Appendix E

Moment Released by Aftershocks

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Part 1: Moment released by aftershocks in the WG99 model

In constructing moment-balanced fault rupture models for the SF Bay region, WG99 needed to account for all the moment released by earthquakes in the region. These include earthquakes on characterized rupture sources, earthquakes on other rupture sources (background), and their aftershocks. Some of the aftershocks will be large, with magnitudes sometimes exceeding 6.7. WG99 assumed that an aftershock of this size would likely break the surface and have a rupture area comparable to one of the characterized fault segments, a fault segment in the background, or one of the floating earthquake sources. These aftershocks probably cannot occur on the same segments that broke in the main shock because they release too much moment. An aftershock of this size is often referred to as a *triggered earthquake*.

WG99 considered triggered earthquakes to be among the events included among the characterized rupture sources on the faults or in the background. Furthermore, it was felt that both the first earthquake and the triggered one usually would be included in any future assessment of the model s probability estimate. For example, if a M=7.5 earthquake on the Hayward fault were followed weeks, months or even a few years later, by a M=7.0 earthquake on the Rodgers Creek fault, or in the background, or on another characterized fault, WG99 would consider both earthquakes to be the output of the average regional model, despite their temporal relationship. Another example (if a WG99-like approach were applied to southern California) would be the 1992 M=7.2 Landers earthquake and the 1999 M=7.0 Hector Mine earthquake.

The smaller magnitude aftershocks tend to occur on or very near the fault segments that ruptured in the main shock. These events may fill in patches of low slip within and around the rupture, or smooth the high stress concentrations created around it by the main shock. These aftershocks help to complete the moment release on the ruptured segments. Here, we estimate the fraction of regional moment that is released in the WG99 model through M<6.7 aftershocks with the use of a generic model (Reasenberg and Jones, 1990, 1994). The model parameters are estimated using only northern California aftershock sequences. See Part 2 for a description of this model. We apply the model to each fault, using the largest WG99-characterized rupture on the fault as the model s main shock. From such models for all the faults and background, we calculate the total long-term moment rate associated with the M<6.7 aftershocks, and express this quantity as a fraction of the total regional moment rate. We find that over the long term the moment released by M<6.7 aftershocks in the region amounts to 3%–2% of the total moment released in the region. Those who go on to read Part 2 in its entirety will find that a generic northern California sequence contains approximately 10% of its moment in aftershocks. However, applying this model to all the faults and keeping only the M<6.7 aftershocks leads to the 3% result.

Part 2: Moment released by aftershocks in a generic northern California earthquake sequence

The moment in an aftershock sequence can be expressed as a function of the magnitude of the mainshock and the a-value and b-value of the aftershock magnitude distribution. I calculate the moment sum for aftershocks in a generic aftershock sequence (as determined empirically for northern California), and express this as a fraction of the total moment for the sequence. Because the generic aftershock model is self-similar, *f* is independent of the main shock magnitude, M_m, and depends only on the model parameters.

Definitions:

M_m	Magnitude of main shock.
W_m	Moment of main shock
$W(M)$	Moment of an aftershock of magnitude *M*.
a, b	Parameters of the cumulative form of the G-R relation
a', b	Parameters of the interval form of the G-R relation
a^*	Parameter in the R&J (1989, 1994) formulation
f	Fraction of moment in earthquake sequence released in aftershocks

All logs are base 10 and R&J stands for Reasenberg and Jones (1989, 1994)

The magnitude distribution of the aftershocks follows the Gutenberg-Richter relation

$$\log N(m \geq M) = a - bM \tag{1}$$

In the incremental form of the G-R relation, the number of earthquakes $dN(M)$ within a magnitude increment ΔM centered on M is

$$\log dN(M) = \log N\left(M - \frac{\Delta M}{2} \leq m < M + \frac{\Delta M}{2}\right) = a' - bM \tag{2}$$

We will work with the incremental form, calculate the moment in each magnitude interval, and sum these to get the total moment in the aftershock sequence.

