Comparison of Two Approaches for Determining Ground-Water Discharge and Pumpage in the Lower Arkansas River Basin, Colorado, 1997–98

By Russell G. Dash, Brent M. Troutman, and Patrick Edelmann

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CONVERSION FACTORS AND ABBREVIATIONS

Multiply	By	To obtain
acre-foot (acre-ft)	1,233	cubic meter
foot (ft)	0.3048	meter
gallons per minute (gal/min)	0.00379	cubic meter per minute
inch (in.)	2.54	centimeter
kilowatthour (kWh)	3,600,000	joule
kilowatthour per acre-foot	2,919	joule per cubic meter

The following terms and abbreviations are used in this report:

Power Conversion Coefficient (PCC)
Totalizing Flowmeter (TFM)
Colorado Division of Water Resources (CDWR)
U.S. Geological Survey (USGS)

Method of Portable Flowmeter:

C (Collins flowmeter)
M (McCrometer flowmeter)
P (Polysonic flowmeter)

Make of Inline Totalizing Flowmeter:

M (new McCrometer TFM)
S (new Signet TFM)
X (existing McCrometer TFM)
B (existing Badger TFM)
R (existing Rockwell TFM)

Type of Discharge Distribution System:

O (open)
L (low-pressure)
S (sprinkler)
C (complex)

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Several sections of this report contain detailed mathematical derivations and statistics. To facilitate reading and use of this report, the report is organized in a manner that presents the primary results first, then the detailed mathematical derivations and statistics in the sections that follow titled "Details of Analysis and Results". For those readers who are interested only in the primary results, rather than the derivations and details, they may wish to read the sections titled "Primary Results" and skip the sections titled "Details of Analysis and Results".

EXECUTIVE SUMMARY

Introduction

In March 1994, the Colorado Division of Water Resources (CDWR) adopted "Rules Governing the Measurement of Tributary Ground Water Diversions Located in the Arkansas River Basin" (Office of the State Engineer, 1994); these initial rules were amended in February 1996 (Office of the State Engineer, 1996). The amended rules require users of wells that divert tributary ground water to annually report the water pumped monthly by each well. The rules allow a well owner to report the pumpage measured by a totalizing flowmeter (TFM) or pumpage determined from electrical power data and a power conversion coefficient (PCC) (Hurr and Litke, 1989).

Opinions by representatives of the State of Kansas, presented before the Special Master hearing a court case [State of Kansas v. State of Colorado, No. 105 Original (1996)] concerning post-Compact well pumping, stated that the PCC approach does not provide the same level of accuracy and reliability as a TFM when used to determine pumpage.

In 1997, the U.S. Geological Survey (USGS), in cooperation with the CDWR, began a 2-year study to compare ground-water pumpage estimates made using the TFM and the PCC approaches. The study area was along the Arkansas River between Pueblo, Colorado, and the Colorado-Kansas State line (fig. 1).

The two approaches for estimating ground-water discharge and pumpage were compared for more than 100 wells completed in the alluvial aquifer of the Arkansas River Basin. The TFM approach uses an inline flowmeter to directly measure instantaneous discharge and the total volume of water pumped at a well. The PCC approach uses electrical power consumption records and a power conversion coefficient to estimate the pumpage at ground-water wells.

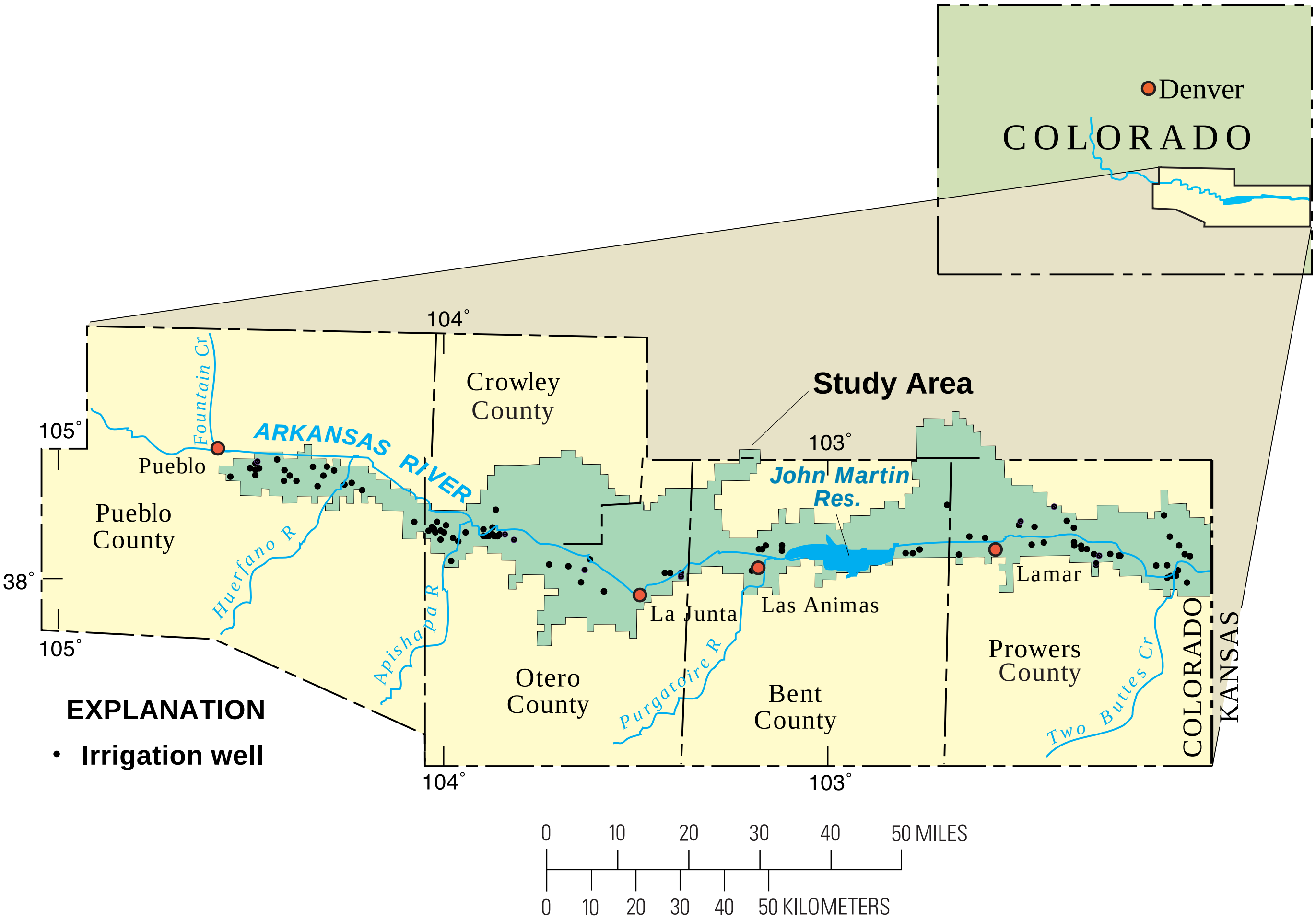

Figure 1. Map showing location of study area and irrigation-wells used in the study, 1997–98.

This executive summary describes the results of the comparison of the two approaches. Specifically, (1) the differences in instantaneous discharge measured with three portable flowmeters and measured with an inline TFM are evaluated, and the statistical differences in paired instantaneous discharge between the two approaches are determined; (2) short- and long-term variations in the PCC's are presented; (3) differences in pumpage between the two approaches are evaluated, and the statistical differences in pumpage between the two approaches are determined; (4) potential sources of discrepancy between pumpage estimates are discussed; and (5) differences in total network pumpage using the two approaches are presented.

During the irrigation seasons of 1997 and 1998, instantaneous discharge and electrical power demand were measured at randomly selected wells to determine PCC's. At more than 100 wells, the PCC's determined during the 1998 season were applied to total electrical power consumption data that was recorded between the initial and final readings at each network well site in 1998 to estimate total ground-water pumpage.

At each site, an inline TFM was installed in a full-flowing, acceptable test section of pipe on the discharge side of the pump where the measurement of discharge was made. Measurements of instantaneous ground-water discharge also were made using three different types of portable flowmeters. The average velocity multiplied by the cross-sectional area of the discharge pipe was used to compute the discharge in gallons per minute. Whenever possible, discharge measurements were made at each network site using all three types of portable flowmeters.

Comparison of Instantaneous Ground-Water Discharge Measurements

Instantaneous discharges measured using portable flowmeters were compared to instantaneous discharges measured using TFM’s. The analysis is based on 747 paired measurements taken at 105 wells during a 2-year period. A mixed analysis of variance model with both fixed and random effects was applied. The overall mean difference in discharge measurements between portable flowmeters and TFM’s was 0.00 percent, indicating no difference on average between the two approaches for the entire network of wells. More than 80 percent of the differences in the paired discharge measurements were less than 10 percent.

Temporal Variations in Power Conversion Coefficients

Analysis of variations in PCC’s measured during the 1998 irrigation season indicated that 58 percent of 104 wells had less than 10-percent change, and 86 percent of 104 wells had less than 20-percent change in the well PCC’s. Seasonal variations in PCC’s generally were not evident for the measurements made during the 1998 irrigation season. Thirty-seven of the 41 wells with PCC’s measurements in 1997 had at least one PCC in the same range as 1998 PCC measurements. The comparison of the 2 years of data indicate that PCC measurements were similar in 1997 and 1998. About 48 percent of available pre-study State-approved PCC’s made during 1994–97 were within 10 percent of the 1998 site average PCC’s, and about 67 percent of the pre-study State-approved PCC measurements made during 1994–97 were within 20 percent of the 1998 site average PCC’s.

Comparison of Ground-Water Pumpage Estimates

Pumpage estimates computed using the PCC approach were compared to pumpage measured by a TFM at network wells. PCC pumpages were computed by applying each PCC obtained during a site visit in 1998 to the total 1998 electrical power consumption. The analysis was based on 553 paired pumpage estimates at 103 wells. The overall mean difference in pumpage between the TFM and PCC approach was 0.01 percent for the entire network of wells, indicating no significant difference on average between pumpage measured by a TFM and pumpage computed by the PCC approach. About 80 percent of the differences in the paired pumpage estimates were less than 10 percent.

Sources of Discrepancy Between Pumpage Estimates

There are several potential sources of discrepancy between pumpage as measured by a TFM and pumpage as computed by the PCC approach. One potential source is temporal variability of the PCC. The analysis indicated that the year-to-year variance component was about nine times the date-within-year variance component and represented a standard deviation of about 15 percent, indicating that the year-to-year variability was a major component of overall variability for this PCC data set.

Estimation of Total Network Pumpage

Differences in the total or aggregated pumpage for a network of wells was estimated by dividing the range of TFM pumpage into equal subdivisions based on the magnitude of TFM total pumpage. Because the correct number of subdivisions (strata) is not known with information now available, the mean and standard deviation of differences in the total pumpage was determined conditionally for several numbers of strata. For a network of 103 wells and a number of strata greater than 10, the resulting mean and standard deviation indicates that, for any given year, there is a 95-percent probability that the difference in aggregated pumpage between the TFM and PCC approaches would be between about –3.41 and 1.59 percent. The analysis indicates that the difference in aggregated pumpage would be expected to be smaller as the total number of wells becomes larger.

INTRODUCTION

Irrigation is the largest use of water in southeastern Colorado, and ground water is a supplemental source for irrigators in the Arkansas River Basin because surface-water supplies in the basin are inadequate to meet irrigation demand. During the past 40 years, ground-water withdrawals were occasionally measured (Luckey, 1972) but were not routinely metered. Some estimates of ground-water withdrawals were reported (Litke and Appel, 1989). However, the accuracy of the ground-water withdrawal estimates were not known.

In March 1994, the Colorado Division of Water Resources (CDWR) adopted “Rules Governing the Measurement of Tributary Ground Water Diversions Located in the Arkansas River Basin” (Office of the State Engineer, 1994); these initial measurement rules were amended in February 1996 (Office of the State Engineer, 1996). The “Amendments to Rules Governing the Measurement of Tributary Ground Water Diversions Located in the Arkansas River Basin” were approved in June 1996 and require that about 1,600 wells that divert tributary ground water must annually report the water pumped monthly by each well. The rules allow a well owner the option of reporting pumpage measured by a totalizing flowmeter (TFM) or estimated using electrical power consumption data and a power conversion coefficient (PCC) (Hurr and Litke, 1989). The inline TFM and the PCC rating must be checked at least once every 4 years by a person approved by the State Engineer. A TFM is an inline flowmeter that directly measures the total volume of water pumped from the well. The PCC approach uses measurements of instantaneous ground-water discharge, hereinafter referred as instantaneous discharge, and instantaneous electrical power demand, hereinafter referred as power demand, to determine the number of kilowatthours of energy required to pump 1 acre-foot of water. Since 1994 when the rules became effective in the river basin, most well owners have chosen to use the PCC approach to determine ground-water pumpage from their irrigation wells.

Opinions by representatives of the State of Kansas, presented before the Special Master of the U.S. Supreme Court hearing a case (State of Kansas v. State of Colorado, No. 105 Original (1996)) concerning well pumping after approval of the Arkansas River Compact of 1948, stated that the PCC approach does not provide the same level of accuracy and reliability as the TFM’s when used to determine annual ground-water pumpage. Thereafter, the Colorado State Engineer proposed a study to determine the comparability of estimates of ground-water pumpage using the TFM and PCC approaches. In 1997, the U.S. Geological Survey (USGS), in cooperation with the Colorado Department of Natural Resources, Division of Water Resources, Office of the State Engineer (CDWR), began a 2-year study to compare ground-water pumpage estimates made using the TFM and PCC approaches. The study area was the Arkansas River alluvial valley between Pueblo, Colorado, and the Colorado-Kansas State line (fig. 1).

Purpose and Scope

This report provides a comparison of two approaches for determining ground-water discharge and pumpage. Specifically, this report:

1. Evaluates differences in instantaneous discharge between TFM’s and three portable flowmeters used with the PCC approach, and determines if differences in instantaneous discharge for the TFM and PCC approach are statistically significant;
2. Evaluates short- and long-term variations in PCC’s, including whether seasonal variations in PCC’s were evident;
3. Evaluates differences in ground-water pumpage estimated with the TFM and PCC approaches, and determines if differences in ground-water pumpage estimated with the TFM and PCC approaches are statistically significant;
4. Evaluates potential sources of discrepancy between pumpage estimates; and
5. Estimates differences in total network pumpage using the two approaches.

One hundred and six irrigation wells that are powered by electric pumps were selected for this study from about 1,300 irrigation wells in the study area. The network of 106 irrigation wells consisted of 11 wells that had TFM’s installed prior to the study and 95 randomly selected wells that had new TFM’s installed during 1997–98. During the irrigation season of 1997, instantaneous discharge was measured at 46 wells (43 of which had TFM’s in 1997) and, during 1998, at 105 wells. One irrigation well was dropped from the network following the 1997 irrigation season because the well owner had reconfigured the discharge distribution system and combined the plumbing of two wells together. This activity created a complex well that was not suitable under the amended rules for Rule 3.6 analyses (Office of the State Engineer, 1996), making the well unacceptable for the continued application of a PCC to determine ground-water pumpage.

During the study, PCC's were calculated each time a portable flowmeter measurement of the instantaneous discharge and power demand were made at a well. At 104 of the wells, PCC's determined during the 1998 irrigation season were applied to the total electrical power consumption recorded between the initial and final readings at the site in 1998 to estimate total ground-water pumpage for the period. The total pumpage estimate derived using the PCC calculation then was compared to the total pumpage measured using the TFM at 104 wells. However, pumpage data from one well were omitted because it was determined that the existing TFM (make R) was not working properly, which resulted in 103 wells that were used for comparison of ground-water pumpage.

Acknowledgments

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METHODS OF INVESTIGATION

Data collection and analysis consisted of several phases: (1) Identification of potential sites; (2) selection of sites for TFM/PCC comparisons; (3) installation of the TFM's; (4) measurement of instantaneous discharges; (5) determination of PCC's; (6) computation of ground-water pumpage using TFM and PCC approaches; and (7) analysis of data.

Initially, the CDWR identified more than 1,300 large-capacity irrigation wells (wells that discharge more than 50 gal/min) in the Arkansas River Valley between Pueblo, Colorado, and the Colorado-Kansas State line for which the PCC approach might be used to determine ground-water pumpage under the amended rules established by the Office of the State Engineer (1996). This initial list of wells was decreased to about 800 potential sites for TFM/PCC consideration based on the following criteria:

1. The well was reported as active and was connected to a power source.
2. The well used an electric motor, as opposed to an internal combustion engine.
3. The well had at least 10 acre-ft of reported annual pumpage at least once since 1994.

