve a word problem involving dividing fractions (NAEP, 2004).
  - For adults, poor understanding of fractions, decimals, and proportions is associated with poor medical outcomes .

- Preschoolers show an intuitive awareness of fractions based on part-whole relations and sharing.

- Studies also show improvement in performance between ages 4 and 7 but understanding of fractions lags far behind understanding of whole numbers.



Fractions

- As with whole numbers, conceptual and procedural knowledge of fractions reinforce and bootstrap one another and influence such varied tasks as estimation, word problems, and computations.
- A key mechanism linking conceptual and procedural knowledge is the ability to represent fractions on a physical, and ultimately mental, number line.
- On-task time, motivation, working memory, well-learned basic arithmetic skills and reading ability also determine performance on fraction problems.
- Absence of a coherent and empirically supported theory of how children learn and understand fractions is a major stumbling block to developing practical interventions to improve performance in this crucial domain of mathematics.
- Instruction focusing on conceptual knowledge of fractions is likely to have the broadest and largest impact on problem-solving performance provided that it is aimed at accurate solution of specific problems.

Estimation

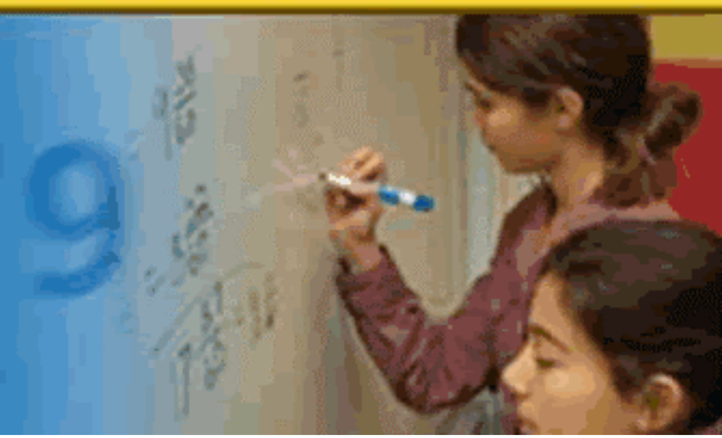
- Numerical estimation is an important part of mathematical cognition, both because it is frequently used in everyday life as well as in scientific, mathematical, and technological professions and because it is closely related to overall math achievement.

- Poor estimation performance often reveals underlying difficulties in understanding of mathematics in general.

- In some classrooms, estimation has been equated with rounding to such an extent that children do not know that the purpose of estimation is to approximate the correct value.

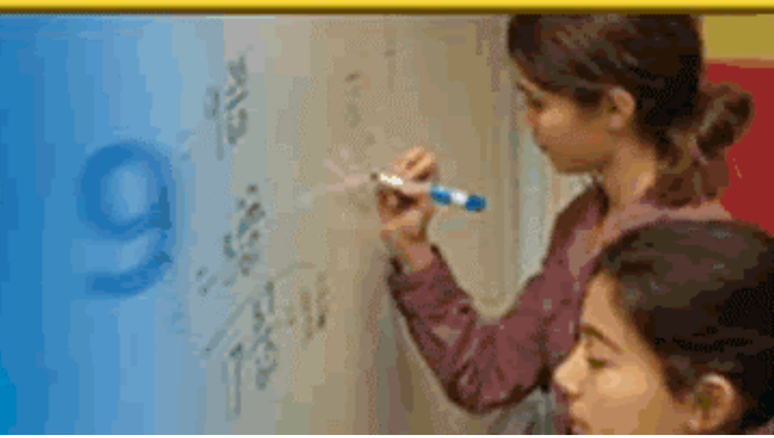
- Children’s estimation of the magnitudes of fractions is especially poor; instructional programs for helping children accurately estimate fractional magnitudes are urgently needed.

Geometry



- Of the five mathematical content areas assessed by the 2003 TIMSS (number, algebra, geometry, measurement, and data), U.S. 8th graders' performance in geometry items was weakest.
    - U.S. 8th graders exhibited no significant improvement in geometry between 1999 and 2003 on the TIMSS, despite significant gains in algebra during this period.
- In comparison to high achieving nations the U.S. devotes only about half as much time to its study of geometry.

Geometry



- The component of geometry most directly relevant for the early learning of algebra is that of *similar triangles.* However, it is difficult to draw firm, scientifically-based conclusions from the empirical research on students' acquisition of similarity and related concepts.