The moment, $W(M)$, of an earthquake is related to its magnitude, M, by

$$\log W(M) = 1.5M + 16.05$$

(Hanks and Kanamori, 1979). The summed moment, $dW(M)$, associated with the events in the magnitude interval centered on M is equal to the number of such events, $dN(M)$ times the mean moment in the interval, $W(M)$. Thus

$$\begin{aligned} \log dW(M) &= \log dN(M) + \log W(M) \\ &= a' + (1.5 - b)M + 16.05 \end{aligned} \tag{3}$$

The relationship between a' and a depends on ΔM and b and is obtained by starting with Andrews and Schwerer, equation (7) (Andrews and Schwerer, 2001):

$$\frac{dN}{dM} = 2.3b\exp[2.3(a - bM)]$$

Then, switching from differentials to finite intervals (Joe Andrews, personal communication),

$$\begin{aligned} \frac{\Delta N}{\Delta M} &= 2.3b\exp[2.3(a - bM)] \\ \Delta N &= 2.3b\Delta M\exp[2.3(a - bM)] \\ &= 2.3b\Delta M 10^{a-bM} \\ \log(\Delta N) &= \log(2.3b\Delta M) + a - bM \end{aligned}$$

so

$$a' = \log(2.3b\Delta M) + a. \tag{4}$$

Plugging this result into (3),

$$\log dW(M) = a + \log(2.3b\Delta M) + (1.5 - b)M + 16.05 \tag{5}$$

Next, we need to relate a to the generic aftershock sequence model. R&J (1994) express the rate of aftershocks with magnitude M or larger at time t after a main shock of magnitude M_m as

$$\lambda(t, M) = 10^{a^* + b(M_m - M)}(t + c)^{-p} \tag{6}$$

where c and p are the Omori constants, and a^* is not to be confused with a *or* a'. The corresponding cumulative number of aftershocks (up to time T) with magnitude M or larger is

$$
\begin{aligned}
N(m \geq M) &= \int_0^T \lambda(t, M)dt \\
&= 10^{a^*+b(M_m - M)} \int_0^T \frac{dt}{(t+c)^p}
\end{aligned} \qquad (7)
$$

where

$$
\int_0^T \frac{dt}{(t+c)^{-p}} = \frac{(T+c)^{(1-p)} - c^{(1-p)}}{1-p} \equiv I \qquad (p > 1)
$$

Equation (7) is in the form of the G-R relation, with

$$
a = \log(I) + a^* + bM_m \qquad (8)
$$

Finally, combining the expressions for the number and moment distribution of aftershocks (equations 2 and 5) with the expressions for a' (equation 4) and the generic aftershock sequence (equations 7 and 8), we have

$$
\begin{aligned}
\log dN(M) &= a^* + b(M_m - M) + \log(2.3b\Delta M) + \log(I) \\
\log dW(M) &= a^* + bM_m + (1.5 - b)M + \log(2.3b\Delta M) + \log(I) + 16.05
\end{aligned} \qquad (9)
$$

Model parameters for northern California

At this point, we have expressed the distribution of aftershock magnitude and moment in terms of the parameters in the R&J generic aftershock model (*a*, b, p , c*) and the magnitude of the mainshock. For 62 sequences in California, R&J found median values *a**= -1.67, *b*=0.91, *p*=1.08 and *c*=0.05. In an unpublished continuation of their 1989 study, R&J estimated mean values of these parameters separately for southern California, northern California and eastern California. While subdividing the data resulted in smaller numbers of sequences in each set, some regional parameter estimates were significantly different from the all-California estimates. A significant difference was found in *a**, which for the 16 northern California sequences averaged *a**=—2.0. They found for Morgan Hill, a* = -3.2; for Coyote Lake, a* = -2.0; and for the Loma Prieta sequence, *a** = -1.7. We adopt the value of *a** estimated by R&J for northern California (*a** = -2.0). For the other parameters, we use the generic California values, since no significant regional differences for these were found by R&J. The curves in Figure 1 depict Equation (9) for these parameters.

Example for a M=7 main shock

Figure 1a shows the cumulative and interval forms of the magnitude (frequency) distribution for aftershocks following a $M_m = 7$ main shock. The numbers and moment are cumulative for the first 1000 days of the sequence. The main shock is represented by the black dot. The a-value is 5.3, $a' = 4.6$. Approximately 1 aftershock of magnitude 5.8

or larger is expected. Figure 1b shows the corresponding cumulative and interval distributions of moment. The moment in the M=7 main shock is $3.55 \bullet 10^{26}$ dyne-cm. The summed moment of aftershocks (in this example, in the magnitude range 3.0 † M † 6.9), expressed as a fraction, *f*, of the total moment (aftershocks + main shock) is *f* = 0.10. In the ranges 3.0†M†3.9, 4.0†M†4.9, 5.0†M†5.9, and 6.0†M†6.9, *f* equals 0.001, 0.005, 0.019 and 0.074, respectively.

