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Regression Equations for Estimating Concentrations of Selected Water-Quality Constituents for Selected Gaging Stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota

Water-Resources Investigations Report 03-4291

U.S. Department of the Interior
U.S. Geological Survey

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By Tara Williams-Sether

In cooperation with the Bureau of Reclamation

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Regression Equations for Estimating Concentrations of Selected Water-Quality Constituents for Selected Gaging Stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota

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Abstract

The Dakota Water Resources Act, passed by the U.S. Congress on December 15, 2000, authorized the Secretary of the Interior to conduct a comprehensive study of future water-quantity and quality needs of the Red River of the North Basin in North Dakota and possible options to meet those water needs. Previous Red River of the North Basin studies conducted by the Bureau of Reclamation used streamflow and water-quality data bases developed by the U.S. Geological Survey that included data for 1931-84. As a result of the recent congressional authorization and results of previous studies by the Bureau of Reclamation, redevelopment of the streamflow and water-quality data bases with current data through 1999 are needed in order to evaluate and predict the water-quantity and quality effects within the Red River of the North Basin. This report provides updated statistical summaries of selected water-quality constituents and streamflow and the regression relations between them.

Available data for 1931-99 were used to develop regression equations between 5 selected water-quality constituents and streamflow for 38 gaging stations in the Red River of the North Basin. The water-quality constituents that were regressed against streamflow were hardness (as $CaCO_3$), sodium, chloride, sulfate, and dissolved solids. Statistical summaries of the selected water-quality constituents and streamflow for the gaging stations used in the regression equations development and the applications and limitations of the regression equations are presented in this report.

Introduction

In 1994, the Bureau of Reclamation (BOR) began a two-phase planning study to investigate and evaluate existing and future municipal, rural, and industrial (MR&I) water use in the Red River of the North Basin in North Dakota. Phase I of the study assessed water needs and Phase II identified preliminary alternatives to address water-quantity and quality needs. Phase I, Part A, was completed in 1998 and appraised the existing and projected MR&I water needs (Bureau of Reclamation, 1998). Phase I, Part B was completed in 1999 and addressed the instream flow issues that pertain to aquatic life and water quality (Bureau of Reclamation, 1999). Parts A and B of the Phase I study used streamflow and water-quality data bases developed by the U.S. Geological Survey (USGS) that included data for 1931-84 (Guenthner and others, 1990; Guenthner, 1991). The BOR needed the data bases to be redeveloped with data through 1999 in order to evaluate and predict water-quantity and quality effects within the Red River of the North Basin.

The Dakota Water Resources Act was passed by the U.S. Congress on December 15, 2000. The Act authorized the Secretary of the Interior to conduct a comprehensive study of future water-quantity and quality needs of the Red River of the North Basin in North Dakota and possible options to meet those water needs. The BOR will administrate the comprehensive study. The BOR has formed a Technical Team and a Study Review Team. The Technical Team will develop plans for the study, perform technical evaluations, draft portions of the Needs and Options Report, and perform other day-to-day study activities. The Study Review Team will review draft products and provide input into study processes and products.

The USGS conducted a study in cooperation with the BOR to provide needed information for the comprehensive study. The study will provide updated historic and unregulated monthly streamflow for selected gaging stations in the Red River of the North Basin for 1931-99 and will develop regression equations between selected water-quality constituents and streamflow. Thus, the study will update the previous reports of Guenthner and others (1990) and Guenthner (1991). The report by Emerson and Dressler (2002) updates the streamflow report by Guenthner and others (1990). This report updates the water-quality and streamflow relations report by Guenthner (1991).

Purpose and Scope

This report provides regression equations for estimating monthly mean concentrations of selected water-quality constituents that can be used for input to and calibration of the water-quality model selected by the BOR for the Red River Valley water-supply project studies. Specific objectives were to (1) provide relations with streamflow for estimating concentrations of selected water-quality constituents, and (2) provide statistical summaries of selected water-quality data used in developing the regression equations.

The equations were developed by regressing 5 selected water-quality constituents against streamflow for 38 selected gaging stations in the Red River of the North Basin (fig. 1). The water-quality constituents were hardness (as $CaCO_3$), sodium, chloride, sulfate, and dissolved solids. Calcium, magnesium, nitrate, total organic carbon, iron, manganese, pH, field conductivity, alkalinity, and turbidity also were considered for the regression equations. However, data were too few or were considered unusable because changes in sampling techniques and laboratory analyses made comparison over time unrealistic. Data bases from the Environmental Protection Agency, North Dakota Department of Health, North Dakota State Water Commission, and various agencies from Minnesota and Canada (Macek-Rowland and Dressler, 2002, p. 6-7) were considered for the regression equations but were not used because of changes in sampling techniques and laboratory analyses and because of the absence of streamflow data. Available water-quality and streamflow data for 1931-99 were retrieved from the USGS computerized National Water Information System and from North Dakota, Minnesota, and South Dakota USGS data bases. Period of records for available water-quality and streamflow data varied within the Red River of the North Basin.

Description of Study Area

The Red River of the North Basin is part of the Hudson Bay drainage system. Parts of Montana, South Dakota, North Dakota, and Minnesota in the United States, and parts of Saskatchewan and Manitoba in Canada are drained by the Red River of the North. The North Dakota-Minnesota boundary is formed by the Red River of the North. Drainage area of the Red River of the North at Emerson, Manitoba, gaging station, which is 0.8 mile downstream from the international boundary, is 40,200 square miles (U.S. Geological Survey, 2000, p. 196). The river flows northward 394 miles to the United States-Canadian boundary. From the international boundary, the Red River of the North flows north about 155 miles and discharges into Lake Winnipeg. The Red River of the North Basin upstream from the international boundary is the only part of the basin included in the study area.

The Red River of the North begins at the confluence of the Otter Tail and Bois de Sioux Rivers in Wahpeton, N. Dak., and Breckenridge, Minn. The Otter Tail River is regulated by Orwell Reservoir, which began operation in 1953. The reservoir provides 13,100 acre-feet of storage for multi-uses. Numerous other controlled lakes and ponds and several power plants affect the flow of the Otter Tail River. Lake Traverse and Mud Lake are natural instream lakes near the headwaters of the Bois de Sioux River. In 1942, Reservation Dam on Lake Traverse and White Rock Dam on Mud Lake were completed. The total storage capacity for Lake Traverse and Mud Lake at an elevation of 981.0 feet is 153,700 acre-feet.

The Red River of the North flows over lacustrine deposits of glacial Lake Agassiz through its entire length in North Dakota. The slope of the river is extremely flat. The river falls only about 200 feet in its 394-mile course from Wahpeton, N. Dak., to the international boundary (Miller and Frink, 1984). Water quality in the river is affected by the lacustrine deposits, by inflow from major tributaries from North Dakota and Minnesota, by ground-water discharge, and by municipal, agricultural, and industrial effluents.

The Sheyenne River is one of the major tributaries to the Red River of the North. The Sheyenne River has a drainage area of about 6,910 square miles (not including the closed Devils Lake Basin). From its headwaters near Harvey, N. Dak., the Sheyenne River, which is about 500 miles long, flows eastward about 150 miles, southward about 200 miles, and then northeastward to its confluence with the Red River of the North, north of Fargo, N. Dak. (Souris-Red-Rainy River Basins Commission, 1972, p. D-50).

The Sheyenne River Basin lies in two distinct physiographic areas (fig. 2). The drift prairie area extends from the headwaters to the vicinity of Lisbon, N. Dak., and the glacial Lake Agassiz area extends from the vicinity of Lisbon, N. Dak., to the confluence of the Sheyenne River and the Red River of the North. A hilly delta topography exists from Valley City, N. Dak., to the vicinity of Kindred, N. Dak. Most of the Sheyenne River valley from the headwaters to Kindred, N. Dak., is incised into glacial till. The valley from Sheyenne, N. Dak., to Kindred, N. Dak., ranges from 100 to 200 feet in depth and 0.2 to 2 miles in width. Average gradient of the river is 1.5 feet per mile in the drift prairie and hilly delta areas and about 1 foot per mile in the glacial Lake Agassiz area.

No flow has been recorded at times in the upper reaches of the Sheyenne River. Flow in the lower reaches of the river is regulated partly by releases from Baldhill Dam, which began operation in 1949. Lake Ashtabula, formed by Baldhill Dam, has a capacity of 69,100 acre-feet between the invert of the outlet conduit and the normal pool elevation. Lake Ashtabula is operated for flood control (capacity at maximum pool elevation of 1,273.2 feet is 116,500 acre-feet), municipal water supply, recreation, and stream-pollution abatement through low-flow augmentation.

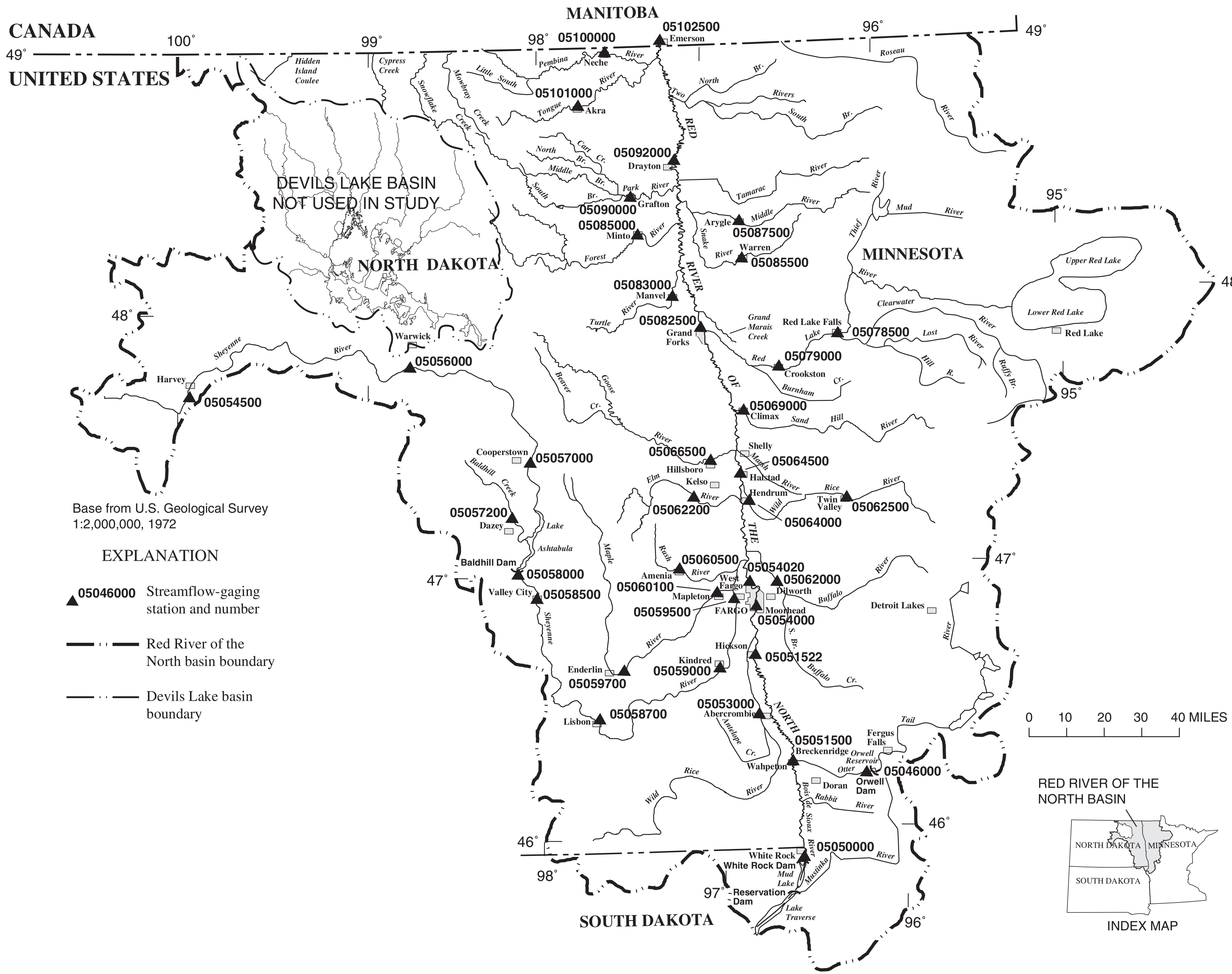

Figure 1. Location of selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.

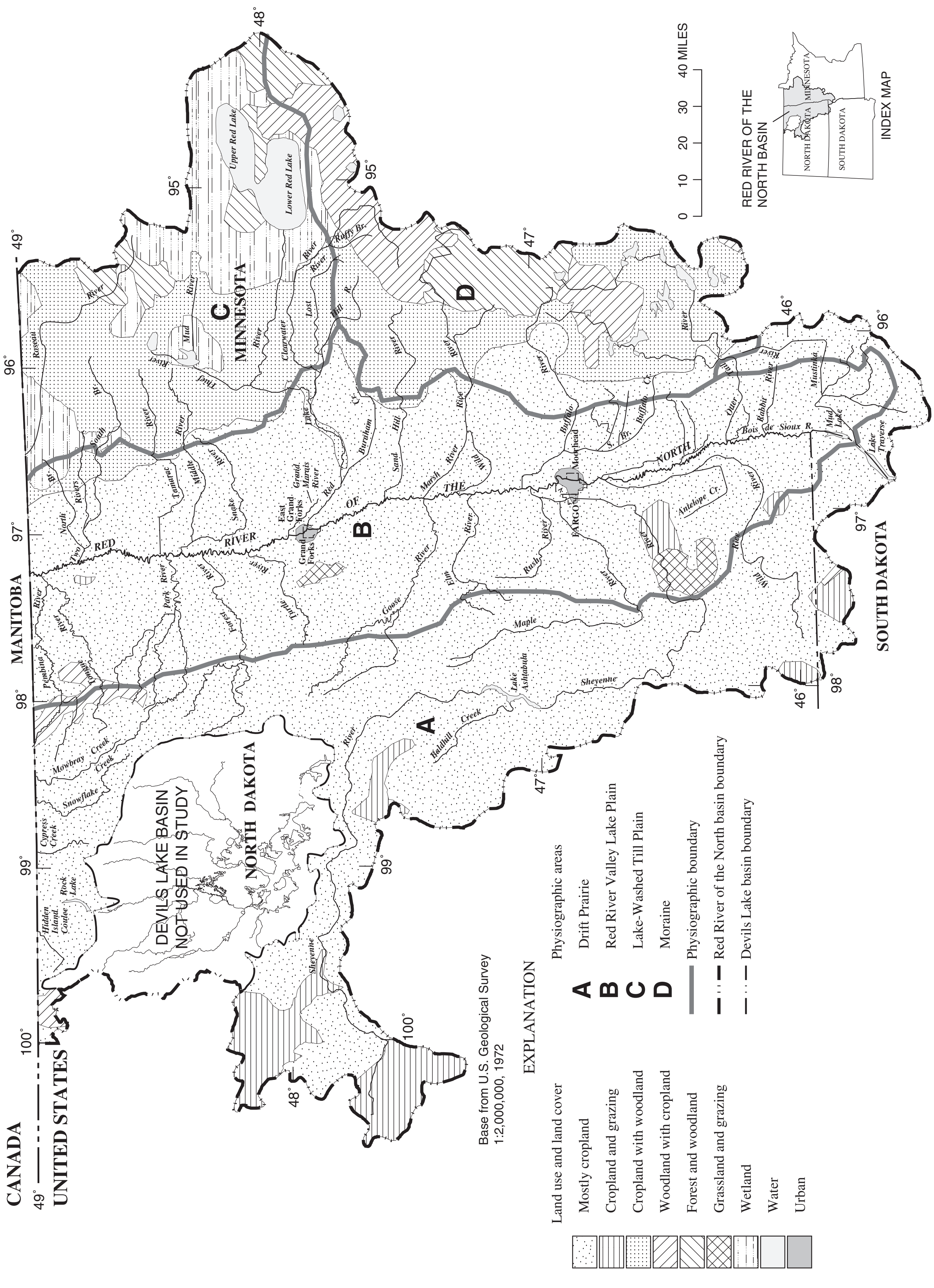

Figure 2. Land use and land cover and physiographic areas in the Red River of the North Basin. (Modified from Tornes and others, 1997).

