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DEPOSITIONAL SETTINGS OF SAND BEACHES ALONG WHITEWATER RIVERS[†]

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ABSTRACT

The numbers and sizes of sand beaches suitable for recreation along selected whitewater rivers in the western United States depend on sand concentrations, range of discharge and the size, frequency and type of depositional settings. River-width expansions downstream from constrictions are the predominant depositional setting for sand beaches in the upper Grand Canyon and along five Wild and Scenic Rivers in Idaho, but not along other rivers. Beaches located upstream from constrictions are rare, in general, except in the Grand Canyon. Beaches found in expansions without constrictions dominate depositional sites along the Yampa and Green Rivers, are fairly common along the rivers in Idaho, but are relatively rare in the Grand Canyon. The magnitude of flow expansion is a reliable predictor of beach size. Beaches located on the inside of curves are uncommon, in general, but can be important recreation sites. The mid-channel bar setting is the least important from a recreation standpoint because that setting is rare and beaches there are typically small, and emergent only at low flow.

The frequency of beaches is highly variable among rivers and the concentration of sand in transport is only partially responsible. Of the rivers studied, the unregulated Yampa River carries the highest concentrations of suspended sand and has among the most beaches $(1.2 \text{ beads km}^{-1})$. Emergent sand beaches are essentially nonexistent along the Deschutes River and are rare along other Oregon rivers, yet these rivers transport some sand. Sand beaches are fairly common (0.8–1.1 beaches km^{-1}) along the regulated Colorado River, but are comparatively rare $(0.6$ beaches km⁻¹) along the unregulated Middle Fork Salmon River. The suspended sand concentrations in study reaches of these two rivers are similar, and the difference in the frequency of beaches may be largely because the processes that create beach-deposition settings are less active along the Middle Fork Salmon. Published in 2008 by John Wiley & Sons, Ltd.

key words: river sand beaches; depositional settings; whitewater recreation

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INTRODUCTION

This report describes a study of the depositional settings of sand beaches, undertaken in order to help understand why some rivers have numerous and/or large beaches whereas others rivers do not. We discuss the requirements for deposition and maintenance of riverine sand beaches. Sand beaches are the primary sites of recreation and camping along whitewater rivers, because they are open areas with soft substrate and have other unique aesthetic qualities. Along some rivers, they are the only sites suitable for camping. Along some regulated rivers these recreational sites have historically diminished in size or number (Brian and Thomas, 1984; Kearsley et al., 1994, 1999; Schmidt et al., 1995), and for these reasons sand beaches have received considerable attention from land managers and scientists.

In this paper, we evaluate the number and frequency of occurrence of five physical settings where sand beaches are deposited along rivers in mountains and canyons (Figure 1). The discussion is restricted to whitewater rivers, which are typically gravel bedded. Sand-bedded rivers may be dominated by islands composed of sand or point bars created by bedload transport of sand. Sand beaches along gravel-bedded rivers, in contrast, are generally created where sand carried in suspension is advected into eddies of recirculating flow where it is deposited (Schmidt, 1990; Schmidt and Rubin, 1995; Andrews and Vincent, 2007). Thus, this is a study of the occurrence of recirculating eddies.

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Figure 1. Map showing the locations of river reaches investigated for this study, and other rivers mentioned in the text

We describe beaches in terms of the physical settings that result in recirculation zones. Our objective is not to emphasize the classification scheme, but rather to compare the frequency of occurrence of sand beaches among rivers organized by one criterion—physical setting. One setting, flow-width 'expansion downstream from constriction' (Figure 2), has been studied in various ways, and much of that work has been done in the Grand Canyon (Leopold, 1964; Howard and Dolan, 1981; Kiefer, 1985; Rubin et al., 1990; Schmidt and Graf, 1990; Schmidt, 1990; Webb, 1996; Andrews et al., 1999). This is the predominant depositional setting in the Grand Canyon, but not along other whitewater rivers. The large volume of scientific work done on the Colorado River in Grand Canyon (Webb et al., 1999) may bias the view of whitewater rivers in general. In particular we question the prevailing notion that sand supply is more important than other factors that control the occurrence, construction and maintenance of riverine beaches. High discharges, a range in discharge and physical sites conducive to recirculating flow are equally important.

This study was conducted in two parts. Fifty-two sites were selected for detailed study, where beaches were emergent at moderately high flow within seven relatively short reaches of five rivers. This was done in conjunction with another study (Andrews and Vincent, 2007) of beaches along Wild and Scenic Rivers of central Idaho,

Figure 2. Map showing a sand beach in an expansion downstream from a constriction at river mile 28.7 along the Main Salmon River near Shoup, Idaho

including reaches of the Middle Fork and Main Salmon, the Middle Fork Clearwater, the Lochsa and the Selway Rivers (Figure 1). General information for those reaches is found in Table I. The 52 sites were mapped at low flow with two results. First, the detailed mapping allowed us to recognize the variety of physical settings of recirculating flow. Second, the maps allowed measurement of the geometric properties of beaches and their settings. The second part of this study was an inventory of beach settings in long reaches of four whitewater rivers in the western United

Table I. General information for reaches of Idaho Wild and Scenic Rivers where beaches were selected during moderately high-flow conditions for detailed study

Two reaches were studied on both the Main Salmon and Middle Fork Salmon Rivers. All reach locations are given in the river miles depicted on USGS topographic maps. All discharges are given in cubic meters per second (cms).

