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Correction to "A general power equation for predicting bed load transport rates in gravel bed rivers"

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1. Typographical Errors

[1] In the paper "A general power equation for predicting bed load transport rates in gravel bed rivers" by Jeffrey J. Barry et al. (*Water Resources Research*, *40*, W10401, doi:10.1029/2004WR003190, 2004), the y axis for Figures 5 and 10 was incorrectly labeled and should have read "$\log_{10}$ (predicted transport) − $\log_{10}$ (observed transport)." In addition, flow depth (D) is incorrectly shown in the denominator of equation (A9) of *Barry et al.* [2004] and should be replaced by d_i, the mean particle diameter for the ith size class as shown below:

$$F_{gri} = \frac{u^{*n}}{\left(gd_i \frac{\rho_s - \rho}{\rho}\right)^{1/2}} \left[\frac{V}{\sqrt{32}\log\left(\frac{10D}{d_i}\right)}\right]^{1-n} \quad (1)$$

[2] Similarly, equation (A24) of *Barry et al.* [2004] incorrectly includes the modal grain size from the subsurface material (d_{mss}) which should be replaced with d_{mqb}, the modal grain size of a given bed load transport observation. The correct equation is

$$\omega_0 = 5.75[(\rho_s - \rho)0.04]^{3/2}\left(\frac{g}{\rho}\right)^{1/2} d_{mqb}^{3/2} \log\left(\frac{12D}{d_{mqb}}\right) \quad (2)$$

2. Dimensions

[3] We also correct two dimensional inconsistencies in equation (6) of *Barry et al.* [2004], (1) with drainage area (A) expressed in units of m^2 rather than km^2 and (2) by scaling discharge by the 2-year flood (Q_2), which gives α constant units of kg m^{-1} s^{-1} and improves the overall performance of our bed load transport equation [*Barry et al.*, 2005]:

$$q_b = \alpha(Q/Q_2)^{\beta} = 8.13 \times 10^{-7} A^{0.49} (Q/Q_2)^{\left(-2.45q^{*}+3.56\right)} \quad (3)$$

The units of the drainage area coefficient (8.13×10^{-7}) depend on the site-specific regression between α and A; in our case, the units are kg m$^{-1.98}$ s^{-1}.

3. Sensitivity of Equation Performance

[4] We have also further tested the sensitivity of our results to selection of ε, the value that was added to bed load transport rates in order to include incorrect zero transport predictions in our log-transformed assessment of formula performance ($\log_{10}(P + \varepsilon) - \log_{10}(O + \varepsilon)$, where P and O are predicted and observed transport rates, respectively) [*Barry et al.*, 2004, section 4.1.3]. Incorrect zero predictions can occur at low to moderate flows for transport equations that contain a threshold for the onset of bed load transport (i.e., the *Meyer-Peter and Müller* [1948], *Ackers and White* [1973], and *Bagnold* [1980] equations) [*Gomez and Church*, 1989; *Habersack and Laronne*, 2002; *Barry et al.*, 2004]. We originally suggested that ε should be set equal to the lowest nonzero predicted transport rate at a given study site (1×10^{-15} kg m^{-1} s^{-1} for our analysis), recognizing that equation performance and degree of underprediction for threshold equations would be influenced by the selected ε value when those equations erroneously predict zero transport [*Barry et al.*, 2004, paragraph 34]. Initial sensitivity analyses showed that ε influenced absolute performance in terms of the magnitude of underprediction reported for threshold equations when they predicted large numbers of incorrect zero transport rates [*Barry et al.*, 2004, Figure 5], but ε did not affect relative performance amongst the transport equations [*Barry et al.*, 2004, paragraph 38].

