Multifactor productivity: a new BLS measure

New annual indexes for private business show that advances in the output per unit of labor and capital input account for most of the growth of output per hour of all persons during 1948–81

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The Bureau of Labor Statistics now publishes three measures of productivity: (1) the familiar index of labor productivity, which relates output to hours of all persons involved in the production process; (2) a new index of capital productivity, which relates output to capital inputs; and (3) a new index of multifactor productivity, which relates output to inputs of labor and capital.

The new annual measures help explain that, between 1948 and 1981, when private business sector output grew by 3.4 percent annually, the growth was due about equally to increases in labor and capital inputs (such as hours of all persons and plant and equipment) and to more productive use of these resources, as measured by multifactor productivity.

This article reports on the development of the multifactor and capital productivity measures and shows how the new measures can be used to analyze the long-term trend and the post-1973 productivity slowdown.

Three objectives

Unlike the familiar BLS productivity measures for the business sector, the new ones for private business exclude government enterprises. (See exhibit 1.) Each of the productivity measures has its own purposes; the multifactor productivity series has at least three. First, it is an important indicator of progress in the U.S. economy because it shows the rise in private business output obtained from a fixed quantity of resource inputs. For example, as a result of the growth in multifactor productivity, the private business sector produced 65 percent more output from a fixed amount of resource inputs in 1981 than it did in 1948,[1] the initial year of the new series.

Among a host of factors contributing to the rise in multifactor productivity were changes in technology and in the skill composition of the work force, changes in resource utilization resulting from shifts in aggregate demand, differences in effort per worker, changes in energy costs, economies of scale, and research and development expenditures.

A second, and closely related, purpose of the multifactor productivity measure is to help explain the long-term growth—and post-1973 slowdown—in output per hour of all persons (labor productivity). In effect, changes in output per hour are divided into changes in the contribution of capital services per hour (capital intensity) and changes in multifactor productivity. For example, between 1948 and 1981, output per hour of all persons in the private business sector grew at an average annual rate of 2.5 percent; the rise in capital services per hour accounted for roughly 40 percent of this growth and the gain in multifactor productivity, for the remaining 60 percent. The rate of growth of capital services per hour decelerated after 1973, helping to slow the growth rate of output per hour, but most of the sluggish advance resulted from a falloff in the growth rate of multifactor productivity.

A third purpose of the multifactor productivity measure is to help analyze cost and price movements. The Bureau regularly publishes annual and quarterly measures showing

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Exhibit 1. Productivity measures for major sectors of the economy

Measure	Inputs	Frequency	Period
Output per hour of all persons			
Business[1]	Labor	Quarterly	1947 to present
Nonfarm business	Labor	Quarterly	1947 to present
Nonfinancial corporations	Labor	Quarterly	1947 to present
Manufacturing	Labor	Quarterly	1947 to present
Durable	Labor	Quarterly	1947 to present
Nondurable	Labor	Quarterly	1947 to present
Output per unit of capital			
Private business	Capital[2]	Annually	1948 to present
Private nonfarm business	Capital[2]	Annually	1948 to present
Manufacturing	Capital[2]	Annually	1948 to present
Multifactor productivity			
Private business	Labor and capital	Annually	1948 to present
Private nonfarm business	Labor and capital	Annually	1948 to present
Manufacturing	Labor and capital	Annually	1948 to present

[1]Includes government enterprises.

[2]In constant dollars (1972).

NOTE: In 1981, business accounted for 78 percent of the gross national product in 1972 dollars; nonfarm business, 75 percent; nonfinancial corporations, 59 percent; manufacturing, 24 percent; durable goods, 14 percent, and nondurable goods, 10 percent. Private business accounted for 76 percent of the gross national product; private nonfarm business, 74 percent; and manufacturing, 24 percent.

the relationship between unit labor cost, hourly compensation, and output per hour. Unit labor cost is directly related to hourly compensation but inversely related to output per hour. Hence, increases in labor productivity help to offset rises in hourly compensation, dampening increases in unit labor cost.

There is a more comprehensive but also simple relationship between prices and multifactor productivity: The changes in the price of net output (that is, the sector's implicit price deflator) are directly related to changes in both hourly compensation and the price of capital services, but inversely related to changes in multifactor productivity.[2] Thus, increases in multifactor productivity help to offset rises in input prices so that increases in output prices are moderated.

As noted, the multifactor productivity index measures changes in output per combined units of labor and capital inputs. To construct this index, the Bureau resolved several major measurement issues.[3] These involved (1) determining the appropriate output measure, (2) establishing the maximum coverage that could be meaningfully obtained, (3) developing the appropriate capital input measure, (4) developing the appropriate labor input measure, and (5) aggregating the capital and labor inputs into a composite input measure. The formal model underlying the multifactor productivity measure is shown in the appendix.

Output measure

In general, the analysis uses a net output measure which is the value of final goods and services produced, adjusted for price change, less the value of purchased materials and services, also adjusted for price change. The output measure includes capital depreciation, as in the more familiar BLS output-per-hour indexes; it is consistent with the gross national product (GNP) concept. Is it appropriate to include capital depreciation in the output measure? Some private researchers developing multifactor productivity measures have, like the Bureau, done so, while others have not.

In deriving the multifactor productivity measures, the Bureau included capital depreciation in output, in part, for consistency with existing measures, but, more importantly, in order to have the productivity measures consistent within a framework for examining changes in prices, costs, and productivity, all of which include depreciation.

Extent of coverage

The coverage was based on two considerations: First, whether the output data available (in this case from the national income and product accounts) are measured by inputs; and, second, whether there are labor and capital input measures that correspond to the available output measures.

In some sectors of the national accounts, because of the unavailability of suitable alternatives, output is measured essentially by labor compensation, which is extrapolated by changes in employment. Because this method implies no change in productivity, such output measures are not useful for productivity measurement and were excluded from the BLS measures. The method is used primarily for the general government, households, and nonprofit institutions components of the national accounts.

For other sectors—such as rest-of-world and owner-occupied housing—the output data are derived independently of the labor input data, but there are no corresponding labor input measures available. Therefore, these sectors have also been excluded from the Bureau's productivity measures.

Government enterprises were also excluded from the multifactor productivity measures because there are no data available for measuring capital's share of output, and it would be extremely difficult to estimate.

Capital input

The capital input series attempts to measure the flow of services derived from the stock of physical assets. In the measurement of capital input, three major issues had to be addressed: (1) the definition of capital, (2) whether gross or net stock should be used, and (3) how to aggregate the stock measures.

With regard to the first issue, a broad definition including equipment, structures, land, and inventories was used. Equipment and structures were assigned to 47 asset classes to take into account differences among types of capital goods. Financial assets are presently not included.

The question of whether capital should be measured in terms of gross or net stock is a difficult empirical issue. For productivity measurement, the appropriate concept is "productive" capital stock, which represents the stock used to produce the capital services employed in current production. To measure the productive stock, it is necessary, for each type of asset, to take account of possible loss of efficiency of the asset as it ages. That is, assets of different vintages

Table 1. Productivity indexes and related measures, percent change from 1981 to 1982			
Measure	Private business[1]	Private nonfarm business[1]	Manufacturing
Productivity:			
Output per hour of all persons	-0.1	-0.1	1.2
Output per unit of capital	-5.1	-5.2	-8.4
Multifactor productivity[2]	-1.9	-1.9	-1.3
Output	-2.8	-2.8	-6.9
Inputs:			
Hours of all persons	-2.8	-2.8	-8.0
Capital services	2.4	2.5	1.6
Combined units of labor and capital input[3]	-1.0	-1.0	-5.7
Capital services per hour of all persons	5.3	5.4	10.4

[1]Excludes government enterprises.

[2]Output per unit of combined labor and capital input.