River	Middle Fork Salmon	Colorado, upper Grand Canyon	Yampa, Dinosaur National Monument	Green, Dinosaur National Monument	
State Idaho		Arizona	Colorado	Colorado. Utah	
Observation period	$8/7 - 13/2002$	11/3-10/1999	$6/23 - 25/2003$	$6/25 - 27/2003$	
Observation discharge, cms	$17 - 18$	$467 - 527$	$105 - 116$	$113 - 127$	
Reach, river miles	$95.6 - 0$	$6 - 88$	$47-0$	$225 - 199.5$	
Reach length, km	153.8	131.9	75.6	41.0	
Number of beaches	86	103	87	61	
Beaches per km	0.6	0.8	1.2	1.5	
Average distance between beaches, km	1.8	1.3	0.9	0.7	

Table II. General information for rivers where beaches were inventoried during moderately low or low-flow conditions.

States, where sand was emergent at moderately low or low flow. These rivers (Figure 1) include the Middle Fork of the Salmon River in Idaho, the Yampa River and the Green River below the Yampa confluence within Dinosaur National Monument of Colorado and Utah and the Colorado River in the upper half of the Grand Canyon in Arizona. General information for those reaches is found in Table II. The inventories were done to compare the number of beaches, and their settings, among rivers. The two parts of our study share the Middle Fork Salmon in common in that the long reach inventoried at low flow includes the two short reaches within which beaches were studied in detail. The data for the Middle Fork Salmon allow us to evaluate the influence of stage on the number of emergent beaches and on the relative frequency of different types of settings. Published information for the Deschutes River in Oregon and the Snake River in Hells Canyon of Idaho and Oregon (Figure 1) is also analyzed. The John Day and Owyhee Rivers in Oregon are mentioned briefly.

Because of our emphasis on the recreational aspects of beaches, we limited our study to riverside sand deposits devoid of vegetation and with an area large enough for camping by a party of two.

BACKGROUND

Recirculation zones

Nearly all of the sand beaches we observed are at sites where water recirculates in an eddy at some stage of flow, typically high flow. Some recirculation zones are located mid-channel, at the downstream end of a gravel bar or large rock. Most recirculation zones that create beaches of recreational value, however, are located along the channel margin (e.g. Figure 2), where the channel is physically or effectively wider than it is either up- or downstream. Recirculation zones on the channel margin contain what is called a 'primary eddy'. In addition, they may contain one or more 'secondary eddies' as well as localized areas of low velocity or stagnant water (Schmidt and Graf, 1990; Schmidt, 1990). Water in an eddy circulates about a near-vertical axis with velocity slower than that in the main downstream-directed current. The eddy is separated from the main current by a von Kármán vortex street (Schlichting, 1968) that is often called an eddy fence by river runners: a narrow and near-vertical zone of hydraulic shear across which there is an abrupt change in velocity magnitude and locally opposed flow direction. Although floating objects tend to enter and remain in an eddy, there is a continual exchange of water between the main current and the eddy. River water typically enters at the downstream end of the recirculation zone especially near the bed, and exits at the upstream end near the surface.

The flux of river water into an eddy is quite large. For the size of eddies considered in this study, the mean residence time of water in the eddy is typically on the order of 60–300 s (Andrews and Vincent, 2007). That is, one half of the water in an eddy is replaced in 1–5 min. The mean residence time of water is somewhat longer than the time required for a medium-sized sand grain to settle through the water column to the eddy bottom. For example, a sand grain with a diameter of 0.25 mm, which is typical of eddy sand deposits (Schmidt and Graf, 1990), will settle 2 m in 80 s (Andrews and Vincent, 2007). Consequently, most of the sand that enters an eddy settles to the eddy bottom rather than being exchanged out of the eddy back into the primary downstream current. As a result, eddies

are very effective sediment traps. Where two eddies are present, sedimentation within the primary eddy results in what is called a 'reattachment deposit', which grows in the up-river direction. Sedimentation within the secondary eddy results in a 'separation deposit' (Schmidt and Graf, 1990; Schmidt, 1990). The secondary eddy is located up-river of the larger primary eddy. The two deposits are separated by a 'return-current channel'. In a recirculation zone on the right side of the river (looking downstream) for example, the primary eddy rotates clockwise and the secondary eddy rotates counterclockwise. Where only one eddy is present sand can deposit at the up-river end of the eddy near what is called the 'separation point'.

Beach classification schemes

Previous beach inventories (e.g. Schmidt et al., 1995) defined some beaches according to their deposit type, but defined others according to their location (such as 'channel margin') without indicating either the deposit type or why the recirculation zone occurred. At some locations, separation deposits and reattachment deposits are easily identified after they become exposed. We have found, however, that it is not always obvious whether the sand deposit resulted from a flow separation or reattachment eddy. This is, in part, because sand beaches can be the result of complex shifting of the locus of deposition, or erosion, during a flood or through the years. Consequently, we did not inventory beaches by deposit type, rather we organized the data by the physical features that result in recirculation zones.

STUDY RIVERS

Rivers evaluated for the study represent both unregulated and regulated flow, and are surrounded by terrain with differing rock type, vegetation and topography. The central Idaho rivers evaluated for this study (Figure 1) are unregulated, and are surrounded by hillslopes composed of resistant plutonic and metamorphic rocks. The hillslopes are typically forested and bedrock cliffs are relatively rare except in isolated reaches. The Yampa River is unregulated, and although the Green River is regulated the reach of that river within Dinosaur National Park does receive high flows and sediment from the Yampa. The Yampa and Green River reaches are surrounded by sparsely vegetated hillslopes composed of competent cliff-forming sandstones and limestones, with thinner interbeds of marine shale. Unlike the other sites discussed here, both of these river reaches have 'parks' where the canyon floor is broad, the gradient is low and sand islands are abundant. The parks have combined length of 22.5 km. The remaining 94 km are canyons with abundant steep rock cliffs and narrow bottoms, and are predominantly gravel bedded. The Colorado River in Grand Canyon is regulated by Glen Canyon Dam, which restricts both high flows and sand supply. The river is surrounded by largely unvegetated hillslopes composed of competent cliff-forming sandstones and limestones, and thick sections of erodible shale. Grand Canyon is young, narrow, deep and steep.

