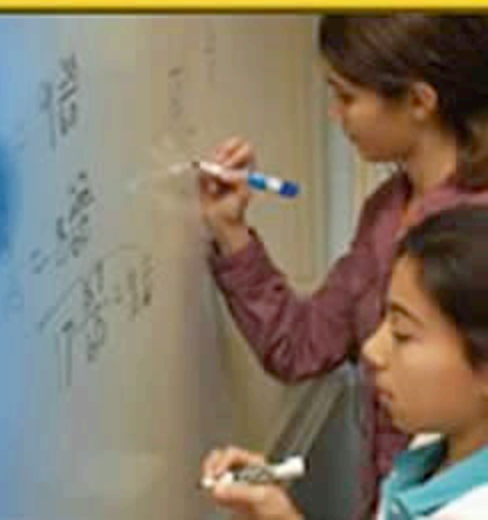
National Mathematics Advisory Panel

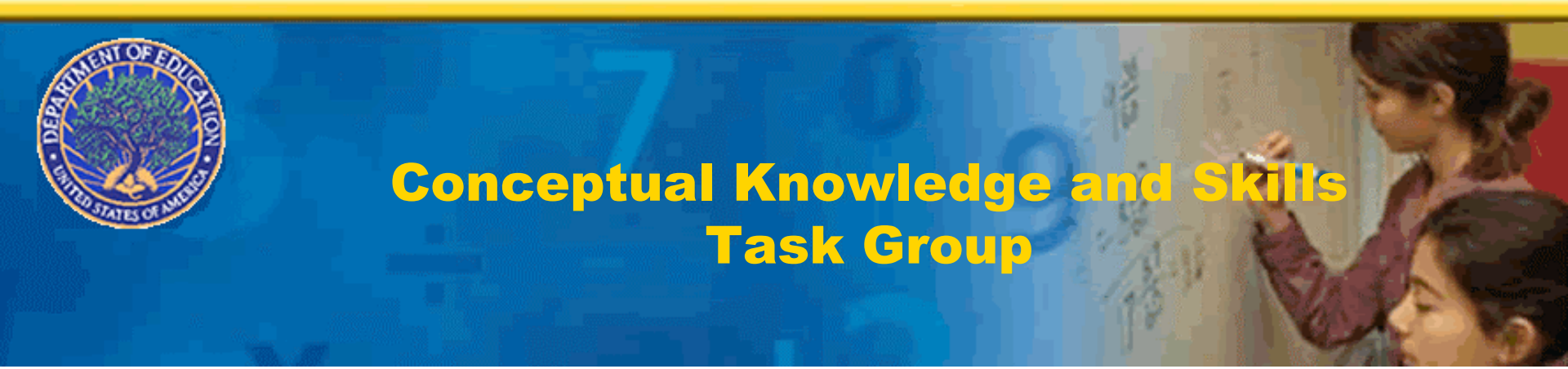
Conceptual Knowledge
&
Skills Task Group