Water-Quality Summaries

Statistical summaries were computed for the 5 selected water-quality constituents and for streamflow using data collected at 38 selected gaging stations in the Red River of the North Basin (table 1). The statistical summaries include number of samples; average annual sampling frequency; number of sample years; descriptive statistics of the maximum, minimum, mean, and standard deviation; and values for the 75th, 50th, and 25th percentiles. The percentiles are the percentage of samples in which the values or concentrations were less than or equal to the value given. The 75th percentile values may be used as indicators of concentration during low-flow conditions, and the 25th percentile values may be used as indicators of concentrations during high-flow conditions. Periods of available data for the water-quality constituents for each gaging station also are listed in table 1. The available water-quality data for each gaging station varies considerably throughout the Red River of the North Basin. Of the 38 gaging stations, about 5 percent had data available during 1940 through 1999, about 16 percent had data available during 1950 through 1999, about 37 percent had data available during 1960 through 1999, about 37 percent had data available during 1970 through 1999, about 2.5 percent had data available during 1980 through 1999, and about 2.5 percent had data available during 1990 through 1999. The sampling frequency of the water-quality constituents varied considerably at 11 of the 38 gaging stations but was more consistent at the remaining 27 gaging stations.

Instantaneous streamflow was the type of gaged streamflow desired for the regression equations because instantaneous streamflow provides a direct link to the water-quality constituents sampled at the same time. However, the records of instantaneous streamflow in the water-quality data base were not always complete. Daily mean streamflow records were more complete than instantaneous streamflow records and were readily available for most of the gaging stations; therefore, the daily mean streamflow records were used to develop the regression equations with the water-quality constituents when instantaneous streamflow records were too few. At some gaging stations, instantaneous and daily mean streamflow records were combined to complete the discharge record needed when only a few instantaneous streamflow records were incomplete.

Daily mean streamflow provides a streamflow measurement averaged over a day interval. Instantaneous streamflow provides a streamflow measurement at a point in the day. Daily mean streamflow may provide bias in regression equation development. The bias may be high or low depending upon the type of sample and when the sample was collected. For example, a high-flow sample will be collected near a peak flow for the day. Using a daily mean instead of instantaneous would under estimate the flow at the time of sampling. The reverse would be true for low-flow samples.

Regression Equations for Estimating Water-Quality Concentrations

Several types of regression equations were considered to model the relation between water-quality constituents and streamflow in the Red River of the North Basin. In order to determine which regression equation might be best suited for each gaging station in the basin, a linear least-squares curve-fitting plot and a lowess (locally weighted scatter-plot smoother) plot were made for each water-quality constituent and for streamflow at each gaging station. Locally weighted least-squares regression is used to obtain the values for the lowess plots that display a smoothed line along with a scatter plot of associated points. The lowess plots are helpful in picking up trends in the data that a linear plot may miss. A linear and a lowess fit plot of sulfate and streamflow at the Red River of the North at Grand Forks, N. Dak., (05082500) gaging station are shown in figure 3. Many of the gaging stations had similar looking plots. After reviewing the plots, it was apparent that a simple linear regression equation between the logarithm of concentration and the logarithm of streamflow would not be adequate to develop a relation between water-quality constituents and streamflow for most gaging stations. The linear fit graph in figure 3 shows the sinuous look of the data and the wide spread away from the linear line. Therefore, two general equations were used to model the relation between water-quality constituents and streamflow.

The first equation assumed that the base-10 logarithm of a water-quality constituent concentration is a piece-wise linear, continuous function of the base-10 logarithm of streamflow. The breakpoints (the values of base-10 logarithm of streamflow at which slope changes occur) can be selected to obtain the best statistical fit to the data. For example, the lowess fit in figure 3 indicates that a piece-wise linear relation between log-transformed sulfate concentration and log-transformed streamflow may be a reasonable assumption for the gaging station. It appears that two breakpoints, one near base-10 logarithm of streamflow equals three and one near base-10 logarithm of streamflow equals four, might be appropriate for the data in figure 3. For the piece-wise linear model with K breakpoints, log-transformed concentration can be computed using $K + 1$ linear equations. For example, for two breakpoints, log-transformed concentrations can be expressed as:

$$\log C = \begin{cases} a_0 + \dfrac{a_1 - a_0}{b_1 - b_0}(\log Q - b_0), \text{ if } b_0 \le \log Q < b_1 \\ a_1 + \dfrac{a_2 - a_1}{b_2 - b_1}(\log Q - b_1), \text{ if } b_1 \le \log Q < b_2 \\ a_2 + \dfrac{a_3 - a_2}{b_3 - b_2}(\log Q - b_2), \text{ if } b_2 \le \log Q < b_3 \end{cases} \quad (1)$$

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.

[mg/L, milligrams per liter; --, no data]

Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Otter Tail River below Orwell Dam near Fergus Falls, Minn., gaging station 05046000, October 12, 1960, through August 24, 1995										
00060	Discharge (cubic feet per second)	41	--	--	1,220	82	698	313	911	687	449
00900	Hardness, total (mg/L as $CaCO_3$)	39	3.5	11	260	190	212	16	220	210	200
00930	Sodium, dissolved (mg/L as Na)	39	3.5	11	11	6.5	8.6	1.1	9.2	8.3	7.9
00945	Sulfate, dissolved (mg/L as SO_4)	41	3.7	11	32	9.3	17	5.6	20	16	12
00940	Chloride, dissolved (mg/L as Cl)	41	3.7	11	14	2.7	8.3	2.9	10	9.0	6.9
70301	Dissolved solids, calculated, sum of constituents (mg/L)	36	3.3	11	290	214	238	18	250	235	227
	Bois de Sioux River near White Rock, S. Dak., gaging station 05050000, October 30, 1963, through November 2, 1995										
00060	Discharge (cubic feet per second)	15	--	--	869	0.20	168	289	180	13	1.0
00900	Hardness, total (mg/L as $CaCO_3$)	15	3.0	5	1,600	350	663	330	720	580	480
00930	Sodium, dissolved (mg/L as Na)	15	3.0	5	140	25	63	33	76	55	40
00945	Sulfate, dissolved (mg/L as SO_4)	20	3.3	6	1,400	200	510	280	600	460	340
00940	Chloride, dissolved (mg/L as Cl)	15	3.0	5	35	6.8	17	7.7	21	17	10
70301	Dissolved solids, calculated, sum of constituents (mg/L)	15	3.0	5	2,360	487	977	493	1,050	828	684
	Red River of the North at Wahpeton, N. Dak., gaging station 05051500, May 2, 1973, through July 15, 1999										
00060	Discharge (cubic feet per second)	53	--	--	11,000	35	1,470	2,170	1,520	511	291
00900	Hardness, total (mg/L as $CaCO_3$)	53	2.0	27	390	110	240	47	270	230	210
00930	Sodium, dissolved (mg/L as Na)	53	2.0	27	33	4.5	13	5.1	16	13	11
00945	Sulfate, dissolved (mg/L as SO_4)	53	2.0	27	230	15	70	48	95	64	32
00940	Chloride, dissolved (mg/L as Cl)	53	2.0	27	22	1.7	10	4.3	13	11	7.0
70301	Dissolved solids, calculated, sum of constituents (mg/L)	53	2.0	27	563	154	297	70	336	292	243
	Red River of the North at Hickson, N. Dak., gaging station 05051522, November 3, 1975, through September 22, 1999										
00060	Discharge (cubic feet per second)	116	--	--	12,000	3.0	1,010	1,830	1,000	486	239
00900	Hardness, total (mg/L as $CaCO_3$)	116	4.6	25	800	92	270	80	300	260	230
00930	Sodium, dissolved (mg/L as Na)	116	4.6	25	92	6.8	16	9.7	19	15	11
00945	Sulfate, dissolved (mg/L as SO_4)	116	4.6	25	340	5.4	80	60	110	66	34
00940	Chloride, dissolved (mg/L as Cl)	116	4.6	25	44	1.0	11	5.7	13	9.8	7.6
70301	Dissolved solids, calculated, sum of constituents (mg/L)	112	4.5	25	1,150	146	339	121	382	316	271

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Wild Rice River near Abercrombie, N. Dak., gaging station 05053000, June 20, 1966, through September 3, 1999										
00060	Discharge (cubic feet per second)	277	--	--	9,250	0.01	217	895	71	11	1.5
00900	Hardness, total (mg/L as $CaCO_3$)	277	8.1	34	1,300	51	510	250	620	490	330
00930	Sodium, dissolved (mg/L as Na)	277	8.1	34	420	5.3	120	77	160	100	60
00945	Sulfate, dissolved (mg/L as SO_4)	240	7.1	34	1,200	11	380	220	500	360	210
00940	Chloride, dissolved (mg/L as Cl)	224	6.6	34	180	2.3	46	31	62	38	23
70301	Dissolved solids, calculated, sum of constituents (mg/L)	224	6.6	34	2,660	76	926	492	1,200	874	536
	Red River of the North at Fargo, N. Dak., gaging station 05054000, May 16, 1949, through July 13, 1999										
00060	Discharge (cubic feet per second)	502	--	--	25,800	0	1,210	2,450	1,020	450	192
00900	Hardness, total (mg/L as $CaCO_3$)	502	11.4	44	420	85	250	50	280	250	220
00930	Sodium, dissolved (mg/L as Na)	499	11.6	43	43	5.2	15	5.5	18	14	11
00945	Sulfate, dissolved (mg/L as SO_4)	438	10.0	44	270	13	73	44	99	60	38
00940	Chloride, dissolved (mg/L as Cl)	237	5.4	44	39	0.01	7.0	4.5	8.0	6.2	4.6
70301	Dissolved solids, calculated, sum of constituents (mg/L)	235	5.5	43	608	130	316	77	362	301	270
	Red River of the North below Fargo, N. Dak., gaging station 05054020, July 16, 1969, through September 16, 1986										
00060	Discharge (cubic feet per second)[1]	180	--	--	17,300	2.0	705	1,520	716	336	162
00900	Hardness, total (mg/L as $CaCO_3$)	180	10.0	18	510	120	260	48	290	260	230
00930	Sodium, dissolved (mg/L as Na)	149	8.3	18	110	6.8	24	19	24	20	15
00945	Sulfate, dissolved (mg/L as SO_4)	180	10.0	18	330	19	83	51	100	69	49
00940	Chloride, dissolved (mg/L as Cl)	168	9.3	18	96	4.4	15	13	14	11	8.7
70301	Dissolved solids, calculated, sum of constituents (mg/L)	134	8.9	15	741	170	357	104	396	334	291
	Sheyenne River above Harvey, N. Dak., gaging station 05054500, April 17, 1972, through August 25, 1999										
00060	Discharge (cubic feet per second)	179	--	--	690	0.19	22	64	11	3.2	1.2
00900	Hardness, total (mg/L as $CaCO_3$)	179	6.4	28	550	58	190	86	240	180	140
00930	Sodium, dissolved (mg/L as Na)	179	6.4	28	480	20	240	92	300	250	180
00945	Sulfate, dissolved (mg/L as SO_4)	179	6.4	28	560	37	220	78	250	210	170
00940	Chloride, dissolved (mg/L as Cl)	179	6.4	28	54	2.2	17	6.8	21	17	14
70301	Dissolved solids, calculated, sum of constituents (mg/L)	178	6.4	28	1,610	140	863	247	1,000	892	791