A computer program (Scott, 1990) was used to randomly select one primary and four alternative sites for each potential well in the TFM/PCC network. Each primary site was evaluated by CDWR and inventoried to determine its suitability for inclusion in the TFM/PCC study. If a primary site was rejected, a randomly selected alternative site was evaluated and so on down the list of alternatives until a suitable site was found. During 1997, CDWR evaluated 107 wells for potential TFM installation; in 1998, CDWR evaluated 122 wells for additional TFM installations. The most common reasons for rejection and the total number of well sites rejected during 1997–98 were as follows:

1. The site was determined to be a complex system and was found unsuitable for Rule 3.6 analyses, or the site was determined to be a compound system, or the owner indicated future modifications were planned that would make the site unsuitable for continued application of the PCC approach. Compound system means that more than one electrical device is being operated from the same electrical power meter. (38 wells rejected)
2. The discharge pipe was in poor physical condition, the pump surged or was unable to maintain a full pipeline of flow at a measurement section, or

there was inadequate upstream or downstream distances available to correctly install a TFM. (32 wells rejected)

3. The well owner declined to participate in the TFM/PCC study. (19 wells rejected)
4. The well had less than 10 acre-ft of pumpage reported the previous year. (12 wells rejected)
5. The well appeared to be inactive, and the owner indicated it was not used. (9 wells rejected)
6. The discharge pipe was not a correct size for installation of a Signet TFM (one of the brands of TFM used in the study). Pipe was smaller than 8-inch diameter during 1997, or smaller than 6-inch diameter during 1998, or was larger than 12-inch diameter during either year. (24 wells rejected)

In 1997, permission to measure discharge at 46 wells was obtained, including 11 wells that had pre-existing TFM's and 35 wells where new TFM's were planned to be installed during the 1997 irrigation season. During 1997, discharge measurements of installed TFM's were made at 43 of the 46 wells in the monitoring network. One new TFM was not installed until the end of the 1997 irrigation season, and two of the new TFM's were returned to the factory for calibration and were not reinstalled until after the 1997 irrigation season. One pre-existing TFM well was reconfigured to a complex system after the 1997 irrigation season and was dropped from the study. During 1998, permission to install TFM's and measure discharge at 60 additional wells was obtained. The changes resulted in a final monitoring network of 105 wells having TFM's. However, upon evaluation of the data, an electric power meter at one site was found to be malfunctioning, resulting in 104 wells being used for analysis of variations in PCC's; and a TFM was found to be malfunctioning at another site, resulting in 103 wells being used to compare ground-water pumpage.

Each well in the network was visited to identify discharge system characteristics and to confirm that the PCC approach could be properly applied at the well in accordance with the amended rules (Office of the State Engineer, 1996). When possible, well owners and operators were interviewed and information was collected about normal operating conditions, flow ranges and pressures, and number of discharge distribution outlet locations. Well-identifying data were recorded from the motor, pump, and electrical meter nameplates during the visit.

The CDWR made an onsite identification of the type of discharge distribution system at each of the wells in the network, based on a visual observation of the discharge plumbing during the initial visit, which was confirmed before making subsequent field measurements. For this study, four major types of discharge distribution systems were identified. The well network included 65 open-discharge, 18 low-pressure, 10 sprinkler, and 12 complex discharge distribution systems. Hereinafter, the open-discharge distribution system type is referred to as type O, the low-pressure discharge distribution system type is referred to as type L, the sprinkler discharge distribution system type is referred to as type S, and the complex discharge distribution system type is referred to as type C.

According to the CDWR, well sites that are classified as complex systems will vary the total dynamic head (TDH) at the pump during the irrigation season. The change in TDH may result from wells that discharge into a pipeline with multiple outlet locations, multiple wells that discharge into one common pipeline, or wells where the method of water delivery changes between different types of distribution systems, such as open-discharge and sprinkler systems. The complex discharge sites that were included in the study network were sites where the wells discharged into a pipeline with more than one point of discharge (multiple outlet locations). As such, these sites qualified for use of the PCC approach pursuant to Rule 3.6 of the amended rules (Office of the State Engineer, 1996). For such sites, a PCC measurement was determined under the high TDH discharge point and a second PCC measurement determined under the low TDH discharge point; and a system PCC was calculated that was weighted on the basis of the PCC's at the discharge points and the expected crop water demand at each discharge point.

Totalizing Flowmeter Measurements

The accuracy of many factory-calibrated TFM's is reportedly 2 to 3 percent of discharge (M.H. Noffke, Great Plains Meter, Inc., written commun., 1998). To obtain an accuracy of 2 to 3 percent of discharge, a

TFM must be installed correctly, following the manufacturer's specifications. At each selected well, a TFM was installed inline in a full-flowing, acceptable test section of pipe on the discharge side of the pump where the measurement of water velocity was made. The flowmeter location was in a straight, constant-diameter length of pipeline without turbulence-inducing obstructions (elbows, valves, pumps, and changes in pipe diameter) for a certain distance upstream and downstream from the flowmeter installation point. The distances required usually were related to the diameter of the discharge pipe at the measurement location. The desired distance upstream for any flowmeter without a straightening vane installed was 10 pipe diameters and for flowmeters with a straightening vane was 5 pipe diameters. At some wells, slight plumbing modifications, such as adding a pipe elbow, were made to the discharge pipe downstream from the flowmeter measurement location to maintain the required full-flowing condition in the pipe.

Two types of TFM's installed during this study were: (1) the propeller flowmeter manufactured by McCrometer, hereinafter referred to as make M; and (2) the rotating-blade flowmeter manufactured by Signet Scientific Corporation, hereinafter referred to as make S. The pre-existing types of TFM's were: (1) the propeller flowmeter manufactured by McCrometer, hereinafter referred to as make X; (2) the propeller flowmeter manufactured by the Badger Corporation, hereinafter referred to as make B; and (3) the propeller flowmeter manufactured by the Rockwell Corporation, hereinafter referred to as make R.

Twenty of the TFM's installed during this study were a prototype, rotating-blade flow sensor developed by the Signet Scientific Company (Tim Quinlin, George Fischer Inc., oral commun., 1999). Because of design-development limitations, the 10 Signet TFM's installed in 1997 were in irrigation wells that had a discharge pipe with a diameter of 8 in. or more, and the 10 installed in 1998 were in wells that had a discharge pipe with a diameter of 6 in. or more.

The cumulative volume pumped, as indicated by readings of the TFM's, was recorded on an irregular basis. During a site visit, a well discharge measurement was made by reading the register dials of the TFM and timing the index wheel for one complete revolution, then dividing the indicated volume by the elapsed time; the procedure was repeated nine more times; the recorded discharge was the average of the 10 values. The volume of water pumped between site visits was determined by recording the register dials of the TFM at the beginning of each visit. The total volume of water pumped at a study site during 1998 was determined as the difference between TFM readings made at the beginning and the end of the monitoring period.

Portable Flowmeter Measurements

During each site visit, electrical power measurements and other onsite information were recorded, and measurements of instantaneous discharge were made using as many as three different types of portable flowmeters—a manometer, an ultrasonic flowmeter, and a propeller-type meter. These portable flowmeters provided three different methods to determine the average velocity of water flowing through the discharge pipe. The average velocity, multiplied by the cross-sectional area of the discharge pipe, was used to compute the discharge in gallons per minute. Whenever possible for the PCC tests, instantaneous discharge measurements were made using all three portable flowmeters during each site visit. All PCC test measurements were made after the drawdown of the pumping water level had stabilized.

To compute well discharge for two of the three portable flowmeter types (manometer and ultrasonic flow meters), the inside pipe diameter was needed; therefore, throughout the study, inside pipe-dimension measurements were made consistently. The pipe-wall thickness was measured during each site visit using an ultrasonic thickness gage. The outside circumference of the discharge pipe was determined using a thin, flexible metal tape.

The first type of portable flowmeter, a manometer, measures differences in water pressure in an upstream and downstream direction and could be used in all the discharge pipe sizes in this study. A device referred to as a "Collins Meter", hereinafter referred to as method "C", was used to determine the average water-velocity distribution across the inside of the discharge pipe. A pitot tube that had two orifices (one oriented upstream and one oriented downstream) was inserted across the diameter of the discharge pipe and a manometer used to measure the pressure difference between the dynamic (upstream) and static (downstream) orifices at two different points in the pipe's cross section. The measured pressure difference is proportional to the water velocity, and mean water velocity multiplied by the cross-sectional area of the pipe is the instantaneous discharge.

The second type of portable flowmeter was an ultrasonic flowmeter. Typical accuracy of an ultrasonic flowmeter is reportedly 1 to 5 percent (Omega Engineering Inc., 1992). An ultrasonic flowmeter manufactured by Polysonic, hereinafter referred to as method “P”, was used in this study and uses the transit-time method for flow measurement. Two transducers were mounted on the outside of the discharge pipe and functioned alternately as a transmitter and a receiver of ultrasonic signals sent upstream and downstream through the pipe. The time difference between the signals, averaged in the upstream and downstream directions, is proportional to the velocity of water flow. The flowmeter was programmed to process the information and output a discharge value every minute. Generally, 10 or more of the discharge readings were averaged to obtain the instantaneous discharge. Diagnostic menus were used to determine the acceptability during each test. Diagnostic parameters such as signal strength and a difference count were supplied by the equipment and had to be within specified limits to be a valid well discharge measurement.

The third portable flowmeter was a typical propeller-type flowmeter manufactured by McCrometer, hereinafter referred to as method “M”. The propeller-type flowmeter was mounted to the end of a section of plastic pipe with sufficient upstream length and attached with a rubber coupler to the open end of the discharge pipe to make a discharge measurement. During each site visit, well discharge measurements were made with a method M portable flowmeter by reading registers dials of the TFM and timing the index wheel for one complete revolution, and dividing the indicated volume by the elapsed time. Generally, 10 readings were made at each site and the recorded discharge was the average of the 10 values.

Power Conversion Calculations and Computations of Pumpage

The PCC is defined as the number of kilowatthours required to pump 1 acre-ft of water. Electrical power meters contain a disk that revolves as electricity passes through the meter. During a site visit, the meter disk was timed with a stopwatch for 10 complete disk revolutions to measure the rate per revolution. This rate measurement was repeated three times and used to determine the average rate of a disk revolution. Power demand, in kilowatts, was calculated from the equation:

$$\text{power demand} = (\text{rate}) \times (3.6) \times (\text{Kh factor}), \qquad (1)$$

where

rate = average time of disk revolution, in revolutions per second,
3.6 = conversion factor (kilowatt seconds per watthour), and
Kh factor = watthours per revolution (imprinted on the front of power meter).

Determining the PCC combines a concurrent measurement of well discharge (in gallons per minute) with the power demand of the pump (in kilowatts).

The PCC, in kilowatthours per acre-foot, is then calculated from the equation:

$$\text{PCC} = (\text{power demand}) \times (5433)/(\text{well discharge}), \qquad (2)$$

where

5433 = conversion factor (in gallon hours per acre-foot minutes), and
well discharge = instantaneous ground-water discharge, in gallons per minute.

A PCC was computed for every instantaneous discharge measurement that was made at a well. The PCC’s derived in 1997 and in 1998 were used to evaluate temporal variations in the PCC data. However, because the majority of PCC’s were measured late in the 1997 irrigation season, only the PCC’s determined from the 1998 measurements were used to compute ground-water pumpage estimates for each well and to compare differences in total pumpage between the TFM and PCC approaches.

Pumpage estimates were calculated using every PCC measurement made at a well during 1998. This was done by dividing the total 1998 power consumed, in kilowatthours, by each unique PCC measurement made at the well during 1998. The number of kilowatthours used between onsite visits was determined by reading the electric meter at the beginning of a site visit. The total electrical power used was determined from readings of the electrical meter at the beginning and end of a monitoring period. The same TFM monitoring period was used with each PCC in 1998 for determining the TFM pumpage at each site.

Quality Control of Data

Data for this study were collected by CDWR personnel and transmitted to the USGS in electronic and paper files for data analysis. Several procedures were used to check the quality of the data. Quality-control checks consisted of developing a form (referred to as a field form) to be completed onsite during each site visit, making periodic site visits with CDWR personnel to observe onsite data collection, reviewing field forms for completeness, and comparing electronic data to written data recorded on the field forms.

Personnel from the USGS visited the sites to ensure that TFM's were installed according to the manufacturer's specifications. In addition, USGS personnel periodically visited selected sites with CDWR personnel to ensure that field techniques were being used correctly. During these visits, USGS personnel checked that (1) site information and essential test information were documented on field forms, (2) multiple water-level measurements were made to confirm that the pumping water level had not changed more than 10 percent in the hour prior to making a well discharge measurement and collecting the PCC data, (3) portable flowmeter discharge measurements were done properly, (4) consistent methods were used in measuring TFM discharge, and (5) electrical power meter measurements were consistently determined.

Field forms were used to document various characteristics of network wells. Site identifier, test date, and test methods used at each well during a PCC measurement also were recorded on field forms. Other data recorded on the field forms included a description of the discharge test procedures used and any type of problem during the measurement, instantaneous discharge (pumping rate), static and pumping water-level measurements, and PCC's determined for each portable flowmeter method used during a site visit. Personnel from the USGS reviewed the field forms for completeness, tabulations, and consistency with established collection procedures. About 10 percent of the electronic data were verified against copies of the original field forms, and all electronic data were scrutinized for anomalous data.

In addition to these quality-control measures, the three types of portable flowmeters used in the study were tested at the Great Plains Meter, Inc., facility in Aurora, Nebraska, before the start of the 1998 irrigation season. The accuracy of the method P portable flowmeter was checked by releasing a known volume of water three times through the test apparatus at the facility, while total elapsed time was measured to calculate an average rate of discharge. The discharge measured by the method P portable flowmeter for each timed release ranged from 99 to 101 percent of the known discharge. The accuracy of the method C portable flowmeter was checked by maintaining a constant flow of water through the test section at the facility. The method C portable flowmeter was installed in a straight length of pipe, and manometer readings were taken at two points in the cross section of the pipe. The instantaneous discharge measured by the method C portable flowmeter ranged from 103 to 104 percent of the discharge measured by a flowmeter installed in the test section at the facility. The test facility did not make any calibration adjustments to either the method P or the method C portable flowmeters. Because the measurements using method P and method C portable flowmeters were within 5 percent of known values, no adjustments were made to the well discharge data collected with these portable flowmeters.

The accuracy of each method M portable flowmeter was checked using a one-point flow test and then calibrated using a three-point flow test. The rate of flow used during these tests ranged from about 100 gal/min for the 4-in. flowmeter, to about 3,000 gal/min for the 10-in. flowmeter. After calibration adjustments, the flows measured by the method M portable flowmeters ranged from 98 to 102 percent of the known flows.

Overview of the Statistics Used for Comparing Discharge and Pumpage

A statistical procedure known as analysis of variance was used to make comparisons of well discharge and pumpage made using the TFM's and the PCC approaches. These comparisons were made by computing the differences in well discharge and pumpage between the two different approaches. The analysis of variance evaluates whether the average or mean difference in values is statistically different and identifies the sources of variation in the data set (Iman and Conover, 1981). A necessary assumption about the analysis of variance model is that the probability distribution of the data is normal. This is a common assumption made when applying statistical models, but it is an assumption that may not be true for many water-resources data sets. One reason a normality assumption is useful is that the normal distribution is characterized by the mean and variance (which is the standard deviation squared). The mean is a measure of central tendency of the random variable, and the variance is a measure of magnitude of random variability. Given the mean and variance, probability statements may be expressed in terms of these parameters; for example, a normally distributed random variable is with probability 0.95 within 1.96 standard deviations of the mean. Another necessary assumption about the analysis of variance model is that the variances are constant.

During data analysis, differences for every well discharge and pumpage estimate initially were computed by subtracting the well discharge or pumpage estimates associated with the PCC approach at each well from the well discharge or pumpage associated with the TFM measured at the same well on the same date. An analysis of the differences computed in this manner indicated that the assumptions of normality and equal variances were not met. Therefore, a transformation of the differences was done by subtracting the natural logarithm of well discharge or pumpage associated with the PCC approach from the natural logarithm of the well discharge or pumpage associated with the TFM. The resulting differences were normally distributed, and the variances were equal for well discharge. However, the differences in pumpage were not normally distributed. Thus, a rank transformation was performed on the differences in pumpage. This consisted of ranking all of the individual differences, and then applying the analysis of variance model to the ranks. The rank transformation for a sample of *n* observations replaces the smallest observation by the integer 1 (called the rank), the next smallest by rank 2, an so on until the largest observation is replaced by rank *n*. Using ranks diminishes the influence of the outlying values on the final results. A consequence of doing this is that the final results of the analysis reflect the behavior of the majority of the data points, but the influence of the outlying values has been diminished. An inverse rank transformation (linear approximation) to the results of the analysis of variance was then done, resulting in estimates of the mean or central tendency of the distribution of differences in pumpage. However, data outliers may well have a significant effect in situations for which properties of the probability distribution other than central tendency are important.