St. Louis Meeting

Progress Report

September 7, 2007

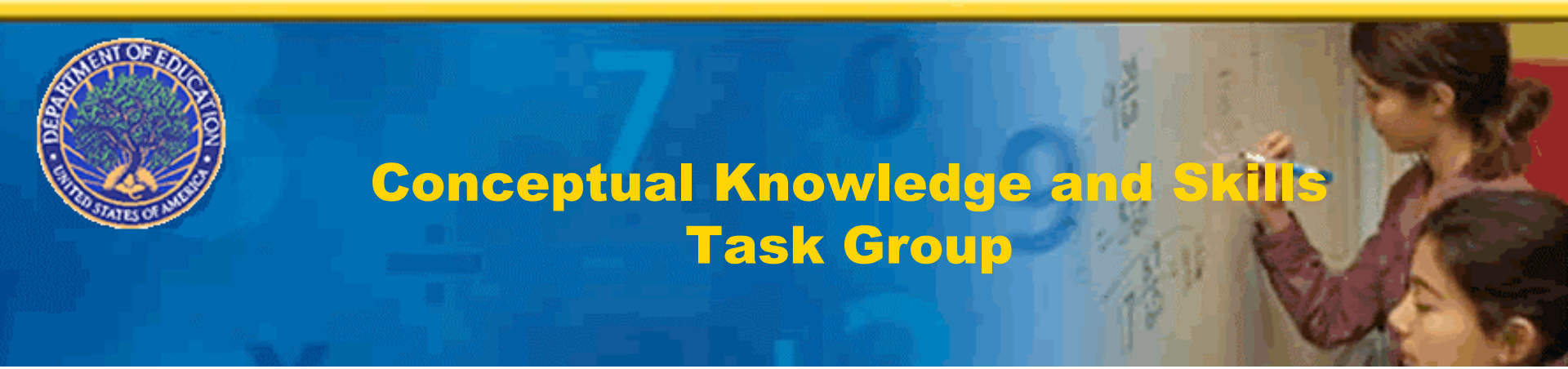




Conceptual Knowledge and Skills Task Group

Task Group Members

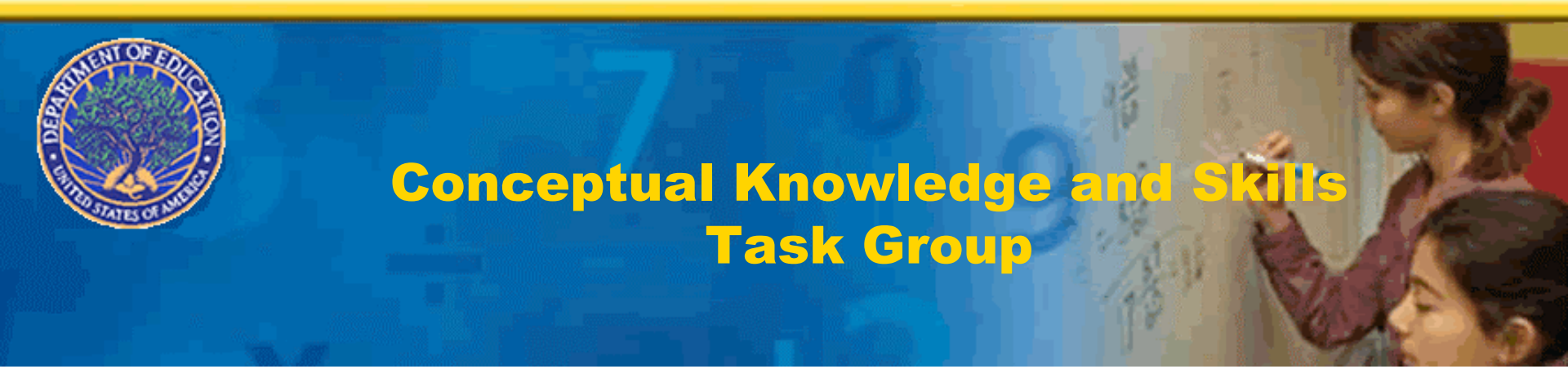
Francis (Skip) Fennell, chair
Larry Faulkner
Liping Ma
Wilfried Schmid
Sandra Stotsky
Hung Hsi Wu, ex-officio
Tyrrell Flawn, staff associate



Conceptual Knowledge and Skills Task Group

To guide its inquiry and deliberations, the Conceptual Knowledge and Skills task group formulated three major questions:

1. What are the major topics of school algebra?
2. What are the essential mathematical concepts and skills that lead to success in algebra and that should be learned as preparation for algebra?
3. Does the sequence of topics prior to formal algebra coursework or for formal algebra coursework effect achievement in algebra?

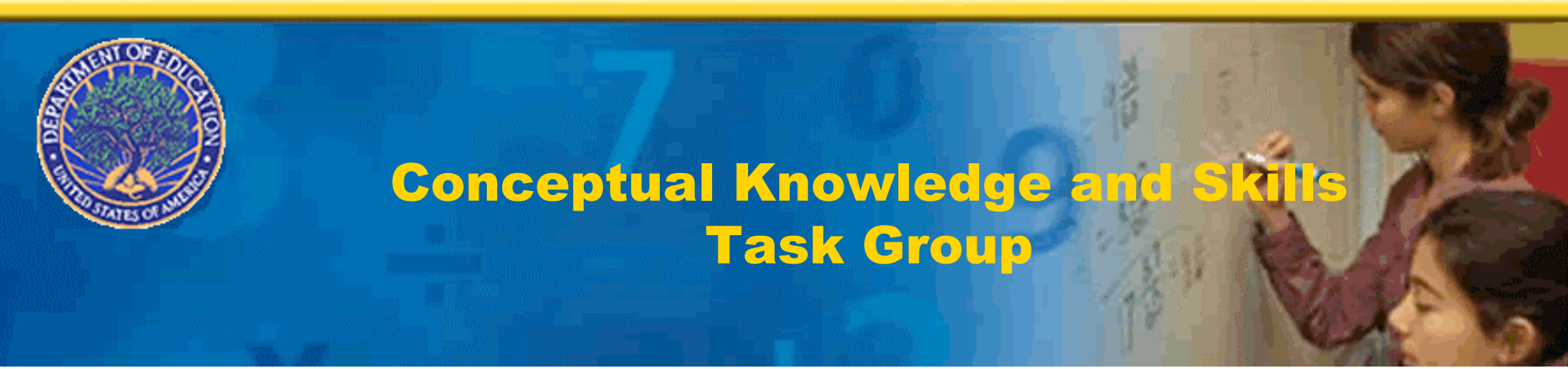


Conceptual Knowledge and Skills Task Group

Methodological Approach

•Panel charged with determining how to use "the results of research relating to proven-effective and evidence-based mathematics instruction" and making recommendations "based on the best available scientific evidence."

•Literature review yielded some peer-reviewed and published studies that met standards of methodological quality and were relevant to the work of this Task Group, especially with respect to its third question (i.e. sequence of topics prior to algebra or for algebra coursework affecting achievement in algebra).

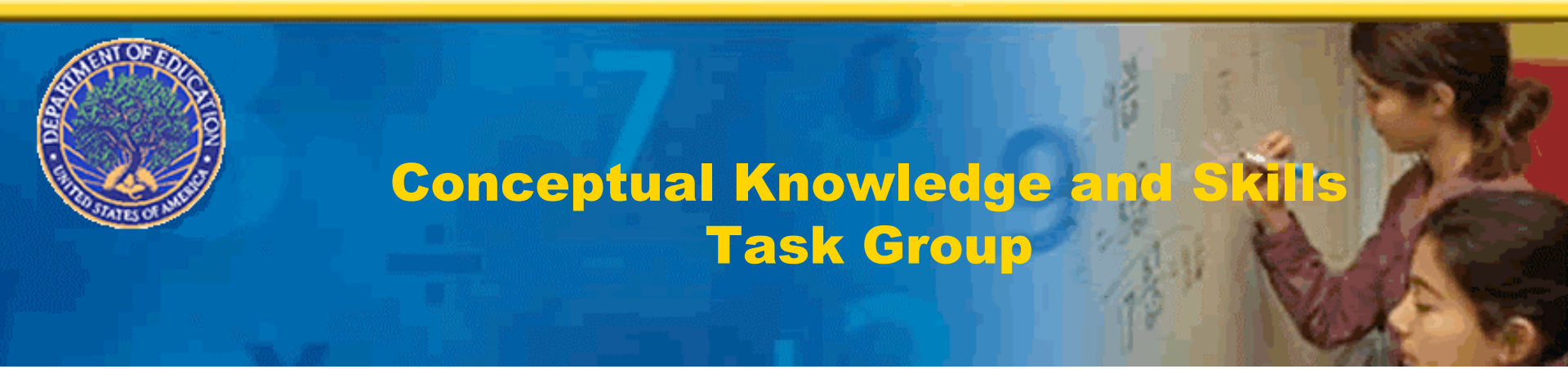


Conceptual Knowledge and Skills Task Group

Methodological Approach

•Small number of such studies; also included reports that presented the best available scientific evidence on the topic of the conceptual knowledge and skills needed for success in algebra.

