Estimation of local spatial scale

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The concept of local scale asserts that for a given class of psychophysical measurements, performance at any two visual field locations is equated by magnifying the targets by the local scale associated with each location. Local scale has been hypothesized to be equal to cortical magnification or alternatively to the linear density of receptors or ganglion cells. Here, we show that it is possible to estimate local scale without prior knowledge about the scale or its physiological basis.

THE CONCEPT OF LOCAL SCALE

Visual spatial sensitivity varies across the visual field in an orderly way, generally declining with distance from the point of fixation.[1–4] This variation has complicated our understanding of spatial vision, since whatever we discover to be so at one point in the visual field may not be so at another. For those who wish to model spatial sensitivity, this variation has introduced an additional dimension along which all parameters of the model might conceivably vary. However, a concept has been introduced by Koenderink *et al.*[4] and Rovamo *et al.*[5] that might simplify at least some of these complexities. I will call this the concept of local scale. It states that spatial processing, as manifest in some class of psychophysical measurements, is homogeneous everywhere across the visual field except for a change of scale. Thus there is a number, the local scale, associated with each point in the visual field. If an experiment of the specified class is conducted in two regions of the visual field, and if the spatial dimensions of the target used in each region are in proportion to the corresponding scale, then the results should be equivalent.

Rovamo *et al.*[5] attributed the local scale to the amount of cortical area dedicated to a given visual field area; hence their term cortical magnification. Other studies have related local scale to the linear density of retinal elements.[6] However, the question of local scale can be addressed independently of any particular physiological interpretation. Although we may wish ultimately to relate the scale to the spatial structure of the underlying physiology, it is useful to be able to examine the issue of local scale without prejudice about its physiological basis.

This point is of special importance, since previous methods of examining local scale have required prior assumptions about the value of local scale, usually based on a particular physiological substrate (e.g., cortical magnification). In such studies,[6–9] local scale has been equated to the inverse of cortical magnification, or alternatively cone density. In theory, if the scale is indeed equal to the corresponding physiological measure, then performance will be equated for the targets at various eccentricities. But if the data do not obey this rule, there is no simple way to estimate the local scale from the data.

The purpose of this paper is to show that local scale for a given task can be estimated without any prior estimates of cortical or retinal magnification and, indeed, without any prejudice about its physiological basis.

We emphasize that the thrust of this paper is a method, not an assertion of whether local scale holds true for one or another psychophysical task or of what its physiological basis might be. Answers to these latter questions will be facilitated, however, by a reliable method of estimating local scale.

A METHOD OF ESTIMATING LOCAL SCALE

Consider the set of targets shown in Fig. 1. They are Gabor functions of various sizes and spatial frequencies, all with the particular property that they are magnified versions of each other. In this particular set they are magnified by a factor of 2 at each step, yielding a set of targets with a constant number of cycles (and hence constant log bandwidth) and spatial frequencies decreasing by factors of 2 from left to right.

Suppose that contrast-detection thresholds are measured for this set of targets when each target is centered at the fovea. The result will be a contrast-sensitivity function, as illustrated schematically in Fig. 2A. Now suppose that thresholds are measured for the same set of targets centered on some eccentric point (Fig. 2B). For clarity, let us imagine that the local scale at this eccentric point is 2, with the foveal scale defined as 1. A local scale of 2 means that we could recreate the foveal contrast thresholds by magnifying each target by a factor of 2. However, this is equivalent to using the original set, except that data for foveal target n should agree with data for eccentric target $n + 1$. This agreement between the two sets of data can be achieved graphically by shifting the peripheral curve to the right by one step (Fig. 2C). More generally, if local scale holds for this task, the contrast-sensitivity functions measured in this way at any two points in the visual field should superimpose when shifted horizontally on a log scale. Furthermore, the shift required is a direct estimate of the ratio of local scales for the two points. Let us call this the shift rule for estimating local scale.[10]

A SAMPLE EXPERIMENT

To illustrate this method of estimating local scale, we collected contrast thresholds for a set of targets such as those in

