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Nonequilibrium Nonlinear-Dynamics in Solids: State of the Art

State of the Art in Nonequilibrium Dynamics

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Abstract

Dynamic nonlinear elastic behavior, *nonequilibrium dynamics*, first observed as a curiosity in earth materials has now been observed in a great variety of solids. The primary manifestations of the behavior are characteristic wave distortion, and *slow dynamics*, a recovery process to equilibrium that takes place linearly with the logarithm of time, over hours to days after a wave disturbance. The link between the diverse materials that exhibit nonequilibrium dynamics appears to be the presence of soft regions, thought to be primarily "damage" at many scales, ranging from order 10^{-9} m to 10^{-1} m at least. The regions of soft matter may be distributed as in a rock sample, or isolated, as in a sample with a single crack. The precise physical origin of the behavior is clear in some cases such as granular media where the source of the nonequilibrium dynamics, grain-to-grain interaction, is understood. In other materials, it appears that the origin must be due fundamentally to shear sliding, related to crack and possibly dislocation dynamics, as well as less clear origins. Because the physical origins of the behavior are related to damage, damage diagnostics in solids, Nonlinear NonDestructive Evaluation, follows naturally. Nonequilibrium dynamics also plays a significant role in other areas such as earthquake strong ground motion and potentially to earthquake dynamics.

Keywords: Nonlinear Elastic Wave Spectroscopy (NEWS), NRUS, NWMS, SDD, Time Reversal, Earthquake triggering, nonlinear NDE, nonequilibrium dynamics, strong ground motion

1. Introduction

Over the last two decades, studies of nonlinear dynamics in materials, known as *non-classical* or *anomalous* that include rock, damaged materials some ceramics, sintered metals, granular media etc., have increased markedly (Ostrovsky and Johnson, 2001, Guyer and Johnson, 1999). These materials exhibit what we term *nonequilibrium dynamics* at elevated strain amplitudes ($>\sim 10^{-6}$). Specifically, when the material is disturbed by a wave, the modulus decreases. We call this *nonlinear fast dynamics*. Following this, it takes tens of minutes to hours to return to its equilibrium state. This is called *slow dynamics* (Johnson et al., 1996; TenCate and Shankland, 1996). Further, the apparent mixture of fast and slow dynamics known as *conditioning* that takes place during nonlinear fast dynamics provides additional complexity not observed in materials whose nonlinearity is due to anharmonicity. The nonequilibrium dynamics is due to mechanically "soft" inclusions (soft matter) in a "hard" matrix (e.g., Ostrovsky and Johnson, 2001). For instance, a crack in a solid will induce nonequilibrium dynamics, but a void will not; a sandstone exhibits nonequilibrium dynamics due to distributed soft inclusions, also known as the bond system, but a bar of aluminium does not. Experimental methods and theory have been developed to interrogate nonequilibrium dynamics in solids.

In this paper we briefly address underlying theory, then provide an overview of the primary methods to interrogate nonequilibrium dynamical behavior, termed Nonlinear Elastic Wave Spectroscopy (NEWS) (Johnson, 1999). We outline Nonlinear Resonant Ultrasound Spectroscopy (NRUS), Slow Dynamics Diagnostics (SDD), Nonlinear Wave Modulation Spectroscopy (NWMS) and Time Reversal Nonlinear Elastic Wave Spectroscopy (TR NEWS). Following the description of these methods, we will briefly describe nonlinear imaging methods currently in development and then provide an overview of new areas of research where nonequilibrium dynamics may be important. Next we address unsolved problems related to the origin of elastic nonlinear behavior, and briefly look into the future.

2. Theory

Fundamentally, elastic nonlinearity implies that the stress-strain relation (also known as the equation of state, EOS) is nonlinear. For such a relation, the one-dimensional stress (σ)-strain (ε) can be described by

$$\sigma = K_o\varepsilon(1 + \beta\varepsilon + \partial\varepsilon^2 + \ldots), \tag{4.1}$$

where K_o is the linear modulus, and β and δ are the first and second order classical nonlinear parameters, normally of order 1–10 in value. At low dynamic wave amplitudes (strains of less than order 10^{-6} under ambient pressure), there is evidence that all (or at least most) solids behave in a manner according to the above equation (TenCate et al, 2004; Pasqualini, this volume). At ambient pressure and temperature conditions, for wave amplitudes above approximately 10^{-6} strain, the material EOS is thought to be hysteretic. A hysteretic EOS relation is,

$$\sigma = K_o\varepsilon(1 + \beta\varepsilon + \alpha\varepsilon, f(\partial u/\partial x)), \tag{4.2}$$

where α is the hysteretic nonlinear parameter and is dependent on the strain derivative $\partial u/\partial x$ due to the hysteresis (e.g., Guyer et al., 1997). Eq. (4.2) is a practical estimate of the dynamics, especially for NDE applications, but does not capture the entirety of nonequilibrium dynamics: the slow dynamics and material conditioning (TenCate and Shankland, 1996; Johnson and Sutin, 2005) as outlined in Fig. 4.1. As previously noted, slow dynamics means the material takes time to return to its rest state modulus K_o relaxing as the logarithm of time. An example of conditioning is as follows: if a

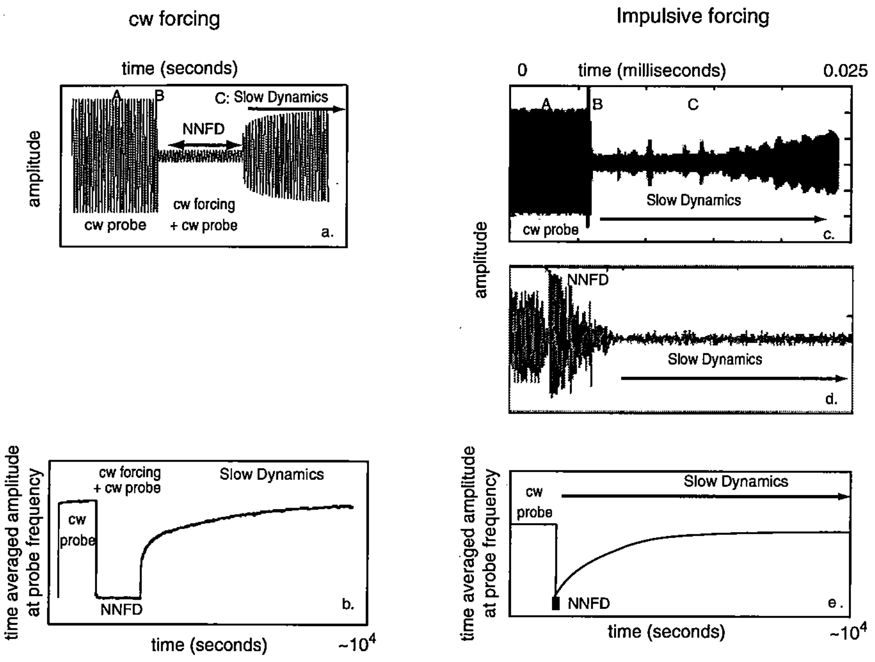

Fig. 4.1. *Nonequilibrium dynamics for two types of forcing.* The figure illustrates the full nonequilibrium dynamics that includes *nonlinear fast dynamics* (also known as *nonclassical* or *anomalous* nonlinear dynamics), *conditioning*, and *slow dynamics*. Figures (a) and (b) show how nonequilibrium dynamics are manifest when a low-amplitude, continuous-wave (*cw*) probe-wave is input into a sample in the presence of a large amplitude vibration. One sees in (a) the undisturbed probe wave (time A) and the corresponding time-average amplitude of the signal in (b) ["*cw* probe"]. At time B, a high-amplitude vibration begins and the probe-wave amplitude changes due to material nonlinearity (see Slow Dynamics Diagnostics section of this paper and Figure 4.5 for more). From the time the vibration is turned on until it is turned off, nonlinear fast dynamics, including conditioning, take place ("Nonclassical Nonlinear Fast Dynamics NNFD" in (a) and (b)). As soon as the large amplitude wave is terminated, one sees in (a) and (b) an instantaneous, partial recovery of the amplitude, and then a longer term recovery that is linear with the logarithm of time where slow dynamics is the sole process acting in the system. Figures (c-e) show the situation where the sample is disturbed by an impact, such as a tap, in the presence of the probe (time B in (c)). Figure (d) shows a zoom of (c) where one can observe the onset of the tap-induced vibration and its ring down ("NNFD" in (d-e)). After the vibration energy has dissipated, slow dynamics is the sole process operating in the system, the onset of which is shown in (c-d), and the long term behaviour is seen in (e).