•Supplemented literature review with reports by national organizations and government agencies, and with analyses and comparisons of state curriculum frameworks and school textbooks.



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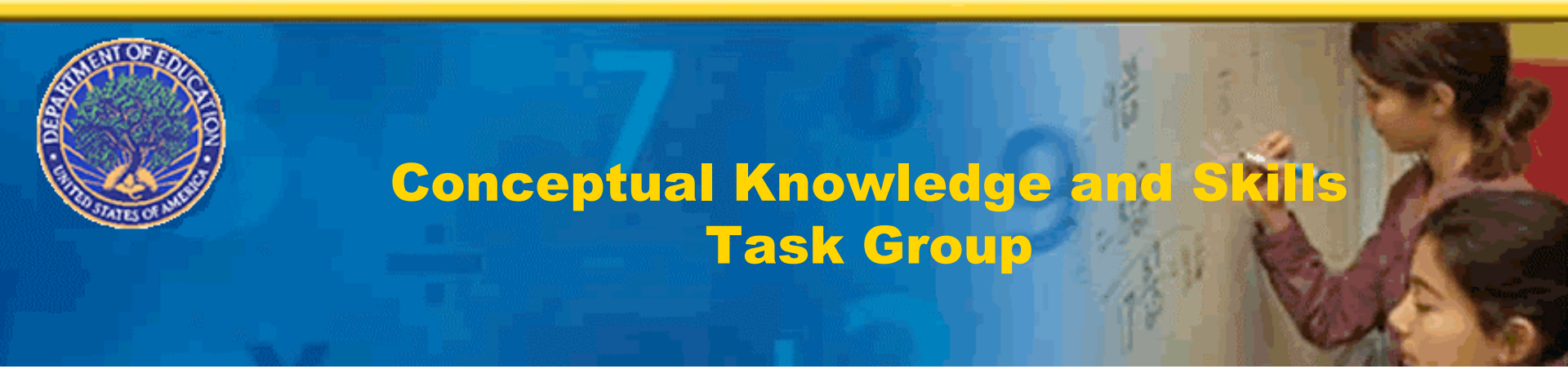
Methodological Approach

•Differences among these reports, studies, or analyses in coding categories or in the clarity of their criteria is noted.

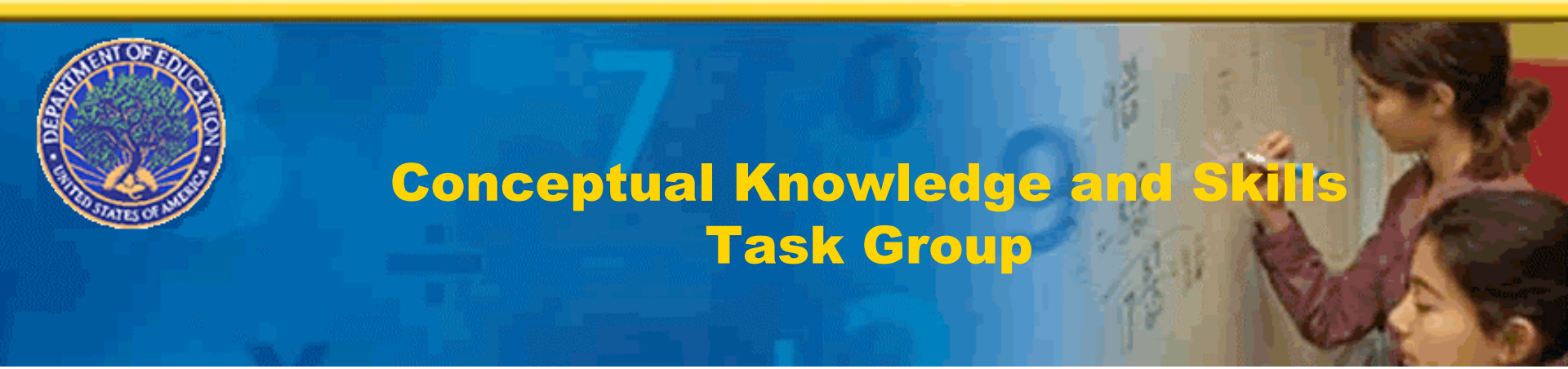
•Recommendations on matters of definition and mathematical content guided by professional judgment.



Conceptual Knowledge and Skills Task Group

The Report Outline

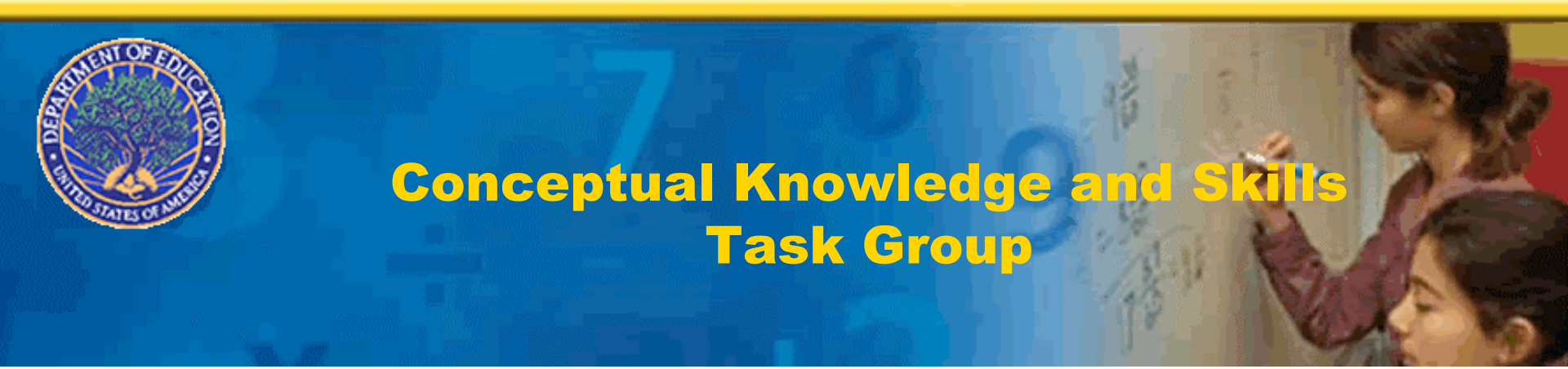
I. Introduction
II. Methodology
III. Student Achievement in Mathematics
IV. What are the Major Topics in School Algebra?
    A. The Major Topics of School Algebra
        1. Introduction to the Topics
        2. Overview of School Algebra
    B. Algebra Topics in Curriculum Sources
        1. State Standards for Algebra I and II
        2. Algebra I and II Textbooks
        3. Singapore's Mathematics Curriculum for Grades 7-10



