SINGULAR VALUE DECOMPOSITION

PURPOSE

Compute the singular value decomposition of a matrix.

DESCRIPTION

If X is a matrix with row and column dimensions n and p respectively, then an n by n orthogonal matrix U and a p by p orthogonal matrix V can be found such that:

$$U^T X V = \begin{bmatrix}\Sigma \\ 0\end{bmatrix} \qquad \textbf{(EQ 4-74)}$$

where Σ is a m by m diagonal matrix (m is the minimum of n and p). The diagonal elements of Σ are the singular values of X and they are stored from largest to smallest. The above assumes that n >= p. A right hand side becomes [Σ 0] if N < p. Singular values of zero (or near zero) indicate that the matrix is singular (i.e., not of full rank) or ill-conditioned. Chapters 2 and 14 of the Numerical Recipes book describe some applications of the SVD.

Since U and V are orthogonal (and so their inverses are equal to their transpose), the above equation can also be written as:

$$X = U\begin{bmatrix}\Sigma \\ 0\end{bmatrix}V^T \qquad \textbf{(EQ 4-75)}$$

For large matrices, it can be impractical to compute U (which is n by n). However, U can be partitioned into

U = (U1, U2)

where U1 is n by p. Then

X = U1ΣV’

is called the singular value factorization of X. Several multivariate statistical techniques are based on this factorization.

SYNTAX

LET <u> <s> <v> = SINGULAR VALUE DECOMPOSITION <mat> <SUBSET/EXCEPT/FOR qualification>
where <mat> is a matrix for which the singular values are to be computed;
<u> is an n by n matrix where U is saved;
<s> is a variable where the singular values are saved (length is minimum of n and p);
<v> is an p by p matrix where V is saved.
and where the <SUBSET/EXCEPT/FOR qualification> is optional and rarely used in this context.

EXAMPLES

LET U S V = SINGULAR VALUE DECOMPOSITION A

NOTE 1

DATAPLOT uses the LINPACK routine SSVDC to calculate the singular value decomposition.

NOTE 2

DATAPLOT will calculate the singular value decomposition even if N <= p. However, in practice this is almost never done.

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

MATRIX EIGENVALUES	=	Compute the matrix eigenvalues.
MATRIX EIGENVECTORS	=	Compute the matrix eigenvectors.
MATRIX MULTIPLICATION	=	Perform a matrix multiplication.

MATRIX SOLUTION	=	Solve a system of linear equations.
CORRELATION MATRIX	=	Compute the correlation matrix of a matrix.
VARIANCE-COVA MATRIX	=	Compute the variance-covariance matrix of a matrix.
SINGULAR VALUES	=	Compute the singular values of a matrix.
SINGULAR VALUE FACT	=	Compute the singular value factorization of a matrix.

REFERENCE

"LINPACK User's Guide," Dongarra, Bunch, Moler, Stewart. Siam, 1979.

"Numerical Recipes: The Art of Scientific Programming (FORTRAN Version)," Press, Flannery, Teukolsky, and Vetterling, Cambridge University Press, 1989 (chapter 2).

APPLICATIONS

Linear Algebra, Multivariate Analysis

IMPLEMENTATION DATE

93/8

PROGRAM

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DIMENSION 100 COLUMNS
SKIP 25
COLUMN LIMITS 20 132
READ MATRIX AUTO79.DAT X
LET U S V = SINGULAR VALUE DECOMPOSITION X
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