rock sample is driven at fixed amplitude for a period of time, the modulus will decrease immediately with the onset of the wave, but then continue to decrease slightly to a new equilibrium value as long as the drive is maintained (TenCate and Shankland, 1996). Conditioning is a small effect in most materials as can be seen in Fig. 4.1b. It may or may not be correct to think of conditioning as a mix of fast and slow dynamics. In any case, Eq. (4.2) has been applied broadly to describe the material elastic nonlinearity. The rate-dependent effect of conditioning appears to have only a minor influence on estimates of α (e.g., Johnson and Sutin, 2005).

3. Nonlinear Elastic Wave Spectroscopy (NEWS)

3.1 Nonlinear Resonant Ultrasound Spectroscopy (NRUS)

Nonlinear Resonant Ultrasound Spectroscopy (NRUS) is based on the measurement of resonance frequency shift and material damping as a function of resonance peak amplitude for one or more resonance modes (e.g., Winkler et al., 1979; Johnson et al., 1996; Johnson, 1999). This method is an extension of linear Resonant Ultrasound Spectroscopy (RUS) that is used in industrial NDE (Migliori and Sarrao, 1999). In this type of measurement, the change in resonance frequency of a mode with drive amplitude is a measure of the wavespeed and modulus change. For instance, in a simple geometry such as a cylindrical bar driven at the fundamental mode, the wavespeed c is,

$$c = f\lambda = 2fL = \sqrt{\frac{K}{\rho}} \tag{4.3}$$

where f is resonance frequency of the fundamental mode, λ is the wavelength, L is the bar length, K is modulus and ρ is density. The equation becomes correspondingly more elaborate for more complicated sample geometries. A typical resonance experimental configuration is shown in Fig. 4.2. Figure 4.3 shows an NRUS result from two concrete samples, one virtually "intact" measured in the undamaged state, and one damaged. The hysteretic nonlinear parameter α can be extracted from the change in frequency with strain amplitude,

$$\frac{\Delta f}{f_0} = \alpha\varepsilon \tag{4.4}$$

where f_0 is the equilibrium frequency, Δf is the change in resonance frequency, α is the nonlinear parameter in Eq. (4.2), and ε is strain. α ranges from approximately

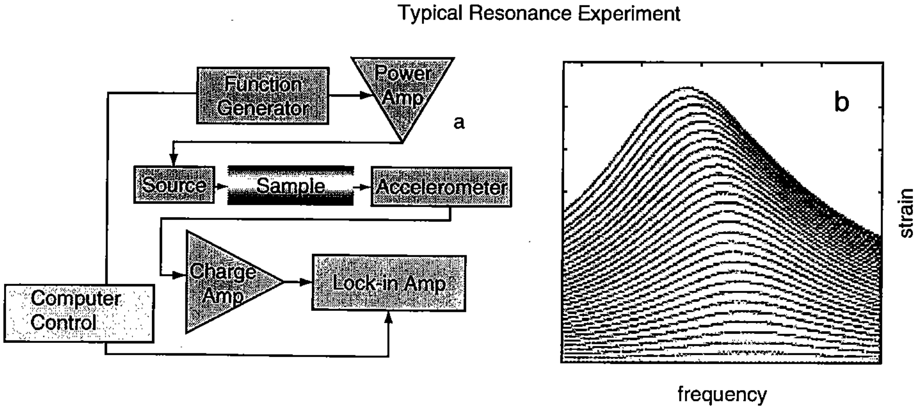

Fig. 4.2. (a) Typical NRUS experimental configuration and (b) resonance curves obtained from a nonclassical material. The source drives at a sequence of frequencies stepping from below to above a resonance mode. A lock-in amplifier is used to extract the time average amplitude of the detected signal. The drive level is increased and the procedure is repeated over a number of drive levels.

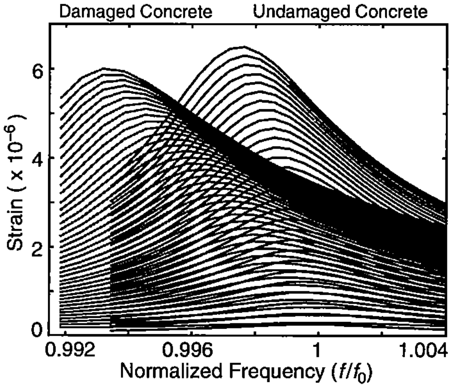

Fig. 4.3. NRUS measurements from intact and damaged concrete samples. Note that the undamaged sample exhibits a small amount of peak shift, meaning it is somewhat elastically nonlinear in its original state. The damaged sample is significantly more nonlinear in that the peak shift is pronounced and the material dissipation characteristics are increased, seen by an increase in the resonance peak width. The frequency axis is normalized to the low amplitude, equilibrium (linear elastic) value (figure courtesy of L. Byers and J. TenCate).

10-10^4. Simultaneous to modal frequency shift, nonlinear damping increases (e.g., Johnson and Sutin, 2005).

3.2 Slow Dynamics Diagnostics (SDD)

The phenomenon of slow dynamics (SD) were first observed in relatively homogeneously elastically nonlinear materials, such as rock and concrete, that have distributed nonlinear sources e.g. (Johnson et al., 1996; TenCate and Shankland, 1996), and has more recently been show to exist in a broad range of solids with both distributed and localized nonlinear sources [cracks, delaminations] (Johnson and Sutin, 2005). The process of SD recovery can be observed by applying RUS measurements at successive times after large-amplitude wave excitation, as well as by observation of the pure tone signal variation. Both methods are described below.

In the RUS variation of the SDD method we take advantage of both the amplitude and frequency of the recovery of a sample mode. In SDD, the equilibrium, low amplitude (linear) amplitude frequency response of the sample is first measured. The sample is then driven at large strain amplitude (order 5 microstrains) to induce material softening. Immediately upon termination of the drive, the RUS measurement recommences at very low strain amplitude ($\sim 10^{-7}$) for probing the recovery. An example of SDD in steel is shown in Fig. 4.4. We see in the left hand figure results from an undamaged sample. The results for the damaged sample are shown in the right-hand side of Fig. 4.4, evident by the initial change in frequency and the successive recovery. The sample recovery time shown is 141 seconds. Full recovery took approximately one hour.

For quick application, a variation of the SDD method known as the *slope amplifier* is useful. Figure 4.5 describes how the SDD slope amplifier works, and Fig. 4.6 shows SDD, slope-amplifier results obtained from an automotive bearing cap.