[3]Hours of all persons combined with capital service input index, weighted by labor and capital shares.

have to be aggregated. Some analysts have used measures of the gross stock, in which an asset shows no decline in efficiency until it is discarded. Others have used a net concept which shows the asset's efficiency declining as it ages. Those who have used net capital stock have assumed different age/efficiency patterns. After carefully considering the alternatives, BLS chose a concave form (slower declining efficiency during earlier years) and used available empirical evidence to confirm its shape. In addition, some members of the Bureau's Business Research Advisory Council canvassed companies they represent to confirm the "reasonableness" of using a concave form. We shall discuss the choice of an age/efficiency pattern in more detail later when we report a sensitivity analysis comparing the BLS method of measuring capital stock with methods used by others.

Finally, in combining the various types of capital stock, the weights applied were implicit rental prices of each type of asset. The implicit rental price can also be viewed as a "user cost" of capital. It reflects the implicit rate of return to capital, the rate of depreciation, capital gains, and taxes. Its use as a weight is based on the principle that capital services inputs should be combined with weights that reflect their marginal productivity—and rental price is the appropriate price.[4] The final capital input measure then is a weighted sum of the percent changes in net capital stocks by asset type. The weights are the averages of the respective rental prices for the current and past year; the measure is a Tornqvist index.

Labor input

The Bureau's measures of output per hour of all persons used in the multifactor productivity indexes are primarily derived from the Current Employment Survey and, in general, refer to hours paid. Although it would be desirable to have a measure based on hours worked, suitable historical data are not now available. We shall discuss changes in the ratio of hours at work to hours paid based on sparse information and recent BLS surveys.

Hours data for the multifactor productivity index, which are aggregated for all persons—namely, production workers, nonproduction workers, self-employed and unpaid family workers—are not differentiated in terms of the composition of the work force (age, sex, education, experience, and so on).

Aggregating capital and labor inputs

Before the overall input and hence multifactor productivity measures could be developed, the labor and capital shares for weighting the factor inputs had to be derived. Data are available for employees' labor compensation and for corporate capital income, but they are not available separately for proprietors' income. Thus, the labor share of proprietors' income had to be estimated.

Various assumptions can be made to do this. For example, production worker earnings can be imputed to the self-employed, but this frequently results in negative nonlabor proprietor income (which is obtained as a residual). Conversely, the rate of return on capital in the corporate sector can be applied to the proprietors' capital, but this frequently implies negative proprietor labor income.

In the Bureau measures, proprietor and unpaid family worker hours were assigned the same average wages received by paid employees, and capital income was measured by assigning noncorporate capital the same rental price as corporate capital. This computed value was compared with reported noncorporate income in the national income accounts, and both the labor and capital income totals were scaled to agree with those levels. With these scaled weights, labor and capital inputs were combined using the Tornqvist index number formula.

Recent developments

In 1982, the most recent year for which data are available, multifactor productivity fell 1.9 percent in the private business sector (table 1). This reflected a 2.8-percent drop in output, the largest annual decline since 1948, coupled with a 1.0-percent decrease in combined labor and capital inputs. There was a 2.4-percent rise in capital services and a 2.8-percent decline in hours, entailing a 5.3-percent increase in the amount of capital per hour.

Output per hour of all persons in the private business sector, the more familiar measure of productivity, declined only 0.1 percent compared with the 1.9-percent decrease in multifactor productivity. This difference was due to the increase in the amount of capital per hour (5.3 percent) which, when multiplied by capital's share of output, indicates that the increased capital per hour offset 1.8 percentage points of the decline in multifactor productivity. Output per unit of capital services (capital productivity) in the private business sector dropped 5.1 percent in 1982. This reflects a reduction in capacity utilization, among other things.

The percent changes in the output, input, and productivity measures in 1982 were virtually the same in private nonfarm

Table 2. Average annual rates of growth in productivity indexes and related measures by major sector, 1948 to 1981[1]
[In percent]

Measure	Private business[2]			Private nonfarm business[2]			Manufacturing		
	1948 to 1981	1948 to 1973	1973 to 1981	1948 to 1981	1948 to 1973	1973 to 1981	1948 to 1981	1948 to 1973	1973 to 1981
Productivity indexes:									
Output per hour of all persons	2.5	3.0	0.8	2.1	2.5	0.6	2.6	2.9	1.6
Output per unit of capital services	-0.1	0.2	-0.9	-0.1	0.2	-1.0	-0.2	0.6	-2.6
Multifactor productivity[3]	1.5	2.0	0.2	1.3	1.7	0.1	1.8	2.2	0.6
Output	3.4	3.7	2.3	3.5	3.9	2.2	3.4	4.0	1.3
Inputs:									
Hours of all persons	0.9	0.7	1.4	1.4	1.3	1.6	0.7	1.1	-0.2
Capital services	3.5	3.6	3.2	3.6	3.6	3.3	3.5	3.4	3.6
Combined labor and capital inputs[4]	1.8	1.7	2.0	2.1	2.1	2.2	1.5	1.8	0.8

[1]Average annual rates based on compound rate formula using data in the appendix tables.
[2]Excludes government enterprises.
[3]Output per unit of combined labor and capital input.
[4]Hours of all persons combined with capital service inputs index, weighted by labor and capital shares.

business as in the private business sector.

Multifactor productivity in the manufacturing sector decreased 1.3 percent in 1982, somewhat less than in the other two sectors. This reflected sharp decreases in both output (− 6.9 percent) and combined inputs of labor and capital (− 5.7 percent). Capital services increased only 1.6 percent, the smallest percent rise since 1972, and hours declined 8.0 percent, the largest relative decrease since 1975.

Output per hour actually increased in the manufacturing sector by 1.2 percent in 1982. This was because the increase in capital per hour (10.4 percent), when multiplied by capital's share, resulted in a 2.5-percentage-point offset to the decline in multifactor productivity. Output per unit of capital services fell 8.4 percent in manufacturing in 1982.

Table 3. Average annual rates of growth in output per hour of all persons, contribution of capital services per hour, and multifactor productivity, by major sector, 1948 to 1981[1]
[In percent]

Measure	1948 to 1981 (1)	1948 to 1973 (2)	1973 to 1981 (3)	Slowdown (4) (Col. 3– Col. 2)
Private business				
Output per hour of all persons	2.5	3.0	0.8	-2.2
Minus: Contribution of capital services per hour[2]	1.0	1.0	0.6	-0.4
Equals: Multifactor productivity[3]	1.5	2.0	0.2	-1.8
Private nonfarm business				
Output per hour of all persons	2.1	2.5	0.6	-1.9
Minus: Contribution of capital services per hour[2]	0.8	0.8	0.5	-0.3
Equals: Multifactor productivity[3]	1.3	1.7	0.1	-1.6
Manufacturing				
Output per hour of all persons	2.6	2.9	1.6	-1.3
Minus: Contribution of capital services per hour[2]	0.8	0.7	1.0	0.3
Equals: Multifactor productivity[3]	1.8	2.2	0.6	-1.6

[1]Average annual rates based on compound rate formula using data in the appendix tables.
[2]Change in capital per unit of labor weighted by capital's share of total output.
[3]Output per unit of combined labor and capital input.

Long-term trends

Productivity varies over the business cycle and, in order to measure trends, average annual rates of change are calculated between periods of peak activity in the cycle. The year 1981 is used as the last year in the comparison of long-term trends because it is the most recent peak year of a business cycle as designated by the National Bureau of Economic Research.

Table 2 summarizes average annual rates of change of the new BLS measures for the private business, private nonfarm business, and manufacturing sectors. Between 1948 and 1981, output in the private business sector, which accounted for about three-fourths of gross national product in 1981, grew at an average rate of 3.4 percent per year. Of this increase, 1.8 percentage points resulted from increases in combined labor and capital inputs, and the remaining 1.5 percentage points was due to growth of multifactor productivity.