The natural logarithmic transformation that was applied to the data has another useful property that makes it appropriate for analyzing this data set. Differences in logarithmically transformed variables are equivalent to relative or fractional differences rather than to absolute differences. Relative differences are an informative way to evaluate differences in well discharge and pumpage. In essence, for small differences, the relative differences, which is the difference in natural log transformed variables, multiplied by 100 times, is nearly equivalent to percent difference. Tornqvist and others (1985) provide a more complete discussion of the advantages of using the log transformation to evaluate relative differences.

During data analysis, various site characteristics, hereinafter called fixed effects (method of discharge measurement, make of TFM, and discharge distribution type) were identified as sources of variation. Additionally, the site, date, and random error, hereinafter called random effects, were identified as sources of variation. Therefore, it was necessary to take these additional sources of variation into consideration when making comparisons of well discharge and pumpage.

COMPARISON OF INSTANTANEOUS GROUND-WATER DISCHARGE MEASUREMENTS

A comparison of the instantaneous discharge measurements using the TFM’s to those using the three portable flowmeters was made by evaluating the differences between the measurements and by determining whether the differences are statistically significant. Because it was determined that the method of discharge measurement, make of TFM, discharge distribution type, and the site, date, and random error were identified as sources of variation, an additional level of data analysis was required.

This section of the report presents (1) the magnitude in differences in well discharge; (2) an estimate of the overall mean difference in well discharge and whether the overall mean difference is significantly different from zero; (3) an estimate of the mean differences for each combination of portable flow meter, make of TFM, and discharge distribution type, and whether these mean differences are significantly different from zero; and (4) how much of the variation in the differences is attributable to the site-to-site, date, and random error components. The comparison of ground-water discharge measurements was based on 747 paired measurements taken at 105 wells during a 2-year period.

Primary Results

Analysis of variance was used to evaluate logarithmically transformed differences between instantaneous discharge measured with portable flowmeters and instantaneous discharge measured with a TFM. The analysis was applied to 747 paired discharge measurements made at 105 wells during the 2-year period. More than 80 percent of the differences were less than 10 percent. The overall mean difference was 0.0 percent, indicating no difference on average between portable flowmeter and TFM discharge measurements. For varying site characteristics (the method of portable flowmeter, the make of TFM, and type of discharge distribution system), mean differences range from –4 percent to 4 percent.

Details of Analysis and Results

For each paired discharge measurement, the difference in well discharge (*diffQ*) was computed as:

$$diffQ = \log(\tilde{Q}) - \log(Q), \tag{3}$$

where $\tilde{Q}$ denotes an instantaneous discharge measurement made using a portable flowmeter at a particular site on a particular date, and Q denotes a corresponding (paired) instantaneous discharge measurement made using a TFM at the same site on the same day. (All logarithms in this report are base e.)

The relation between *diffQ* and Q is shown in figure 2*A*, and the relation between differences in the untransformed discharge, $\tilde{Q} - Q$, and Q is shown in figure 2*B*. There is a marked tendency in figure 2*B* for variability in differences to increase as Q increases. That is, although untransformed differences generally tend to be centered around an average value of zero, the variance of untransformed differences tends to increase with the magnitude of the discharge. In contrast, the differences in log-transformed discharges have variance that is much more nearly constant for the entire range of well discharge values (fig. 2*A*).

As mentioned earlier in the report, the natural logarithmic transformation of the discharges allows *diffQ* to be interpreted as a relative or fractional difference between discharges, and for small differences between $\tilde{Q}$ and Q,

$$diffQ \approx \frac{\tilde{Q} - Q}{Q} \approx \frac{\tilde{Q} - Q}{\tilde{Q}}. \tag{4}$$

Thus, *diffQ* multiplied by 100 may be interpreted as a percent difference.

Each measurement of Q and $\tilde{Q}$ is made under certain conditions; changes in these conditions may cause the distribution (that is the mean and variance) of *diffQ* to change in a systematic way. Each discharge measurement $\tilde{Q}$ is made with a particular type of flowmeter. There are three portable flowmeters used, resulting in three “levels” associated with this factor. Likewise, the TFM’s made by different manufacturers may affect the distribution of *diffQ*. Finally, each pair of measurements is made on a particular type of discharge distribution system, so any systematic effect of this factor also may be important. Therefore, the effects associated with these three factors: portable flowmeter method, make of the TFM, and type of discharge distribution system were included in the analysis of variance. (These three factors will hereinafter be referred to as simply method, make, and type.)

In addition to method, make, and type, there are two other conditions that can affect *diffQ*; these are site and date. For example, it is important to know whether *diffQ* at a certain site tends to be consistently larger or smaller than values at other sites. Similarly, there may a tendency for *diffQ* to be larger or smaller on certain dates at a given site. In analysis of variance, effects may be treated as either random or fixed. The site and date effects are treated as random, whereas the method, make, and type effects are treated as fixed,

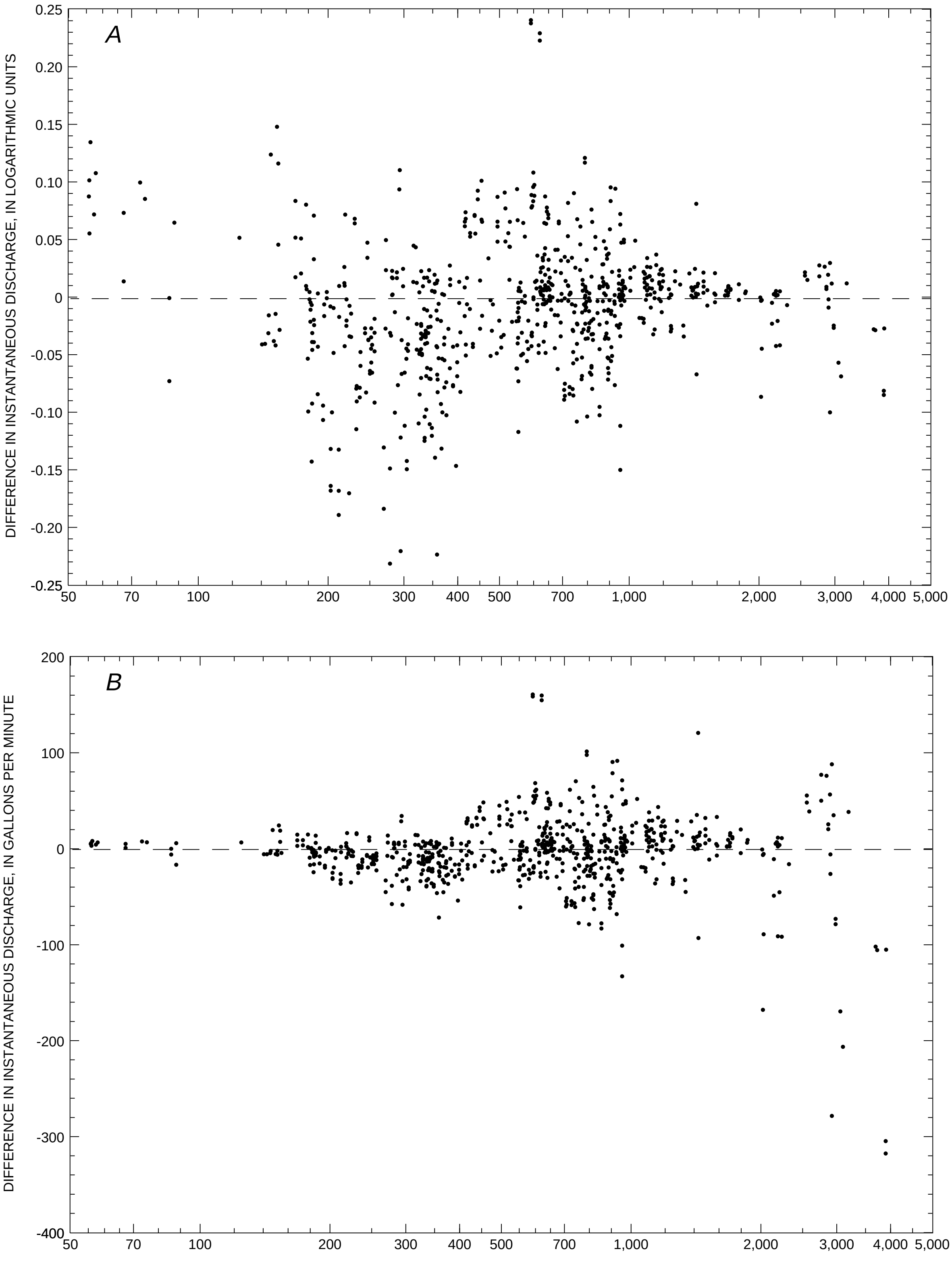


Figure 2. Graphs showing relation of instantaneous discharge measurements from totalizing flowmeter to the differences in instantaneous discharge measurements between portable flowmeters and totalizing flowmeters, expressed (*A*) in logarithmic units and (*B*) in gallons per minute.

and the overall model for *diffQ* is, therefore, known as a mixed model. (See, for example, Snedecor and Cochran (1967) for a more detailed discussion of the distinction between fixed and random effects.) The random effects associated with site and date each have a variance (known as variance components), and the variance of *diffQ* thus is the sum of three constituent terms: the site variance, the date variance, and an error variance, which represents variability (such as measurement error) that is not accounted for by any known factors.

Therefore, a mixed analysis of variance model with both fixed and random effects was applied as follows: The three fixed (nonrandom) effects of interest were: (1) method, with levels P, C, and M; (2) make, with levels M, S, X, and B; and (3) type, with levels O, L, S, and C. The eight values for make R were not included in the analysis because the differences in instantaneous discharge were so much greater in magnitude than all the other values. Boxplots for all the discharge data pooled and for each level of the three fixed effects are shown in figure 3. More than 80 percent of the differences in the paired discharge measurements for the entire network of wells were less than 10 percent, more than 50 percent of the differences were less than 5 percent, and the median difference was less than 1 percent (fig. 3*A*). The distribution of the differences varied among the three fixed effects (method, make, and type) (figs. 3*B*, 3*C*, and 3*D*).

In addition to the fixed effects, two random effects were included in the analysis: (4) site and (5) date. The sites were classified as to make and type; for example, each site was associated with one and only one make and type. Thus, random factor site (4) is said to be nested under fixed effects make (2) and type (3). Likewise, random factor date (5) was nested under fixed factor site (4). The portable flowmeter methods [factor (1)] were applied at all sites, and often two or more methods were applied at the same well on the same date, so there was no nesting used for this factor. This analysis of variance design is referred to as a split-plot design, with “plots” corresponding to a given site on a given day. Snedecor and Cochran (1967) and Helsel and Hirsch (1992) provide more in-depth discussion of fixed and random effects and of nested (or hierarchical) designs.

The mathematical model for *diffQ* may be written as

$$\begin{aligned} diffQ_{ijkmn} &= \mu + \alpha_i + \beta_j + \gamma_k \\ &\quad + S_{jkm} + C_{jkmn} + e_{ijkmn}, \end{aligned} \tag{5}$$

where

μ	is the intercept term,
α_i	is the effect (fixed) for the portable flowmeter method *i*,
β_j	is the effect (fixed) for totalizing flowmeter make *j*,
γ_k	is the effect (fixed) for distribution system type *k*,
S_{jkm}	is the effect (random) for site *m* of wells with make *j* and type *k*,
C_{jkmn}	is the effect (random) for make *j* and type *k* on day *n* at site *m*, and
e_{ijkmn}	is a random error term.

In this model, the random terms *S*, *C*, and *e* are assumed to be independent and normally distributed with mean 0 and variances σ_S^2, σ_C^2, and σ^2, respectively. The analysis of variance provides estimates of the fixed effects and of the magnitudes of these three variances (known as “variance components” because they constitute a partitioning of the random variability of *diffQ*) as well.

The three fixed effects were included in order to determine if average values of *diffQ* tend to change systematically with method, make, or type. The random effects for site and date were included to account for the correlation among measurements taken at the same site and on the same day. In most cases, more than one portable flowmeter method was used at a given site on the same day. In many cases, the well discharge measurements made at the same site on the same day by portable flowmeters clustered together and exhibited similar deviation from the TFM discharge. This clustering tendency is shown in figure 4, which shows how *diffQ* varies with site. The magnitude of the tendency for differences to cluster is evaluated by the site-and date-variance components. The site variance σ_S^2 is a measure of the tendency for all the measurements made at a well to exhibit a systematic discrepancy between portable

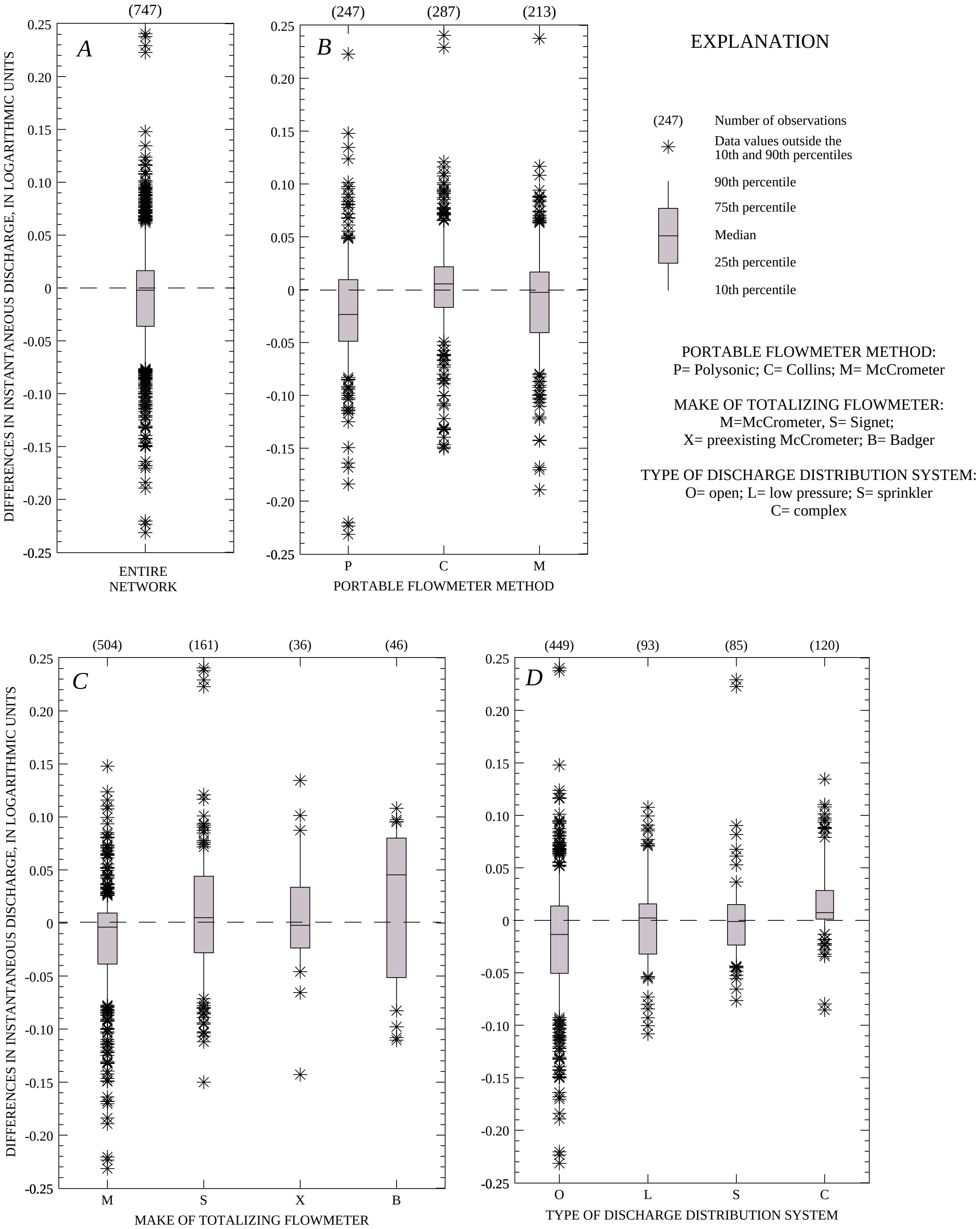


Figure 3. Boxplots showing differences in instantaneous ground-water discharge between portable flowmeters and totalizing flowmeters (*A*) for the entire network, (**B**) by portable flowmeter method, (*C*) by make of totalizing flowmeter, and (*D*) by type of discharge distribution system, 1997–98.