Other rivers mentioned in this report include the following. The Snake River in Hells Canyon is regulated by dams that do not appreciably influence high flows but do restrict the sand supply. The Snake River is surrounded by largely unvegetated hillslopes composed of competent volcanic rocks. Hells Canyon is young, narrow, deep and steep. The Deschutes River is also regulated by dams that do not appreciably influence high flows but do restrict the sand supply. This river is groundwater fed and has a remarkably low range in discharge. The river is surrounded by sparsely vegetated hillslopes composed of volcanic rocks. Cliffs, where present, are low. Both the John Day and Owyhee Rivers are unregulated and surrounded by sparsely vegetated hillslopes, and cliffs, where present, are low. Interbedded volcanic and sedimentary rocks surround the John Day. The Owyhee is surrounded by volcanic rocks.

METHODS

Beaches studied in detail

The reaches within which sites were selected for detailed study are 17–47 km long (Table I). A streamflow gaging station is located within each reach, and the reaches were defined so that discharges measured at a gage were representative of the entire reach. This was done so that a stage/discharge rating curve could be constructed for each beach, as part of a related project (Andrews and Vincent, 2007). Within each reach, 5–10 beach sites were selected during river trips at moderately high flow in 1998, and these represented all of the beaches emergent at the time of the river trips, with one exception. Within the lower reach of the Main Salmon a few beaches were not selected for detailed study because camping parties occupied them at the time of that river trip. Most of the surveyed beaches are recognized camping areas by either the land management agency or published river guides. Fifty-one places were selected and mapped, but one map encompassed two separated depositional settings, which resulted in 52 sites.

The beaches and their surroundings were mapped during a low-water field trip in August 1998. In addition to the sand beaches, adjacent surficial deposits were mapped as well as the river margins, and topographic features and roughness elements (like large boulders, trees and shrubs) that might have influenced sand deposition. Examples of the results are shown on Figures 2, 4 and 5 in this report. For survey purposes a fabric tape was laid out along the long axis of the sand deposit. A second tape was oriented perpendicular to the first and extended from the waters edge to the back of the beach, and topographic profiles were surveyed along that tape using a self-levelling level and rod. Together the two tapes created a measurement 'grid' for mapping. The locations of features beyond the tapes were determined by pacing the distance from a tape to the feature. The margins of sand deposits are located to 1 m. The locations of other mapped features are less accurate, river widths in particular. The distance to the water's edge

Figure 3. Relations between measured river discharge and suspended sand concentrations for river reaches investigated by this study. The data were collected either by the US Geological Survey, or the US Forest Service, using standard methods and equipment (Andrews and Vincent, 2007; and http://waterdata.usgs.gov/nwis/). This figure is available in colour online at www.interscience.wiley.com/journal/rra

Figure 4. Map showing a sand beach in an expansion without a constriction at river mile 11.9 along the Lochsa River near Lowell, Idaho

on the opposite side of the river was estimated by visual comparison to a length reference provided by the tapes, thus our river widths could be in error by as much as 50%. The ratio of widths made at individual sites, such as the constriction ratio, should be reliable for comparison to meet the needs of this study.

The field maps were used to develop the beach setting categories discussed in this report. In addition, a variety of measurements were determined from the maps, including expansion amount or expansion 'abruptness'. These parameters are defined where the results are presented.

Beach inventories

Inventories of beaches emergent at moderately low to low flows along reaches of four rivers were conducted during subsequent river trips (Table II). The length of these reaches ranged from 41 to 154 km. The low-flow inventory reach of the Middle Fork Salmon River encompassed the two shorter reaches within which beaches emergent at moderately high flow were selected for detailed study. Notes and sketch maps were made to aid classification of all observed beaches large enough for two people to camp on. While conducting the lower flow

Figure 5. Map showing a sand beach inside of curve at river mile 62.8 along the Middle Fork Salmon River at the confluence of Marble Creek, Idaho

beach inventories, most observations were made while floating past beaches and a few beaches may have been missed. Most observations were made from a solo craft, not as a passenger on a raft, and distractions such as the necessity to navigate rapids occurred occasionally. Where there were multiple threads of flow around bars, only one passage was navigated so some beaches on mid-channel bars were not visible, but this was likely only a problem in the parks along the Yampa and Green Rivers. Some sand deposits screened by vegetation, and some elevated beaches, are not always visible from river level, but care was taken to inspect sites officially designated as camps. Some sandbars are densely vegetated, but the data we present are for beaches that were not densely vegetated at the times indicated on Table II.

Suspended sediment

The concentration of sand suspended in streamflow has been sampled over a range of discharge at gaging stations located within each of the ten river reaches discussed in this report. These concentrations describe the effective supply of sand available to enter recirculation zones. The variation of suspended sand concentration with river discharge was determined by fitting a linear relation to the log-transformed values. The ten sand transport relations are compared in Figure 3. The length of the fitted curves indicates the range of river discharges over which the suspended sand concentrations were sampled. Reservoir storage and regulation affect river flows and sand transport in the Colorado and Green River study reaches, and the relations presented on Figure 3 were developed using data collected after dam construction. The Idaho Wild and Scenic River reaches, and the Yampa River reach, are not affected to any appreciable degree by reservoirs and flow regulation. The reaches where the beach inventories were conducted are fairly long and sediment transport relations may not be uniform, and for that reason the locations of suspended sediment sampling sites within those reaches are listed on Figure 3 and explained here. The Middle Fork Salmon at Middle Fork Lodge is near the middle of that reach and Middle Fork Salmon at mouth is at the downstream end of that reach. Yampa River at Deer Lodge is at the upstream end of that reach. Green River near Jensen is at Split Mountain at the downstream end of that reach. Colorado River above Little Colorado River is near the middle of that reach, and Colorado River at Grand Canyon is at the downstream end.