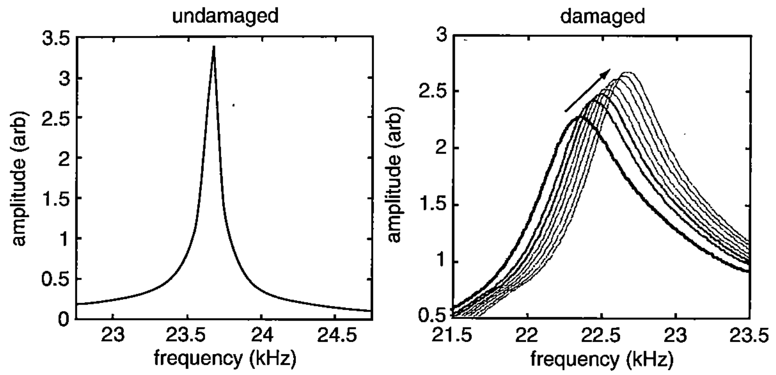

Fig. 4.4. Resonance response of a mode in an undamaged sample and the recovery process of SD in a damaged sample.

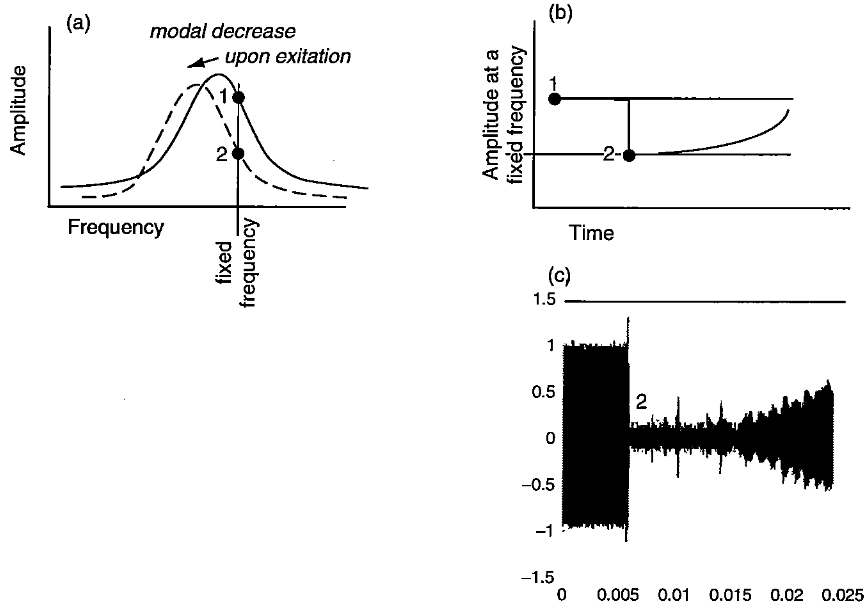

Fig. 4.5. Variation on the SDD technique: the slope amplifier. (a) A low-amplitude signal probes the sample near a modal peak. The signal amplitude [1] is controlled by the modal structure. The time-average amplitude behavior of the time signal is shown in (b). The sample is disturbed by a large amplitude signal induced by a tap for instance, and the modal peak shifts downward causing the probe wave amplitude to change in amplitude to position [2] and (a) and (b). Slow dynamics keeps the modal peak diminished in frequency and thus in amplitude. An actual example of SDD applied to a damaged solid is shown in (c).

One can follow the onset and partial or full recovery of a sample by applying the slope amplifier, capturing more detail than RUS which requires a minute or more for each resonance sweep. For example, Fig. 4.7 shows the onset and several hundred seconds of recovery in four materials.

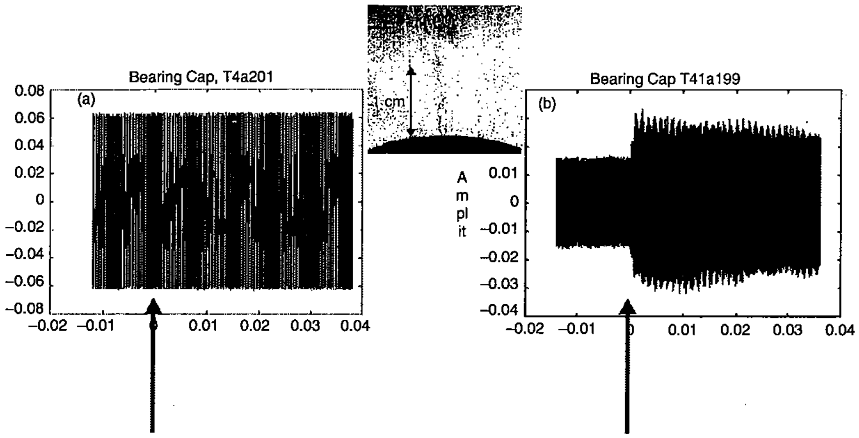

Fig. 4.6. Slope amplifier results in an undamaged (a) and damaged (b) automotive bearing cap. The sample crack is shown in (c). Note that in this experiment, the probe frequency was lower than the resonance peak frequency and therefore the amplitude increases when impacted the source. Arrows point to the time of impact. The data were high-pass filtered.

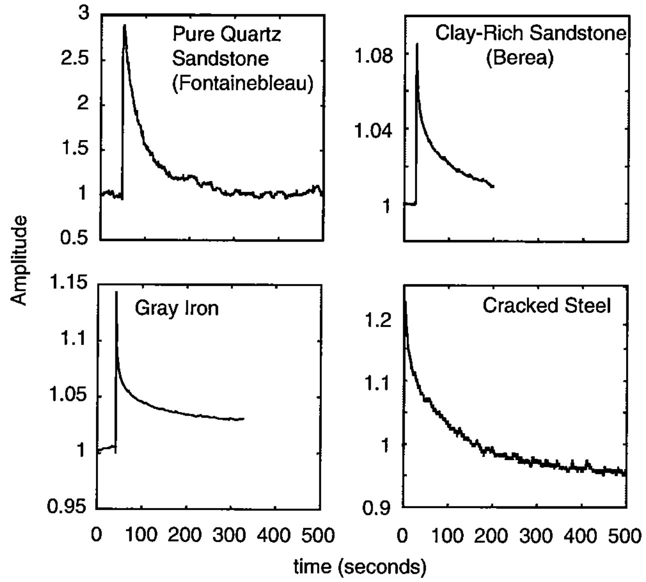

Fig. 4.7. Slow dynamics recovery in four materials applying the *slope amplifier*. Each curve represents the time-averaged amplitude of the low-amplitude, continues-wave probe, at a frequency just below a resonance mode. This method provides the means to capture all of the physical characteristics of the recovery.

3.3 Nonlinear Wave Modulation Spectroscopy (NWMS)

One of the simplest ways to evaluate nonlinear elastic properties of a material is to measure the modulation of an ultrasonic wave by low-frequency vibration. This method is termed here nonlinear wave modulation spectroscopy (NWMS) pioneered

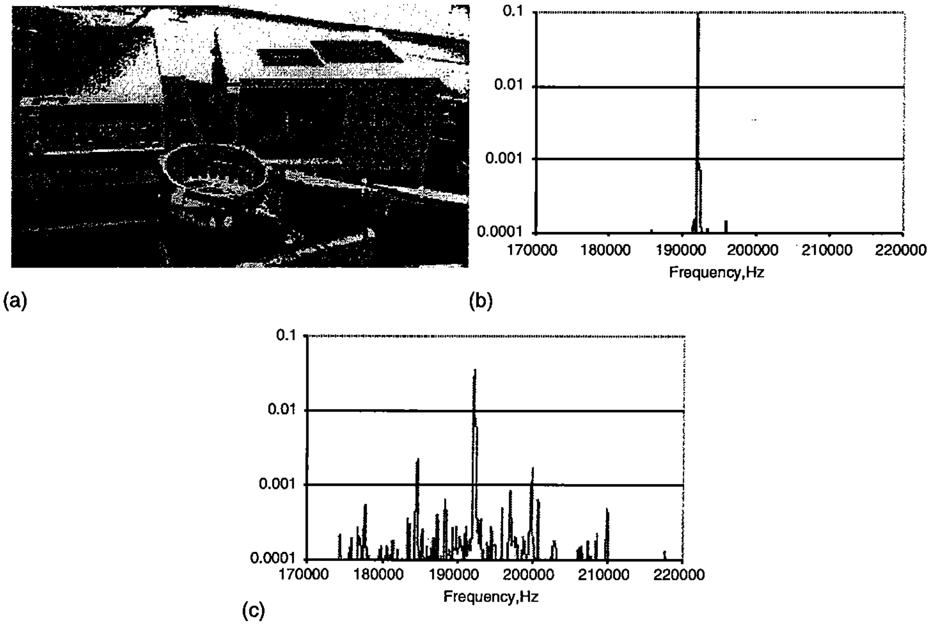

Fig. 4.8. NWMS test in an alternator housing. (a) Experimental configuration. (b) Result in undamaged sample. (c) Result from the damaged sample.