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Sheyenne River near Warwick, N. Dak., gaging station 05056000, January 8, 1951, through August 5, 1999										
00060	Discharge (cubic feet per second)	530	--	--	2,310	0.10	99	272	62	15	3.2
00900	Hardness, total (mg/L as $CaCO_3$)	530	11.5	46	570	71	240	71	280	240	200
00930	Sodium, dissolved (mg/L as Na)	530	11.5	46	230	10	73	40	100	68	40
00945	Sulfate, dissolved (mg/L as SO_4)	277	6.2	45	230	28	97	44	120	88	60
00940	Chloride, dissolved (mg/L as Cl)	247	5.5	45	37	0.6	12	5.7	17	12	8.2
70301	Dissolved solids, calculated, sum of constituents (mg/L)	230	5.3	43	902	136	460	160	588	464	328
	Sheyenne River near Cooperstown, N. Dak., gaging station 05057000, October 11, 1959, through July 20, 1999										
00060	Discharge (cubic feet per second)	376	--	--	4,150	0.02	206	510	156	31	12
00900	Hardness, total (mg/L as $CaCO_3$)	367	9.2	40	680	74	290	87	340	300	250
00930	Sodium, dissolved (mg/L as Na)	367	9.2	40	920	10	83	52	97	83	63
00945	Sulfate, dissolved (mg/L as SO_4)	320	8.0	40	360	21	140	44	170	140	120
00940	Chloride, dissolved (mg/L as Cl)	315	7.7	41	39	0.1	16	5.7	19	16	12
70301	Dissolved solids, calculated, sum of constituents (mg/L)	306	7.6	40	1,360	122	552	167	648	582	476
	Baldhill Creek near Dazey, N. Dak., gaging station 05057200, May 2, 1972, through July 20, 1999										
00060	Discharge (cubic feet per second)	96	--	--	2,450	0.01	171	389	122	5.8	1.3
00900	Hardness, total (mg/L as $CaCO_3$)	86	3.2	27	660	81	290	120	360	300	200
00930	Sodium, dissolved (mg/L as Na)	86	3.2	27	150	4.7	55	36	75	58	22
00945	Sulfate, dissolved (mg/L as SO_4)	86	3.2	27	300	17	160	72	210	170	110
00940	Chloride, dissolved (mg/L as Cl)	96	3.4	28	51	2.1	16	11	20	15	8.0
70301	Dissolved solids, calculated, sum of constituents (mg/L)	86	3.2	27	1,020	124	492	213	628	520	318
	Sheyenne River below Baldhill Dam, N. Dak., gaging station 05058000, June 5, 1959, through July 14, 1999										
00060	Discharge (cubic feet per second)	122	--	--	4,710	3.6	375	824	255	40	13
00900	Hardness, total (mg/L as $CaCO_3$)	112	3.3	34	380	96	230	57	260	220	200
00930	Sodium, dissolved (mg/L as Na)	112	3.3	34	120	20	64	19	76	64	51
00945	Sulfate, dissolved (mg/L as SO_4)	112	3.3	34	240	48	120	39	140	120	94
00940	Chloride, dissolved (mg/L as Cl)	122	3.5	35	26	4.7	14	4.4	17	13	11
70301	Dissolved solids, calculated, sum of constituents (mg/L)	112	3.3	34	741	175	440	112	500	422	364

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Sheyenne River at Valley City, N. Dak., gaging station 05058500, May 3, 1972, through April 6, 1999										
00060	Discharge (cubic feet per second)[1]	25	--	--	3,520	9.1	808	1,090	1,230	335	15
00900	Hardness, total (mg/L as $CaCO_3$)	27	1.3	21	390	140	250	62	280	240	200
00930	Sodium, dissolved (mg/L as Na)	27	1.3	21	100	24	63	19	74	63	51
00945	Sulfate, dissolved (mg/L as SO_4)	27	1.3	21	260	81	140	41	160	130	120
00940	Chloride, dissolved (mg/L as Cl)	27	1.3	21	24	5.1	15	4.7	20	15	12
70301	Dissolved solids, calculated, sum of constituents (mg/L)	27	1.3	21	687	244	457	118	515	455	382
	Sheyenne River at Lisbon, N. Dak., gaging station 05058700, August 2, 1956, through July 21, 1999										
00060	Discharge (cubic feet per second)	609	--	--	4,190	0.40	259	552	197	68	29
00900	Hardness, total (mg/L as $CaCO_3$)	611	13.9	44	530	100	270	64	320	270	230
00930	Sodium, dissolved (mg/L as Na)	611	13.9	44	560	13	76	31	91	76	59
00945	Sulfate, dissolved (mg/L as SO_4)	547	12.4	44	450	39	160	56	200	160	130
00940	Chloride, dissolved (mg/L as Cl)	333	7.6	44	110	8.0	30	15	38	26	20
70301	Dissolved solids, calculated, sum of constituents (mg/L)	330	7.5	44	1,040	192	531	137	621	520	447
	Sheyenne River near Kindred, N. Dak., gaging station 05059000, May 4, 1972, through September 2, 1999										
00060	Discharge (cubic feet per second)	222	--	--	4,070	19	366	697	292	118	50
00900	Hardness, total (mg/L as $CaCO_3$)	222	7.9	28	430	120	300	61	330	310	270
00930	Sodium, dissolved (mg/L as Na)	222	7.9	28	110	9.5	64	18	76	64	54
00945	Sulfate, dissolved (mg/L as SO_4)	222	7.9	28	290	50	150	39	170	150	130
00940	Chloride, dissolved (mg/L as Cl)	222	7.9	28	74	5.7	28	12	35	26	20
70301	Dissolved solids, calculated, sum of constituents (mg/L)	222	7.9	28	776	191	523	107	590	531	480
	Sheyenne River at West Fargo, N. Dak., gaging station 05059500, September 16, 1969, through August 31, 1999										
00060	Discharge (cubic feet per second)	58	--	--	3,050	5.9	447	688	562	104	56
00900	Hardness, total (mg/L as $CaCO_3$)	58	2.1	28	400	130	280	75	340	290	220
00930	Sodium, dissolved (mg/L as Na)	58	2.1	28	94	23	57	17	69	60	46
00945	Sulfate, dissolved (mg/L as SO_4)	58	2.1	28	230	8.6	130	44	170	140	100
00940	Chloride, dissolved (mg/L as Cl)	58	2.1	28	57	7.8	27	12	36	27	18
70301	Dissolved solids, calculated, sum of constituents (mg/L)	58	2.1	28	701	219	474	126	571	512	401

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

					Descriptive statistics				Percentage of samples in which values were less than or equal to those shown		
Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Maximum	Minimum	Mean	Standard deviation	75	50 (Median)	25
	Maple River near Enderlin, N. Dak., gaging station 05059700, May 3, 1972, through July 22, 1999										
00060	Discharge (cubic feet per second)	79	--	--	3,500	1.3	226	545	130	5.4	2.9
00900	Hardness, total (mg/L as $CaCO_3$)	79	2.8	28	910	100	550	220	720	640	440
00930	Sodium, dissolved (mg/L as Na)	79	2.8	28	180	12	76	36	94	79	60
00945	Sulfate, dissolved (mg/L as SO_4)	79	2.8	28	650	42	370	160	480	450	260
00940	Chloride, dissolved (mg/L as Cl)	79	2.8	28	140	2.5	50	30	65	47	27
70301	Dissolved solids, calculated, sum of constituents (mg/L)	79	2.8	28	1,540	176	881	348	1,120	1,010	700
	Maple River below Mapleton, N. Dak., gaging station 05060100, March 18, 1995, through July 19, 1999										
00060	Discharge (cubic feet per second)	9	--	--	6,620	0.79	1,410	2,140	1,520	480	39
00900	Hardness, total (mg/L as $CaCO_3$)	9	1.8	5	530	140	350	150	490	390	200
00930	Sodium, dissolved (mg/L as Na)	9	1.8	5	110	13	53	32	73	46	28
00945	Sulfate, dissolved (mg/L as SO_4)	9	1.8	5	440	73	250	140	400	250	130
00940	Chloride, dissolved (mg/L as Cl)	9	1.8	5	59	7.4	30	18	46	22	17
70301	Dissolved solids, calculated, sum of constituents (mg/L)	9	1.8	5	959	204	580	281	837	599	329
	Rush River at Amenia, N. Dak., gaging station 05060500, May 3, 1972, through July 15, 1999										
00060	Discharge (cubic feet per second)	48	--	--	2,310	0.01	141	397	70	8.8	0.95
00900	Hardness, total (mg/L as $CaCO_3$)	48	1.7	28	770	100	420	200	580	480	210
00930	Sodium, dissolved (mg/L as Na)	48	1.7	28	200	7	52	39	70	50	19
00945	Sulfate, dissolved (mg/L as SO_4)	48	1.7	28	670	37	270	180	370	250	110
00940	Chloride, dissolved (mg/L as Cl)	48	1.7	28	120	2.3	25	21	34	20	9.8
70301	Dissolved solids, calculated, sum of constituents (mg/L)	48	1.7	28	1,390	146	655	336	866	700	308
	Buffalo River near Dilworth, Minn., gaging station 05062000, April 10, 1962, through March 12, 1991										
00060	Discharge (cubic feet per second)	12	--	--	5,250	6.1	596	1,520	86	48	22
00900	Hardness, total (mg/L as $CaCO_3$)	11	1.4	8	600	120	380	140	440	420	340
00930	Sodium, dissolved (mg/L as Na)	11	1.4	8	27	4.5	17	6.6	20	18	15
00945	Sulfate, dissolved (mg/L as SO_4)	11	1.4	8	230	36	130	60	150	120	95
00940	Chloride, dissolved (mg/L as Cl)	11	1.4	8	15	0	6.0	3.9	7.4	6.4	3.6
70301	Dissolved solids, calculated, sum of constituents (mg/L)	11	1.4	8	718	156	474	168	547	508	420

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Elm River near Kelso, N. Dak., gaging station 05062200, February 25, 1981, through March 31, 1988										
00060	Discharge (cubic feet per second)[2]	11	--	--	705	0.01	182	214	283	140	3.35
00900	Hardness, total (mg/L as $CaCO_3$)	11	1.4	8	450	120	230	120	280	170	160
00930	Sodium, dissolved (mg/L as Na)	11	1.4	8	59	5.9	23	18	32	16	11
00945	Sulfate, dissolved (mg/L as SO_4)	11	1.4	8	210	35	99	67	140	70	48
00940	Chloride, dissolved (mg/L as Cl)	11	1.4	8	41	4.5	17	11	22	13	9.2
70301	Dissolved solids, calculated, sum of constituents (mg/L)	11	1.4	8	666	182	343	179	414	260	232
	Wild Rice River at Twin Valley, Minn., gaging station 05062500, September 21, 1966, through April 30, 1997										
00060	Discharge (cubic feet per second)	173	--	--	4,330	2.8	348	668	372	99	43
00900	Hardness, total (mg/L as $CaCO_3$)	171	14.2	12	480	110	270	53	300	260	240
00930	Sodium, dissolved (mg/L as Na)	170	14.2	12	31	2.4	10	4.7	13	9.4	7.3
00945	Sulfate, dissolved (mg/L as SO_4)	170	14.2	12	85	10	34	14	42	31	23
00940	Chloride, dissolved (mg/L as Cl)	170	14.2	12	50	0.6	4.5	4.2	4.7	3.8	2.8
70301	Dissolved solids, calculated, sum of constituents (mg/L)	170	14.2	12	570	147	305	64	338	294	261
	Wid Rice River at Hendrum, Minn., gaging station 05064000, October 1, 1962, through June 22, 1994										
00060	Discharge (cubic feet per second)	118	--	--	9,110	0.09	444	1,240	222	77	28
00900	Hardness, total (mg/L as $CaCO_3$)	7	1.2	6	490	170	310	100	360	290	260
00930	Sodium, dissolved (mg/L as Na)	7	1.2	6	31	5.2	15	9.0	19	10	9.0
00945	Sulfate, dissolved (mg/L as SO_4)	8	1.1	7	85	28	49	17	52	48	39
00940	Chloride, dissolved (mg/L as Cl)	118	11.8	10	31	0	5.1	3.6	5.8	4.2	3.4
70301	Dissolved solids, calculated, sum of constituents (mg/L)	7	1.2	6	593	217	366	126	422	326	293
	Red River of the North at Halstad, Minn., gaging station 05064500, July 8, 1961, through September 21, 1999										
00060	Discharge (cubic feet per second)	163	--	--	66,200	109	3,200	6,750	2,900	1,000	471
00900	Hardness, total (mg/L as $CaCO_3$)	163	4.7	35	480	120	290	65	330	290	250
00930	Sodium, dissolved (mg/L as Na)	163	4.7	35	77	7.6	31	13	38	30	22
00945	Sulfate, dissolved (mg/L as SO_4)	163	4.7	35	240	36	110	42	130	100	80
00940	Chloride, dissolved (mg/L as Cl)	163	4.7	35	52	4	18	8.4	22	16	12
70301	Dissolved solids, calculated, sum of constituents (mg/L)	163	4.7	35	843	164	410	104	474	404	343

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

Param-eter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Goose River at Hillsboro, N. Dak., gaging station 05066500, September 15, 1969, through August 20, 1999										
00060	Discharge (cubic feet per second)	93	--	--	5,360	0.07	409	980	221	18	3.8
00900	Hardness, total (mg/L as $CaCO_3$)	93	3.0	31	930	120	520	210	660	550	360
00930	Sodium, dissolved (mg/L as Na)	93	3.0	31	330	8.5	98	71	120	84	54
00945	Sulfate, dissolved (mg/L as SO_4)	93	3.0	31	800	49	370	170	480	400	240
00940	Chloride, dissolved (mg/L as Cl)	93	3.0	31	310	5.4	70	63	90	50	24
70301	Dissolved solids, calculated, sum of constituents (mg/L)	93	3.0	31	2,060	184	897	412	1,100	929	585
	Sand Hill River near Climax, Minn., gaging station 05069000, November 1, 1966, through December 9, 1991										
00060	Discharge (cubic feet per second)	2	--	--	18	10	14	--	--	--	--
00900	Hardness, total (mg/L as $CaCO_3$)	2	1.0	2	410	360	380	--	--	--	--
00930	Sodium, dissolved (mg/L as Na)	2	1.0	2	19	14	16	--	--	--	--
00945	Sulfate, dissolved (mg/L as SO_4)	2	1.0	2	78	69	74	--	--	--	--
00940	Chloride, dissolved (mg/L as Cl)	2	1.0	2	15	13	14	--	--	--	--
70301	Dissolved solids, calculated, sum of constituents (mg/L)	1	1.0	1	435	--	--	--	--	--	--
	Clearwater River at Red Lake Falls, Minn., gaging station 05078500, October 25, 1963, through July 18, 1995										
00060	Discharge (cubic feet per second)	12	--	--	1,300	40	452	481	721	274	54
00900	Hardness, total (mg/L as $CaCO_3$)	12	2.4	5	310	120	250	56	300	260	230
00930	Sodium, dissolved (mg/L as Na)	12	2.4	5	11	3.4	7.9	1.9	9.0	8.2	7.1
00945	Sulfate, dissolved (mg/L as SO_4)	12	2.4	5	74	16	40	19	48	39	28
00940	Chloride, dissolved (mg/L as Cl)	12	2.4	5	7.5	2.8	4.5	1.3	5.0	4.5	3.8
70301	Dissolved solids, calculated, sum of constituents (mg/L)	12	2.4	5	346	141	290	58	334	297	268
	Red Lake River at Crookston, Minn., gaging station 05079000, April 11, 1962, through April 22, 1997										
00060	Discharge (cubic feet per second)	168	--	--	15,000	70	1,420	1,900	1,620	987	514
00900	Hardness, total (mg/L as $CaCO_3$)	167	5.4	31	390	100	210	43	240	210	180
00930	Sodium, dissolved (mg/L as Na)	168	5.4	31	15	2.3	5.8	2.4	6.6	5.1	4.3
00945	Sulfate, dissolved (mg/L as SO_4)	168	5.4	31	120	7.0	37	24	50	32	18
00940	Chloride, dissolved (mg/L as Cl)	168	5.4	31	12	0	4.6	2.4	5.6	3.9	3.0
70301	Dissolved solids, calculated, sum of constituents (mg/L)	167	5.4	31	474	68	237	53	270	232	202