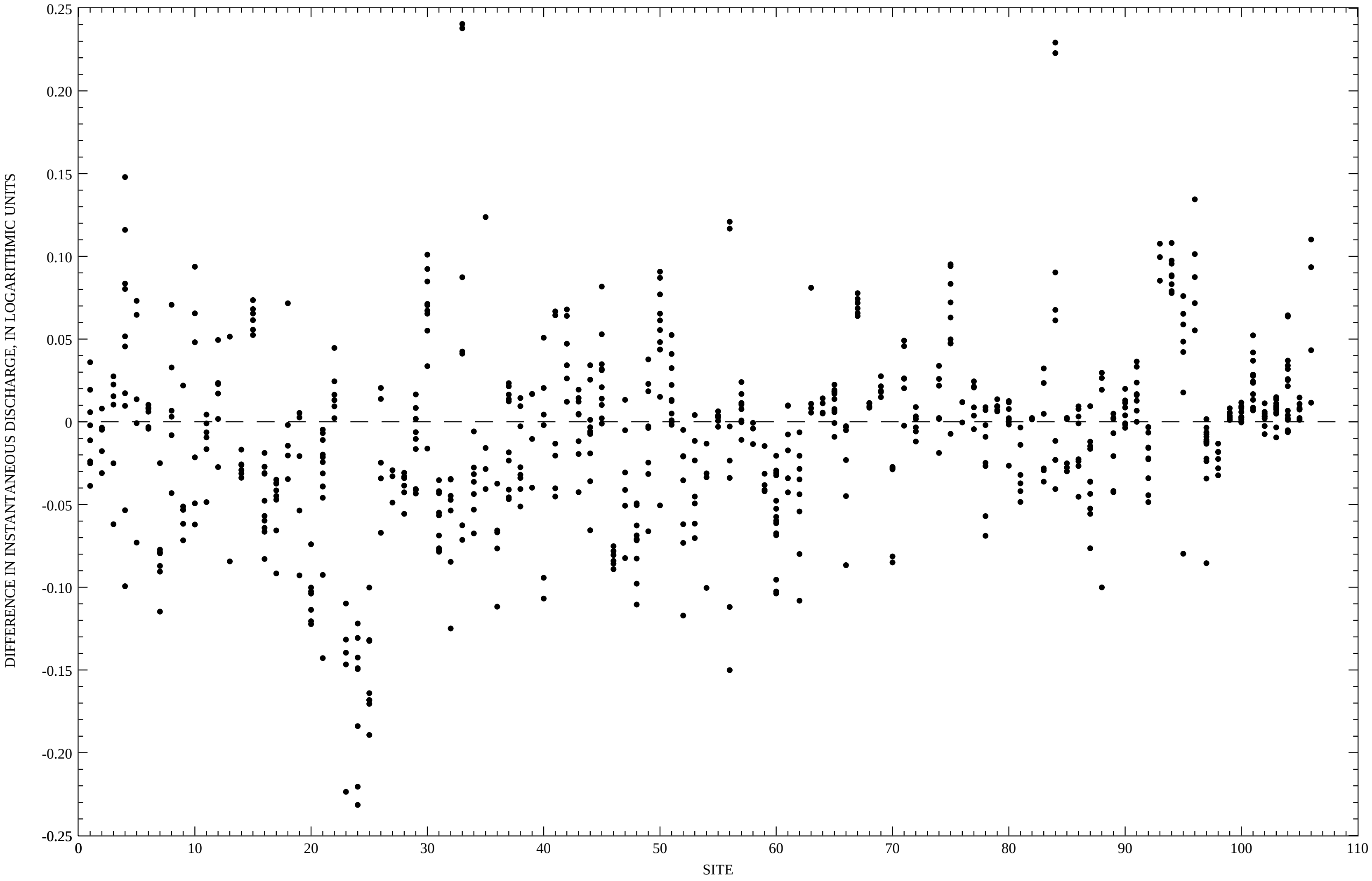


Figure 4. Distribution of differences in instantaneous ground-water discharge between portable flowmeters and totalizing flowmeters for network sites during 1998.

flowmeter and TFM discharge measurements, and the date variance σ_C^2 is a measure of the deviation of TFM discharge from the average of the discharges for portable flowmeters at the same site on the same day. The error variance σ^2 is a measure of the internal consistency of discharge measurements by different portable flowmeters at the same site and same day. If the consistency of measurements among portable flowmeters at the same site on the same day indicates an accurate estimate of true discharge, the magnitudes of the variances σ_S^2 and σ_C^2 may be interpreted as reflecting inaccuracy in the TFM discharge measurement value relative to the true value.

The initial analysis of variance indicated a significant difference (at the 5-percent level) between all pairs of portable flowmeter methods, between makes M and S, and between types O and C. To assess appropriate pooling of the different makes, makes B and X were compared to make M and make S using an estimated difference divided by the standard error of the difference, revealing that the makes B and X data could be pooled with the make S data. This pooling resulted in two levels for the make factor: M and other (B, S, X). Similarly, it was determined that types L and S could be pooled with type C, resulting in two levels of type: O and other (C, L, S). The analysis was redone using the same mathematical model but with only two make levels, M and (B, S, X), and two type levels, O and (C, L, S). Diagnostic plots were examined following the analysis, including a plot of residuals versus fitted and normal quantile-quantile plots for the three random terms in the model. These plots indicated no serious violation of model assumptions that would adversely affect final results.

Final estimates of the means and the differences in means associated with the fixed effects are presented in tables 1–3. [See Graybill (1976) for a discussion of important technical estimability issues associated with these estimates]. A standard error is given for each of the values in these tables, and values that are significantly different from zero (i.e., greater than 2 standard errors from zero) at the (approximately) 5-percent level are noted.

Final estimates of the grand mean (overall average difference of *diffQ*) and fixed effects are listed in table 1. The grand mean is 0.0000; the uncertainty in this number as measured by the standard error is 0.0045 or 0.45 percent. The mean difference for method C is about 1.1 percent, for method M is 0.0 percent, and for method P is about –1.1 percent. The positive sign on the mean for method C indicates that instantaneous discharge measured by portable flowmeters tends to be greater than instantaneous discharge measured by TFM’s, and the opposite holds for method P. The mean differences for each method are very comparable to the differences measured during the quality-control checks done at the Great Plains Meter facility (see “Quality Control of Data” section).

Table 1. Estimates of mean differences in instantaneous ground-water discharge between portable flowmeters and totalizing flowmeters for the grand mean and fixed effects of method, make, and type

[NS, mean is not significantly different from zero at the 5-percent significance level; S, mean is significantly different from zero at the 5-percent significance level; the mean and the standard error can be expressed as a percent difference by multiplying the respective value by 100]

Mean differences	Mean	Standard error	Significance at the 5-percent level
Grand mean	0.0000	0.0045	NS
Method of portable flowmeter (fixed)			
C	.0109	.0047	S
M	.0000	.0048	NS
P	–.0108	.0047	S
Make of totalizing flowmeter (fixed)			
M	–.0152	.0047	S
BSX	.0152	.0075	S
Type of discharge distribution system (fixed)			
O	–.0130	.0054	S
CLS	.0131	.0067	NS

Table 2. Estimates of mean differences in instantaneous ground-water discharge between portable flowmeters and totalizing flowmeters among fixed effects of method, make, and type

[NS, mean is not significantly different from zero at the 5-percent significance level; S, mean is significantly different from zero at the 5-percent significance level; the mean and the standard error can be expressed as a percent difference by multiplying the respective value by 100]

Mean Differences	Mean	Standard error	Significance at the 5-percent level
Method of portable flowmeter			
M–C	–0.0109	0.0026	S
P–C	–.0217	.0025	S
M–P	.0108	.0027	S
Make of totalizing flowmeter			
BSX–M	.0304	.0088	S
Type of discharge distribution system			
CLS–O	.0261	.0082	S

Table 3. Estimates of mean differences in instantaneous ground-water discharge between portable flowmeters and totalizing flowmeters for each combination of fixed effects of method, make, and type

[NS, mean is not significantly different from zero at the 5-percent significance level; S, mean is significantly different from zero at the 5-percent significance level; the mean and the standard error can be expressed as a percent difference by multiplying the respective value by 100]

Method	Mean	Standard error	Significance at the 5-percent level	Method	Mean	Standard error	Significance at the 5-percent level
Discharge distribution type = O Make of totalizing flowmeter = M				Discharge distribution type = O Make of totalizing flowmeter = BSX			
C	–0.0174	0.0060	S	C	0.0130	0.0081	NS
M	–.0283	.0060	S	M	.0021	.0080	NS
P	–.0391	.0060	S	P	–.0087	.0080	NS
Discharge distribution type = CLS Make of totalizing flowmeter = M				Discharge distribution type = CLS Make of totalizing flowmeter = BSX			
C	.0088	.0067	NS	C	.0392	.0093	S
M	–.0022	.0070	NS	M	.0282	.0094	S
P	–.0130	.0068	NS	P	.0174	.0093	NS

Estimates of differences among fixed effects are all less than 5 percent and are listed in table 2. The means in this table may be obtained by computing differences using the means in table 1. All the differences in table 2 are significant at the 5-percent level.

Estimates of combined effects (that is, effects associated with each different combination of levels of the fixed factors) are listed in table 3. For example, for type O distribution systems and make M TFM’s, method P portable flowmeters have a mean difference of about –3.9 percent, and mean differences are negative for other methods as well. Differences for make (B, S, X) and type (C, L, S), however, are all positive, with a mean difference for method C portable flowmeters of about 3.9 percent. Overall, for particular combinations of method, make, and type, mean differences range from about –4 percent to 4 percent.

Estimates of the variance components (variances of the site, date, and error random terms) are listed in table 4. The sum of the variance components is 0.002639. The relative magnitude of the three variance components indicates what fraction of the variance of *diffQ* is associated with each of the random terms in equation 5. Site-to-site variability accounts for about 53 percent (100 × 0.001399/ 0.002639) of the sum of the variance components, the date within site variability accounts for about 27 percent of the sum of the variance components, and the random error terms accounts for the remaining 20 percent.

Table 4. Estimates of the variances of the site, date, and error random terms in discharge measurements

Random terms	Variance
Site	0.001399
Date	.000701
Error	.000539
Sum	0.002639

The total variance of *diffQ* around the overall mean (that is, the variance of *diffQ* without a model) is 0.003037, which indicates that the fixed effects account for about 13 percent [equals 100 × (0.003037 – 0.002639) / 0.003037] of the variance of *diffQ*. This is a relatively small part of the total variability, but the data set is large enough to result in the statistically significant differences listed in tables 1 through 3. Similarly, the site, date, and error variance components expressed as a percent of the total variance are 46 percent, 23 percent, and 18 percent, respectively. Overall, the largest portion of the variance of *diffQ* is accounted for by site-to-site variability.

The random error variance (0.000539 in table 4) measures the amount of variability among different portable flowmeter measurements applied on the same day at the same site. The error variance can be used to determine the range in expected differences between (logarithmically transformed) instantaneous discharge measured using two different portable flowmeters. The estimated variance of the difference will be 2 × 0.000539 because the variance of the difference between two independent random variables is the sum of their variances. This translates into a standard deviation of about 3.28 percent. When this measure of the random component of the difference is considered in conjunction with the systematic differences in table 2 for different portable flowmeter methods, an estimate of the total error can be determined. For example, if measurements are made using P and M portable flowmeters, the systematic bias (M–P) is 1.08 percent with a standard deviation of 3.28 percent. If normality is assumed, about 95 percent of the differences between the measurements taken with the two portable flowmeters will be between –5.48 percent and 7.64 percent.

The small size of the random error variance component is indicated by the precision with which differences among portable flowmeters can be estimated in table 2. The standard errors for portable flowmeter differences range from 0.25 to 0.27 percent, which is considerably smaller than standard errors for make differences (0.88 percent) or type differences (0.82 percent). A strength of the design for this data collection was the application of multiple portable flowmeter methods at the same well on the same date during a short period of time.

TEMPORAL VARIATIONS IN POWER CONVERSION COEFFICIENTS

The use of PCC's to estimate ground-water pumpage from wells is most accurate when the relation of well discharge to power consumption remains stable. However, over time, hydrologic and pump operating conditions may change, thus altering the PCC relation to well discharge and power consumption. As examples, depth to ground water may increase after an extended period of pumping or pump efficiency may decrease as the irrigation pump ages. Any well operation that results in significant variations in the PCC over time can result in errors when using the PCC approach to estimate ground-water pumpage.

Short-Term Variations in Power Conversion Coefficients

Multiple PCC measurements repeated at the well sites during 1997 and 1998 are used to indicate the temporal variability in PCC's during one and two irrigation seasons. The range in PCC's at 104 sites during 1998 is shown in figure 5*A*. The PCC's for most sites (86 percent) did not fluctuate more than 20 percent throughout the 1998 irrigation season; however, for unknown reasons, a wide range in PCC's occurred at about 14 percent of the network sites. At some wells, a lower than expected PCC measurement (site 5) or several lower than expected PCC measurements (site 27) resulted in the wide range in PCC's that were measured. The percent difference for the

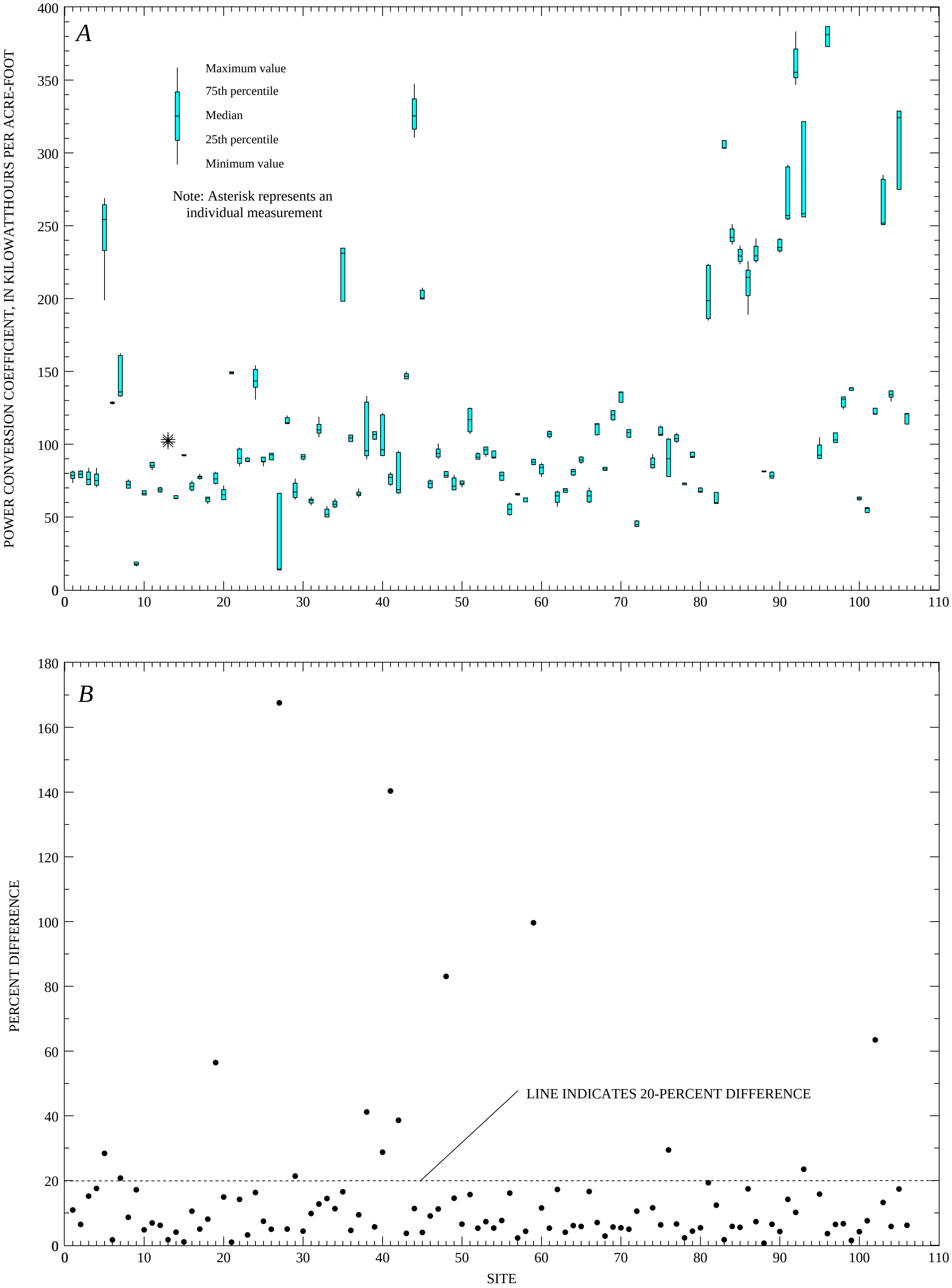


Figure 5. Power conversion coefficients (PCC) determined at network sites during 1998: (*A*) range in PCC and (*B*) percent difference.

total range in PCC's determined during 1998 is shown in figure 5*B*. The equation used to determine percent difference shown in figure 5*B* is:

$$\text{Percent difference} = 100 \times (\text{maximum PCC} - \text{minimum PCC})/\text{site average PCC}. \qquad (6)$$

Percent differences in PCC data at wells ranged from less than 1 percent (sites 21 and 88) to more than 150 percent (site 27). The data indicated that 58 percent of the site comparisons had less than a 10-percent change and 86 percent of the site comparisons had less than a 20-percent change in PCC's throughout the 1998 irrigation season.