by the group at the Institute of Applied Physics in Nizhny Novgorod, Russia. The following experiment demonstrates a variation of NWMS in testing an alternator housing. The experimental setup is shown in Fig. (4.8a). Ultrasonic transducers were glued to the sides of each sample, and low-frequency, broadband vibration was generated with an instrumental hammer. The spectrum of the signal for an intact part is compared with that from a part with a tiny crack in Figs. 4.8b and 4.8c. Clearly the modulation (sideband components) identifies the damaged sample. Many groups have developed a multitude of variations of the technique, for instance Solodov's group at Moscow State University who have focused on interfaces and disbonding.

Imaging Nonlinear Scatterers

3.4 Imaging Applying NWMS

Nonlinear imaging is in its infancy in solids. One method, the NWMS nonlinear imaging method, described in detail in (Kazakov et al., 2002) is presented here followed by a method based on Time Reversal. In the NWMS method, a low frequency, continuous wave (*cw*) excitation is applied to the specimen simultaneous to a group of high frequency tonebursts (rather than a *cw* probe as in NWMS). In the experiment, the wavefield scattering from a hole, created by drilling, and a crack, created by cyclic loading, were measured in a small steel plate (Fig. 4.9a). Ultrasonic pulses with frequency 3MHz were used for imaging, and a low-frequency vibration of 10 Hz was produced by a shaker. Figure (4.9b) shows how the method works. Figure (4.10)

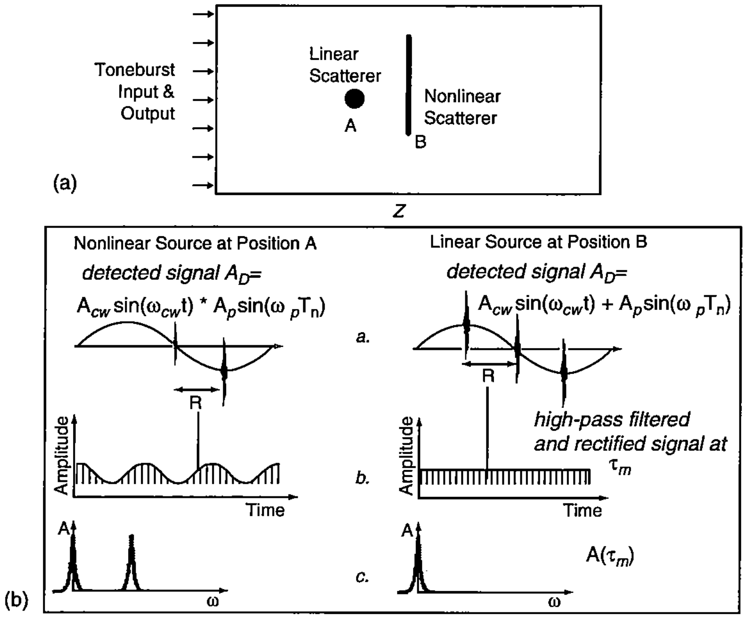

Fig. 4.9. Conceptual view of the data collection and the processing in the nonlinear imaging technique. (a) shows a sample with a hole [A] and a crack [B]. In (b) the left panel shows signals reflected from the crack at different times. The first pulse is reflected at the moment when the applied vibration stress reaches the maximum, minimizing the acoustic impedance of the crack. At this time the crack is compressed and the signal reflected from the crack has minimal amplitude. The second pulses is shown at time when the vibration stress is small and there is no change in the reflected impulse. The third pulse shows the extension phase of the vibration at the moment the crack is maximally opened, minimizing the acoustic impedance of the crack, causing the reflection signal to be higher than in previous phases. Thus, the pulse train is modulated. An FFT of the modulated pulse train gives the time delay of the reflection from the crack. Going through the same process for the hole we observe no modulation.

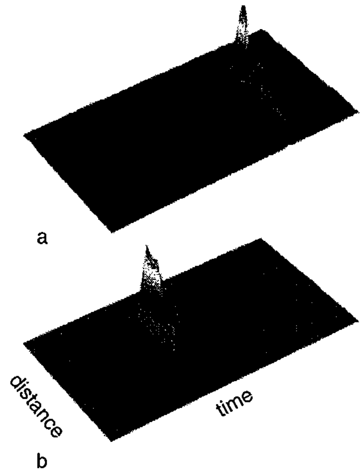

Fig. 4.10. Results of the pulse NWMS method in a sample of steel. The source and receiver are located on the left side (the axis marked “distance”. (a) Image of crack applying the nonlinear method. Note hole is not imaged. (b) Standard pulse-echo result showing hole (large amplitude response) and crack, which is much smaller in amplitude do to the shadowing effect of the hole. Distance is obtained from the time axis by using the wavespeed.

shows the measurements, comparing a standard, pulse-echo measurement to the nonlinear method. The method provides the means to isolate a nonlinear scatterer. It has not been demonstrated in three-dimensions, however.

3.5 Time Reversal Nonlinear Elastic Wave Spectroscopy (TR NEWS)

Much of the seminal research in Time Reverse Acoustics (TRA) has been carried out by the group located at the University of Paris VII (Laboratoire Ondes et Acoustique, ESPCI) (e.g., Fink, 1997). A significant aspect of TR in regards to elastic nonlinearity is that it provides one the ability to focus an ultrasonic wave, regardless of the position of the initial source and of the heterogeneity of the medium in which the wave propagates. Currently, we are exploiting the focusing properties of TR and the elastic nonlinear properties of cracks together to develop methods for crack and damage location (e.g., Sutin and Johnson, 2005; Ulrich et al., 2006).

Figure 4.11 shows an experimental configuration for demonstration experiments of TR conducted in a sample of sandstone, and TR NEWS in a glass parallelepiped sample (Sutin et al, 2004; Sutin and Johnson, 2004). The method is described as follows. A pulse was applied to the first transmitter. The detected signal measured from the opposite side of the sample was measured by a laser vibrometer. The recorded signal was time reversed as shown in Fig. 4.12a. The TR signal was then re-radiated. The TRA focused signal was recorded by the laser vibrometer and analyzed. A typical TRA focused signal is shown in Fig. 4.12b. The spatial distribution of the focused signal is shown in Fig. (4.12c).

The significant elastic nonlinearity due to the presence of a crack can be used for crack location. By scanning the surface using the laser vibrometer in tandem with TR focusing at each scan point, then analyzing for nonlinear response at that point, one can determine if damage exists in the scanned area. The feasibility of this technique was evaluated in an experiment where the TRA focusing was conducted along a single line scan in the glass sample with and without damage present. Figure 4.13 presents the results. A small, 3mm crack oriented parallel to the glass surface is located at the

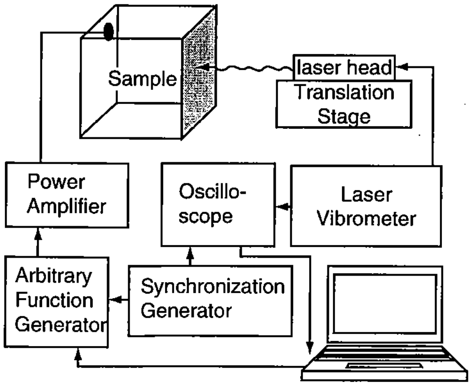

Fig. 4.11. TR NEWS experimental configuration.