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Red River of the North at Grand Forks, N. Dak., gaging station 05082500, June 22, 1949, through August 23, 1999										
00060	Discharge (cubic feet per second)	532	--	--	105,000	156	5,620	9,900	5,460	2,120	1,160
00900	Hardness, total (mg/L as $CaCO_3$)	532	11.8	45	830	120	250	67	280	250	220
00930	Sodium, dissolved (mg/L as Na)	531	11.8	45	43	2.9	18	6.4	20	17	14
00945	Sulfate, dissolved (mg/L as SO_4)	464	10.3	45	200	18	74	30	95	70	51
00940	Chloride, dissolved (mg/L as Cl)	251	5.6	45	34	0.1	10	5.0	12	9.1	6.9
70301	Dissolved solids, calculated, sum of constituents (mg/L)	248	5.5	45	1,890	166	327	121	367	313	277
	Turtle River at Manvel, N. Dak., gaging station 05083000, October 15, 1971, through April 13, 1990										
00060	Discharge (cubic feet per second)[1]	39	--	--	1,450	0.20	217	340	338	51	8.4
00900	Hardness, total (mg/L as $CaCO_3$)	39	3.0	13	1,800	210	590	330	730	520	340
00930	Sodium, dissolved (mg/L as Na)	39	3.0	13	2,100	63	390	380	460	280	180
00945	Sulfate, dissolved (mg/L as SO_4)	39	3.0	13	1,600	100	390	270	420	330	240
00940	Chloride, dissolved (mg/L as Cl)	39	3.0	13	3,600	89	610	650	730	430	280
70301	Dissolved solids, calculated, sum of constituents (mg/L)	39	3.0	13	8,120	449	1,750	1,410	2,020	1,370	927
	Forest River at Minto, N. Dak., gaging station 05085000, October 12, 1971, through August 16, 1999										
00060	Discharge (cubic feet per second)	91	--	--	6,140	0.10	203	725	64	16	4.6
00900	Hardness, total (mg/L as $CaCO_3$)	91	3.1	29	850	110	330	100	370	330	290
00930	Sodium, dissolved (mg/L as Na)	91	3.1	29	170	6	41	19	48	39	32
00945	Sulfate, dissolved (mg/L as SO_4)	91	3.1	29	290	36	150	50	170	140	120
00940	Chloride, dissolved (mg/L as Cl)	91	3.1	29	270	1.3	32	31	38	24	18
70301	Dissolved solids, calculated, sum of constituents (mg/L)	91	3.1	29	1,370	152	496	157	554	496	428
	Snake River at Warren, Minn., gaging station 05085500, September 16, 1953, through July 29, 1955										
00060	Discharge (cubic feet per second)	15	--	--	206	0.10	29	56	17	8.0	5.6
00900	Hardness, total (mg/L as $CaCO_3$)	16	5.3	3	440	130	360	89	430	400	320
00930	Sodium, dissolved (mg/L as Na)	16	5.3	3	83	7.4	25	19	28	18	16
00945	Sulfate, dissolved (mg/L as SO_4)	16	5.3	3	200	57	140	34	150	140	120
00940	Chloride, dissolved (mg/L as Cl)	16	5.3	3	93	0.5	14	23	13	5.8	2.8
70301	Dissolved solids, calculated, sum of constituents (mg/L)	15	7.5	2	684	195	492	126	574	518	438

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

					Descriptive statistics				Percentage of samples in which values were less than or equal to those shown		
Parameter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Maximum	Minimum	Mean	Standard deviation	75	50 (Median)	25
	Middle River at Argyle, Minn., gaging station 05087500, April 12, 1954, through September 24, 1968										
00060	Discharge (cubic feet per second)	23	--	--	122	0.10	44	39	65	28	13
00900	Hardness, total (mg/L as $CaCO_3$)	23	7.7	3	470	120	290	81	340	310	240
00930	Sodium, dissolved (mg/L as Na)	23	7.7	3	14	4.3	7.7	2.6	9.6	7.6	5.8
00945	Sulfate, dissolved (mg/L as SO_4)	23	7.7	3	89	25	64	18	78	64	49
00940	Chloride, dissolved (mg/L as Cl)	23	7.7	3	11	0	3.0	2.9	4.5	2.0	1.0
70301	Dissolved solids, calculated, sum of constituents (mg/L)	23	7.7	3	533	180	357	89	424	367	306
	Park River at Grafton, N. Dak., gaging station 05090000, September 22, 1969, through August 16, 1999										
00060	Discharge (cubic feet per second)	80	--	--	4,840	0.01	274	790	170	6.0	0.78
00900	Hardness, total (mg/L as $CaCO_3$)	80	2.7	30	610	120	320	120	410	320	240
00930	Sodium, dissolved (mg/L as Na)	80	2.7	30	370	12	100	69	130	98	46
00945	Sulfate, dissolved (mg/L as SO_4)	80	2.7	30	420	53	210	89	260	210	130
00940	Chloride, dissolved (mg/L as Cl)	80	2.7	30	410	6.2	97	79	130	81	31
70301	Dissolved solids, calculated, sum of constituents (mg/L)	80	2.7	30	1,430	193	666	284	826	718	401
	Red River of the North at Drayton, N. Dak., gaging station 05092000, May 9, 1972, through August 17, 1999										
00060	Discharge (cubic feet per second)	55	--	--	91,000	169	12,300	17,700	21,300	2,900	1,430
00900	Hardness, total (mg/L as $CaCO_3$)	55	2.0	28	480	120	250	69	290	240	210
00930	Sodium, dissolved (mg/L as Na)	55	2.0	28	130	7.3	39	28	46	30	22
00945	Sulfate, dissolved (mg/L as SO_4)	55	2.0	28	220	35	89	36	120	86	64
00940	Chloride, dissolved (mg/L as Cl)	55	2.0	28	160	4.8	46	39	52	31	20
70301	Dissolved solids, calculated, sum of constituents (mg/L)	55	2.0	28	878	169	386	137	442	367	298
	Pembina River at Neche, N. Dak., gaging station 05100000, May 9, 1972, through August 12, 1999										
00060	Discharge (cubic feet per second)	53	--	--	10,300	5.6	1,050	1,910	1,100	164	61
00900	Hardness, total (mg/L as $CaCO_3$)	53	1.9	28	570	98	280	110	370	280	200
00930	Sodium, dissolved (mg/L as Na)	53	1.9	28	59	19	39	12	49	41	28
00945	Sulfate, dissolved (mg/L as SO_4)	53	1.9	28	250	56	150	48	190	160	110
00940	Chloride, dissolved (mg/L as Cl)	53	1.9	28	34	3.4	14	6.9	18	14	7.8
70301	Dissolved solids, calculated, sum of constituents (mg/L)	53	1.9	28	761	204	447	144	572	498	318

Table 1. Statistical summaries of selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[mg/L, milligrams per liter; --, no data]

Param-eter code	Property or constituent	Number of samples	Average annual sampling frequency	Number of sample years	Descriptive statistics: Maximum	Minimum	Mean	Standard deviation	Percentage of samples in which values were less than or equal to those shown: 75	50 (Median)	25
	Tongue River at Akra, N. Dak., gaging station 05101000, May 11, 1972, through August 12, 1999										
00060	Discharge (cubic feet per second)	77	--	--	659	0.02	64	131	38	8.2	2.4
00900	Hardness, total (mg/L as $CaCO_3$)	78	2.8	28	350	100	240	56	270	240	200
00930	Sodium, dissolved (mg/L as Na)	78	2.8	28	37	11	26	5.6	29	26	22
00945	Sulfate, dissolved (mg/L as SO_4)	78	2.8	28	140	5.6	84	22	97	87	72
00940	Chloride, dissolved (mg/L as Cl)	78	2.8	28	40	2.2	10	4.5	12	9.6	7.8
70301	Dissolved solids, calculated, sum of constituents (mg/L)	78	2.8	28	505	170	339	67	381	342	309
	Red River of the North at Emerson, Manitoba, gaging station 05102500, July 30, 1977, through February 4, 1999										
00060	Discharge (cubic feet per second)	143	--	--	62,800	222	5,200	8,890	4,590	1,780	1,000
00900	Hardness, total (mg/L as $CaCO_3$)	142	6.4	22	500	160	280	57	310	280	250
00930	Sodium, dissolved (mg/L as Na)	142	6.4	22	190	7.5	44	30	50	34	28
00945	Sulfate, dissolved (mg/L as SO_4)	143	6.4	22	230	6.0	97	39	120	94	70
00940	Chloride, dissolved (mg/L as Cl)	143	6.4	22	240	9.8	50	43	62	35	24
70301	Dissolved solids, calculated, sum of constituents (mg/L)	142	6.4	22	1,060	197	435	131	477	413	355

[1]Discharge is combined instantaneous and daily mean streamflow.

[2]Discharge is instantaneous streamflow.

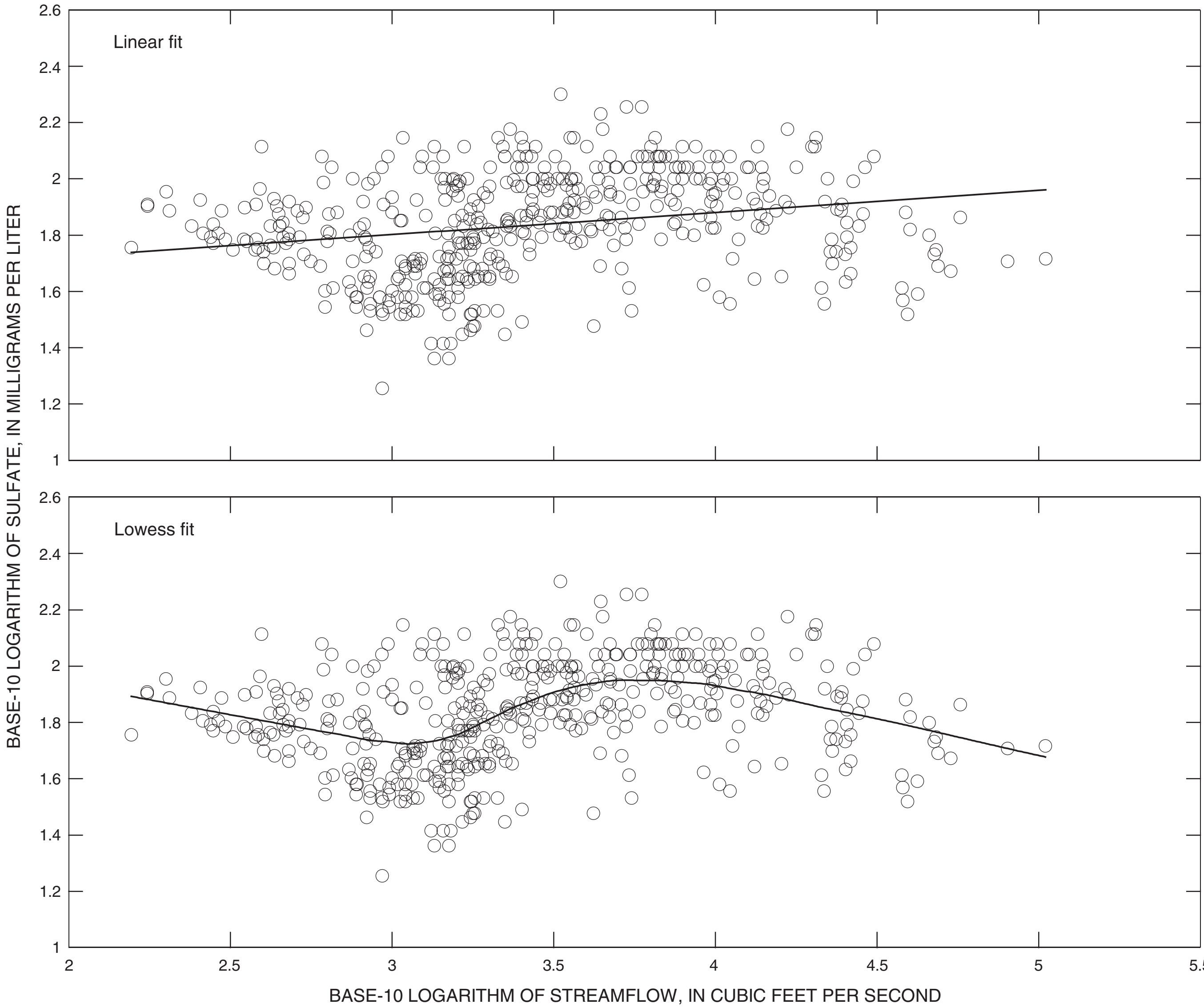

Figure 3. Linear and lowess relations between sulfate and streamflow for the Red River of the North at Grand Forks, North Dakota, gaging station.

where

log is the base-10 logarithm;
C is concentration, in milligrams per liter;
Q is streamflow, in cubic feet per second;
$a_0, \ldots, a_3$ are regression coefficients, where a_k is the value of $\log C$ when $\log Q = b_k$; and
$b_0 < b_1 < b_2 < b_3$ are fixed values for $\log Q$, indicating intervals for which the linear equations are used to compute $\log C$.

The values of b_1 and b_2 correspond to the two breakpoints at which slope changes occur and the values of b_0 and b_3 correspond to the minimum and maximum values of $\log Q$ for which the regression equation should be used to compute concentration.

If $\log Q$ is less than b_0 or greater than b_3, the regression equation should not be used to estimate concentration because the equation should not be used to extrapolate too far outside of the data used to fit the equation. For example, in the lowess fit in figure 3, b_0 might be two and b_3 might be five, and the regression equation could be used with confidence for streamflow ranging from 100 to 100,000 cubic feet per second.