References

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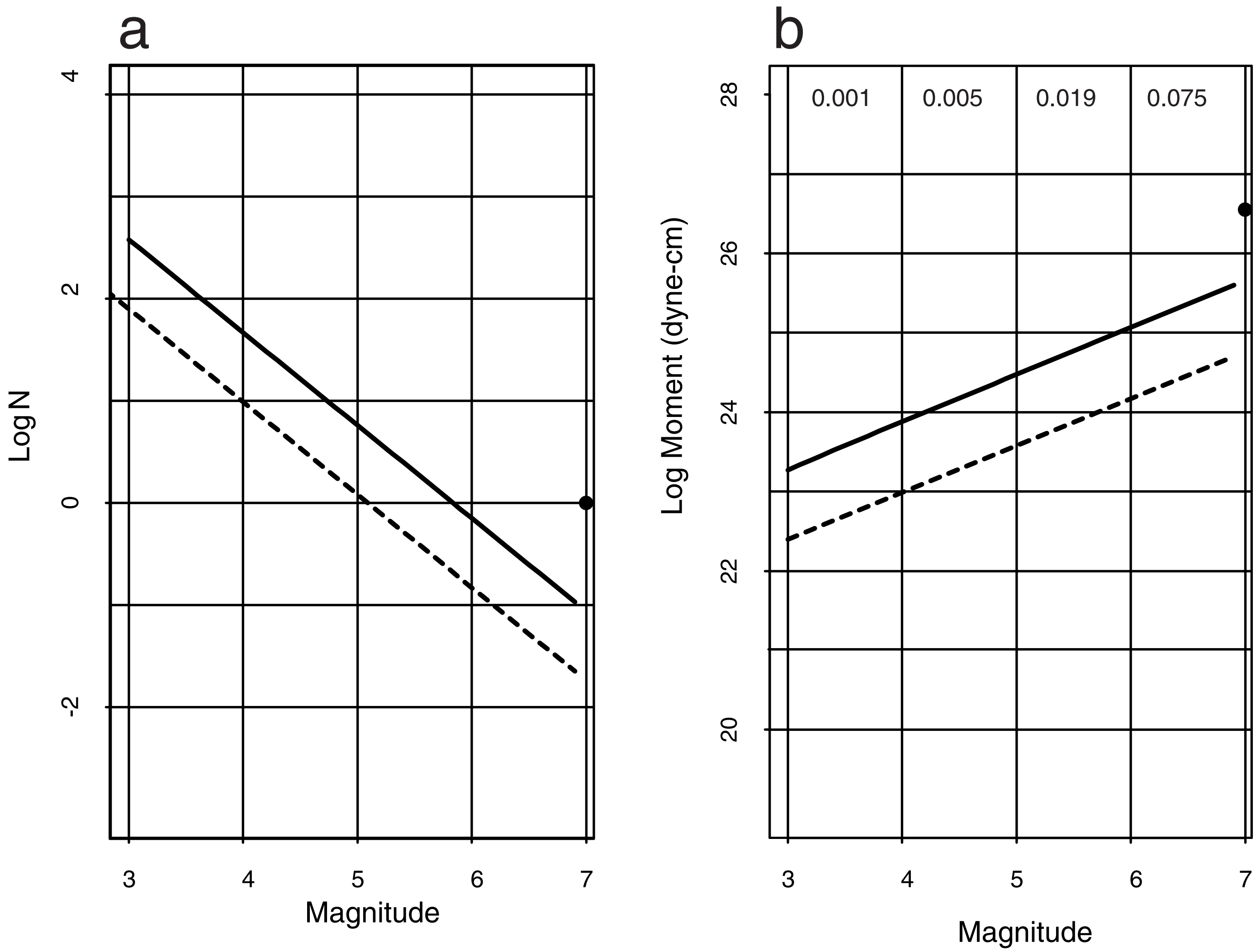

Plots of Equation (9) for an aftershock sequence following a M=7 main shock, using model parameters (a*=-2.0, p=1.08, b=0.91, c=0.05). These parameters are representative of aftershock sequences observed in northern California. (a) Magnitude distribution of aftershocks. Solid dot represents the main shock (N=1). Solid line is the cumulative distribution; dashed line is the interval distribution, for magnitude interval width 0.1. (b) Distribution of moment. Solid dot is the moment of the main shock. Cumulative distribution of aftershock moment (solid line) is obtained by summing the interval distribution (dashed line). 10% of the total moment in this model sequence (main shock plus aftershocks) is released in $3.0 \le M \le 6.9$ aftershocks. Numbers at top are the fraction, f, of moment in the magnitude ranges $3.0 \le M \le 3.9$, $4.0 \le M \le 4.9$, $5.0 \le M \le 5.9$, and $6.0 \le M \le 6.9$.

Appendix E - Figure 1