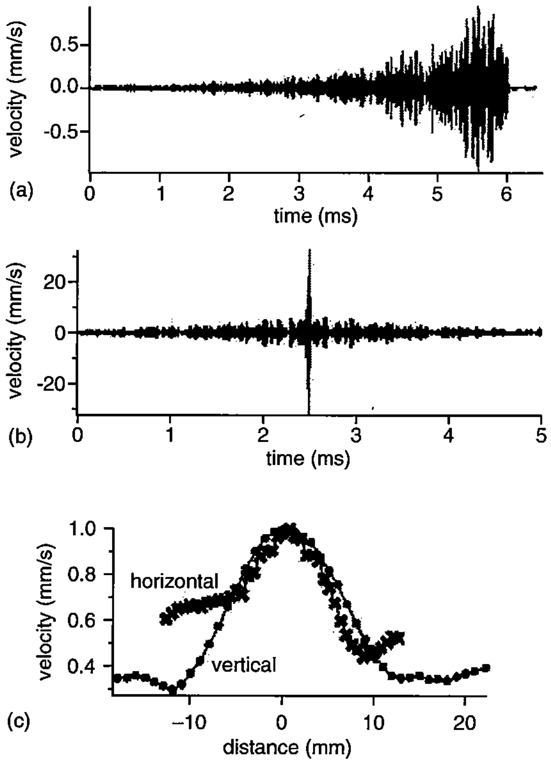

Fig. 4.12. Linear Time Reversal (TR) results in a sandstone sample. (a) Detected, time-reversed signal. (b) re-emitted and detected TR signal. (c) Spatial distribution of TR detected signal in the sample measured along perpendicular lines crossing at the TR focal point. Amplitudes are normalized to the maximum (from Sutin et al., 2004).

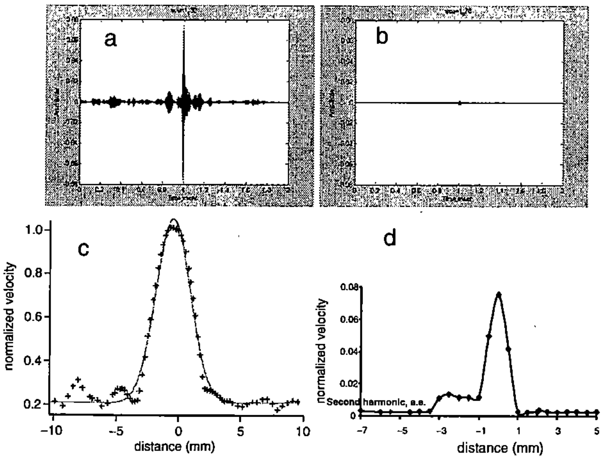

Fig. 4.13. TR NEWS results in a cracked solid. (a) The TR signal bandpass-filtered around the second harmonic, detected above the crack with a laser vibrometer and, (b), 30 mm away from the crack. The fact that a signal is observed means that strong nonlinear response exists at the crack as we expect. By scanning along the crack, it can be imaged. (c) Spatial distribution of the fundamental and (d), the second harmonic TR signal above the crack, normed to the maximum amplitude. The crack was about 2 mm in diameter (see Sutin and Johnson, 2005).

surface. For observation of nonlinear effects, narrow band filtering was used to detect the second harmonic of the TRA focused signal (Fig. 4.13). It can be seen that that the amplitude of the second harmonic of the signal detected above the crack is much higher than the amplitude of the harmonic from the intact surface signal. This experiment demonstrates the feasibility of the application of TR NEWS for crack imaging (see Sutin et al., 2004, for details). Other methods of nonlinear imaging are in development as well, in particular one using modal analysis (Van Den Abeele, this volume).

4. Some New Areas of Study and Application

4.1 Granular Media and Strong Ground Motion

Much recent effort has gone into nonlinear studies of granular media by the group at Université du Mans (France) and a collaborative effort between Los Alamos and Université de Marne-la-Vallée (France). Granular media is another member of the large class exhibiting nonequilibrium dynamics. It is interesting material from the perspective of earth processes, particularly strong ground motion and earthquake physics, and because it is a much simpler system to understand than many others of the class: One can apply Hertz-Mindlin theory (or some variation thereof) in order to understand its elastic behavior.

In earthquake strong-ground-motion, broad-frequency band waves propagate from hypocentral depths of order 10 km to the earth's surface. Sediments at the surface, composed of granular media, can respond by ringing at their resonance modes. The shear modes are particularly dangerous because, if they couple into building (or other structure) modes, damage or failure of the structure can take place.

Predicting the elastic linear and nonlinear behavior of near surface sediments during an earthquake is a large field of study in itself. Some years ago, it was demonstrated that significant elastic nonlinear behavior manifest by changes in surface-layer resonances may take place during large earthquakes, due to nonlinear response. For instance, a 75% decrease in resonance frequency was observed at one site in the Los Angeles Basin during the 1994 Parkfield, California earthquake (Field et al., 1997). The magnitude of this change came as a significant surprise to the seismic community. Currently, in collaboration with the United States Geological Society, the University of Memphis, the University of Texas at Austin, the University of Massachusetts at Amherst and the Massachusetts Institute of Technology, we are applying an active, large-vibrator source to *in situ* characterization of the near surface layers in an attempt to induce and measure nonlinear response. We observe significant nonlinear response of a near-surface layer from a recent, preliminary experiment at Garner Valley California, located near the San Andreas Fault southeast of Los Angeles. Figure 4.14 shows results of one experiment where the vibrator source was driven in compression in an NRUS-type experiment. A decrease in resonance frequency of order 25% over a strain interval of approximately 10^{-6}-10^{-4} was observed. Slow dynamics appeared to be present as well; however due to experimental difficulties, the observation was unconvincing. Future experiments are planned, and at least one follow-on experiment

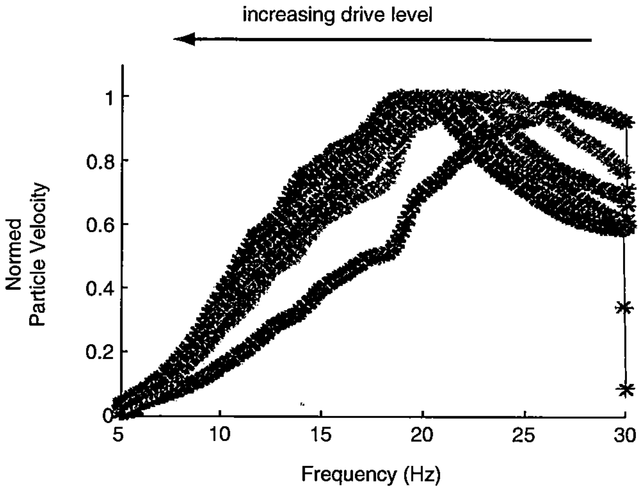

Fig. 4.14. NRUS field experiment at Garner Valley, California. The curves are more complex than in lab studies; however, the dominant frequency peak shift is significant, from approximately 27 to 19.5 Hz over a strain interval of about 10^{-6} to 10^{-4}. The resonating layer is 3-m thick. The shear wave resonance shift is even larger (from Pearce et al., 2004).

will have been completed by the time this paper goes to press. Note that the results indicate that nonequilibrium dynamics takes place over frequencies from order 1 Hz to hundreds of kHz if we compare them to laboratory experiments described above.