There was a sharp slowdown in the rate of growth of output between 1948–73 and 1973–81 which coincided with an even greater slackening in multifactor productivity growth. Nearly all of the growth in output after 1973 came from increases in combined labor and capital inputs. This reflected a moderate slowdown in the annual rate of growth of capital inputs and a doubling of the rate of growth of hours of all persons between the two periods.

In private nonfarm business, multifactor productivity hardly grew after 1973; virtually all of the annual rise in output (2.2 percent) came from increases in labor and capital inputs. There was also a moderate slowdown in the annual rate of growth of capital services coupled with only a small rise in inputs of hours of all persons. The much smaller increase, after 1973, in the annual growth rate of hours of all persons in nonfarm business, compared with that for all private business, is due to a large shift of workers from the farm to nonfarm sector during 1948–73.

The picture is essentially the same in manufacturing. Over the three decades, growth in multifactor productivity and combined labor and capital inputs contributed about equally to the growth in output. And, a slowdown in the growth rate of output after 1973 was accompanied by a falloff in productivity growth. Manufacturing differed from the other two sectors in that capital services rose at a faster rate after 1973, while hours of all persons showed an absolute decline. This means that all of the growth in hours in the nonfarm business sector after 1973 occurred outside manufacturing and outside farming.

Table 2 also shows average annual rates of growth of the new BLS measures of output per unit of capital services (capital productivity). This series exhibited only a negligible downward trend, between −0.1 and −0.2 percent per year, in each of the three sectors during 1948–81. In effect, there was no saving in capital per unit of output over the three decades.

As shown in chart 1 for the private business sector, the annual movements in output per unit of capital services were largely cyclical.[5] Output per hour of all persons and multifactor productivity also exhibited cyclical patterns. Although the numbers differ somewhat, the analysis for private nonfarm business and manufacturing is essentially the same.

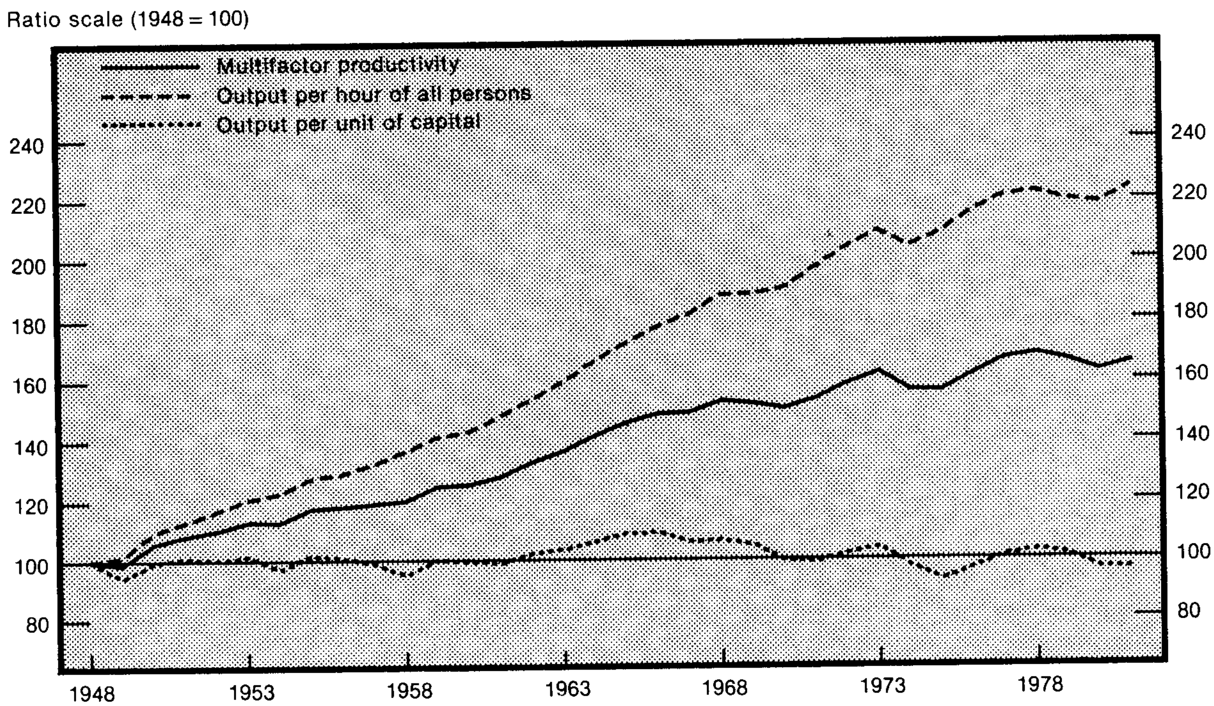
Table 3 summarizes the relationship between average annual rates of growth of output per hour, capital per hour, and multifactor productivity. In this form, it extends the Bureau's work toward explaining the growth and post-1973 slowdown in labor productivity.

From 1948 to 1981, output per hour of all persons in the private business sector grew at an average annual rate of 2.5 percent. The growth of capital services per hour contributed 1.0 percentage points to the growth in labor productivity, and multifactor productivity accounted for the balance. From 1973, after the trend rate slowed, to 1981, output per hour of all persons grew at an annual rate of 0.8 percent compared with 3.0 percent between 1948 and 1973, a falloff of 2.2 percentage points per year. There was also a slowdown in the annual rate of growth of capital services per hour. However, this contributed only 0.4 percentage point to the deceleration in labor productivity; the falloff in the rate of growth of multifactor productivity—1.8 percentage points—accounted for most of the slowdown.

The picture was essentially the same for private nonfarm business. The major share of the growth of output per hour from 1948 to 1981 was accounted for by growth in multifactor productivity; the opposite occurred after 1973, with growth in the contribution of capital services also slowing.

The experience in manufacturing differed somewhat from that in the other two sectors. In contrast to private business and private nonfarm business, capital services per hour in manufacturing grew at a faster annual rate after 1973 than before and, consequently, the slowdown in the annual rate of growth was somewhat less for output per hour than for multifactor productivity.

Chart 1. Indexes of output per hour of all persons, output per unit of capital, and multifactor productivity in the private business sector, 1948 to 1982



Some sensitivity analyses

Only about 18 percent of the slowdown in the rate of growth of output per hour in the private business sector between 1948–73 and 1973–81 can be explained by the slowdown in the growth rate of capital per hour. (See table 3.) The fraction is slightly smaller (16 percent) for the private nonfarm sector and, in the case of manufacturing, the higher rate of growth of capital per hour after 1973 helped to offset part of the multifactor productivity slowdown.

Given the importance of this result, it is useful to address the following quantitative question: How sensitive is this finding to some frequently debated measurement issues? Specifically, is the broad conclusion about the relative importance of capital to the slowdown in output per hour significantly affected by the following:

(1) the choice of terminal years after 1973;

(2) the inclusion of land, inventories, or tenant-occupied residential structures, or all, as part of the aggregate capital service measure; or

(3) the use of different age/efficiency functions in computing the productive capital stock.

Effect of changing the terminal year

In general, there are at least two considerations in selecting specific intervals when measuring productivity growth rates. First, we want a period that is long enough to "establish" a statistical trend. Second, we want to select end points which represent similar points of the economic cycle and thus minimize the effects of cyclical changes. The most common method is to select peaks of business cycles as the end points. The presumption is that labor and capital are fully—or at least about equally—used during both periods. Given these criteria, we selected the periods 1948 through 1973 and 1973 through 1981. Each of the terminal years includes a cyclical peak designated by the National Bureau of Economic Research.[6]

To examine whether the choice of a different end year would significantly affect the explanation of the productivity slowdown, we analyzed the slowdown by looking at periods varying from 1973 to 1977, 1978, 1979, 1980 and 1981. In 1981, the annual index of business output reached a peak in July. The year 1979 was also a somewhat higher year than the two earlier ones but not as high as 1981. The other three years (1977, 1978, and 1980) are included only for comparison. (See table 4.)