The second equation used to model the relation between concentration and streamflow was a multiple linear regression model that incorporated month and streamflow as explanatory variables to allow the intercept and/or the slope to change depending upon the time of year. This model will be referred to as a linear model with seasonal coefficients and can be expressed as:

$$\log C = a(m) + b(m)\log Q \qquad (2)$$

where

$a(m)$ is the intercept for month m;
$m = 1, 2, \ldots, 12$ designates the month, with $m = 1$ corresponding to January and $m = 12$ to December; and
$b(m)$ is the slope for month m.

Because the historical data for most gaging stations were not sufficient to estimate a separate intercept and slope for every month, the coefficients were expressed as linear combinations of various trigonometric functions of the month (Helsel and Hirsch, 1992).

The intercept and the slope were allowed to take different forms depending upon the constituent or gaging station being analyzed. If no seasonality was detected in the intercept or the slope, the regression coefficients in equation 2 would be:

$$a(m) = a_o$$

and

$$b(m) = b_o$$

where

a_o and b_o are regression coefficients.

If significant seasonality was detected in the slope but not the intercept, the regression coefficient in equation 2 would be:

$$a(m) = a_o$$

and

$$b(m) = b_o + b_1\cos\left(\frac{2\pi m}{12}\right) + b_2\sin\left(\frac{2\pi m}{12}\right)$$

where

a_o, b_o, b_1, and b_2 are regression coefficients.

If seasonality was detected in the intercept and the slope, the regression coefficient in equation 2 would be:

$$a(m) = a_o + a_1\cos\left(\frac{2\pi m}{12}\right) + a_2\sin\left(\frac{2\pi m}{12}\right)$$

and

$$b(m) = b_o + b_1\cos\left(\frac{2\pi m}{12}\right) + b_2\sin\left(\frac{2\pi m}{12}\right)$$

where

a_o, a_1, a_2, b_o, b_1, and b_2 are regression coefficients.

If seasonality was detected in the intercept but not the slope, the regression coefficient in equation 2 would be:

$$a_o + a_1\cos\left(\frac{2\pi m}{12}\right) + a_2\sin\left(pd\frac{2\pi m}{12}\right) + a_3\cos\left(\frac{2\pi m}{6}\right) + a_4\sin\left(\frac{2\pi m}{6}\right)$$

and

$$b(m) = b_o$$

where

$a_o, \ldots, a_4, b_o$ are regression coefficients.

The piece-wise linear model and the linear model with the various forms for the intercept and slope were fitted using available data for 1931-99. Regression equations could not be developed for the Maple River below Mapleton, N. Dak., (05060100); the Buffalo River near Dilworth, Minn., (05062000); the Elm River near Kelso, N. Dak., (05062200); and the Sand Hill River near Climax, Minn., (05069000) gaging stations because of too few available data. The Otter Tail River below Orwell Dam near Fergus Falls, Minn., (05046000) gaging station had eight data pairs sampled during the 1960's that possibly were of a different data population or in error. A scatter plot of the data showed the eight data pairs grouped separately from the rest of the data. The eight data pairs were removed prior to regression runs. Instantaneous streamflow and daily mean streamflow were combined to complete the discharge records at the Red River of the North below Fargo, N. Dak., (05054020); the Sheyenne River at Valley City,

N. Dak., (05058500); and the Turtle River at Manvel, N. Dak., (05083000) gaging stations.

Each of the models defined by equations 1 and 2 can be expressed as a multiple linear regression model with $\log C$ as the response variable and various functions of $\log Q$ and month as explanatory variables. The regression coefficients were estimated using the ordinary-least-squares multiple linear regression. The best equation for each water-quality constituent concentration at each gaging station was selected based on review of the residual plots, coefficient of determination, residual standard error of the regression, p-value, and F-statistic. The residual plots were evaluated to determine if the regression resulted in equal variance of the residuals for the full range of predicted values. Maximization of the coefficient of determination and F-statistic and minimization of the residual standard error of the regression and p-value were desired for the best equation. A step-wise approach was used for equations 1 and 2 for each water-quality constituent concentration at each gaging station, and the best equation was selected based on residual variance and maximization and minimization of the various regression statistics. If no clear choice could be made, the simplest equation form was selected.

Final regression equations developed between water-quality constituent concentrations and streamflow for each gaging station are listed in table 2. The number of data pairs used in the equation development, p-value, coefficient of determination (R^2), residual standard error, and range of standard error of actual concentration also are listed in table 2. The p-value is the probability that the tabulated F-statistic is greater than the calculated F-statistic and is desired to be as close to zero as possible. The F-statistic shows how well the regression equation explains the variance. The p-values ranged from zero to 0.244. The coefficient of determination is the percentage of the total variation in the base-10 logarithm of water-quality constituent concentration that is explained by the regression equation. Coefficient of determination ranged from 0.03 to 0.97. The residual standard error is the estimated standard deviation of the residuals (the deviations of actual log-transformed concentrations from fitted log-transformed concentrations). The residual standard error ranged from 0.019 to 0.310.

A small standard error indicates that the difference between the fitted and actual values of log-transformed concentration tend to be small. In particular, about 70 percent of the fitted values of log-transformed concentrations will be within plus or minus one standard error of the actual values. Equivalently, about 70 percent of the fitted values for untransformed concentrations (obtained by taking the anti-log of the fitted values for log-transformed concentrations) will satisfy the following inequality:

$$10^{-s} - 1 < \frac{\hat{c} - c}{c} < 10^{s} - 1 \qquad (3)$$

where

s is the residual standard error of the regression equation for log-transformed concentration,

$\hat{c}$ is the fitted concentration using the regression equation, and

c is the actual concentration.

Thus, about 70 percent of the fitted concentrations will be within $100(10^{-s} - 1)$ percent and $100(10^{s} - 1)$ percent of the actual concentrations. The standard error of the estimate, as a percentage of the actual concentration, is given in table 2 for the cases when concentration is underestimated, $100(10^{-s} - 1)$, or overestimated, $100(10^{s} - 1)$.

Because the range of standard error of actual concentration shows how far off the model predictive values can be from the fitted values, it is desirable to have a narrow range of standard error of actual concentration. Ranges of the standard error of actual concentration vary considerably by station and by water-quality constituent (table 2).

Equation Applications and Limitations

Because the equations in table 2 were developed by either using mean daily streamflow or by using a combination of instantaneous and mean daily streamflow and because of the possible equation usage, water-quality load calculations may be determined various ways. Equations will have very poor predictive ability and large error when using regression equations with very low R^2 (less than 0.20), with high p-values, with high residual standard error values, and with a broad range of standard error of actual concentration (such as -40 to 68). An alternative to such poor regressions would be to use the mean value (table 1) for the water-quality constituent being considered.

The following equation may be used to obtain monthly flow-weighted water-quality concentrations (FWC):

$$\mathrm{FWC} = \frac{\sum_{i=1}^{n} c_i q_i}{\sum_{i=1}^{n} q_i} \; x \; \mathrm{BCF} \qquad (4)$$

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm			Range of standard error of actual concentration (percent)
			p-value	Coefficient of determination (R^2)	Residual standard error	
Otter Tail River below Orwell Dam near Fergus Falls, Minn., gaging station 05046000 Range of fit: 208 ≤ Q ≤ 1,220 (without 1960's data)						
$\log CaCO_3 = 2.37 - (0.015 - 0.006C_1 - 0.008S_1)\log Q$		29	3.18×10^{-4}	0.52	0.019	-4.2 to 4.4
$Na = 130Q^{-0.445}$ $Na = 1.68Q^{0.262}$ $Na = 1{,}580Q^{-0.782}$ $Na = 1.68Q^{0.229}$ $Na = 189Q^{-0.452}$	$208 \le Q \le 468$ $468 < Q \le 708$ $708 < Q \le 871$ $871 < Q \le 1{,}020$ $1{,}020 < Q \le 1{,}220$	27	5.05×10^{-7}	0.81	0.022	-4.9 to 5.1
$\log SO_4 = 0.887 + (0.084 - 0.024C_1 + 0.038S_1)\log Q$		32	0.001	0.42	0.108	-22 to 28
$\log Cl = 1.63 - (0.227 - 0.020C_1 - 0.017S_1)\log Q$		31	1.66×10^{-8}	0.76	0.043	-9.4 to 10
$\log DS = 2.43 - (0.021 - 0.005C_1 - 0.008S_1)\log Q$		28	0.001	0.50	0.020	-4.5 to 4.7
Bois de Sioux River near White Rock, S. Dak., gaging station 05050000 Range of fit: 0.20 ≤ Q ≤ 869						
$\log CaCO_3 = 2.95 - (0.136 + 0.041C_1 + 0.061S_1)\log Q + 0.041C_1 + 0.176S_1$		12	0.010	0.88	0.063	-14 to 16
$\log Na = 1.92 - 0.128 \log Q + 0.034C_1 + 0.034S_1 + 0.030C_2 + 0.068S_2$		14	0.004	0.85	0.101	-21 to 26
$\log SO_4 = 2.82 - 0.136 \log Q + 0.013 C_1 + 0.090S_1 + 0.022C_2 + 0.081S_2$		12	0.060	0.77	0.115	-23 to 30
$\log Cl = 1.22 - 0.064\log Q + 0.126C_1 - 0.083S_1 + 0.081C_2 + 0.046S_2$		14	6.90×10^{-5}	0.95	0.055	-12 to 14
$\log DS = 3.15 - (0.145 + 0.017C_1 + 0.071S_1)\log Q + 0.007C_1 + 0.180S_1$		12	0.016	0.86	0.073	-15 to 18
Red River of the North at Wahpeton, N. Dak., gaging station 05051500 Range of fit: 35 ≤ Q ≤ 11,000						
$\log CaCO_3 = 2.68 - 0.024\log Q + 0.332C_1 + 0.019S_1 + 0.218C_2 + 0.024S_2$		50	0.085	0.19	0.088	-18 to 22
$\log Na = 1.41 - (0.116 - 0.015C_1 - 0.028S_1)\log Q$		51	0.065	0.14	0.157	-30 to 44
$SO_4 = 250Q^{-0.407}$ $SO_4 = 0.563Q^{0.696}$ $SO_4 = 308Q^{-0.146}$	$35 \le Q \le 251$ $251 < Q \le 1{,}780$ $1{,}780 < Q \le 11{,}000$	48	8.81×10^{-6}	0.44	0.232	-41 to 70
$\log Cl = 0.814 + (0.062 + 0.211C_1 - 0.236S_1)\log Q - 0.678C_1 + 0.734S_1$		45	0.008	0.32	0.174	-33 to 49
$\log DS = 2.56 - (0.041 - 0.006C_1 - 0.019S_1)\log Q$		49	0.008	0.23	0.071	-15 to 18

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm			Range of standard error of actual concentration (percent)
			p-value	Coefficient of determination (R^2)	Residual standard error	
Red River of the North at Hickson, N. Dak., gaging station 05051522 **Range of fit: $3.0 \le Q \le 12{,}000$**						
$CaCO_3 = 760Q^{-0.218}$	$3.0 \le Q \le 126$	115	8.85×10^{-14}	0.45	0.080	-17 to 20
$CaCO_3 = 344Q^{-0.054}$	$126 < Q \le 708$					
$CaCO_3 = 56.8Q^{0.221}$	$708 < Q \le 2{,}510$					
$CaCO_3 = 27{,}910Q^{-0.571}$	$2{,}510 < Q \le 12{,}000$					
$Na = 39.4Q^{-0.177}$	$3.0 \le Q \le 631$	114	2.59×10^{-6}	0.23	0.141	-28 to 38
$Na = 2.78Q^{0.234}$	$631 < Q \le 2{,}510$					
$Na = 1{,}090Q^{-0.528}$	$2{,}510 < Q \le 12{,}000$					
$SO_4 = 379Q^{-0.379}$	$3.0 \le Q \le 501$	115	7.05×10^{-8}	0.28	0.286	-48 to 93
$SO_4 = 0.142Q^{0.891}$	$501 < Q \le 2{,}510$					
$SO_4 = 10{,}050Q^{-0.536}$	$2{,}510 < Q \le 12{,}000$					
$Cl = 24.1Q^{-0.206}$	$3.0 \le Q \le 126$	114	0.115	0.05	0.225	-40 to 68
$Cl = 6.83Q^{0.054}$	$126 < Q \le 2{,}510$					
$Cl = 261Q^{-0.411}$	$2{,}510 < Q \le 12{,}000$					
$DS = 6{,}670Q^{-0.931}$	$3.0 \le Q \le 22.4$	111	6.90×10^{-13}	0.44	0.091	-19 to 23
$DS = 462Q^{-0.072}$	$22.4 < Q \le 501$					
$DS = 113Q^{0.155}$	$501 < Q \le 2{,}510$					
$DS = 12{,}960Q^{-0.451}$	$2{,}510 < Q \le 12{,}000$					
Wild Rice River near Abercrombie, N. Dak., gaging station 05053000 **Range of fit: $0.01 \le Q \le 9{,}250$**						
$\log CaCO_3 = 2.76 - (0.088 + 0.041C_1 + 0.125S_1)\log Q + 0.061C_1 + 0.138S_1$		275	0	0.50	0.167	-32 to 47
$\log Na = 2.14 - (0.134 + 0.012C_1 + 0.166S_1)\log Q + 0.028C_1 + 0.177S_1$		275	0	0.50	0.236	-42 to 72
$\log SO_4 = 2.64 - (0.102 + 0.021C_1 + 0.171S_1)\log Q + 0.009C_1 + 0.206S_1$		238	0	0.46	0.233	-41 to 71
$\log Cl = 1.73 - (0.111 - 0.039C_1 + 0.082S_1)\log Q$		222	0	0.47	0.230	-41 to 70
$\log DS = 3.06 - (0.118 + 0.027C_1 + 0.154S_1)\log Q + 0.022C_1 + 0.192S_1$		222	0	0.60	0.164	-32 to 46