Report Outline

IV. What are the Major Topics in School Algebra? (cont.)

- C. Algebra Topics in Assessment Sources
  - 1. National Assessment of Educational Progress Test Objectives
  - 2. American Diploma Project Benchmarks and Test Objectives
- D. Comparisons
- E. Observations Regarding Rigor

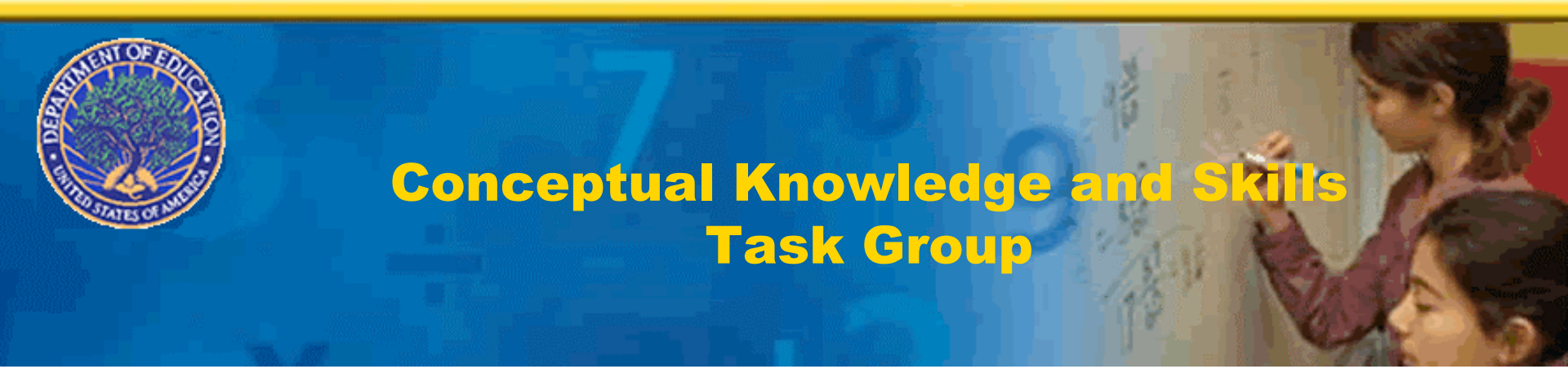


Conceptual Knowledge and Skills Task Group

Report Outline

V. What are the Essential Mathematical Concepts and Skills that Lead to Success in Algebra and Should be Learned as Preparation for Algebra?

- A. International Approaches to Pre-Algebra Education
  1. Mathematical Topics Taught in Grades 1-8 in the Top-Performing TIMSS Countries
  2. Differences in Curriculum Approaches in Top-Performing TIMSS Countries
- B. National Approaches to Pre-Algebra Education
  1. NCTM's Curriculum Focal Points
  2. Skills and Concepts in the Six Highest-Rated Curriculum Frameworks



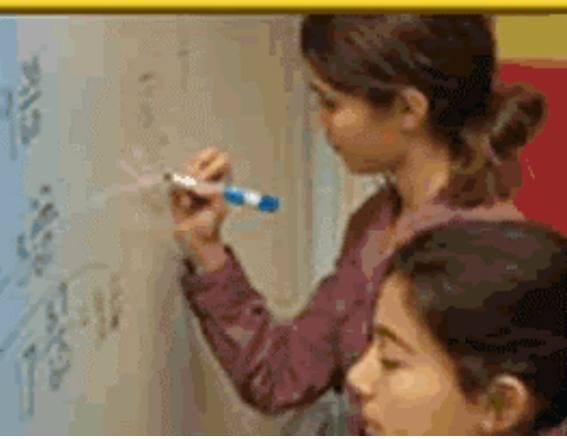
Conceptual Knowledge and Skills Task Group

Report Outline

V. What are the Essential Mathematical Concepts and Skills that Lead to Success (cont.)
- C. Surveys of What College and Secondary Teachers See as Essential Concepts and Skills
  1. Findings from the ACT Curriculum Survey
  2. Findings from the National Mathematics Advisory Panel Survey
- D. Critical Foundations for Success in Algebra

VI. Does the Sequence of Mathematics Topics Prior to and During Formal Algebra Coursework Affect Algebra Achievement?
- A. Benefits of an Integrated or Single-Subject Approach for the Study of Algebra
- B. Research on the Timing of Formal Algebra Coursework

VII. Conclusions and Recommendations



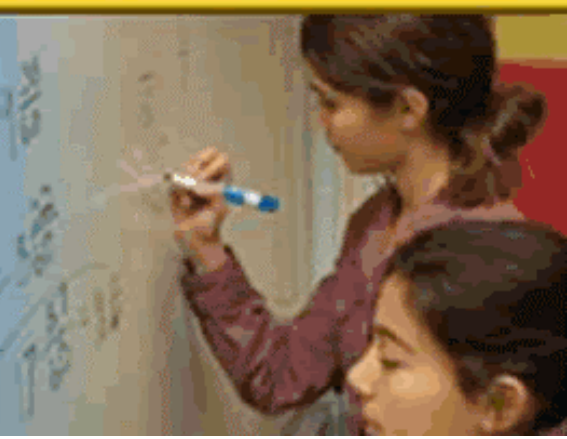
What are the major topics of school algebra?