Table 4. Contributions to the slowdown in the annual growth rate of output per hour of all persons, by major sector, for selected periods compared with 1948–73
[In percent]

Measure	1973 to 1977	1973 to 1978	1973 to 1979	1973 to 1980	1973 to 1981
Private business					
Output per hour of all persons	-1.6	-1.8	-2.2	-2.4	-2.2
Minus: Contribution of capital services per hour	-0.2	-0.5	-0.6	-0.4	-0.4
Equals: Multifactor productivity	-1.4	-1.3	-1.6	-2.0	-1.8
Percent of slowdown:					
Capital services per hour	12	28	27	17	18
Multifactor productivity	88	72	73	83	82
Private nonfarm business					
Output per hour of all persons	-1.3	-1.4	-1.9	-2.1	-1.9
Minus: Contribution of capital services per hour	-0.1	-0.3	-0.5	-0.3	-0.3
Equals: Multifactor productivity	-1.2	-1.1	-1.4	-1.8	-1.6
Percent of slowdown:					
Capital services per hour	8	21	26	14	16
Multifactor productivity	92	79	74	86	84
Manufacturing					
Output per hour of all persons	-1.1	-1.3	-1.4	-1.6	-1.3
Minus: Contribution of capital services per hour	0.3	0.0	0.1	0.3	0.3
Equals: Multifactor productivity	-1.4	-1.3	-1.5	-1.9	-1.6
Percent of slowdown:					
Capital services per hour	-27	0	-7	-19	-23
Multifactor productivity	127	100	107	119	123

When 1979 and 1981 are used as terminal years, the slowdown in the annual growth rate of output per hour is the same—2.2 percentage points. However, for the 1973–79 period, 27 percent of the slowdown in labor productivity is attributable to a slower rate of growth in the capital-labor ratio and 73 percent to a deceleration in multifactor productivity. As previously indicated, the respective proportions based on 1973–81 are about 18 percent and 82 percent. The proportions for the other 3 years are approximately within the range of those for 1979 and 1981. These patterns are similar for the nonfarm business sector.

The story in manufacturing is somewhat different. Although there was a slowdown in the rate of growth of output per hour for each of the five periods compared, there was none during which a falloff in the growth in capital per hour was a contributing factor. In fact, in 4 of the 5 comparisons, the rate of growth of capital per hour accelerated in the later period, so that the slowdown in multifactor productivity was actually larger than that for output per hour.

Therefore, for private business and nonfarm business, there is some change in the relative importance of capital in explaining the slowdown in output per hour when the terminal year is changed from 1981 to 1979 or other years. However, in the case of manufacturing, changes in the capital-labor ratio did not contribute to the productivity slowdown in any of the five periods.

Regardless of the periods selected, the smaller growth in the capital-labor ratio never accounts for the bulk of the slowdown in output per hour and, at most, accounts for less than 30 percent, while multifactor productivity accounts for at least 70 percent. This applies to all three categories: private business, private nonfarm business, and manufacturing.

The capital services measure

The second measurement issue concerns the composition of the capital service measure. The BLS measure is designed to gauge the flow of capital services to the production process and comprises business structures and equipment, tenant-occupied residential structures, inventories, and land. Scholars working on productivity generally agree that inventories and land should be counted in capital inputs, but there is a question about how these nondepreciable assets should be combined with the depreciable ones—that is, business structures and equipment. (BLS aggregates different asset types using rental prices; the rental prices for depreciable assets include depreciation.) A question has also been raised about whether tenant-occupied structures should be included because owner-occupied dwellings are excluded.

To judge the sensitivity of the results to these questions, we excluded tenant occupied dwellings, inventories, and land individually and together from the measure of the productive capital stock. In the case of the private business sector, excluding land or inventories has only a negligible effect on the annual rates of growth of capital services per hour during both 1948–73 and 1973–81. **(See table 5.)** Excluding tenant-occupied residential structures has a larger effect on the growth rates of the capital-labor ratio, but the differences are too small to significantly affect capital's contribution to the growth rates of output per hour during the two subperiods. This is because the contribution is measured by weighting the growth in the capital-labor ratio by capital's share of output, which was about 35 percent.

The net result of these experiments for the private business sector is that changing the composition of the capital input measure would alter the contribution of the capital-labor ratio to the falloff in output per hour by no more than 0.1 percentage point. The results are the same for the private nonfarm business sector; and the earlier conclusions for manufacturing remain unchanged.[6]

The age/efficiency function

The third and last sensitivity analysis with regard to capital involves the choice of the age/efficiency function. To measure the productive capital stock, BLS used the so-called perpetual inventory method, which is simply a weighted sum of past investments. The weights are based on an age/efficiency function which describes the pattern of services derived from the capital good as it ages. Unfortunately, the best available empirical evidence does not provide a clear answer on the shape of the function. In fact, different researchers have used different forms based largely on their own observations.

BLS and some private researchers have assumed that assets lose efficiency at a slow rate early in their life and at a much faster rate as they age.[7] Other researchers assume that an asset's efficiency decreases at a constant rate throughout its life,[8] and others assume a function in which an asset loses no efficiency until the end of its life, followed by a 100-percent loss.[9] The Bureau of Economic Analysis of the U.S. Department of Commerce uses a straight-line decay function for developing its measures of capital wealth for the National Income and Product Accounts.

Table 5. Effects of excluding selected assets from published measures for private business, selected periods

[In percent]

Period	All assets[1]	All assets excluding: Land	Inventories	Residential	Residential, land, and inventories
	Contribution of capital services[2]				
1948–1981	0.9	0.9	0.9	1.0	1.1
1948–1973	1.0	1.1	1.0	1.1	1.2
1973–1981	0.6	0.6	0.6	0.7	0.8
Slowdown	-0.4	-0.5	-0.4	-0.4	-0.4
	Multifactor productivity[3]				
1948–1981	1.5	1.5	1.5	1.4	1.3
1948–1973	2.0	1.9	2.0	1.9	1.8
1973–1981	0.2	0.2	0.2	0.1	0.0
Slowdown	-1.8	-1.7	-1.8	-1.8	-1.8

[1]All assets include equipment structures, rental residential capital, inventories, and land.

[2]Rate of growth of capital services per hour weighted by capital's share of output.

[3]Output per unit of combined labor and capital inputs where the combined input is a weighted average of capital and labor (hours of all persons) inputs. The respective weights are capital's share (approximately 35 percent during the period) and labor's share (approximately 65 percent during the period).

BLS calculated the contribution of the growth of the capital-labor ratio and the growth rates of multifactor productivity under each assumption and concluded that the choice of function had very little effect on either the multifactor productivity growth rates or the contribution of capital services per hour to the growth rate of output per hour. (See **table 6.**) In fact, the differences in the annual growth rate of multifactor productivity are at most 0.1 percentage point regardless of the form of the function or the period.

In sum, selecting a different terminal year for the post-1973 productivity slowdown, changing the composition of the capital input measure, or choosing a different age/efficiency function would not significantly alter the broad findings that most of the slowdown in output per hour after 1973 is attributable to factors affecting the growth in multifactor productivity.

