The Nuts and Bolts of Falloff Testing

Sponsored by EPA Region 6

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Purpose of a Falloff Test

- Satisfy regulatory requirements
- Measure injection and static reservoir pressures
  - Downhole pressure
  - Surface pressure: requires measurement or estimation of specific gravity of the injectate to calculate bottomhole pressure
- Obtain reservoir properties
  - Calculate transmissibility, kh/
- Provide data for Area of Review (AOR) calculations
- Characterize the nature of the injection zone
- Observe and identify reservoir anomalies
  - Faults or boundaries (multiple or single)
  - Dual porosity (naturally fractured)
- Evaluate completion conditions
  - Skin factor
- Identify completion anomalies
  - Partial penetration
  - Layering
  - Presence of a hydraulic fracture

Background and Definition

UIC Class 1 Well Regulatory Requirements

- §146.13 Operating, monitoring and reporting requirements

  (d)(1) …At a minimum, the Director <u>shall</u> require monitoring of the pressure buildup in the injection zone annually, including at a minimum, a shut down of the well for a time sufficient to conduct a valid observation of the pressure falloff curve.

- Hazardous wells:§146.68 Testing and monitoring requirements

  (e)(1) …At a minimum, the Director <u>shall</u> require monitoring of the pressure buildup in the injection zone annually, including at a minimum, a shut down of the well for a time sufficient to conduct a valid observation of the pressure falloff curve.

Requirements for Hazardous Wells Injecting Restricted Hazardous Waste

- §148.21 Information to be submitted in support of petitions

  (b)(1) Thickness, porosity, <u>permeability</u> and extent of the various strata in the injection zone.

  (b)(4) <u>Hydrostatic pressure</u> in the injection zone

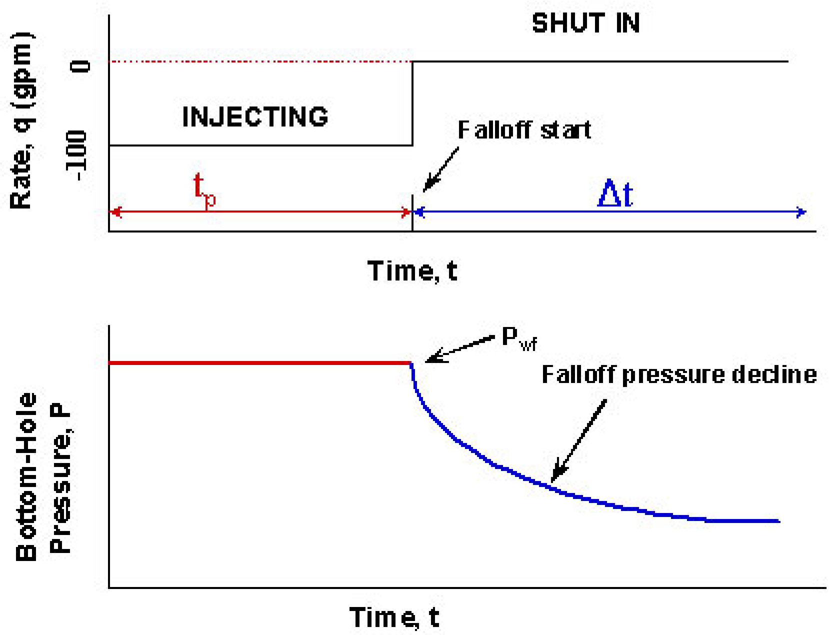
Though the regulations may not require a falloff test for Class II wells, the Director can request additional testing to assure protection of the USDW prior to issuing a permit.

Additional Testing Requirement of Any Class of Injection Well
- §146.8 Mechanical integrity8.21
  (f) The Director may require additional or alternative tests if the results presented by the owner or operator under §146.8(e) are not satisfactory to the Director to demonstrate that there is no movement of fluid into or between USDWs resulting from the injection activity.

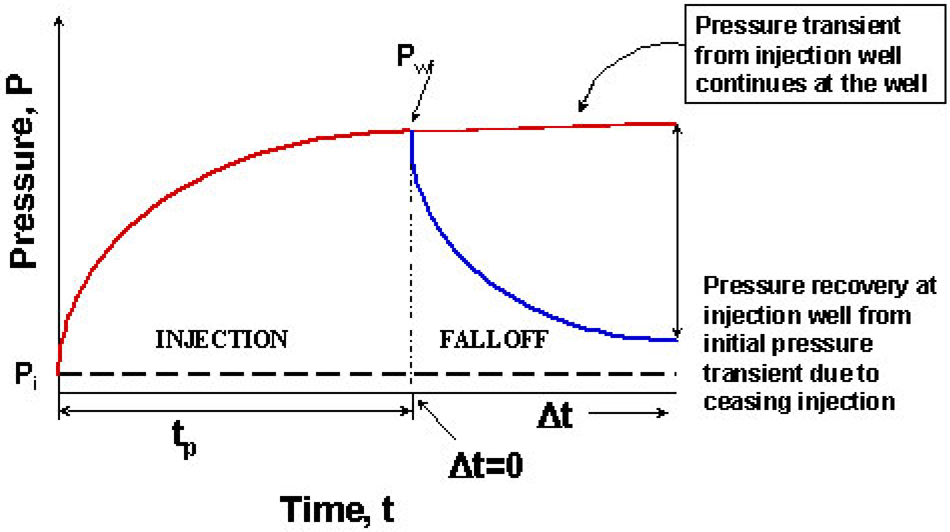
Falloff testing is part of pressure transient theory that involves shutting in an injection well and measuring the pressure falloff
- Equivalent to a pressure buildup test in a producing well
- Analyzed using the same pressure transient analysis techniques used for pressure buildup and drawdown tests

Sequence of Events During a Falloff Test

The falloff is replay of the injection portion of the test. Therefore the injection period controls what is seen on the falloff. A falloff test tends to be less noisy than an injectivity test because there is no fluid passing by the gauge.



Effects of Injection and Falloff



Pressure Transients

- Any injection rate change in the test well or offset well creates a pressure transient in the reservoir
- Simplify the pressure transients in the reservoir
  - Do not shut-in two wells simultaneously
  - Do no change the rate in two wells simultaneously
  - e.g., shut-in test well and increase rate in offset well during the falloff test

Falloff Test Planning

<u>General Planning</u>

- Successful welltests involve considerable pre-planning
- Most problems encountered are within the operator's control and are avoidable
  - Allow adequate time in both injection and falloff periods
  - Injection at a constant rate during the injection period preceding the falloff

<u>Reservoir Considerations</u>

- Reduce the wellbore damage, if necessary, with a stimulation prior to conducting the test
- Type of reservoir:

Sandstone or carbonate (naturally fractured)
Single or multiple injection intervals

Operational Considerations

- Injection well construction
  - wellbore diameters, changing dimensions
- Type of completion
  - Perforated, screen and gravel packed, or open hole
  - Downhole condition of the well that may impact the gauge depth
  - e.g., wellbore fill, liner, junk in the hole
- Wellhead configuration
  - Installation of the pressure gauge without shutting in the well
  - e.g., install a crown valve
- Shut-in valve should be located near the wellhead
  - Minimizes the portion of the test dominated by wellbore hydraulics instead of the reservoir
- Surface Facility Constraints
  - Adequate injection fluid to maintain a constant injection rate prior to the falloff
  - Availability of plant waste
  - Brine brought in from offsite: Location of storage frac tanks
  - Combination of both
- Adequate waste storage for the duration of the falloff test
  - Tests are often ended prematurely because of waste storage issues

Offset Well Considerations

Locate any offset wells completed and operating in the same injection interval

- Obtain a map with offset well distances relative to the injection well
- Shut-in offset well prior to and during the test
  - Requires additional waste storage capabilities
- Maintain a constant injection rate must be maintained both prior to and during the falloff test if not shut-in (Same rate both before and during the test)
- Confirm that diverting waste from the test well does not impact the offset well rate

Recordkeeping

- Maintain an accurate record of injection rates
  - Adequate rate metering system
  - Injection well - prior to shut-in
  - Offset wells - prior to and during the falloff test
- Obtain viscosity measurements of the injectate fluid
  - Confirms the consistency of the waste injected

Rule of Thumb:

At a bare minimum, maintain injection rate data equivalent to twice the length of the falloff

Instrumentation

Pressure Gauges

- Use two, one serving as a backup
  - The backup gauge does not have to be an identical type gauge
- Pressure span of the gauge should not grossly exceed the expected test pressures
- Accuracy and resolution is usually based on a % of the full range of the gauge
- Calibration
  - Ask to see the vendor calibration sheet

Types of Pressure Gauges

- Mechanical downhole gauges
  - Amerada/Kuster: chart recorder with bourdon tube
    - Wind up clock is not reliable for long test periods
  - Typical resolution is approximately 0.05% of full range
- Mechanical surface gauges
  - Surface chart recorders (cheap, but not better)
    - Bourdon tube
    - Can be difficult to read with any accuracy
  - Echometer
  - Pressure gauge
    - requires someone to take pressure readings
- Electronic downhole
  - Quartz crystal
  - Torque capacitance
    - Panex/McAllister/Terratek/HP
    - Much better resolution, approximately 0.0002% of full range
  - Temperature compensated
- Electronic surface
  - Spidr gauge
    - Internal data logging
    - Good for hostile environments
  - Plant transducer
    - Questionable resolution

Pressure Gauge Selection

- Surface readout (SRO) versus downhole memory gauges
  - SRO enables tracking of the downhole pressures in real time
  - More expensive than a memory gauge
- Pressure gauge selection checklist
  - Surface gauge may be impacted by ambient temperature (sunrise to sunset)
  - Wellbore configuration or wastestream may prevent the use of a downhole gauge
  - Surface gauges are insufficient if the well goes on a vacuum
  - Pressure gauge must be able to measure the pressure changes at the end of the test
    - Confirm the accuracy and resolution of the gauge is sufficient for the pressure changes anticipated throughout the welltest
    - Ideally, the maximum test pressure should be at least 50% of the gauge pressure limit
    - Typical electronic downhole pressure gauge limits: 2000/5000/10000 psi

Example: What pressure gauge is necessary to obtain a good falloff test for the following well?

Operating surface pressure: 500 psia
Injection interval: 5000'
Specific gravity of injectate: 1.05
Past falloff tests have indicated a high permeability reservoir of 500 md
Injection well goes on a vacuum toward the end of the test
Expected rate of pressure change during the radial flow is 0.5 psi/hr

1. Calculate the flowing bottomhole pressure to pick a pressure gauge range:
   500 psi + (0.433 psi/ft)(1.05)(5000') = 2773 psi neglecting tubing friction
2. Select a pressure gauge type and range:
   2000 psi gauge is too low
   5000 psi and 10000 psi gauges may both work

Check resolution levels: Mechanical gauge: 0.05% of full range
Electronic gauge: 0.0002% of full range

Mechanical gauges:
5000(0.0005) = 2.5 10000(0.0005) = 5 psi
Electronic gauges:
5000(0.00002) = 0.01 10000(0.00002) = 0.02 psi

The mechanical gauges do no provide enough resolution for the 0.5 psi/hr anticipated at the end of the test. Both the 5000 and 10000 psi electronic gauges provide adequate resolution.

Select the 5000 psi electronic gauge so that more of the full range of the pressure gauge is utilized during the test.

Falloff Test Design

Questions that must be addressed prior to conducting the test:
- How long must the injection period last?
- How long must the well remain shut-in?
- Is there a need to look for a boundary or "x" distance in the reservoir?

The answer to these questions requires making some preliminary assumptions and calculations. If appropriate software is available, it is good to simulate the falloff test using the assumed parameters.

The ultimate objective of the falloff test is to reach radial flow during the injection and falloff portions of the test. The radial flow portion of the test is the basis for all pressure transient calculations.

For wells that have been injecting with no previous falloff data:
- Review the historical well pressure and rate data from plant monitoring equipment
- Look for "pressure falloff" periods when the well was shut-in
- This information *may* provide some information that can be used to design the fallfoff test.

Wellbore Storage: The initial portion of the test when the pressure response at the well is governed by wellbore hydraulics instead of the reservoir.
Radial Flow: Follows the wellbore storage and transition period. The pressure response is only controlled by reservoir conditions during radial flow.
Transition Period: The time period between identifiable flow regimes.

It is necessary to calculate the time to reach radial flow during both the injectivity and falloff periods.

Test Design Calculations

<u>Wellbore Storage Coefficient</u>

To calculate the time to reach radial flow, first estimate the wellbore storage coefficient, C in bbl/psi. There are two different equations to calculate C depending on whether the well goes on a vacuum or maintains a positive pressure at the surface throughout the duration of the test.

For a fluid filled well with positive pressure at the surface during the falloff test:

$$C = V_w \cdot c_{waste}$$

where, V_w is the total wellbore volume, bbls

c_{waste} is the injectate compressibility, psi^{-1}

For a falling fluid level or well that goes on a vacuum during the falloff test:

$$C = \frac{V_u}{\dfrac{\rho \cdot g}{144 \cdot g_c}}$$

where, V_u is the wellbore volume per unit length, bbls/ft

is the injectate density, lb/ft^3 or psi/ft

These empirically derived equations can be used with limitations:

- If C is small, the well is connected with the reservoir within a short timeframe if the skin factor is not excessively large
- If C is large, a longer transition time is warranted for the well to display a reservoir governed response
    - High skin prolongs wellbore storage
    - Some carbonate reservoirs contain vugs which cause larger C values
    - C can be minimized by downhole shut-in

<u>Time to Reach Radial Flow</u>

The equations used to calculate the time to reach radial flow, $t_{radial\ flow}$, are different for the injectivity and falloff portions of the test. The $t_{radial\ flow}$ can be approximated using the following equations:

To calculate the time to reach radial flow for an injectivity test use:

$$t_{radial\ flow} > \frac{(200000 + 12000s) \cdot C}{\dfrac{k \cdot h}{\mu}} \quad hours$$

To calculate the time to reach radial flow during the falloff test use:

$$t_{radialflow} > \frac{170000 \cdot C \cdot e^{0.14s}}{\frac{k \cdot h}{\mu}} \quad hours$$

Note: Skin factor, s, influences the falloff more than the injection period

Example: What injection and falloff timeframes are necessary to reach radial flow given the following injection well conditions? Assumption is that the well maintains a positive wellhead pressure during the test.

Reservoir	Wellbore
h=120'	7" tubing (6.456" ID)
k=50 md	9 5/8" casing (8.921" ID)
s=15	Packer depth: 4000'
=0.5 cp	Top of the injection interval: 4300'
c_w= 3e-6 psi^{-1}	

1. Calculate the wellbore volume, V_w:
   Tubing volume+casing volume below the packer

$$V_w = \left[\pi\left(\frac{6.456}{2 \cdot 12}\right)^2 (4000) + \pi\left(\frac{8.921}{2 \cdot 12}\right)^2 (300)\right]\left(\frac{1\,bbl}{5.615\,ft^3}\right) = 185.1\,bbls$$

2. Calculate the wellbore storage coefficient, C
   Fluid filled wellbore: $C=V_w\ c_{waste}$

$$C = 185.1\,bbls \cdot \frac{3x10^{-6}}{psi} = 5.5x10^{-4}\,\frac{bbl}{psi}$$

Note: Assume the
wellbore storage coefficient is the same for both the injection and falloff periods
C is small since the wellbore is fluid-filled

3. Calculate the minimum time to reach radial flow during the injection period

$$t_{radial\ flow} > \frac{(200000 + 12000s) \cdot C}{\frac{k \cdot h}{u}} \qquad hours$$

$$t_{radial\ flow} > \frac{(200000 + 12000 \cdot 15) \cdot 5.5x10^{-4}}{\frac{50 \cdot 120}{0.5}} = 0.017\,hours$$

Note: The test should not only reach radial flow, but also sustain a timeframe sufficient for analysis of the radial flow period.

4. Calculate the minimum time to reach radial flow during the falloff period

$$t_{radial\ flow} > \frac{170000 \cdot C \cdot e^{0.14 \cdot s}}{\frac{k \cdot h}{\mu}} \qquad hours$$

$$t_{radial\ flow} > \frac{170000 \cdot 5.5x10^{-4} \cdot e^{0.14(15)}}{\frac{50 \cdot 120}{0.5}} = 0.064\,hours$$

The time to radial flow is still short, but the falloff needed four times the time the injection period needed to reach radial flow.

Use with caution! This equation tends to blow up in large permeability reservoirs or wells with high skin factors

Test Design Criteria

- Decide on test objectives
  - Completion evaluation
    - Need to reach radial flow to calculate the skin factor which indicates the condition of the well
  - Determining the distance to a fault or boundary
  - Seeing "x" distance into the reservoir to confirm geology
    - Use the radius of investigation, r_i, to calculate time
- Determine the type of test needed to produce analyzable results
  - Falloff, multi-rate, or interference test

- Simulate the test using estimated parameters
    - Sensitivity cases can evaluate the effects of varying reservoir parameters
- Review earlier test data, if available

Data Needed to Analyze a Falloff Test

- Time and pressure data
    - Surface and bottomhole pressure measurements can be used
- Rate history prior to the falloff
    - Include rate history of offset injection or production wells if completed into the same interval
- Basic reservoir and fluid information
- Wellbore and completion data
    - Wellbore radius, r_w
- Record sufficient pressure data to analyze
- Consider recording pressures more frequently earlier in the test
    - More frequent data with an electronic gauge generally provides a better quality derivative curve, by providing more points to average when calculating the slope
    - Consider plotting data while the test is in progress to monitor the test
- Net thickness, h (feet)
    - Obtain from well log, cross-sections, or flow profile surveys
- Permeability, k (md)
    - Obtain from core data or previous well tests
- Porosity, (fraction)
    - Obtain from well log or core data
- Viscosity of reservoir fluid, $_f$ (cp)
    - Direct measurement or correlations
- Total system compressibility, c_t (psi^{-1})
    - Correlations, core measurement, or welltest
- Viscosity of reservoir fluid, $_w$ (cp)
    - Direct measurement or correlations
- Specific gravity, s.g., of injectate
    - Direct measurement
- Rate, q (bpd)
    - Direct measurement

<u>Test Design Checklist</u>

- Wellbore construction: Prepare a wellbore schematic for completion depths, well dimensions, obstructions, fill depth, injection interval depths

- Injection rate period: Constant rate if possible, minimum duration, injection history, waste storage capacity, offset well rates

- Falloff period: Time and pressure data, rate history, duration to radial flow, offset well rates, waste storage capacity
- Instrumentation: Resolution of the gauge, surface versus bottomhole gauge, backup gauge, rate measurments
- General reservoir and waste information: h, , c_t, $_f$, $_w$
- Area geology Boundaries, net thickness trends, type of formation (sandstone or carbonate)

Pressure Transient Theory Overview

Pressure Transient (P-T) theory attempts to correlate well pressures and rates as a function of time in terms of reservoir, fluid, and well completion parameters. P-T theory is the basis for drawdowns, buildup, injectivity testing, interference or pulse tests, falloffs, step-rate tests, multi-rate tests, drill stem tests, slug tests, inflow performance, and decline curve analysis. P-T theory is used in petroleum engineering, groundwater hydrology, solution mining, waste disposal, and geothermal projects.

P-T theory involves working the problem backwards:

- From the measured pressure response, determine the reservoir parameters
- Start at the wellbore and work out to the reservoir boundaries
- Late time data is a pressure response from farther in the reservoir

Start with what you know:

Well and completion history
Geology
Test conditions

Pressure responses show dominant features called flow regimes

<u>P-T Theory Applied to Falloff Tests</u>

Falloff testing is part of P-T theory. Falloff tests are analyzed in terms of flow models which are derived from basic concepts to obtain pressure-rate behavior as a function of time. Flow models are analytical solutions to the flow equations or numerical simulators.

The starting point is a partial differential equation (PDE) based on Darcy's Law and the material balance equation. The PDE is solved for drawdown for a variety of boundary conditions to calculate pressure or rate as a function of time and distance.

For non-steady state flow, the PDE is:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial P}{\partial r} = \frac{1}{0.000264} \cdot \frac{\phi \cdot \mu \cdot c_t}{k} \cdot \frac{\partial P}{\partial t}$$

This equation assumes an infinite, homogeneous, isotropic reservoir with a slightly compressible fluid and c_t, k, , , are independent of pressure, P.

These equations and assumptions provide a model for injection well behavior and an analysis approach for the evaluation of reservoir parameters. The equations are only applicable during the radial flow period of the falloff test.