The PCC measurements made during the 1998 irrigation season were evaluated for systematic seasonal variations. Figure 6 shows that for the majority of instances, there are no evident seasonal patterns in the PCC measurements made during 1998. Comparisons of PCC's to depth to ground water did not reveal any systematic relation between changes in PCC's and depth to water.

The PCC measurements made at 41 network sites during 1997 were compared to PCC measurements made during 1998 at the same 41 sites (fig. 7) to evaluate temporal variations during two irrigation seasons. Thirty-seven sites (90 percent) had at least one PCC measurement made in 1997 that was less than the range of PCC's made in 1998 (fig. 7); 16 of the sites (39 percent) had all 1997 PCC's less than the range of PCC's made during 1998. Only sites 83 and 87 had a large difference between the 2 years of data. Overall, the 2 years of data indicate that the PCC measurements were similar between 1997 and 1998.

Long-Term Variations in Power Conversion Coefficients

State-approved PCC measurements collected at the network sites during 1994 to 1997 for compliance with State rules (Office of the State Engineer, 1994 and 1996) were used to evaluate temporal variability that occurred in PCC's during the 4-year period. The long-term variability between PCC's for wells in the 1998 network and corresponding State-approved PCC's during 1994–97 is shown in figure 8*A*. Implicit in this comparison is the assumption that the State-approved PCC's determined during 1994–97 are of the same quality as the PCC's determined during this study, including the removal of the cases where the PCC's change under Rule 3.5 (Office of the State Engineer, 1996) due to a change in pump or motor. The equation used to compute the percent differences shown in figure 8*B* is:

$$\text{Percent difference} = 100 \times (\text{State-approved PCC} - \text{site average PCC})/\text{site average PCC}, \qquad (7)$$

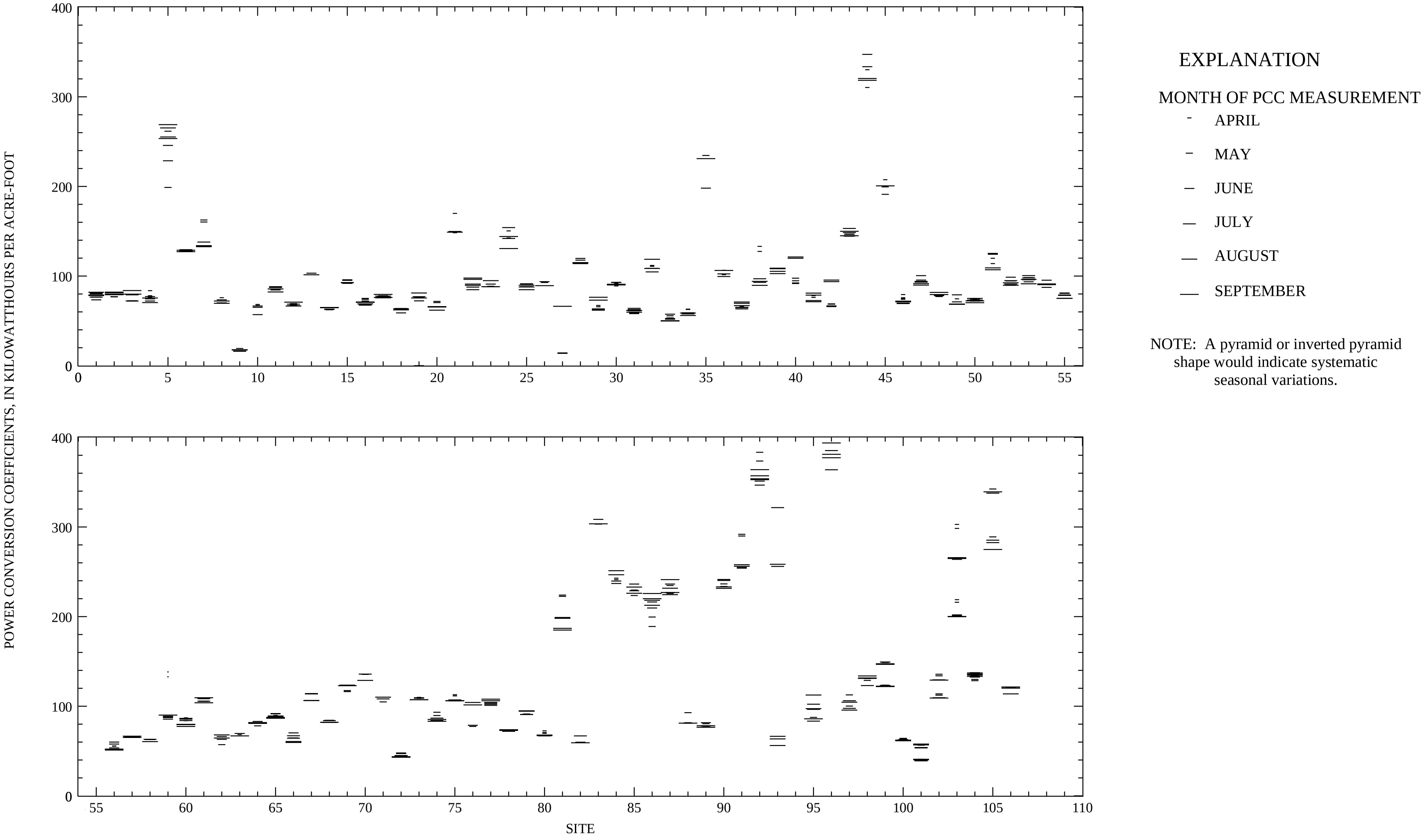
where

site average PCC = the arithmetic mean of all PCC's determined at each site in 1998.

Fifty comparisons of 103 PCC measurements (about 48 percent) had less than 10-percent difference between the State-approved PCC's and the site average PCC measured during 1998 and about 67 percent of the State-approved PCC measurements were less than 20 percent of the PCC's during 1998 (table 5). Twenty-one of the 103 site comparisons indicated a positive percent difference of more than 20 percent in PCC's, and 13 site comparisons indicated a negative percent difference of more than 20 percent. A positive percent difference indicated that the 1994–97 State-approved PCC was greater than the site average PCC in 1998.

The percent difference between the State-approved 1994–97 PCC's and the average 1998 PCC ranged from about –57 to 211 (table 5). The largest range in percent difference was between the State-approved PCC's measured in 1995 and the 1998 PCC's. A comparison of the percent differences computed using the State-approved PCC's from 1997 to the average 1998 PCC's indicated that 78 percent of the sites were within 10 percent and 89 percent of the sites were within 20 percent.

During well operation, the PCC is generally constant for a specific discharge pressure and a stable pumping water level. Because the water level in a well often declines rapidly during the initial period of pumping, the PCC also changes rapidly until the pumping water level stabilizes. A potentially important change made in the Colorado amended rules in 1996 (Office of the State Engineer, 1996) required PCC measurements be made only after the pumping water level had not changed more than 10 percent in the hour prior to making the PCC measurement.



NOTE: A pyramid or inverted pyramid shape would indicate systematic seasonal variations.

Figure 6. Seasonal distribution of power conversion coefficients (PCC) for network sites during 1998.

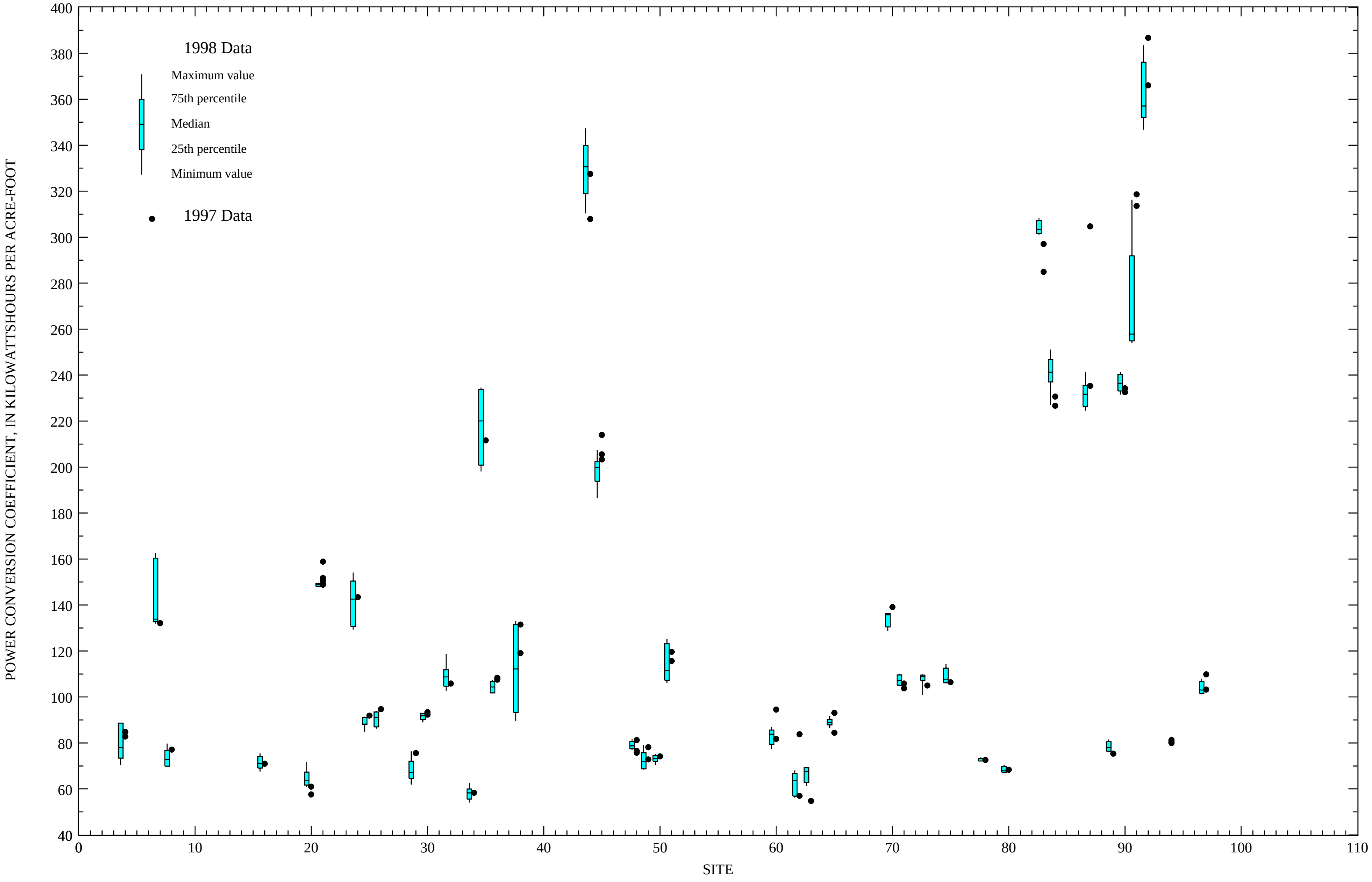


Figure 7. Power conversion coefficients (PCC) determined at selected network sites during 1997 and 1998.

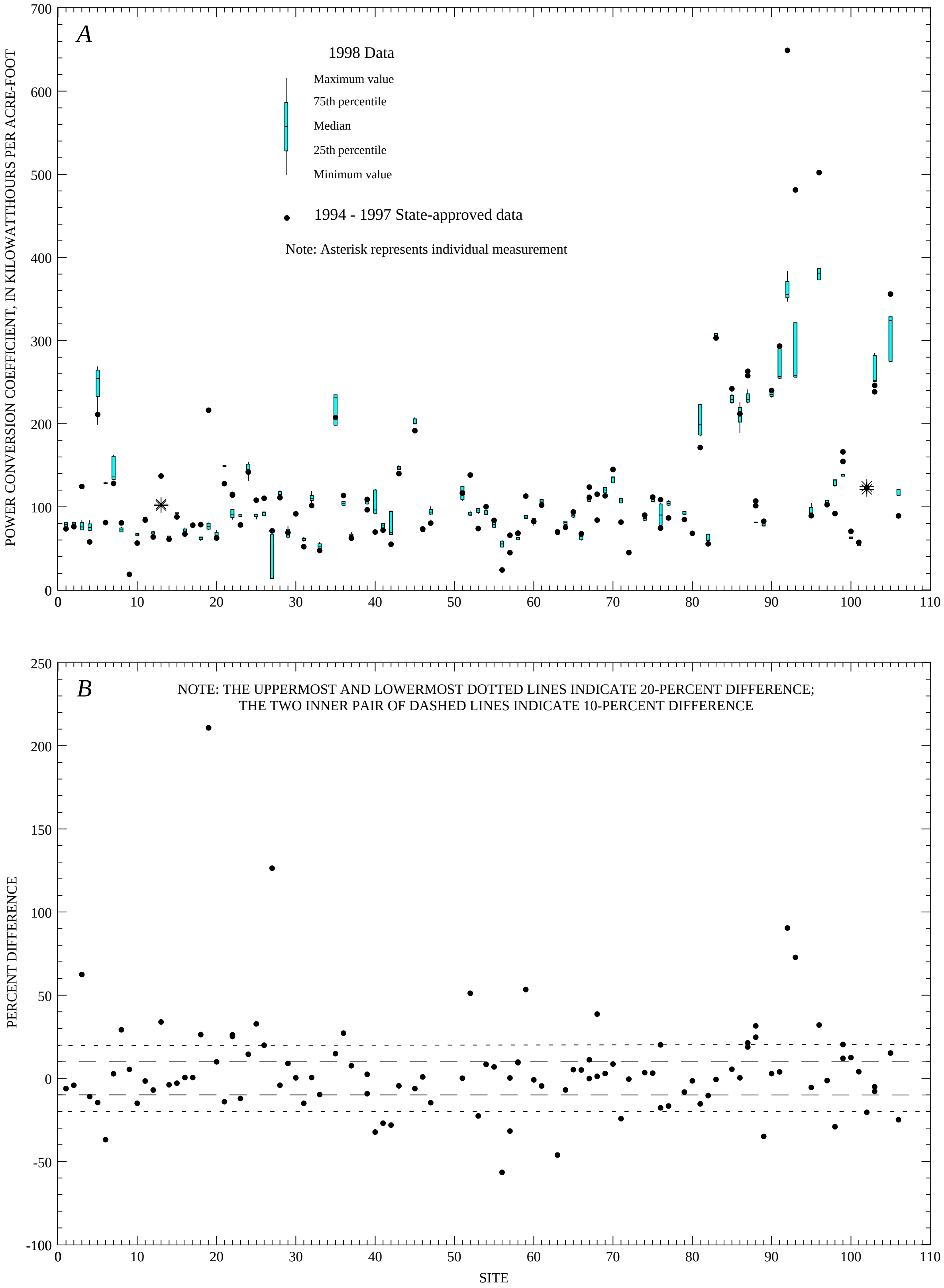


Figure 8. Power conversion coefficients (PCC) at network sites: (*A*) 1994–97 State-approved PCC and 1998 PCC, and (*B*) percent difference between 1994–97 State-approved PCC and 1998 PCC.