Symbols and Expressions

- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Relations

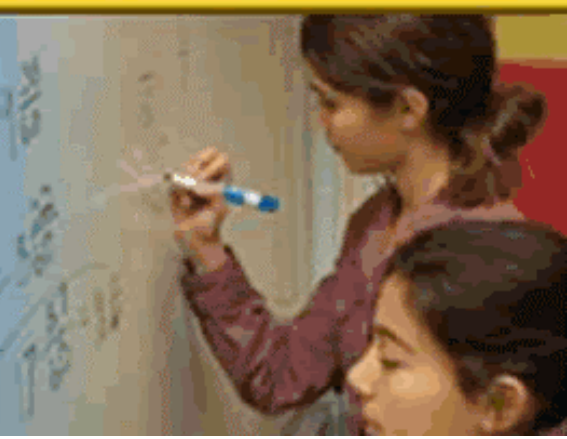
- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations



What are the major topics of school algebra? (cont.)

Quadratic Relations

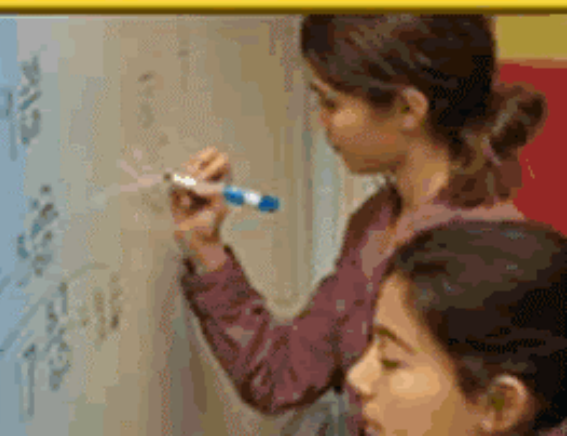
- Factors and factoring of quadratic polynomials with integer coefficients.
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations



What are the major topics of school algebra? (cont.)

Functions

- Linear functions
- Quadratic functions – word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g.. square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data



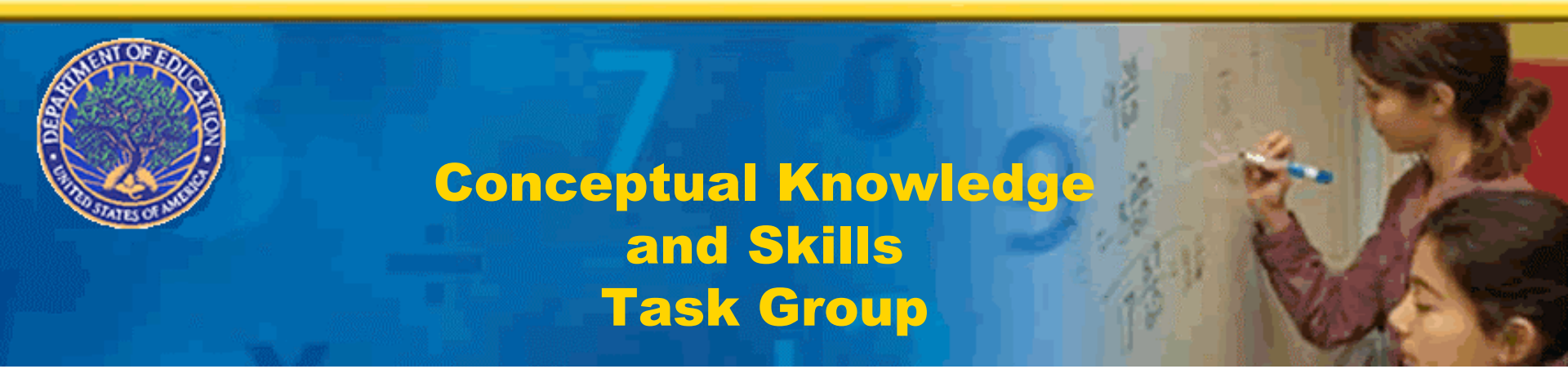
What are the major topics of school algebra? (cont.)

Algebra of Polynomials

- Roots and factorization of polynomial forms
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's triangle)
- Mathematical induction and the binomial theorem

Applications

- Combinatorics and Finite Probability



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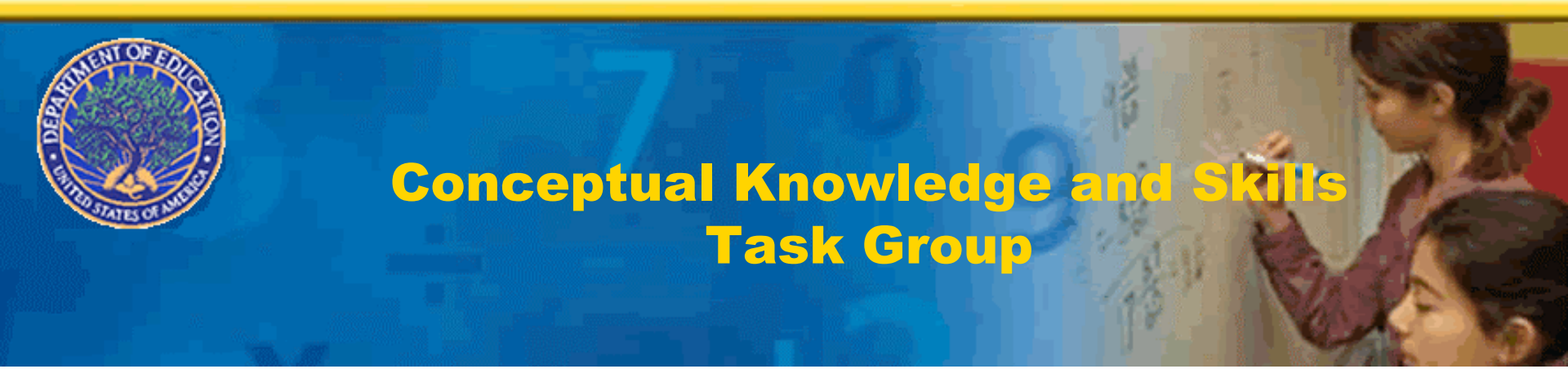
What are the essential mathematics concepts and skills that lead to success in algebra and that should be learned as preparation for formal algebra coursework?