To solve the PDE some equation constraints must be assumed both near and away from the well to obtain a flow model. For a typical falloff analysis the following constraints are assumed:

- Inner (near the well constraints)
  - Wellbore has a finite well radius
  - Inject rate is constant prior to the falloff, at time t=0
- Outer (out in the reservoir constraints)
  - Infinite-acting reservoir
  - Welltest reaches radial flow
  - Isotropic reservoir properties
  - Reservoir is at a uniform initial pressure, P_i

The exact solution to the PDE is in terms of cumbersome Bessel functions. Fortunately an approximate solution based on the exponential integral, Ei function, gives almost identical results. The solution using the Ei function is:

$$P = P_i + 70.6\,\frac{q \cdot \mathrm{B} \cdot \mu}{k \cdot h} \cdot Ei\left(\frac{-948 \cdot \phi \cdot \mu \cdot c_t \cdot r_w^2}{k \cdot t}\right)$$

where,

$$Ei(-x) = -\int_x^{\infty} \frac{e^{-u}}{u}\,du$$

- Ei Function
  - Tabulated and easy to use
  - Valid until boundaries affect the data
  - Give the pressure in the reservoir as a function of both time and distance from the well center

The Ei function can be simplified further with a logarighmic approximation which is the basis for all radial flow analyses:

$$Ei = \ln(1.781 \cdot x)$$

This approximation for the Ei function leads us to our flow model for falloff test analysis to predict the pressure buildup in the well using the PDE solution.

$$P_{wf} - P_i = \frac{141.2 \cdot q \cdot B_w \cdot \mu}{k \times h} \cdot (P_D + s)$$

where,

$$P_D = -\frac{1}{2} \cdot Ei\left[-\frac{r_D^2}{4 \cdot t_D}\right] \cong \frac{1}{2} \cdot \left\{\ln\left[\frac{t_D}{r_D^2}\right] + 0.809\right\}$$

and

$$t_D = \frac{0.0002637 \cdot k \cdot t}{\phi \cdot \mu \cdot c_t \cdot r_w^2} \qquad r_D = \frac{r}{r_w}$$

Note these equations use dimensionless variables, P_D, t_D, and r_D

Example: Estimate the injection pressure of a well located in an infinite acting reservoir with no skin (s=0). The well has injected 100 gpm for 2 days. Other reservoir data are:

P_i = 2000 psi	h = 50'
k = 200 md	B_w = 1 rvb/stb
$_f$ = 0.6 cp	c_t = 6e-6 psi^{-1}
= 30%	r_w = 0.4'

1. After converting to the appropriate units, calculate r_D, t_D, and P_D:

$$q = \left(\frac{100\,gal}{\min}\right)\left(\frac{bbl}{42\,gal}\right)\left(\frac{1440\min}{day}\right) = 3428.6\,bpd$$

$$t = (2\,days)\left(\frac{24\,hrs}{day}\right) = 48\,hrs$$

Since we're calculating the pressure at the well, $r = r_w$ and $r/r_w = r_D=1$

$$t_D = \frac{0.0002637 \cdot k \cdot t}{\phi \cdot \mu \cdot c_t \cdot r_w^2}$$

$$t_D = \frac{0.0002637 \ (200\ md)(48\ hours)}{(0.3)(.6\ cp)(6e-6\ psi^{-1})(0.4^2\ ft^2)}$$

$$t_D = 14.65x10^6$$

P_D can be looked up on the following graph taken from Figure C.2 from SPE Monograph 5. At t_D= 1.465x10^7 and r_D=1, P_D= 8.5

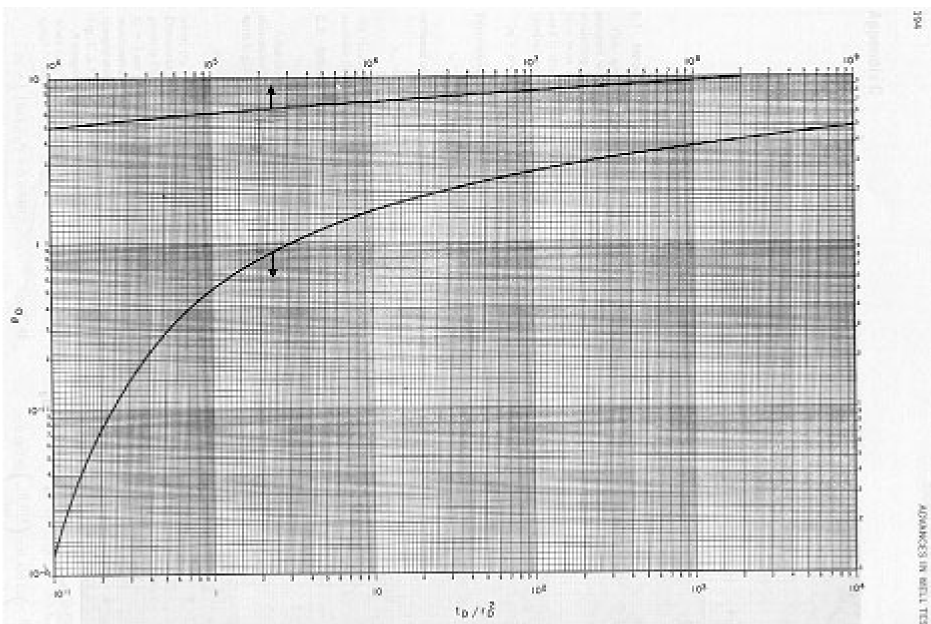


Fig. C.2 Dimensionless pressure for a single well in an infinite system, no wellbore storage, no skin. Exponential-integral solution.

P_D can also be calculated:

$$P_D \cong \frac{1}{2} \cdot \left\{ \ln\left[\frac{14650000}{1^2}\right] + 0.809 \right\}$$

$$P_D \cong 8.65$$

2. Now calculate the pressure increase at the well, P_{wf}:

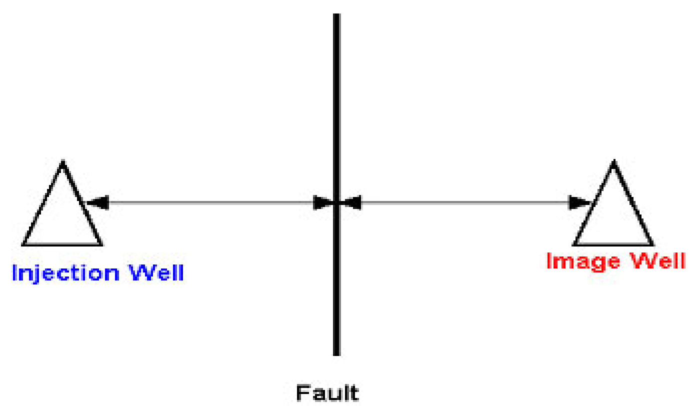
$$P_{wf} - P_i = \left(\frac{141.2 \cdot q \cdot B_w \cdot \mu}{k \times h}\right) \cdot (P_D + s)$$

$$P_{wf} - 2000 = \left(\frac{141.2(3428.6)(1)(0.6)}{(200)(50)}\right) \cdot (8.65 + 0)$$

$$P_{wf} = 2251\, psi$$ (a pressure increase of 251 psi)

The assumptions that the reservoir is infinite or the injection rate is constant are not always valid. The solution to the PDE is linear so that Ei solutions can be added together to account for boundaries and rate changes in the test well or offset well.

The boundaries are handled by representing them as virtual boundaries with the use of fictitious "image" wells. Pressure contributions of the real injector and image wells are summed together to account for the boundary.



$$\Delta P_{total} = \Delta P_{injection\ well} + \Delta P_{image\ well}$$

Where, $P_{injection\ well}$ is
the pressure buildup at the injection well due to injection
$P_{image\ well}$ is the pressure buildup at the injection well due to the fault
P_{total} is the measured pressure buildup at the injection well

For a single boundary, each injector has an offset image well. In the case of multiple boundaries, boundaries are treated similarly, but image well location determination and number is more complex due to interactions of the boundaries and mirroring of the image wells.

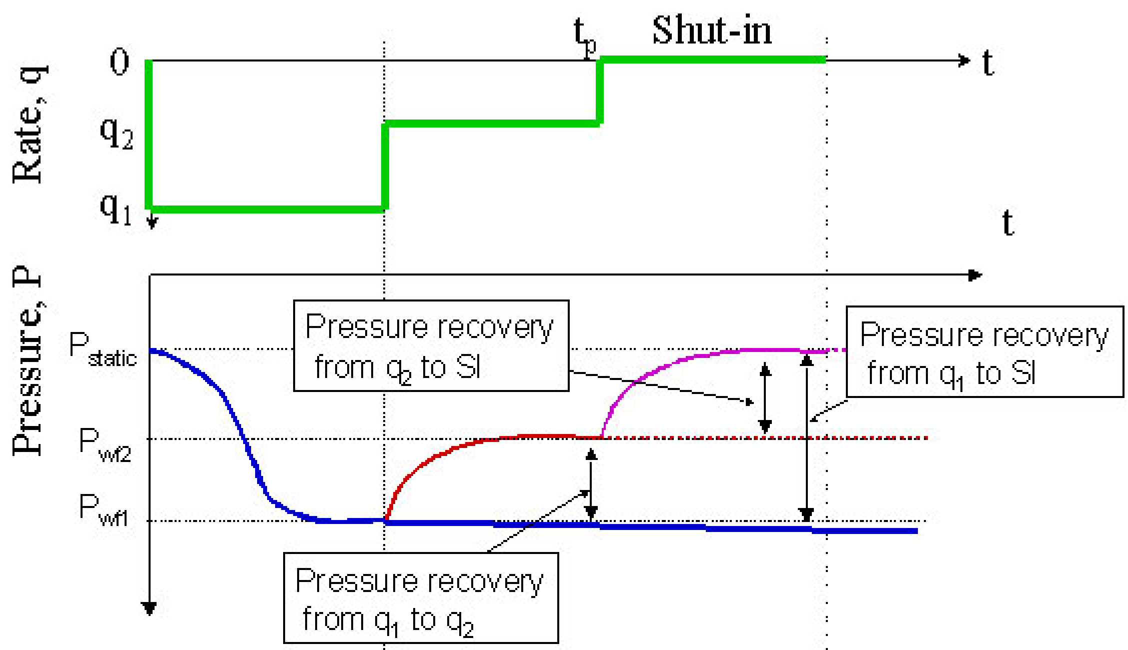
If the injection rate in the test well or offset wells varies prior to the falloff, each rate change can be accounted for using the PDE solution. Each rate causes a new pressure response to be added to the previous response. Each rate change is accounted for by using an image well at the same location as the injector with a time delay and summing image well pressure contributions.

$$\Delta P_{total} = \Delta P_{injector} + \sum \text{Image well for each rate change with time lag}$$

In dimensionless terms for any point in time, t, the equations results in the following:

$$\Delta P_{total} = \frac{141.2 \cdot \mu}{k \cdot h} \cdot \sum_{j=1}^{N} \left\{ \left(q_j \cdot \beta_j - q_{j-1} \cdot \beta_{j-1} \right) \cdot \left[P_D \cdot \left(\left[t - t_{j-1} \right]_D \right) + s \right] \right\}$$

Superposition is the method of accounting for the effects of rate changes on a single point in the reservoir from anywhere and anytime in the reservoir including at the point itself using the PDE solution.



The “Kitchen Sink” solution to the PDE to account for all wells and potential boundaries (image wells) in a reservoir, the pressure change at any point could be given by:

$$p(x,y,t) = p_o + \sum_{j=1}^{N} \frac{70.6 q_1^j \mu}{kh} Ei\left(\frac{-39.5\phi\mu c_t\left[(x-x_j)^2 + (y-y_j)^2\right]}{k\,t} \right)$$
$$+ \sum_{j=1}^{N} \sum_{i=1}^{n_j-1} \frac{70.6\left[(q_{i+1}^j - q_i^j)\mu\right]}{k\,h} Ei\left(\frac{-39.5\phi\mu c_t\left[(x-x_j)^2 + (y-y_j)^2\right]}{k(t-t_i^j)} \right)$$

This is essentially what an analytical reservoir simulator does!

PDE Solution at the Injector

The PDE can give the pressure at any distance from the wellbore using dimensionless variables. This is useful for area of review (AOR) calculations.

At the wellbore, r_D=1 so:

$$P_{wf} = P_i - \left[\frac{162.6 \cdot q \cdot B \cdot \mu}{k \cdot h}\right] \cdot \left[\log(t) + \log\left(\frac{k}{\phi \cdot \mu \cdot c_t \cdot r_w^2}\right) + 3.23 + 0.87s\right]$$

Note: This equation leads to the use of the semilog plot

Semilog Plot

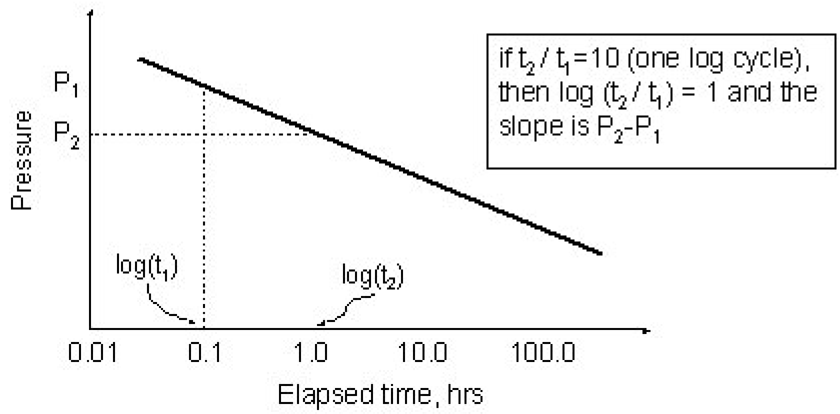
The semilog plot is *only* used during the radial flow portion of the test. By grouping the slope and intercept terms together, the solution to the PDE can be written in the following form, used to define a straight line, which is the basis for the semilog plot.

$$P_{wf} = m \cdot \log\ (t) + P_{1hr}$$

where, m is the slope of the semilog plot and defined as:

$$m = -\frac{162.6 \cdot q \cdot \mathrm{B} \cdot \mu}{k \cdot h}$$

The semilog slope, m, can be determined from the semilog plot:



$$\text{slope} = \frac{\Delta P}{\Delta \log(\Delta t)} = \frac{P_2 - P_1}{\log(t_2) - \log(t_1)} = \frac{P_2 - P_1}{\log\left(\dfrac{t_2}{t_1}\right)}, \quad \text{psi/logcycle}$$

There are four different semilog plots typically used in pressure transient analysis:

- Miller Dyes Hutchinson (MDH) Plot
  - Pressure vs log t
- Horner Plot
  - Pressure vs log (t_p+ t)/ t
- Agarwal Time Plot
  - Pressure vs log equivalent time
- Superposition Time Plot
  - Pressure vs log superposition time function
  - Pressure/rate vs log superposition time function

<u>MDH Plot</u>

- Semilog plot of pressure versus log t, where t is the elapsed shut-in time of the falloff period.
- Applies to wells that have reached psuedo-steady state during injection. Psuedo-steady state means the response from the well has encountered all the boundaries around the well.
- Only applicable to wells with *very* long injection periods at a constant rate.

Note: EPA Region 6 does not recommend the use of the MDH Plot.

<u>Horner Plot</u>

- Semilog plot of pressure versus log $(t_p+\ \ t)/\ \ t$, where t_p is the time of the injection period preceding the falloff
- Used only for a falloff preceded by a constant rate injection period.
- Calculate the injection time, t_p:

  $t_p = V_p/q$ hours

  where, V_p = injection volume since the last pressure stabilization

  V_p is often calculated as the cumulative injection volume since completion

Caution: Horner time can result in significant analysis errors if the injection rate varies prior to the falloff

<u>Agarwal Time Plot</u>

- Semilog plot of pressure versus log equivalent time, t_e
- Calculate equivalent time, t_e:

  $t_e = \log((t_p\ \ \ t)/(t_p+\ \ t)$ where t_p is defined above for a Horner Plot
- Similar to a Horner plot except the time function is scaled to make the falloff look like the injectivity portion of the test. In the case with a short injection period and long falloff period, the equivalent time function will compress the falloff time to that of the injection period.

<u>Superposition Time Function</u>

- Semilog plot of pressure or normalized pressure versus a superposition time function
- The superposition time function can be written several ways. Below is for a drawdown or injectivity test:

$$\Delta t_{sp} = \left[\sum_{j-1}^{n} \left(\frac{q_j - q_{j-1}}{q_n} \right) \log\left[\Delta t - \Delta t_{j-1} \right] \right]$$

- Used to account for rate variations. Pressure function can be modified for the rate preceding the falloff by the following:

$$\Delta P_{sp} = \frac{\left(P_{initial} - P_{wf}\right)}{q_n}$$

Which Time Function is Correct?

The correct time function to use is dependent on the available information and software.

- If no rate history or cumulative injection total, use elapsed time on a MDH plot.
- If there is not rate history other than a single rate and cumulative injection, use Horner time on a Horner plot.
- If the injection period is shorter than the falloff test and only a single rate is available, use Agarwal equivalent time.
- If you have a variable rate history use superposition when possible. As alternative to superposition, use Agarwal equivalent time on the log-log plot to identify radial flow. The semilog plot can be plotted in either Horner or Agarwal time if radial flow is observed on the log-log plot.
- Horner is a single rate superposition and *may* substitute for superposition if:
  - The rate prior to shut-in lasts twice as long as the previous rate **and**
  - The rate prior to shut-in lasts as long as the falloff period

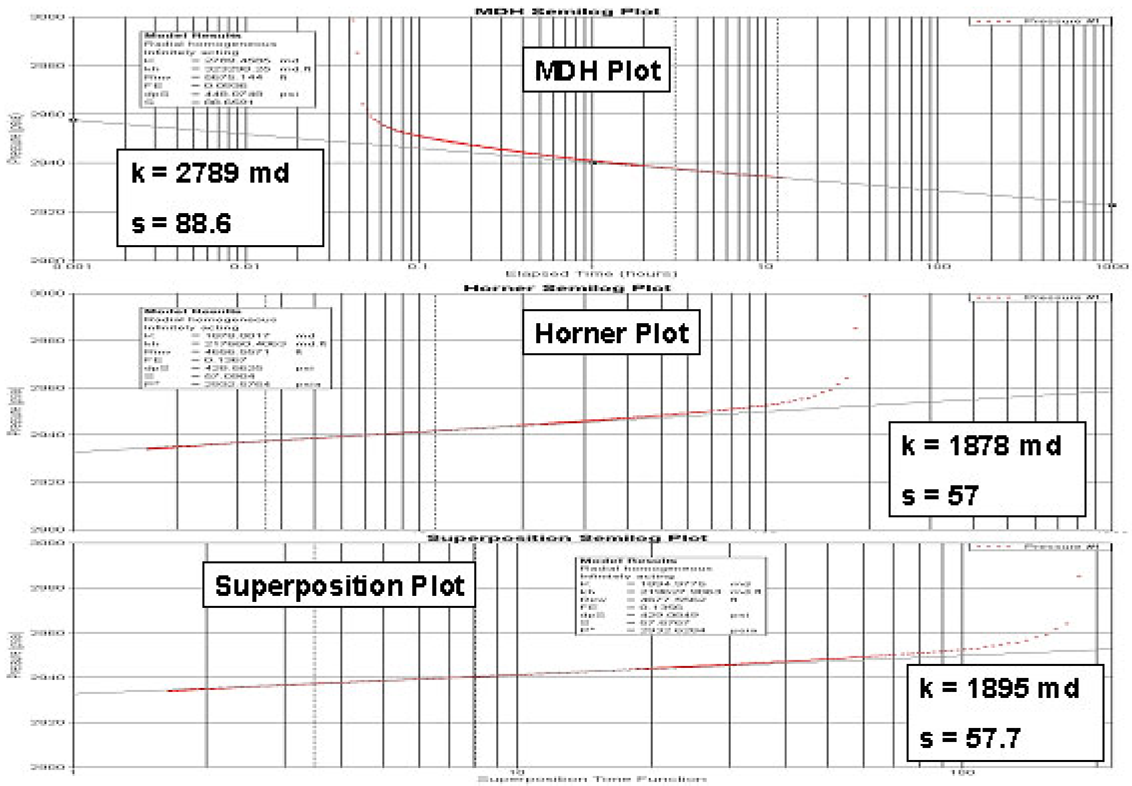
Agarwal, Horner, or MDH plots can be generated in a spreadsheet, however, the superposition time function is usually done with welltest software.

Rule of Thumb:

Use MDH time only for *very* long injection times (not recommended)
Use Horner time when you lack rate history or software capability to compute the superposition function
Superposition is the *preferred* method if a rate history is available

Example of the same falloff test plotted using three semilog methods:

The test consisted of a 24 hour injection period followed by a 24 hour falloff. Notice the invalid permeability and skin values calculated by the MDH plot.



Other Uses for the Semilog Plot

- Calculating the radius of investigation, r_i
- Providing completion evaluation by the skin factor, s
- Effective wellbore radius, r_{wa}
- Determining the skin pressure drop, P_{skin}
- Calculating the false extrapolated pressure, P^*
- Calculating the injection efficiency

<u>Radius of Investigation</u>

The radius of investigation, r_i, is the distance a pressure transient has moved into a formation following a rate change in a well (definition taken from Well Testing by Lee). The appropriate time is needed to calculate r_i. For a falloff time longer than the injection period, use equivalent time, t_e, or the length of the injection period preceding the falloff to calculate r_i. There are numerous equation that exist to calculate r_i. They are all square root equations based on cylindrical geometry, but each has its own coefficient that results in slightly different results (from OGJ, Van Poollen, 1964)

Two equivalent equations to calculate r_i in feet are taken from SPE Monograph 1 (Eq 11.2) and Well Testing by Lee (Eq 1.47):

$$r_i = \sqrt{0.00105 \frac{k\,t}{\phi\,\mu\,c_t}} \quad \equiv \quad \sqrt{\frac{k\,t}{948\phi\,\mu\,c_t}}$$

where,
k = permeability, md
= viscosity, cp
c_t = total system compressibility, psi^{-1}
= porosity, fraction
t = time, hours (depends on the falloff and injection periods of the test)

Wellbore Skin and Skin Factor

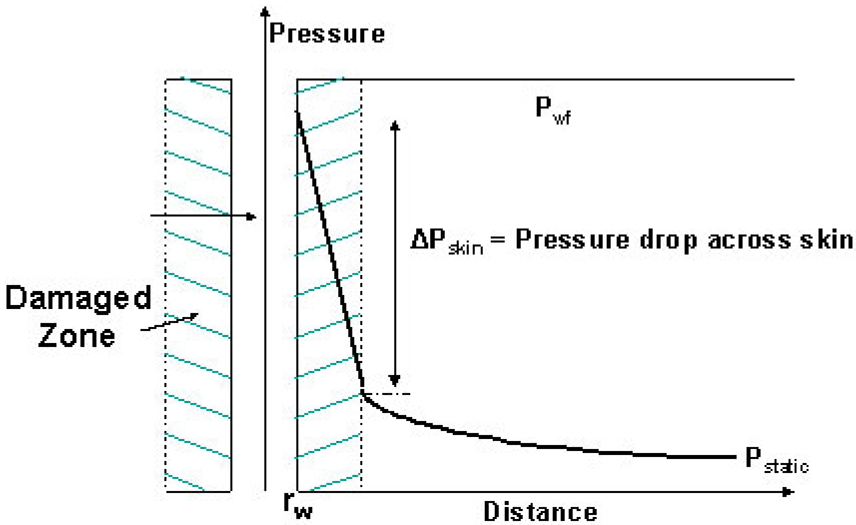
The skin factor, s, is included in the PDE. Wellbore skin is the measurement of damage near the wellbore, i.e., completion condition. The skin factor is calculated from the radial flow portion of the welltest using the following equation:

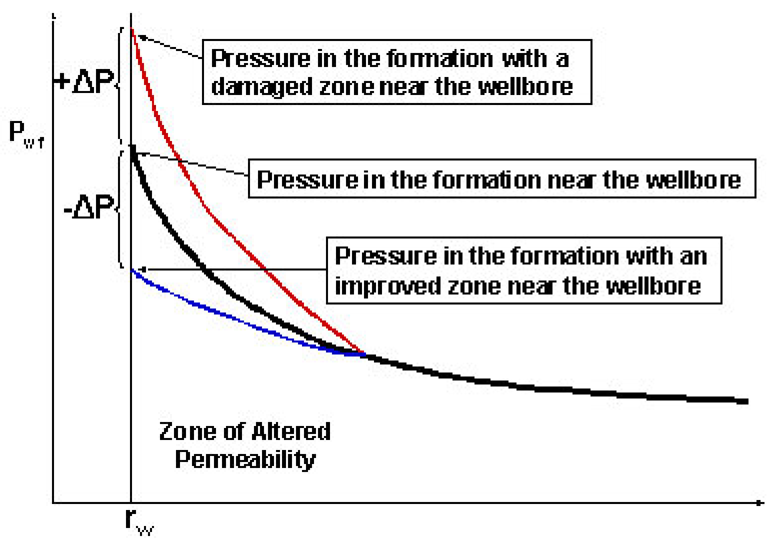
$$s = 1.1513\left[\frac{p_{1hr} - p_{wf}}{m} - \log\left(\frac{k\,t_p}{(t_p+1)\phi\,\mu\;c_t\,r_w^2}\right) + 3.23\right]$$

The slope of the semilog straight line, the injection pressure prior to shut-in, and the pressure value of the extended semilog straight line at a t = 1hr are used to calculate the skin factor.