RESULTS

Beach depositional settings

We recognized five depositional settings for classifying the occurrence of riverine sand beaches along whitewater rivers (Table III). In almost all cases these settings are sites of flow recirculation involving an actual or effective change in flow width.

Many recirculation zones are found up- or downstream from a constriction of the flow width and cross-sectional area of the main channel (Schmidt and Graf, 1990). The constrictions are often the result of debris fans emanating from the mouths of tributaries, but may also be the result of talus cones, rock fall blocks, log jams, bedrock that projects out into the river, or man-made features. Constrictions generally result from some encroachment on the river from one side of the valley, and thus usually recirculation zones are present on the side of the river from which the constriction was derived. Although constrictions are common, we emphasize here that it is the channel widening that is the critical feature for formation of most recirculation zones.

Expansion downstream from constriction. Eddies can form where a river widens downstream from a constriction (Figure 2), and the reader is probably most familiar with this setting because such sites are common and can be large (Schmidt, 1990). Where streamflow encounters a constriction in the width of the channel, which is often accompanied by shallowing, it accelerates and the jet of high velocity flow extends downstream from the constriction until the momentum is diffused (Kiefer, 1985). An eddy develops in the space created by the abrupt widening, between the jet of streamflow and the bank. Where the constriction causes a substantial fall in the water surface a rapid is present and the jet of flow can extend into a pool or scour hole if those features are present. The jet entering a pool may augment the deposition of sand in the adjacent eddy, because the increased turbulence tends to suspend sand higher in the water column (Schmidt, 1990). Our observations indicate, however, that neither a rapid nor a pool is required for sand to deposit in this kind of eddy.

Table III. Sand beaches along several western US rivers presented in terms of their spatial (river length) occurrence, and organized by the frequency of their physical Table III. Sand beaches along several western US rivers presented in terms of their spatial (river length) occurrence, and organized by the frequency of their physical settings

The frequency of types of settings is expressed as the percentage of all beaches within the study reach, rounded to the nearest whole number. The frequency of types of settings is expressed as the percentage of all beaches within the study reach, rounded to the nearest whole number.

Upstream from constriction. Eddies can develop upstream from flow constrictions and result in what have been called 'upper pool' deposits by Schmidt and Graf (1990). These deposits occur where water on the margin of the channel circulates adjacent to the flow that is guided into the constriction. The resulting deposits are often small in size. Because these beaches were not found at our detailed study sites we cannot define the physical parameters, such as constriction magnitude or convergence angle, that led to them.

Expansion without constriction. Eddies can develop where the channel widens abruptly without a nearby flow constriction (Figure 4). Sites of widening include bedrock alcoves, the mouths of tributaries, gaps in foot-slope talus sheets, or at the ends of long rows of rock-fall blocks. Figure 4 illustrates an example where, at some point in the past, the river eroded an alcove in a river terrace. At some locations, a minor constriction was located just upstream/downstream from the expansion, illustrating an aspect of subjectivity inherent in most classification schemes. For example, we classified the Redwall Cavern alcove in Grand Canyon as an expansion without constriction, even though there is a low debris fan located upstream. During commonly occurring flows the fan constitutes a constriction, and an eddy is present in front of the Cavern with the eddy fence located towards the middle of the river. Our classification of the site within the Cavern is justified by the following observations made by Gary Bolton (written comm., 2002), who visited the site as a river guide when the discharge was about 2400 cms (where cms stands for cubic meters per second) in June of 1983. At that flow level, the fan upstream was submerged, the eddy was contained within the cavern and the eddy fence was located at the drip line of the canyon wall. The beach in the cavern was covered by more than 1.5 m of water, and the strong eddy current reworked the sand deposit.

Inside of curve. Eddies can develop where a tight curve forces the core of high velocity flow to the outside allowing a recirculation zone to form on the inside and downstream end of the curve (Figure 5). As mentioned previously we are not discussing sandy point bars, which are uncommon on gravel-bedded whitewater rivers. Inside of curve sites are typically located where the flow impinges on curved bedrock cliffs, which force the flow to turn abruptly. This depositional setting is similar to settings involving physical flow-width expansions in that the downstream-directed main current occupies only a portion of the channel cross-section, except that the flow jet is created by channel curvature. At expansions downstream from constrictions there are commonly jets of flow, but the velocity of the jet need not be exceptionally high and, as mentioned previously, an actual rapid is not required for sand to deposit in the adjacent expansion. At expansions without constrictions there may be no acceleration of the downstream-directed main current. At inside of curve sites, in contrast, a jet of high velocity flow is likely required for the eddy to develop.

Mid-channel bar. Sand beaches also are found on mid-channel bars or islands. Along most of the study rivers, these beaches are exclusively located at the downstream ends of cobble bars. The bars are likely constructed and maintained by very high flows, but during flows that do not overtop the bars eddies develop in the lee of the obstructions and sand is deposited. Beaches in this setting usually occur in locally wide reaches, and are found in greatest abundance in the park-like reaches of the Yampa and Green Rivers. Typically, however, beaches found in the mid-channel bar setting are rare (Table III), small and emergent only at low flow.

Factors that influence results

Designating a particular beach site as belonging to one setting category or another can be subjective. This is because more than one factor (width changes, curvature, large roughness elements) can contribute to the formation of an eddy and it can be difficult to identify the most important factor. We believe that our designations 'upstream from constriction' and 'inside of curve' categories involve the most subjectivity. For example, along the Idaho rivers about half of the mapped beaches classified as occurring downstream from a constriction were situated between a pair of constrictions. A different observer might have classified some of these sites as upstream from a constriction. In most cases, however, the beaches were located closer to the upstream constriction although the second constriction may have influenced the formation of the eddy. The first author made all beach inventories and setting classifications, so any 'operator bias' is at least consistent within the results developed by this study.