We should note that there is another, possibly significant, measurement issue. In the brief statement on the age/efficiency function, we observed that the BLS and all other measures of capital input for productivity analysis assume a fixed pattern of efficiency loss as assets age. Some analysts have hypothesized that the slowdown in output per hour after 1973 may have been caused by a decrease in the services of capital relative to the *measured* capital stock.[10] Presumably, the principal reason is increased obsolescence as a result of the sharp rise in oil prices in 1973 and 1979 and the shift of part of capital spending to energy-saving techniques. This hypothesis has been much debated in the literature. It is an important issue, and the Bureau has undertaken research to measure its significance.

Sources of change in multifactor productivity

As we have indicated, many factors have influenced the long-term growth and the post-1973 slowdown in the BLS measure of multifactor productivity. We will briefly review several of the more empirically manageable sources of these changes.[11] These include (1) intersectoral shifts in resources; (2) compositional changes in the workforce; (3) changes in capacity utilization; (4) growth of research and development (R&D) outlays; and (5) changes in *hours at work* relative to *hours paid*. While these factors help to explain part of the longer term annual growth rate of multifactor productivity and its falloff after 1973, the part left unexplained remains uncomfortably large.

Long-term growth. Improved allocation of labor and capital among sectors obviously results in increased multifactor productivity. The most dramatic shift during the postwar period was the movement of labor from the farm to the nonfarm sector of the economy. In 1948, the number of persons engaged in farming accounted for about 16 percent of the total number engaged in the private business sector; by 1973, the ratio had dropped to 5 percent, and by 1981, to 4 percent. In fact, the shift was virtually completed by the mid-1960's. According to BLS estimates, this reallocation of labor contributed about 0.1 percentage point to the multifactor productivity growth rate from 1948 to 1981.

The BLS measure of multifactor productivity is based on hours of all persons and assumes that their skills are homogeneous. Consequently, shifts from less to more skilled labor are not reflected in the BLS measure of labor input but, instead, are attributed to growth in multifactor productivity. The change in the composition of the labor force—particularly in higher educational attainment—has been one of the most important sources of growth in multifactor productivity between 1948 and 1981. Increases in the efficiency of an hour's work resulting from a shorter workweek, as well as increased work experience (at least as suggested by changes in the age-sex composition of the labor force) have also contributed to changes in the BLS measure of multifactor productivity. Based on estimates made by Edward F. Denison, the sum of these compositional changes—mainly increased education—contributed about 0.4 percentage point per year to the growth of multifactor productivity over the 33 years.[12]

Available information on capacity utilization for manufacturing indicates that the rates were about the same in 1948 and 1981. This at least suggests that changes in the rate of capital utilization probably did not affect the long-term trend in the BLS measure of multifactor productivity.

Technological improvements in production are generally viewed as one of the major sources of growth in multifactor productivity. Consequently, research and development have been a major area of study in connection with multifactor productivity. Judging from estimates made by Zvi Griliches for the mid-1960's and 1970's and by Nestor Terleckyj from the late 1940's to the early 1980's, R&D may have contributed between 0.2 and 0.3 percentage points to the annual growth in multifactor productivity from 1948 to 1981.[13]

The BLS series on labor inputs is based on hours paid rather than hours worked and therefore includes paid vacations and sick leave. For productivity measurement, it would be more appropriate to use an hours worked measure, but the necessary data are not now available.[14] The Bureau has experimented with varied sources of data on leave practices and so on for 1952, 1972, and 1977 to obtain a rough approximation to the trend in the ratio of hours at work to hours paid for all employees in the private nonfarm business sector. According to these rough estimates, the ratio decreased by 0.1 percent per year between 1952 and 1977. Therefore, adjusting the BLS measure of hours paid to an hours at work concept would reduce the average annual rate of growth of labor inputs by 0.1 percent per year during the 15-year period and, consequently, raise the annual rate of growth of multifactor productivity by somewhat less than 0.1 percentage point.[15] (Estimates for manufacturing suggest that the decrease in hours at work relative to hours paid was somewhat larger (−0.2 percent per year) during the same period, 1972–77, and therefore the upward adjustment in the growth rate of multifactor productivity would be somewhat more than 0.1 percentage point.)

Adding the effects of the five sources we have briefly discussed indicates that, together, they explain about 0.6-percentage point of the 1.5-percent average annual rate of growth in multifactor productivity in the private business sector during 1948–81. That is, these measured factors explain about 40 percent of the long-term rise in multifactor productivity—about 60 percent remains unexplained.

The post-1973 slowdown. The measured sources account for an even smaller fraction of the post-1973 multifactor productivity slowdown. As indicated, the shift of workers out of farming had virtually come to an end by 1965 and this contributed 0.2 percentage points to the productivity slowdown after 1973. Compositional changes in the labor force occurred at about the same rate before and after the slowdown and consequently were not a contributing factor. There was a slowdown in the rate of growth of R&D during the 1970's and this could have been a factor, but probably did not contribute more than 0.1 percentage points. And, using hours paid rather than hours at work in measuring hours of all persons could have contributed another 0.1 percentage point to the measured productivity slowdown.

The effects of these four sources, taken together, account for 0.4 percentage points—or about 22 percent—of the 1.8-percent-per-year falloff in multifactor productivity growth in the private business sector between 1948–73 and 1973–81. Data are not available for measuring changes in capacity utilization for private business but, judging from an analysis of manufacturing, changes in the rates of capacity utilization could account for a significant proportion of the multifactor productivity slowdown in private business after 1973. Even with this additional adjustment, the percentage left unexplained would probably still be large.

Table 6. Sensitivity of multifactor productivity measure, and the contribution of the capital-labor ratio to output per hour for selected age/efficiency functions in private business

[In percent]

Year	BLS (Hyperbolic)	Hulten/Wykoff (Best geometric approximation)[1]	Gross (One-hoss-shay)	Straight line
	Multifactor productivity			
1948–1981	1.5	1.6	1.5	1.5
1948–1973	2.0	2.0	2.0	1.9
1973–1981	0.2	0.3	0.2	0.3
Slowdown	−1.8	−1.7	−1.8	−1.6
	Contribution of capital services per hour			
1948–1981	1.0	0.9	1.0	1.0
1948–1973	1.0	1.0	1.0	1.1
1973–1981	0.6	0.5	0.6	0.5
Slowdown	−0.4	−0.5	−0.4	−0.6

[1] Charles R. Hulten and Frank C. Wykoff, "The Measurement of Economic Depreciation," in Charles R. Hulten, ed., *Depreciation, Inflation and the Taxation of Income from Capital* (Washington, The Urban Institute Press, 1981), pp. 81–125.

Summary

As we pointed out in the beginning, the new BLS measures of capital service inputs and multifactor productivity extend the Bureau's work in measuring the causes of the growth of labor productivity and its slowdown after 1973. The major conclusions at this stage are that, between 1948 and 1981, about two-fifths of the growth of output per hour of all persons in the private business sector resulted from increases in the amount of capital per hour used in production and about three-fifths came from the growth of multifactor productivity, or economic progress. Although the growth rate of capital per hour slowed between 1948–73 and 1973–81, most of the labor productivity deceleration reflected a falloff in multifactor productivity growth.

These findings virtually prescribe the Bureau's future research in this area. It includes trying to determine whether the method of measuring capital stock has tended to overstate its growth, particularly after 1973, because of unaccounted-for increases in obsolesence rates due to the sharp rises in energy prices in 1973 and 1979. The Bureau is also attempting to measure the sources of growth and the slowdown of multifactor productivity, including the sources we have discussed. And, in addition, BLS is constructing multifactor productivity measures at the two-digit Standard Industrial Classification (SIC) level in manufacturing which will relate gross output to inputs of energy, other purchased materials, and purchased services, as well as to inputs of capital services and labor. These disaggregated measures will make it possible to measure the direct and indirect effects of changes in energy and other materials prices on the growth and slowdown of multifactor productivity.[16] □

FOOTNOTES

[1] Part of the increase in output per unit of combined capital and labor inputs in the private business sector reflects gains from resources employed in other sectors of the economy. These include, for example, resources used by government and nonprofit institutions for education and training programs. The Bureau of Labor Statistics presently treats education of the work force as a source of growth of multifactor productivity. The Bureau is currently developing measures showing the compositional changes in the labor force that reflect, among other things, the resources used in education and training. These will be used to adjust the hours series in order to obtain a more comprehensive measure of labor input.