[5] Further analyses demonstrate that significant numbers of incorrect zero predictions make the critical error, e*, a function of ε, rather than an indicator of actual formula performance (Figure 1; see caption for e* definition). This is particularly evident for the *Meyer-Peter and Müller* [1948] and *Bagnold* [1980] equations (Figure 1) because of their high number of incorrect zero predictions at our study sites [*Barry et al.*, 2004, paragraph 62]. In contrast, the *Ackers and White* [1973], *Parker* [1990] and *Barry et al.* [2004] equations predict some degree of transport at most discharges, which makes their e* values less susceptible to choice of ε (at least up to values of 1×10^{-5} kg m^{-1} s^{-1}; Figure 1). The decline in prediction error toward zero as ε increases beyond 1×10^{-5} kg m^{-1} s^{-1} is an artifact of ε becoming larger than the majority of the observed and predicted transport rates at our test sites. As ε becomes large, it masks the actual prediction error, with our assessment of formula performance effectively comparing the

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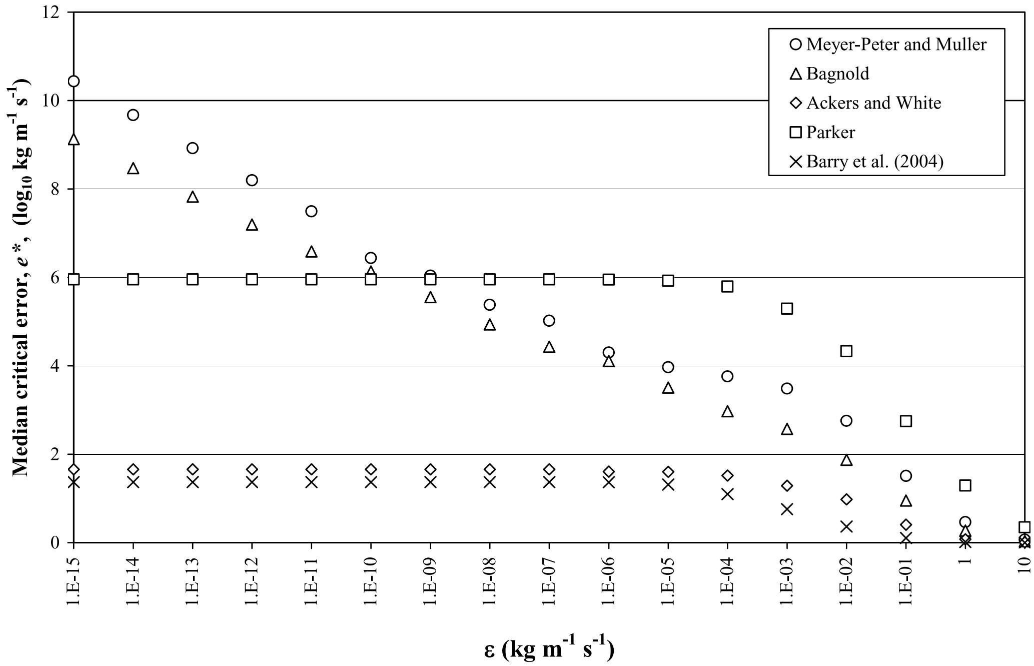

Figure 1. Sensitivity of median critical error, e^*, to changes in ε (constant added to preclude taking the logarithm of 0 when predicted transport rates are zero) at the 17 test sites. Sites are described elsewhere [*Barry et al.*, 2004, section 3]. Here e^* is the amount of error that one would have to accept for equivalence between observed and predicted transport rates using *Freese*'s [1960] χ^2 test as modified by *Reynolds* [1984], $e^* = \sqrt{\frac{1.96^2}{\chi^2}\sum_{i=1}^{n}[\log(P_i+\varepsilon)-\log(O_i+\varepsilon)]^2}$, where P_i and O_i are the ith predicted and observed transport rates, respectively; n is the number of observations; 1.96 is the value of the standard normal deviate corresponding to a two-tailed probability of 0.05; and χ^2 is the two-tailed chi-square statistic with n degrees of freedom.

logarithmic difference of two very large numbers set by the magnitude of ε, resulting in vanishingly small differences and correspondingly small critical errors ($\log_{10}(P+\varepsilon) - \log_{10}(O+\varepsilon) \Rightarrow 0$ when $\varepsilon \gg P$ and O). In summary, Figure 1 shows that the median critical errors, e^*, of the *Meyer-Peter and Müller* [1948] and *Bagnold* [1980] equations are influenced by ε regardless of its magnitude, while the selected value of ε only begins to influence the critical errors of the other equations when ε is greater than 1×10^{-5} kg m^{-1} s^{-1}.

