BIVARIATE INTERPOLATION

PURPOSE

Perform a bivariate interpolation of a series of gridded data points.

DESCRIPTION

Two dimensional interpolation takes a series of (x,y,z) points and generates estimated values for z's at new (x,y) points. Interpolation is used when the function that generated the original (x,y,z) points is unknown. Interpolation is related to, but distinct from, fitting a function to a series of points. In particular, an interpolated function goes through all the original points while a fitted function may not.

There are two distinct types of 2d interpolation. In the first, data is available for a rectangular grid of points and interpolation is performed for points off the grid. In the second, data is available for a random set of points and the interpolation is generated on a rectangular grid. This second form can be used to generate a contour or surface plot when the data do not form a grid.

The BILINEAR INTERPOLATION and BIVARIATE INTERPOLATION commands are used for the first type. The bilinear interpolation is analogous to linear interpolation. The bivariate interpolation uses an interpolating function that is a piecewise polynomial function that is represented as a tensor product of one-dimensional B-splines. That is,

$$S(x,y) = \sum_{i=1}^{NX} \sum_{j=1}^{NY} a(i,j)U(x)V(y) \qquad \textbf{(EQ 3-25)}$$

where U(i) and V(j) are one-dimensional B-spline basis functions and the coefficients a(i,j) are chosen so that the interpolating function equals the z axis input values at the grid points.

The 2D INTERPOLATION command is used for the second type of interpolation. See the documentation for the BILINEAR INTERPOLATION and the 2D INTERPOLATION commands for more details on these methods.

SYNTAX

LET <z2> = BIVARIATE INTERPOLATION <z1> <y1> <x1> <y2> <x2> <SUBSET/EXCEPT/FOR qualification>
where <z1> is a variable containing the z-axis data points;
<y1> is a variable containing the vertical axis data points;
<x1> is a variable containing the horizontal axis data points;
<x2> is a variable containing the horizontal points where the interpolation is to be performed;
<y2> is a variable containing the vertical points where the interpolation is to be performed;
<z2> is a variable (same length as <x2>) where the interpolated values are stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

EXAMPLES

LET Z2 = BIVARIATE INTERPOLATION Z1 Y1 X1 Y2 X2

NOTE 1

The <z1>, <y1>, and <x1> arrays should be the same size. In addition, <y1> and <x1> must form a rectangular grid. The rectangular grid must be complete. No provision is made for missing data values on the grid. The <x2> and <y2> arrays must be the same size. An error message is printed if any of these conditions is not met.

NOTE 2

The interpolation points (i.e., <x2> and <y2>) must be within the range of the original data points (i.e., <x1>). An error message is generated if this is not the case.

NOTE 3

The original data do not have to be in sorted order.

NOTE 4

The degree of the polynomial in each direction can be specified with the following commands:

LET XDEGREE = <value>
LET YDEGREE = <value>

where <value> is 3 for cubic polynomials, 2 for quadratic polynomials, and 1 for linear polynomials. The default is cubic polynomials.

NOTE 5

DATAPLOT uses the B2INK and B2VAL routines written by Dr. Ron Boisvert of the National Institute of Standards and Technology. These routines are available in the NIST CMLIB (Core Mathematical Library) library.

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

INTERPOLATION	=	Compute a cubic spline interpolation of a series of points.
LINEAR INTERPOLATION	=	Compute a linear interpolation of a series of points.
BILINEAR INTERPOLATION	=	Compute a bilinear interpolation from a grid to random points of a 2D series of points.
2D INTERPOLATION	=	Compute a bivariate interpolation from a 2D series of points to a rectangular grid.
FIT	=	Perform a least squares fit.

REFERENCE

"A Practical Guide to Splines," Carl de Boor, Springer-Verlang, 1978.

"Efficient Computer Manipulation of Tensor Products," Carl de Boor, ACM Transaction of Mathematical Software, Vol. 5, 1979 (pp. 173-182).

"Numerical Recipes: The Art of Scientific Computing (FORTRAN Version)," Press, Flannery, Teukolsky, and Vetterling, Cambridge University Press, 1989 (pages 170-172).

APPLICATIONS

Mathematics

IMPLEMENTATION DATE

94/6

PROGRAM

```
LET X = SEQUENCE -4 1 4 FOR I = 1 1 81
LET Y = SEQUENCE -4 9 1 4
LET Z = X**2 + Y**2 - X*Y
READ X2 Y2
-3.5 -2.8
-1.5 -0.8
-0.6 3.2
2.1 -3.1
3.8 3.2
END OF DATA
LET Z2 = BIVARIATE INTERPOLATION Z Y X Y2 X2
SET WRITE DECIMALS 3
PRINT X2 Y2 Z2
```

The following output is generated.

```
VARIABLES--X2             Y2             Z2

          -3.500         -2.800         10.290
          -1.500         -0.800          1.690
          -0.600          3.200         12.520
           2.100         -3.100         20.530
           3.800          3.200         12.520
```