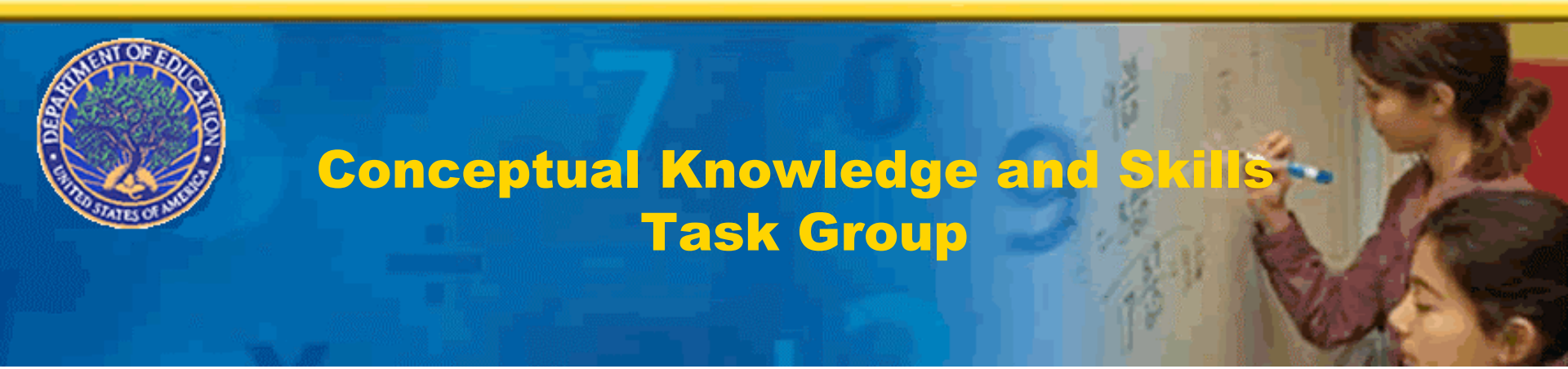
- Fluency with whole numbers
- Fluency with fractions
- Selected topics in geometry and measurement



Conceptual Knowledge and Skills Task Group

Findings and Recommendations

- The Task Group affirms that algebra is the gateway to more advanced mathematics and to most postsecondary education.
- All schools and teachers must concentrate on providing a sound and strong mathematics education to all elementary and middle school students so that all of them can enroll and succeed in algebra.
- It is much more important for our students to be soundly prepared for algebra and then well taught in algebra than to study algebra at any particular grade level.
- To improve the teaching of algebra, the Task Group proposes the following six recommendations:



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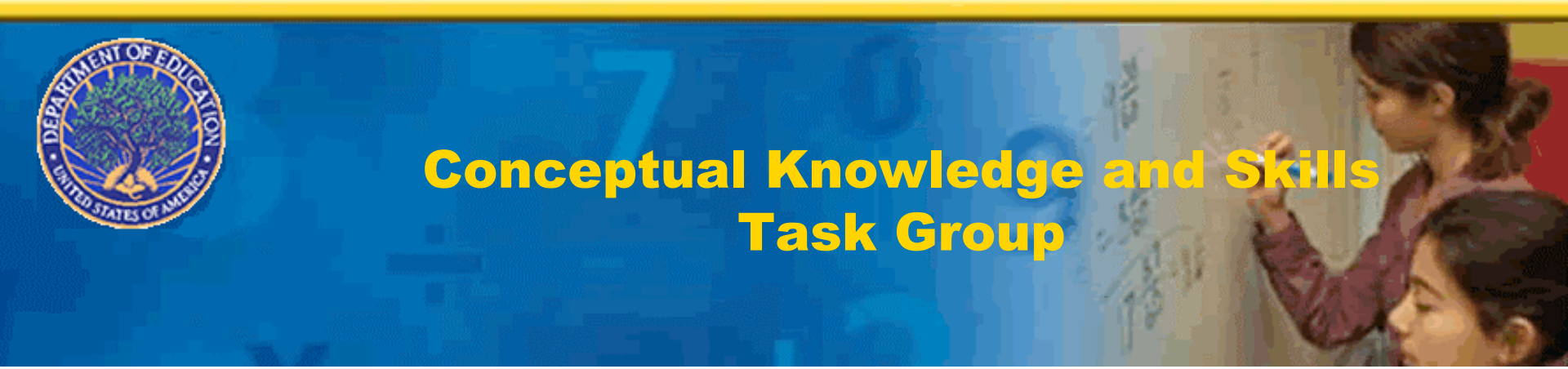
1. The Task Group recommends that school algebra be consistently understood in terms of the list of Major Topics of School Algebra (MTSA) provided in this report.