The number of subaerial beaches that can be observed within a given reach is dependent on water level (e.g. Kearsley *et al.*, 1999). Observations made at high flow obviously underestimate the number of sand bars present along a river, because most sand deposits are submerged at high flow. To address this issue, we compare the number

Figure 6. Relation between the number of beaches emergent above water level as a function of discharge, for two reaches of the Middle Fork Salmon River in Idaho

DISCHARGE (%Qpk), as PERCENT OF 1997 PEAK

40

60

80

20

 $\mathbf 0$

of beaches observed at three different stages of flow in the two study reaches of the Middle Fork Salmon River (Figure 6). We assume that no beaches are exposed during flows equivalent to the 1997 peak flow because at our mapped sites no unvegetated sand was observed higher than the high-water marks left by that event. The 1997 peak discharges (Table I) in the Idaho rivers have recurrence intervals of about 20 years, and were the highest during the past 30 years (Andrews and Vincent, 2007). We consider the 1997 flood to be the practical-maximum beach building flow along the Idaho rivers, and express other discharges as a percentage of that maximum. During the 1998 river trip, the flow was moderately high, with discharge equaled or exceeded about 8% of the time. In the two study reaches combined, 17 beaches were observed during that trip and we are confident that there were no other exposed sand deposits of any appreciable size. During the 2002 river trip, the flow was much lower, with discharge equaled or exceeded about 50% of the time. Along the Middle Fork Salmon River, the dependence of beach exposure on discharge is strongly nonlinear (Figure 6), and the relation is steepest for low flows. If similar relations exist for other rivers, the time series results of beach inventories would be confounded if the observations were made at even slightly differing flow levels.

Beach statistics depend in part on exactly how and what was counted. This observation applies prominently to the comparison of results by different investigators. We counted settings, but had we counted distinct sand deposits the results would have been different. Of 52 mapped sites, for example, 33 contained a single expanse of sand, although at many of those sites there were multiple subhorizontal sand platforms located at different levels and separated by erosional escarpments in sand. At 15 sites, 2 disconnected patches of sand were present, and at the remaining 4 sites there were 3 or 4 isolated sand patches. The number of individual sand deposits exceeds the number of sites with recirculating flow by 45%. Most previous studies have inventoried beaches by deposit type, with the two principal deposits being separation and reattachment deposits (Schmidt, 1990). As mentioned previously, both can develop in the same recirculation zone if two eddies are present, and at some locations these can comprise parts of one expanse of sand. One recirculation zone has been counted as one setting in our scheme, even if two deposits were present. We attempt to reconcile the approaches by interpreting the deposit inventory data in Schmidt and Graf (1990; their Appendix A). They used high-resolution aerial photographs taken when the water level was low, and thus avoided two limitations of our float trip census: the possibility of missing sites that were hidden from view or submerged by higher water. Our reconciliation process involved two assumptions. Where they noted both 'reattachment' and 'separation' bars at the same 'river mile' site, we assumed these were located in the same recirculation zone. Where they noted either a 'channel margin' or 'upper pool' deposit at a site of an expansion-related deposit, we assumed this indicated two independent beach settings. The few sites labelled as 'marsh' were not counted.

100

Date	1973				1984	1999	
Source		Schmidt and Graf (1990)			Schmidt and Graf (1990)	this study	
observation discharge, cms	$167 - 343$ 1:7,200 aerial photograph				$147 - 164$	$467 - 527$	
observation type					1:3,000 aerial photograph from river		
No. of sand deposits	129				147	103	
Beaches per km	0.97				1.11	0.78	
Average distance between beaches, km	1.0				0.9	1.3	
Deposit type or location, Schmidt and Graf (1990)	Number of beaches				Beach setting, this study		
				71	Expansion downstream from constriction		
Separation or reattachment deposits		75	84				
				4	Expansion without constriction		
Upper pool		16 21		22	Upstream from constriction		
				3	Inside of curve		
Point bar		6	5				
Channel margin		32	37				
				3	Mid-channel bar		

Table IV. Comparison of inventories of beach settings along the Colorado River in Grand Canyon, between Six Mile Wash (river mile 6) and Phantom Ranch (river mile 88)

As discussed in the text, the inventories were made at various discharges, using various tools, and with different perspectives. See Kearsley et al. (1999) for discussion of changes in beaches due to dam closure and the occurrence of floods.

The results of Schmidt and Graf (1990) along with our inventory are shown in Table IV for the reach of the Colorado River between Six Mile Wash (near Lees Ferry) and Phantom Ranch. The average distance between observed beaches ranged between 0.9 km (in 1984) and 1.3 km (in 1999). This is fairly good agreement considering three qualifications. Our 1999 float trip inventory was conducted at a discharge of more than three times that of the 1984 survey. The 1984 survey used aerial photographs of exceptionally detailed scale that were taken just after a very large beach-building flood. Lastly, since 1984 there has been a net loss in the number of beaches (Kearsley et al., 1999) in the Grand Canyon. This comparison (Table IV) illustrates the uncertainties involved with beach inventories, but our main point is to emphasize that our method did not overestimate the occurrence of beaches in the Grand Canyon. Therefore, we have concluded that beaches were still abundant in the Grand Canyon in 1999, despite the presence of Glen Canyon Dam upstream, compared to the Middle Fork Salmon and other rivers (Table III).