Table 5. Comparison of State-approved power conversion coefficient (PCC) measurements (1994–97) to the site average PCC measurements made during 1998

[PCC, power conversion coefficient]

State-approved PCC measurement year	Number of PCC comparisons	Minimum and maximum percent difference	Number of PCC comparisons within 10 percent of 1998 site average PCC		Number of PCC comparisons within 20 percent of 1998 site average PCC	
			Number	Percent	Number	Percent
1994	37	–57 90	18	49	25	68
1995	26	–37 211	8	31	11	42
1996	22	–46 27	10	45	17	77
1997	18	–18 73	14	78	16	89
1994–97	103	–57 211	50	48	69	67

COMPARISON OF GROUND-WATER PUMPAGE ESTIMATES

A comparison of ground-water pumpage measured using TFM's to ground-water pumpage estimates determined by the PCC approach was made by evaluating the differences in pumpage and determining whether the differences are statistically significant. The pumpage estimates were calculated using PCC measurements made at network sites during 1998. This was done by dividing the total 1998 power consumed at each site during the monitoring period, in kilowatthours, by each unique PCC measurement made at that same well site during 1998. The TFM derived pumpage measurement at each well was determined as the difference between TFM readings made at the beginning and the end of the monitoring period. The monitoring period was the same for the TFM and PCC approach. Because it was determined that the method of portable flowmeter, make of TFM, discharge distribution type, and the site, date, and random error were identified as sources of variation, an additional level of data analysis was required.

This section of the report presents (1) the magnitude in differences in ground-water pumpage between TFM and PCC estimates; (2) an estimate of the overall mean difference in pumpage and whether the overall mean difference is significantly different from zero; (3) an estimate of the mean differences for each combination of portable flow meter, make of TFM, and discharge distribution type, and whether these mean differences are significantly different from zero. The comparison of ground-water pumpage was based on 553 paired measurements made at 103 wells during 1998.

Primary Results

The analysis of variance on the differences in pumpage was performed using a rank transformation on 553 paired pumpage measurements made at 103 wells during 1998. About 80 percent of the differences in pumpage between the TFM and PCC approach were less than 10 percent. The overall mean difference in pumpage was 0.01 percent, indicating no significant difference on average between pumpage as measured by TFM and pumpage as computed by the PCC approach. For varying site characteristics (the method of portable flowmeter, the make of TFM, and type of discharge distribution system), mean differences in pumpage were generally less than ±3 percent and, for most instances, the mean differences in total pumpage were not significantly different from zero at the 5-percent level.

Details of Analysis and Results

For each paired pumpage measurement made at a well, the difference in ground-water pumpage, *diffP*, was computed as:

$$diffP = \log(\tilde{V}) - \log(V), \qquad (8)$$

where $\tilde{V}$ denotes estimated pumpage as calculated by the PCC approach, and V denotes a corresponding total pumpage as measured by a TFM. As with instantaneous discharge measurements, a log transformation was used, so that the variable of interest is the difference between the log-transformed values.

Because *diffP* is a random variable like *diffQ*, the probability distribution must be characterized. However, as mentioned earlier in the report, unlike *diffQ*, the distribution for *diffP* deviated significantly from normality. There are a number of data values found outside the range of the majority of data values, and such a deviation from normality can cause serious problems with analysis of variance. Therefore, a rank transformation was performed on the data before performing the analysis, and an inverse rank transformation (linear approximation) to the results of the analysis of variance provided estimates of the central tendency of the distribution of *diffP*. Use of the rank transformation in analysis of variance is discussed by Iman and Conover (1981), Helsel and Hirsch (1992), Kepner and Wackerly (1996) and Hora and Iman (1988). Rank transformation does not render the test truly nonparametric, but asymptotic normal theory should be more applicable than would be the case if using untransformed data. Rank transformation minimizes the influence of very large outliers so that the analysis better reflects the central tendency of the data. Evaluating the data without the influence of extreme outliers was essential in understanding the data, and the results of this analysis indicated the types of errors in estimation of pumpage expected at a typical site under typical circumstances. Because a typical-site analysis is inadequate when analyzing aggregated pumpage for a number of wells, a separate analysis of this problem is discussed later in the report in the section titled “Estimation of Total Network Pumpage”.

The overall pattern of differences between V and $\tilde{V}$ are illustrated in figure 9*A*, which is a plot of *diffP* versus *V*, and in figure 9*B*, which is a plot of the difference in untransformed pumpage versus *V*. These plots are analogous to the plots in figure 2*A* and 2*B* for discharge. Variability about the mean tends to be more nearly constant in figure 9*A* than in figure 9*B* for most of the data, so making a logarithmic transformation on the variables is reasonable. These plots also show clearly that there is a small proportion of the differences for which *diffP* tends to be outside the range of the majority of the data.

Boxplots of *diffP* for all the data pooled and for each level of method, make, and type are shown in figure 10. About 80 percent of the differences in pumpage estimates between the TFM and PCC approach were less than 10 percent, more than 50 percent of the differences were less than 6 percent, and the median difference was about 1 percent (fig. 10*A*). The distribution of the differences varied somewhat depending on method, make, and type (figs. 10*B*, 10*C*, and 10*D*).

The analysis of variance model was first applied using all levels of each of the fixed factors: method, make, and type. Significant differences occurred between all pairs of methods, but not between different makes or types. However, pooling the pumpage data in the same manner as the discharge data allows direct comparisons between results of the two analyses. Such comparisons are useful and can be used to determine how errors in instantaneous discharge measurements affect errors in pumpage calculations. Thus, the analysis was redone using the same pooling described in the “Comparison of Instantaneous Ground-Water Discharge Measurements” section. Diagnostic plots again indicated satisfactory adherence to the analysis of variance assumptions.

Final results of the analysis of variance are listed in tables 6 through 8 and are analogous to the results for the instantaneous discharge data presented in tables 1 through 3. As stated earlier, results from the analysis of variance is in terms of ranks, so a linear approximation to the rank-transformation curve near the median was used to back-transform and obtain results in terms of *diffP*. Differences in estimates of the mean differences listed in tables 6 through 8 that are more than 2 standard errors from zero again are indicated as being statistically significant.

The overall grand mean difference for all possible pairs of pumpage in table 6 is 0.0001 (0.01 percent), again almost zero. The estimates for the portable flowmeter method effects were: for method C, 0.73 percent; for method M, 0.22 percent; and for method P, –0.93 percent. These effects are comparable in magnitude to portable flowmeter method effects for the well discharge data in table 1. Similarly, signs of the make and type effects are the

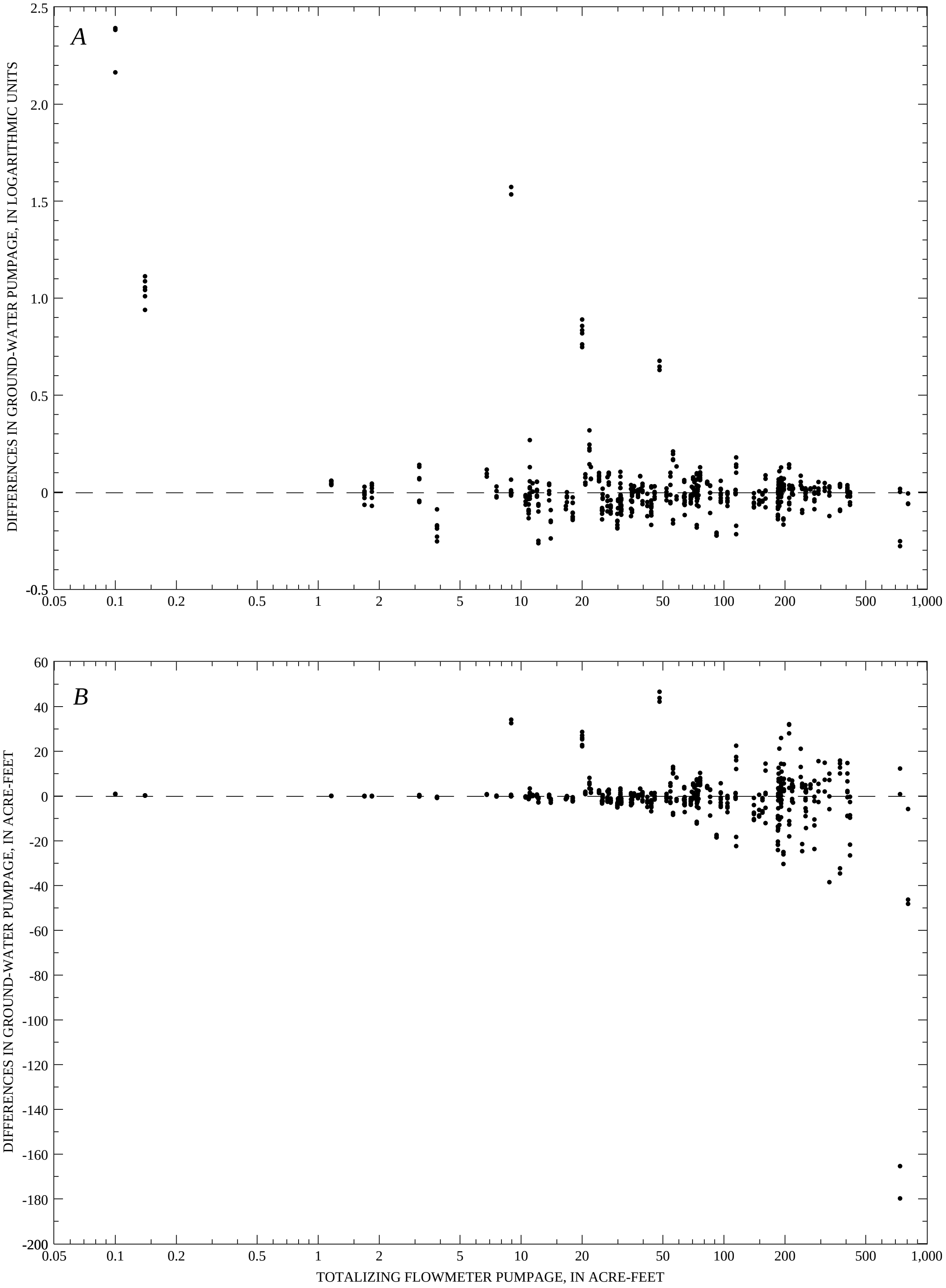


Figure 9. Relation of ground-water pumpage from inline totalizing flowmeter to the differences in pumpage estimates between power conversion coefficient approach and totalizing flowmeters, expressed (*A*) in logarithmic units and (*B*) in acre-feet.

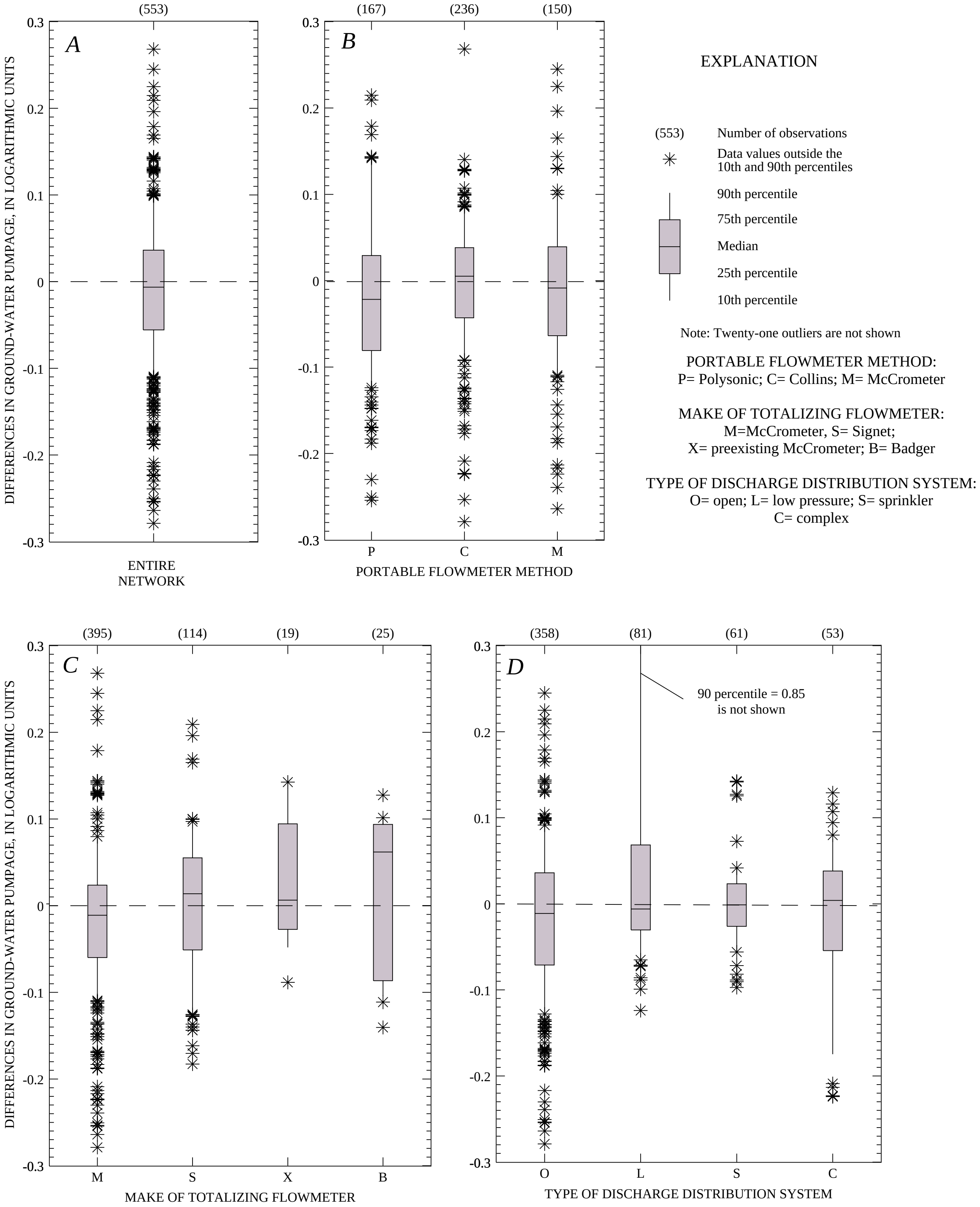


Figure 10. Boxplots showing differences in ground-water pumpage estimates between power conversion coefficient approach and totalizing flowmeters (*A*) for the entire network, (*B*) by portable flowmeter method, (*C*) by make of totalizing flowmeter, and (*D*) by type of discharge distribution system, 1998.

same as for the discharge analysis, but the magnitude of the effects for pumpage estimates is somewhat smaller. Most of the differences shown in table 6 are not statistically significant.

In table 7, estimates of mean differences among the three fixed effects are less than ±2.04 percent. The small standard errors for these differences reflect the increase in precision due to use of more than one portable flowmeter method at the same well on the same day.

The largest positive value for the combined effects (table 8) is 2.44 percent for type (CLS), make (BSX), and method C. The most negative value in table 8 is −2.63 percent for type O, make M, and method P. Both of these extreme values are statistically significant, but most other combined effects in table 8 are not.

A linear approximation for the back-transformation from ranks provided good results for the estimated mean values in tables 6 through 8 because these values were all near zero. However, such a linearization technique applied to variances is questionable because of increasing effects of non-linearity for errors far from zero. Therefore, estimates of the variance components for the pumpage analysis are not presented.