Note: The term $t_p/(t_p+\ t)$, where t=1 hr, appears in the log term and this term is assumed to be 1. For short injection periods, e.g., drill stem tests, this term could be significant.

The assumption that the skin exists as a thin sheath is not always valid for injection wells. This is not a serious problem in the interpretation of the falloff test, but can impact the calculation for correcting the reservoir injection pressure for skin effects. Wellbore skin creates a pressure change immediately around the wellbore. The effect may be a flow enhancement or impediment.





Wellbore skin is quantified by the skin factor:

- \+ positive value indicates a damaged completion. The magnitude is dictated by the transmissibility of the formation
- \- negative value indicates a stimulated completion. Negative value results in a larger effective wellbore and therefore a lower injection pressure
  - -4 to - 6 generally indicates a hydraulic fracture
  - -1 to -3 typical of an acid stimulation results in a sandstone reservoir.

Wellbore skin increases the time needed to reach radial flow in a falloff. Too high a skin may require excessively long injection and falloff periods to establish radial flow. The larger the skin, the more the pressure drop is due to the skin.

There are several causes or sources of skin damage. Some impediments may include mud invasion and partial penetration, whereas an enhancement may come from an acid or a frac job. The total skin calculated from the welltest may be a combination of several skin components, for example:

$$s_{total} = s_d + s_{pp} + s_s + s_{pt} + s_{ft} + s_e$$

where, s_d is skin due to damage or stimulation
s_{pp} is skin due to partial penetration
s_s is skin due to a slanted wellbore
s_{pt} is skin due to perforation turbulence
s_{ft} is skin due to formation turbulence
s_e is skin due to equipment upstream of pressure gauge

Effective Wellbore Radius

The calculation for the effective wellbore radius, r_{wa}, ties in the skin factor. The r_{wa} is also referred to as the wellbore apparent radius.

$r_{wa} = r_w \; e^{-s}$ where, r_w - wellbore radius, in
s - skin factor, dimensionless

Example: A 5.5" cased well had a skin of +5 prior to stimulation and -2 following the acid job. What was the effective wellbore radius before and after stiumlation?

Before: r_{wa} = (5.5 in)(e^{-5}) = 0.037 in
After: r_{wa} = (5.5 in)($e^{-(-2)}$) = 40.6 in

A little bit of skin makes a big difference in the effective wellbore radius!

Skin Pressure Drop

The skin factor is converted to a pressure loss using the skin pressure drop equation.

P_{skin} = 0.868 m s
where, P_{skin} = pressure drop due to skin, psi
m = slope of the semilog plot, psi/cycle
s = skin factor, dimensionless

This equation quantifies what portion of the total pressure drop in a falloff test is due to formation damage.

Corrected Injection Pressure

The following equation is used to calculate the injection pressure with the skin effects removed:

$$P_{corrected} = P_{inj} - \Delta P_{skin}$$

where, $P_{corrected}$ = adjusted bottomhol pressure, psi
P_{inj} = measured injection pressure prior to shut-in at t=0, psi
P_{skin} = pressure drop due to skin, psi

The corrected injection pressure, $P_{corrected}$, is based on the pressure loss through the formation only. This term is used for comparison to modeled pressures in a no migration petition.

False Extrapolated Pressure versus Average Reservoir Pressure

False extrapolated pressure, P^*, is the pressure obtained from the Horner or superposition semilog time of 1 as illustrated from Figure 5.5 taken from SPE Monograph 5.

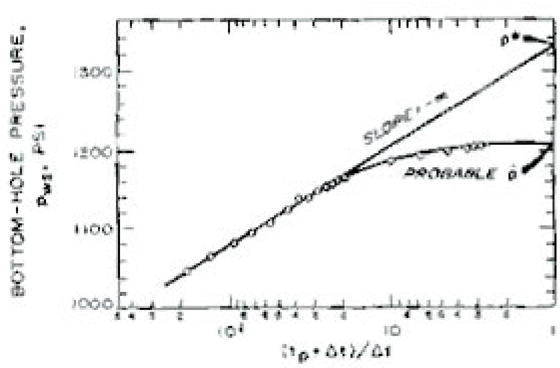


Fig. 6.1 Typical pressure buildup curve for a well in a finite reservoir. After Matthews and Russell.[1]

For a new well in an infinite acting reservoir, P^* represents the initial reservoir pressure. Whereas for existing wells, P^* must be adjusted to the average reservoir pressure, p. This requires an assumption of reservoir size, shape, injection time, and well position within the shape.

For long injection times, P^* will differ significantly from p. P^* to p conversions are based on one well reservoirs with simple geometry or specific waterflood patterns.

Rule of Thumb:

EPA Region 6 does not recommend using P*. Use the final measured shut-in pressure if the well reaches radial flow for the cone of influence calculations.

Injection Efficiency

Injection efficiency calculation is identical to the flow efficiency equation:

$$FE = \frac{\overline{P} - P_{wf} - \Delta P_{skin}}{\overline{P} - P_{wf}}$$

This equation requires an estimation of the average reservoir pressure, $\overline{P}$.

Identifying Flow Regimes

To identify the radial flow portion of the test, the falloff data is first plotted on a master diagnostic plot called the log-log plot. The log-log plot identifies the various stages and flow regimes that can be present in a falloff test.

Key stages and flow regimes found on the log-log plot include wellbore storage, partial penetration, radial flow, and boundary effects. Not all stages and flow regimes are observed on every falloff test.

The *critical* flow regime is radial flow, from which all analysis calculations are performed. Therefore identifying the radial flow portion of the test is necessary before any reservoir parameters or well completion conditions can be determined.

Individual flow regimes have characteristic slopes and a sequencing order on the log-log plot. These dominant features are a result of the pressure responses observed during the welltest.

Log-log Plot

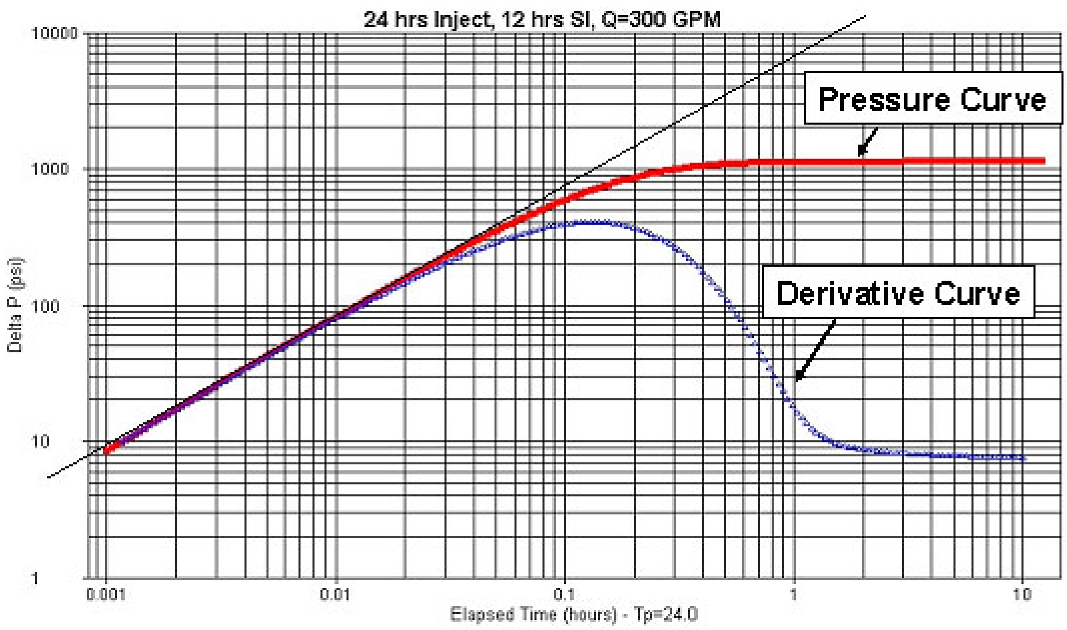
The log-log plot contains two curves:

- Pressure curve
    - Plot of measured pressures from start of the test on the Y-axis versus the appropriate time on the X-axis

- Derivative curve

  Plot of the slope of the semilog pressure function on the Y-axis versus the appropriate time function on the X-axis

Example log-log plot:



<u>Log-log Plot Pressure Functions</u>

Rate variations in the test well prior to shut-in determine how pressure will be plotted on the Y-axis.

- Constant rate: Plot pressure
- Variable rate: Normalize pressure (P/q term) using the rate data

<u>Log-log Plot Time Functions</u>

As with the semilog plot, injection rate variations prior to the falloff period dictate the log-log plot time function. The time function is plotted on the X-axis

- Elapsed Time, t

  Use if the injection rate preceding the falloff is constant *and* the injection period preceding the falloff is significantly longer than the falloff period

  Calculate as: $t = t_{shut\text{-}in} - t_{each\ data\ point}$

- Agarwal Equivalent Time, t_e
  - Use if the injection period is short
  - Calculate as the following for each test point, t:

$$t_e = \frac{t_p \, \Delta t}{t_p + \Delta t}$$

where, $t_p = V_p/q$, hours
V_p = injection volume since last pressure equalization
V_p = often taken as the cumulative injection volume since completion
q = injection rate prior to shut-in

- Superposition Time
  - Use if the injection rate varied prior to the falloff and rate history is available
  - Calculate as the following for each test point, t:

$$\Delta t_{sp} = \left[\sum_{j-1}^{n} \left(\frac{q_j - q_{j-1}}{q_n} \right) \log\left[\Delta t - \Delta t_{j-1}\right] \right]$$

Most rigorous time function for the log-log plot

<u>Log-log Plot Derivative Function</u>

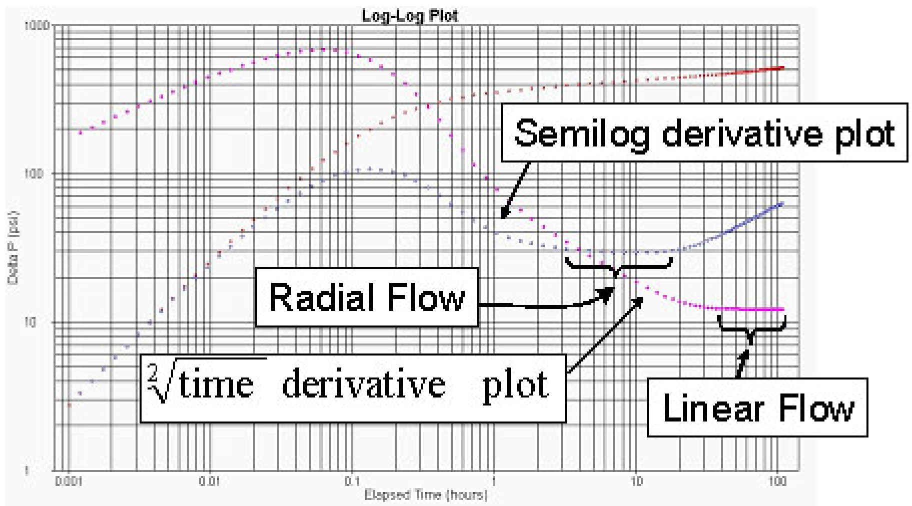
The derivative function is graphed on the log-log plot with the pressure change trend (slope). Its main use is to magnify small changes in pressure trends to identify flow regimes, boundary effects, layering, or natural fractures. This methodology has been popular since 1983 when an article by Bourdet was published in World Oil in May 1983. The derivative for a specific flow regime is independent of the skin factor, while the pressure is not.

The derivative essentially combines a semilog plot with a log-log plot. It calculates the running slope of the MDH, Horner, equivalent time, or superposition time semilog plots. Derivatives amplify reservoir signatures and noise so the use of a good pressure recording device is critical.

Derivative curves are usually based on the semilog pressure plot, but it can be calculated based on other plots such as the following. Some flow regimes are easily identified when plotted with one of these time functions.

- Cartesian plot
- square root of time plot
- 1/square root of time
- quarter root of time

Example: Well in a channel - well observes linear flow after reaching the channel boundaries



The logarithmic derivative is defined by:

$$P' = \frac{d[P]}{d[\ln(\Delta t)]} = \Delta t \cdot \frac{d[P]}{d[\Delta t]}$$

For an infinite acting reservoir with radial flow:

$$P_D = 0.5(\ln[t_D] + 0.80907)$$

When dealing with dimensionless variables, the derivative is always 0.5

$$P'_D = t_D \cdot \frac{d[P_D]}{d[t_D]} = 0.5$$

For cases when a reservoir is in radial flow and infinite acting and dimensionless variables are not used, the derivative will plot as a *constant value* which is graphically depicted as a *flat spot* on the derivative curve.

At any t during the wellbore storage period, the pressure changes, P, and derivative, P', are given by:

$$\Delta P = \frac{q \cdot B}{24 \cdot C} \cdot \Delta t \quad \text{and} \quad P' = \Delta t \cdot \frac{d[\Delta P]}{d[\Delta t]} = \frac{q \cdot B}{24 \cdot C} \cdot \Delta t$$

taking logs of both sides of the pressure change equation:

$$\log[\Delta P] = \log\left[\frac{q \cdot B}{24 \cdot C}\right] + \log[\Delta t]$$

The above equation plots on a log-log plot as a slope of 1. This is known as the "unit slope" during wellbore storage. Since the pressure derivative is described by the same equation during wellbore storage, it overlays the pressure change trend with the same unit slope on the log-log plot.

For linear flow:

$$P_D = \sqrt{\pi \cdot t_D}$$

therefore,

$$\log[P_D'] = \log[0.5] + 0.5 \cdot \log[\pi] + 0.5 \cdot \log[t_D]$$

so a log-log plot will have a slope of 0.5 (half slope)
The derivative, P':

$$P_D' = t_D \cdot 0.5 \cdot \frac{\pi^{0.5}}{t_D^{0.5}} = 0.5 \cdot \pi^{0.5} \cdot t_D^{0.5}$$

for t_D =1.0, P_D = 0.5 log[ ] = 0.248

$$\log[P_D'] = 0.5 \cdot \log[\pi] + 0.5 \cdot \log[t_D]$$

again we get a slope of 0.5, but the line is lower because when t_D = 1, log [P'] = -0.1

On the log-log plot, flow regimes are characterized by specific slopes and trends for the pressure, P, and derivative, P', curves as well as specific separation between P and P' curves.

Recent type curves make use of the derivative by matching both the pressure and derivative curves simultaneously to get one match for the parameter evaluation.

Specific Flow Regimes

Flow regimes are characterized by mathematical relationships between pressure, rate, and time. They provide a visualization of what goes on. Flow regimes have readily identifiable signatures on diagnostic log-log plots or specialized plots. A test can show several flow regimes with “late time” responses correlating to distances farther from the wellbore.

Examine the well completion history and wellbore fill to determine what flow regimes may be present in and near the wellbore during the early time behavior.

Examine the reservoir geology, logs, etc., to determine late time behavior. Typical late time flow regimes may include faults, layering, or natural fractures.

<u>Wellbore Storage</u>

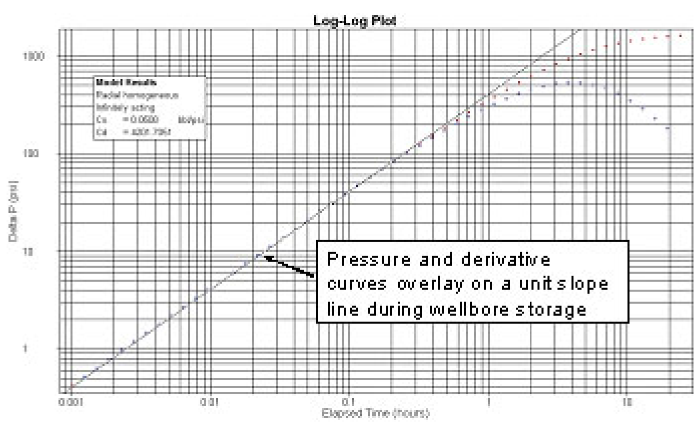
Occurs during the early portion of the test. It is caused by the shut-in of the well being located at the surface rather than the sandface resulting in afterflow as fluid continues to fall down the well after it is shut-in. The location of the shut-in valve away from the wellhead will also prolong the wellbore storage period.

The pressure responses governed by wellbore conditions, e.g., wellbore storage, are not representative of reservoir behavior. Wellbore skin or low permeability reservoirs results in a slower transfer of fluid from the well to the formation extending the duration of the wellbore storage period.

A wellbore storage dominated test is unanalyzable.

Identifying characteristics:

- Log-log plot: unit slope for both the pressure and derivative curves
- Cartesian plot: straight line for the pressure curve

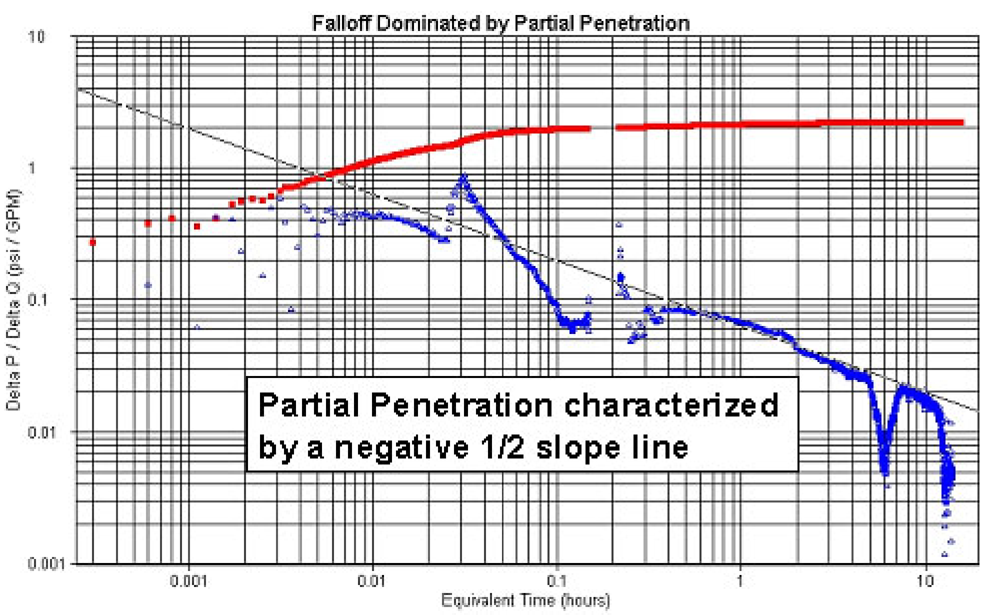


Linear Flow
Results from injection into a channel sand, a well being located between parallel faults, or a well with a highly conductive fracture.

Identifying characteristics:
- Log-log plot: half slope on both the pressure and derivative curves with the derivative curve appoximately 1/3 of a log cycle lower than the pressure curve
- Square root time plot: straight line for the pressure curve

Spherical Flow
Results from wellbore fill covering the injection interval or only a portion of a larger injection interval is completed.

Identifying characteristics:
- Log-log plot: negative half slope on the derivative curve
- 1/square root time plot: straight line for the pressure curve

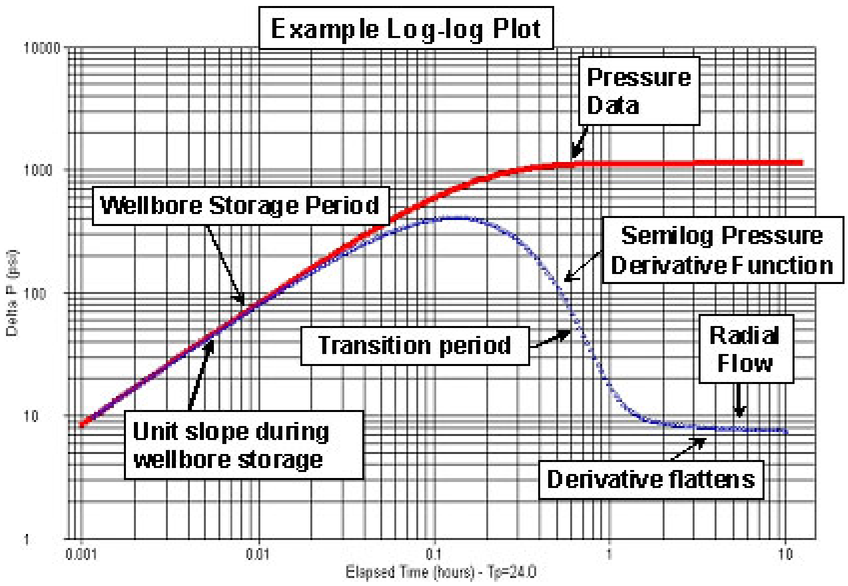


Radial Flow

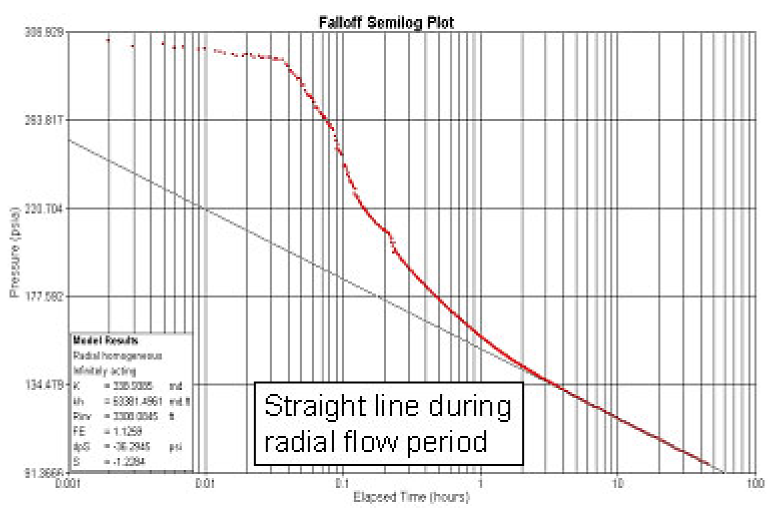
The *critical* flow regime from which all analysis calculations are performed. This flow regime is used to derive key reservoir parameters and completion conditions.