The relative frequencies of the various depositional settings is little influenced by the river stage during which the observations were made, despite the fact that the number of beaches exposed is strongly dependent on stage. This is evident along the Middle Fork Salmon (Table V), where both the mapped beaches and the low-flow inventory indicate that 71% of the Middle Fork Salmon River beach sites are located in expansions downstream from constrictions, and that beaches located upstream from constrictions or on mid-channel bars are very rare. Both datasets also indicate that beaches in the inside of curves are uncommon. In the Grand Canyon (Table IV), the dependence of exposure on stage may be a partial explanation for the differences in the number of beaches counted in three surveys of the same reach, but the frequencies of setting results are similar. According to all three surveys, sites downstream from constrictions (containing separation and/or reattachment deposits) are dominant, and sites upstream from constrictions are fairly common.

DISCUSSION

Geometric aspects of expansions and constrictions

The magnitude of channel-width expansions extends over a large range, and this variable provides a reasonable predictor of beach size or sand area (Figure 7). Based on our maps, we estimated the cross-sectional width of the recirculation zone, where it was widest during high flow, and refer to that as the expansion amount. This analysis

The two sampled reaches are 62.9–45.6 and 18.0–0.0 river miles (as denoted on USGS maps). One dataset consists of beaches selected at high flow (Andrews and Vincent, 2007), and the other consists of beaches observed from river level during low-flow conditions (this study).

Upstream from constriction 0 0 0 2 4 Expansion downstream from constriction 12 71 35 71 Expansion without constriction $\begin{array}{cccc} 0 & 0 & 6 & 12 \\ 5 & 29 & 6 & 12 \end{array}$ Inside of curve $\begin{array}{ccccccc} 5 & 29 & 6 & 12 \end{array}$ Mid-channel bar 0 0 0 0

was done for both expansions downstream from constrictions and expansions without constrictions. For the mapped beaches along the Idaho rivers, the expansions averaged 15 m and ranged between 2 and 46 m. Even a 2-m expansion can produce a beach that two people can camp on. A straight-line fit to the observations indicates that the mapped extent of sand deposits was proportional to the expansion time 100 min. The correlation coefficient was

Figure 7. Relations between the area of sand mapped at low flow as a function of channel-width expansion amount, a proxy for eddy width, at high flow. Data for sites along five Wild and Scenic Rivers in Idaho from this study are shown with data for sites along the Colorado River in Grand Canyon (Schmidt and Graf, 1990). Data for sites where the downstream length of the recirculation zone was restricted (open symbols) were not included in the correlation. This figure is available in colour online at www.interscience.wiley.com/journal/rra

Discharge, cms

Total sand beaches

0.61 and the line did not pass through the origin. The length of recirculation zones also should influence the sizes of sand deposits, but obstacles were present at the downstream end of the expansions at a variety of sites and appeared to restrict the length of the recirculation zones. For example, an eddy within a narrow tributary mouth might be longer if not confined by solid objects at its downstream end. The 14 sites for Idaho rivers where the eddy length appeared to be limited by the presence of a downstream obstacle were isolated from the rest of the data for this comparison and thus from statistical correlations. For the remaining 27 Idaho river sites, the eddy length was predominantly controlled by the hydraulic interplay of the flow in the eddy with the downstream-directed main current. For this subset, the correlation of sand area and expansion amount was substantially improved ($R² = 0.77$). Similar data from the Grand Canyon are available and were included in the analysis on Figure 7. Schmidt and Graf (1990; their Table 14) reported the area of select sand deposits exposed at flows of about 170 cms. Where they noted both reattachment and separation bars at the same 'river mile' site, we have assumed this was one expansion setting and added the areas of the two deposits. We approximated the expansion magnitude at these sites by using the channel top width of the constrictions and the expansion ratio reported for a discharge of 1130 cms (Schmidt and Graf, 1990; their Table III). Three of the Grand Canyon sites appeared to have restricted length and were excluded from the statistical analysis. Using the data for combined Idaho and Grand Canyon sites with apparently unrestricted lengths, the correlation is further improved (Figure 7). A polynomial curve provides the best results $(R^2 = 0.94)$, and we conclude that expansion amount largely explains beach size. Assuming that expansion amount is a proxy for eddy width, the relation (Figure 7) also implies that smaller eddies are proportionally long compared to large eddies. For example, a site with expansion of 70 m has mean eddy length of about twice eddy width, whereas a site with expansion of 10 m has mean eddy length of about nine times eddy width.

Channel-width expansions are generally fairly abrupt, and there is a minimum divergence angle for the formation of recirculation zones. We defined an expansion 'abruptness' parameter, by dividing the expansion amount by the downstream distance over which the channel widened to its fullest, as estimated for high flow. The average value of expansion abruptness was 1 (divergence angle of 45°), and the range was 0.1 (6°) –4 (76°). It appears that eddies do not form where the downstream distance over which the widening occurs is greater than 10 times the widening amount. In other words, where the channel margin diverges more than 6° from the down-river axis, an eddy will develop next to that channel margin. For sites of recirculation at high flow, Schmidt and Graf (1990, their Table III) observed the divergence angles to be greater than 11° with one exception of 1° . The mapped size of sand deposits in our study showed a poor correlation with abruptness ratio.

The abruptness ratio is measured at the scale of the eddy, but a similar parameter reflects the scale of the whole river. Schmidt (1990) and Schmidt and Graf (1990) discussed expansions downstream from constrictions in terms of an expansion ratio (river width at the expansion divided by width at the constrictions), where a value of 2 means that the river was twice as wide downstream from the constriction as it was in the constriction. Expansion ratio values generally ranged from 1.05 to 1.65 for the Idaho beaches that were mapped in detail, and ranged from 1.3 to nearly 5 for sites in the Grand Canyon. For Idaho beaches at least, the expansion ratio is a poor predictor of beach area $(R^2 = 0.3)$.