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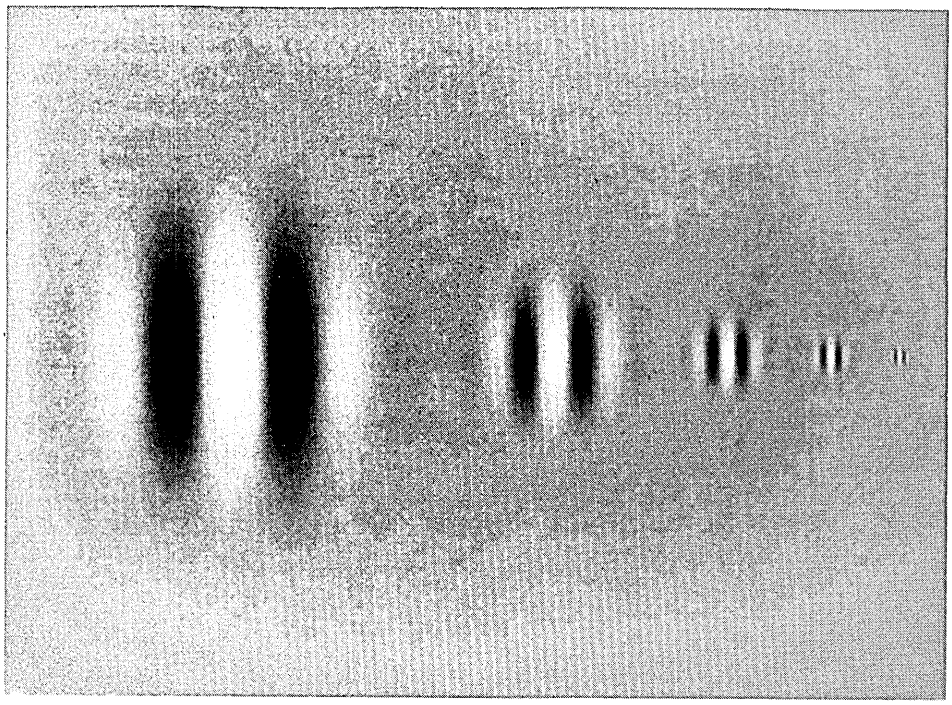

Fig. 1. A set of size-scaled Gabor stimuli that increase in size by factors of 2.

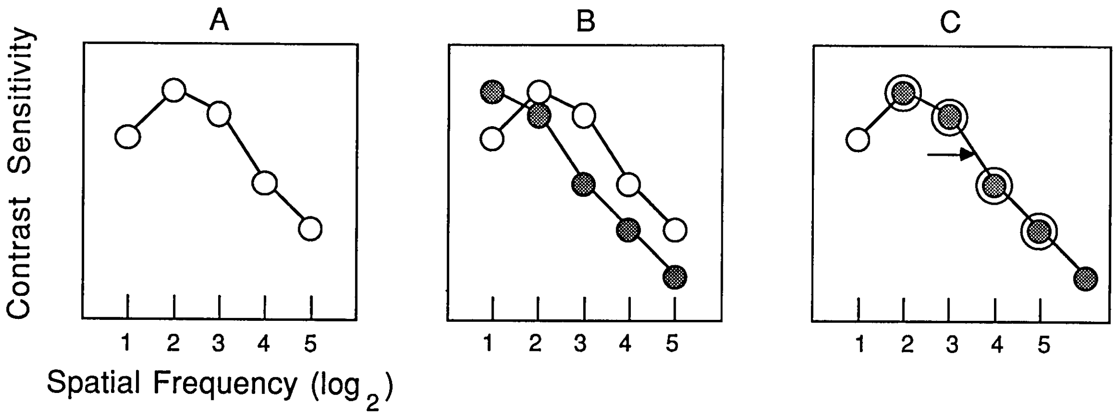

Fig. 2. A method of estimating local scale. A, A contrast-sensitivity function measured with size-scaled stimuli centered at the fovea. B, Comparison of the same measurements made at the fovea and a peripheral location (filled symbols). C, The peripheral curve has been slid to superimpose the foveal curve. The horizontal shift required is an estimate of local scale at the peripheral location.

Fig. 1. Each was a Gabor function with a width at $1/e$ of 1.6 cycles of the underlying sinusoid. The frequencies used were 0.25, 0.5, 1, 2, 4, 8, and 16 cycles/deg. Eccentricities tested were 0 and 3 deg from the point of fixation. The time course was a Gaussian with a width at $1/e$ of 500 msec. All thresholds were collected with a two-alternative forced-choice QUEST staircase.[11] Staircase data were fitted with a Weibull function to estimate thresholds.[12] Targets were displayed on a monochrome cathode-ray tube driven by an Adage RDS-3000 raster frame buffer, using methods described elsewhere.[13] Mean luminance was 100 cd/m^2.