Identifying characteristics:

- Log-log plot: flattening of the derivative curve



- Semilog plot: straight line for the pressure curve



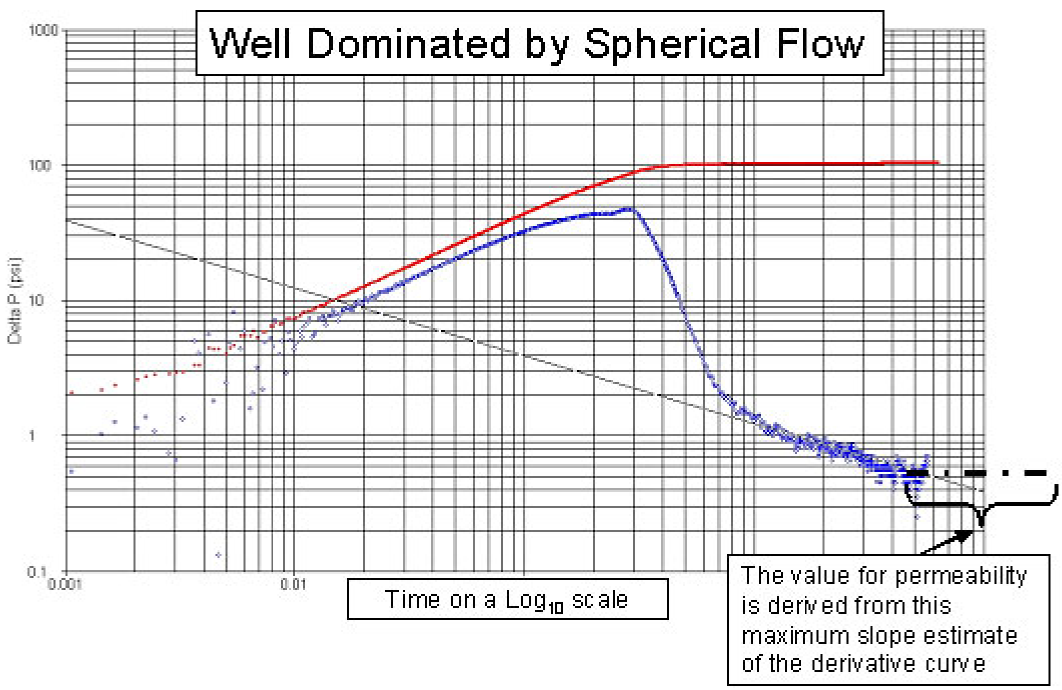
In tests where the derivative did not reach a plateau (i.e. radial flow), a minimum estimate for transmissibility can be obtained from either log-log plot derivative or semilog plot slope. The transmissibility obtained at this point in the test is a minimum because the derivative has not reached its minimum value. The derivative reaches its minimum value at the radial flow plateau, resulting in a smaller slope value and, consequently, a larger transmissibility.

The minimum value for transmissibility is estimated as follows:

$$\frac{k \cdot h}{\mu} = \frac{162.6 \cdot q \cdot B}{m_{test\ end}}$$

where m is determined from drawing a straight line at the end of the semilog plot or by taking the antilog of the derivative value at the test end as follows:

$$m_{test\ end} = 10^{P'_{test\ end}}$$



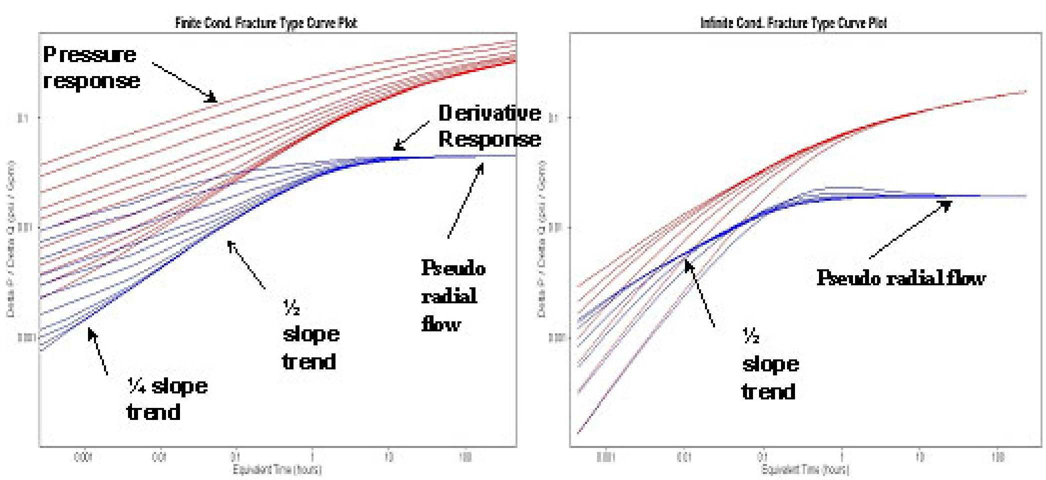
<u>Hydraulically Fractured Well</u>

Typical flow regimes and identifying characteristics:

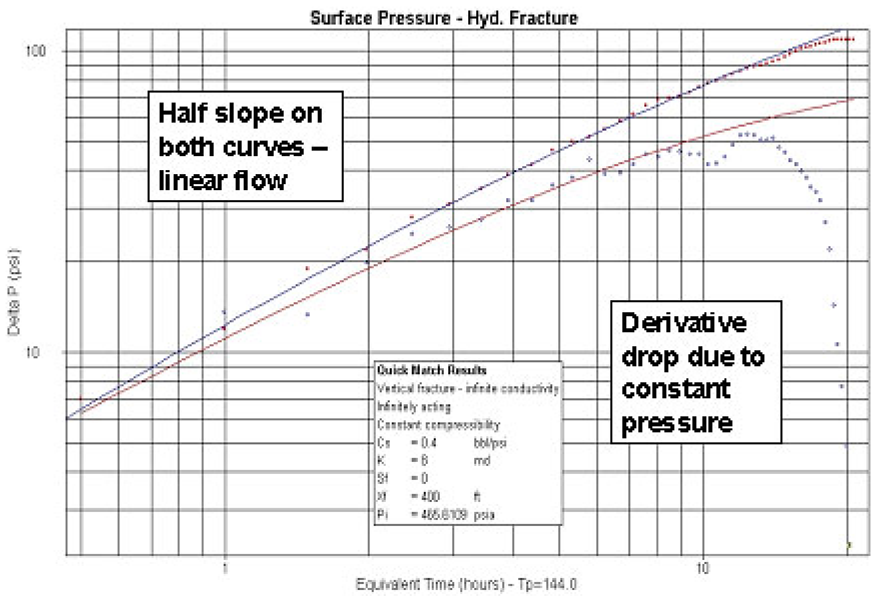
- Wellbore storage
  - log-log plot - unit slope of both derivative and pressure curves
- Fracture linear flow
  - Usually hidden by wellbore storage
- Bilinear flow
  - Result of simultaneous linear flows in the fracture and from the formation into the fracture
  - Log-log plot - quarter slope on the derivative curve
  - Quarter root plot - straight line for the pressure curve

  Formation linear flow
  - Linear flow from formation into fractures
  - Log-log plot - half slope on both the pressure and derivative curves
  - Square root time plot - straight line for pressure curve
- Psuedo-radial flow
  - Log-log plot - horizontal line (flattening) of derivative
  - Log-log plot type curve - dervative will fall about a dimensionless derivative value of 0.5
  - Semilog plot: straight line for pressure curve
  - Semilog plot valid for determining reservoir parameters and fracture characteristics

Hydraulic Fracture Type Curve Responses:

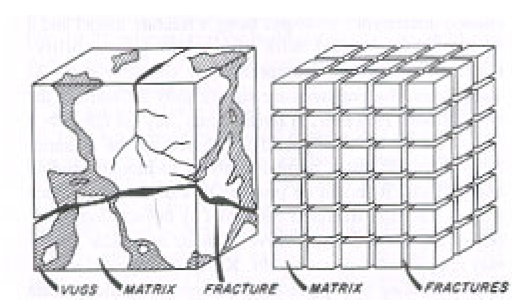


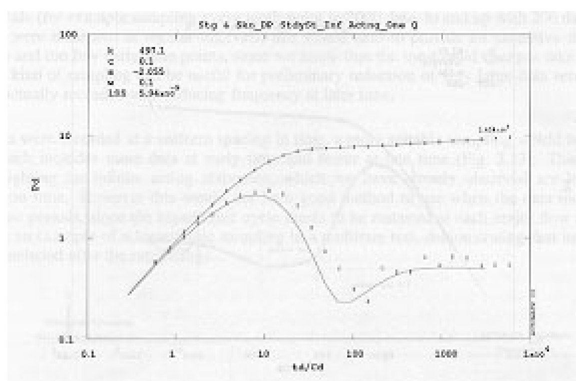
Example from a fractured injection well



<u>Naturally Fractured Rock</u>

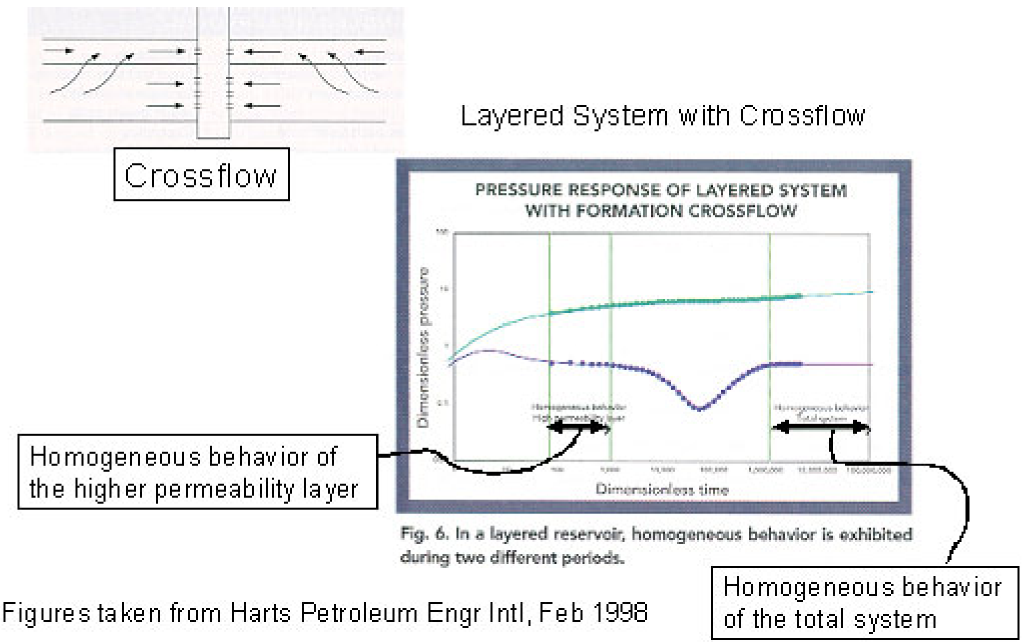
- Fracture system will be observed first on the falloff followed by the total system comprised of the fracture and tight matrix rock.
- Analysis is complex. The derivative trough indicates the level of communication between the fracture and matrix rock.



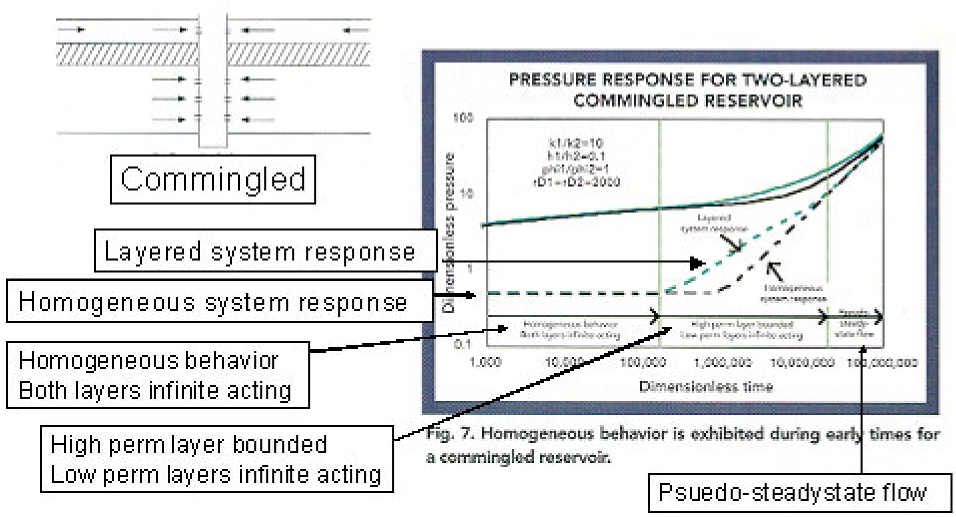


<u>Layered Reservoir</u>

Analysis of a layered reservoir is complex because different boundaries may exist for each layer. The falloff objective for UIC purposes is to get a total transmissibility from the whole reservoir system.



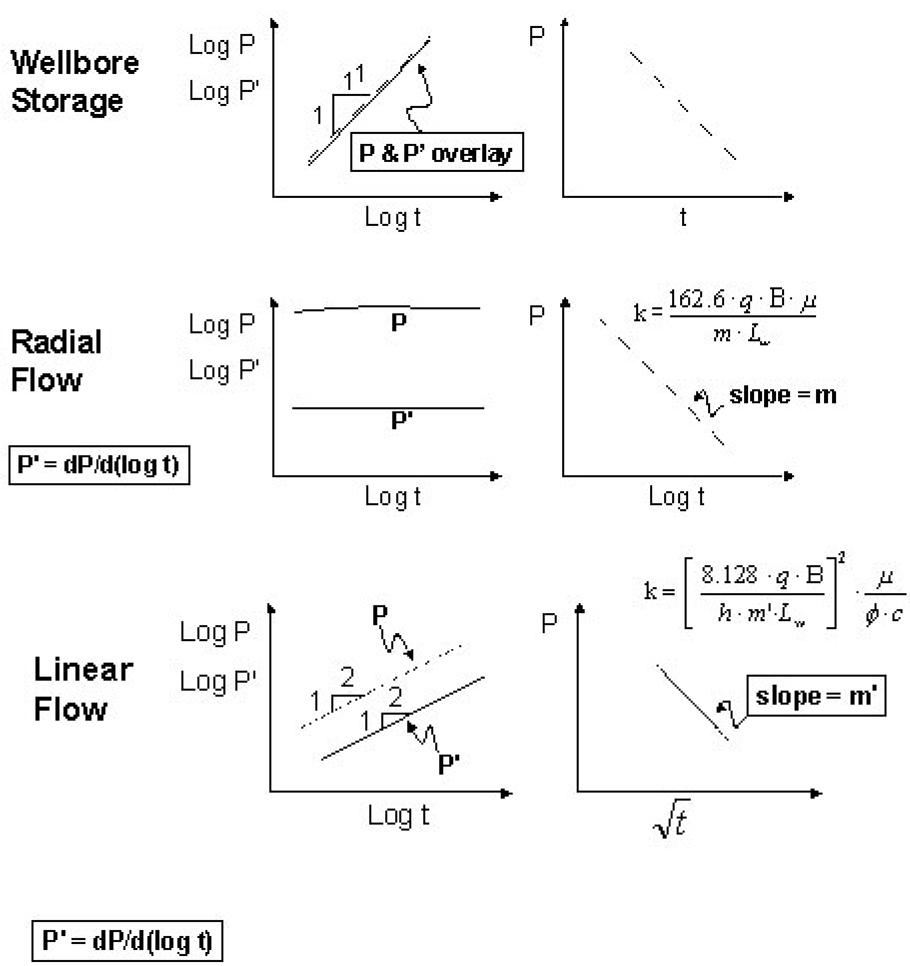
Figures taken from Harts Petroleum Engr Intl, Feb 1998



Figures taken from Harts Petroleum Engr Intl, Feb 1998

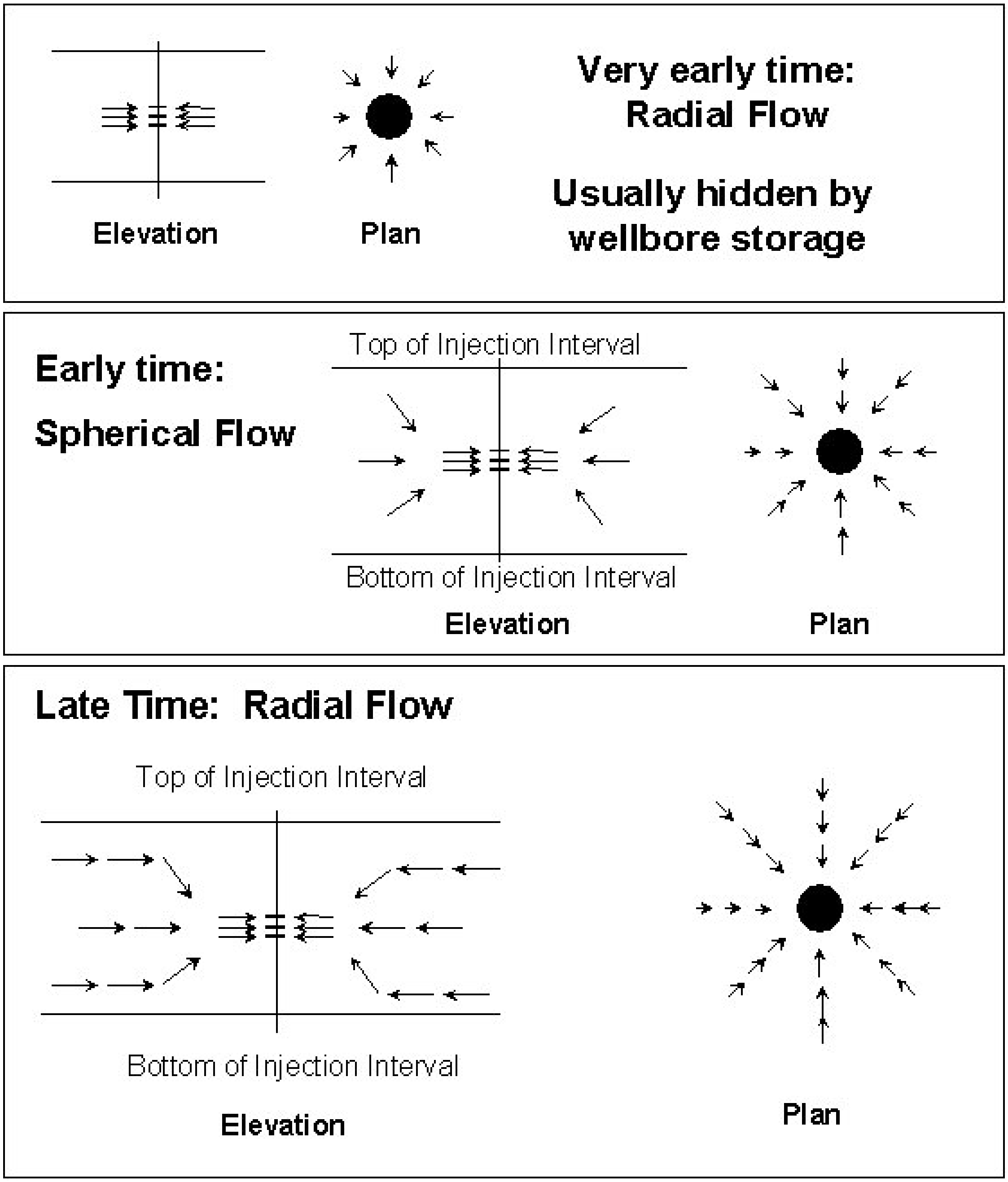
Typical Derivative Flow Regime Patterns

Flow Regime	Derivative Pattern
Wellbore storage	Unit slope
Radial flow	Flat plateau
Linear flow	Half slope
Bilinear flow	Quarter slope
Partial penetration	Negative half slope
Layering	Derivative trough
Dual porosity	Derivative trough
Boundaries	Upswing followed by plateau
Constant pressure	Sharp derivative plunge



Example: Partial Penetrating Well

Partial interval perforated in a block sand injection interval. Can predicted the pressure response based on the completion and injection interval thickness.



Falloff Test Evaluation Procedure

- Data acquisition
    - Well information from well schematic
        - well radius, r_w
        - type of completion
    - Get reservoir and injectate fluid parameters
        - porosity,
        - total system compressibility, c_t
        - viscosity, $_f$ and $_w$
    - Estimate reservoir thickness, h
        - use flow profile surveys
        - well log or cross-section
    - Obtain rate histories
        - test well prior to the test
        - offset wells prior to and during the test
    - Time sync injection rate data with pressure data
- Prepare a Cartesian plot of pressure and temperature versus time
    - Confirm stabilization of the pressure prior to shut-in
    - Look for anomalous data
        - missing data
        - pressure rise or jump in data
        - fluctuations in temperature that may impact pressure
- Prepare a log-log plot of the pressure and the derivative
    - Use appropriate time scale
    - Identify the radial flow period - flattening of derivative curve
    - If there is no radial flow period, try type curve matching
- Make a semilog plot
    - Use the appropriate time function
    - Draw a straight line through the points located within the equivalent time interval where radial flow is indicated on the log-log plot
    - Determine the slope, m, and P_{1hr} from the semilog straight line
    - Calculate reservoir and completion parameters
        - transmissibility, kh/
        - skin factor, s
        - radius of investigation, r_i, based on the Agarwal equivalent time, t_e
- Check results using type curves (optional)

Example Gulf Coast Falloff Test

Well parameters:

r_w = 0.4 ft

cased hole perforated completion

6020'-6040'

6055'-6150'

6196'-6220'

Depth to fill - 6121'

Gauge depth - 6100' (Panex 2525 SRO)