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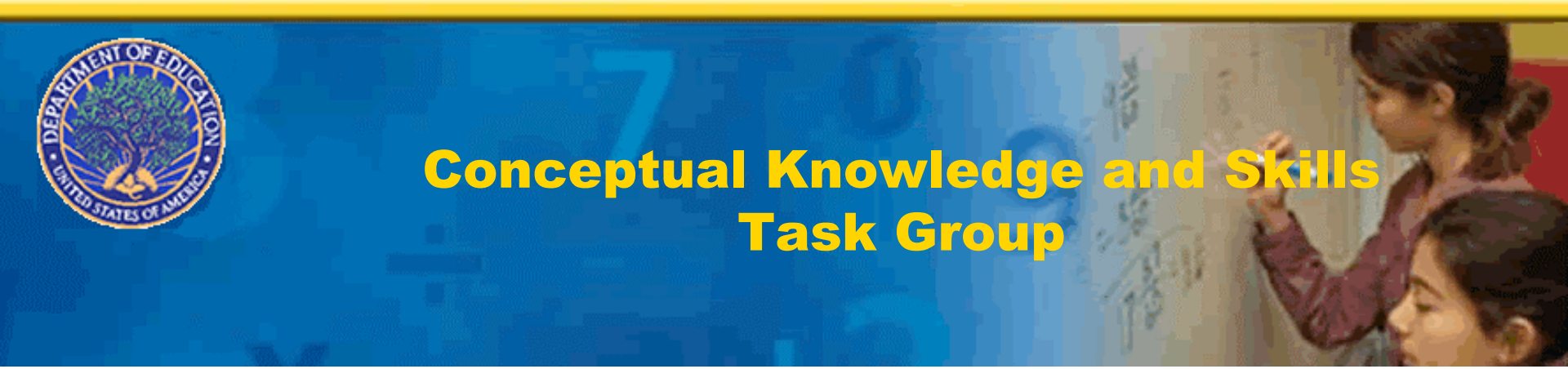
2. The list of Major Topics of School Algebra, accompanied by a thorough elucidation of the mathematical connections among these topics, should be the main focus of Algebra I and Algebra II standards in state curriculum frameworks, in Algebra I and Algebra II courses, in textbooks for these two levels of algebra whether integrated or otherwise, and in end-of-course assessments of these two levels of algebra.

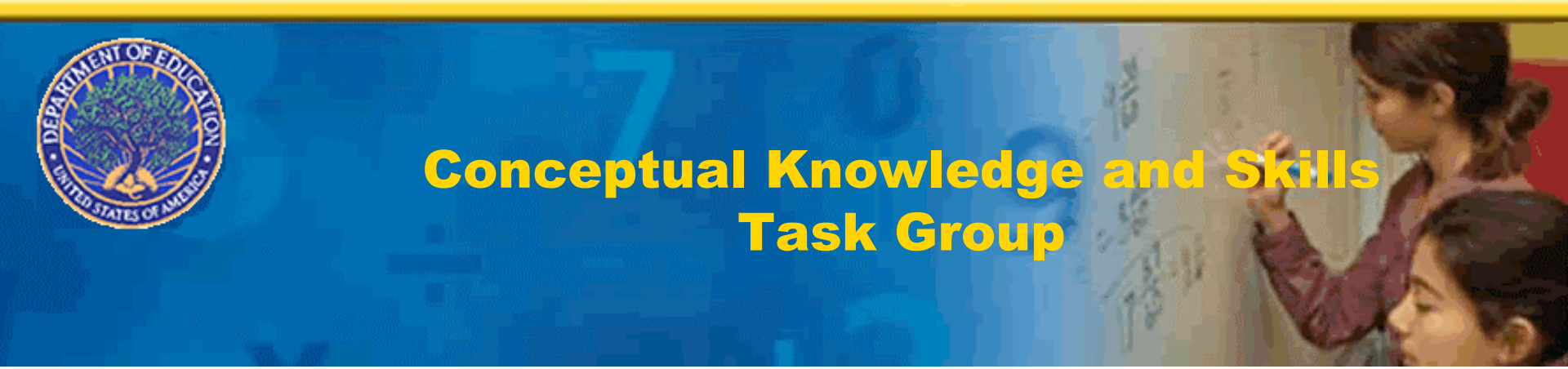
   The Task Group also recommends use of the list of Major Topics of School Algebra in revisions of mathematics standards at the high school level in state curriculum frameworks, in high school textbooks organized by an integrated approach, and in grade-level state assessments using an integrated approach at the high school, by Grade 11 at the latest.



Conceptual Knowledge and Skills Task Group

3. Proficiency with whole numbers, fractions, and particular aspects of geometry and measurement are the Critical Foundation of Algebra. Emphasis on these essential concepts and skills must be provided at the elementary and middle grade levels.

The coherence and hierarchical nature of mathematics dictate the foundational skills that are necessary for the learning of algebra. By the nature of algebra, the most important among them is proficiency with fractions (including decimals, percent, and negative fractions). The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected.



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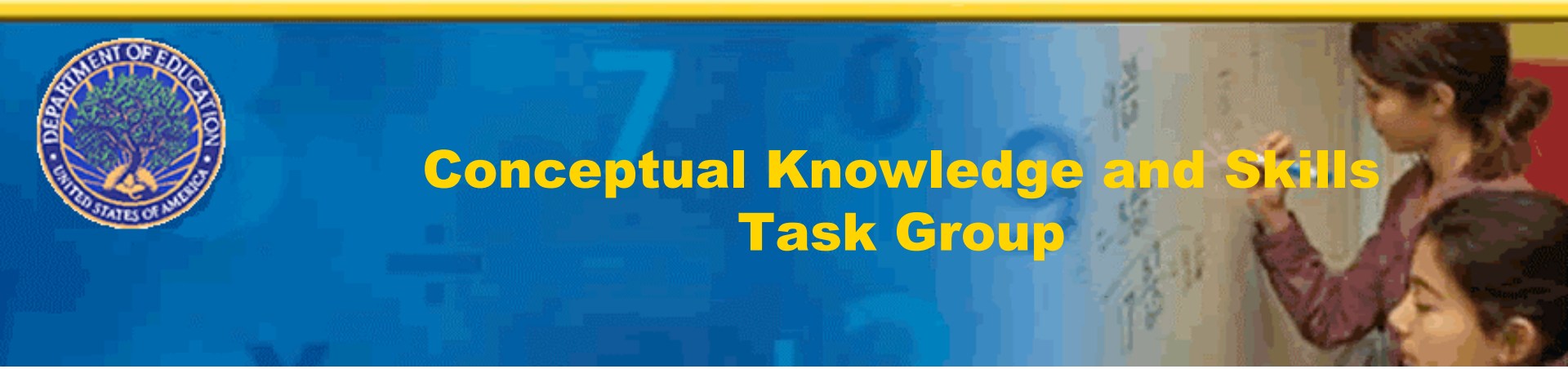
4. International studies show that high achieving nations teach for mastery in a few topics, in comparison with our mile-wide-inch-deep curriculum. A coherent progression, with an emphasis on mastery of key topics, should become the norm in elementary and middle school curricula. There should be a de-emphasis on a spiral approach that continually revisits topics year after year without closure.