4.2 Granular Media and Earthquake Triggering

Recently, we speculated that a phenomenon known as *dynamic earthquake triggering* (Gomberg et al. 2001; 2004) could be due, at least in part, by nonequilibrium dynamics (Johnson and Jia, 2005). Normally, an earthquake exhibits precursors known as foreshocks, followed by a main shock (the magnitude of which is reported for an earthquake—the associated smaller earthquakes are not normally reported to the public), followed by aftershocks. Under certain, and apparently rare conditions (Gomberg et al., 2001), some of the aftershocks can take place at hundreds of kilometers from a mainshock at the time or soon after the seismic wave from the mainshock impinges on a distant fault. This is the phenomenon of *dynamic earthquake triggering*. It has been a puzzle for a number of years as to why dynamic triggering takes place because wave strains at these distances tend to be order 10^{-6} and is it difficult to understand how such small strains could be responsible for this phenomenon.

We speculate that if the conditions are right, triggering may be due to nonlinear softening and weakening of the fault core (the gouge material, granular in nature, that is created by a fault as it progressively slips over the history of the fault). Our conceptual model is that the dynamic wave temporarily reduces the core modulus. The modulus reduction is accompanied by a material strength reduction sufficient to induce fault slip, thereby triggering events. Figure 4.15 shows how this may happen. Taking Eq. 1, (or any nonlinear material softening theory for that matter) we relate the material

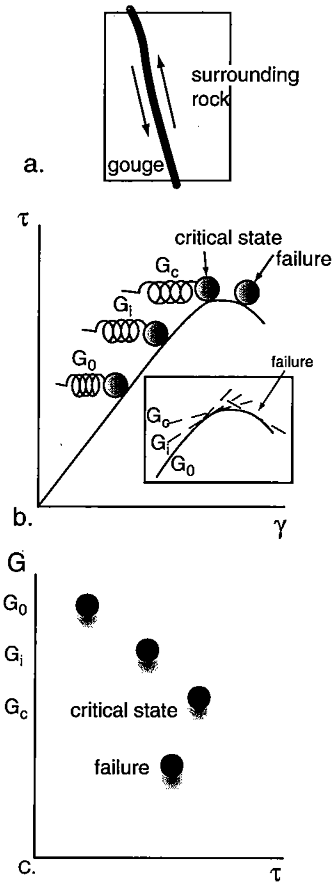

Fig. 4.15. How nonlinear dynamics takes a fault in a critical state to failure. (a) the physical system. (b-c) shear-stress τ, shear-strain curve γ and shear modulus G of the fault gouge. A wave with nonlinear amplitudes causes the modulus to decrease. If the gouge is in a critical state, near failure, the seismic wave can cause modulus softening and failure.

softening to weakening. We consider the competent fault blocks containing the softer fault core in Fig. 4.15a, and the effect of a seismic wave impinging on the system that is in a *critical state*, near failure (Figs. 4.15b, 4.15c). In order for triggering to take place, we speculate that the *triggering conditions* must be met: (a) experiments indicate that strain amplitude must be above 10^{-6}; (b) the fault must be in a *critical state*, near failure; and (c) the confining pressure in the fault core must be small. Small confining pressures are shown to be necessary based on numerous tests that indicate elastic nonlinearity decreases with confining pressure (e.g., Zinszner et al., 1997). Confining pressures could be low at earthquake nucleation depths (order 10 km), if fluid pressures are high. There is observational evidence that suggests this is the case in some faults (e.g., Nur and Booker, 1992; Miller, 1996). We suggest that this is why most seismic waves, even from large events, do not cause triggering (except near the earthquake source)—the triggering conditions are not met: their strain amplitudes tend to be 10^{-7}-10^{-6} at regional distances, and the other conditions may be quite rare. Only large events that focus sufficiently large strains cause triggering beyond what is traditionally deemed the aftershock zone. Field observations support this (J. Gomberg, personal communication, 2005). In these cases, the fault core must meet the triggering conditions, where the fault core can be instantaneously taken through instability to failure by the impingement of the seismic wave.

5. On the origin on Nonequilibrium Dynamics

5.1 Regimes of nonlinear dynamics

Many aspects of nonequilibrium dynamics remain to be understood. One aspect is the process of slow dynamics and how it relates to nonlinear fast dynamics and conditioning. That issue is progressively being addressed by various groups (e.g., Tencate et al., 2004; Pasqualini, this volume). We now understand that there are clear regimes of elastic behavior, contrary to what was thought by some by some of us in the past. These claims were based on the fact that no linear elastic regime seemed to be observed (e.g., Zinszner et al., 1997, Guyer and Johnson, 1999). This erroneous interpretation was due to thermal contamination at very low strain levels that masked the dynamic elastic behavior. In the lowest amplitude regime, the materials behave linearly—there is no modulus dependence on strain amplitude (Pasqualini, this volume). In the next regime, the materials act as a classical nonlinear oscillator that can be described by Landau theory (up to strains of roughly 1-3x10^{-6} at ambient conditions), and above this, nonequilibrium dynamics emerges. Figure 4.16 shows observations for many rocks over under

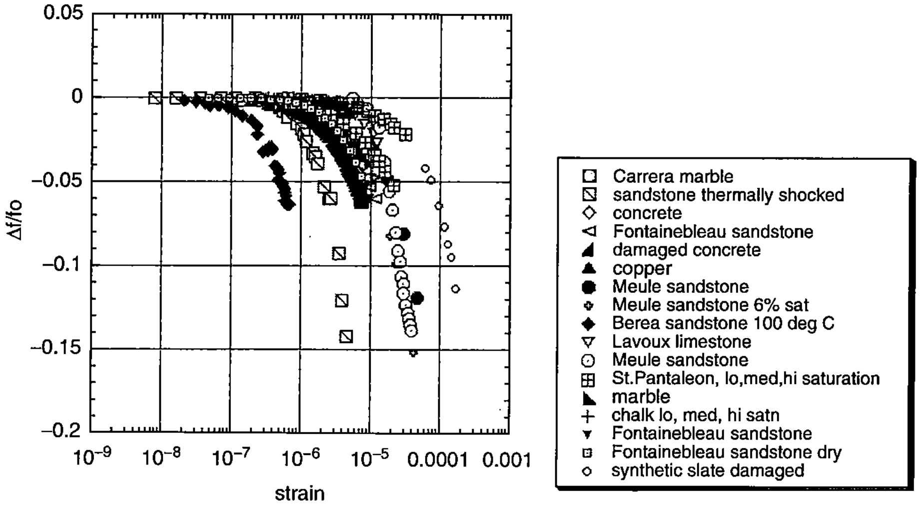

Fig. 4.16. NRUS experimental data taken over a range of saturation, temperature, and "damage" conditions. Based on recent, very careful experiments in Berea and Fontainebleau sandstones, it is currently believed that there are three regimes on elastic behavior (TenCat et al., 2004). One, at low amplitude, that is elastically linear. Following this is elastic behavior that is Landau in nature, and, above this strain, nonequilibrium behavior exists (data taken by B. Zinszner M. Masson, P. Rasolofasoan and P. Johnson at the Institut Francais du Petrole; Slate data from K. Van Den Abeele, Catholic University Leuven Campus Kortrijk).

many saturation conditions extracted from data presented in Johnson et al., (1996). The regimes are material dependent as one might guess, and this is clear from Fig. 4.16 as well. This last and most interesting regime (nonequilibrium) remains to be carefully understood in terms of what is physically taking place (e.g., is it simply a mix of nonlinear fast dynamics an slow dynamics?), and to develop a verifyable physics-based theory that describes all aspects of it. The original P-M Space theory does not account for conditioning or slow dynamics (e.g., Guyer et al., 1997). The variations of the P-M space theory that include conditioning and slow dynamics, although effective at modeling observed behaviors, are *ad hoc*, based on thermal fluctuations (e.g., Scalerandi et al., 2003). Physical based models such as a recently-proposed ratchet-model (Vakhnenko et al., 2004) are physically-based but the physics are as yet verified by experiment and will be hard to do so. Models indicating that thermal heating and diffusion are the source of nonequilibrium dynamics are questionable if one invokes three dimensional thermal diffusion, which is the case (Zaitsev et al., 2002), as shown by Pasqualini (in preparation, 2006).