The following analysis considers the extent to which beach size, or sand area, is correlated with river discharge. It is evident that small beaches can occur on rivers of all sizes. An eddy and sand deposit could form behind a 3-m wide rock-fall block along any whitewater river navigable by rafts. For that reason, it is not surprising that we found that local river width (estimated for the 1997 flood stage and excluding the constrictions) is a very poor predictor of beach size $(R^2 = 0.1)$. Furthermore, it also seems clear that very large beaches cannot occur along small rivers because the space available is insufficient. To evaluate this relation we selected a discharge proxy for river size. As mentioned previously, we assume the 1997 peak discharge is the practical-maximum beach building flow on the Idaho rivers. For the Colorado River in Grand Canyon we assume the practical maximum is the 1983 peak flow, given the current flow regulation. The beaches along Idaho Wild and Scenic Rivers selected for detailed study ranged in size (at low flow) from 28 to 5500 m², most (31 out of 52) were smaller than 1000 m² and very few (4 out of 52) were larger than 3000 m². Our sample of beaches probably does not represent a reliable size distribution, but we are confident that we sampled the largest beaches occurring in the study reaches. The data available for the Grand Canyon also do not represent a reliable size distribution, but we suspect Schmidt and Graf (1990) selected the sites because of their importance, which includes their large size. Most (8 out of 11) of their beaches were larger than 3000 m². The combined observations indicate that the largest beaches in a given reach are larger on larger

Figure 8. Relation between the area of sand beaches mapped at low flow as a function of the river discharge considered to be the practical maximum for the construction and maintenance of sand beaches. For the Idaho rivers the 1997 peak discharges, which have recurrence intervals of about 20 years, are considered the practical maximum because no unvegetated sand deposits were observed higher than the high-water marks left by that flow. For the Colorado River, the 1983 peak discharge is considered the practical maximum given the current flow regulation. Data for sites along the Colorado River in Grand Canyon are from Schmidt and Graf (1990)

rivers (Figure 8). In addition, some of the beaches in the Grand Canyon are exceptionally large even given the size of that river.

The frequency at which flow obstacles are rebuilt likely influences beach size. For example, if a tributary debris fan does not receive additional material over an extended period of time, it will likely be reduced in size by the main river and constitute less of a constriction compared to a tributary debris fan that frequently receives fresh debris. Figure 2 illustrates an example of a dramatically trimmed fan. Schmidt et al. (1995) suggested a geometric parameter to represent debris fan activity. Flow constrictions have been described using a constriction ratio (width at constriction/width upstream) where a value of 0.6 indicates that 40% of the channel width was obstructed (Schmidt and Graf, 1990; Schmidt et al., 1995). The ratio values are dependent on discharge, and, in addition, Melis et al. (1994) have shown that there are significant differences in the mean discharge required for overtopping debris fans in different reaches of Grand Canyon. Following previous investigators, we use measurements made at low flow, in the hope that these better reflect downstream changes in mean cross-sectional width at high flow. The comparison of results is shown on Table VI. The mean constriction ratio of large rapids in Grand Canyon is about 0.5 (Kiefer, 1985; Schmidt and Graf, 1990). The mean constriction ratio of all debris-fan constrictions that form rapids in Hells Canyon is 0.6, determined from aerial photographs taken in 1973 (Schmidt et al., 1995). The mean constriction ratio also is about 0.6 for obstacles found along the Idaho rivers evaluated by this study. Debris fans constrict the Colorado River to a greater degree than do obstacles along Wild and Scenic Rivers in Idaho, suggesting that debris fans in Grand Canyon are more active than those in Idaho. For example, of 147 sites in Grand

Canyon depicted in early photographs, 84 were modified by debris flows between 1890 and 1983 (Griffiths et al., 2004). In addition, results of monitoring at 740 sites between 1984 and 2003 indicate that about 5 debris flows reach the Colorado River annually, modifying the associated debris fans and/or rapids. This observation suggests that for the 400 identified debris fans in Grand Canyon about 500 debris flows have occurred over the past 100 years (Peter Griffiths, written comm., 2005). The frequency of debris fan deposition along the Idaho rivers is likely less than in the Grand Canyon. A long-term empirical record of debris fan activity along the Idaho rivers does not exist. The frequency of fan deposition, however, may be inferred. Debris fans along the Idaho rivers commonly have soil A-horizons, which develop slowly and in the absence of sediment deposition or erosion. Many tributary fans do not currently have well-defined channels, which suggest they have been inactive for some time. In addition, Mazama ash deposited nearly 7000 years ago was observed in debris fan deposits at a number of locations, thus indicating the general antiquity of at least some of the deposits. Presumably, the active nature of debris fans in Grand Canyon explains why some beaches are exceptionally large, and may partially explain why they are more numerous here then along the Idaho rivers.

Variation of beach occurrence among rivers

The frequency of beaches along a reach of given length is variable among whitewater rivers (Table III). Beaches are quite frequent along the Yampa and Green Rivers in Dinosaur National Monument. Our measurements of the average distance between beaches is 0.9 km along the Yampa and 0.7 km along the Green, or 0.8 km for the combined reaches within the Monument whether or not the wide areas called parks were excluded (Table III). These observations were made at a moderately low discharge (Table II), and even more beaches likely would be emergent at a lower flow. Beaches are common along the Colorado River in Grand Canyon between Six Mile Wash (near Lees Ferry) and Phantom Ranch (Tables III and IV). Depending on the methods used, and, in particular, the discharge at the time of observation, the average distance between observed beaches ranged from 0.9 to 1.3 km. Beaches are less frequent along the Middle Fork Salmon River, with average distance between beaches of 1.8 km, as observed at a very low flow (Table III). Beaches are exceedingly rare along other whitewater rivers, such as on the John Day and Owyhee Rivers in Oregon, based on casual observations by the authors. Beaches are essentially absent along a few rivers such as the Deschutes River in Oregon (Curran and O'Connor, 2003).