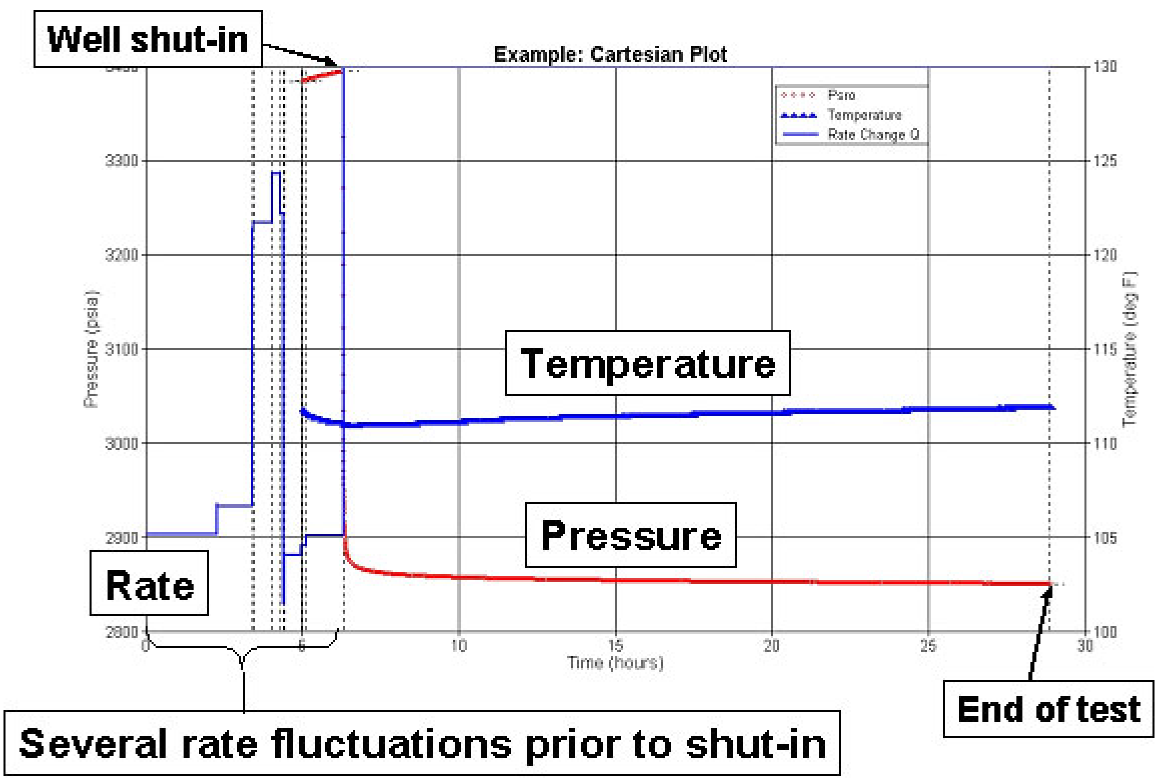
Reservoir parameters

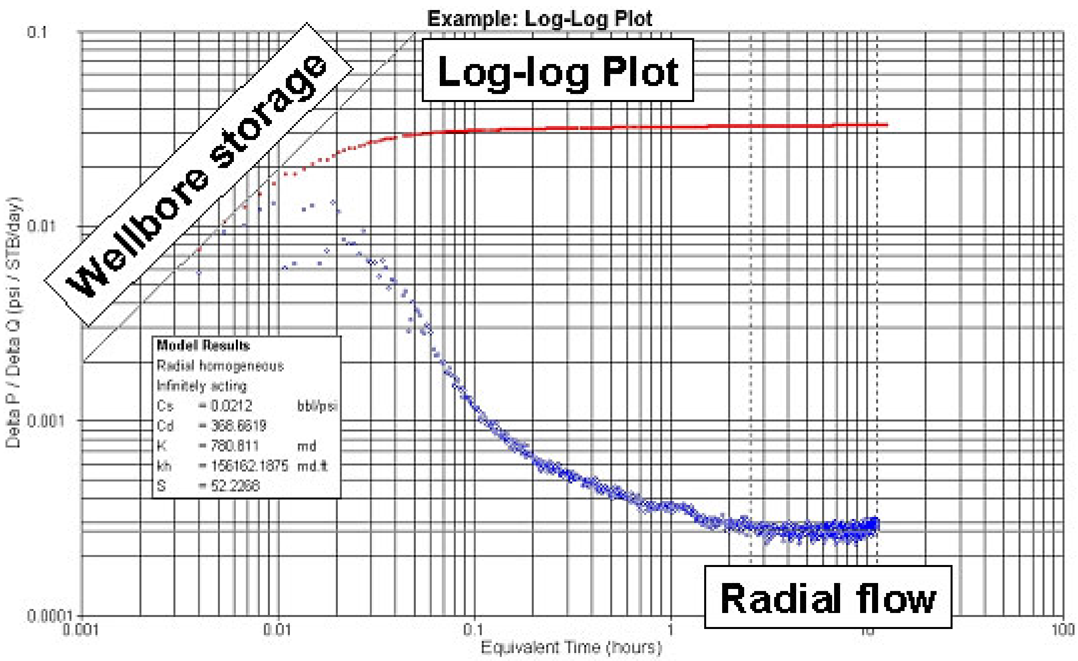
h = 200'

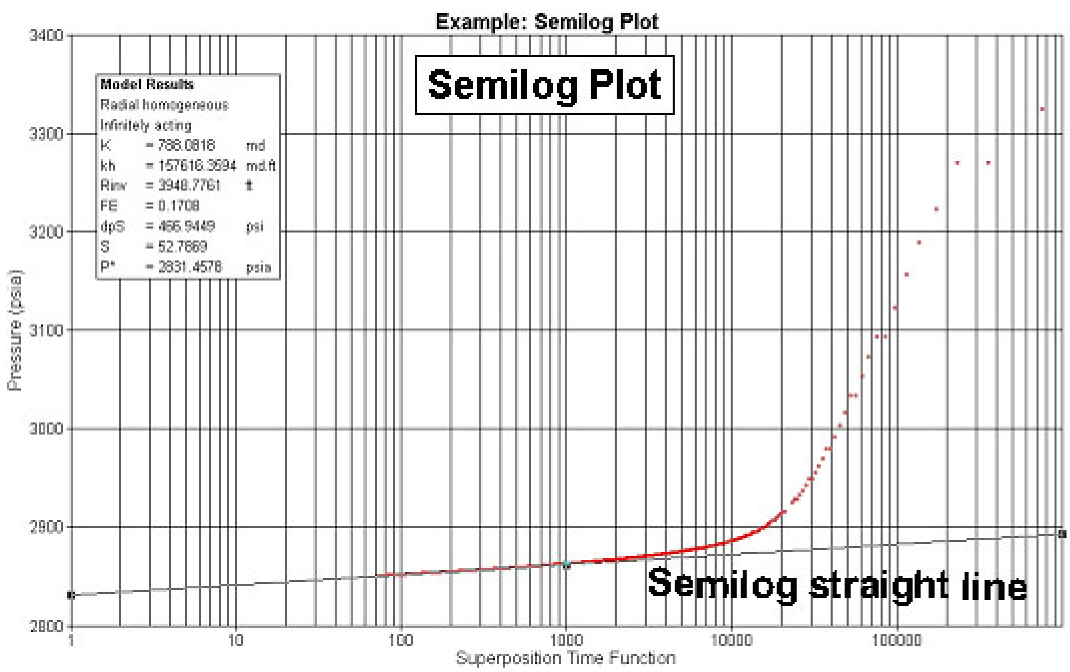
= .28

c_t = 5.7 e-6 psi^{-1}

$_f$ = 0.6 cp



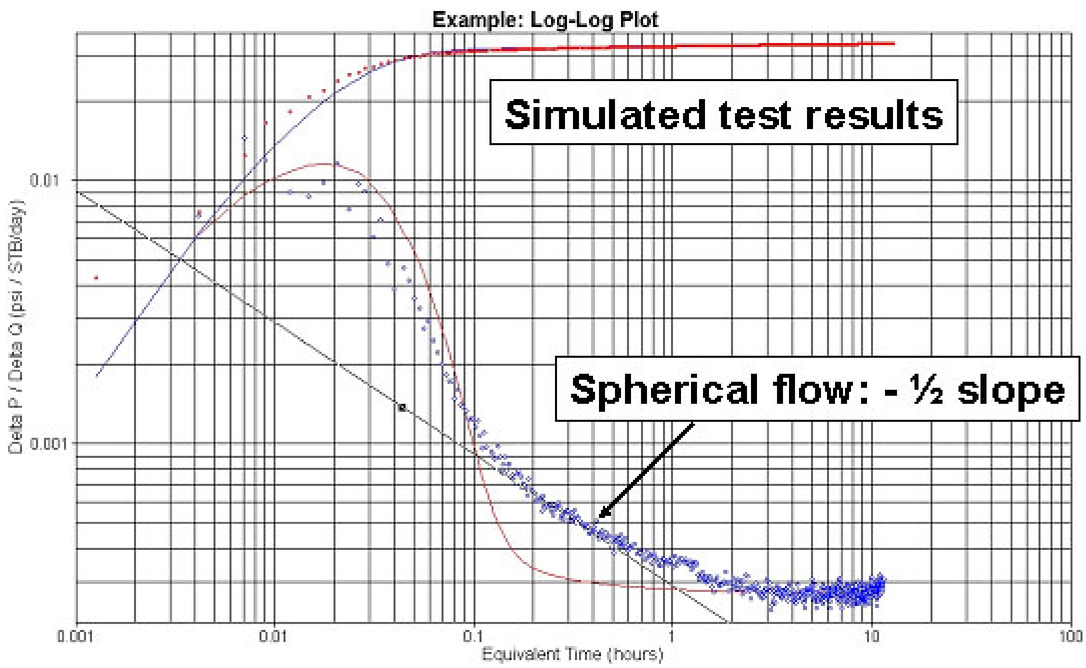

Example: Log-Log Plot
Log-log Plot
Wellbore storage
Radial flow
Model Results
Radial homogeneous
Infinitely acting
Cs = 0.0212 bbl/psi
Cd = 368.6619
K = 780.811 md
kh = 156162.1875 md.ft
S = 52.2268
Delta P / Delta Q (psi / STB/day)
Equivalent Time (hours)
0.1
0.01
0.001
0.0001
1
10
100


Example: Semilog Plot
Semilog Plot
Semilog straight line
Model Results
Radial homogeneous
Infinitely acting
K = 788.0818 md
kh = 157616.3594 md.ft
Rinv = 3948.7761 ft
FE = 0.1708
dpS = 466.9449 psi
S = 52.7869
P* = 2831.4578 psia
Pressure (psia)
Superposition Time Function
3400
3300
3200
3100
3000
2900
2800
1
10
100
1000
10000
100000

Example Gulf Coast Well Falloff Test Results:

$k = 780$ md
$s = 52$
$m = -10.21$ psi/cycle
$P_{1hr} = 2861.7$ psi
$P^* = 2831$ psi

Simulated test results:

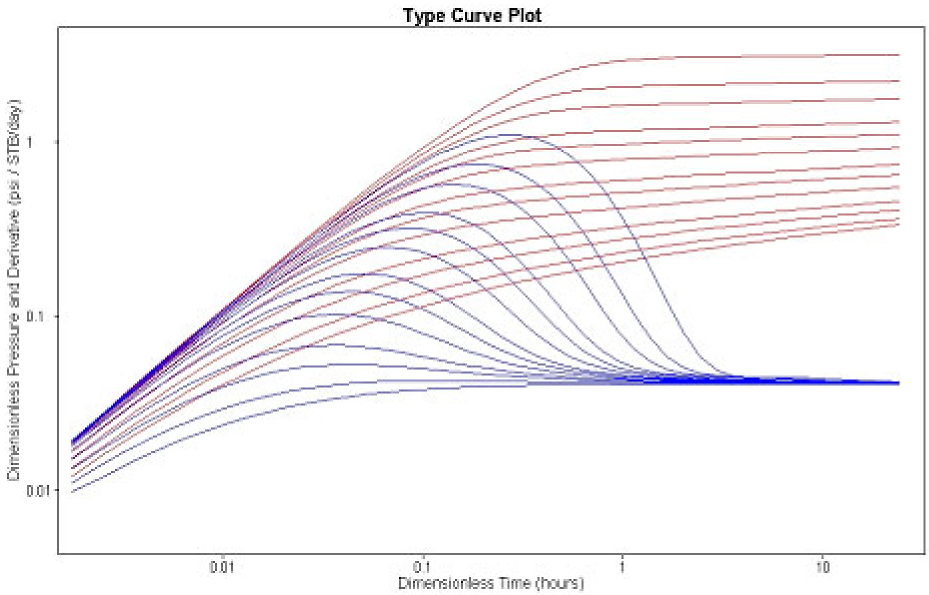


Type Curves

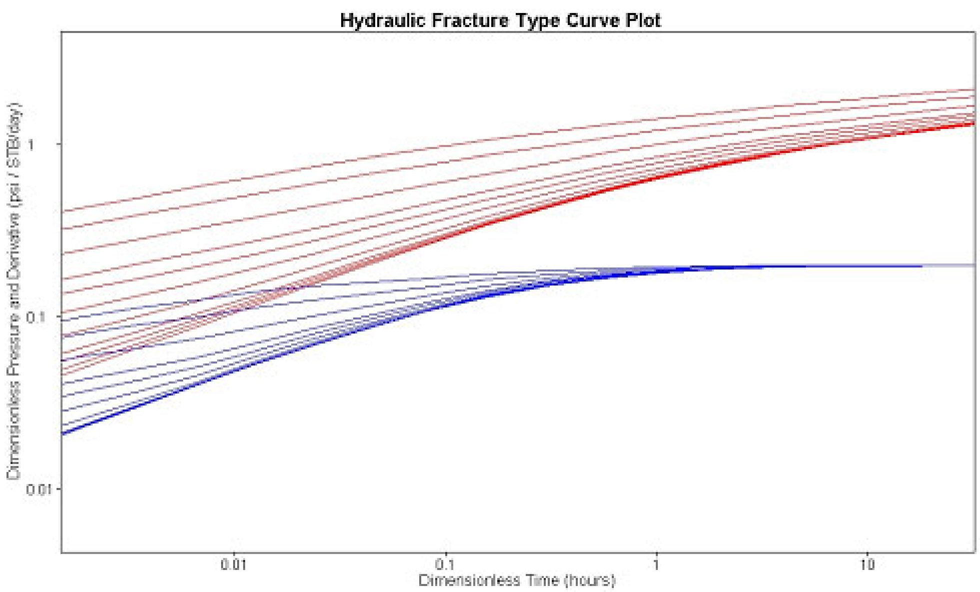
Type curves are graphs of dimensionless variables, P_D vs t_D for various solutions to the pressure transient PDE that provide a “picture” of what a solution to the PDE looks like for a certain set of boundary conditions. The curves can be determined from either analytical or numerical solutions and cover a wide range of parameter combinations. Type curves may work even when specialized plots do not readily identify flow regimes.

The process of applying the curves to field data is called type curve matching. It involves overlaying existing or simulated data to obtain a best fit or match. The reservoir parameters used to generate the matched curve can be applied to the field data. Type curves are generally based on the drawdown or injectivity tests and may require plotting the test data with specialized time functions to use correctly.

Homogeneous Reservoir Type Curves



Hydraulic Fracture Type Curves



Notice the hydraulic fracture type curves do not much of a unique shape as the homogeneous reservoir type curves. Software is now available that can provide a type curve, i.e., simulate, a given set of parameters and boundary conditions. The software can also account for rate fluctuations.

Key Falloff Variables

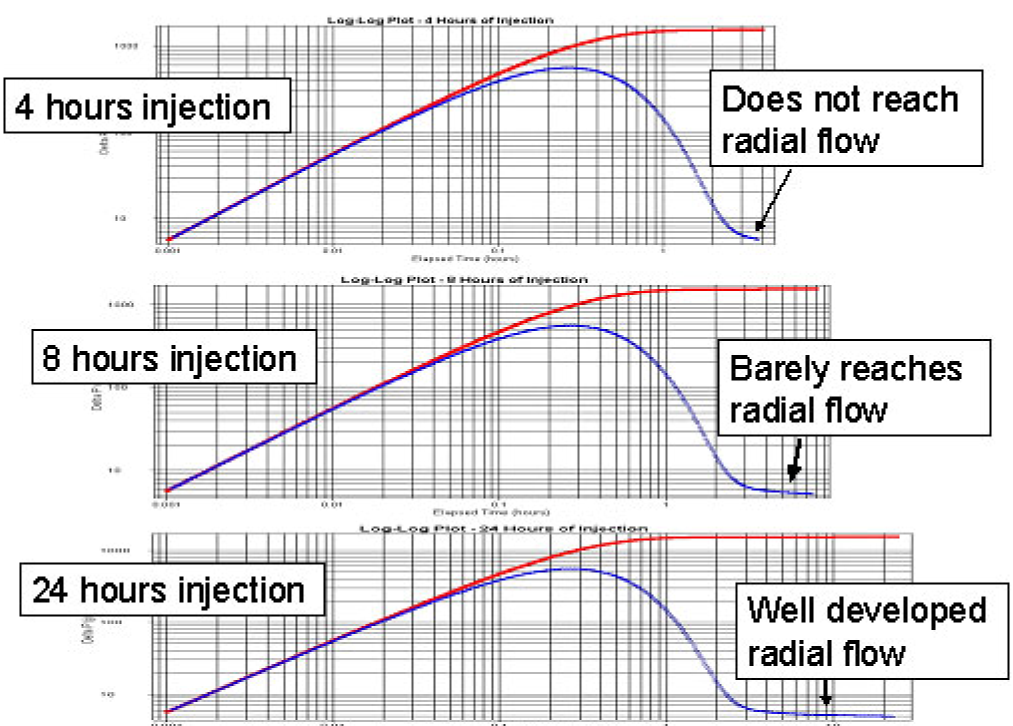
1. Length of injection time
2. Injection rate
3. Length of shut-in time
4. Wellbore storage and skin factor

1. <u>Effects on the Length of the Injection Time</u>

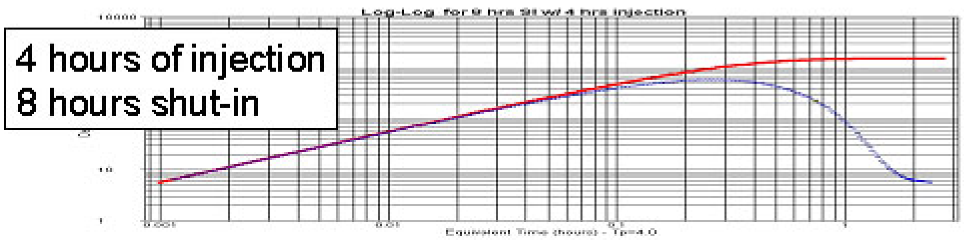
The length of injection period controls the radius of investigation of the falloff test since the falloff is a "replay" of the preceding injection period. Since the falloff cannot see any further out into the reservoir than the injection period, the injection period should last long enough to establish radial flow prior to shutting in the well.

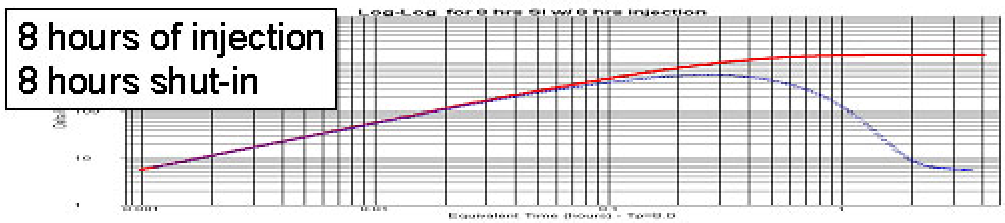
The injection time may need to be increased if the intent of the test is to observe the presence of faults or boundary effects or lack thereof. In this instance it is suggested to calculate the time needed to reach a certain distance away from the injection well during the planning portion of the test.

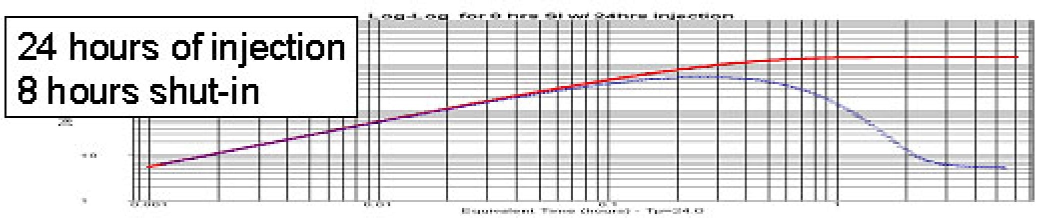
The following three plots indicate the results of simulated injection and falloff periods that were conducted using the same reservoir properties. These tests are for the injectivity portion of the test with varying injection times.



The following plots indicate the results of simulated injection and falloff periods that were conducted using the same reservoir properties. These three log-log plots are for the 8 hour falloff portion of the test following varying length injection periods.







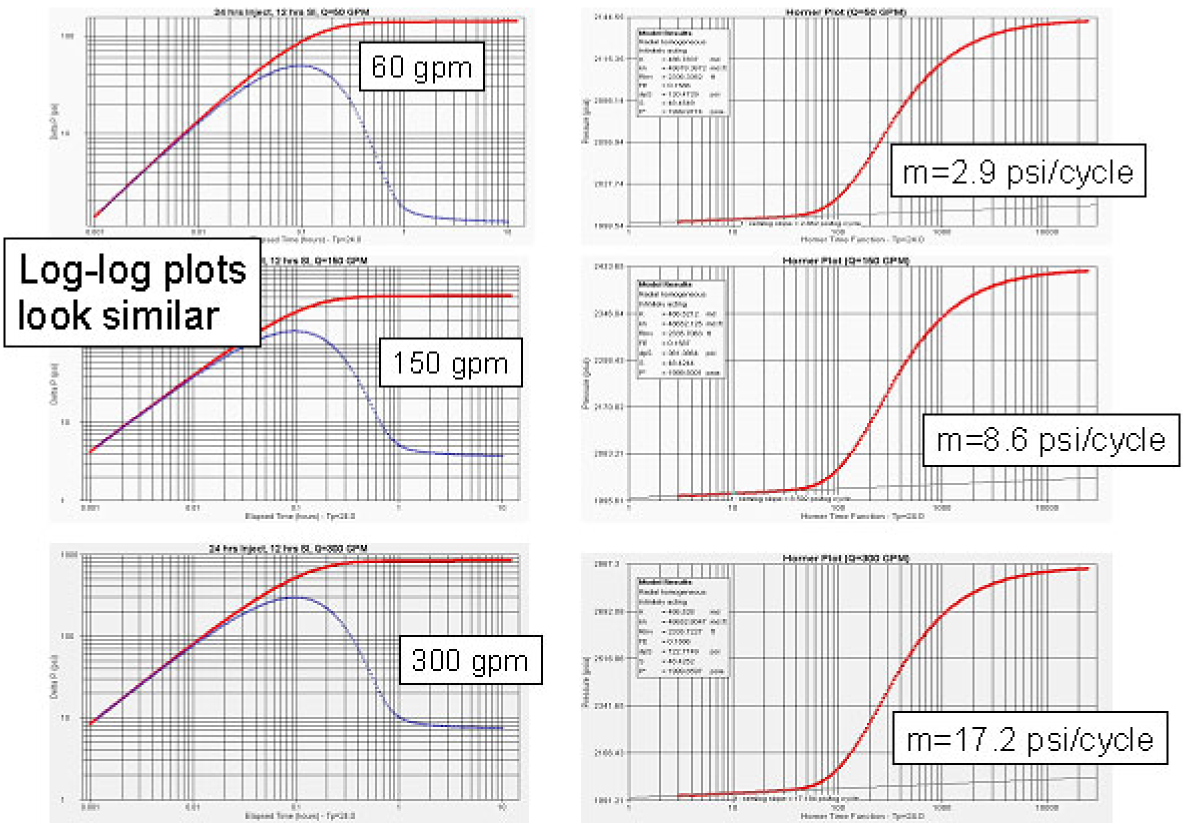
Summary of the effects on the length of the injection time:

- When the injection time is shorter than the falloff, the result is that the falloff response is compressed on the log-log plot when using the correct time function.
- Extending the injection time extends the falloff response
- When the injection time is very long relative to the falloff time, it has little effect on the falloff response.

2. <u>Effects of the Injection Rate</u>

The injection rate determines the magnitude of the pressure rise during the injectivity portion and therefore, the amount of pressure falloff during the shut-in period of the well. Too small of a rate can minimize the degree of pressure change measured during a falloff test. The rate limit during a test may be constrained by permit or petition limits, formation transmissibility, or skin factor. Other operational considerations may include available injectate capacity, type of wastestream, pumping capacity, waste storage capacity, or the pressure gauge resolution.

The following plots indicate the results of simulated injection and falloff periods that were conducted using the same reservoir properties. These plots illustrate that increasing the injection rate does not make a change on the log-log plot, however the resulting slope of the semilog straight line is greatly impacted by the injection rate. The greater the slope, the easier the pressure change is to measure and is less dependent on the resolution of the pressure gauge.