Some have suggested that fluids are responsible for nonequilibrium dynamics. Van Den Abeele et al. (2000) have shown that fluids act to modify the internal forces in porous media and thereby influence the nonlinear behavior, but are not responsible for the underlying behavior. In any case, some materials in the class are dry with no means for fluid penetration (e.g., gray iron, alumina ceramic are two examples described in Johnson and Sutin, 2005).

A very important issue that has not been explored experimentally is whether shear sliding is the fundamental cause, at least in some cases, of nonequilibrium dynamics, as some of us currently believe. Many controlled experiments have been conducted with longitudinal or bulk modes. For instance, there is evidence suggesting the nonlinear response in shear is larger than in bulk mode but experiments aimed at isolating shear from other effects have not been conducted to our knowledge. Because I believe the physical origin is shear sliding, such experiments would aid tremendously in helping verify such a hypothesis and developing theory.

5.2 Slow and Nonlinear Fast Dynamics Including Dissipation

Results were recently reported of the first systematic study of nonlinear fast dynamics and slow dynamics in a number of solids (Johnson and Sutin, 2005). Observations were presented from seven diverse materials showing results of nonlinear fast dynamics and slow dynamics (see Fig. 4.17). The materials include samples of gray iron, alumina ceramic, quartzite, cracked Pyrex glas, marble, sintered metal, and perovskite ceramic. It was shown that materials that exhibit nonequilibrium behavior have very similar ratios of amplitude-dependent internal-friction to the resonance-frequency shift as a function of strain amplitude. The ratios range between 0.28 and 0.63 (except for cracked Pyrex glass, which exhibits a ratio of 1.1), and the ratio appears to be a material characteristic. The ratio of internal friction to resonance frequency shift as a function of time during slow dynamics is time independent, ranging from 0.23 - 0.43 for the materials studied (within the error bars they are approximately the same). The above relations relating nonlinear attenuation and frequency shift in slow and fast dynamics demand

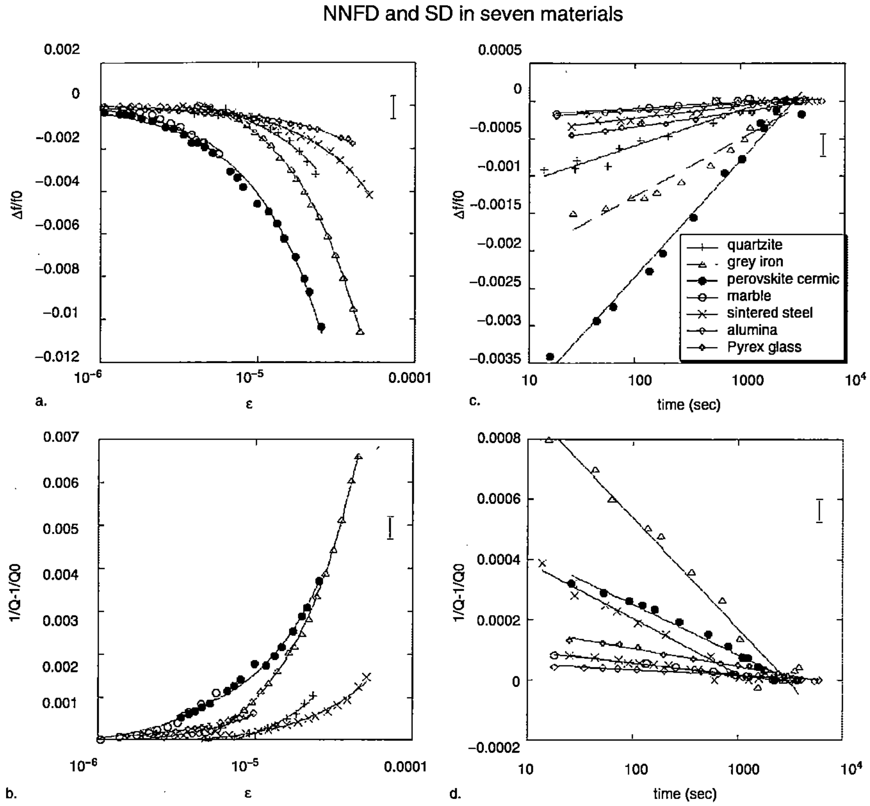

Fig. 4.17. Nonequilibrium behavior in seven materials obtained from NRUS measurements (from JS). (a) and (c) show the change is resonance frequency normalized to the linear resonance frequency for nonlinear fast and slow dynamics respectively. (b) and (d) show the behavior in wave dissipation, 1/Q, for nonlinear fast and slow dynamics respectively, where Q_0 is the linear value and Q is the nonlinear value. The error bars shown are based are extremely conservative: they are obtained from the frequency sampling rate of the experiment, carried through all of the calculations. The lines in the figures are present to guide the eye (they are linear fits in the case of fast dynamics and logarithmic fits for slow dynamics).

more study to see if the relation between nonlinear dissipation and frequency shift (modulus change) are always material dependent, and what that implies for a theoretical description.

We note that some characteristics of slow dynamics have yet to be studied. The slope amplifier offers a means to carefully study the recovery process. For instance, we may see physical processes that could aid in theory development that have been overlooked in the past. Figure 4.18 hints at one interesting behavior where we may be observing something like isolated cascade slip events during recovery.

5.3 On developing a physics-based model of nonequilibrium dynamics

One suggested approach to addressing a generalized, physics-based model is to start with specific systems and look for similarities in the underlying physics. For instance,

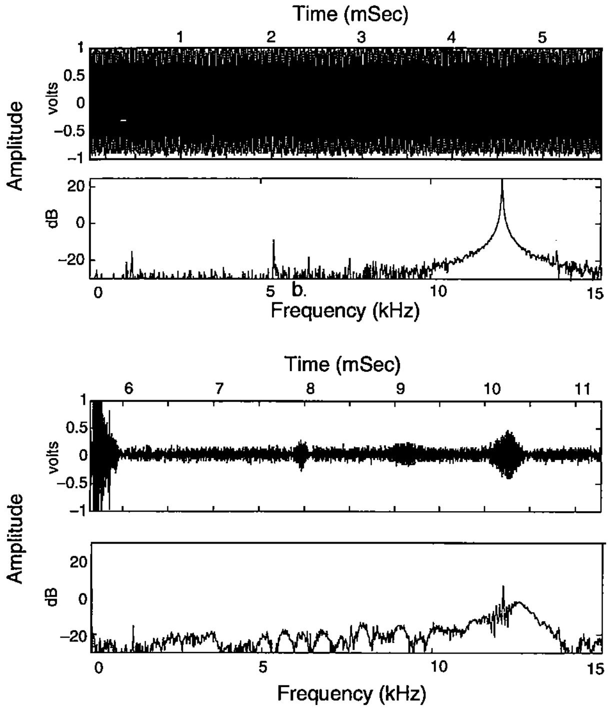

Fig. 4.18. Interesting characteristics of slow dynamics in Fontainebleau sandstone applying a low amplitude probe and an impulse. The sample was a bar of sandstone approximately 2x20 cm in dimension. A low amplitude probe was applied at one end and detected at the other. A tap was introduced to induce slow dynamics. The top two panels show the probe time-series and corresponding Fourier transform before the tap. The bottom panels show the time and frequency response during, and for a number of milliseconds after, the tap. Note the "humps" in amplitude. What are these? Could they be large slip events or some other as yet, unidentified process that takes place during slow dynamics? This general topic requires more study and may aid in a new theoretical description of nonequilibrium dynamics.

in granular media, Hertz-Mindlin theory can be applied as noted above; for cracks in metals, crack dynamics and dislocation-point defect interaction may be applied. The underlying shear-mechanisms may ultimately be related in a broader model. Incorporating in slow dynamics may be the most difficult aspect (see below). Such an approach could presumably evolve to a generalized nonequilibrium theory for all, or most of the materials in the class.