The relative importance of the different physical settings also is variable among rivers. Expansions downstream from constrictions (Figure 2) are the predominant depositional setting for sand beaches in the Grand Canyon and along the Idaho rivers, and perhaps along whitewater rivers in general, but they are not the predominate setting along some rivers including the Yampa and Green rivers (Table III). Beaches located upstream from constrictions are fairly common in the Grand Canyon, but only 2–6% of beaches along the other rivers are found in this setting (Table III). Beaches found in expansions without constrictions (Figure 4) dominate depositional sites along the Yampa and Green Rivers, are fairly common along the rivers in Idaho, but are proportionally rare in the Grand Canyon (Table III). Beaches located in the inside of curves (Figure 5) are uncommon, in general, and are insignificant in the Grand Canyon. Beaches located in this setting, however, do occur in significant numbers along the Idaho rivers and along the Yampa and Green study reaches (Table III). Some of these sites are important for recreation. For example, along the Idaho rivers, where beaches are widely spaced, many camping beaches created in the inside of curves are popular because they are strategically located or large. In addition, beaches occupying this setting along the Middle Fork Salmon River are available for recreation almost all of the time. This conclusion is based on the observation that five out of the six beaches found in the inside of curves during the low-flow trip also were above water during the trip where the flow was moderately high. Along the Middle Fork Salmon, beaches located in the inside of curves are constructed during high flow (Andrews and Vincent, 2007). The mid-channel bar setting is the least important from a recreation standpoint because beaches located there are rare (Table III), small and emergent only at low flow. In addition, these beaches usually occur in locally wide reaches, such as the park-like reaches of the Yampa and Green Rivers, where more preferable campsites are available.

Requirements for formation of beaches

There are three requirements for the emplacement and maintenance of riverine beaches: sand supply, high and variable discharges and sites of recirculating flow. Sand supply and high discharge requirements have been studied,

in part because dams can diminish these two requirements. The historical decrease in the number of beaches along the Snake River in Hells Canyon is probably the clearest example of the consequence of diminished sand supply (Schmidt et al., 1995). Dams upstream have dramatically limited the sand supply, but have not substantially altered the flow regime, and the number of sandbars has decreased by nearly 80% since dam closure. Of our study reaches (Table III), the Yampa and Green rivers have the most beaches in a given reach length, and this may be the result of a large sand supply. The Yampa and nearby Green River tributaries yield large quantities of sediment to the Green River, compared to the headwaters and other tributaries located downstream (Andrews, 1986). The suspended-sand concentrations at a given discharge in the Yampa and Green rivers are larger than at all other sites compared in Figure 3.

Streamflows must be sufficiently high or sand will not be transported in suspension and deposited in recirculation zones, but there are other aspects of streamflow that are important. For example, the range of discharges in the groundwater fed Deschutes river is remarkably narrow (O'Connor et al., 2003). Sand is transported by that river and deposited in recirculating zones, but the stage does not recede sufficiently for the deposits to emerge (Curran and O'Connor, 2003). The consequence of reservoirs and dam operations commonly is to narrow the range of discharge by both decreasing the peaks and increasing the low-flow discharges. The decrease in the number of beaches available for recreation due to a decreased range in discharge can be estimated independent of a reduction in sand supply. The operation of Flaming Gorge Dam has reduced peak discharges in the Green River downstream; flows equal to or exceeded 1% of the time were reduced by 30% (Andrews, 1986). In addition, flows equal to or exceeded 80% of the time were approximately doubled. If the Middle Fork Salmon River were regulated in that way, the influence on the number of beaches present can be estimated using Figure 6. If peak flows were reduced by 30%, at least 10 and perhaps as many as 15% of the current beaches would no longer be overtopped and, thus, would no longer be maintained. If the low flow were doubled, an additional 20% of the current beaches would no longer be emergent. Thus, excluding the influence of impoundment of sand in reservoirs, flow regulation alone can greatly decrease the number of beaches available for recreation along a river.

Lastly, the physical features that allow the formation of recirculation zones must be abundant or beaches will not be abundant along a river. Beaches are fairly abundant in the Grand Canyon, compared to the Middle Fork Salmon River, even though the Colorado River flow is regulated. Although the sand supply to the Colorado River in Grand Canyon has been reduced by dams, the suspended-sand concentration relations for the Colorado River and the Middle Fork Salmon River are similar (Figure 3). As noted above, the active nature of debris fans in Grand Canyon, which create sites of sand deposition, likely explains why beaches are so numerous, and why some beaches are exceptionally large.

CONCLUSIONS

This paper has considered several characteristics affecting the size and frequency of sand beaches along whitewater rivers. The occurrence of emergent sand beaches along whitewater rivers spans a large range. At low flow, emergent beaches range from essentially nonexistent to at least 1.5 beaches per km. Beaches located in expansions downstream from constrictions are, in general, the most common. Along some rivers, however, beaches located in other depositional settings can be important for recreation because they are frequent, relatively large and/or accessible over a wide range in discharge. Although there are three requirements for the formation of any beach, the numbers of beaches emergent along an individual river seems to be disproportionately influenced by one of the three. In general, beaches are most frequent along rivers that carry a relatively large concentration of sand. Nevertheless, where debris fans or other flow obstructions are particularly active or frequently replaced, a large number of beaches can exist even if the sand supply is limited or peak flows have been reduced. Beaches will be few in number where the range in flow is limited, either naturally or resulting from flow regulation. In rivers affected by flow regulation, decreased peak flow discharges limit the number of beach sites that are constructed and maintained. Such beaches become eroded or vegetated and are lost as a recreational resource. Where regulation increases low-flow discharges, fewer emergent beaches result and the reduction may equal or exceed that resulting from decreases in peak flow. Lastly, sand area is set by the geometry of flow expansion, which is dominated by the expansion of channel width; sand flux influences only the rate of deposition.

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