RESULTS

Results for two eccentricities are shown in Fig. 3. The foveal data exhibit a shape not too different from a more traditional contrast-sensitivity function collected with extended gratings. Comparing the data at the two eccentricities, we observe that sensitivity to the higher spatial frequencies declines with increasing eccentricity, in agreement with previous findings.[3,4,8]

Figure 4 shows application of the shift rule to these data. The high-frequency limbs of the two curves in Fig. 3 are shifted into agreement. The factor by which the peripheral curve must be shifted is 1.72, which is then our estimate of the local scale at 3 deg of eccentricity.

In Fig. 4 and in other data that we have collected, we note that the lowest frequencies fail to agree. This is a violation of the simple prediction from local scale embodied in the shift rule. Does this mean that local scale does not hold for this task? A problem with the simple prediction is that while local scale applies to a point in the visual field, spatial targets occupy some extended region of visual space. The larger the target, the less it is a measure of local point sensitivity and the more it is a measure of some average sensitivity over a spatial region. (What constitutes a large target will of course be relative to the rate at which sensitivity changes over space.) Thus we might expect the simple prediction to fail for the largest targets, which in this case also means the lowest spatial frequencies. If this argument is correct, then the shift rule must be amended to give precedence to the high spatial frequencies or small spatial targets, as we have done in Fig. 4.

Furthermore, if our argument is correct, it should be possible to construct a more elaborate prediction embodying local scale that agrees with the data in Fig. 3.

SIMULATION OF A SCALE-VARIANT MODEL

To produce this more elaborate prediction we made use of a model of spatial detection and discrimination presented elsewhere.[14] In this model the image is cross correlated with an array of sensors, each with a receptive field given by a Gabor function of a particular size, spatial frequency, phase, orientation, and position. There is a discrete number of spatial frequencies at the fovea, but each type grows in size (and declines in frequency) with eccentricity at the same rate. The magnification of a sensor at eccentricity e, relative to the size at the fovea, is given by a scale s:

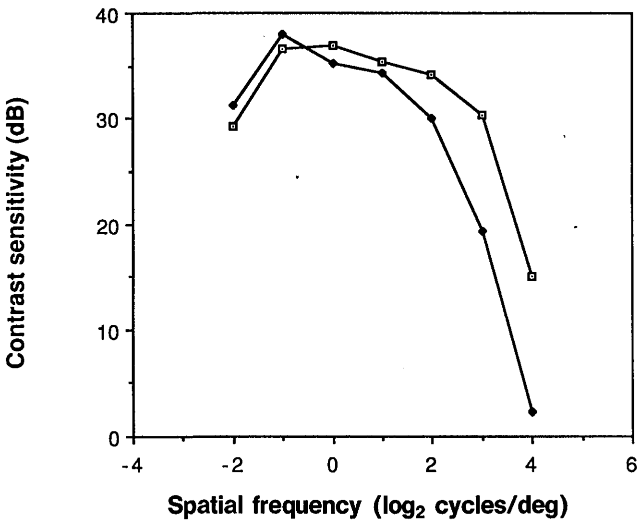

Fig. 3. Contrast-detection thresholds for size-scaled Gabor stimuli at eccentricities of 0 deg (⊡) and 3 deg (♦).

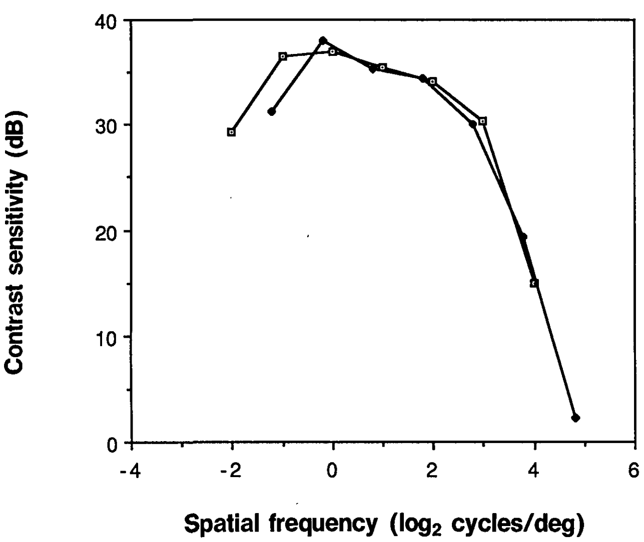

Fig. 4. Data from Fig. 3, with peripheral data shifted to the right to superimpose the foveal data at high spatial frequencies.