- Piaget theorized that the representation of space develops from topological to projective to Euclidean. The mathematical inaccuracies of this hypothesis along with the mounting negative empirical evidence suggest that it should no longer inform the design of instructional approaches in geometry.

- One of the challenges to effective learning in geometry is the persistence of misconceptions and their resistance to instruction:
  - "illusion of linearity" where students incorrectly believe that if the perimeter of a geometric figure is enlarged *k* times, its area (and/or volume) is enlarged *k* times as well

Geometry

- Young children appear to possess at least an implicit understanding of basic facets of Euclidean concepts, although formal instruction is needed to ensure that children adequately build upon and make explicit this core knowledge so they can learn formal geometry.

- Despite the widespread use of mathematical manipulatives such as geoboards, dynamic software, and so forth, evidence regarding their usefulness in helping children learn geometry is tenuous at best.

- Students must eventually transition from concrete (hands-on) or visual representations to internalized abstract representations. The crucial steps in making such transitions are not clearly understood at present.

- Studies are needed to demonstrate whether and to what extent knowledge about similar triangles enhances the understanding that the slope of a straight line is the same regardless of the two points chosen, thus leading to a mathematical understanding of linearity.

Algebra

- Cognitive studies of algebra have focused on linear equations and word problems and reveal that many students in high-school algebra classes are woefully unprepared for learning the basics of algebra.
- The errors students make when solving algebraic equations reveal many do not have a firm understanding of basic principles of arithmetic (e.g., commutativity, distributivity, laws of exponents), and many do not understand the concept of mathematical equality.
- Students have difficulty grasping the syntax or structure of algebraic expressions and do not understand procedures for transforming equations (e.g., adding or subtracting the same value from both sides of the equation) or *why* transformations are done the way they are.
- There are many gaps in our current understanding of how students learn algebra and the preparation that is needed by the time they enter the algebra classroom. Research efforts to close these gaps are recommended.

Individual and Group Differences: Learning Disabilities

- Between 5% and 10% of students will experience a significant learning disability or learning difficulty in mathematics before completing high school.
- The corresponding cognitive deficits include a compromised working memory system, and difficulties with basic concepts. These contribute to difficulties with whole number arithmetic learning.
- At the same time, much less is known about how these difficulties are related to learning fractions, estimation, geometry, and algebra.
- Funding of longitudinal and brain imaging studies that assess cognitive mechanisms underlying learning disabilities in core mathematical domains are recommended.
- Promising intervention studies are in progress and funding for additional studies is recommended.

Individual and Group Differences: Gifted Students

- The few cognitive studies of the sources of the accelerated learning of mathematically gifted students suggest an enhanced ability to remember and process numerical and spatial information.

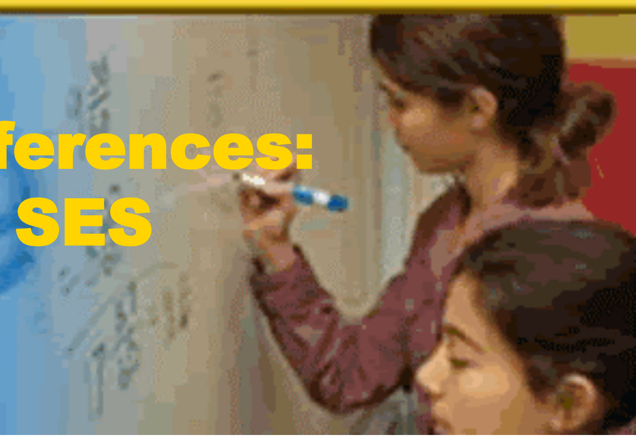
- Cognitive and brain imaging studies of the mechanisms that underlie their accelerated learning are needed to better understand how to help these students achieve their full potential.

Individual and Group Differences: Gender

- For nationally representative samples, the average mathematics scores of boys and girls are very similar; when differences are found they are small and typically favor boys.

- There are consistently more boys than girls at both the low and high ends of mathematical performance on standardized tests, though the differences at the high end have decreased significantly.

- Media attention to the overrepresentation of boys at the high end of mathematical performance has obscured the fact that relative to high achieving countries the achievement of both boys and girls in the U.S. is poor.