Table 6. Estimates of mean differences in pumpage between power conversion coefficient approach and totalizing flowmeter for the grand mean and fixed effects of method, make, and type

[NS, mean is not significantly different from zero at the 5-percent significance level; S, mean is significantly different from zero at the 5-percent significance level; the mean and the standard error can be expressed as a percent difference by multiplying the respective value by 100

Mean differences	Mean	Standard error	Significance at the 5-percent level
Grand mean	0.0001	0.0046	NS
Method of portable flowmeter (fixed)			
C	.0073	.0047	NS
M	.0022	.0048	NS
P	−.0093	.0047	NS
Make of totalizing flowmeter (fixed)			
M	−.0101	.0048	S
BSX	.0103	.0077	NS
Type of discharge distribution system (fixed)			
O	−.0068	.0053	NS
CLS	.0070	.0069	NS

Table 7. Estimates of mean differences in pumpage between power conversion coefficient approach and totalizing flowmeter among fixed effects of method, make, and type

[NS, mean is not significantly different from zero at the 5-percent significance level; S, mean is significantly different from zero at the 5-percent significance level; the mean and the standard error can be expressed as a percent difference by multiplying the respective value by 100]

Mean differences	Mean	Standard error	Significance at the 5-percent level
Method of portable flowmeter			
M–C	−0.0052	0.0021	S
P–C	−.0166	.0020	S
M–P	.0114	.0023	S
Make of totalizing flowmeter			
BSX–M	.0204	.0089	S
Type of discharge distribution system			
CLS–O	.0138	.0082	NS

Table 8. Estimates of mean differences in pumpage between power conversion coefficient approach and totalizing flowmeter for each combination of fixed effects of method, make, and type

[NS, mean is not significantly different from zero at the 5-percent significance level; S, mean is significantly different from zero at the 5-percent significance level; the mean and the standard error can be expressed as a percent difference by multiplying the respective value by 100]

Method	Mean	Standard error	Significance at the 5-percent level	Method	Mean	Standard error	Significance at the 5-percent level
Discharge distribution type = O Make of totalizing flowmeter = M				Discharge distribution type = O Make of totalizing flowmeter = BSX			
C	–0.0097	0.0058	NS	C	0.0106	0.0080	NS
M	–.0149	.0059	S	M	.0055	.0080	NS
P	–.0263	.0059	S	P	–.0060	.0080	NS
Discharge distribution type = CLS Make of totalizing flowmeter = M				Discharge distribution type = CLS Make of totalizing flowmeter = BSX			
C	.0040	.0068	NS	C	.0244	.0094	S
M	–.0011	.0070	NS	M	.0192	.0096	NS
P	–.0126	.0069	NS	P	.0078	.0095	NS

SOURCES OF DISCREPANCY BETWEEN PUMPAGE ESTIMATES

The analysis of variance procedures applied to instantaneous discharge and pumpage data provided information on the mean differences in well discharge (*diffQ*) and pumpage (*diffP*) and on the variance of *diffQ*. It is clear, however, that these analyses are not independent of each other. Part of the discrepancy between total pumpage computed by the PCC approach and pumpage measured by the TFM comes from differences between measurements made by portable flowmeters and TFM's and differences between these meters are reflected in differences between the paired instantaneous discharge measurements. In other words, one would expect that part of the variability in *diffP* is being caused by variability in *diffQ*. However, there are other possible sources of discrepancy between total pumpage obtained by the two approaches. The following section of the report enumerates several possible sources of discrepancy. For most of these sources, data are not available to estimate exactly how much of the discrepancy is coming from each source. Nevertheless, it is important to explicitly discuss what the possible sources of error are, possibly providing guidance for future data-collection efforts. One important source of potential discrepancy that is discussed in some detail is temporal variability of the PCC. Some data are available to obtain an estimate of the contribution of this component to the difference between pumpage by the two approaches.

Specifically, this section of the report discusses (1) possible sources of discrepancy that result in differences between ground-water pumpage as measured by a TFM and ground-water pumpage as obtained by the PCC approach; and (2) with available data, how might the temporal variability of PCC's effect the differences in pumpage.

Primary Results

There are several potential sources of discrepancy between pumpage as measured by a TFM and pumpage as computed by the PCC approach. These include errors in instantaneous discharge as measured by a TFM and a portable flowmeter, TFM pumpage errors, errors in the electrical power meter, and temporal variability of the PCC. Each may account for a portion of the discrepancy between pumpage as measured by a TFM and pumpage as computed by the PCC approach. It is not possible with data currently available to give reliable estimates of the magnitude of each of the components of pumpage error. Additional data and evaluation of these data are needed to define long-term temporal variations in PCC's and TFM's, as well as defining other sources of discrepancy in pumpage estimates.

Limited data are available to provide an estimate of errors caused by temporal variability of PCC's. The standard deviation associated with year-to-year variability of these PCC's was estimated to be about

15 percent, and the year-to-year variance was about nine times the date-within-year variance. This indicates that year-to-year variability of the PCC may make a significant contribution to errors in the PCC approach for estimating pumpage. The conclusions based on this analysis are based on an assumption that the State-approved PCC's made from 1994–97 are of the same quality as the 1998 study PCC's.

Details of Analysis and Results

To determine total pumpage for some specified period of time, such as one pumping season, at a given well, the following terms are used:

V_T = true total pumpage volume for the monitoring period,
V = pumpage volume as measured by a TFM,
$\tilde{V}$ = pumpage volume as estimated by the PCC approach,
A_T = true electrical power consumption for the period,
A = total electrical power consumption as measured by a meter,
p_T = true PCC for the period, and
$\tilde{p}$ = estimated PCC

True values in these definitions cannot be measured directly, but are still assumed to exist. Total pumpage estimated by the PCC approach ($\tilde{V}$) is computed using metered power consumption (A) and the estimated PCC ($\tilde{p}$) from the equation

$$\tilde{V} = \frac{A}{\tilde{p}} . \qquad (9)$$

(The conversion factor to account for different units of measure will for simplicity be taken to be unity in this section.)

At this point, no assumption is made about how $\tilde{p}$ is obtained. The true PCC (p_T) is the value that, by definition, yields a correct value for total pumpage when divided into true power consumption, or

$$p_T = \frac{A_T}{V_T} ; \qquad (10)$$

however, the true values A_T and V_T are generally unknown, so p_T also is unknown.

Again, logarithmic transformations are used to express all errors; that is, an error is the difference between a log-transformed quantity that is measured or estimated and the log transform of the corresponding true value. Hence the errors are defined as:

$U_1 = \log(V) - \log(V_T)$ = TFM pumpage error,
$U_2 = \log(A) - \log(A_T)$ = electrical power meter error,
$U_3 = \log(\tilde{p}) - \log(p_T)$ = error in the estimated PCC.

One relation of interest is the error in the PCC approach, given by

$$\log(\tilde{V}) - \log(V_T) = U_2 - U_3 , \qquad (11)$$

the difference between power meter error and PCC error. This relation is derived using the definitions and equations 9 and 10.

If the estimated PCC is obtained using a measured instantaneous discharge, as in the data set analyzed in the section "Comparison of Ground-Water Pumpage Estimates", and if measurements are made at some single time, *t*, the following can be defined:

$Q_T(t)$ = true instantaneous discharge at time *t*,
$Q(t)$ = instantaneous discharge as measured by a TFM,
$\tilde{Q}(t)$ = instantaneous discharge as measured by a portable flowmeter,
$a_T(t)$ = true instantaneous electrical power consumption at time *t*,
$a(t)$ = instantaneous electrical power consumption determined from a power meter,
$p_T(t)$ = true instantaneous PCC, and
$\hat{p}$ = PCC estimated with one instantaneous discharge measurement.

Thus, $\hat{p}$ is calculated by

$$\hat{p} = \frac{a(t)}{\tilde{Q}(t)} \qquad (12)$$

and total pumpage estimated by the PCC approach is equation 9 with $\hat{p}$ used for $\tilde{p}$. The true instantaneous PCC is defined to be

$$p_T(t) = \frac{a_T(t)}{Q_T(t)} . \qquad (13)$$

Using instantaneous measurements introduces four new errors:

$U_4 = \log[Q(t)] - \log[Q_T(t)]$ = error in instantaneous discharge measured with TFM,

$U_5 = \log[\tilde{Q}(t)] - \log[Q_T(t)]$ = error in instantaneous discharge measured with a portable flowmeter,

$U_6 = \log[a(t)] - \log[a_T(t)]$ = error in instantaneous power meter reading,

$U_7 = \log[p_T(t)] - \log(p_T)$ = error in instantaneous PCC, or the difference between true instantaneous PCC, and true PCC for the period.

Therefore, when the PCC is estimated using equation 12, the PCC error U_3 may be broken down into three components,

$$U_3 = U_6 - U_5 + U_7 , \qquad (14)$$

which again is shown using the definitions of the various errors. Combining equations 11 and 14 gives the final expression for the error in total pumpage as estimated by the PCC approach,

$$\log(\tilde{V}) - \log(V_T) = U_2 - U_6 + U_5 - U_7 . \qquad (15)$$

The difference in log-transformed instantaneous discharge (*diffQ*) as measured by a portable flowmeter and a TFM may be expressed as the difference of two errors,

$$diffQ = \log[\tilde{Q}(t)] - \log[Q(t)] = U_5 - U_4 . \qquad (16)$$

Similarly, the difference between log-transformed pumpage computed by the PCC approach and TFM approach (*diffP*) may be computed by subtracting the TFM error (U_1) from both sides of equation 15 to yield

$$\begin{aligned} diffP &= \log(\tilde{V}) - \log(V) \\ &= U_2 - U_6 + U_5 - U_7 - U_1. \end{aligned} \qquad (17)$$

The expression for *diffP* in equation 17 has one additional component, namely U_1, that is not contained in the actual error for the PCC approach (that is, the error relative to true total pumpage) given by equation 15. That is, TFM errors in an actual application of the PCC approach would not be observed.

The differing signs in these expressions indicate that some of the errors can be compensating. A positive error in one term may cancel a negative error in another, giving a smaller overall error. While such cancellation may hold for certain pairs of terms, other pairs of errors may be independent of each other. For example, U_5, the error in instantaneous discharge measured with a portable flowmeter, would not be expected to be related to U_6, the error in instantaneous power meter reading. For variables that are uncorrelated, the signs make no difference in the contribution to total variance, because the variance of a difference of two uncorrelated random variables is the same as the variance of the sum.

The errors U_4 (error in instantaneous discharge measured with a TFM) and U_5 (error in instantaneous discharge measured with a portable flowmeter) represent deviations of instantaneous discharge from true discharge. Because true discharge is unknown, there is no estimate of size of these component errors. Data are available only for *diffQ*, which, in equation 16, is the difference between these two individual errors. The values in table 4 indicate bounds on the variance of U_5 under different conditions. If consistency between two (or more) portable flowmeter methods is an indication that the methods are both accurate, in the sense of being a good estimate of the true instantaneous discharge, then the error variance (0.000539) from table 4 would be a good estimate of the variance of U_5. In this case, the site- and date-variance components in table 4 would be mostly attributable to error in the TFM, U_4. However, an upper bound for the variance of U_5 would be the sum of variance components in table 4, or 0.002639. Use of this value as an estimate of the variance of U_5 would assume that the TFM is error-free.

The error U_7 (error in instantaneous PCC) in equation 17 represents deviation of the instantaneous PCC from some long-term true value, which is assumed to be constant. Thus, the average magnitude of U_7 depends on how much temporal variability exists in the time series $\{p_T(t)\}$. An in-depth study of temporal variability of the PCC, including trends, seasonality, and magnitude of serial correlation, would need detailed data on $\{p_T(t)\}$, which are not available for any of the sites. Therefore, a simpler approach was used to obtain an idea of the short- and long-term variabilities in PCC. This approach uses the State-approved PCC's made from 1994–97 together with the PCC's made in 1998 as part of this study. One potential fallacy in this approach is the implicit assumption that the quality of the State-approved PCC's made from 1994–97 is the same as the quality of the PCC's made during this study. If this assumption is accepted, then the temporal variability in PCC can be evaluated. If the PCC data are not of the same quality, then errors not associated with temporal variability could be attributed to the errors in temporal variability in the following analyses, resulting in an inflated estimate of the year-to-year variability.

A nested variance-components analysis using random terms for site, year within site, and date within year and site, was performed using all the log-transformed PCC values, including the 1998 values and 106 State-approved PCC's from 1994–97. Once again, fixed effects (method, make, and type) were not included in the analysis. Such a nested model that has terms representing variability at different time scales is one way of modeling temporal correlation. The estimate of the variance for the year component was 0.02297, for the date-within-year component was 0.00254, and for the residual variance was 0.00077. The year component represents about a 15-percent standard deviation (obtained by taking the square root of the variance and multiplying by 100). This indicates that the year-to-year variability could be a major component of variability for this PCC data set; the year variance component is about nine times the variance of date-within-year component. The PCC values used in this analysis contained uncertainty due to errors in instantaneous discharge as measured by the portable flowmeter as well as errors in instantaneous power meter reading (see eq. 14). This means that an estimate of the variance of U_7 (error in instantaneous PCC) using this analysis is inflated somewhat. Based on the estimates given in the preceding paragraph, however, errors in discharge as measured by a portable flowmeter would not account for much of the year-to-year variability in the PCC (fig. 8*B*). To accurately quantify the temporal variability in the PCC, long-term time series PCC data are needed.

Errors U_1 (TFM pumpage error) and U_2 (electrical power meter error) represent errors in the long-term integrated values of discharge and power consumption, respectively. The first error (U_1) would result from a TFM that is malfunctioning and providing consistently biased readings, and the second error (U_2) would result from a malfunctioning electrical meter. Although no data are available for evaluating the magnitude of these errors, one or both may be at least partly responsible for the extreme differences in pumpage (*diffP*) seen in figure 9*A*. Component U_1 (TFM pumpage error) would not be present when comparing PCC-estimated pumpage to true pumpage (eq. 15). Finally, errors U_2 and U_6, integrated and instantaneous power meter error, may somewhat compensate for each other if the errors result from a malfunctioning power meter.

ESTIMATION OF TOTAL NETWORK PUMPAGE

The analysis presented earlier in the report, in the “Comparison of Ground-Water Pumpage Estimates” section, provided estimates of the mean or average differences between the log-transformed PCC-estimated total pumpage and TFM-measured total pumpage, *diffP*, at a well. However, it also is important to quantify the differences in the total or aggregated pumpage for a network of wells.

Primary Results

An analysis of the pumpage data was done to determine differences in the total or aggregated pumpage between the TFM and PCC approach for a network of wells. The difference in pumpage between the TFM and PCC approach varied with the volume of water pumped during the 1998 monitoring period. Some wells that recorded small pumpage exhibited larger percent differences than wells with larger pumpage. Because of these unequal differences with respect to total pumpage, it was necessary to group or stratify the data based on the magnitude of total pumpage for the 1998 monitoring period. Because the correct number of groupings, or strata, is not known with the information available, the mean and standard deviation of differences in the total pumpage was determined conditionally for several numbers of strata. For a network of 103 wells and a number of strata greater than 10, the resulting mean and standard deviation leads to a conclusion that, for any given year, there is a 95-percent probability that the difference in aggregated pumpage between the TFM and PCC approach would be between about –3.41 and 1.59 percent. The analysis indicates that the difference in aggregated pumpage would be expected to be smaller as the number of wells becomes larger. Assuming the distribution of total TFM pumpage is the same for 1998 data set, there is a 95-percent probability that the difference in aggregated pumpage between the TFM and the PCC approach for any given year for a network of 1,000 wells would be between –1.71 and –0.11 percent. This assumes that the large differences in pumpage are confined to wells with smaller pumpage. It also is important to emphasize that only 1998 pumpage data were used for this analysis, so the effect of temporal variations (over a period greater than 1 year) of PCC's on total network pumpage is not known.

Details of Analysis and Results

The difference in total pumpage between the PCC and TFM approaches for n wells, D_n, is denoted as,

$$D_n = \sum_{i=1}^{n} \tilde{V}_i - \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} (\tilde{V}_i - V_i) \quad . \tag{18}$$

where $\tilde{V}_i$ denotes the PCC-estimated total pumpage at well i ($i = 1, 2, ..., n$), and V_i is the corresponding value of TFM-measured total pumpage at well i.

To determine the difference in total pumpage for n wells, D_n, it may be assumed that D_n is approximately normally distributed. Once the mean of D_n and the standard deviation of D_n are defined, probability statements may be made on the likely magnitude of network differences from year to year. It is assumed that TFM-measured pumpage values V_i are fixed (non-random), and the mean of D_n and the standard deviation of D_n are expressed relative to total network TFM-measured pumpage.

Complications arise in computing the mean and standard deviation of D_n primarily because of the nonnormality of the individual well differences, *diffP*, and the fact that these differences appear to have a tendency to vary in magnitude depending on how large ground-water pumpage, V, is. This variation necessitates using a stratification scheme. The effect of using the logarithmic transformation also must be considered. Specifically, analysis of how the errors (differences) at individual wells is propagated to total network errors (differences) requires that three relevant issues be considered in some detail: the effect of the logarithmic transformation, the effect of changes of the distribution of differences depending on volume pumped at a well, and the effect of nonnormality of the distribution of differences between (logarithmically transformed) TFM and PCC pumpage volumes.The effect of the logarithmic transformation becomes an issue because, when computing total network pumpage for a number of wells, it is the untransformed values that need to summed. Thus, results from analyses using logarithmically transformed data first need to be back transformed. This back-transformation results in a so-called transformation bias. If the differences between the log-transformed pumpage volumes were identically and normally distributed, then estimating the magnitude of this bias would be straightforward. However, as shown in figure 9*A*, there is indication of a tendency of the distribution of differences to change depending on total pumpage and of nonnormality. Therefore, stratification is used to account for changes in the distribution of differences, and a parameter-estimation procedure that does not rely on an assumption of normality is used.

Much of the problem is associated with the relatively few number of paired measurements that have a much larger difference in pumpage than most of the data (fig. 9*A*). The rank transformation that was used in the analysis of variance down-weighted the effect of these differences and, therefore, produced results that are representative of the central tendency of the data. However, when summing volumes over all wells in a network, the small number of data that have large differences will be included; therefore, the potential effect of these data cannot be ignored. The data associated with the large differences were examined, and a valid reason for deleting them from the analysis was not found. In addition, the nature of the data did not lend itself to fitting a common probability distribution or to description of the exact pattern of the nonuniform variations in the distribution with respect to pumpage. Thus, the approach taken below is essentially nonparametric and should be viewed as an attempt to explore the sensitivity of total network pumpage to these large errors (differences).