[2] Technically speaking, the relationship between the price of net output, factor prices, and multifactor productivity is the "dual" of the relationship between net output, labor and capital service inputs, and multifactor productivity.

[3] The methodology and sources of data underlying the measures of productivity are discussed in detail in *Trends in Multifactor Productivity, 1948–81*, Bulletin 2178 (Bureau of Labor Statistics, 1983).

[4] Dale W. Jorgenson and Zvi Griliches, "The Explanation of Productivity Change," *The Review of Economic Studies*, July 1967, pp. 249–83.

[5] Changes in the BLS measures of output per unit of capital services were closely correlated with changes in the Federal Reserve Board index of capacity utilization in manufacturing. For 1948–81, the correlation coefficient was 0.90.

[6] The choice of these terminal years was also based on an analysis of BLS quarterly data on output per hour of all persons. For the detailed discussion, see *Trends in Multifactor Productivity*.

[7] The BLS calculations for private nonfarm business and for manufacturing are reported in *Trends in Multifactor Productivity*. See also Edward F. Denison, *Accounting for Slower Economic Growth* (Washington, The Brookings Institution, 1979); and *Capital Stock Estimates for Input-Output Industries: Method and Data* (Bureau of Labor Statistics, 1979). These estimates were mainly developed by Jack Faucett Associates.

[8] Barbara Fraumeni and Dale Jorgenson, "The Role of Capital in U.S. Economic Growth, 1948–76," in George M. von Furstenberg, ed., *Capital Efficiency and Growth* (Cambridge, Mass, Ballinger Publishing Co., 1980).

[9] John Kendrick and Elliot Grossman, *Productivity in the United States* (Baltimore, Md., The Johns Hopkins University Press, 1980).

[10] Martin Neil Baily, "Productivity and the Services of Capital and Labor," *Brookings Papers on Economic Activity*, Vol. 1, 1981, pp. 1–66; and E. R. Berndt and D. O. Wood, "Engineering and Econometric Interpretations of Energy-Capital Complementarity," *American Economic Review*, June 1979, pp. 342–54.

[11] For a more detailed discussion of factors affecting the BLS measure of multifactor productivity, see *Trends in Multifactor Productivity*. For analyses of possible sources contributing to the productivity growth and slowdown besides those discussed in this section, see Edward F. Denison, "The Interruption of Productivity Growth in the United States," *Economic Journal*, March 1983, pp. 1–22, and references cited there.

[12] Edward F. Denison has kindly made his estimates through 1981 available to us. For a discussion of his methodology in arriving at these estimates, see Edward F. Denison, *Accounting for United States Economic Growth, 1929–69* (Washington, The Brookings Institution, 1974).

[13] Zvi Griliches, "R&D and the Productivity Slowdown," *American Economic Review*, May 1980, pp. 343–48; and Nestor E. Terleckyj, "R&D, Innovation and the Economy: What do Economists Know?" Remarks delivered at the White House Conference on Productivity, held in San Diego, Calif., July 20, 1983.

[14] The BLS started a survey in 1981 which collects statistics on hours at work, and this will make it possible in the future to adjust the hours measure to a more appropriate one. At the time of this writing, the survey data for 1982 are being processed. An article showing the findings and the methodology will be published in the *Monthly Labor Review*.

[15] The contribution of the decline in the ratio to multifactor productivity growth is measured by multiplying labor's share of total output (0.65) by the annual rate of decline in the ratio of hours at work to hours paid.

[16] Dale W. Jorgenson, "Energy Prices and Productivity Growth," in Jerome M. Rosow, ed., *Productivity Prospects for Growth* (New York, Van Nostrand Reinhold Co., 1981), pp. 35–53; and E. R. Berndt and D. O. Wood, "Engineering and Econometric Interpretations of Energy-Capital Complementarity."

APPENDIX: The multifactor productivity model

As indicated in the text, the BLS multifactor productivity measure includes capital in addition to labor inputs. It also incorporates recent theoretical developments in productivity measurement using an index number framework based on a fairly flexible form of the production function.

The production function underlying the multifactor productivity measure assumes Hicks' neutral technical change and constant returns to scale (which is used later in the analysis). The general form of the function can be written as,[1]

$$(1) \qquad Q(t) = A(t) f [K(t), L(t)]$$

where,

$Q(t)$ = *real net output at time t;*
$K(t)$ = *input of capital services at time t;*
$L(t)$ = *input of labor services at time t; and*
$A(t)$ = *index of Hicks' neutral technical change or multifactor productivity at time t.*

Differentiating (1) with respect to time, t, and with some algebraic manipulations, the derived "sources of growth" equation (with t omitted) is,[2]

$$(2) \qquad \frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \left[\left(\frac{\partial Q}{\partial K} \frac{K}{Q} \right) \frac{\dot{K}}{K} + \left(\frac{\partial Q}{\partial L} \frac{L}{Q} \right) \frac{\dot{L}}{L} \right]$$

where a dot over the variable indicates the derivative of the

Table A-1. Productivity and related measures in private business, 1948–82[1]
[1977 = 100]

Year	Productivity: Output per hour of all persons	Productivity: Output per unit of capital	Productivity: Multifactor productivity[2]	Output[3]	Inputs: Hours of all persons[4]	Inputs: Capital[5]	Inputs: Combined units of labor and capital inputs[6]	Capital per hour of all persons
1948	45.3	99.0	60.0	36.8	81.3	37.2	61.3	45.7
1949	46.0	93.5	59.3	36.1	78.6	38.6	60.9	49.2
1950	49.7	98.6	63.6	39.5	79.5	40.1	62.1	50.4
1951	51.2	100.1	65.1	41.8	81.8	41.8	64.3	51.1
1952	52.9	99.3	66.3	43.2	81.8	43.5	65.2	53.2
1953	54.6	100.6	68.0	45.1	82.6	44.9	66.4	54.3
1954	55.6	96.2	67.7	44.3	79.8	46.1	65.5	57.7
1955	57.8	100.9	70.7	47.9	82.9	47.5	67.8	57.3
1956	58.5	100.0	70.9	49.2	84.2	49.2	69.4	58.5
1957	60.0	97.9	71.6	49.7	82.9	50.7	69.4	61.2
1958	61.8	94.3	72.0	48.9	79.0	51.9	67.9	65.6
1959	63.9	99.3	74.9	52.5	82.1	52.9	70.0	64.4
1960	64.8	98.5	75.4	53.3	82.2	54.1	70.7	65.8
1961	67.0	98.0	76.9	54.2	80.9	55.3	70.5	68.4
1962	69.6	101.2	79.7	57.2	82.2	56.6	71.8	68.8
1963	72.2	102.6	82.0	59.7	82.7	58.2	72.9	70.4
1964	75.3	105.2	84.9	63.3	84.0	60.2	74.5	71.6
1965	78.0	107.8	87.6	67.6	86.7	62.7	77.2	72.4
1966	80.4	108.0	89.3	71.3	88.7	66.0	79.9	74.5
1967	82.3	104.9	89.6	72.9	88.6	69.5	81.4	78.5
1968	85.1	105.5	91.7	76.7	90.1	72.7	83.7	80.7
1969	85.3	103.7	91.2	78.9	92.5	76.1	86.5	82.3
1970	86.1	98.5	90.2	78.3	90.9	79.4	86.8	87.4
1971	89.2	98.1	92.2	80.6	90.4	82.2	87.5	91.0
1972	92.3	101.0	95.2	86.0	93.2	85.2	90.4	91.5
1973	94.7	103.0	97.5	91.8	96.9	89.1	94.1	92.0
1974	92.4	96.5	93.8	89.9	97.2	93.1	95.8	95.8
1975	94.5	92.0	93.6	88.0	93.1	95.7	94.0	102.8
1976	97.6	96.1	97.1	93.7	95.9	97.5	96.5	101.6
1977	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1978	100.6	101.8	101.0	105.5	104.9	103.6	104.4	98.8
1979	99.3	100.3	99.7	107.9	108.6	107.5	108.2	99.0
1980	98.8	95.5	97.7	106.4	107.7	111.4	108.9	103.4
1981	101.2	95.8	99.3	109.8	108.4	114.6	110.5	105.7
1982	101.1	90.9	97.4	106.6	105.4	117.3	109.4	111.3