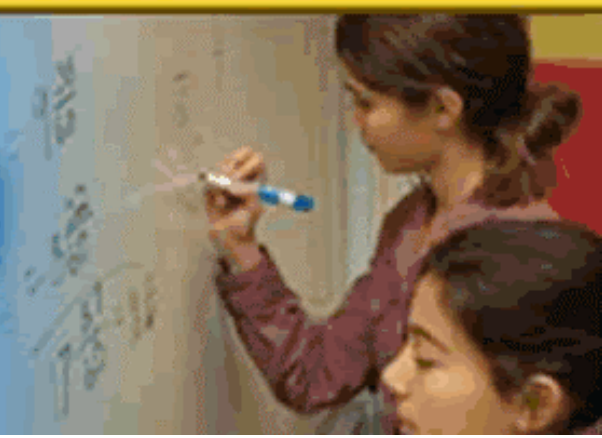


Individual and Group Differences: Race, Ethnicity and SES

In preparation

Brain Sciences and Mathematics Learning

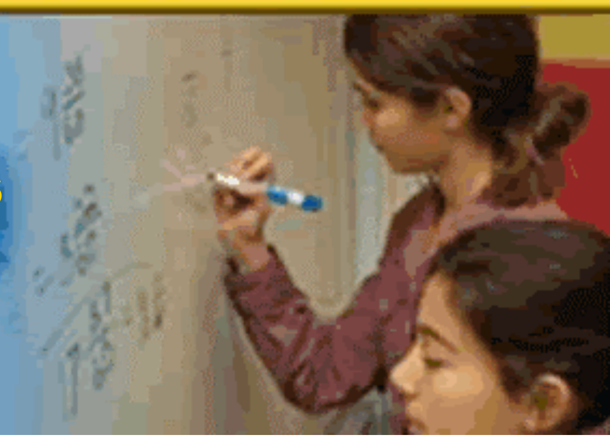
- Brain sciences research has potential for contributing unique knowledge regarding mathematical learning and cognition and for eventually informing educational practice.
- Funding of brain imaging studies that focus on children’s learning in core mathematical domains is recommended.
- At the same time, the application of research in the brain sciences to classroom teaching and student learning in mathematics is premature.
- Instructional programs in mathematics that claim to be based on brain sciences research remain to be validated.



General Conclusions

<u>Research</u>

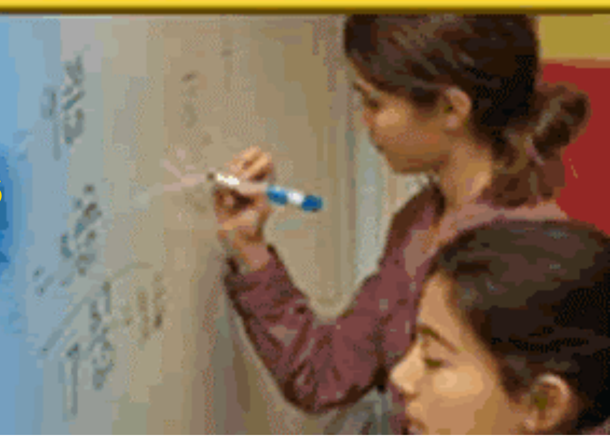
- For all areas, a pipeline of research must be funded that extends from the basic science of learning to field studies in classrooms.
- Incentives should be provided to encourage partnerships between basic and applied researchers.
- Many interventions demonstrated to be effective in experiments should be scaled up and evaluated in classrooms.
- Research is essential to discover mechanisms that contribute to emergence of formal competencies in school, including linking earlier intuitive understanding with later formal problem solving.



General Conclusions

Research

- Educational research must be integrated with basic research in cognition, motivation, neuroscience, and social psychology.
- Educationally relevant research need not be conducted in classrooms. Research conducted in laboratories under carefully controlled conditions can often be directly applied in classrooms.
- Incentives are needed to encourage more scientists to perform instructionally relevant research and to participate in research teams that will translate basic science findings into instructional interventions.
- More research is needed that specifically links cognitive, theory-driven research to classroom contexts. At the same time, cognitive research on learning needs to take into account more facets of classroom settings if it is to eventually have a greater impact on instruction.



General Conclusions

<u>Teacher Training Should Include</u>

- instruction in the scientific method and in evaluating research evidence
- comprehensive courses on contemporary cognitive science research on children's learning

<u>Curricula Should</u>

- provide sufficient time on task to ensure acquisition and long term retention of both conceptual and procedural knowledge
- be based on results from contemporary, rigorous, empirical research on learning