[6] To avoid the analytical artifacts introduced by use of ε, we suggest that e^* values be determined from nonzero transport predictions (obviating the need for ε), but with results qualified by the proportion of incorrect zero predictions (Figure 2; see caption for e^* definition). Because equation performance varies with discharge [*Barry et al.*, 2004, Figure 2], we show results binned by discharge as a percentage of Q_2. Furthermore, in this analysis we use the improved version of our bed load transport equation given in section 2 above. Results show that equation (3) and the *Ackers and White* [1973] equation still outperform the others, with median critical errors near 1 across most discharge bins (Figure 2a). The performance of both the *Meyer-Peter and Müller* [1948] and *Bagnold* [1980] equations improves compared to that reported by *Barry et al.* [2004], with median e^* values typically between 3 and 4 for the *Meyer-Peter and Müller* [1948] equation and between 2 and 4 for the *Bagnold* [1980] equation. However, the frequency of incorrect zero predictions should be considered when evaluating the performance of those equations; both the *Meyer-Peter and Müller* [1948] and the *Bagnold* [1980] equations incorrectly predicted zero transport approximately 40% of the time at flows as large as 40–50% of Q_2 (Figure 2b). The performance of the *Parker* [1990] equation is essentially unchanged. The principal drawback of the approach shown in Figure 2 is that the user must select both an acceptable e^* value and an acceptable percentage of incorrect zero predictions. Nevertheless, having to do so highlights the frequently neglected error of threshold equations in terms of incorrectly predicting zero transport at low and moderate discharges.

[7] An alternative method for evaluating equation performance that includes incorrect zero predictions is to compare ratios of untransformed values of predicted versus observed transport rates (P/O) [*Gomez and Church*, 1989; *Reid et al.*, 1996; *Habersack and Laronne*, 2002]. However, this method skews the results to those equations that underpredict bed load transport. In terms of percentages, the maximum underprediction is 100% (incorrect zero prediction), while the percentage of overprediction can be infinite. Consequently, this approach creates a skewed error distribution that tends to favor equations that under predict.

[8] Another method for evaluating equation performance that does not bias results is presented by *Bravo-Espinosa et al.* [2003]. They use an inequality coefficient, U, which can vary from 0 to 1, for evaluating equation performance on the basis of untransformed values of predicted and observed