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm: p-value	In base-10 logarithm: Coefficient of determination (R^2)	In base-10 logarithm: Residual standard error	Range of standard error of actual concentration (percent)
Red River of the North at Fargo, N. Dak., gaging station 05054000						
Range of fit: $0 \le Q \le 25{,}800$ for Na and $CaCO_3$ and $14 \le Q \le 25{,}800$ for SO_4, Cl, and DS						
$\log CaCO_3 = 2.55 - (0.061 + 0.052C_1 + 0.092S_1)\log Q + 0.129C_1 + 0.256S_1$		501	0	0.27	0.082	-17 to 21
$Na = 10.5Q^{0.093}$	$0 \le Q \le 56.2$	498	0	0.18	0.139	-27 to 38
$Na = 28.9Q^{-0.159}$	$56.2 < Q \le 251$					
$Na = 2.74Q^{0.267}$	$251 < Q \le 1{,}580$					
$Na = 290Q^{-0.365}$	$1{,}580 < Q \le 25{,}800$					
$SO_4 = 157Q^{-0.269}$	$14 \le Q \le 200$	437	0	0.36	0.202	-37 to 59
$SO_4 = 2.21Q^{0.536}$	$200 < Q \le 1{,}580$					
$SO_4 = 1{,}025Q^{-0.298}$	$1{,}580 < Q \le 25{,}800$					
$\log Cl = 0.956 - 0.076\log Q - 0.025C_1 + 0.052S_1 - 0.056C_2 - 0.014S_2$ (Alternate option: use mean value of 7.02 instead of equation.)		235	0.171	0.03	0.310	-51 to 104
$DS = 234Q^{0.082}$	$14 \le Q \le 70.8$	233	4.84×10^{-13}	0.24	0.094	-19 to 24
$DS = 538Q^{-0.114}$	$70.8 < Q \le 251$					
$DS = 142Q^{0.128}$	$251 < Q \le 1{,}580$					
$DS = 3{,}620Q^{-0.312}$	$1{,}580 < Q \le 25{,}800$					
Red River of the North below Fargo, N. Dak., gaging station 05054020						
Range of fit: $2.0 \le Q \le 17{,}300$						
$\log CaCO_3 = 2.55 - (0.057 + 0.054C_1 + 0.031S_1)\log Q + 0.132C_1 + 0.100S_1$		178	2.36×10^{-8}	0.22	0.064	-14 to 16
$\log Na = 2.09 - (0.323 + 0.198C_1 - 0.002S_1)\log Q + 0.418C_1 + 0.038S_1$		149	0	0.64	0.135	-27 to 37
$\log SO_4 = 2.14 - (0.143 + 0.303C_1 - 0.099S_1)\log Q + 0.631C_1 - 0.245S_1$		177	3.91×10^{-14}	0.34	0.213	-39 to 63
$\log Cl = 1.90 - (0.332 + 0.139C_1 - 0.022S_1)\log Q + 0.322C_1 + 0.005S_1$		167	0	0.59	0.150	-29 to 41
$\log DS = 2.84 - (0.125 + 0.109C_1 + 0.026S_1)\log Q + 0.241C_1 + 0.093S_1$		133	2.39×10^{-14}	0.43	0.084	-17 to 21
Sheyenne River above Harvey, N. Dak., gaging station 05054500						
Range of fit: $0.19 \le Q \le 690$						
$\log CaCO_3 = 2.19 + (0.128 - 0.083C_1 - 0.144S_1)\log Q + 0.038C_1 + 0.033S_1$		178	2.40×10^{-9}	0.24	0.165	-32 to 46
$\log Na = 2.48 - (0.184 - 0.002C_1 + 0.139S_1)\log Q - 0.012C_1 + 0.030S_1$		177	0	0.69	0.135	-27 to 37
$SO_4 = 198Q^{0.099}$	$0.19 \le Q \le 7.9$	177	0	0.38	0.143	-28 to 39
$SO_4 = 174Q^{0.163}$	$7.9 < Q \le 20$					
$SO_4 = 1{,}360Q^{-0.523}$	$20 < Q \le 690$					
$\log Cl = 1.29 - (0.084 - 0.018C_1 + 0.113S_1)\log Q + 0.007C_1 + 0.040S_1$		178	0	0.40	0.146	-29 to 40
$\log DS = 2.99 - 0.117\log Q + 0.007C_1 - 0.038S_1 + 0.092C_2 - 0.027S_2$		177	0	0.58	0.106	-22 to 28

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; *Q*, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation	Number of data pairs	In base-10 logarithm			Range of standard error of actual concentration (percent)
		p-value	Coefficient of determination (R^2)	Residual standard error	
Sheyenne River near Warwick, N. Dak., gaging station 05056000 Range of fit: 0.10 ≤ *Q* ≤ 2,310 for Na and $CaCO_3$ and 0.26 ≤ *Q* ≤ 2,310 for SO_4, Cl, and DS					
$\log CaCO_3 = 2.44 - (0.043 + 0.013C_1 + 0.126S_1)\log Q + 0.001C_1 + 0.138S_1$	529	0	0.45	0.106	-22 to 28
$\log Na = 1.69 + (0.142 + 0.041C_1 - 0.233S_1)\log Q - 0.126C_1 + 0.137S_1$	529	0	0.36	0.216	-39 to 64
$\log SO_4 = 1.89 + (0.066 - 0.022C_1 - 0.155S_1)\log Q - 0.066C_1 + 0.118S_1$	275	0	0.32	0.171	-33 to 48
$\log Cl = 1.04 + (0.068 + 0.046C_1 - 0.234S_1)\log Q - 0.104C_1 + 0.255S_1$	245	0	0.40	0.176	-33 to 50
$\log DS = 2.67 + (0.012 - 0.005C_1 - 0.172S_1)\log Q - 0.048C_1 + 0.155S_1$	229	0	0.49	0.121	-24 to 32
Sheyenne River near Cooperstown, N. Dak., gaging station 05057000 Range of fit: 0.02 ≤ *Q* ≤ 4,150					
$\log CaCO_3 = 2.60 - (0.093 + 0.100C_1 + 0.130S_1)\log Q + 0.147C_1 + 0.161S_1$	366	0	0.63	0.095	-20 to 24
$\log Na = 2.05 - 0.106\log Q - 0.017C_1 - 0.050S_1 + 0.095C_2 + 0.023S_2$	365	0	0.40	0.151	-29 to 42
$\log SO_4 = 2.24 - (0.070 + 0.110C_1 + 0.091S_1)\log Q + 0.137C_1 + 0.081S_1$	319	0	0.41	0.118	-24 to 31
$\log Cl = 1.35 - (0.105 + 0.077C_1 + 0.116S_1)\log Q + 0.118C_1 + 0.146S_1$	314	0	0.60	0.114	-23 to 30
$\log DS = 2.87 - (0.089 + 0.079C_1 + 0.123S_1)\log Q + 0.089C_1 + 0.144S_1$	305	0	0.61	0.099	-20 to 26
Baldhill Creek near Dazey, N. Dak., gaging station 05057200 Range of fit: 0.01 ≤ *Q* ≤ 2,450					
$\log CaCO_3 = 2.57 - (0.091 + 0.070C_1 + 0.140S_1)\log Q + 0.042C_1 + 0.049S_1$	84	0	0.85	0.088	-18 to 22
$\log Na = 1.86 - (0.206 + 0.057C_1 + 0.147S_1)\log Q$	84	0	0.85	0.157	-30 to 44
$\log SO_4 = 2.29 - (0.158 + 0.112C_1 + 0.106S_1)\log Q$	84	0	0.78	0.138	-27 to 37
$\log Cl = 1.32 - (0.171 + 0.037C_1 + 0.065S_1)\log Q$	95	0	0.68	0.179	-34 to 51
$\log DS = 2.80 - (0.116 + 0.057C_1 + 0.116S_1)\log Q + 0.024C_1 + 0.020S_1$	84	0	0.85	0.094	-19 to 24

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; *Q*, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm			Range of standard error of actual concentration (percent)
			p-value	Coefficient of determination (R^2)	Residual standard error	
Sheyenne River below Baldhill Dam, N. Dak., gaging station 05058000						
Range of fit: 3.6 ≤ *Q* ≤ 4,710						
$\log CaCO_3 = 2.36 - 0.012\log Q - 0.003C_1 + 0.070S_1 - 0.048C_2 + 0.034S_2$		111	5.13×10^{-9}	0.36	0.084	-18 to 21
$\log Na = 1.85 - 0.046\log Q + 0.009C_1 + 0.074S_1 - 0.062C_2 + 0.039S_2$		111	4.73×10^{-7}	0.30	0.114	-23 to 30
$\log SO_4 = 2.06 - 0.005\log Q - 0.016C_1 + 0.063S_1 - 0.053C_2 + 0.047S_2$		111	1.27×10^{-4}	0.21	0.119	-24 to 31
$\log Cl = 1.20 - 0.049\log Q + 0.016C_1 + 0.079S_1 - 0.058C_2 + 0.024S_2$		121	4.40×10^{-8}	0.31	0.114	-23 to 30
$\log DS = 2.65 - 0.021\log Q + 0.001C_1 + 0.067S_1 - 0.055C_2 + 0.036S_2$		111	3.70×10^{-8}	0.33	0.090	-19 to 23
Sheyenne River at Valley City, N. Dak., gaging station 05058500						
Range of fit: 9.1 ≤ *Q* ≤ 3,520						
$CaCO_3 = 106Q^{0.314}$ $CaCO_3 = 1,160Q^{-0.264}$ $CaCO_3 = 53.1Q^{0.252}$ $CaCO_3 = 148,800Q^{-0.860}$	$9.1 \le Q \le 63.1$ $63.1 < Q \le 398$ $398 < Q \le 1,260$ $1,260 < Q \le 3,520$	22	2.65×10^{-5}	0.77	0.057	-12 to 14
$\log Na = 2.38 - (0.440 + 0.122C_1 - 0.416S_1)\log Q + 0.229C_1 - 0.422S_1$		22	5.68×10^{-4}	0.72	0.082	-17 to 21
$\log SO_4 = 2.30 - (0.103 - 0.045C_1 - 0.064S_1)\log Q$		23	4.68×10^{-4}	0.60	0.074	-16 to 19
$\log Cl = 1.43 - (0.156 - 0.026C_1 - 0.095S_1)\log Q$		22	2.64×10^{-5}	0.73	0.065	-14 to 16
$DS = 208Q^{0.297}$ $DS = 1,860Q^{-0.232}$ $DS = 117Q^{0.229}$ $DS = 576,600Q^{-0.961}$	$9.1 \le Q \le 63.1$ $63.1 < Q \le 398$ $398 < Q \le 1,260$ $1,260 < Q \le 3,520$	22	1.37×10^{-5}	0.79	0.059	-13 to 15
Sheyenne River at Lisbon, N. Dak., gaging station 05058700						
Range of fit: 0.40 ≤ *Q* ≤ 4,190						
$\log CaCO_3 = 2.62 - 0.098\log Q + 0.021C_1 + 0.016S_1 + 0.031C_2 + 0.008S_2$		608	0	0.41	0.081	-17 to 20
$\log Na = 2.17 - 0.165\log Q + 0.040C_1 + 0.015S_1 + 0.033C_2 + 0.020S_2$		608	0	0.50	0.116	-24 to 31
$\log SO_4 = 2.49 - (0.153 + 0.048S_1)\log Q - 0.020C_1 + 0.099S_1$		544	0	0.38	0.126	-25 to 34
$\log Cl = 1.94 - 0.260\log Q + 0.026C_1 + 0.011S_1 + 0.020C_2 - 0.001S_2$		330	0	0.62	0.134	-27 to 36
$\log DS = 2.92 - 0.106\log Q + 0.027C_1 + 0.013S_1 + 0.025C_2 + 0.025S_2$		327	0	0.45	0.087	-18 to 22

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; *Q*, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation	Number of data pairs	In base-10 logarithm: p-value	In base-10 logarithm: Coefficient of determination (R^2)	In base-10 logarithm: Residual standard error	Range of standard error of actual concentration (percent)
Sheyenne River near Kindred, N. Dak., gaging station 05059000 Range of fit: $19 \le Q \le 4,070$					
$\log CaCO_3 = 2.67 - (0.088 - 0.008C_1 + 0.062S_1)\log Q - 0.001C_1 + 0.133S_1$	220	0	0.43	0.073	-15 to 18
$\log Na = 1.98 - (0.074 - 0.046C_1 + 0.094S_1)\log Q - 0.034C_1 + 0.209S_1$	220	0	0.39	0.107	-22 to 28
$\log SO_4 = 2.29 - (0.044 - 0.001C_1 + 0.102S_1)\log Q + 0.018C_1 + 0.220S_1$	222	1.53×10^{-10}	0.22	0.104	-21 to 27
$\log Cl = 2.01 - 0.276\log Q + 0.011C_1 + 0.016S_1 + 0.010C_2 + 0.008S_2$	220	0	0.61	0.121	-24 to 32
$\log DS = 2.90 - (0.082 - 0.007C_1 + 0.068S_1)\log Q + 0.010C_1 + 0.148S_1$	221	0	0.41	0.076	-16 to 19
Sheyenne River at West Fargo, N. Dak., gaging station 05059500 Range of fit: $5.9 \le Q \le 3,050$					
$\log CaCO_3 = 2.49 - (0.004 - 0.125C_1 + 0.155S_1)\log Q - 0.202C_1 + 0.253S_1$	55	1.11×10^{-16}	0.81	0.061	-13 to 15
$\log Na = 1.88 - (0.034 - 0.134C_1 + 0.189S_1)\log Q - 0.195C_1 + 0.349S_1$	55	3.77×10^{-13}	0.74	0.083	-17 to 21
$\log SO_4 = 2.26 - 0.059\log Q + 0.096C_1 - 0.027S_1 + 0.004C_2 + 0.073S_2$	55	4.19×10^{-6}	0.48	0.104	-21 to 27
$\log Cl = 2.06 - (0.297 - 0.038C_1 + 0.004S_1)\log Q$	54	0	0.81	0.097	-20 to 25
$\log DS = 2.84 - (0.055 - 0.079C_1 + 0.165S_1)\log Q - 0.120C_1 + 0.299S_1$	54	1.66×10^{-15}	0.80	0.064	-14 to 16
Maple River near Enderlin, N. Dak., gaging station 05059700 Range of fit: $1.3 \le Q \le 3,500$					
$\log CaCO_3 = 2.92 - (0.185 + 0.067C_1 + 0.052S_1)\log Q$	77	0	0.85	0.098	-20 to 25
$\log Na = 2.03 - (0.100 + 0.002C_1 + 0.188S_1)\log Q - 0.052C_1 + 0.141S_1$	78	0	0.78	0.139	-27 to 38
$\log SO_4 = 2.74 - (0.192 + 0.098C_1 + 0.070S_1)\log Q$	75	0	0.78	0.136	-27 to 37
$\log Cl = 1.90 - (0.218 + 0.049C_1 + 0.044S_1)\log Q$	77	0	0.69	0.179	-38 to 51
$\log DS = 3.13 - (0.179 + 0.099C_1 + 0.084S_1)\log Q + 0.044C_1 + 0.056S_1$	77	0	0.86	0.093	-19 to 24
Maple River below Mapleton, N. Dak., gaging station 05060100 Range of fit: $0.79 \le Q \le 6,620$					
Insufficient data for regression development.					