Even though it is not assumed that the differences at individual sites, $\tilde{V}_i - V_i$, have any particular distribution, the network difference (D_n), which is the sum of a number of independent random variables, will, under some general conditions, be approximately normally distributed. This follows from central limit theory, and, because of the stratification that is applied below, central limit results for random variables that are not identically distributed need to be used. Experiments at randomly selecting values from the stratified population of total well pumpage to estimate network pumpage indicate that normality is a good approximation for total network pumpage. Given that D_n has an approximately normal distribution, only the mean and variance (or standard deviation) need evaluation. The main purpose of the analysis that follows is to obtain expressions for the mean and standard deviation.

Assume that the V_i are a set of fixed (nonrandom) values, and that the deviation of $\tilde{V}_i$ from V_i is described by a random error. The difference between log-transformed PCC-estimated pumpage and log-transformed TFM-measured pumpage at well *i* is

$$diffP_i = \log(\tilde{V}_i) - \log(V_i) . \tag{19}$$

These errors are all assumed to be associated with different wells, so they will be assumed throughout to be independent.

Exponentiating both sides of equation 19 gives the relation

$$\tilde{V}_i = V_i e^{diffP_i} \tag{20}$$

between the untransformed variables. The additive error on the log-transformed variables becomes a multiplicative error on the untransformed variables. The mean difference between the PCC and the TFM pumpage volume for well *i* is

$$\begin{aligned} E(\tilde{V}_i - V_i) &= V_i E e^{diffP_i} - V_i \\ &= V_i E(e^{diffP_i} - 1). \end{aligned} \tag{21}$$

where *E* denotes mathematical expectation, or mean, and V_i is assumed to be fixed. If the mean deviation is expressed as a fraction of TFM pumpage V_i, it is

$$\frac{E(\tilde{V}_i - V_i)}{V_i} = E(e^{diffP_i} - 1) . \tag{22}$$

If $EdiffP_i = 0$, then it may be shown that $Ee^{diffP_i} > 1$, or $E(e^{diffP_i} - 1) > 0$. Thus, even if the errors in the log-transformed variables have mean zero, there is a positive bias when looking at untransformed variables. This is important because, when looking at network-wide aggregates, the untransformed variables need to be summed, so the absence of bias in the log-transformed variables does not automatically translate into a lack of bias for network-wide aggregates. Bias in the present situation, however, is not limited to bias caused by the logarithmic transformation. Additional bias is introduced by large positive errors that reflect nonnormality of $diffP_i$ (fig. 9*A*), and the variance of these errors changes with V_i, which motivates the need for the stratification that follows.

The mean, or expected, difference (also referred to as bias) is given by

$$ED_n = \sum_{i=1}^{n} V_i E(e^{diffP_i} - 1) , \tag{23}$$

and the variance is given by

$$Var(D_n) = \sum_{i=1}^{n} V_i^2 Var(e^{diffP_i}) . \tag{24}$$

To deal with the error distribution dependence on total pumpage, the population of wells is stratified with respect to the magnitude of total pumpage at a well, V_i, and it is assumed that the errors within each stratum are identically distributed. Equation 23 leads to

$$ED_n = \sum_{k=1}^{K} B_k \mu_k \tag{25}$$

where *K* is the number of strata, B_k is the sum of the V_i for all wells in the *k*th stratum, and $\mu_k = E(e^{diffP_i} - 1)$ for each well *i* in the *k*th stratum. Likewise, equation 24 yields

$$Var(D_n) = \sum_{k=1}^{K} T_k \sigma_k^2 \qquad (26)$$

where T_k is the sum of the V_i^2 for all wells in the *k*th stratum and $\sigma_k^2 = Var(e^{diffP_i})$ for each well *i* in the *k*th stratum.

If the number of strata *K*=1, that is, if the assumption of identical distribution holds, equation 25 gives

$$\frac{ED_n}{\sum_{i=1}^{n} V_i} = E(e^{diffP} - 1) \ . \qquad (27)$$

The important implication in this equation is that, if the differences have the same distribution, bias in the difference in total network pumpage relative to the magnitude of total (TFM) network pumpage is the same magnitude as relative bias for an individual well given in equation 22. For example, a 5-percent bias per well translates into a 5-percent bias for the total network. If *K* is greater than 1, then according to equation 25, network relative bias is a pumpage-weighted average of the individual stratum biases $\mu_1, \mu_2, \ldots, \mu_K$.

Likewise, if *K*=1, the standard deviation of total network error as a fraction of total network pumpage is given by

$$\frac{SD(D_n)}{\sum_{i=1}^{n} V_i} = \frac{\sqrt{\sum_{i=1}^{n} V_i^2}}{\sum_{i=1}^{n} V_i} SD(e^{diffP}) \ . \qquad (28)$$

In this equation, the ratio involving V_i on the right-hand side tends to decrease as the number of wells (*n*) increases. The rate of decrease is in proportion to $1/\sqrt{n}$. Thus, the random component of difference in total network pumpage tends to decrease and become less important compared to the bias component, represented in equation 25, which does not diminish with number of wells, *n*. If *K*>1, it may be shown that the standard deviation of D_n , computed from equation 26, relative to total network pumpage, will still tend to grow smaller as number of wells (*n*) increases, again roughly in proportion to $1/\sqrt{n}$.

Use of equations 25 and 26 requires estimates of the parameters μ_k and σ_k . Let n_k be the number of measurements from the *k*th stratum, and denote these measurements by $diffP_{k1}, diffP_{k2}, \ldots, diffP_{kn_k}$. If it can be assumed that these observations are normally distributed, there are special widely used techniques based on this assumption that can be used to estimate the parameters. Because the normal assumption is not a good one, however, the parameters are estimated by

$$\hat{\mu}_k = \frac{1}{n_k} \sum_{j=1}^{n_k} (e^{diffP_{kj}} - 1) \ . \qquad (29)$$

and

$$\hat{\sigma}_k = \sqrt{\frac{1}{n_k} \sum_{j=1}^{n_k} (e^{diffP_{kj}} - 1 - \hat{\mu}_k)^2} \ . \qquad (30)$$

Equations 29 and 30 are the ordinary sample mean and sample standard deviation of the $e^{diffP} - 1$ values in the *k*th stratum. These estimates are essentially the “smearing estimates” for nonparametric retransformation discussed by Duan (1983) in the context of regression.

The *K* strata for this analysis are formed by dividing the range of $\log(V_i)$ values for the 553 paired-pumpage measurements for 1998 into *K* equal intervals. The number of wells was *n* = 103. The $diffP_{kj}$ used for estimating the mean and the standard deviation in equations 29 and 30 consist of the differences $\log(\tilde{V}_i) - \log(V_i)$ for all the $\log(V_i)$ in the *k*th stratum. The correct or most appropriate value of *K* is not known, so computations in equations 25 and 26 were done using parameter estimates from equations 29 and 30, for *K* ranging from 1 to 50. As *K* increases, the outcome of this analysis is essentially equivalent to randomly selecting a PCC-estimated value at each well for computing pumpage at that site. Results are shown in figure 11. When *K* = 1, the tendency for error magnitude to diminish for larger pumpage is ignored, so that large pumpage could conceivably have errors as large as 239 percent (the maximum value in the data set; see fig. 9*A*), an error that, in the actual data set, is associated with a well that contributes little to total network pumpage. The results of permitting large errors to be associated with wells having large pumpage are severe, yielding a mean of about 9.3 percent and standard deviation of about 11.9 percent for *K* = 1 (fig. 11). For *K* = 2, the mean decreases to 2.70 percent; for *K* = 3, it decreases to 0.77 percent; for K = 4, it decreases to less than 0 percent; for *K* greater than about 10, the mean tends to level off at approximately –0.91 percent. Likewise, the standard deviation levels off for *K* greater than 4 at about 1.25 percent. The fact that the mean becomes negative when *K* is greater than 4 indicates that the large positive errors at a small number of wells have little effect on total network pumpage; it is instead the influence of negative errors for large-pumpage wells (see figure 9) that is causing the mean to become negative as *K* increases. Imposing the restriction that number of strata *K* be larger than 4 prevents the few very large positive errors from being associated with wells that have large pumpage.

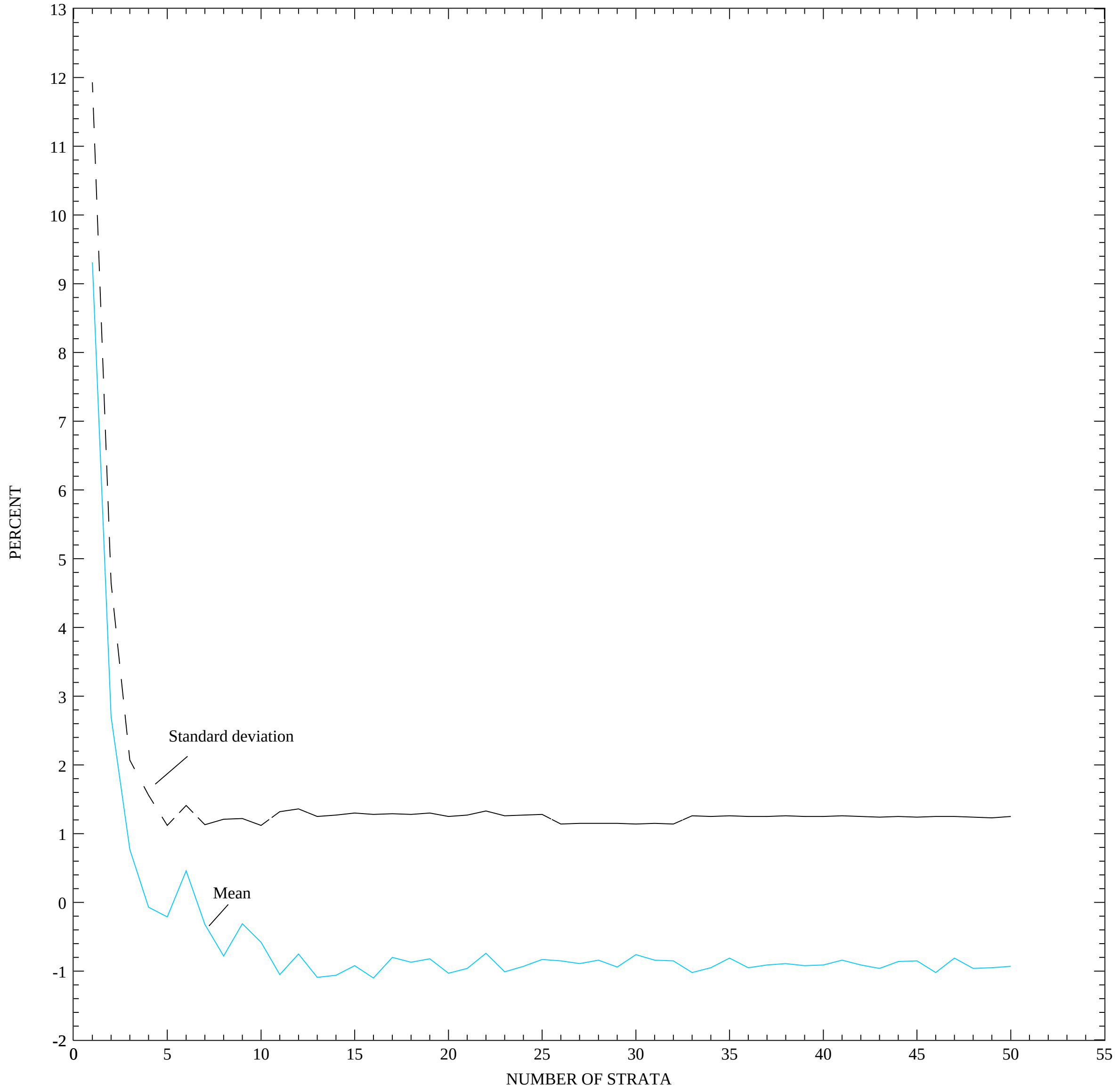


Figure 11. Graph showing relation of the mean and standard deviation of total network pumpage, in percent, to the number of strata.

The approximate normality of the difference in network-aggregated pumpage D_n can be used with the mean and the standard deviation to make probability statements about likely differences in total network pumpage obtained by TFM and PCC approach. Using a mean of –0.91 percent and a standard deviation of 1.25 percent, for example, results in a conclusion that, for any given year, there is a 95-percent probability that the difference in aggregated pumpage between the TFM and PCC approach would be between about –3.41 and 1.59 percent for a network of 103 wells.

To predict the difference in aggregated pumpage for a larger network, the distribution of total TFM pumpage (the values of V_i) will be assumed to be the same, the estimate of the mean remains the same (–0.91 percent), but the standard deviation decreases in proportion to the square root of the ratio of numbers of wells. For $n = 1{,}000$ wells, the 1.25 percent standard deviation for 103 wells decreases by a factor of $\sqrt{\frac{103}{1000}}$, resulting in an estimated standard deviation of 0.40 percent. Therefore, for a network of 1,000 wells, there is a 95-percent probability that the difference in aggregated pumpage between the TFM and the PCC approach for any given year would be between –1.71 and –0.11 percent.

CONCLUSIONS

This report compares two approaches for determining instantaneous ground-water discharge and pumpage. The data collected and analyzed as part of this study included (1) logarithmically transformed differences of well discharge computed from 747 paired discharge measurements made at 105 wells during 1997 and 1998; (2) power conversion coefficients (PCC's) derived for 104 wells during 1997 and 1998; (3) ranked, logarithmically transformed differences of pumpage computed from 553 paired pumpage comparisons made at 103 wells during 1998, and (4) State-approved PCC's that were made from 1994–97.

Given the data analysis presented in this report, the main conclusions are:

1. More than 80 percent of the differences in well discharge were less than 10 percent. The overall mean difference in well discharge for all sites was 0.0 percent, indicating no difference on average between TFM's and portable flow-meter instantaneous discharge measurements. For varying site characteristics, mean differences in well discharge range from a –4 percent to 4 percent.
2. Variations in PCC's measured during the 1998 irrigation season indicated that 58 percent of the wells had less than 10-percent change, and 86 percent of the wells had less than 20-percent change. Systematic seasonal variations in PCC's generally were not evident for the measurements made during the 1998 irrigation season.
3. Ninety percent of the sites had at least one PCC measured during 1997 that was less than the range of PCC's measured in 1998, indicating the range in PCC's measured at majority of sites between 1997 and 1998 were similar.
4. About 48 percent of the State-approved PCC's made between 1994 through 1997 were within 10 percent of the 1998 site average PCC's and about 67 percent of the State-approved PCC measurements made between 1994 through 1997 were within 20 percent of the 1998 site average PCC's.
5. About 80 percent of the differences in pumpage between the TFM and PCC approaches were less than 10 percent. The overall mean difference in pumpage was 0.01 percent, indicating no significant difference on average between pumpage as measured by TFM's and pumpage as computed by the PCC approach. For varying site characteristics, mean differences in pumpage were generally less than ±3 percent and, for most instances, the mean differences in pumpage were not significantly different from zero at the 5-percent significance level.
6. There are several potential sources of discrepancy between pumpage as measured by a TFM and pumpage as computed by the PCC approach. With data currently available, it is not possible to give reliable estimates of the magnitude of each of the potential sources of pumpage error. However, using available data, an estimate of errors caused by temporal variability of PCC's can be made. The year-to-year variance was about nine times the date-within-year variance, indicating that year-to-year variability of the PCC's may make a significant contribution to error in the PCC approach for estimating pumpage. This conclusion is based on an assumption that the State-approved PCC's from 1994–97 are of the same quality as the 1998 PCC's.

7. For a network of 103 wells and a number of strata (logarithms of TFM total pumpage divided into equal subdivisions) greater than 10, the resulting mean and standard deviation indicates that, for any given year, there is a 95-percent probability that the difference in aggregated pumpage between the TFM and PCC approach would be between about –3.41 and 1.59 percent.
8. The difference in aggregated pumpage would be expected to be smaller as the number of well sites becomes larger. Assuming the distribution of total TFM pumpage is the same for 1998 data set, there is a 95-percent probability that the difference in aggregated pumpage between the TFM and the PCC approach, for any given year, for a network of 1,000 wells would be between –1.71 and –0.11 percent. This assumes that the large differences in pumpage are confined to wells with smaller pumpage. It also is important to emphasize that only 1998 pumpage data were used for this analysis, so the effect of temporal variations of PCC's on total network pumpage is not known.

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