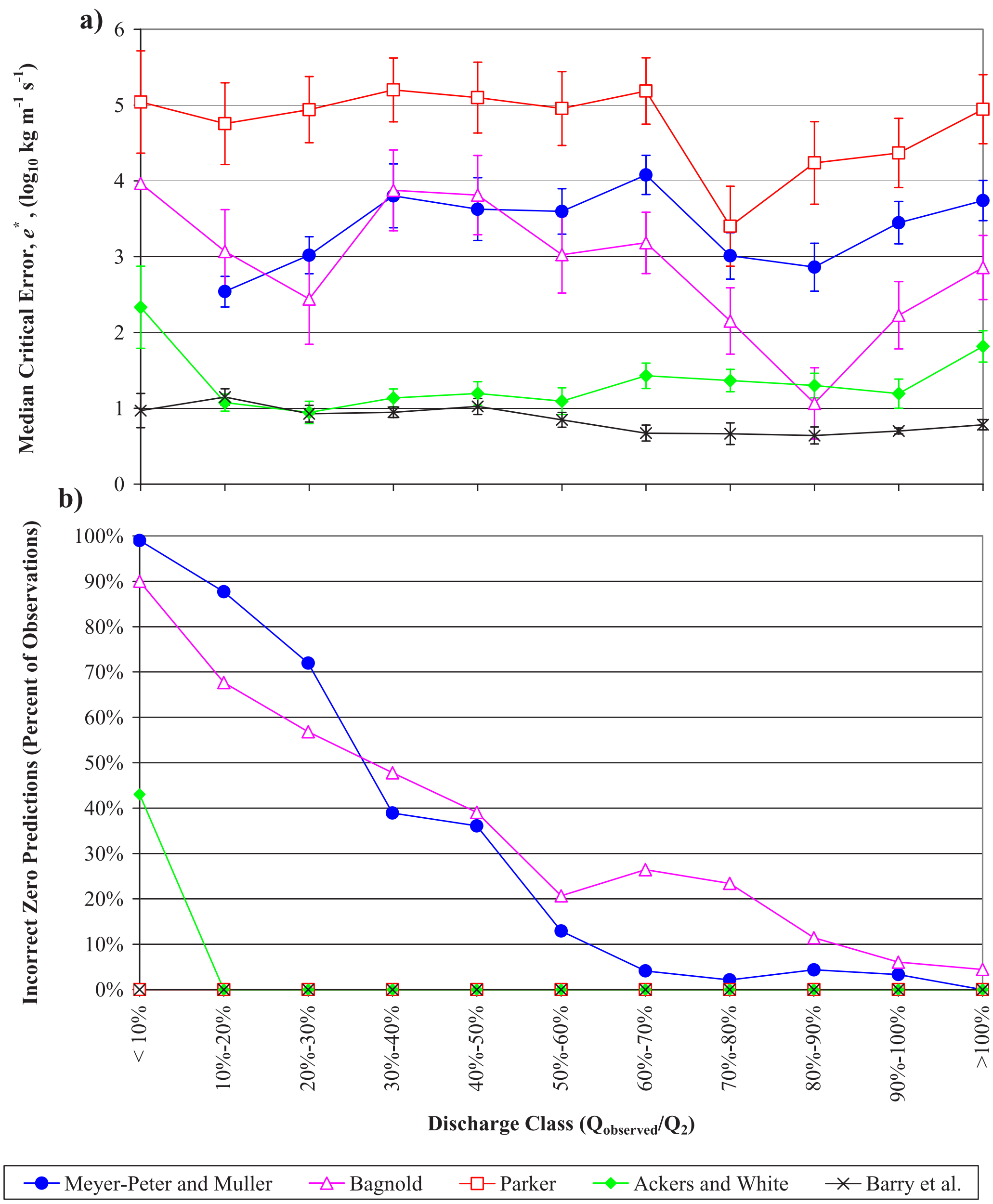

Figure 2. (a) Median critical error, e^*, for nonzero predictions of bed load transport as a function of discharge scaled by the 2-year flow, Q_2, and (b) frequency of incorrect zero predictions for same. Here $e^* = \sqrt{\frac{1.96^2}{\chi^2}\sum_{i=1}^{n}(\log P_i - \log O_i)^2}$, with parameters defined in the caption for Figure 1. Whiskers in Figure 2a indicate 95% confidence intervals around e^*. The *Meyer-Peter and Müller* [1948] equation predicted zero transport for all but one observation during flows <10% of Q_2; consequently, no median e^* value is shown in Figure 2a for those flows.

transport rates. $U = 0$ corresponds with perfect agreement between observed and predicted values, while $U = 1$ indicates a complete lack of predictive power. The authors assume that equation performance is acceptable when $U \leq 0.5$, however the basis for this value is not given and the relative significance of different U values is uncertain. For example, how much better is $U = 0.4$ versus 0.5?

[9] To our knowledge, there is no ideal method for assessing equation performance that allows for inclusion of incorrect zero predictions without either biasing results (i.e., creating a skewed error distribution) or requiring subjective qualification of results (i.e., determination of acceptable U values or acceptable percentages of incorrect zero predictions).

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