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm			Range of standard error of actual concentration (percent)
			p-value	Coefficient of determination (R^2)	Residual standard error	
Rush River at Amenia, N. Dak., gaging station 05060500 Range of fit: 0.01 ≤ *Q* ≤ 2,310						
$\log CaCO_3 = 2.77 - (0.140 + 0.117C_1 + 0.113S_1) \log Q$		45	0	0.87	0.096	-20 to 25
$\log Na = 1.82 - (0.237 + 0.138C_1 + 0.149S_1)\log Q - 0.106C_1 + 0.142S_1$		46	0	0.88	0.131	-26 to 35
$\log SO_4 = 2.47 - (0.274 + 0.201C_1 + 0.052S_1)\log Q - 0.177C_1 + 0.105S_1$		44	2.01×10^{-14}	0.85	0.138	-27 to 37
$\log Cl = 1.71 - 0.212\log Q + 0.398C_1 - 0.008S_1 + 0.333C_2 - 0.241S_2$		46	7.19×10^{-9}	0.68	0.222	-40 to 67
$\log DS = 2.97 - (0.145 + 0.121C_1 + 0.124S_1)\log Q$		45	0	0.88	0.101	-21 to 26
Buffalo River near Dilworth, Minn., gaging station 05062000 Range of fit: 6.1 ≤ *Q* ≤ 5,250						
Insufficient data for regression development.						
Elm River near Kelso, N. Dak., gaging station 05062200 Range of fit: 0.01 ≤ *Q* ≤ 705						
Insufficient data for regression development.						
Wild Rice River at Twin Valley, Minn., gaging station 05062500 Range of fit: 2.8 ≤ *Q* ≤ 4,330						
$\log CaCO_3 = 2.62 - (0.091 + 0.055C_1 + 0.062S_1)\log Q + 0.156C_1 + 0.139S_1$		168	0	0.72	0.045	-9.8 to 11
$\log Na = 1.53 - (0.272 + 0.066C_1 + 0.038S_1)\log Q + 0.135C_1 + 0.091S_1$		169	0	0.83	0.077	-16 to 20
$SO_4 = 64.9Q^{-0.159}$	$2.8 \le Q \le 12.6$	168	0	0.42	0.138	-27 to 37
$SO_4 = 115Q^{-0.386}$	$12.6 < Q \le 44.7$					
$SO_4 = 691Q^{-0.857}$	$44.7 < Q \le 70.8$					
$SO_4 = 0.266Q^{0.988}$	$70.8 < Q \le 100$					
$SO_4 = 9.18Q^{0.220}$	$100 < Q \le 794$					
$SO_4 = 29.9Q^{0.043}$	$794 < Q \le 4{,}330$					
$\log Cl = 0.555 + 0.020\log Q + 0.052C_1 + 0.018S_1 - 0.058C_2 + 0.010S_2$ (Alternate option: use mean value of 4.48 instead of equation.)		168	0.006	0.10	0.186	-35 to 53
$\log DS = 2.70 - (0.100 + 0.056C_1 + 0.043S_1)\log Q + 0.163C_1 + 0.112S_1$		168	0	0.73	0.045	-9.9 to 11

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation	Number of data pairs	In base-10 logarithm: p-value	In base-10 logarithm: Coefficient of determination (R^2)	In base-10 logarithm: Residual standard error	Range of standard error of actual concentration (percent)
Wild Rice River at Hendrum, Minn., gaging station 05064000 Range of fit: 0.09 ≤ *Q* ≤ 9,110 for Cl and 4.7 ≤ *Q* ≤ 2,160 for $CaCO_3$, Na, SO_4, and DS					
$CaCO_3 = 628Q^{-0.169}$	7	7.38×10^{-5}	0.97	0.030	-6.6 to 7.1
$Na = 48.6Q^{-0.305}$	7	8.00×10^{-5}	0.96	0.055	-12 to 13
$\log SO_4 = 2.01 - (0.170 + 0.091C_1 + 0.026S_1)\log Q$	8	0.227	0.63	0.114	-23 to 30
$\log Cl = 0.913 - 0.127\log Q + 0.053C_1 + 0.074S_1 - 0.008C_2 - 0.005S_2$	114	1.11×10^{-15}	0.52	0.128	-25 to 34
$DS = 728Q^{-0.165}$	7	1.51×10^{-4}	0.95	0.034	-7.4 to 8.0
Red River of the North at Halstad, Minn., gaging station 05064500 Range of fit: 109 ≤ *Q* ≤ 66,200					
$\log CaCO_3 = 2.75 - 0.098\log Q + 0.032C_1 + 0.014S_1 + 0.047C_2 - 0.019S_2$	163	0	0.43	0.084	-18 to 21
$\log Na = 2.16 - 0.229\log Q + 0.028C_1 + 0.009S_1 + 0.053C_2 - 0.046S_2$	162	0	0.53	0.136	-27 to 37
$\log SO_4 = 2.16 - 0.047\log Q + 0.027C_1 - 0.028S_1 + 0.083C_2 - 0.064S_2$	161	7.76×10^{-7}	0.21	0.159	-31 to 44
$\log Cl = 1.95 - 0.243\log Q + 0.027C_1 + 0.016S_1 + 0.039C_2 - 0.051S_2$	161	0	0.53	0.138	-27 to 38
$\log DS = 2.94 - 0.109\log Q + 0.024C_1 + 0.010S_1 + 0.047C_2 - 0.024S_2$	161	0	0.43	0.087	-18 to 22
Goose River at Hillsboro, N. Dak., gaging station 05066500 Range of fit: 0.07 ≤ *Q* ≤ 5,360					
$\log CaCO_3 = 2.88 - (0.101 + 0.044C_1 + 0.140S_1)\log Q + 0.060C_1 + 0.168S_1$	90	0	0.80	0.100	-20 to 26
$\log Na = 2.29 - (0.239 + 0.072C_1 + 0.167S_1)\log Q + 0.134C_1 + 0.262S_1$	89	0	0.86	0.136	-27 to 37
$\log SO_4 = 2.75 - (0.124 + 0.057C_1 + 0.154S_1)\log Q + 0.075C_1 + 0.194S_1$	90	0	0.78	0.122	-26 to 35
$\log Cl = 2.21 - (0.335 + 0.066C_1 + 0.145S_1)\log Q + 0.149C_1 + 0.315S_1$	91	0	0.86	0.164	-31 to 46
$\log DS = 3.16 - (0.142 + 0.057C_1 + 0.140S_1)\log Q + 0.088C_1 + 0.192S_1$	89	0	0.86	0.095	-20 to 24
Sand Hill River near Climax, Minn., gaging station 05069000 Range of fit: 10 ≤ *Q* ≤ 18 for $CaCO_3$, Na, SO_4, and Cl and 18 ≤ *Q* ≤ 18 for DS					
Insufficient data for regression development.					

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm: p-value	In base-10 logarithm: Coefficient of determination (R^2)	In base-10 logarithm: Residual standard error	Range of standard error of actual concentration (percent)
Clearwater River at Red Lake Falls, Minn., gaging station 05078500						
Range of fit: $40 \leq Q \leq 1,300$						
$\log CaCO_3 = 2.73 - (0.158 + 0.143C_1 + 0.134S_1)\log Q + 0.249C_1 + 0.279S_1$		12	0.244	0.60	0.102	-21 to 26
$\log Na = 1.15 - (0.103 + 0.004C_1 - 0.013S_1)\log Q$		11	0.047	0.66	0.050	-11 to 12
$\log SO_4 = 1.36 + 0.098\log Q - 0.048C_1 - 0.092S_1 + 0.102C_2 - 0.148S_2$		11	0.073	0.80	0.125	-25 to 33
$\log Cl = 0.716 + (0.009 + 0.475C_1 - 0.072S_1)\log Q - 1.02C_1 + 0.266S_1$		11	0.194	0.69	0.078	-16 to 20
$\log DS = 2.49 + (0.008 + 0.018C_1 + 0.038S_1)\log Q + 0.033C_1 - 0.009S_1$		10	0.194	0.76	0.036	-8.0 to 8.6
Red Lake River at Crookston, Minn., gaging station 05079000						
Range of fit: $70 \leq Q \leq 15,000$						
$\log CaCO_3 = 2.62 - (0.106 + 0.079C_1 + 0.044S_1)\log Q + 0.216C_1 + 0.112S_1$		166	3.33×10^{-16}	0.40	0.067	-14 to 17
$\log Na = 1.54 - 0.276\log Q - 0.017C_1 - 0.017S_1 + 0.014C_2 - 0.031S_2$		167	0	0.62	0.100	-21 to 26
$\log SO_4 = 1.80 - 0.115\log Q - 0.195C_1 - 0.96S_1 + 0.032C_2 - 0.163S_2$		166	0	0.43	0.219	-40 to 66
$Cl = 41.6Q^{-0.365}$	$70 \leq Q \leq 631$	165	3.09×10^{-13}	0.33	0.166	-32 to 47
$Cl = 40.1Q^{-0.359}$	$631 < Q \leq 1,260$					
$Cl = 0.147Q^{0.427}$	$1,260 < Q \leq 3,980$					
$Cl = 33.7Q^{-0.229}$	$3,980 < Q \leq 15,000$					
$\log DS = 2.68 - (0.110 + 0.081C_1 + 0.036S_1)\log Q + 0.212C_1 + 0.089S_1$		166	2.33×10^{-15}	0.38	0.072	-15 to 18
Red River of the North at Grand Forks, N. Dak., gaging station 05082500						
Range of fit: $156 \leq Q \leq 105,000$						
$\log CaCO_3 = 2.70 - 0.092\log Q - 0.012C_1 + 0.007S_1 + 0.051C_2 - 0.013S_2$		528	0	0.40	0.066	-14 to 16
$\log Na = 1.88 - (0.203 + 0.116C_1 + 0.048S_1)\log Q + 0.336C_1 + 0.200S_1$		528	0	0.35	0.126	-25 to 34
$\log SO_4 = 1.86 - 0.015\log Q - 0.116C_1 - 0.015S_1 + 0.068C_2 - 0.061S_2$		464	0	0.30	0.153	-30 to 42
$\log Cl = 1.59 - (0.185 + 0.001C_1 - 0.013S_1)\log Q$		249	2.06×10^{-12}	0.21	0.178	-34 to 51
$DS = 1,670Q^{-0.244}$	$156 \leq Q \leq 1,260$	246	0	0.32	0.079	-17 to 20
$DS = 103Q^{0.146}$	$1,260 < Q \leq 5,010$					
$DS = 2,230Q^{-0.215}$	$5,010 < Q \leq 105,000$					

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; $\leq$, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm: p-value	In base-10 logarithm: Coefficient of determination (R^2)	In base-10 logarithm: Residual standard error	Range of standard error of actual concentration (percent)
Turtle River at Manvel, N. Dak., gaging station 05083000						
Range of fit: $0.20 \leq Q \leq 1,450$						
$\log CaCO_3 = 2.94 - (0.097 - 0.006C_1 + 0.075S_1)\log Q$		36	2.33×10^{-10}	0.77	0.104	-21 to 27
$\log Na = 2.83 - (0.163 + 0.016C_1 + 0.104S_1)\log Q$		36	6.02×10^{-9}	0.72	0.172	-33 to 49
$\log SO_4 = 2.72 - (0.080 - 0.013C_1 + 0.078S_1)\log Q$		36	3.93×10^{-7}	0.63	0.136	-27 to 37
$\log Cl = 2.96 - (0.120 + 0.002C_1 + 0.124S_1)\log Q$		37	4.31×10^{-8}	0.67	0.194	-36 to 56
$\log DS = 3.41 - (0.101 - 0.017C_1 + 0.094S_1)\log Q$		38	1.61×10^{-9}	0.72	0.139	-27 to 38
Forest River at Minto, N. Dak., gaging station 05085000						
Range of fit: $0.10 \leq Q \leq 6,140$						
$\log CaCO_3 = 2.63 - (0.083 + 0.058C_1 + 0.097S_1)\log Q + 0.071C_1 + 0.104S_1$		90	0	0.69	0.082	-17 to 21
$Na = 56.0Q^{-0.178}$ $Na = 23.2Q^{0.118}$ $Na = 164Q^{-0.269}$	$0.10 \leq Q \leq 20.0$ $20.0 < Q \leq 158$ $158 < Q \leq 6,140$	88	4.50×10^{-9}	0.39	0.126	-25 to 34
$SO_4 = 149Q^{-0.125}$ $SO_4 = 87.6Q^{0.259}$ $SO_4 = 271Q^{-0.257}$ $SO_4 = 80.6Q^{0.148}$ $SO_4 = 1,440Q^{-0.397}$	$0.10 \leq Q \leq 4.0$ $4.0 < Q \leq 9.0$ $9.0 < Q \leq 20.0$ $20.0 < Q \leq 200$ $200 < Q \leq 6,140$	91	5.70×10^{-6}	0.31	0.138	-27 to 37
$Cl = 52.1Q^{-0.152}$ $Cl = 84.8Q^{-0.505}$ $Cl = 15.7Q^{0.057}$ $Cl = 207Q^{-0.430}$	$0.10 \leq Q \leq 4.0$ $4.0 < Q \leq 20.0$ $20.0 < Q \leq 200$ $200 < Q \leq 6,140$	89	0	0.76	0.122	-25 to 33
$DS = 596Q^{-0.132}$ $DS = 531Q^{-0.033}$ $DS = 2,170Q^{-0.298}$	$0.10 \leq Q \leq 3.0$ $3.0 < Q \leq 200$ $200 < Q \leq 6,140$	89	4.44×10^{-16}	0.58	0.086	-18 to 22
Snake River at Warren, Minn., gaging station 05085500						
Range of fit: $0.10 \leq Q \leq 206$						
$\log CaCO_3 = 2.40 - (0.012 - 0.087C_1 + 0.137S_1)\log Q - 0.299C_1 + 0.072S_1$		13	0.001	0.92	0.036	-7.9 to 8.6
$\log Na = 2.54 - (0.623 + 0.516C_1 + 0.393S_1)\log Q + 1.17C_1 + 0.104S_1$		13	0.004	0.88	0.060	-13 to 15
$\log SO_4 = 2.24 - (0.698 + 0.628C_1 - 0.220S_1)\log Q$		14	1.44×10^{-4}	0.86	0.059	-13 to 15
$Cl = 12.7Q^{-0.362}$ $Cl = 30.6Q^{-0.788}$	$0.10 \leq Q \leq 7.94$ $7.94 < Q \leq 206$	13	1.56×10^{-5}	0.89	0.185	-35 to 53
$\log DS = 2.77 - (0.146 + 0.073C_1 + 0.143S_1)\log Q - 0.025C_1 + 0.067S_1$		13	0.005	0.87	0.041	-9.0 to 9.9