[1]The private business sector includes all of gross national product except the rest-of-world sector, the rental value of owner-occupied real estate, the output arising in nonprofit organizations, the rental value of real estate occupied by nonprofit organizations, the output of paid employees of private households, government, and the statistical discrepancy in preparing the national income accounts. The private nonfarm business sector also excludes farms but includes agricultural services.

[2]Output per unit of combined labor and capital inputs.

[3]Gross Domestic Product originating in the sector, in constant dollars.

[4]Paid hours of all employees, plus the hours of proprietors and unpaid family workers engaged in the sector.

[5]A measure of the flow of capital services used in the sector.

[6]Hours of all persons combined with capital input, using labor and capital shares of output as weights.

SOURCE: Output data are from Bureau of Economic Analysis (BEA), U.S. Department of Commerce, and the Federal Reserve Board. Compensation and hours data are from the Bureau of Labor Statistics, and BEA. Capital measures are based on data supplied by BEA and the U.S. Department of Agriculture.

Table A-2. Productivity and related measures in private nonfarm business, 1948–82[1]

[1977 = 100]

Year	Productivity: Output per hour of all persons	Productivity: Output per unit of capital	Productivity: Multifactor productivity[2]	Output[3]	Inputs: Hours of all persons[4]	Inputs: Capital[5]	Inputs: Combined units of labor and capital inputs[6]	Inputs: Capital per hour of all persons
1948	51.2	97.9	64.6	35.6	69.6	36.4	55.2	52.3
1949	52.3	92.7	64.2	34.9	66.8	37.7	54.5	56.4
1950	55.6	98.2	68.1	38.3	69.0	39.0	56.3	56.6
1951	56.6	100.4	69.5	40.8	72.2	40.7	58.8	56.3
1952	58.0	99.6	70.4	42.2	72.8	42.4	60.0	58.2
1953	59.0	100.8	71.4	44.1	74.7	43.7	61.7	58.5
1954	59.9	96.1	71.0	43.2	72.1	44.9	60.9	62.3
1955	62.3	100.9	74.1	46.8	75.1	46.4	63.2	61.8
1956	62.5	100.0	74.0	48.1	77.0	48.1	65.1	62.5
1957	63.6	98.0	74.3	48.7	76.6	49.7	65.6	64.9
1958	65.1	94.0	74.3	47.8	73.4	50.8	64.3	69.3
1959	67.4	99.5	77.5	51.6	76.6	51.9	66.6	67.7
1960	67.9	98.4	77.6	52.3	77.0	53.2	67.4	69.0
1961	70.0	98.0	78.9	53.3	76.1	54.4	67.5	71.4
1962	72.5	101.3	81.7	56.4	77.8	55.7	69.0	71.6
1963	74.9	102.7	83.8	58.9	78.6	57.4	70.3	73.0
1964	77.8	105.6	86.7	62.7	80.5	59.4	72.3	73.7
1965	80.3	108.2	89.2	67.0	83.5	62.0	75.1	74.2
1966	82.2	108.7	90.7	71.0	86.4	65.3	78.3	75.6
1967	83.8	105.3	90.7	72.5	86.5	68.9	79.9	79.6
1968	86.6	106.0	92.9	76.4	88.2	72.1	82.3	81.7
1969	86.4	104.1	92.1	78.7	91.1	75.6	85.4	83.0
1970	86.8	98.6	90.6	77.8	89.7	78.9	85.9	88.0
1971	89.7	98.0	92.4	80.1	89.3	81.8	86.7	91.5
1972	93.0	101.1	95.7	85.8	92.2	84.8	89.7	92.0
1973	95.3	103.2	97.9	91.7	96.2	88.8	93.6	92.3
1974	92.9	96.5	94.1	89.7	96.6	93.0	95.4	96.3
1975	94.7	91.7	93.6	87.6	92.5	95.6	93.6	103.4
1976	97.8	96.1	97.2	93.6	95.7	97.4	96.3	101.8
1977	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1978	100.6	101.9	101.1	105.7	105.1	103.7	104.6	98.7
1979	99.0	100.1	99.4	108.0	109.0	107.9	108.6	99.0
1980	98.3	95.2	97.3	106.4	108.2	111.7	109.4	103.2
1981	100.2	95.0	98.4	109.3	109.0	115.1	111.1	105.5
1982	100.2	90.0	96.6	106.2	106.0	118.0	110.0	111.2

NOTE: See table A-1 for sources and footnotes.

variable with respect to time $\left(\text{i.e., } \dot{Q} = \dfrac{dQ}{dt}\right)$.

Equation (2) shows the rate of change of output as the sum of (a) the rate of change of multifactor productivity, $\left(\dfrac{\dot{A}}{A}\right)$, and (b) a weighted average of the rates of change of capital and labor inputs, the terms in brackets. Conceptually, multifactor productivity indicates the changes in output resulting from shifts of the production function whereas the terms in brackets measure changes in output resulting from movements along the production function (that is, from increases in combined capital and labor inputs).

The terms in brackets that measure the movements along the production function have a straightforward interpretation: the first term in parenthesis, $\left(\dfrac{\partial Q}{\partial K}\dfrac{K}{Q}\right)$, is the elasticity of output with respect to the input of capital services, that is, the percent change in output per 1-percent change in the input of capital service. This is multiplied by the percent change in capital input, $\dfrac{\dot{K}}{K}$, so that the product, $\left(\dfrac{\partial Q}{\partial K}\dfrac{K}{Q}\right)\left(\dfrac{\dot{K}}{K}\right)$, is simply the percent change in output resulting from the relative increase in capital services—holding labor inputs constant. The interpretation of the terms for labor input shown in the brackets is the same as that for capital services. Thus, the sum of the terms in brackets measures the contribution of changes in both capital service and labor inputs to changes in output. It shows the change in output that would be realized if there were no change in multifactor productivity.

Transferring the term for the relative change in multifactor productivity in (2) to the lefthand side of the equation, we have,

$$(3) \quad \frac{\dot{A}}{A} = \frac{\dot{Q}}{Q} - \left[\left(\frac{\partial Q}{\partial K}\frac{K}{Q}\right)\frac{\dot{K}}{K} + \left(\frac{\partial Q}{\partial L}\frac{L}{Q}\right)\frac{\dot{L}}{L}\right]$$

In this expression, multifactor productivity can be seen as a measure of economic progress; it shows the rate of growth in output in excess of the increases simply due to increases in labor and capital inputs. This is the first major purpose of the multifactor productivity measure referred to in the introduction.