5.4 Universal behavior?

A discussion has taken place in the literature as to whether or not the observed behaviours of nonequilibtium dynamics are universal (see, e.g., Guyer and Johnson, 1999; Hirsekorn and Delsanto, 2004; Hirsekorn and Delsanto this volume). I and others would argue that they are not in the sense of critical phenomena such as a phase change, but they are in the sense that the nonlinear signatures are identical across a large number of very different materials (Johnson and Sutin, 2005).

6. Summary and Conclusions

In summary, we know much from experiments regarding the behaviours and the breath of the class of materials that exhibit nonequilibrium dynamics. Fundamentally, we believe that the nonlinear response is attributable to the damage features in the material at many scales. A proper theory containing the underlying physics is still to be addressed, however. Nonetheless, many applications based on nonlinear methods, including applications to earthquake strong ground motion and elsewhere, but especially in regard to NonDestructive testing have been developed. One can determine whether or not a material exhibits nonequilibrium behaviour and simultaneously if it is damaged, if there is disbonding, etc. However, without knowledge of the physical basis, the relation between the nonlinear response (often in the form of the hysteretic nonlinear parameter α) and damage quantity or other features responsible for the nonlinearity as in granular media, must be obtained empirically. With a physics-based model we will have the means to relate the nonlinear response to damage quantity, with the caveat that, in solids with macroscale damage, only some portion actively contributes to the nonlinear response (we know this from the NWMS imaging experiment mentioned above where crack tips seemed to make the primary contribution to the nonlinear response). Imaging of nonlinear scatterers is in its infancy and should progress significantly in the near future. Much work remains in regards to understanding the relation of slow and fast dynamics, the details of slow dynamics. There are enormous and fascinating opportunities for new research that can provide insight into nonequilibrium dynamics.

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References

Field, E. H., Johnson, P. A., Beresnev, I. and Zeng, Y., 1997, Nonlinear ground-motion amplification be sediments during the 1994 Northridge earthquake, *Nature* **390**: 599–602.

Fink, M., 1997, Time reversed acoustics *Physics Today* **50**: 34–40.

Gomberg, J., Reasenberg, P. A., Bodin P. and Harris R.A., 2001, Earthquake triggering by seismic waves following the Landers and Hector Mine earthquakes, *Nature* **411**: 462–466.

Gomberg, J., Bodin, P., Larson K., and Dragert, H., 2004, Earthquake nucleation by transient deformations caused by the M = 7.9 Denali, Alaska earthquake, *Nature* **427**: 621–624.

Guyer, R. A., and Johnson, P. A., 1999, The astonishing case of mesoscopic elastic nonlinearity, *Physics Today* **52**: 30–35.

Guyer, R. A., McCall, K. R., Boitnott, G. N., Hilbert, L. B., and Plona, T. J., 1997, Quantitative implementation of Preisach-Mayergoyz space to find static and dynamic elastic moduli in rock, *J. Geophys. Res.* **102**: 5281–93.

Hirsekorn, S., and Delsanto, PP, 2004, On the universality of nonclassical nonlinear phenomena and their classification, *Applied Phys. Lett.* **84**: 1413–1415.

Johnson, P., 1999, The new wave in acoustic testing, *Materials World, the J. Inst. Materials* **7**: 544–546.

Johnson, P.A., and Jia, X, 2005, Nonlinear dynamics, granular media and dynamic earthquake triggering, *Nature, in review.*

Johnson, P. and Sutin, A., 2005, Slow dynamics in diverse solids, *J. Acoust. Soc Am.* **117**: 124–130.

Johnson, P. A. and A. Sutin, 2005, QNDE 2004, Nonlinear elastic wave NDE I. Nonlinear resonant ultrasound spectroscopy (NRUS) and slow dynamics diagnostics (SDD), Review of Progress in Quantitative Nondestructive Evaluation, Volume 24B, Thompson, D and Chimenti D., Eds., 377–384.

Johnson, P. A., Zinszner, B., and P. N. J. Rasolofosaon, 1996, Resonance and nonlinear elastic phenomena in rock, *J. Geophys. Res.* **101**: 11553–11564.

Kazakov, V. V., Sutin, A. and Johnson, P. A., 2002, Sensitive imaging of an elastic nonlinear wave source in a solid, *Applied Physics Letters* **81**, 646–648.

Korotkov, A.S., Slavinsky, M.M., Sutin, A.M. *Acoustical Physics.* **40**, 84 (1994).

Migliori, A. M.. and Sarrao, J., 1997, "Resonant Ultrasound Spectroscopy", John Wiley.

Miller, S. A., 1996, Fluid-mediated influence of adjacent thrusting on the seismic cycle at Parkfield, *Nature* **382**: 799–802.

Maev, RG; Solodov, I.Yu., 2000, Nonlinear acoustic spectroscopy of cracked flaws and disbonds: fundamentals, techniques and applications, Review of Progress in Quantitative Nondestructive Evaluation, 25–30 July 1999, Montreal, Que., Canada, AIP Conference Proceedings 2000, **509-B**, 1409–1416.

Nur, A. and Booker, J., 1977, Aftershocks caused by pore fluid flow?, *Science* **175**: 885–887.

Ostrovsky, L. and Johnson, P. A., 2001, Dynamic nonlinear elasticity in geomaterials, *Rivista del Nuovo Cimenta* **24**, 1–46.

Pasqualini, D., this volume.

Pasqualini, D., 2006, The role of thermal diffusion in nonequilibrium dynamics, *in preparation.*

Pearce, F., Bodin, P. Brackman, T., Lawrence, Z., Gomberg, J., Steidl, J., Menq, F., Guyer, R., Stokoe, K., and Johnson, P. A., 2004, Nonlinear soil response induced in situ by an active source at Garner Valley, *Eos Trans. American Geophysical Union* **85**, Fall Meeting Supplement, Abstract S42A-04.

Sutin, A., TenCate, J. and Johnson, P. A., 2004, Single-channel time reversal in elastic solids, *J. Acoust. Soc. Am.*, **116**, 2779–2784.

Sutin, A. and Johnson, P. A., 2005, Nonlinear elastic wave NDE II. Nonlinear wave modulation spectroscopy and nonlinear time reversed acoustics, Review of Progress in Quantitative Nondestructive Evaluation, Volume 24B, Thompson, D and Chimenti D., Eds., 385–393.

TenCate J. and Shankland T., 1996, Slow dynamics in the nonlinear response of Berea sandstone, *J. Geophys. Res.* **23**, 3019–3022.

Ulrich, T. J., Sutin, A. and Johnson, P. A., 2006, Imaging Nonlinear Scatterers Applying the Time Reversal Mirror, *J. Acoust. Soc. Am.* **119:** 1514–1518.

Van Den Abeele, K. E.-A., Johnson, P. A. and Sutin, A., 2000, Nonlinear Elastic Wave Spectroscopy (NEWS) techniques to discern material damage. Part I: Nonlinear Wave Modulation Spectroscopy (NWMS), *Research on NonDestructive Evaluation* **12**: 17–30.

Van Den Abeele, K. E.-A., Carmeliet, J., TenCate, J. and Johnson P. A., 2000, Nonlinear Elastic Wave Spectroscopy (NEWS) techniques to discern material damage. Part II.

Zaitsev, V., Gusev, V. and Castagnede, B., 2002, Thermoelastic mechanism for logarithmic slow dynamics and memory in elastic wave interactions with individual cracks, *Phys. Rev. Lett.* **84**: 159–162.