Summary of the effects of the injection rate:

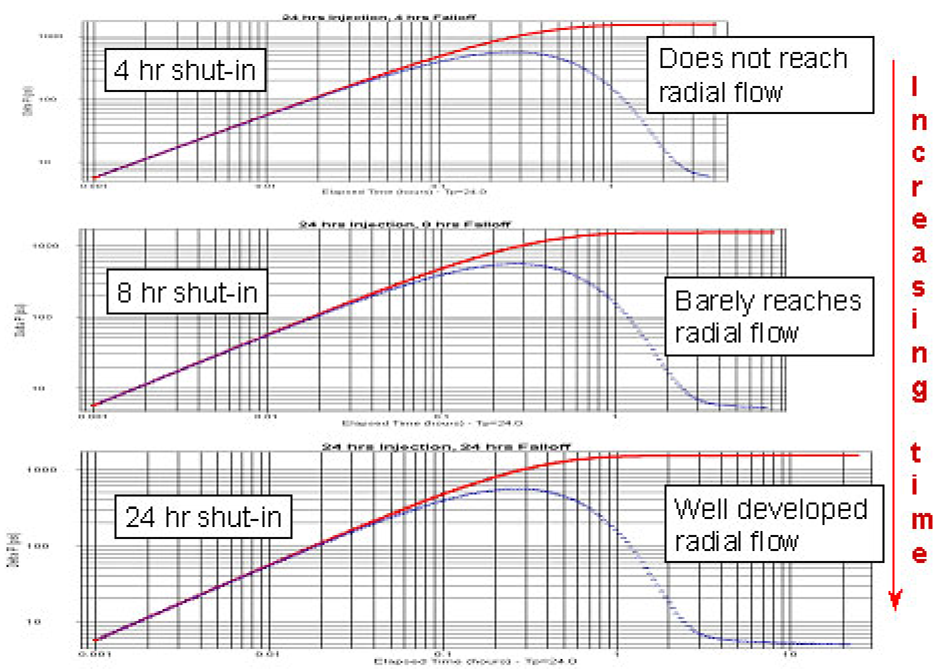
- Injection rate impacts the amount of pressure buildup during the injection period
- A higher injection rate results in:
  - A higher injection pressure and greater total falloff pressure change
  - A larger slope of the semilog straight line during radial flow
- An increased semilog slope enables a more reliable measurement of radial flow

3. <u>Effects of the Length of the Shut-in Time</u>

Too short of a shut-in time prevents the falloff from reaching radial flow making it unanalyzable. A shut-in time exceeding the injection period length is compressed when plotted with the proper time function on the log-log plot.

Falloff test data should be plotted on the log-log and semilog plots using the appropriate time functions to account for the effects of the injection period which were discussed earlier. Increase the falloff time to observe the presence of faults and boundary effects if the preceding injection period was long enough to encounter them.

The following log-log plots indicate the effects of the length of the falloff period for identical injection and reservoir conditions:



Summary of the effects of the shut-in time:

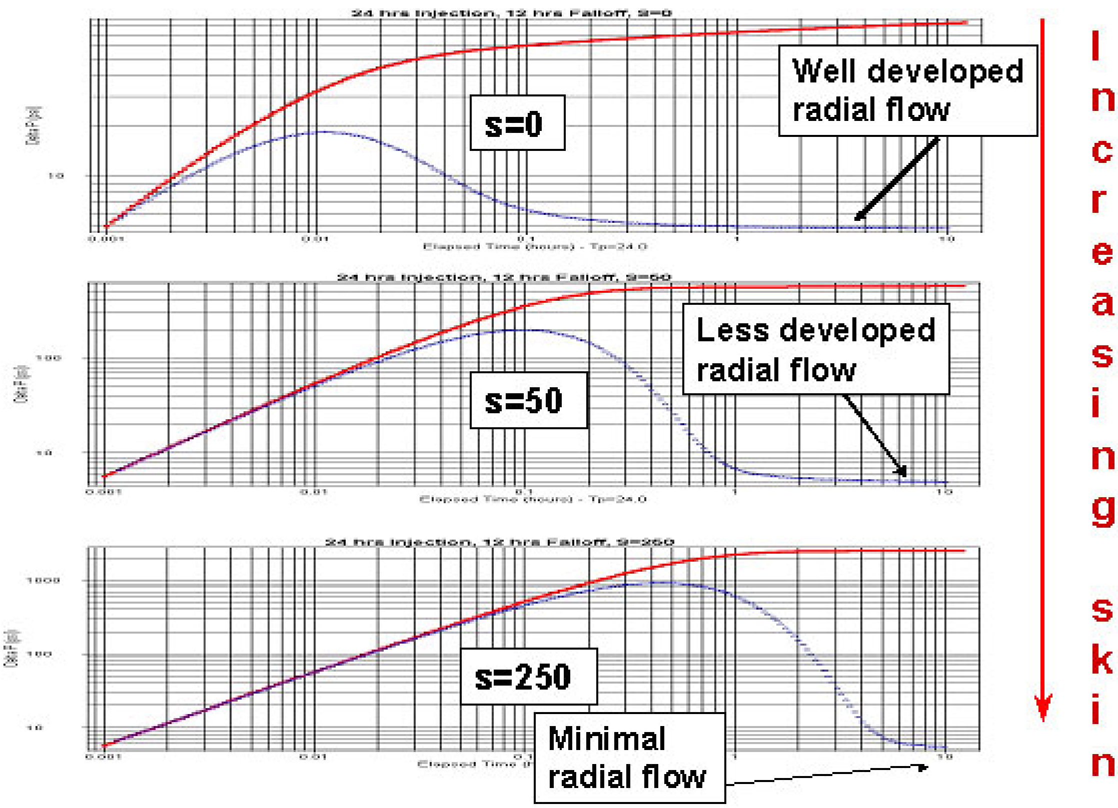
- Too short of a shut-in time may result in the test not reaching radial flow
- Shut-in time may be dictated by the preceding injection time since the falloff is a replay of the injection period
- Wellbore storage, positive skin factor, and the need to observe a boundary condition may increase the required shut-in time for a test

4. <u>Effects of Wellbore Storage and Skin Factor</u>

A positive skin factor indicates a damaged completion and increases the time needed to reach radial flow in a welltest. A negative skin is indicative of a stimulated completion and reduced the time to reach radial flow.

A large wellbore storage coefficient may be caused by a well going on a vacuum, formation vugs, the presence of fractures, or a large wellbore volume. A large wellbore storage coefficient increases the time needed for a test to reach radial flow.

The following log-log plots compare the effects of increasing skin on identical injectivity and falloff conditions:



Summary of the effects of wellbore skin:

- The larger the skin factor, the longer the wellbore storage period and time it takes for the falloff test to reach radial flow.
- The derivative hump size increases with the skin factor
- A wellbore storage dominated test is unanalyzable

Boundary Effects

Falloff tests can provide information concerning the number of boundaries, shape of the boundaries, and the position of the well relative to the boundary. A composite reservoir can give a similar test response signature to a conventional boundary. The area geology should always be checked to see if a sealing boundary is feasible or if a net thickness change may be present.

The type of injectate may also impact the test. A mobility change may be observed if a viscous waste is injected, whereas a composite reservoir may exist in the case of an acid waste stream being injected into a carbonate formation.

To see a boundary, both the injection and falloff periods must last long enough to encounter it. Most pressure transients are too short to see boundaries. Additional falloff time is required to observe a fully developed boundary on the test past the time needed to just reach the boundary.

If radial flow develops before the boundary effects are observed, the distance to the boundary can be calculated. Additionally, when planning the falloff test, the time to reach a boundary can be calculated from the radius of investigation equation:

$$t_{boundary} = \frac{948 \cdot \phi \cdot \mu \cdot c_t \cdot L_{boundary}}{k}$$

where, $L_{boundary}$ = distance to the boundary, feet
$t_{boundary}$ = time, hours

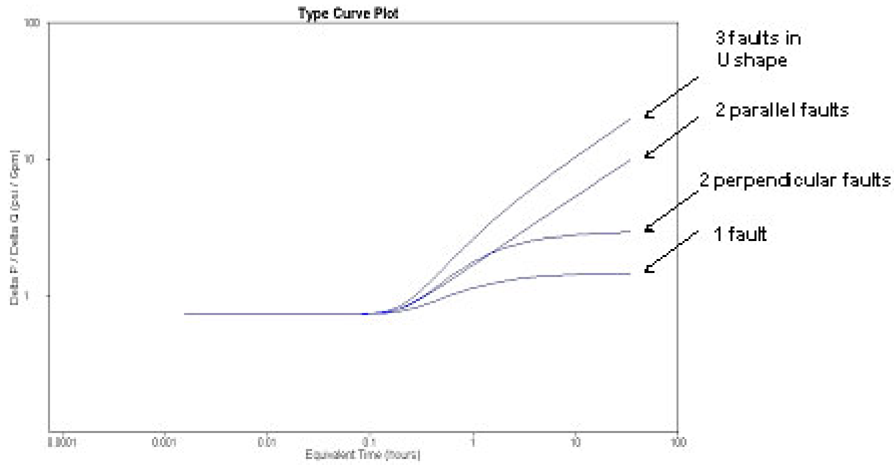
Rule of Thumb:

Allow at least five time the time to reach the boundary to see it fully developed on a log-log plot

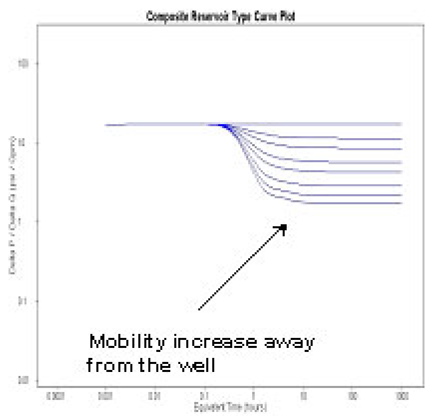
The shape of the derivative response on the log-log plot can indicate shape will double for each sealing boundary observed. The derivative response is is a result of the doubling of the slope of the semilog straight line. However, this slope change is easier to identify on the derivative curve on the log-log plot.

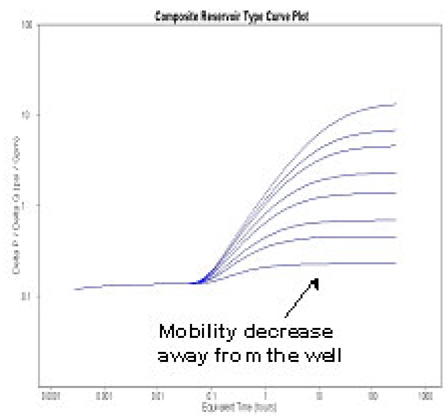
A single sealing fault causes the semilog slope to double while 2 perpendicular faults cause the slope to quadruple if fully developed.

Log-log plot derivative patterns from sealing fault boundaries:



The log-log plot derivative patterns resulting from boundary effects from a composite reservoir can be similar to the sealing fault cases.



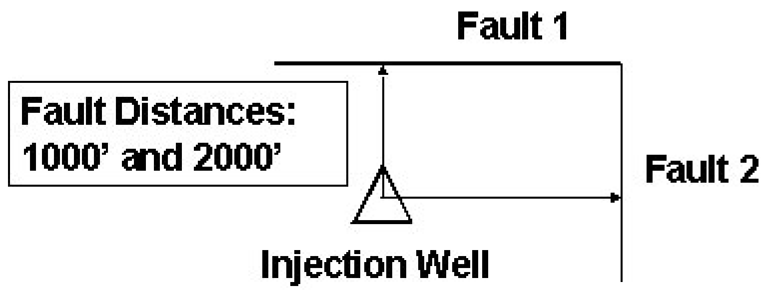


The geology must be checked to confirm what type of boundary may be reasonable for a site.

Summary of boundary effects:

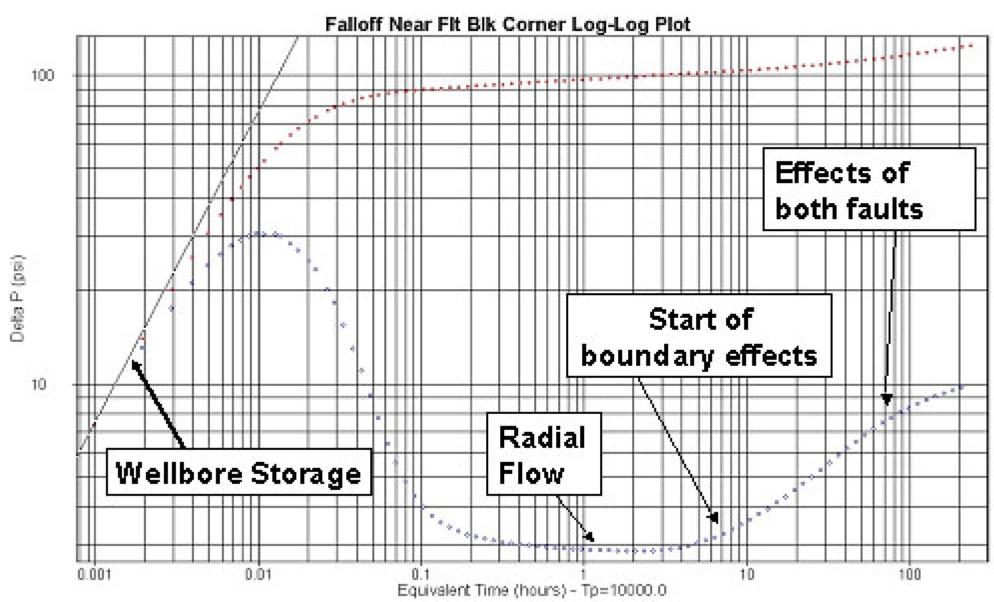
- Use the log-log plot as a “master test picture” to see the response patterns
- Look for changes in pressure and pressure derivative curves to identify boundary effects
- Inner boundary conditions such as wellbore storage, partial penetration, and hydraulic fractures are typically observed first
- Hopefully outer boundary effects show up after radial flow occurs so that the distance to the boundary can be calculated

Example: A falloff test is conducted in a well located near two perpendicular faults

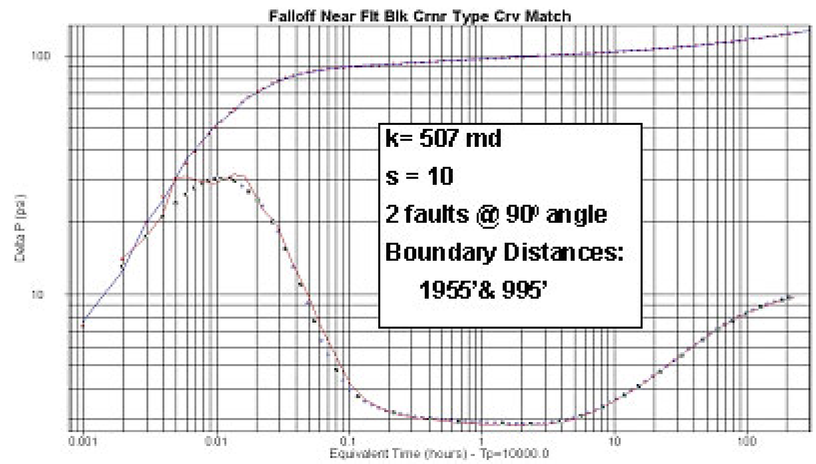


- Falloff test consists of injecting 2000 bpd for 10,000 hrs and then the well is shut-in for 240 hrs.
- The reservoir is a high permeability sandstone and there are no mobility differences

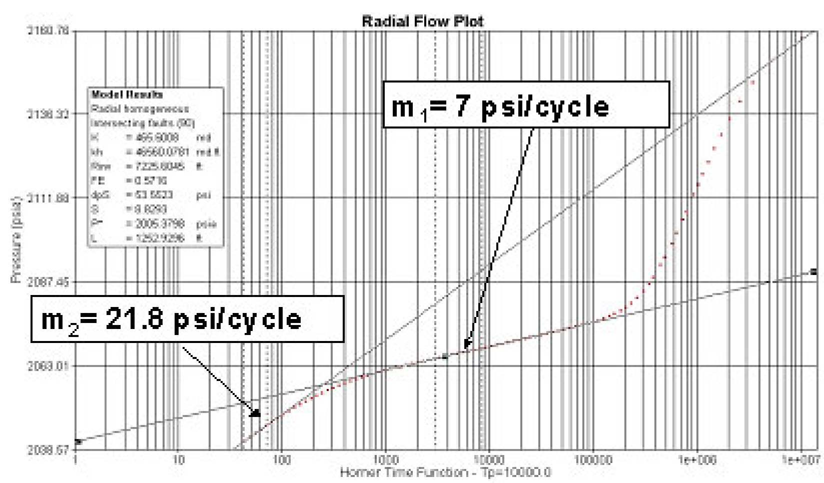
The following log-log plot shows that radial flow is observed prior to reaching the first fault. Though the faults are located at different distances from the injection well, the plateau from the 1000' fault is not observed prior to seeing the effects of the second fault.



The type curve analysis of the falloff test:



The slope changes are also observed on the semilog plot:



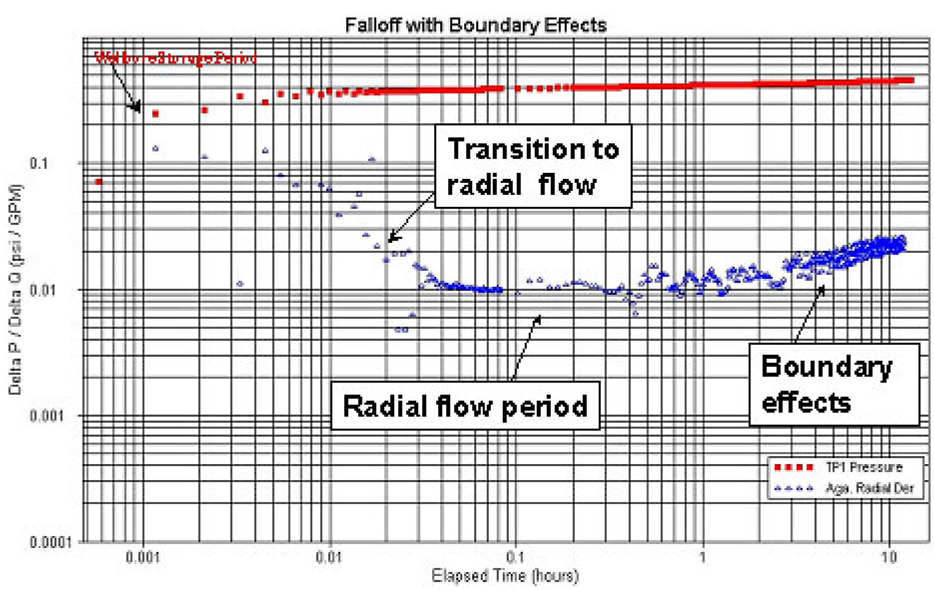
m_2 indicates more than 1 boundary

Typical Outer Boundary Patterns

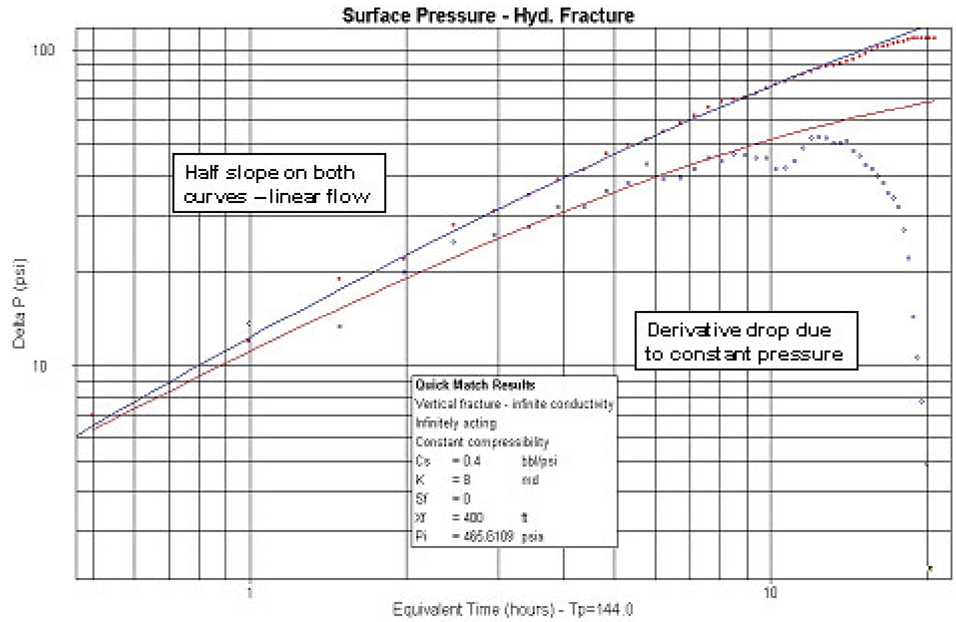
- Infinite acting
  - no outer boundary is observed
  - only radial flow is observed on the log-log plot
- Composite reservoir
  - change in transmissibility, kh/ , or mobility. k/
  - derivative can swing up or down depending on the mobilitiy change and replateau
- Constant pressure boundary
  - derivative plunges sharply
- Sealing boundary
  - derivative upswing followed by a plateau
  - multiple boundaries cause variations in shape and degree of the upswing
- Pseudosteady-state
  - all boundaries around the well are reached - injection well is in a closed reservoir
  - derivative swings up to a unit slope

Gallery of Falloff Log-log Plots

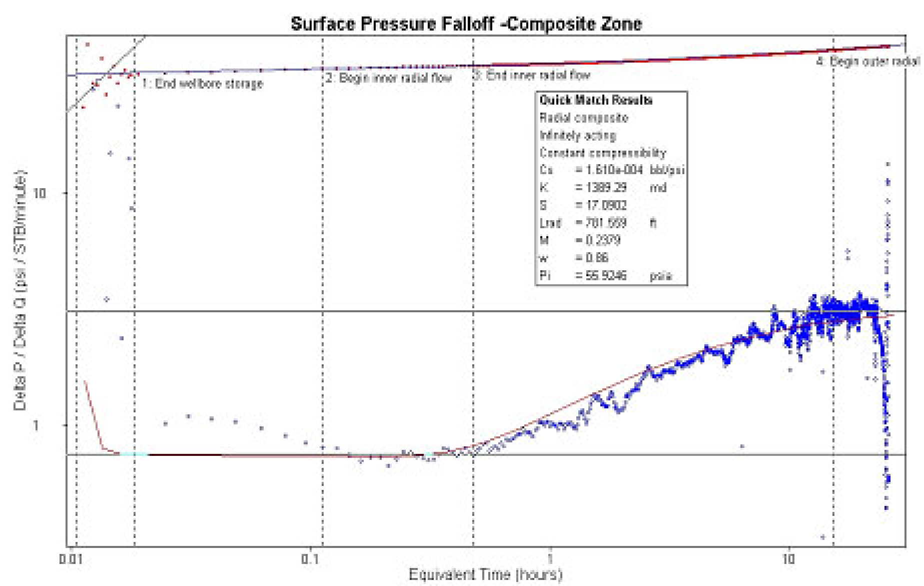
Radial flow with single fault boundary effects:



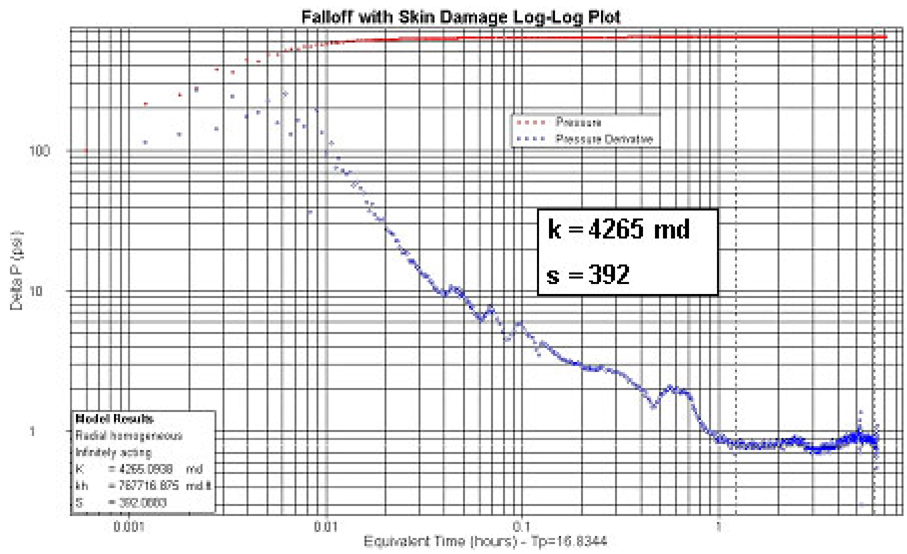
Hydraulically fractured well with surface gauge showing constant pressure at test end:



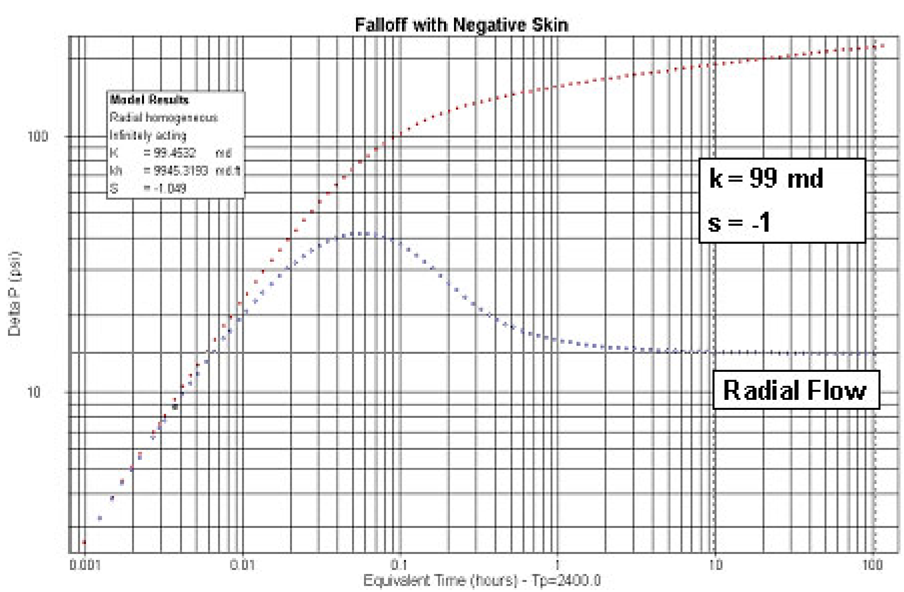
Composite reservoir



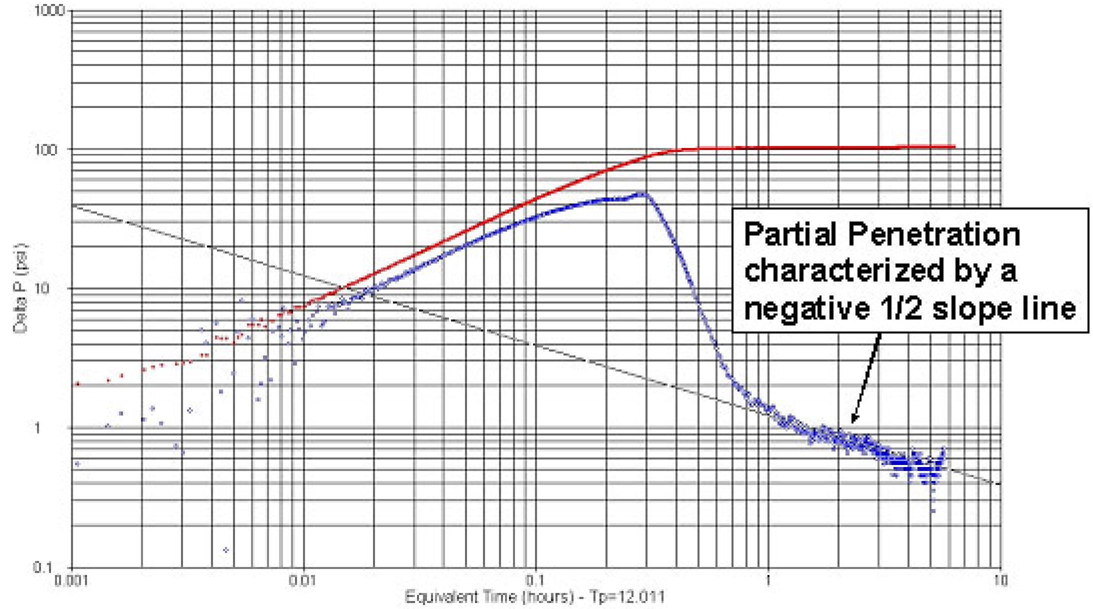
Skin damaged completion



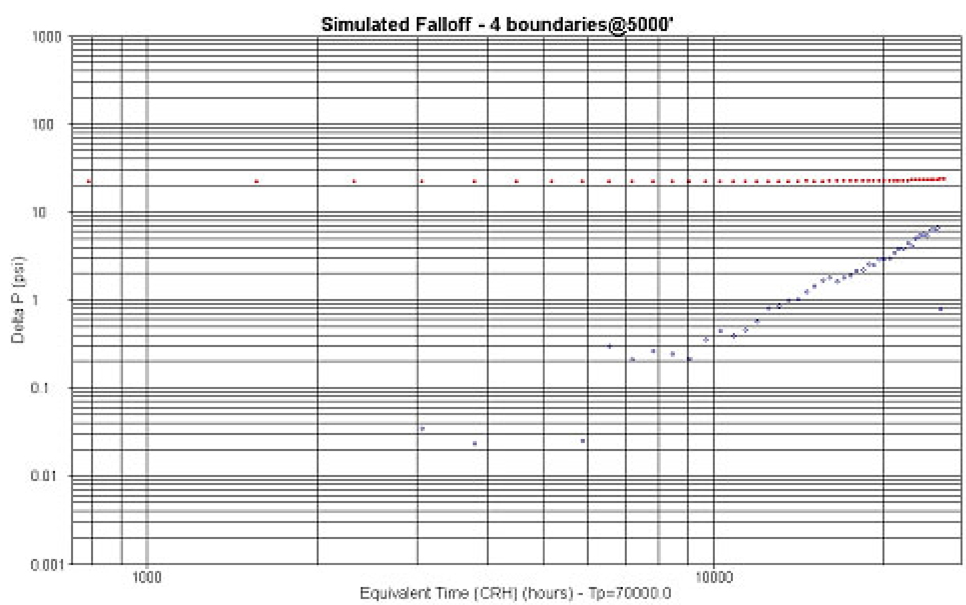
Negative skin



Spherical flow



Simulated test in pseudosteady-state



Other Types of Pressure Transient Tests

Injectivity test
Following a stabilization period, an injectivity test involves recording the pressure and time data from the start of an injection period.

Pros:
- Well does not have to be shut-in
- Usually maintain surface pressure so less wellbore storage
- Less impact from skin

Cons:
- Data is usually noisy due to fluid velocity by the pressure gauge
- Rates may fluctuate during the test so an accurate rate history is important

Multi-rate injection test
Involves recording the pressure and time data through at least two constant injection periods. The first injection period should reach radial flow prior to changing the rate. The injection rate may be increased or decreased, but the rate change should be significant enough to produce a pressure change at the injection well.

Pros:
- Rate can be increased or decreased and the injection well does not have to be shut-in
- Minimizes wellbore storage, especially with a rate increase
- Provides two sets of time, pressure, and rate data for analysis
- Decreasing the rate provides a signal falloff without shutting in the well

Cons:
- Noisy data due to fluid velocity by the pressure gauge
- First rate period needs to reach radial flow

Interference test
Involves the use of two wells, a signal and observer well. The signal well undergoes a rate change which causes a pressure change at the observer well. This pressure change at the observer well is measured over time and then analyzed using an Ei type curve. If radial flow is reached, a semilog plot can be used

Pros:
- Test can yield the transmissibility and a porosity-compressibility product of the reservoir between the wells tested
- May give analyzable results when a falloff doesn't work

Cons:
- Generally involves a small pressure change so accurate an surface or bottomhole gauge is needed
- Observable pressure change decreases as the distance between the two wells increases
- The analysis is complex if more than two injectors are active
- The test rate should be constant at the signal well.

Pulse test
Similar to an interference test except the rate changes at the observer well are repeated several times

Pros:
- Test results in multiple data sets to analyze
- Verifies the communication between wells

Cons:
- Difficult to analyze using SPE Monograph 5 methodology without welltest software
- Requires careful control of the signal well rate

Interference Test Design

The best design approach for both an interference test and pulse test is to use a welltest simulator. Interference tests can be designed using the Ei type curve.

Test design information needed:
- Distance between the signal and observer wells
- Desired pressure change to measure - may be pressure gauge dependent
- Desired injection rate
- Estimates of c_t, , , k, h, r_w

Example: Interference test design
Two injection wells are located 500' apart (r=500'). Both wells have been shut-in for a month so previous injection is not a factor. An interference test is planned with an injection rate, q, of 87.5 gpm (3000 bpd). How long will the test need to be run to see a 3 psi pressure change at the observer well assuming no skin?

The estimated reservoir parameters are:

k = 50 md	h = 100'	= 20%
$_f$ = 1 cp	c_t = $6x10^{-6}$	r_w = 0.3 ft

Calculate P_D and r_D:

$$P_D = \frac{\Delta P \cdot k \cdot h}{141.2 \cdot q \cdot \mu} \qquad r_D = \frac{r}{r_w}$$

The resulting values for P_D and r_D for a 3 psi pressure change:

P_D = 0.0354 and r_D = 1666.7

Find t_D/r_D^2 from the corresponding P_D value on the Ei type curve located in Figure C.2 in SPE Monograph 5: t_D/r_D^2 = 0.15

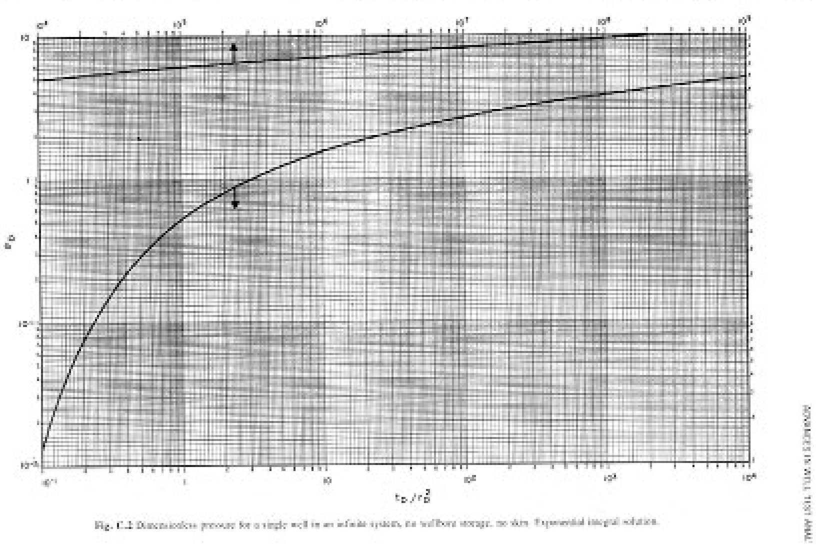
Solve for t_D: t_D = 416683

Then solve for t=$t_{interference}$ by substituting for t_D:

$$t_D = \frac{0.0002637 \cdot k \cdot t}{\phi \cdot \mu \cdot c_t \cdot r_w^2}$$

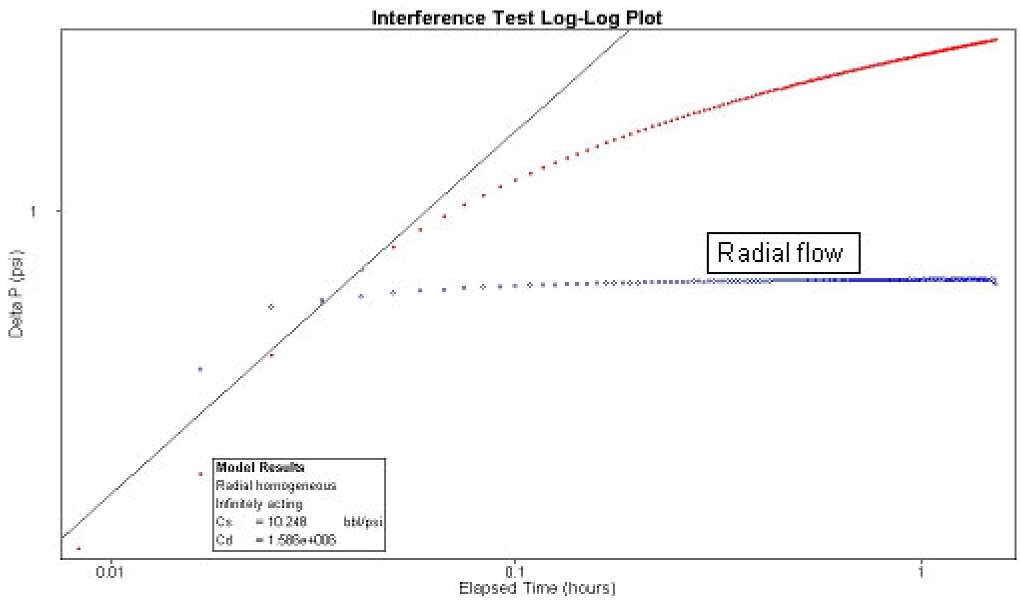
$t_{interference}$ = 3.4 hours

Ei Type Curve: from Figure C.2 in SPE Monograph 5

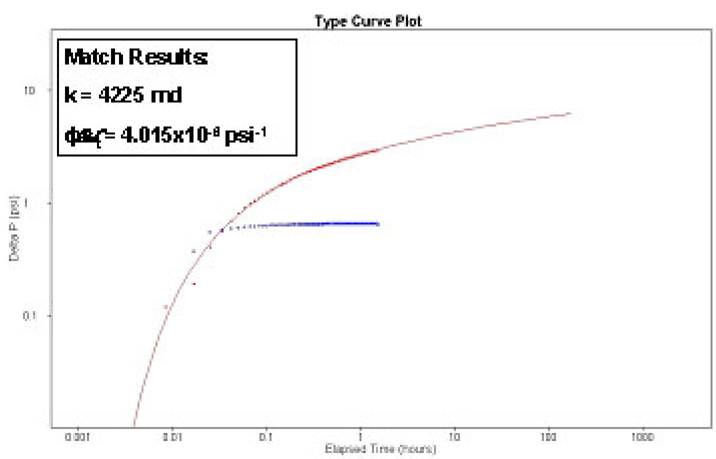


Example: Interference test analysis
An interference test was conducted between two injection wells located at a Gulf Coast area facility. The two wells are 150' apart.
reservoir conditions: h = 55', = 28%, $c_t = 6x10^{-6}\ psi^{-1}$
Well data: r_w = 0.25 ft, q = -120 gpm

Prepare a log-log plot of the measured pressure data at the observer well:



Type curve match the pressure data using the Ei type curve:



The type curve match results in a permeability and porosity-compressibility product.

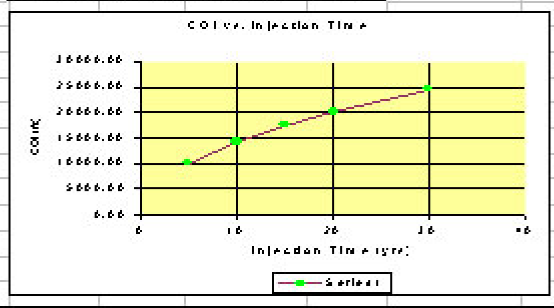
Falloff Test Impact on an Area of Review Evaluation

The transmissibility obtained from the falloff test and the solution from the PDE can be used to project the pressure increase due to injection at the injection well or a distance away from the well. The PDE solution can also be used to estimate the cone of influence location. Both the pressure buildup projection and cone of influence location estimates can be set up in a spreadsheet.

Example Pressure Buildup Projection Spreadsheet:

Input Parameters	
Facility:	Example
Pi (initial res. pressure in psia):	1200
h (ft):	50
porosity:	0.2
rw (ft):	0.3
ct (1/psi):	8.00E-06
viscosity (cp):	1.00
Depth to USDW base (ft):	300
Depth to Groundwater (ft):	10
Reservoir fluid SG:	1.040
Min. aband. well diameter (in.):	9.000
Min. aband. well mud wt. (lb/gal):	8.90
Top of injection interval (ft):	3000

Critical Pressure Calculations	
Critical pressure rise - brine filled borehole (psi):	141.43
Critical pressure rise - mud filled borehole (psi):	362.34
Critical pressure rise basis (enter mud or brine):	brine



COI Calculations Inj. Rate (bpd)	Inj. Rate (gpm)	Falloff k (md)	Injection Time (hrs)	Injection Time (yrs)	Dimensionless Time	Critical Pressure (psi)	Dimensionless Pressure	Dimensionless Radius	Total Pressure Increase at Injection Well (psi)	COI Radius (ft)
1714.29	50	20	43800	5	1.6042E+08	141.43	0.53	3342.23	2663.23	10038.67
1714.29	50	20	87600	10	3.2084E+08	141.43	0.53	4732.74	2747.12	14196.82
1714.29	50	20	131400	15	4.8126E+08	141.43	0.53	5758.29	2796.19	17387.49
1714.29	50	20	175200	20	6.4167E+08	141.43	0.53	6624.46	2831.01	20077.34
1714.29	50	20	262800	30	9.6251E+08	141.43	0.53	8155.39	2880.08	24589.62

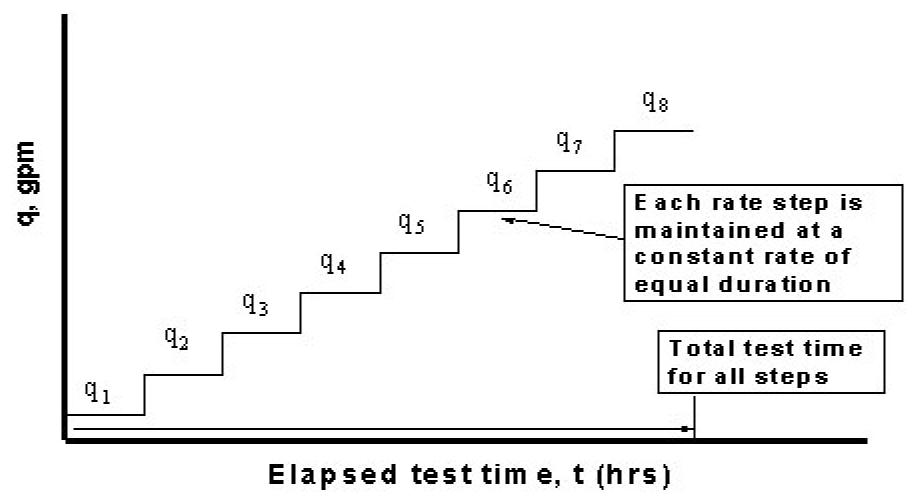
Determination of Fracture Pressure

Fracture pressure usually varies with depth, lithology, and geographical region. Specifically, fracture pressure increases with depth because the compaction of the formation tends to increase with depth and requires higher pressures to initiate a fracture. The rock type and composition are also important factors in determining how brittle the rock is and ultimately the pressure necessary to part or fracture the rock.

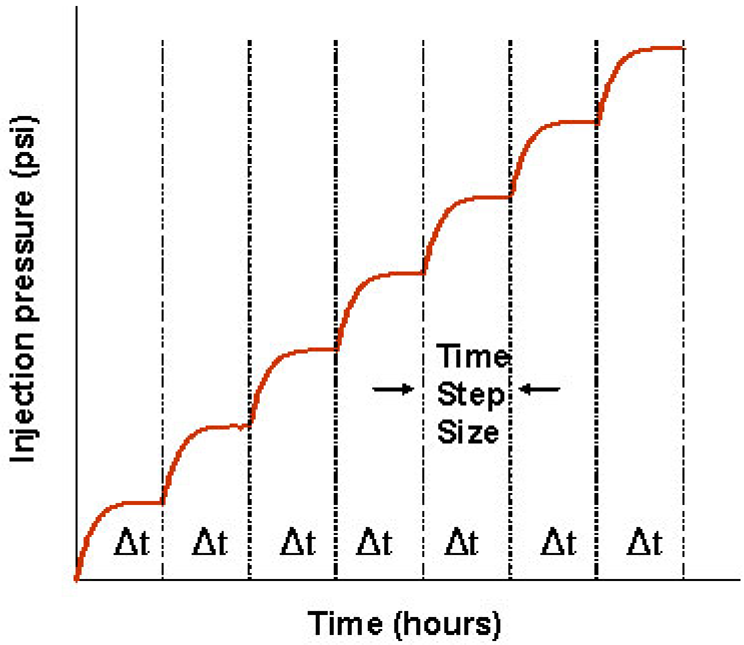
The fracture gradient is typically estimated from correlations, (e.g. Hubbert and Willis, Eaton). Another method of determining fracture pressures is from a step-rate test.

<u>Step-Rate Test</u>
A step-rate test consists of a series of pressure transient tests caused by rate increases at the injection well. Each rate change creates a pressure transient in the reservoir. Data is analyzed using log-log and linear plots. The linear plot is used to estimate fracture pressure, also called the formation parting pressure. The log-log plot is used to verify that fracturing occurs and to estimate kh/u and skin.

Ideally, the sequence of events for a step-rate test consists of a series of constant rate injection over an equal time duration and the length of each step is of sufficient duration to reach radial flow. Practically, each rate is not maintained long enough to reach radial flow. In fact, maintaining a constant injection rate at each step is itself a challenge since the reservoir pressure and therefore the injection pressure typically increases with the increase in rate and duration of the test. Pump trucks are often used to conduct the step-rate test. As a result, injection volumes may be limited and maintaining a constant rate as injection pressures increase is difficult. Preplanning is important so that an adequate injection volume is available and constant rates can be maintained.



Each step increase of the injection rate will result in a corresponding change in pressure behavior.