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm			Range of standard error of actual concentration (percent)
			p-value	Coefficient of determination (R^2)	Residual standard error	
Middle River at Argyle, Minn., gaging station 05087500 **Range of fit: 0.10 ≤ *Q* ≤ 122**						
$\log CaCO_3 = 2.50 - (0.085 + 0.108C_1 + 0.106S_1)\log Q$		23	1.46×10^{-5}	0.72	0.080	-17 to 20
$Na = 12.8Q^{0.042}$ $Na = 23.8Q^{-0.857}$ $Na = 1.16Q^{0.602}$ $Na = 16.0Q^{-0.214}$ $Na = 23.1Q^{-0.299}$	$0.10 \le Q \le 2.0$ $2.0 < Q \le 7.94$ $7.94 < Q \le 25.1$ $25.1 < Q \le 70.8$ $70.8 < Q \le 122$	22	0.003	0.64	0.100	-20 to 26
$\log SO_4 = 1.43 - (0.183 + 0.292C_1 - 0.165S_1)\log Q - 0.286C_1 - 0.077S_1$		22	0.012	0.57	0.107	-22 to 28
$\log Cl = 3.01 - 0.562\log Q + 3.10C_1 + 0.171S_1 + 1.14C_2 - 0.218S_2$		18	3.93×10^{-5}	0.88	0.124	-25 to 33
$\log DS = 2.56 - (0.071 + 0.121C_1 + 0.091S_1)\log Q$		23	1.21×10^{-5}	0.73	0.069	-15 to 17
Park River at Grafton, N. Dak., gaging station 05090000 **Range of fit: 0.01 ≤ *Q* ≤ 4,840**						
$\log CaCO_3 = 2.59 - (0.045 + 0.020C_1 + 0.100S_1)\log Q + 0.020C_1 - 0.001S_1$		78	0	0.81	0.080	-17 to 20
$Na = 115Q^{-0.066}$ $Na = 121Q^{0.052}$ $Na = 177Q^{-0.220}$ $Na = 263Q^{-0.344}$	$0.01 \le Q \le 1.58$ $1.58 < Q \le 3.98$ $3.98 < Q \le 25.1$ $25.1 < Q \le 4{,}840$	79	0	0.81	0.142	-28 to 39
$\log SO_4 = 2.41 - (0.062 + 0.014C_1 + 0.113S_1)\log Q$		79	0	0.77	0.105	-21 to 27
$Cl = 123Q^{-0.050}$ $Cl = 184Q^{-0.270}$ $Cl = 379Q^{-0.472}$	$0.01 \le Q \le 6.31$ $6.31 < Q \le 35.5$ $35.5 < Q \le 4{,}840$	79	0	0.78	0.199	-37 to 58
$\log DS = 2.93 - (0.074 - 0.019C_1 + 0.105S_1)\log Q - 0.021C_1 + 0.024S_1$		79	0	0.83	0.093	-19 to 24

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm			Range of standard error of actual concentration (percent)
			p-value	Coefficient of determination (R^2)	Residual standard error	
Red River of the North at Drayton, N. Dak., gaging station 05092000						
Range of fit: $169 \le Q \le 91{,}000$						
$CaCO_3 = 540Q^{-0.091}$	$169 \le Q \le 1{,}260$	55	8.80×10^{-8}	0.56	0.086	-18 to 22
$CaCO_3 = 943Q^{-0.169}$	$1{,}260 < Q \le 3{,}160$					
$CaCO_3 = 185Q^{0.033}$	$3{,}160 < Q \le 7{,}080$					
$CaCO_3 = 259Q^{-0.005}$	$7{,}080 < Q \le 15{,}850$					
$CaCO_3 = 15{,}210Q^{-0.426}$	$15{,}850 < Q \le 91{,}000$					
$\log Na = 3.13 - (0.455 - 0.035S_1)\log Q$		53	1.34×10^{-13}	0.72	0.151	-29 to 42
$SO_4 = 175Q^{-0.074}$	$169 \le Q \le 1{,}000$	54	0.015	0.22	0.156	-30 to 43
$SO_4 = 1{,}010Q^{-0.328}$	$1{,}000 < Q \le 3{,}160$					
$SO_4 = 5.45Q^{0.320}$	$3{,}160 < Q \le 7{,}940$					
$SO_4 = 874Q^{-0.245}$	$7{,}940 < Q \le 91{,}000$					
$Cl = 487Q^{-0.261}$	$169 \le Q \le 1{,}000$	53	3.33×10^{-15}	0.78	0.174	-33 to 49
$Cl = 305{,}350Q^{-1.19}$	$1{,}000 < Q \le 2{,}000$					
$Cl = 137Q^{-0.179}$	$2{,}000 < Q \le 17{,}780$					
$Cl = 223{,}070Q^{-0.935}$	$17{,}780 < Q \le 91{,}000$					
$DS = 1{,}890Q^{-0.193}$	$169 \le Q \le 1{,}120$	53	8.11×10^{-11}	0.66	0.092	-19 to 24
$DS = 5{,}540Q^{-0.346}$	$1{,}120 < Q \le 2{,}820$					
$DS = 282Q^{0.029}$	$2{,}820 < Q \le 10{,}000$					
$DS = 5{,}980Q^{-0.303}$	$10{,}000 < Q \le 91{,}000$					
Pembina River at Neche, N. Dak., gaging station 05100000						
Range of fit: $5.6 \le Q \le 10{,}300$						
$\log CaCO_3 = 2.68 - (0.106 + 0.013C_1 + 0.044S_1)\log Q$		52	1.11×10^{-16}	0.80	0.082	-17 to 21
$\log Na = 1.73 - (0.051 + 0.008C_1 + 0.085S_1)\log Q - 0.002C_1 + 0.106S_1$		52	2.15×10^{-11}	0.71	0.079	-17 to 20
$\log SO_4 = 2.40 - (0.109 + 0.108C_1 + 0.071S_1)\log Q + 0.146C_1 + 0.064S_1$		52	1.02×10^{-9}	0.66	0.096	-20 to 25
$\log Cl = 1.54 - (0.198 + 0.047C_1 + 0.034S_1)\log Q$		52	1.33×10^{-15}	0.78	0.107	-22 to 28
$\log DS = 2.85 - (0.093 + 0.021C_1 + 0.041S_1)\log Q$		52	0	0.82	0.066	-14 to 16

Table 2. Relations between selected water-quality constituents and streamflow (discharge) for selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota.—Continued

[log, base-10 logarithm; $CaCO_3$, hardness concentration, in milligrams per liter; C_1, cos (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; S_1, sin (2π month/12) where month = 1, ..., 12 and 1 = January, ..., 12 = December; Q, streamflow, in cubic feet per second; Na, sodium concentration, in milligrams per liter; ≤, less than or equal to; <, less than; SO_4, sulfate concentration, in milligrams per liter; Cl, chloride concentration, in milligrams per liter; DS, dissolved-solids concentration, in milligrams per liter; C_2, cos (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December; and S_2, sin (2π month/6) where month = 1, ..., 12 and 1 = January, ..., 12 = December]

Equation		Number of data pairs	In base-10 logarithm: p-value	In base-10 logarithm: Coefficient of determination (R^2)	In base-10 logarithm: Residual standard error	Range of standard error of actual concentration (percent)
Tongue River at Akra, N. Dak., gaging station 05101000						
Range of fit: $0.02 \le Q \le 659$						
$\log CaCO_3 = 2.44 - (0.052 + 0.028C_1 + 0.068S_1)\log Q + 0.063C_1 + 0.066S_1$		75	3.84×10^{-11}	0.56	0.071	-15 to 18
$\log Na = 1.44 - (0.017 + 0.025C_1 + 0.053S_1)\log Q + 0.038C_1 + 0.006S_1$		76	1.97×10^{-6}	0.38	0.079	-17 to 20
$\log SO_4 = 1.96 - (0.037 + 0.069C_1 + 0.051S_1)\log Q + 0.039C_1 + 0.012S_1$		75	0.001	0.26	0.098	-20 to 25
$\log Cl = 0.960 + (0.040 + 0.031C_1 - 0.076S_1)\log Q - 0.051C_1 + 0.041S_1$		75	0.008	0.20	0.107	-22 to 28
$\log DS = 2.59 - (0.044 + 0.038C_1 + 0.062S_1)\log Q + 0.052C_1 + 0.054S_1$		74	7.43×10^{-10}	0.52	0.065	-14 to 16
Red River of the North at Emerson, Manitoba, gaging station 05102500						
Range of fit: $222 \le Q \le 62{,}800$						
$\log CaCO_3 = 2.72 - (0.087 + 0.061C_1 + 0.053S_1)\log Q + 0.208C_1 + 0.181S_1$		142	1.62×10^{-11}	0.35	0.073	-15 to 18
$Na = 5{,}130Q^{-0.685}$	$222 \le Q \le 1{,}780$	140	0	0.58	0.156	-30 to 43
$Na = 10.3Q^{0.145}$	$1{,}780 < Q \le 3{,}980$					
$Na = 357Q^{-0.282}$	$3{,}980 < Q \le 62{,}800$					
$\log SO_4 = 2.23 - 0.081\log Q - 0.031C_1 - 0.026S_1 + 0.096C_2 - 0.077S_2$		141	2.35×10^{-9}	0.30	0.144	-28 to 39
$Cl = 17{,}210Q^{-0.847}$	$222 \le Q \le 1{,}780$	142	0	0.59	0.198	-37 to 58
$Cl = 7.52Q^{0.187}$	$1{,}780 < Q \le 3{,}550$					
$Cl = 723Q^{-0.372}$	$3{,}550 < Q \le 62{,}800$					
$DS = 3{,}820Q^{-0.311}$	$222 \le Q \le 2{,}000$	141	0	0.48	0.085	-18 to 22
$DS = 46.9Q^{0.268}$	$2{,}000 < Q \le 3{,}980$					
$DS = 1{,}670Q^{-0.163}$	$3{,}980 < Q \le 62{,}800$					

where

FWC is monthly flow-weighted water-quality concentration, in milligrams per liter;
c_i is daily water-quality concentration determined from regression equation or mean value in table 1, in milligrams per liter;
q_i is mean daily streamflow, in cubic feet per second;
n is number of days in month;
i is i^{th} day of month; and
BCF is regression equation bias correction factor because of base-10 log-transformation and determined using $10^{1.15s^2}$, where s is the residual standard error.

After the FWC is determined, a load may be calculated using the following equation:

$$\text{Load} = \text{FWC}\ x\ \text{cf}\ x\ \sum_{i=1}^{n} q_i \qquad (5)$$

where

Load is monthly constituent load, dependent upon units desired (for example, tons per acre-foot);
cf is conversion factor to change FWC into desired units (for example, cf = 0.00136 to change FWC from milligrams per liter to tons per acre-foot); and
$\sum_{i=1}^{n} q_i$ is total streamflow for month, dependent upon units desired (for example, acre-foot).

An estimate of FWC that occurs at the mean flow for a given month may be determined using equation 4. Instead of using mean daily streamflow values, a single monthly mean streamflow value is used for q_i. This method is not accurate for proper load estimates, but may be adequate if a point-value for a given month is desired. However, additional monthly and/or seasonal bias would need to be determined to more accurately estimate the load because this method could over or under estimate the actual monthly flow-weighted water-quality concentration.

Summary

The Dakota Water Resources Act was passed by the U.S. Congress on December 15, 2000. The Act authorized the Secretary of the Interior to conduct a comprehensive study, administrated by the Bureau of Reclamation, of future water-quantity and quality needs of the Red River of the North Basin in North Dakota and possible options to meet those water needs. To provide needed information for the comprehensive study, the U.S. Geological Survey conducted a study in cooperation with the Bureau of Reclamation. This report provides updated statistical summaries of selected water-quality constituents and streamflow and the regression relations between them.

Available data for 1931-99 were used to develop regression equations between 5 selected water-quality constituents and streamflow for 38 selected gaging stations in the Red River of the North Basin, North Dakota, Minnesota, and South Dakota. The water-quality constituents that were regressed against streamflow were hardness (as $CaCO_3$), sodium, chloride, sulfate, and dissolved solids.

Daily mean streamflow records were more complete than instantaneous streamflow records and were readily available for most of the gaging stations; therefore, daily mean streamflow records were used to develop the regression equations with the water-quality constituents when instantaneous streamflow records were too few. At some gaging stations, instantaneous and daily mean streamflow records were combined to complete the streamflow record needed. Statistical summaries of the 5 selected water-quality constituents and streamflow for the 38 selected gaging stations used in the regression equations development are presented in this report. The statistical summaries include number of samples; average annual sampling frequency; number of samples years; descriptive statistics of the maximum, minimum, mean, and standard deviation; and values for the 25th, 50th, and 75th percentiles.

Two general equations were considered to model the relation between water-quality constituents and streamflow for the selected gaging stations in the Red River of the North Basin. The first equation assumed that the base-10 logarithm of a water-quality constituent concentration is a piece-wise linear, continuous function of the base-10 logarithm of streamflow. The second equation was a multiple linear regression model that incorporated month and streamflow as explanatory variables to allow the intercept and/or slope to change depending upon the time of year. The best equation for each gaging station was selected based on review of the residual plots, coefficient of determination, residual standard error of the regression, p-value, and F-statistic. The final regression equations, number of data pairs used in the equation development, p-value, coefficient of determination, residual standard error, and range of standard error of actual concentration for each gaging station are presented in this report. The applications and limitations of the regression equations also are presented in this report.

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