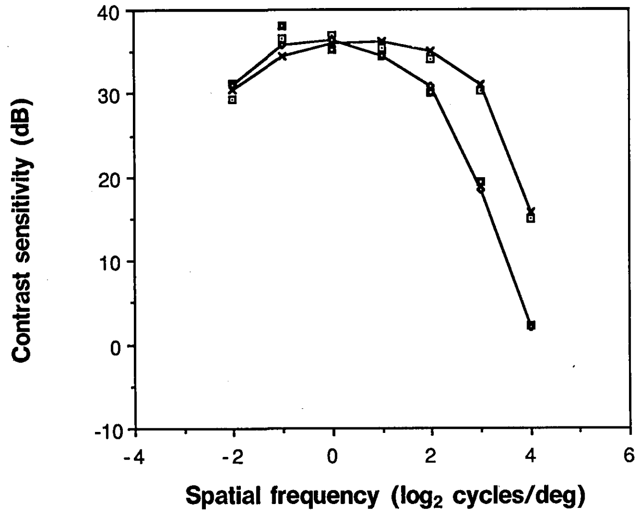

Fig. 5. Data from Fig. 3 fitted by a model of contrast detection embodying local scale proportional to eccentricity. Symbols: ⊡, data for 0 deg; ×, fit for 0 deg; ◻, data for 3 deg; ◇, fit for 3 deg.

$$s = 1 + \kappa e, \quad (1)$$

where κ is the rate of scale change. Likewise, the sampling density of each type of sensor declines with eccentricity at the same rate. Thus the model incorporates the concept of local scale in the sense that, if a portion of the sensor array were cut from one region of the visual field and compared with a portion from some other region, the two arrays would differ only in scale with respect to the sizes of the sensors, their spatial frequencies, and the distances between them. The outputs of the sensors form a feature vector **v**, which is examined by an ideal observer subject to spatial uncertainty. The contrast threshold c for this model can be approximated by

$$c = \left(\sum_{i=0}^{N} |v_i|^\beta \right)^{-1/\beta}, \quad (2)$$

where N is the dimension of the vector (the number of sensors) and β is about 3.5.[14,15]

The model has five adjustable parameters; four describe the gains of the eight types of different frequency sensors at the fovea, and one describes the rate κ at which local scale changes with eccentricity. These parameters have been allowed to vary in order to find the best fit of the model to the data.

This best fit is shown in Fig. 5. The reasonable fit of the model demonstrates two things. First, it shows that a model that explicitly incorporates the concept of local scale does not lead to the simple prediction embodied in the shift rule, because large, low-frequency targets are not local relative to the rate of change of scale over space. Second, it shows that human data, including the departures from the shift rule at low spatial frequencies, are consistent with a model incorporating local scale.

A final question remains. Were we justified in suspecting, although the shift rule is not entirely true, that when applied to the high-frequency limb of the data curves it still yields an estimate of local scale? This amounts to asking whether the estimate of local scale obtained by the amended shift rule is the same as that in the best-fitting model. The shift rule yields a value of $\kappa = 0.24$. The model estimate is $\kappa = 0.22$.

CONCLUSIONS

As noted above, a particular hypothetical local scale is associated with a particular set of tasks. We have not addressed the question of which tasks share the same local scale as contrast sensitivity. There are reports of tasks whose local scale differs from that of contrast sensitivity.[6,16] In many cases the estimation method described here can be extended to these other tasks. For example, vernier acuity performance could be measured with a set of scaled targets, and the threshold offset could be expressed as a proportion of target size.[17] Curves for different eccentricities could then be shifted to determine local scale for that task. The important point is that local scale can be estimated without any preconception as to its value.

One further important point is that an experiment in which a target of fixed dimensions is used in the fovea and the periphery does not suffice to measure local scale. For example, suppose that both resolution acuity and vernier acuity were measured at two eccentricities with targets of fixed size. Suppose further that vernier performance declines more with eccentricity than does resolution.[16] This result does not in itself show different local scales for the two tasks. In effect, local scale means that moving a target to the periphery is equivalent to minifying it at the fovea. Minification may have different effects on the two tasks, without prejudice to the question of whether they have different local scales.

For example, imagine two psychophysical measures that grow as the first and second powers, respectively, of the size of foveal targets. Suppose further that both tasks share the same local scale of 2 at some eccentric point. In that case, moving the target for either task to the eccentric point is equivalent to minifying it by a factor of 2 at the fovea. This will reduce the first measure by a factor of 2 but will reduce the second measure by a factor of 4. Thus the performance with targets of fixed size changes at different rates as a function of eccentricity, despite a common local scale. In this light, it is especially important that the possibility of different local scales for different tasks be assessed by using the method described here.

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