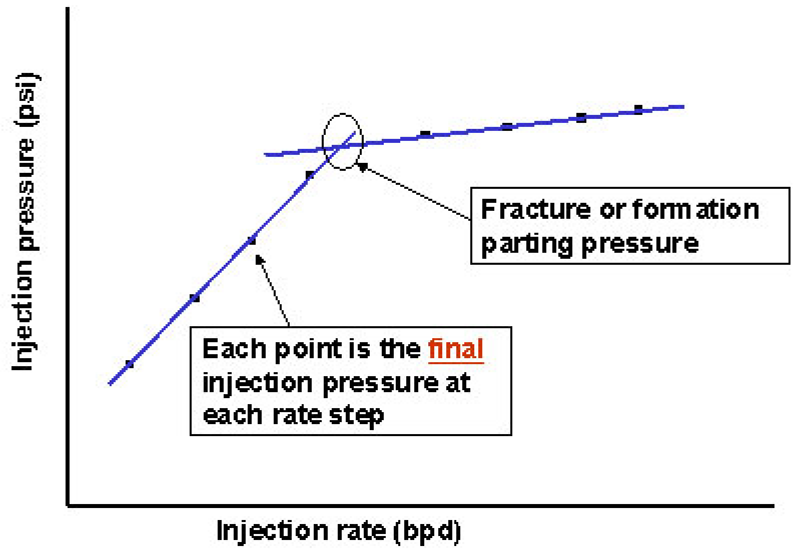


Both log-log and linear plots are used to analyze the step-rate test. The log-log plot can verify that fracturing occurs by observing a half slope on both the pressure and derivative curves. The log-log plot can also identify if radial flow is observed during a time step, t, by observing a flattening of the derivative curve. The radial flow portion of the test can then be analyzed to obtain the transmissibility, kh/ product, and skin factor.

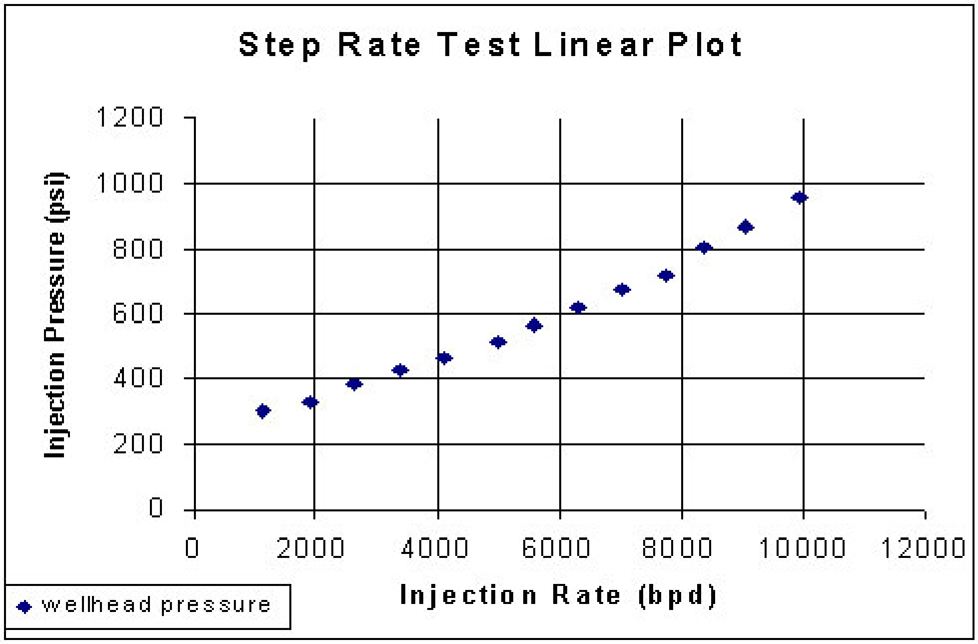
The linear plot is typically the plot associated with step rate tests. This plot is used to estimate the fracture pressure or formation parting pressure. This pressure is estimated at the intersection of two lines drawn through the final injection pressure at each time step. If a slope change is not observed, the step-rate test was either initiated above the fracture pressure, or the rate increases did not result in the fracturing of the formation. If the test is initiated above the fracture pressure, the log-log plot should show indications of a fracture.

For the linear plot, the injection pressure at the end of each injection rate is plotted on the y-axis at the corresponding injection rate located on the x-axis. For this pressure versus rate plot to be of use, the data obtained should not be dominated by wellbore storage, identified by a unit slope on a log-log plot or a concave upward curve on the pressure versus rate plot.

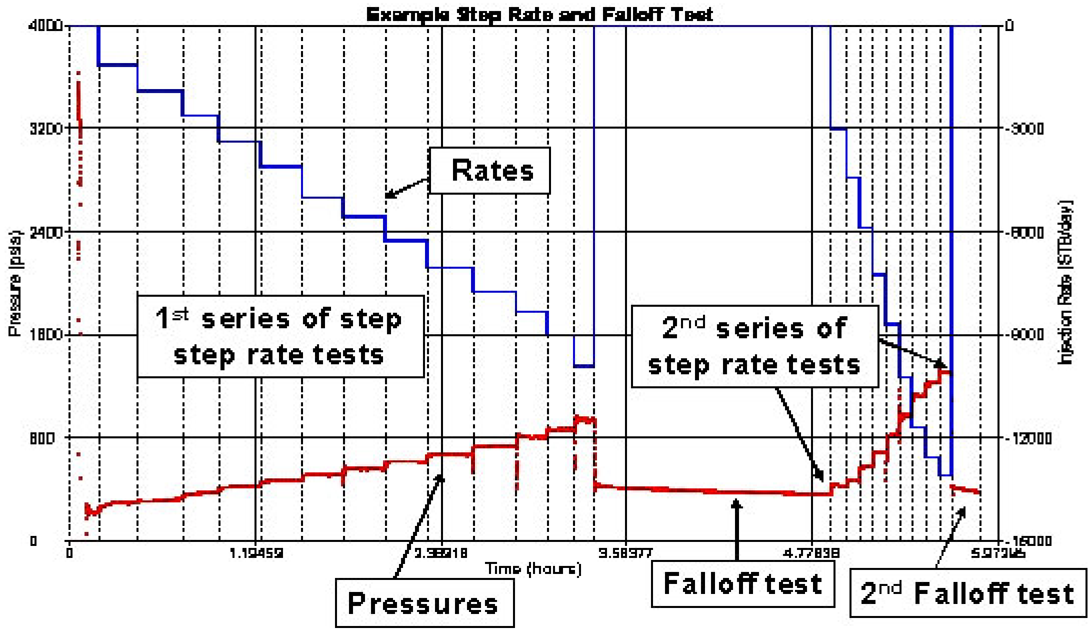
Linear plot example with fracture observed:



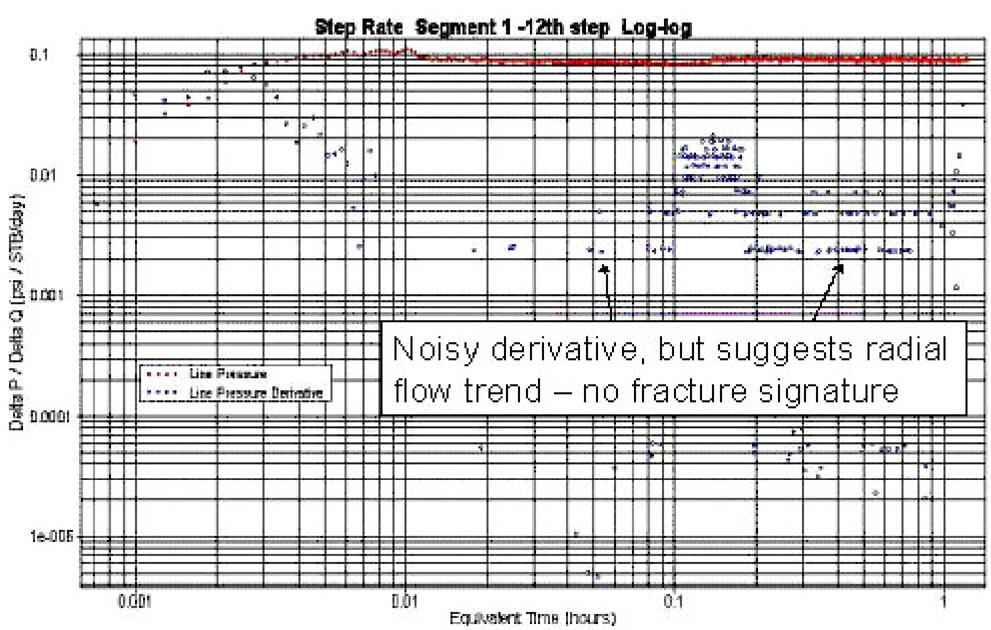
Linear plot example with no fracture observed:



Here is an example of a combination of step rate tests and falloff tests conducted in an injection well.



Below is the log-log plot for the 12th step of the first series of step rate tests:



Other Uses of Injection Rates and Pressures

Though step-rate testing is the principal method used for calculating the reservoir fracture pressure and establishing a maximum injection rate, there are other methods for evaluating the condition of an injection well. One method was developed by Hall in 1963 and a second method was published by Hearn in 1983. Both the Hall and Hearn methods require injection rate and wellhead injection pressure data. This information should be readily available for Class I wells since continuous monitoring is a regulatory requirement.

The Hall method involves plotting the cumulative change in bottom hole pressure times the change in time (P* t) versus the cumulative injection volume in barrels. The Hearn method involves a semilog plot of the inverse injectivity index, i.e., change in pressure divided by the injection rate (P/q), versus the cumulative injection volume plotted on a logarithmic axis in 1000 barrel units. As with the step-rate test, these plots identify well conditions and fracturing of the formation by slope changes on the plot.

Both the Hall and Hearn plots assume piston-like displacement of fluid, steady-state, radial single phase, single-layer flow. The Hearn plot is applicable to a Class II injection well prior to reservoir fill-up. The Hall plot is used after fill-up and is best suited for Class I injection well projects or Class III wells in mature water injection projects. The pressure at the external drainage radius, P_e must be estimated in the calculations for both plots. The initial reservoir pressure should be a reasonable approximation for P_e if there are no nearby pressure sinks or sources that would impact the reservoir pressure.

The slope, m, calculated from each plot has unique units and both are different than the slope, m, calculated from the semilog plot.

Hall Plot

The Hall plot offers the advantage of using operational data to provide continuous monitoring methods for injection well operations. The method is based on the use of the steady-state form of the Darcy flow equation. The only data required are injection rate, injection pressure, and an estimate of P_e, the reservoir pressure.

For a Hall plot, the P function can be calculated several different ways. The function is described rigorously by the following equation:

$$\int P_{wf}\,dt = \frac{141.2\, B_w \mu_w \left[\ln\left(r_e / r_{wa}\right) + s\right]}{k\,h} W_i + \int P_e\,dt$$

where:

- P_{wf} = Bottomhole Injection Pressure, psi
- B_w = Formation volume factor, rvb/stb
- $_w$ = Viscosity of formation fluid, cp
- r_e = External drainage radius, ft
- r_w = Wellbore radius, ft
- s = Skin factor, dimensionless
- k = effective permeability to water, md
- h = formation thickness, ft
- W_i = cumulative injection, bbl
- P_e = Pressure at external radius, psi

with $P_{wf} = P_{tf} - \Delta P_f + \left(\rho g D\right)$

where:

- P_{tf} = Surface injection pressure (tubing flowing pressure), psi
- P_f = Pressure due to friction loss, psi
- gD = Pressure of static fluid column, psi
  - g = fluid gradient, psi/ft
  - D = depth to middle of the injection interval, ft

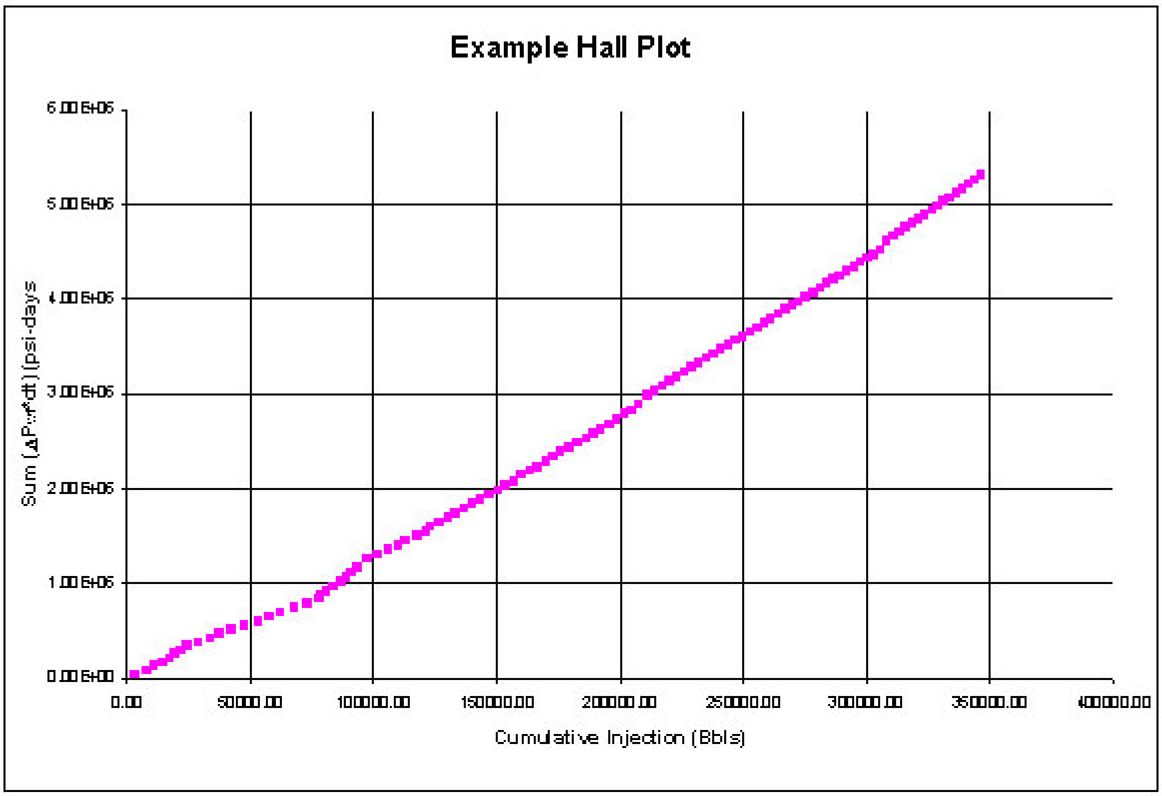
After substituting, the following equation is obtained:

$$\int P_{tf}\,dt = \frac{141.2\,B_w \mu_w \left[\ln\left(r_e / r_{wa}\right) + s\right]}{k\,h} Wi + \int \left(P_e + \Delta P_f - \rho g D\right) dt$$

Typically, to simplify the plot, the integral on the right hand side is dropped and a plot of the summation of the P_{tf}, wellhead pressure, or P_{wf}, bottomhole injection pressure, times delta time is plotted versus W_i, cumulative injection. However, the change in bottomhole pressure, P_{wf}, must be plotted to use the plot for quantitative analysis. The pressure data are plotted along the y-axis of a linear plot. The graph is used to identify changes in injection behavior that occur over an extended time period. An upward slope indicates damage while a flattening of the line indicates some type of stimulation, e.g. fracturing. Slope changes on these types of plots may result from rate changes and the transmissibility or skin factor may not have changed. Therefore, it is recommended to take the additional effort to make a Hall plot using the delta bottomhole pressure for a quantitative analysis.

For quantitative analysis of a Hall plot, i.e., transmissibility and skin factor determination, a value for P_e should be estimated or assumed, P_{wf} calculated, and the integral (cumulative function) of P_{wf}-P_e plotted versus W_i. Remember, if only the wellhead or bottomhole pressure is used, the slope changes observed may only be due to injection rate changes. The use of P_{wf}-P_e eliminates slope changes due to rate changes and smooths the data, but requires a calculation of P_{wf}, the bottomhole injection pressure. Note that the slope of the Hall plot incorporates both skin factor and transmissibility, so that neither variable can be determined independently from the slope. However, for single phase flow, the transmissibility should not change significantly with time and therefore any change in slope will likely be due to skin effects.

Below is an example Hall Plot:



As noted previously, the bottomhole pressure, P_{wf}, can be estimated from surface pressures by subtracting the pressure loss due to friction in the tubing and adding the hydrostatic head at the midpoint of the perforations. For large tubulars, friction loss can be neglected.

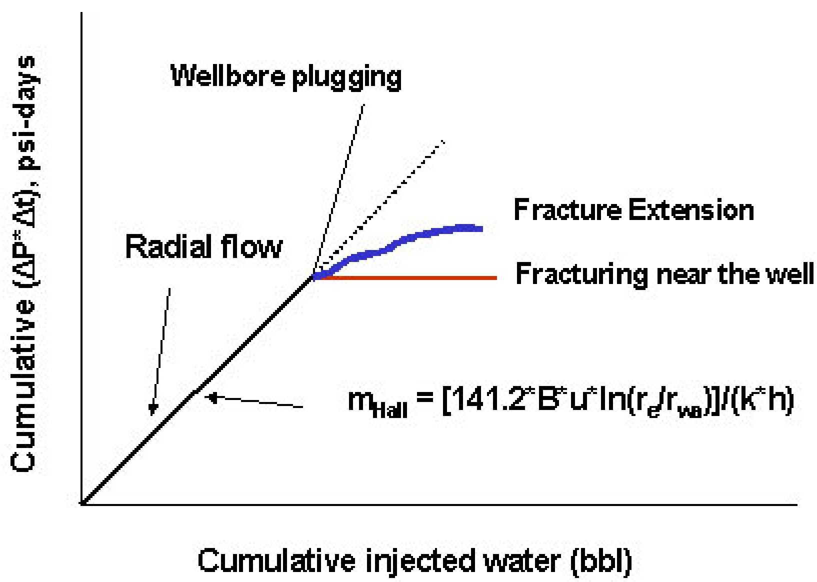
The transmissibility of the formation can be calculated by the straight line slope on the Hall plot. Specifically,

$$m_{Hall} = \frac{141.2 \cdot \beta \cdot \mu \cdot \ln\left(\frac{r_e}{r_{wa}}\right)}{k \cdot h}$$, psi-day/bbl

The Hall plot was developed for use in waterfloods, so the relative permeability of oil and water were a consideration. Additionally, there was a oil bank radius, r_o, and water bank radius, r_{wtr}, resulting from water injection. In Hall's 1963 paper, the permeability k is listed as k_e, the specific water permeability. Since the formations used for injection are assumed to be water wet and the injection is assumed to have characteristics to that of water, the relative permeability to water is 1.0 and therefore the effective formation permeability to water, k can be substituted in place of k_e. The Hall plot also involves an effective radius value, r_e. The effective radius can be approximated by taking the injection volume and calculating the radius influenced by injection. Another option would be to calculate the radius of the injected volume based on volumetrics. The accuracy of r_e/r_{wa} is not critical since this is a log term in the transmissibility equation.

As with the step rate test, the well conditions are indicated by slope changes on the plots:

- Decrease in slope indicates fracturing, i.e., decrease in skin factor
- Increase in slope indicates well plugging, i.e., an increase in skin factor
- Straight line indicates radial flow



<u>Hearn Plot</u>

Another plot that uses operational data is the Hearn plot. This method also based on the steady-state form of Darcy's equation. The Hearn plot P/q function is similarly based upon the flowing bottomhole pressure and an estimate of P_e. The Hearn plot is developed from the Muskat form of the Darcy equation. To simplify the plot, flowing bottomhole pressure is often estimated by adding wellhead pressure and the static fluid column pressure in the injection well while neglecting friction pressure. Friction pressure should be added if the injection rate is extremely high. The Hearn plot's advantage over the Hall plot is that it gives a transmissibility from the slope and a skin factor from the intercept.

Typically, the Hearn plot was developed for use early in the life of an injection well and the Hall plot used after the well has operated for an extended time. The Hearn plot develops a constant slope prior to reservoir fill-up and a second horizontal straight line occurs after fill-up. The Hall plot develops a straight-line slope after fill-up. Prior to reservoir fill-up, the P_e is increasing, resulting in upward curvature in the Hall plot.

Though both the Hall and Hearn plots require the estimate of a few parameters, the results may provide an estimation of the reservoir transmissibility and condition of the wellbore, valuable data when designing or planning a falloff test. Minimal time and costs are needed for the potential data that may be obtained.

Nomenclature

B = formation volume factor, rvb/stb
B_w = formation volume factor of water, rvb/stb
C = wellbore storage coefficient, bbls/psi
c_r = rock compressibility, psi^{-1}
c_t = total compressibility, psi^{-1} (c_t=c_r+c_w)
c_w = formation fluid compressibility, psi^{-1}
c_{waste} = injectate compressibility, psi^{-1}
D = Depth, feet
Ei = Exponential Interval
FE=injection efficiency (flow efficiency in a producing well)
g & g_c: gravitational constants
h = reservoir thickness, feet
k = effective formation permeability to water, md
$L_{boundary}$ = distance to boundary, feet
m = slope of the semilog plot, psi/cycle
m_{Hall} = slope off the Hall plot, psi-day/bbl
P = pressure, psi
P_e = pressure at external radius, psi
$P_{corrected}$ = pressure corrected for wellbore skin effects
P_D = dimensionless pressure
P_i = initial pressure, psi
P_{sp} = superposition pressure function, psi or psi/bbl
P_{static} = pressure at end of falloff or stabilization period, psi
P_{tf} = surface injection pressure, psi (tubing flowing pressure)
P_{wf} = pressure at end of injection period, psi (flowing pressure -producer)
P_{1hr} = pressure intercept along the straight line portion of the Horner Plot or superposition plot at a shut-in time of 1 hr, psi
P = change in pressure, psi
P_f = pressure loss due to friction, psi
P_{skin} = pressure change due to wellbore skin, psi
P^* = false extrapolated pressure, psi
$\overline{P}$ = average reservoir pressure, psi
q = injection rate, bpd or gpm
r = distance into the reservoir. feet
r_D = dimensionless radius
r_e = effective wellbore radius. feet
r_i = radius of investigation, feet
r_w = wellbore radius, feet
r_{wa} = effective wellbore radius, feet (wellbore apparent radius)
s = skin factor, dimensionless

t = injection time or falloff time, hours
$t_{boundary}$ = time to reach a boundary, hours
t_D = dimensionless time
t_e = Agarwal equivalent time, hours
$t_{elasped}$ = shut-in time or real time, hours
$t_{interference}$ = time until interference between wells is observed, hours
t_p = injection time, hours
$t_{radial\ flow}$ = time to reach radial flow, hours
t_{sp} = superposition time function, hrs
t = change in time, hrs
V_w = total wellbore volume, bbls
V_u = wellbore volume per unit length, bbls/ft
V_p = injection volume since last stabilization period, bbls

$_f$ = viscosity of formation fluid, cp
$_w$ = viscosity of injectate, cp
= porosity, fraction
= injectate density, lbm/ft^3
g = pressure gradient, psi/ft

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