Table A-3. Productivity and related measures in the manufacturing sector, 1948–82[1]
[1977 = 100]

Year	Productivity			Output[3]	Inputs			
	Output per hour of all persons	Output per unit of capital	Multifactor productivity[2]		Hours of all persons[4]	Capital[5]	Combined units of labor and capital inputs[6]	Capital per hour of all persons
1948	45.1	93.9	56.1	35.8	79.4	38.1	63.8	48.0
1949	46.9	85.6	55.9	33.9	72.4	39.6	60.7	54.8
1950	49.4	94.5	59.9	38.6	78.2	40.9	64.6	52.3
1951	51.1	99.2	62.2	43.0	84.2	43.4	69.2	51.5
1952	52.0	95.5	62.2	44.5	85.4	46.6	71.5	54.5
1953	52.9	98.4	63.5	47.5	89.8	48.3	74.8	53.8
1954	53.7	89.0	62.2	44.1	82.1	49.6	70.9	60.4
1955	56.4	95.6	65.8	48.9	86.6	51.1	74.2	59.0
1956	56.0	92.4	64.8	49.2	87.9	53.3	76.0	60.6
1957	57.1	89.5	65.1	49.5	86.5	55.3	76.0	63.9
1958	56.9	80.4	62.8	45.2	79.4	56.2	72.0	70.8
1959	59.6	89.1	67.0	50.5	84.7	56.7	75.4	66.9
1960	60.0	88.0	67.0	50.7	84.4	57.5	75.6	68.2
1961	61.6	86.9	68.0	50.7	82.3	58.3	74.6	70.9
1962	64.3	92.9	71.5	55.1	85.6	59.2	77.0	69.2
1963	68.9	98.3	76.3	59.6	86.5	60.7	78.2	70.1
1964	72.3	102.3	79.8	63.9	88.4	62.4	80.0	70.6
1965	74.5	107.3	82.8	69.8	93.6	65.1	84.3	69.5
1966	75.3	108.6	83.7	75.1	99.8	69.2	89.8	69.3
1967	75.3	101.1	81.8	75.0	99.6	74.2	91.7	74.5
1968	78.0	101.1	83.7	79.1	101.4	78.2	94.4	77.1
1969	79.3	100.5	84.6	81.7	103.1	81.3	96.6	78.9
1970	79.1	91.8	82.3	77.0	97.3	83.9	93.6	86.2
1971	83.9	92.3	86.0	78.7	93.7	85.2	91.5	90.9
1972	88.2	99.8	91.1	86.2	97.8	86.4	94.7	88.3
1973	93.0	108.2	96.8	95.9	103.2	88.6	99.1	85.9
1974	90.8	99.6	93.0	91.9	101.2	92.2	98.8	91.1
1975	93.4	89.4	92.2	85.4	91.4	95.5	92.6	104.4
1976	97.5	96.1	97.1	93.6	95.9	97.4	96.4	101.5
1977	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1978	100.8	101.5	101.0	105.3	104.5	103.8	104.3	99.3
1979	101.5	99.5	101.0	108.2	106.6	108.8	107.2	102.1
1980	101.7	90.7	98.7	103.5	101.8	114.1	104.8	112.1
1981	105.3	90.2	101.2	106.5	101.2	118.0	105.2	116.7
1982	106.5	82.7	99.9	99.1	93.0	119.9	99.2	128.8

NOTE: See table A–1 for sources and footnotes.

The assumption of constant returns to scale means that the weights (that is, the elasticities) in brackets sum to unity. Using this, we can obtain the important relationship,

$$(4) \quad \left(\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L}\right) = \frac{\dot{A}}{A} + \left[\left(\frac{\partial Q}{\partial K}\frac{K}{Q}\right)\left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right)\right]$$

This expression shows that the rate of change of labor productivity, $\left(\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L}\right)$, is equal to the sum of the rate of change of multifactor productivity, $\frac{\dot{A}}{A}$, and the contribution of the change in capital per hour (capital intensity) to output, where the contribution is measured by the elasticity of output with respect to the input of capital services, $\left(\frac{\partial Q}{\partial K}\frac{K}{Q}\right)$, times the rate of change of capital services per hour, $\left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right)$. This relationship helps to explain the growth and post-1973 slowdown of labor productivity, the second major purpose of multifactor productivity measurement noted in the introduction.

The elasticities, or weights, in equations (2) through (4) are not observable and, in order to estimate these, it is necessary to make the further assumption that the marginal products of capital and labor are equal to their respective real market prices. This is equivalent to assuming a competitive economy operating at long-run equilibrium. Thus, it is assumed that,

$$(5) \quad \frac{\partial Q}{\partial K} = \frac{C}{P} \text{ and, } \frac{\partial Q}{\partial L} = \frac{W}{P}$$

where,

P = *price of net output;*
C = *rental price of capital services; and*
W = *price of labor services.*

Substituting the expressions in (5) for the marginal productivities in the elasticity equations yields the capital and labor shares, S_K and S_L, respectively.

$$(6) \quad S_K = \frac{CK}{PQ}, \text{ and } S_L = \frac{WL}{PQ}$$

where, $S_K + S_L = 1$.

Equations (2) through (4) can now be written as:

$$(2') \qquad \frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \left[S_K \frac{\dot{K}}{K} + S_L \frac{\dot{L}}{L}\right]$$

$$(3') \qquad \frac{\dot{A}}{A} = \frac{\dot{Q}}{Q} - \left[S_K \frac{\dot{K}}{K} + S_L \frac{\dot{L}}{L}\right]$$

$$(4') \qquad \frac{\dot{Q}}{Q} - \frac{\dot{L}}{L} = \frac{\dot{A}}{A} + S_K \left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right)$$

Equations (2′) through (4′) are *Divisia* indexes with changing weights, and require continuous data. The BLS multifactor productivity indexes are based on the Tornqvist index number formula which is a discrete approximation to the *Divisia* index.[3] More specifically, the discrete index number formula used for measuring (2′) is:

$$(2'') \qquad ln \frac{Q(t)}{Q(t-1)} = ln \frac{A(t)}{A(t-1)} + \left[\bar{S}_{Kt}\, ln \frac{K(t)}{K(t-1)} + \bar{S}_{Lt}\, ln \frac{L(t)}{L(t-1)}\right]$$

where

$$\bar{S}_{Kt} = 1/2\,[S_{Kt} + S_{Kt-1}]; \text{ and}$$

$$\bar{S}_{Lt} = 1/2\,[S_{Lt} + S_{Lt-1}].$$

Tables A-1, A-2, and A-3 present index numbers of the BLS annual measures (of the antilogarithms) of the variables shown in equation (2″) and of the Tornqvist approximations of (3′) and (4′). Thus, table A shows for the private business sector yearly index numbers (1977 = 100) of output, $\frac{Q(t)}{Q(t-1)}$, multifactor productivity, $\frac{A(t)}{A(t-1)}$, and combined units of labor and capital inputs, the antilogarithm of the sum of the terms in brackets.

FOOTNOTES

[1] For simplicity, the analysis is limited to two inputs, capital and labor; more generally, K and L can be viewed as vectors of capital and labor inputs, respectively.

[2] For the derivation of this growth equation and its interpretation, see Robert M. Solow, "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, August 1957, pp. 312–20; and Dale W. Jorgenson and Zvi Griliches, "The Explanation of Productivity Change," *Review of Economics Studies*, July 1967, pp. 249–80.

[3] The Tornqvist quantity index is said to be an *exact* index for the homogeneous translogarithmic production function. This means that the change in output resulting from changes in inputs and input prices as measured by the Tornqvist index is the same as would be obtained by using a homogeneous translogarithmic production function. See W. E. Diewert, "Aggregation Problems in Measurement of Capital," in Dan Usher, ed., *The Measurement of Capital*, Studies in Income and Wealth Vol. 45, National Bureau of Economic Research (Chicago, University of Chicago Press, 1980), pp. 446–52, and cited references.