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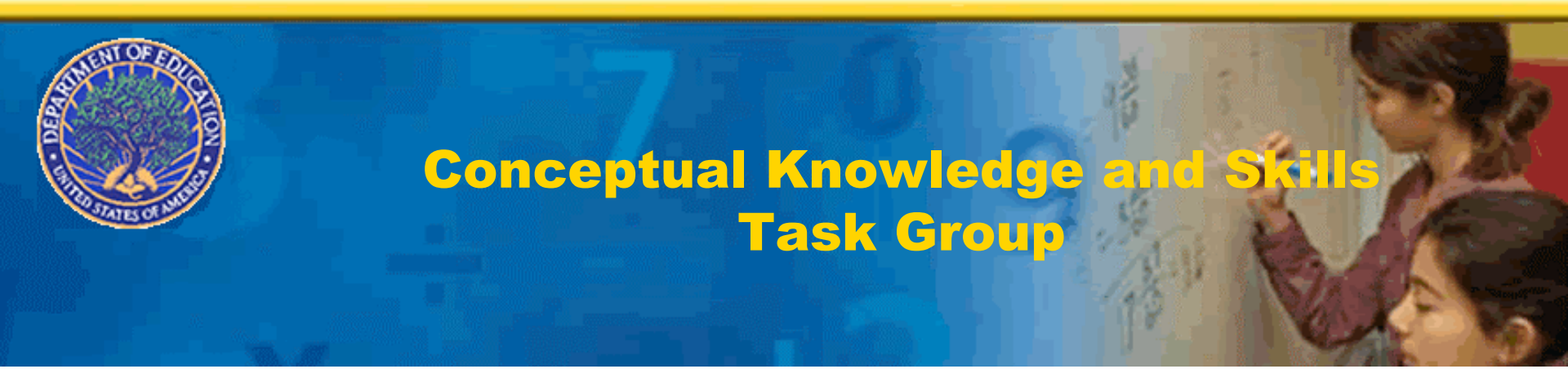
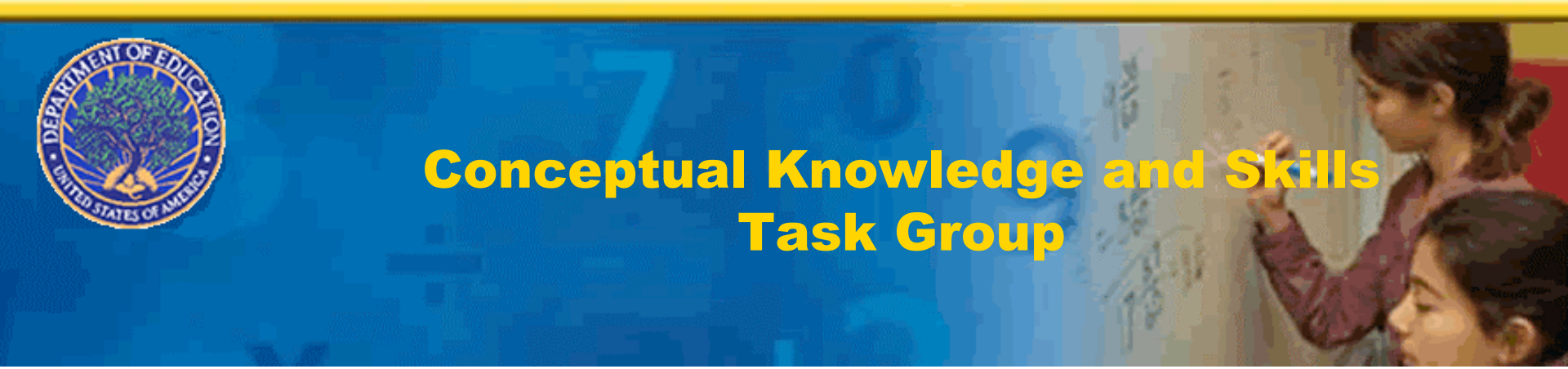
5. Federal and state policies should give incentives to schools to offer an authentic Algebra I course in Grade 8, and to prepare a higher percentage of students to enter the study of algebra by Grade 8. Care must be taken to insure that such a course addresses algebra as described in Recommendation 2, and that students be mathematically prepared for such a course in the sense of Recommendation 3.



Conceptual Knowledge and Skills Task Group

6. Publishers must ensure the mathematical accuracy of their materials. Those involved with developing mathematics textbooks and related instructional materials need to engage mathematicians, as well as mathematics educators, in writing, editing, and reviewing these materials.



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7. Adequate preparation of students for algebra requires their teachers to have a strong mathematics background. To this end, the Major Topics of School Algebra and the Critical Foundations of Algebra must be fundamental in the mathematics preparation of elementary and middle school teachers.

National Mathematics Advisory Panel

Questions?